Harvesting a Single Natural Population

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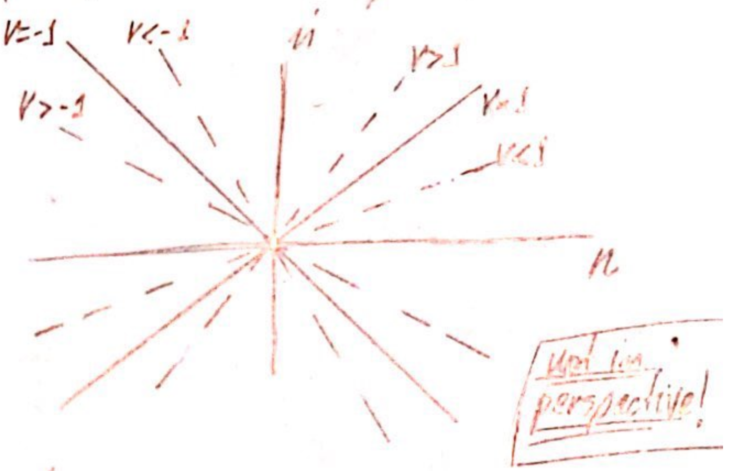
- —Model I predicts a unbounded growth rate, the solution is like $N(t) = N_0 e^{-rt}$, which gives a exponential growth rate.
- — The quadractic term is the competition term, it's try to include in the model the competition for resources in the growth of a population, K is the carrying capacity, a abstract way to include a maximum in the capacity of an population in a envirorment to grow.



- Model I: $\dot{n}(t) = r n(t)$

-fixed point: $rn^* = 0 \Rightarrow \boxed{n^* = 0}$

i) for $r > 0$ gives a line intersecting $x=0$ with a positive slope.

ii) for $r < 0$ gives a line intersecting $x=0$ with a negative slope

(iii) $r=0$ is equals to zero everywhere.

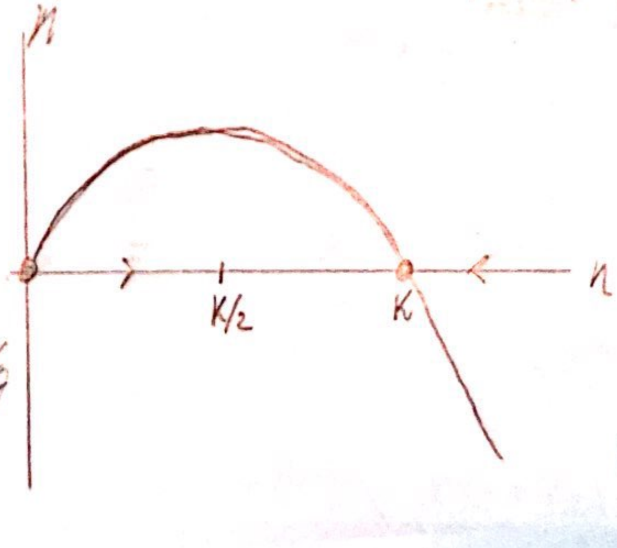
Model II: $\dot{n} = rn\left(1 - \frac{n}{K}\right)$ (logistic model)

- fixed points: $rn^*\left(1 - \frac{n^*}{K}\right) = 0 \Rightarrow \boxed{n^* = 0}$ and $\boxed{n^* = K}$

Only for $N > 0$

- $n = K$ is a stable fixed point
- $n = 0$ is a unstable fixed point.

For different values of r, the stability of fixed changes.



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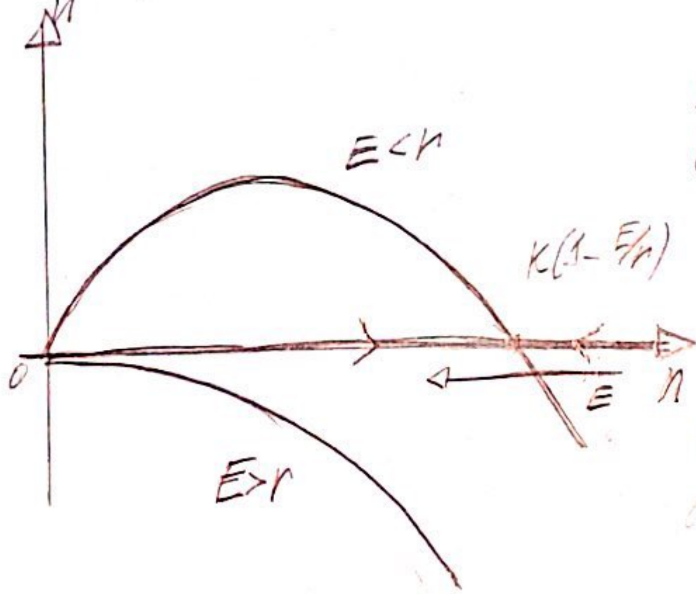
Model III: $\dot{n}(t) = rn(t)\left(1 - \frac{n(t)}{K}\right) - En(t)$

- fixed points: $\dot{n} = 0 \Rightarrow rn^* - \frac{rn^{*2}}{K} - En^* = 0$

$$n^*\left(r - \frac{rn^*}{K} - E\right) = 0 \Rightarrow \boxed{n^* = 0} \text{ or}$$

$$r - E = \frac{rn^*}{K} \Rightarrow \boxed{n^* = K(1 - E/r)}$$

As before for the simple logistic model, we have: Only considering $E > 0$



- If $E > r$, we harvest rapid than the growth rate of population, and always go to extinction
- If $E < r$, we harvest giving time to population growing, and the system turn into a state called logistic equation.

• The yield will be $\Rightarrow Y(E) = En^* = EK(1 - \frac{E}{r})$ and Y_{max} will be $\frac{dY(E)}{dE} = 0 \Rightarrow \boxed{E^* = \frac{r}{2}}$, back with E^* into $Y(E)$ we have $\boxed{Y_{max} = \frac{rK}{4}}$ and finally $\boxed{n^*_{max} = \frac{K}{2}}$ the pop. in the maximum obtained yield.

Extra Bonus: Murray section 1.6.

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