Parte Teorica

1. demostrar que

$$\frac{d^2 f(x_i)}{dx_i^2} \cong \frac{f(x_{i+2}) - 2f(x_i) + f(x_{i-2})}{4h}$$

$$f'(x) \cong \frac{f(x+h) - f(x-h)}{2h} = p(x) \quad (1)$$

- $$p'(x) \cong \frac{p(x+h) - p(x-h)}{2h} \quad (2)$$

$$p(x+h) \cong \frac{f(x+h+h) - f(x-h+h)}{2h} \quad (3)$$

$$p(x-h) \cong \frac{f(x+h-h) - f(x-h-h)}{2h} \quad (4)$$

Reemplazar (4), (3) en (2)

$$\frac{\frac{f(x+2h) - f(x)}{2h} - \frac{f(x) - f(x-2h)}{2h}}{2h} = p'(x) = f''(x)$$

$$\frac{f(x+2h) - f(x) - f(x) + f(x-2h)}{4h^2} = \frac{f(x+2h) - 2f(x) + f(x-2h)}{4h^2} = f''(x)$$

o también

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_i) + f(x_{i-2})}{4h^2}$$

(5) Demostrar

(A) $$D^4 f(x_j) \cong \frac{f(x_j+2) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_j-2)}{h^4}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h^2) = p(x) \quad (1)$$

$$p''(x) = \frac{p(x+h) - 2p(x) + p(x-h)}{h^2} + O(h^2) \quad (2)$$

$$p(x+h) = \frac{f(x+h+h) - 2f(x+h) + f(x-h+h)}{h^2} + O(h^2) \quad (3)$$

$$p(x-h) = \frac{f(x+h-h) - 2f(x-h) + f(x-h-h)}{h^2} + O(h^2) \quad (4)$$

Ahora se reemplaza (1), (3) y (4) en (2)

$$p''(x) = \frac{1}{h^2}\left[\frac{f(\overset{2}{x+2h}) - 2f(\overset{1}{x+h}) + f(\overset{0}{x})}{h^2} + \cancel{O(h)^2} - 2\left(\frac{f(\overset{1}{x+h}) - 2f(\overset{0}{x}) + f(\overset{-1}{x-h})}{h^2} + \cancel{O(h)^2}\right) + \ldots \right.$$
$$\left. \ldots \frac{f(\overset{1}{x}) - 2f(\overset{-1}{x-h}) + f(\overset{-2}{x-2h})}{h^2} + \cancel{O(h^2)}\right] + O(h)^2$$

$$p''(x) = \frac{\left[\dfrac{f(x+2h) - 2f(x+h) + f(x) - 2f(x+h) + 4f(x) - 2f(x-h) + f(x) - 2f(x-h) + f(x-2h)}{h^2}\right]}{h^2} + O(h)^2$$

$$p''(x) = \frac{f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h) + f(x+2h)}{h^4} + O(h)^2 = D^4 f(x)$$

ó

$$D^4 f(x_i) = \frac{f(x_i+2) - 4f(x_i+1) + 6f(x_i) - 4f(x_{i-1}) + f(x_i+2)}{h^4} + \underbrace{O(h)^2}_{\text{orden } h^2}$$

(B) el orden es 2 $(O(h^2))$