Harvesting a Single Natural Population

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- – Model I predicts a unbounded growth rate, the solution is like $N(t)=N_0 e^{-rt}$, which gives a exponential growth rate.

– The quadractic term is the competition term, its try to include in the model the competition for resources in the growth of a population.

- K is the carrying capacity, a abstract way to include a maximum in the capacity of an population in a enviroment to grow.

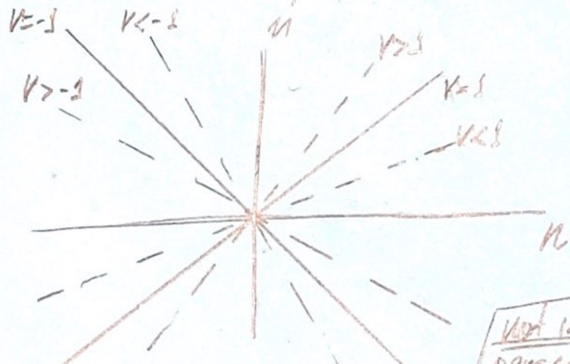
- Model I: $\dot{n}(t) = r n(t)$

- fixed point: $rn^* = 0 \Rightarrow \boxed{n^* = 0}$

i) for $r>0$ gives a line intersecting $x=0$ with a positive slope.

ii) for $r<0$ gives a line intersecting $x=0$ with a negative slope

(iii) $r=0$ is equals to zero everywhere.



not in perspective!

Model II: $\dot{n} = rn\left(1-\frac{n}{K}\right)$ (logistic model)

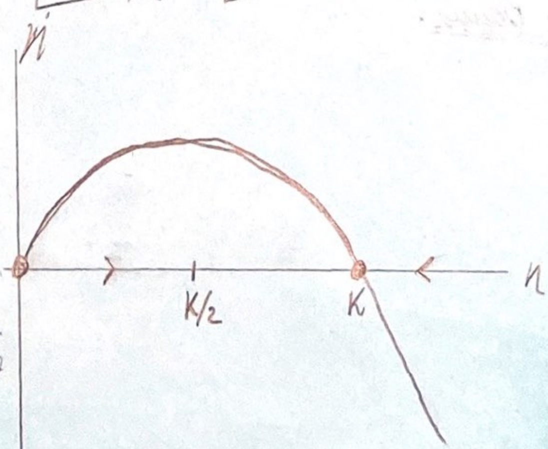
- fixed points: $rn^*\left(1-\frac{n^*}{K}\right)=0 \Rightarrow \boxed{n^*=0}$ and $\boxed{n^*=K}$

Only for $N>0$

- $n=K$ is a stable fixed point

- $n=0$ is a unstable fixed point.

For different values of r, the stability of fixed changes.



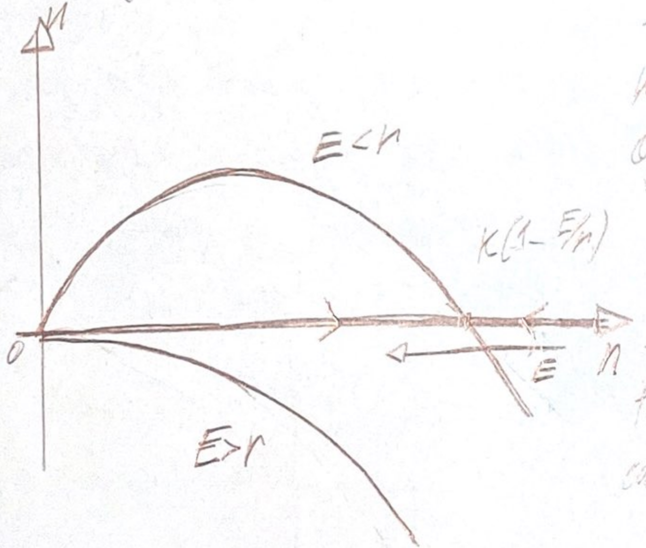
Model III: $\dot{n}(t) = r n(t)\left(1 - \frac{n(t)}{K}\right) - E n(t)$

- fixed points: $\dot{n} = 0 \Rightarrow r n^* - \frac{r n^{*2}}{K} - E n^* = 0$

$$n^*\left(r - \frac{r n^*}{K} - E\right) = 0 \Rightarrow \boxed{n^* = 0} \text{ or}$$

$$r - E = \frac{r n^*}{K} \Rightarrow \boxed{n^* = K(1 - E/r)}$$

As before for the simple logistic model, we have: Only considering $E > 0$



- If $E > r$, we harvest rapid than the growth rate of population, and always go to extinction.
- If $E < r$, we harvest giving time to population grows, and the system turns into a stable logistic system.

- The yield will be $\Rightarrow Y(E) = E n^* = EK\left(1 - \frac{E}{r}\right)$ and Y_{max} will be $\frac{dY(E^*)}{dE} = 0 \Rightarrow \boxed{E^* = \frac{r}{2}}$, back with E^* into $Y(E)$ we have $\boxed{Y_{max} = \frac{Kr}{4}}$

and finally $\boxed{n^*_{max} = \frac{K}{2}}$ the pop. in the maximum sustained yield.

<u>Extra Bonus:</u> Murray section 1.6.