Bead on a Tilted Wire

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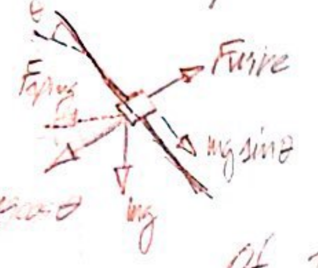
- In the equilibrium all forces must be sum zero.



$\boxed{F_{wire} = mg\cos\theta}$ $\quad mg\sin\theta = F_{spring,x}$

$F_{spring} = K(H - L_0)$, where H is the hypotenuse of the triangle

$H = \sqrt{x^2+a^2}$

$F_{spring,x} = F_{spring}\,\text{sen}\,\alpha$

$$\text{sen}\,\alpha = \frac{x}{H} = \frac{x}{\sqrt{x^2+a^2}} \Rightarrow mg\,\text{sen}\,\theta = K(\sqrt{x^2+a^2} - L_0)\frac{x}{\sqrt{x^2+a^2}}$$

$$\boxed{mg\,\text{sen}\,\theta = Kx\left(1 - \frac{L_0}{\sqrt{x^2+a^2}}\right)}$$ c.q.d.

- $\dfrac{mg\sin\theta}{Kx} = 1 - \dfrac{L_0}{\sqrt{x^2+a^2}} \Rightarrow 1 - \dfrac{mg\sin\theta}{Kx} = \dfrac{L_0}{\sqrt{x^2+a^2}}$, the hint of

identifying $\sqrt{1+u^2} = \sqrt{x^2+a^2} \Rightarrow 1+u^2 = x^2+a^2 \Rightarrow 1+u^2 = 1+\dfrac{x^2}{a^2}$ so $\boxed{u = \dfrac{x}{a}}$

now $\dfrac{R}{\sqrt{1+u^2}} = \dfrac{L_0}{\sqrt{x^2+a^2}} \Rightarrow \dfrac{R}{\sqrt{1+\frac{x^2}{a^2}}} = \dfrac{L_0}{\sqrt{x^2+a^2}} \Rightarrow R = L_0\sqrt{\dfrac{1+x^2/a^2}{x^2+a^2}} \Rightarrow$

$R = L_0\sqrt{\dfrac{\cancel{x^2+a^2}}{a^2(\cancel{x^2+a^2})}} \Rightarrow \boxed{R = \dfrac{L_0}{a}}$ and finally $\dfrac{mg\sin\theta}{Kx} = \dfrac{h}{u} \to$

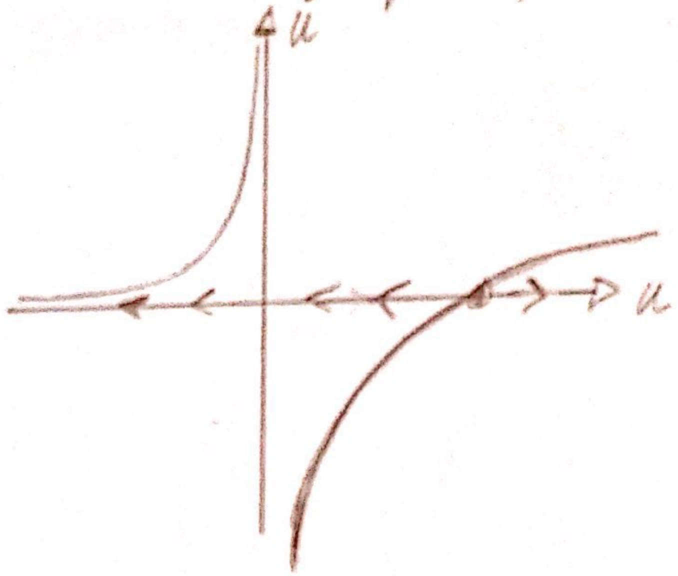
$\dfrac{mg\sin\theta}{K\cancel{x}} = \dfrac{h}{\cancel{x}}a \Rightarrow \boxed{h = \dfrac{mg\sin\theta}{a}}$ c.q.d. So:

$$\boxed{1 - \frac{h}{u} = \frac{R}{\sqrt{1+u^2}}}$$

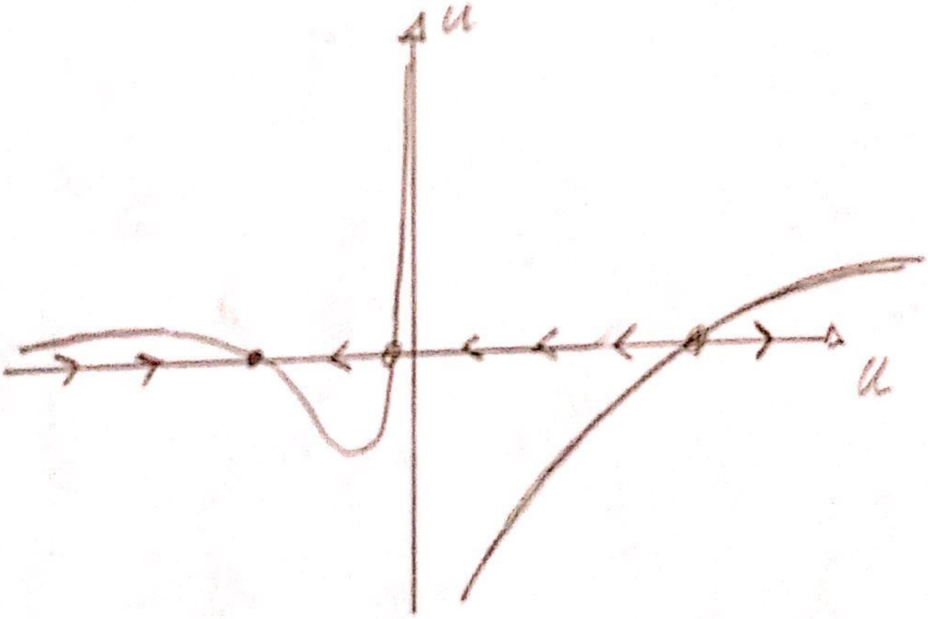
• So u is the displacement from equilibrium position, the dynamical system is formulated as: $f(u) = 1 - h/u - \frac{R}{\sqrt{1+u^2}}$

for simplicity we assume $h=1$ everywhere, $R>0$;

for $0<R<1$:



for $R>1$:



• $h = R - 1 \Rightarrow 1 - \frac{h}{u} = \frac{h+1}{\sqrt{1+u^2}}$, using $\sqrt{1+u^2} \simeq 1 + u^2/2$

$1 - \frac{h}{u} = \frac{h+1}{1+u^2/2} \Rightarrow (u-h)(1+u^2/2) = uh + u \Rightarrow u + \frac{u^3}{2} - h - \frac{u^2 h}{2} = uh + u$

$\Rightarrow h + uh - \frac{u^3}{2} + \frac{u^2 h}{2} \Rightarrow \boxed{h + uh - \frac{u^3}{2} \simeq 0}$

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- $h + ru - \frac{u^3}{2} \approx 0 \rightarrow \frac{d}{du}\left(h + ru - \frac{u^3}{2}\right) = r - \frac{3}{2}u^2 = 0$

$\boxed{u^* = \pm\sqrt{\frac{2}{3}r}} \longrightarrow h(u^*) = -r\left(\pm\sqrt{\frac{2}{3}r}\right) + \frac{1}{\cancel{2}}\left(\pm\sqrt{\frac{2r}{3}}\right)\frac{\cancel{2}}{3}r$

$= \pm\sqrt{\frac{2r}{3}}\left(-r + \frac{r}{3}\right) = \pm\sqrt{\frac{2r}{3}}\left(-\frac{2r}{3}\right)$

$\boxed{h(u^*) = \pm\sqrt{\frac{8}{27}r^3}}$

- h is the relative displacement of the bead due to its own weight component. r is the relative displacement of the bead to the initial length of the spring. For the approximation to small values of h, r, u draws a saddle-node bifurcation what means, it has a stable equilibrium to the original position. Being necessary a greater force perpendicular to the wire to take the bead off the equilibrium point. Only with higher perpendicular forces the bead will move greater in the wire plane.

- **extra bonus:**