Pendulum connected to a torsion spring

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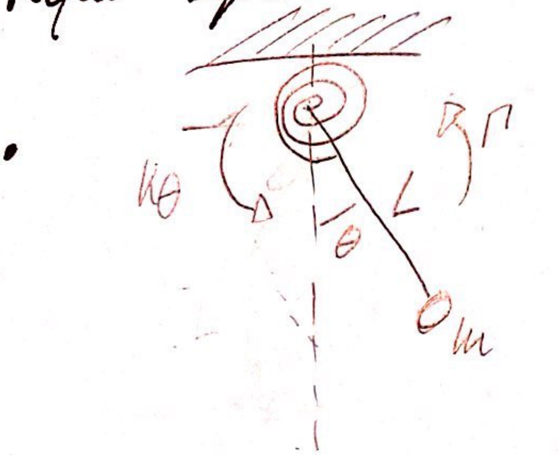


- From textbook, $mL^2\ddot{\theta}$ is a inertial amplitude term, $b\dot{\theta}$ a viscous damping and $mgL\sin\theta$ is as always the gravitational component of the mass m. Differently from the textbook

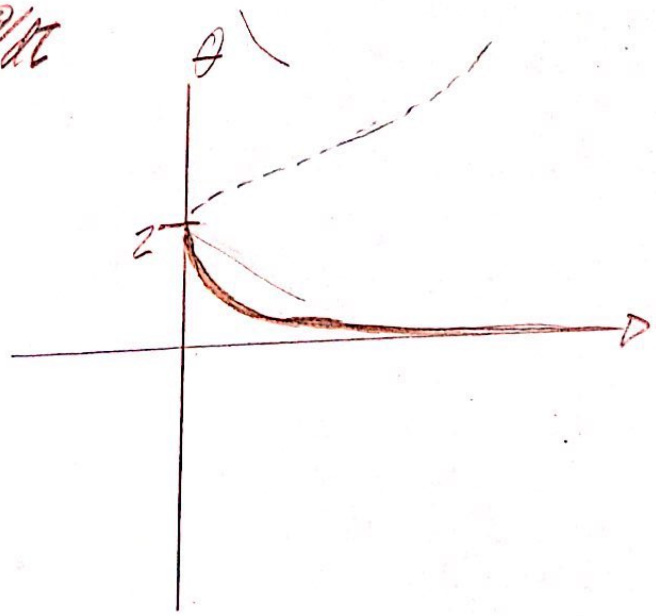
We have other than, Γ, an applied torque that gives a direction to the movement we have too an opposite torque due to torsional spring, the term $-k\theta$.

- For a $b \gg 1$, we can neglect $mL^2\ddot{\theta}$, and we have now:

$$b\dot{\theta} + mgL\sin\theta = \Gamma - k\theta \Longrightarrow \frac{b\dot{\theta}}{mgL} = \frac{\Gamma}{mgL} - \sin\theta - \frac{k}{mgL}\theta$$

$$\tau = \frac{mgL}{b}t, \quad \gamma = \frac{\Gamma}{mgL} \text{ and } k = \frac{k}{mgL} \Longrightarrow \boxed{\theta' = \gamma - \sin\theta - k\theta}$$

$\theta' = d\theta/d\tau$



- **Extra Bonus:**

The Bifurcation shows a stable line for θ less than $\frac{\pi}{2}$ which goes for the ∞. There a unstable points when θ is greater than $\pi/2$, for which gravitional align with the vertical line.

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