Optimal Monetary Response to Tariffs with Heterogeneous Households

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January 6, 2026

1 Introduction

How should an independent monetary authority respond to tariffs to maximize the welfare of its people? Due to a combination of existing inflationary pressures and the Trump administration's April 2nd announcement of sweeping tariffs, this question is of increasing relevance and has an unclear answer.

Among Federal Reserve policymakers, there were two primary schools of thought on the optimal monetary stance. The first, a "look-through" policy was described succinctly by Anna Paulson in a talk at the National Association for Business Economics: "Tariffs will increase the price level, but they won't leave a lasting imprint on inflation... Monetary policy should look through tariff effects on prices." This "look-through" policy is one that views a tariff as a mechanical change in the price level, rather than a phenomenon that leaves substantial, long-lasting effects on inflation. Thus, the fed's stance should be roughly neutral. They should leave rates unchanged and inflation will stabilize naturally.

The countervailing stance views price changes as more persistent and a threat towards the fed's goal of price stability. In an April talk, then-Fed governor Adriana Kugler summarized this stance: "It should be a priority to make sure that inflation doesn't move up...We are very committed still to our 2% target...which should be a priority now". Kugler highlights the risk of more persistent inflation and seems to err towards a contractionary stance to ensure that prices stabilize.

Though the discussion of the effect of tariffs on prices over short to medium horizons is important for the optimal monetary policy response, there is an additional channel under-emphasized by Fed policymakers, the *fiscal revenue channel* and the interactions between monetary and fiscal policy that a tariff shock induces. Tariffs induce changes in fiscal revenue. The spending of this revenue has real effects on the economy and will play a key role in determining the effect of a change in interest rates. In a recent paper, Bianchi and Coulibaly (2025) study the optimal monetary response to tariffs in a *representative agent environment* and find that optimal policy is expansionary, citing this fiscal revenue channel as the primary driver of this result. The tariffs create a fiscal externality that leads the household to consume too little of the foreign good, as they do not internalize the effect of their foreign consumption on fiscal revenues and redistributed transfers. Expansionary policy partially corrects this externality by cutting rates, which increases demand, foreign consumption, and redistributed tariff revenues at the cost of higher inflation.

I study the same question of optimal monetary responses to a tariff shock in a heterogeneous agent New-Keynesian (HANK) environment. The HANK literature has emphasized that monetary transmission (ex: Kaplan et al. (2018)) and therefore optimal policy (ex: Acharya et al. (2023)) differs in an environment with heterogeneous agents, as the planner now has an additional incentive to stabilize cross-sectional consumption volatility and reduce inequality.

Furthermore, the failure of Ricardian Equivalence in this class of models implies that the specification of fiscal policy and how the government's budget adjusts can affect equilibrium outcomes in a way that it cannot in RANK. In the case of tariffs, the revenue can be used to redistribute from rich, high foreign consumption agents to poor, low foreign consumption agents, increasing aggregate demand.

In this paper, I primarily focus on how the fiscal externality induced by a tariff shock affects optimal monetary policy in HANK. I write, calibrate, and solve a heterogeneous agent version of the model of Bianchi and Coulibaly (2025). I use the variational approach pioneered by Nuño and Moll (2018), Nuño and Thomas (2022), and Dávila and Schaab (2023) to solve for timeless, Ramsey-optimal policy numerically and highlight some simple theoretical mechanisms. In particular, I derive the targeting rule of Dávila and Schaab (2023) in my environment and show the new fiscal motive that is present when incorporating tariffs and their redistributed fiscal revenue.

The key insight for optimal policy is that how tariff revenue is used (redistributed) is a major qualitative and quantitative driver of the optimal monetary response to a tariff shock due to the presence of the fiscal externality.

I show analytically that it is comprised of two forces in HANK: an aggregate and a distributional component. The fiscal externality pushes towards a monetary expansion when the aggregate effect of additional revenue is positive (the mechanism of Bianchi and Coulibaly (2025)). It further pushes towards a monetary expansion when the covariance between the planner's marginal value of each household and the income effect of fiscal revenue is positive; this is the distributional effect.

Building off this intuition, I show numerically that when more progressive instruments adjust, such as progressive transfers or labor income taxes, rate cuts are larger. The cost of the inflation this additional expansion induces is small relative to the benefits that increased redistribution induces. However, when more distributionally neutral instruments adjust, such as the debt, rate cuts are smaller, as the redistributive benefits are no longer present.

Relative to the results of Bianchi and Coulibaly (2025), I emphasize that the optimality of expansionary policy and rate cuts comes from the fact that the planner views extra fiscal revenue and any redistribution it induces *positively*. If the planner does not desire additional fiscal revenue or the redistribution it induces, the look through policy, or even one that features rate hikes, will be optimal. Thus, knowledge of how fiscal revenues will be used is paramount for understanding the optimal monetary response to shocks, particular when the fiscal effects are strong (as is the case with tariffs)

Next, since any HANK economy necessarily has an inefficient steady state (unlike in RANK), the monetary response to tariffs will both correct existing distortions (from incomplete markets or suboptimal long run fiscal policy) and respond to new ones. To correct this issue, I use an inverse optimal approach (like the one of McKay and Wolf (2022)) to generate an efficient steady state and thus study how monetary policy should respond only to the *new* inefficiencies induced by a tariff, rather than correcting existing ones.

Again, I find that when the fiscal revenue is used to redistribute towards agents the planner values more (less) highly, they will want to cut rates more (less). Thus, if the revenue is used in a distributionally neutral way (for example, reducing the debt) the optimal monetary response will be nearly identical regardless of the planner weights. However, if the revenue is not spent in a distributionally neutral way, the optimal response may be very different across different sets of planner weights. In particular, when tariff revenue is redistributed lump sum and the planner uses welfare weights that induce an initially efficient steady state, the optimal policy becomes deflationary, as the planner views the fiscal redistribution as an explicit bad.

Like Bianchi and Coulibaly (2025), I then explore some additional environments to better understand how robust my results are to different model specifications. I consider a specification where the planner has an incentive to manipulate the terms of trade and highlight a new motive in the targeting rule. The planner will be less incentivized to cut rates, as doing so worsens the terms of trade. I also consider an environment where there are richer and more realistic structures of fiscal policy, and highlight how progressivity strengthens the incentives to inflate. I then study an environment where there are intermediate inputs subject to a tariff, which allows the tariff to function as a supply shock, rather than just a demand shock. Following other studies of the distributional effects of trade, I study an environment with non-homothetic preferences where households vary in their preferences for foreign consumption, and show the expenditure channel is qualitatively important but weak in practice. Finally, I study an environment where the planner is only subject to a traditional dual mandate rather than a utilitarian social welfare function. I find that these extensions do not meaningfully change my results: in each of them expansion remains optimal due to the fiscal revenue channel.

2 Literature

My work intersects intersects with four major strands of literature. First, I build off of the open economy (optimal) monetary policy literature. I am primarily in conversation with Bianchi and Coulibaly (2025), who emphasizes the role of a fiscal externality in shaping the optimal monetary policy response to *exogenous* tariff shocks in RANK, inspired by the tariff shocks of 2025. I extend this analysis and show how household heterogeneity changes the optimal monetary response to the same tariff shocks, highlighting that the failure of

Ricardian equivalence (or distributional effects more generally) can strengthen or weaken this fiscal externality. Rather than simply issuing a refund to the representative household, tariff revenue in HANK can redistribute from rich households to poor households and increase aggregate demand, strengthening incentives to inflate. Work by Alessandria et al. (2025) emphasizes the strength of this revenue channel in correcting distortions induced by other taxes. Though they primarily analyze how this revenue channel affects the optimal tariff itself, the insight that using tariff revenue to reduce, for example, the distortion from a labor income tax, can be particularly powerful remains in my model, with this motive shaping optimal monetary policy.

Other work in this literature studies jointly optimal (endogenous) tariff and monetary policy. Auray et al. (2024) show that optimal monetary policy can partially thwart attempts for countries to endogenously engage in trade wars by lowering the optimal tariff and Jeanne (2021) studies the interaction between monetary policy instruments and a wide variety of fiscal instruments, including tariffs. Due to the difficulty of studying optimal monetary and fiscal policy jointly with heterogeneous agents, I leave questions of endogenous trade policy (or other endogenous fiscal policy) to future work, though I use an inverse optimal approach to work around the fact that my fiscal policy is chosen suboptimally. Work by Kalemli-Özcan et al. (2025) studies monetary responses to tariffs in a multi-sector RANK model and emphasizes that the positive effects of tariff shocks and the monetary policy responses they induce differ in environments with multiple sectors. Since my fiscal mechanisms remain present regardless of the economy's production structure, I leave analysis of this problem in HANK to future work.

I also contribute to the burgeoning HANK literature, particularly the strand of the literature concerned with optimal monetary policy. Starting from the seminal work of Kaplan et al. (2018), the HANK literature emphasizes that monetary transmission in heterogeneous and representative agent economies differs due to the presence of high MPC, liquidity constrained households who are unaffected by traditional intertemporal substitution effects of interest rate changes but greatly affected by general equilibrium effects on wage or transfer income. This difference extends to the conduct of optimal monetary policy. I primarily follow the analytical and computational framework introduced by Nuño and Moll (2018) and further developed by Nuño and Thomas (2022) and Dávila and Schaab (2023). Like them, I use continuous time and a variational approach with Gâteaux derivatives to characterize the solution to the planner's problem analytically coupled with standard computational methods to solve for optimal policy computationally. To circumvent the Ramsey planner's time 0 problem, I follow the timeless approach discussed by Dávila and Schaab (2023). My paper is the first in this particular substrand of the heterogeneous agent optimal monetary policy literature to emphasize how a realistic fiscal block affects optimal monetary policy. I show theoretically that the addition of non-trivial fiscal policy induces additional motives for the planner to inflate/deflate (expanding on the targeting rule of Dávila and Schaab (2023)) and show it is quantitatively powerful in the case of a tariff shock.

Other substrands of this literature have studied how fiscal and monetary interactions shape optimal policy in environments with limited heterogeneity. For instance, Le Grand et al. (2022) and Le Grand et al. (2025) use a truncation method to study jointly optimal fiscal and monetary policy. In an environment similar to mine, where fiscal instruments are limited, optimal monetary policy has a role to redistribute over the business cycle, though there is no such scope with sufficient rich and optimally chosen fiscal instruments. My work puts particular emphasis on how a fiscal externality (or a fiscal revenue motive) shapes optimal policy and is used to redistribute over the business cycle. I show that the covariance between the effects of fiscal revenue and planner weights is a key determinant of whether the fiscal externality pushes the planner to inflate or deflate. Since fiscal policy in my model is not chosen optimally and optimal longer run fiscal policy is outside the scope of this analysis, I use an inverse optimal approach (see Heathcote and Tsujiyama (2021) or McKay and Wolf (2022)) to start from an efficient steady state with fiscal policy taken as given and isolate the effect of optimal monetary policy on the new inefficiencies induced by the tariff.

Finally, in the only paper that discusses optimal policy in an open economy HANK model, Acharya and Challe (2025) extend their 2023 paper to an open economy, and show that the central bank has an additional motive when conducting monetary policy: manipulation of the terms of trade. They uses CARA preferences and lognormal shocks to obtain closed form solutions to their model. When I allow for endogenous terms of trade, this motive remains present, though I primarily abstract from it and instead focus on the fiscal revenue effects of a tariff.

I also contribute to the open economy HANK literature. Guo et al. (2023) study the positive effects of monetary policy in heterogeneous agent, open economy environments. They show that different international integration/exposure through household employment or access to international markets drives much of the heterogeneity in the positive consequences of international shocks. I show that heterogeneity in exposure through preferences can similarly affect the normative design of monetary policy, though this channel is quantitatively weak. I avoid explicit discussion of heterogeneity in sectoral employment or financial integration in this work, emphasizing that all of my fiscal mechanisms remain in environments with richer labor and financial markets.

Finally, I contribute to the literature discussing the distributional consequences of trade and international shocks. Carroll and Hur (2023) studies the distributional effects of tariff shocks and emphasizes the allocation of tariff revenue is a strong determinant of the positive effects of the shock. I build on this insight, showing that the reallocation of tariff revenue plays a similarly strong role in determining the normative response of policy to a tariff shock, as the planner is incentivized to inflate more when the adjusting instrument improves the welfare of higher marginal value agents. Borusyak and Jaravel (2021) studies the expenditure channel of trade shocks and finds that American households have roughly similar expenditure on foreign, tariffed goods across the income distribution, contrary to past work (for example, that of Carroll and Hur (2020)). I show that these expenditure shares, or preference weights on foreign goods, partially determine the planner's marginal value on each household and in turn the extent to which the planner will inflate or deflate. I use the fact that expenditure shares are roughly constant to justify my baseline assumption of homogeneous expenditure shares over home and foreign goods. However, this expenditure channel does not quantitatively affect optimal policy, even if I use nonhomothetic preferences.

Outline I organize the remainder of the paper as follows. Section 3 introduces my baseline model. Section 4 describes the timeless Ramsey optimal monetary policy problem and highlights some theoretical results. Section 5 describes the baseline quantitative results. Section 6 quantitatively solves the same model with heterogeneous planner welfare weights that generate an ex-ante efficient steady state. Section 7 introduces additional extensions of the baseline model and discusses their properties. Finally, section 8 concludes

3 Model

I adapt the model of Bianchi and Coulibaly (2025) into continuous time and incorporate household heterogeneity in productivity.

Environment Time is continuous and infinite. The economy includes

1. A continuum of ex-ante identical households.
2. A final firm that produces the home good.
3. A continuum of intermediate firms that produce inputs for the home good.
4. A fiscal authority that levies tariffs on the foreign good and distributes subsidies and transfers to households and firms
5. A monetary authority that sets interest rates.

Households are subject to idiosyncratic labor productivity shocks z, which evolve according to a n-state Poisson process, and exponentially distributed life lengths with mean $1/\eta$[1]. These are the only source of uncertainty in the model. Households supply labor to the monopolistically competitive intermediate good firms, who in turn sell goods to a perfectly competitive final firm. Households choose consumption and savings in home denominated bonds subject to a borrowing constraint ϕ. Intermediate firms demand household labor and set prices under monopolistic competition subject to Rotemberg adjustment costs. The government is subject to a period budget constraint, issuing one period non-state contingent debt, levying tariffs, providing an employment subsidy to firms, and distributing tariff revenue to households. The monetary authority optimally chooses interest rates to maximize household welfare.

[1] Random births and deaths are not needed to ensure the existence of the stationary distribution, but does ensure the generator matrix $\mathcal{A}$ has a well behaved inverse, which will be necessary when numerically solving for the monetary authority's optimal policy

3.1 Households

There is a continuum of ex-ante identical households in the home country indexed $i \in [0, 1]$. Households receive disutility from labor $n_{i,t}$, which is divisible and indexed 0 to 1 and receive utility from consumption, aggregated according to the CES aggregator

$$(c_{i,t})^{\frac{\gamma-1}{\gamma}} = \omega_i (c^h_{i,t})^{\frac{\gamma-1}{\gamma}} + (1-\omega_i)\left(c^f_{i,t}\right)^{\frac{\gamma-1}{\gamma}}$$

where $c_{i,t}$ is aggregate consumption, $c^h_{i,t}$ is consumption of the home good, $c^f_{i,t}$ is consumption of the foreign-produced good, ω_i is the preference weight on home goods relative to foreign, and γ is the elasticity of substitution between home and foreign goods (the trade elasticity).

Each instant, households choose consumption, labor supply, and savings in home and foreign bonds, subject to a borrowing constraint ϕ. Household productivity z_i evolves according to the n-state Poisson process Λ with transition intensity $\lambda_{j,j'}$. Households have exponentially distributed lifetimes with mean $1/\eta$. Each period, mass η of households are born with no wealth and productivity drawn from the stationary distribution of Λ. To simplify notation, in this subsection I avoid explicitly denoting each household's variables with i. Thus, the household's Hamilton Jacobi Bellman equation can be written as:

$$\tilde{\rho} V_t(b_t, z_{jt}) = \max_{c^f_t, c^h_t, n_t} u(c^f_t, c^h_t) - g(n_t) + \frac{\partial V_t}{\partial b_t}\dot{b}_t + \sum_{j' \neq j} \lambda_{j,j'}\left[V_t(j') - V_t(j)\right] + \frac{\partial V_t}{\partial t}$$

subject to

$$\dot{b}_t = (r_t + \eta) b_t + (1-\tau_n) z_t w_t n_t + T_t + D_t - p^h_t c^h_t - p^f_t (1+\tau_t) c^f_t$$

$$b_t \geq \phi$$

where $\tilde{\rho} = \rho + \eta$, c^f and c^h are consumption of the foreign and home goods with prices p^f and p^h, respectively, n is labor supply, b^h and b^f are positions in the risk-free home and foreign bonds with nominal interest rates r and r^f, respectively, z_j is idiosyncratic productivity, w is the nominal wage, T and D are lump sum transfers/taxes and dividends, respectively, τ^n is a labor income tax, and τ is a tariff. Following Nuño and Moll (2018), households purchase bonds from a perfectly competitive intermediary who offers a gross return η in exchange for all of the household's assets at time of death[2]. The relative price $p = \frac{p^f}{p^h}$ is exogenous and optimal tariffs are 0[3].

Now, define the generator $\mathcal{A}$ according to

$$\mathcal{A}V = \begin{bmatrix} \frac{\partial V_1}{\partial b} s_1 + \sum_{j' \neq 1} \lambda_{1,j'} \left[V_{j'} - V_1 \right] \\ \vdots \\ \frac{\partial V_J}{\partial b} s_J + \sum_{j' \neq J} \lambda_{J,j'} \left[V_{j'} - V_J \right] \end{bmatrix}$$

where V_j is shorthand for $V(b, z_j)$ and s_j similar shorthand for the savings function $s(b, z_j) = \dot{b}$. Thus, the HJB can be expressed as

$$\tilde{\rho} V_t = \max_{c_t^f, c_t^h, n_t} u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} \tag{1}$$

which will be useful when deriving the solution to the planner's problem later.

When solving the model numerically, I assume CRRA utility for total consumption and power disutility of labor according to

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma}$$
$$g(n_{i,t}) = \chi \frac{n_{i,t}^{1+\kappa}}{1+\kappa}$$

Household First Order Conditions The first order conditions for the household are

$$c_t^f = c_t^h \left(\frac{\omega}{1-\omega} (1+\tau_t) \frac{p_t^f}{p_t^h} \right)^{-\gamma} \tag{2}$$

$$g'(n_t) = \frac{w_t z_t}{p_t^h} u_h(c_t^h, c_t^f) \tag{3}$$

$$u_h(c_t^h, c_t^f) = \frac{\partial V(b_t, z_t)}{\partial b_t} \tag{4}$$

The first equation governs the optimal split between home and foreign goods. The second and third equations are standard labor supply and Euler equations.

[2] The intermediary has flow revenues ηB and pays flow costs ηb to each household; aggregate profits are thus 0

[3] I lift this assumption in section 7

Sources of Suboptimality The tariff distorts three margins, two of which are present in RANK and HANK, with the third only being present in HANK.

1. First, it directly distorts the home vs foreign consumption margin. As τ increases, c^f decreases.

2. Second, it distorts the labor/leisure margin. The price level the household faces in equilibrium can be written as $\mathcal{P}_{i,t} = P_t^h \left[\omega_i^\gamma + (1-\omega_i)^\gamma (p(1+\tau_t))^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, which is increasing in the tariff. Thus, since consumption is more expensive, it acts as a tax on labor, distorting the labor/leisure margin indirectly.

3. Finally, unlike in a RANK economy, it distorts the intertemporal consumption savings margin, even if tariffs are constant. Since consumption is more expensive, households who are borrowing constrained can afford less consumption.[4] Thus, there is an incentive to engage in more precautionary savings.

Kolmogorov Forward Equation The distribution of households $f(b, z_j, t)$ satisfies the following Kolmogorov Forward Equation

$$\frac{\partial f_t(b_t, z_{jt})}{\partial t} = -\frac{\mathrm{d}}{\mathrm{d}b_t}[s_t(b_t, z_{jt}) f_t(b_t, z_{jt})] - f_t(b_t, z_{jt}) \sum_{j' \neq j} \lambda_{jj'} + \sum_{j' \neq j} \lambda_{j'j} f_t(b_t, z_{jt}) - \eta f_t(b_t, z_{jt}) + \eta \delta_0$$

where s is the savings functions that solve the household problem. The final two terms represent respectively the outflow due to deaths and the inflow of agents with 0 wealth and productivity drawn from the stationary distribution of Λ; δ_0 is the Dirac delta function.

The KFE can similarly be expressed in terms of the adjoint of $\mathcal{A}$, $\mathcal{A}^*$

$$\frac{\partial f}{\partial t} = \mathcal{A}^* f + \eta \Lambda \tag{5}$$

[4]This can be shown mathematically by examining a discrete time version of the model's Euler Equation: $(c_{i,t}^h)^{-\sigma} \mathcal{C}_{i,\tau} = \mathbb{E}\left[\beta R_{t+1} (c_{i,t+1}^h)^{-\sigma} \mathcal{C}_{i,\tau}\right] + \lambda_{i,t}$ where $\mathcal{C}_{i,\tau}$ is a time-invariant scalar that increases with τ. In RANK, or if $\lambda_{i,t} = 0 \forall i$, the scalar $\mathcal{C}_{i,\tau}$ cancels and the consumption savings margin is non-distorted. However, in HANK when the borrowing constraint binds, this is no longer the case and the consumption savings margin is distorted. There will be additional precautionary savings in equilibrium.

where

$$\mathcal{A}^* f = \begin{bmatrix} -\frac{\partial(s^1 f_1)}{\partial b} - f_1 \sum_{j' \neq 1} \lambda_{1j'} + \sum_{j' \neq 1} \lambda_{j'1} f_1 \\ \vdots \\ -\frac{\partial(s_J f_J)}{\partial b} - f_J \sum_{j' \neq J} \lambda_{Jj'} + \sum_{j' \neq J} \lambda_{j'J} f_J \end{bmatrix}$$

with shorthand similar to that defined in the previous section[5].

3.2 Firms

Firms are nearly identical to a standard New Keynsian production block.

Final Firm The output of the final good producer is Y_t, which combines a continuum of intermediate goods $y_{j,t}$, $j \in [0, 1]$ according to

$$Y_t = \left(\int_0^1 (y_{j,t})^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\theta > 0$ is the elasticity of substitution across intermediate goods. The static profit maximization problem of the final firm yields the demand for intermediate goods

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t^h} \right)^{-\epsilon} Y_t$$

where the price index of the home-produced good is $P_t^h = \left(\int_0^1 (p_{j,t})^{1-\epsilon} \, \mathrm{d}j \right)^{\frac{1}{1-\epsilon}}$.

Intermediate Firms The continuum of intermediate good producers each produce one variety $j \in [0, 1]$ according to the linear production technology

$$y_{j,t} = A_t n_{j,t}$$

These producers are monopolistically competitive and set prices a la Rotemberg (1982). Defining producer price inflation as $\pi_t = \frac{\dot{p}_t^h}{p_t}$, the adjustment costs are

$$\Theta(\pi) = \frac{\theta}{2} \pi_t^2 Y_t$$

[5]See Nuño and Moll (2018) for a proof that $\mathcal{A}$ and $\mathcal{A}^*$ are adjoints.

Given the production technology, equilibrium marginal costs are $m_t = \frac{w_t}{P_t^h A_t}$. Assuming firms discount the future at rate r_t^h and omitting j subscripts for clarity, the firm's pricing problem is cast recursively as

$$r_t J(p^h, t) = \max_{\pi} \left(\frac{p_t^h}{P_t^h}(1+S) - m_t \right) \left(\frac{p_t^h}{P_t^h} \right)^{-\epsilon} Y_t - \Theta(\pi) + J_p(p^h, t) p_t^h \pi_t + J_t(p^h, t)$$

where the production subsidy $S = \frac{1}{\epsilon - 1}$ corrects the markup distortion in steady state. Then, in a symmetric equilibrium with $n_{j,t} = N_t = A_t Y_t = A_t y_{j,t}$, $p_{j,t}^h = P_t^h$ the NKPC is

$$(r^h - \frac{\dot{N}_t}{N_t})\pi_t = \frac{\epsilon}{\theta}(m-1) + \dot{\pi} \tag{6}$$

Additionally, this implies (real) dividends are

$$\frac{D_t}{P_t^h} = (1+S)Y_t - m_t Y_t - \Theta(\pi_t) \tag{7}$$

3.3 Government

The fiscal authority issues debt, collects tariff revenue, and distributes it to households after paying for the production subsidy

$$\dot{B}_t^h = r_t B_t^h + T_t + s p_t^h Y_t - \tau_t p^f \int c_t^f \, \mathrm{d}f_t(b, z) - \tau^n w_t \int n_t z_t \, \mathrm{d}f_t(b, z) \tag{8}$$

where $\int c_f \, \mathrm{d}f(b^h, b^f, z)$ is aggregate consumption of the foreign good, $\int nz \, \mathrm{d}f(b^h, b^f, z)$ is aggregate labor, B_t are aggregate bonds, T_t are lump sum transfers, and τ^n are labor income taxes. Since Ricardian equivalence fails, the specification of fiscal policy and which variable adjusts to satisfy the government budget will affect equilibrium outcomes. As a baseline, I assume that transfers adjust, though I present results with bonds and labor taxes adjusting.

3.4 Market Clearing

Market clearing requires

$$N_t = n_{j,t} = \int nz \, \mathrm{d}f(b^h, b^f, z) \tag{9}$$

$$B^h + B^f = \int b^h + b^f \, \mathrm{d}f(b^h, b^f, z) \tag{10}$$

$$0 = Y_t - \frac{\phi}{2}\pi_t^2 Y_t - c_t^h - pc_t^f \tag{11}$$

i.e. labor market clearing, bond market clearing, and a balance of payments condition

3.5 Equilibrium

I now definite a competitive equilibrium.

Definition 1 *Given an initial distribution over bond holdings and idiosyncratic states $f_0(b_0, z_0)$, an exogenous relative price p, sequences of government policy for S, T_t, B_t, $\tau_{n,t}$, and τ_t, a specification for monetary policy r_t, a competitive equilibrium requires finding (sequences of) household policy functions s_t, n_t c_t^h, c_t^f and value functions V_t, solutions for the firms' problem $n_{j,t}$, $y_{j,t}$, $p_{j,t}^h$, Y_t, N_t, prices w_t, and measures $f_t(b_t, z_t)$ such that*

1. *Given prices and interest rates, the HJB holds*
2. *Given the savings function $s_t(b_t, z_t)$, f_t solves the KFE*
3. *Given wages, $n_{j,t}$ solves the firm cost minimization problem and $p_{j,t}^h$ solves the price setting problem*
4. *Given a specification for 3 of S, T_t, B_t, and τ_t^n, the fourth is determined by the government budget constraint*
5. *Markets clear*
   (a) *Labor market:* $N_t = n_{j,t} = \int n \, \mathrm{d}f_t(b_t, z_t)$
   (b) *Bond market:* $B_t = \int b_t \, \mathrm{d}f_t(b_t, z_t)$
   (c) *Goods:* $\underbrace{Y_t - \frac{\theta}{2}\pi_t^2 Y_t - C_t^h}_{Exports} = \underbrace{pC_t^f}_{Imports}$

4 Timeless Ramsey Optimal Monetary Policy Problem

I now describe the timeless Ramsey planner's optimal policy problem from the timeless perspective. The planner's only instrument is the (real) interest rate r_t. Optimal policy from the utilitarian central bank requires picking paths of the instrument r_t, aggregate allocations

N_t, C_t^f , policy functions $c_t^f(.), c_t^h(.), n_t(.)$, prices w_t, π_t, distributions f_t, and value functions V_t subject to satisfying the optimality conditions of all agents and all markets clearing. Using the timeless perspective ensures the planner has no incentive to induce inflation at time 0, a standard issue in the optimal monetary policy literature.

In particular, define time 0 welfare W_0 as

$$W_0[f_0(b,z)] \equiv \mathbb{E}_{f_0(b,z)} V_0(b,z) = \int_0^\infty e^{-\tilde{\rho}t} \mathbb{E}_{f_0(b,z)}[u(c_t^f, c_t^h) - g(n_t)]\, \mathrm{d}t \tag{12}$$

where $\tilde{\rho} = \rho + \eta$, the effective discount rate. Informally, the Ramsey optimal policy problem requires maximizing equation 12 by choosing sequences of $r_t, \pi_t, V_t(.), c_t^f(.), c_t^h(.), n_t(.), f_t(.), w_t$, N_t, C_t^f subject to all of the implementability conditions for the competitive equilibrium: the HJB (eq 1), all household FOC (eq 2-4),the KFE (eq 5), the NKPC (eq 6), the government budget constraint (eq 8), and market clearing (eq 9-11).

As in Nuño and Moll (2018), the optimal value and associated polices are functionals, as they map the infinite dimension initial distribution f_0 into $\mathbb{R}$ (in the case of welfare) or other infinite dimensional distributions (in the case of, for example consumption policy functions).

Proposition 1 (Primal Timeless Ramsey Problem) *The Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{9}} \max_{r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0}, \eta_{2,0}) \tag{13}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ [6] *defined as*

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_{\phi}^{\infty}\left[u(c_t^h, c_t^f) - g(n_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$
$$\eta_{2,t}(b,z)\left[u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right] +$$
$$\eta_{4,t}[b_t^h - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c_t^f - C_t^f]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h, c_t^f)\right]$$
$$\eta_{8,t}(b,z)\left[u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}p_t^h\right] + \eta_{9,t}(b,z)\left[u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}p_t^h(1+\tau)\right] \mathrm{d}b\,\mathrm{d}z\}\,\mathrm{d}t$$

where $\eta_{1,t}, \eta_{2,t}, \eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ *are functional Lagrange multipliers and* $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, *and* $\eta_{6,t}$ *are scalar Lagrange multipliers and* z_1 *and* z_J *are the lower and upper bounds, respectively, on the idiosyncratic shock* z. *Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in* $\mathcal{A}$ *by substituting for transfers.*

The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ *is defined as in Dávila and Schaab (2023) and equals:*

$$\int_{z_1}^{z_J}\int_{\phi}^{\infty}\eta_{2,0}V_0(b,z)\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^9$ *and choice variables* $r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f$ *that solve 13*

In appendix B I derive proposition 1 and all other results in more detail, as well as more mathematical details in appendix A.

Now, I characterize the solution to the optimal policy problem

Proposition 2 (Ramsey Optimality Conditions) *In addition to the implementability conditions, the solution to the Ramsey planner's optimal policy problem is characterized by the following system of equations*

[6] The Lagrangian depends on both time and the infinite dimension initial distribution f_0. To maximize with respect to the choice functions, I follow Nuño and Moll (2018) and employ Gâteaux derivatives, an extension of the derivative from $\mathbb{R}^n$ to infinite dimension function spaces. The Gâteuax derivative of $\mathcal{L}$ with respect to, for example, V_t is defined as

$$\lim_{\alpha\to 0}\frac{\mathcal{L}[v_t + \alpha h, ...] - \mathcal{L}[v_t, ...]}{\alpha} = \frac{\mathrm{d}}{\mathrm{d}\alpha}\mathcal{L}[v_t + \alpha h, ...]|_{\alpha=0}$$

where h is any function in the same space as V_t. More mathematical details can be found in appendix A.

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(n_t - N_t) + \eta_{6,t}(c_t^f - C_t^f) \tag{14}$$

$$\frac{\partial\eta_{2,t}}{\partial t} = \mathcal{A}^*\eta_{2,t} + \frac{\partial\eta_{8,t}}{\partial b}p + \frac{\partial\eta_{9,t}}{\partial b}(1+\tau)p \tag{15}$$

$$0 = u_f(c_t^h, c_t^f) - \frac{\partial\eta_{1,t}}{\partial b}(p(1+\tau)) + \frac{\eta_{9,t}}{f_t}u_{ff}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hf}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{fh}(c_t^h, c_t^f) + \eta_{6,t} \tag{16}$$

$$0 = u_h(c_t^h, c_t^f) - \frac{\partial\eta_{1,t}}{\partial b}p + \frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) \tag{17}$$

$$0 = -g'(n_t) + \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h}\frac{\partial\eta_{1,t}}{\partial b} + \eta_{7,t}\frac{g''(n_t)}{f_t} + z\eta_{5,t} \tag{18}$$

$$\eta_{3,t}\pi_t = \int_{z_1}^{z_J}\int_{\phi}^{\infty}(B_t - b_t^h)\frac{\partial\eta_{1,t}}{\partial b}f_t + (B_t - b_t^h)\frac{\partial V_t}{\partial b}\eta_{2,t}\,\mathrm{d}b\,\mathrm{d}z \tag{19}$$

$$\eta_{3,t}\frac{\epsilon}{\theta} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}(1-\tau^n)(z_t n_t - N_t)\left(\frac{\partial\eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right) - (1-\tau^n)zu_h(c^f, c^h)\eta_{7,t}\,\mathrm{d}b\,\mathrm{d}z \tag{20}$$

$$\dot{\eta}_{3,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial\eta_{1,t}}{\partial b}(N_t\theta\pi_t)f_t + \frac{\partial V_t}{\partial b}(N_t\theta\pi_t)\eta_{2,t}\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,t}(\tilde{\rho} - r + \frac{\dot{N}_t}{N_t}) \tag{21}$$

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial\eta_{1,t}}{\partial b}\left(1 - m_t - \frac{\theta}{2}\pi_t^2 + \tau w\right)f_t + \frac{\partial V_t}{\partial b}\left(1 - m_t - \frac{\theta}{2}\pi_t^2 + \tau w\right)\eta_{2,t}\,\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} \tag{22}$$

$$\eta_{6,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial\eta_{1,t}}{\partial b}\tau p f_t + \frac{\partial V_t}{\partial b}\tau p\eta_{2,t}\,\mathrm{d}b\,\mathrm{d}z \tag{23}$$

with boundary conditions[7]

$$\lim_{T\to\infty} e^{-\tilde{\rho}T}\eta_{1,T} = 0$$

$$\eta_{3,0} = \eta_3$$

$$\eta_{2,0} = \eta_2$$

Equation 14 can be interpreted as the planner's HJB equation, where $\eta_{1,t}(b,z)$ is the social value function. Understanding the difference between the social and private values is best done through the marginal value functions. The marginal social value of wealth for a household with a given state vector $\frac{\partial\eta_{1,t}(b,z)}{\partial b}$ is related to the marginal private value $\frac{\partial V_t(b,z)}{\partial b}$ through the following identity derived in Appendix B

$$\frac{\partial\eta_{1,t}(b,z)}{\partial b} = \frac{\partial V_t(b,z)}{\partial b} + \eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}\frac{\partial n(b,z)}{\partial b} + \eta_6\mathcal{M}^{-1}\frac{\partial c^f(b,z)}{\partial b}$$

[7]These initial conditions are the only place the timeless penalty affects the planner's solution. All first order conditions are identical with or without a timeless penalty. Solving this problem without the timeless penalty would allow both inflation and initial household values to be determined freely by the planner and incentivize the planner to induce inflation at time 0, even in the absence of shocks. Adding the timeless penalty will set the initial conditions to ensure the planner does not do this and make the steady state I initialize my shocks from sensible. See Dávila and Schaab (2023) for a more detailed discussion.

defining the operator .

$$\mathcal{M} = \tilde{\rho} - s_b - \mathrm{d}t - \mathcal{A} \tag{24}$$

The 3 Lagrange multipliers $\eta_{4,t}$, $\eta_{5,t}$, and $\eta_{6,t}$ reflect the addition of pecuniary externalities. The former two terms are not unique to an open economy environment. Individual agents do not internalize how their savings and labor decisions infinitesimally affect market-clearing prices, redistributed profits, or transfers they receive. The planner internalizes these forces, which are captured by the additional terms in their HJB. Additionally, in this open economy model with tariffs, the planner's HJB includes a fiscal externality multiplier $\eta_{6,t}$, similarly reflecting that individual agents do not internalize how their foreign consumption affects fiscal revenue and therefore the transfers they and all other agents receive.

Thus, the difference in the marginal social and private values of wealth $\eta_4 \mathcal{M}^{-1} + \eta_5 \mathcal{M}^{-1} \frac{\partial n(b,z)}{\partial b} + \eta_6 \mathcal{M}^{-1} \frac{\partial c^f(b,z)}{\partial b}$ captures the present discounted value of future savings to aggregate excess bond demand, future labor supply to aggregate excess labor, and foreign consumption to aggregate excess consumption demand caused by an increase in time t assets. Increasing assets today will affect the marginal propensity to consume foreign goods $c^f(b,z)$, the marginal propensity to work $n(b,z)$, and directly affects asset holdings, both today and in the future (as captured by the $\mathcal{M}^{-1}$ term). When one of the Lagrange multipliers η_4, η_5, η_6 are positive (negative), the additional excess savings, labor supply, and foreign consumption are respectively beneficial (harmful). Note that relative to the closed economy model of Dávila and Schaab (2023), the η_6 term is novel and captures the planner's assessment of the value of the fiscal externality when tariffs are non-zero.

Equation 15 can be interpreted as the "planner's KFE", where $\eta_{2,t}$ is the "social density", or a sort of distributional penalty in the language of Dávila and Schaab (2023). If the planner was not constrained by household optimality conditions, they would prefer different individual policies and a different density than the one that prevails in equilibrium. $\eta_{2,t}$ governs this penalty/benefit on the welfare gains/losses for each type of household and works to ensure the planner satisfies the private optimality conditions.

Equations 16-18 govern the planner's choice for individual policies. UThe central bank must obey household optimality conditions. Without explicitly including these constraints, the FOC would simply be the partials of the planner's HJB (equation 14) with respect to c^h, c^f, and n respectively. Thus, optimal household choices would be fully characterized by solving the planner's HJB. However, enforcing the private FOC hold adds $\eta_7(b,z), \eta_8(b,z)$, and $\eta_9(b,z)$ terms, capturing the fact that the planner would prefer levels of individual consumption and labor not consistent with households' optimization.[8]

Equations 19-21 are the first order conditions for optimal interest rates, wages, and inflation, and characterize the Lagrange multiplier on the Phillips curve and the level of optimal inflation. The former states that planner balances a redistributive motive and associated penalty (raising the interest rate redistributes from rich, low social marginal value agents to poor, high social marginal value agents) with tightening/loosening the NKPC by lowering/raising rates. Similarly, the first order condition for inflation reflects the tradeoff that an increase in π reduces household income through an increase in deadweight loss adjustment costs but makes the NKPC more slack.

The Fiscal Externality Equations 22-23 are the planner's first order conditions for optimal aggregate labor and foreign consumption. They characterize the Lagrange multipliers in the planner's HJB equation. Following Nuño and Thomas (2022), I can rewrite the Lagrange multiplier for aggregate foreign consumption, $\eta_{6,t}$ ($\eta_{5,t}$ can be interpreted analogously) as

$$\eta_{6,t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \underbrace{\frac{\partial s_t}{\partial T} \frac{\partial T}{\partial C^f}}_{\text{Marginal Effect of Transfers}} \underbrace{\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right)}_{\text{Net Planner Marginal Value}} f_t \, \mathrm{d}b \, \mathrm{d}z$$

$$\eta_{6,t} = \underbrace{\mathrm{COV}_{f_t} \left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right), \frac{\partial s_t}{\partial T} \frac{\partial T}{\partial C^f} \right]}_{\text{Distributional Effect}} + \underbrace{\mathbb{E}_{f_t} \left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right) \right] \mathbb{E}_{f_t} \left[\frac{\partial s_t}{\partial T} \frac{\partial T}{\partial C^f} \right]}_{\text{Average Effect}}$$

which is a combination of a covariance or distributional effect term and an average effect term.

The covariance term says that the shadow value of additional foreign consumption is higher if increasing aggregate foreign consumption has a larger income effect on the those with a higher net planner marginal value (in general, the poor). The second term says the

[8]For example, with a non-zero tariff, the first best mix of consumption over the two goods satisfies $\frac{1-\omega}{\omega}\left(\frac{c^h}{c^f}\right)^{\frac{1}{\gamma}} = p$ while the private optimality condition is $\frac{1-\omega}{\omega}\left(\frac{c^h}{c^f}\right)^{\frac{1}{\gamma}} = (1+\tau_t)p$. Hence, η_7, η_8, and η_9 can be thought of as the penalty value of the planner being forced to obey the three private optimality conditions.

shadow value of additional foreign consumption is higher if the economy on average will receive more income as a result of increasing aggregate foreign consumption. Under the assumption of flat transfers $\frac{\partial s_t}{\partial T}$ is constant. However, equation 23 still holds if transfers are not flat[9]. If transfers were progressive, the covariance term would be larger and more positive, indicating that the planner would prefer even more foreign consumption. However, even if transfers were regressive, $\eta_{6,t}$ could be positive, as the "average effect" term would remain positive even though the covariance term would be negative.

Note that $\eta_6 > 0$ will push the planner towards cutting rates and positive inflation. Since the planner's instrument is blunt, the only way to induce additional foreign consumption is by cutting rates. This mechanism is present in RANK (see Bianchi and Coulibaly (2025)) but the additional covariance or distributional term in the definition of the fiscal externality strengthens this effect. Now the planner views additional foreign consumption positively since it has positive aggregate effects and (potentially) positive distributional effects.

4.1 Characterizing Optimal Policy

There is no closed form for the optimal level of inflation or interest rate path in this economy with idiosyncratic risk and CRRA preferences. However, I am able to qualitatively characterize some aspects of optimal policy.

First, I briefly discuss the features of the steady state of the economy.

Proposition 3 (Long Run Inflation) *Long-run (or steady state) inflation is 0*

Proof. In a steady state,

$$0 = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial \eta_{1,t}}{\partial b}(N_t \theta \pi_t) f_t + \frac{\partial V_t}{\partial b}(N_t \theta \pi_t)\eta_{2,t} \, \mathrm{d}b \, \mathrm{d}z + \eta_{3,t}(\tilde{\rho} - r)$$

The precautionary motive implies $\tilde{\rho} - r > 0$ (Sargent and Ljungqvist (2012)). To satisfy the first order condition if $\pi \neq 0$, either $\pi > 0$ and $\eta_{3,t} < 0$ or $\pi < 0$ and $\eta_{3,t} > 0$ due to properties of the social and private value functions. Additionally,

$$\eta_{3,t}\pi_t = \int_{z_1}^{z_J} \int_{\phi}^{\infty} (B_t - b_t^h) \frac{\partial \eta_{1,t}}{\partial b} f_t + (B_t - b_t^h) \frac{\partial V_t}{\partial b} \eta_{2,t} \, \mathrm{d}b \, \mathrm{d}z$$

[9] See appendix G for a proof

must hold. However, the RHS of the above is positive since agents with $B_t - b_t^h >> 0$ ($B_t - b_t^h << 0$) will have the highest (lowest) marginal value, so either $\pi > 0$ and $\eta_{3,t} > 0$ or $\pi < 0$ and $\eta_{3,t} < 0$, a contradiction of $\pi \neq 0$ ∎

Thus, with $\tau = 0$ (no fiscal externality) or $\tau > 0$ (fiscal externality present) long run inflation is 0. Inflation or deflation in response to any tariff shock, even if the shock itself is permanent, is temporary. This result echos the same result of Dávila and Schaab (2023) even with a fiscal externality providing an additional incentive to inflate.

Proposition 4 (Steady State Inefficiency With $\tau = 0$) *Even if $\tau = 0$, the solution to the planner's problem remains inefficient, relative to both the first best allocation and the constrained efficient allocation.*

Proof. The planner's solution cannot coincide with the first best allocation since markets are incomplete and state-contingent transfers or Arrow securities are assumed away.

The constrained efficient planner's problem is to maximize social welfare 12 subject only to feasibility: the KFE (eq 5), the NKPC (eq 6), the government budget constraint (eq 8), and market clearing (eq 9-11). Crucially, the constrained efficient planner does not need to respect household first order conditions, only household feasibility.

Suppose the solution to the central bank's problem coincided with the solution of the constrained efficient planner. This would imply the equilibrium values of the multipliers on individual choices satisfy $\eta_{2,t} = \eta_{7,t} = \eta_{8,t} = \eta_{9,t} = 0$ (note that $\eta_{6,t} = 0$ in any environment with $\tau = 0$). If this were the case, the planner's first order condition for individual labor supply would be

$$0 = -g'(n_t) + \frac{(1-\tau^n)z_t w_t}{p_t^h}\frac{\partial \eta_{1,t}}{\partial b} + z\eta_{5,t}$$

while the household's would be

$$0 = -g'(n_t) + \frac{(1-\tau^n)z_t w_t}{p_t^h}\frac{\partial V_t}{\partial b}.$$

Since the central bank must respect household optimality conditions, $\eta_{5,t} = 0$ and $V_t = \eta_{1,t}$ must hold. Using the planner's HJB, the latter implies that $\eta_{4,t} = 0$. Then, the first order condition for aggregate labor coupled with the result of proposition 1 yields

$$0 = \int_{z_1}^{z_J}\int_{\phi}^{\infty} \frac{\partial V_t}{\partial b}\frac{\partial s_t}{\partial N} f_t \,\mathrm{d}b\,\mathrm{d}z$$

which cannot hold since $\frac{\partial V_t(b,z)}{\partial b} > 0 \forall (b,z)$ and $\frac{\partial s_t}{\partial B} \perp (b,z)$ and $\frac{\partial s_t}{\partial B} \neq 0$. ■

Unlike proposition 1 of Bianchi and Coulibaly (2025), the initial (or terminal) steady state of the economy is inefficient: the pecuniary externalities and market incompleteness create inefficiencies beyond those created by tariffs. Thus, when tariffs are increased, monetary policy responds to the new inefficiencies induced by the tariff and to the existing ones. This motivates section 6 where I use welfare weights to construct an initially efficient steady state.

Next, I derive a targeting rule similar in spirit to the one of Dávila and Schaab (2023) that characterizes the motives and penalties faced by the central bank when choosing monetary policy.

Proposition 5 (A Targeting Rule) *Optimal interest rate policy is determined by the following equation*

$$
\begin{aligned}
0 = &\underbrace{\mathbb{E}_{f_t}\left[(1-m_t)\frac{u_h}{p}\right]}_{\textit{Output Gap}} - \underbrace{\mathbb{E}_{f_t}\left[\frac{\theta}{2}\pi_t^2\frac{u_h}{p}\right]}_{\textit{Inflation Costs}} - \frac{\Omega_2}{\Omega_1}\underbrace{\mathbb{E}_{f_t}\left[\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{u_h}{p}\right]}_{\textit{Tariff Fiscal Externality}} + \\
&\underbrace{\mathbb{E}_{f_t}[\frac{\partial s}{\partial T}\frac{\partial IncTax}{\partial N}\frac{u_h}{p}]}_{\textit{Income Tax Fiscal Externality}} + \frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\underbrace{\mathbb{E}_{f_t}\left[(B-b)\frac{u_h}{p}\right]}_{\textit{Redistribution}} + \\
&\underbrace{\int\int\frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)[B-b]\frac{u_h}{p}\eta_{2,t} + \left[\frac{\partial s}{\partial N}\right]\frac{u_h}{p}\eta_{2,t}\,\mathrm{d}b\,\mathrm{d}z - \frac{\Omega_2}{\Omega_1}\int\int\left[\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\right]\frac{u_h}{p}\eta_{2,t}}_{\textit{Commitment Penalties}}
\end{aligned}
$$

where Ω_1 and Ω_2 are (endogenous and generally positive) constants defined in this proposition's proof in Appendix B

A change in interest rates requires balancing the "motives" (an individual term) laid out in proposition 3, all of which have the general form of marginal utility-weighted averages. Now, I describe all of these motives.

The first term is a standard output gap term, present in RANK and HANK. It captures the average welfare effect that a change in interest rates has on the output gap. Under the assumption of uniformly distributed profits, this affects all households equally. A more empirically realistic regressive redistribution of profits would weaken this term, since the benefits of expansion would accrue to richer households.

The second term captures the direct, deadweight loss, costs of inflation, again weighted by household marginal utility. It is present in both RANK and HANK, though in HANK, the cost can vary across households. This is a similar term to the inflation penalty of Dávila and Schaab (2023), capturing the direct efficiency costs of inflation on household welfare

The third term is novel to my environment relative to the previous heterogeneous agent optimal monetary policy literature, and is present in open economy RANK and HANK models with tariffs. It captures the fiscal externality present in the model via the redistribution of tariff revenue. Cutting rates will increase expenditure on both goods, leading to more revenues to redistribute. Again, progressive transfers can strength this effect, as $\frac{\partial s}{\partial T}$ will be decreasing in b, leading to high marginal utility households receiving even more of the tariff revenue. The fourth term is similar to the third in that it reflects fiscal redistribution. However, this term is not novel to an open economy environment and instead reflects the redistribution of fiscal revenue from the income tax. Note that this motive is quantitatively weak in my environment, as income tax rates do not change (only the labor supply), unlike the tariff, which will increase substantially.

The fifth term is exclusive to HANK environments and captures a redistributive motive. Decreasing the interest rate redistributes from high marginal utility agents ($B - b < 0$) to low marginal utility agents ($B - b > 0$), leading to potential welfare gains. This provides an additional incentive for the planner to inflate.

Following the naming convention of Dávila and Schaab (2023), the remaining terms capture commitment penalties. They penalize the welfare gains of certain households. In particular, each of the five policy motives discussed above has an associated penalty term, capturing the necessary penalty ($\eta_{2,t}(b, z)$) assessed to each household for each motive to make the planner respect household optimality conditions on labor and consumption. To fix ideas, suppose $\eta_{2,t}(\phi, z_1) < 0$ (the welfare gains of low productivity households at the borrowing constraint are penalized). This implies that were they not constrained by household first order conditions, the planner would prefer to redistribute more towards these households than would be consistent with individual household optimization. Thus, the gains of these households need to penalized to ensure the planner respects individual optimization.

Next, I discuss more directly how the presence of the fiscal externality affects optimal levels of foreign consumption and how that translates into monetary policy decisions.

Proposition 6 (Fiscal Externality) *The presence of the fiscal externality implies the socially optimal level of foreign consumption is higher than the optimal level of foreign consumption in its absence*

By manipulating the planner's first order conditions, the socially optimal level of foreign consumption satisfies

$$0 = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \left[\underbrace{u_f(c_t^h, c_t^f)}_{\text{Marginal Private Benefit}} - \underbrace{\frac{\partial \eta_{1,t}}{\partial b} p(1+\tau)}_{\text{Marginal Private Cost}} + \underbrace{\frac{\partial s^h}{\partial T} \frac{\partial T}{\partial C^f}}_{\text{Fiscal Revenue}} \left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right) - \underbrace{\mathcal{H}_t^f}_{\text{Promise Keeping}} \right] f_t \, \mathrm{d}b \tag{25}$$

where $\mathcal{H}_t^f \equiv -\frac{\eta_{9,t}}{f_t} u_{ff}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t} u_{hf}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t} u_{fh}(c_t^h, c_t^f)$.

The first two terms in eq 25 are standard and reflect the marginal private benefit and costs of additional foreign consumption, as valued by the planner. The third term in eq 25 represents the fiscal externality. Since tariff revenue is rebated to households, the planner internalizes the marginal effect of total foreign consumption on the transfer received by households. This marginal effect consists of two terms, $\frac{\partial s^h}{\partial T}$ captures how an individual household's transfer changes (capturing potential progressivity in the transfer scheme), while $\frac{\partial T}{\partial C^f}$ captures the revenue gain from additional foreign consumption. $\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right)$ is then the (properly penalized) marginal social value of wealth for each household. The final term reflects that changing foreign consumption loosens or tightens private first order conditions, which the planner must obey.

However, the private household FOC is simply $\frac{\partial V_t}{\partial b} p(1+\tau) - u_f(c_t^h, c_t^f)$. Intuitively, the household fails to internalize that additional foreign consumption increases the transfer received by all households in the economy. Thus, the socially optimal level of foreign consumption is higher than level without the fiscal externality for a utilitarian planner who uses tariff revenue as transfers to households.

To see this, suppose instead tariff revenue is instead spent on a "useless" good G_t that does not affect household utility, their transfers, or aggregate demand ($G_t = \int \int p\tau c^f \, \mathrm{d}f_t(b, z)$).

Then, the fiscal externality is not present and the socially optimal level of foreign consumption satisfies

$$0 = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \left[\frac{\partial \eta_{1,t}}{\partial b} p(1+\tau) - u_f(c_t^h, c_t^f) + \mathcal{H}_t^f \right] f_t \, \mathrm{d}b \, \mathrm{d}z.$$

Since the third term of 25 is positive, satisfying this new equality will require a lower level of C^f all else equal. Decreasing each agent's foreign consumption will increase the marginal utility of foreign consumption and decrease the value of $\frac{\partial \eta_{1,t}}{\partial b}$ (since households will by construction hold more wealth). Thus, the presence of the fiscal externality implies a higher level of foreign consumption. Similarly, a progressive transfer scheme that assigns more transfers to high marginal value households will also increase the optimal level of C^f. In both cases, the existence of this fiscal externality will push the planner towards cutting rates and inducing inflation, as the only way to increase foreign consumption is to cut rates and stimulate aggregate demand.

4.2 RANK vs HANK; Closed vs Open

Now, I use the results of proposition 5 to characterize the differences between my environment (open economy HANK), a closed economy RANK model, an open economy RANK model, and a closed economy HANK model. Table 1 summarizes which policy shaping motives are present in each of these economies

Table 1: Policy Motives in RANK, HANK, Closed, and Open Economies

Motive	Closed RANK	Open RANK	Closed HANK	Open HANK
Output Gap	✓	✓	✓	✓
Inflation Costs	✓	✓	✓	✓
Tariff Fiscal Externality		✓		✓
Redistribution			✓	✓
Penalties			✓	✓

In the standard 3 equation, closed-economy RANK model, only the asset demand motive (direct, IES), output gap motive, and inflation cost motives are present. All penalty terms are 0,($\eta_{2,t} = 0$), the redistributive motive is not present ($B = b$), and there are no tariffs ($\tau = 0$). The standard divine coincidence RANK results hold.

Expanding to the open economy model of Bianchi and Coulibaly (2025) adds the fiscal redistribution motive, though it is trivially the same for all households. It provides an additional incentive to inflate (in the short run): expansion counteracts the contractionary force of the tariff and adds additional fiscal revenue that is redistributed lump sum to households. The tariff only creates a wedge between the private costs of consuming the two goods; the social cost is unaffected due to revenue redistribution, providing an incentive to run expansionary policy.

Now, expanding from the 3 equation RANK model to the closed economy HANK models re-introduces all of the wedges save for the fiscal externality wedge. The corresponding penalty terms are also active and are non-zero. Additionally, all of the previous wedges can be heterogeneous in their magnitude across households if dividends or transfers are redistributed non-uniformly. This will provide stronger or weaker incentives to cut rates.

Finally, all motives and associated penalties are present in the open-economy HANK model.

Interactions Between Tariffs and Heterogeneity Household heterogeneity interacts with the tariffs in two novel ways in this model. First, household heterogeneity and the fiscal redistribution motives interact **directly** when transfers are non-flat ($\frac{\partial s^h}{\partial T}$ depends on the state vector b, z). Relative to RANK, this can either dampen or strengthen the incentive to cut rates after a tariff shock, depending on whether $\frac{\partial s^h}{\partial C^f}$ increases or decreases with wealth.

Tariffs also interact with all of the other motives **indirectly**, through the marginal utility terms, changing the weight placed on each household's welfare. To see this, rewrite the marginal utility of (home) consumption as

$$u_h(c^h_{i,t}, c^f_{i,t}) = (c^h_{i,t})^{-\sigma} \mathcal{C}_{i,\tau}$$

where $\mathcal{C}_{i,\tau}$ is constructed using the household first order condition for the optimal split between home and foreign consumption and is defined as:

$$\mathcal{C}_{i,\tau} = \omega_i \left(\omega_i + (1-\omega_i)\Theta_{i,\tau}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma(1-\sigma)}{\gamma-1}-1}$$
$$\Theta_{i,\tau} = \left(\frac{1-\omega_i}{\omega_i(1+\tau_t)p} \right)^{\gamma}$$

Under reasonable parameter values for the IES σ and trade elasticity γ, Θ_τ is decreasing in the tariff while $\mathcal{C}_{i,\tau}$ and the marginal utility of consumption are increasing in the tariff. If ω_i is constant for all agents, the marginal utility of each agent is simply shifted upwards by the same amount when tariffs increase. Thus, $\mathcal{C}_{i,\tau}$ does not depend on i and the consequences of a tariff shock for optimal policy stem only from the direct effects of the fiscal externality and how tariff revenue is redistributed.

However, suppose the home consumption share ω_i depends on the state vector (b, z). Then, non-zero tariffs effectively change the weight each household is given, increasing the welfare weight on households with higher foreign consumption shares, $1-\omega_i$. In other words, the potentially unequal incidence of tariffs can change which households have higher or lower marginal utility and cause the planner to emphasize the households with higher foreign consumption shares, echoing the message of Guo et al. (2023) where heterogeneity in the exposure to foreign shocks is a major determinant of the positive welfare consequences of the monetary response.

Note that the planner emphasizes the welfare of high *marginal utility* agents rather than agents with a high MPC. In a one good, one asset, closed economy model, these notions would be roughly equivalent; the highest MPC agents will be on the borrowing constraint and have low consumption, thus their marginal utility is high. However, in this model, the highest marginal utility agents do not have to be exactly those with the highest MPCs, particularly if the foreign consumption share $1-\omega_i$ increases in wealth.

5 Numerical Results

5.1 Calibration

I calibrate the model to an annual frequency. For the baseline calibration, I assume that, as in the presentation of the theoretical results, transfers adjust to solve the government's budget constraint. Additionally, I choose to assume that government bonds are a fixed, flat number to simplify computation rather than chosen optimally. This number will be calibrated in the steady state to be a certain fraction of GDP. Since I am not concerned with (optimal) provision of public debt and how the monetary authority internalizes the effects of their policy on the provision of public debt, this assumption simplifies the fiscal block while still maintaining rich monetary/fiscal interactions.

Table 2 summarizes all parameter values for the baseline calibration.

Table 2: Parameter Values

		Parameter	**Value**	**Target/Source**
Household	Effective Discount Factor	$\rho + \eta$	0.0417	4% annual RA interest rate
	Death Rate	η	.02	50 year working lifespan
	Risk Aversion	σ	1	
	Frisch Elasticity	κ	1	
	Borrowing Limit	ϕ	-0.25	One quarter of average labor income
	Labor Disutility Scaling Parameter	χ	1	Steady state output
	Persistence Idiosyncratic Shock	ρ_z	0.914	
	Std. Idiosyncratic Shock	σ_z	0.2063	
	No. of St. Dev Idiosyncratic Shock	m_z	2.0	
	Preference Weight, Home Goods	ω	0.7	Share of Foreign Consumption, US
	Trade Elasticity	γ	2	Estimate from ...
	Relative Price	p	1	
Firms	Intermediate Firm Elasticity	θ	10	Average markups
	Adj. Cost Parameter	ϵ	100	Implied annual slope of 0.1
Gvt.	Debt to GDP ratio	B_{perc}	1.4	FRED
	Labor Tax	τ_n	0.3	Average labor income tax , US
	Firm Subsidy	s	$\frac{1}{\epsilon-1}$	Zero markup steady state

Notes: The table reports the parameter values. The debt to GDP ratio is only for calibration, it is never internalized by the planner and is never adjusted in the transition dynamics.

I assume an effective discount factor for both the planner and the household of $\rho + \eta = \frac{1}{.96} - 1$, standard for an annually calibrated model. The death rate of $\eta = .02$ implies a (working) lifespan of 50 years. I assume that $\sigma = \kappa = 1$ to simplify the computation of the labor/lesiure choice[10]. The borrowing limit $\phi = -.25$ is calibrated to be one quarter of average labor income following Kaplan et al. (2018). The Poisson process for z is a discretization of an AR(1) process, using Rouwenhorst discretization. The transition matrix in continuous time is found by taking the (unique) matrix logarithm of the discrete time transition matrix. The preference weight on home goods ω is set so that the share of foreign consumption is 15% in the steady state, in line with US data. I set the intermediate firm demand elasticity to $\theta = 10$ and the adjustment cost parameter $\epsilon = 100$ to have an annual slope of the Phillips curve of .1, in line with the literature. I set the constant level of debt B to equal 1.4 times GDP. Note that the debt to GDP ratio is never internalized by the monetary authority, from the perspective of the central bank, debt is constant. The labor tax is set to .3, in line with average US labor income taxes. Finally, the firm subsidy is set to eliminate firm markups in the zero inflation steady state.

5.2 Baseline Results

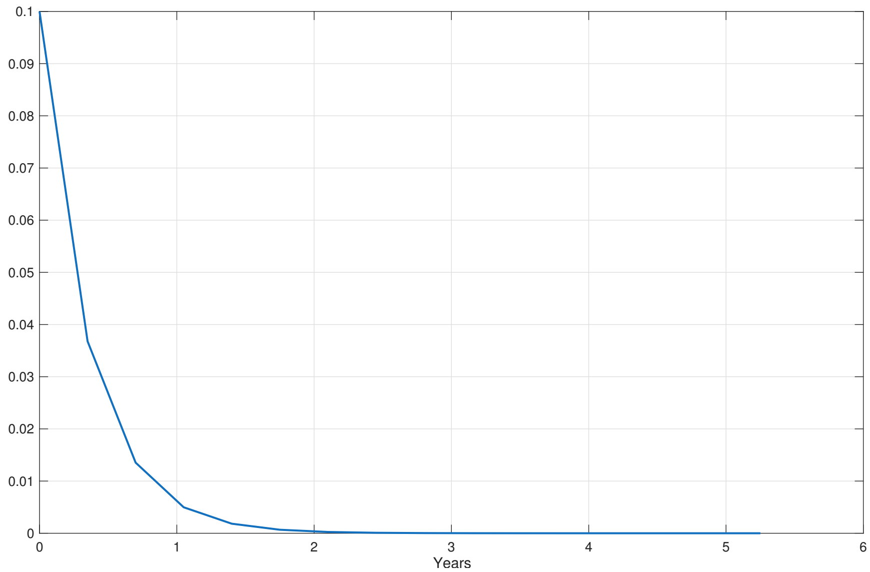
I investigate fully nonlinear, perfect foresight responses to a temporary MIT shock to tariffs[11]. From a zero tariff steady state[12], there is a 10% increase to the tariff rate τ which asymptotes exponentially to 0 after approximately 2 years. Figure 1 plots the tariff shock I use for all of my numerical experiments.

[10]This parameterization ensures that the first order condition for labor is a quadratic equation, ensuring I do not need to use a nonlinear solver to find labor choice.

[11]I discuss the numerical algorithm in appendix C

[12]The use of the timeless approach allows the time 0 steady state to be well behaved

Figure 1: Path of Exogenous Tariff Shock

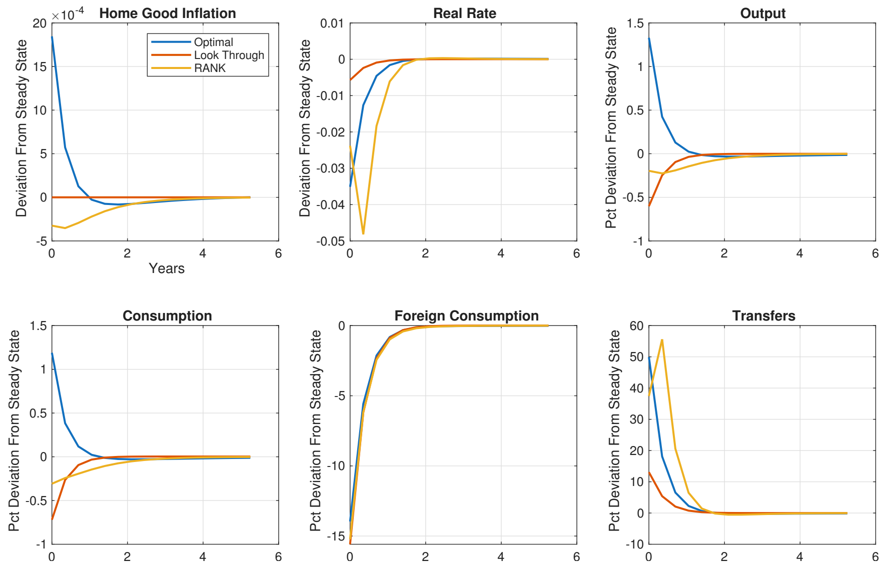


Notes: The path of tariffs satisfies $\tau_t = .1 \cdot e^{-\xi t}$ with $\xi = .35$.

First, I compare aggregate and distributional outcomes for three types of policy: the Ramsey optimal policy in HANK, a "look-through" policy, and the Ramsey optimal policy in RANK[13]. I define a "look-through" policy as one that sets home produced goods' inflation $\pi_t = 0 \quad \forall t$. The central bank allows the tariff to (mechanically) affect the consumer price level, but sets interest rates to keep home goods inflation at 0. Figure 2 plots aggregate outcomes for the optimal and look through policies.

[13]I explicitly define the corresponding RANK economy and its planner problem in appendix K. To fix ideas, it is one with the standard deviation of idiosyncratic shocks set to 0, the death rate set to 0, and the borrowing limit set to its natural level

Figure 2: Optimal vs Look Through Policy, Aggregate Variables

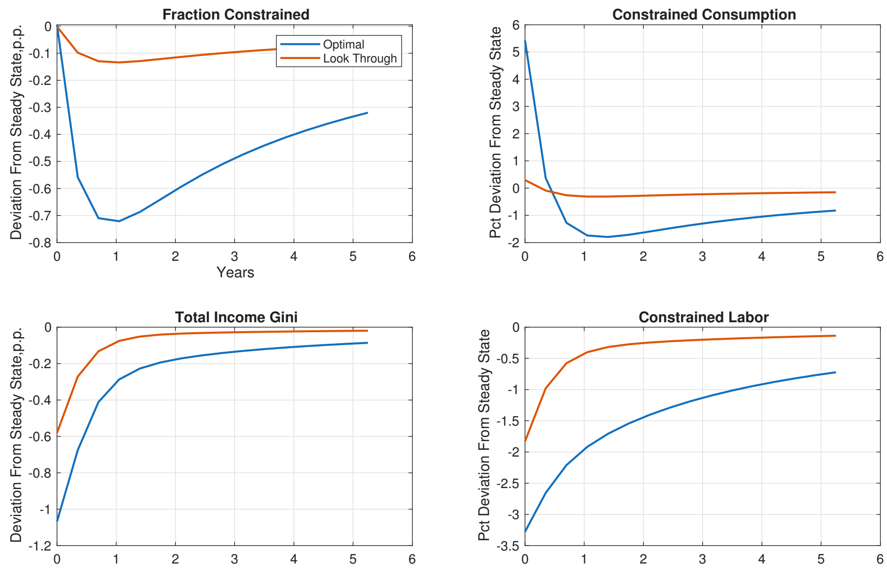


Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state. The optimal policy features positive home goods inflation, while the look through policy sets inflation to 0 by construction. The central bank cuts rates with the optimal policy, increasing output (labor) and consumption. The look through policy necessitates a small rate cut, but output (labor) and consumption both fall.

Similar to Bianchi and Coulibaly (2025), the optimal policy remains inflationary and involves cutting rates. However, the planner has additional or stronger motives to inflate in the HANK environment: they have a redistributive motive and a stronger fiscal motive (transfers are more valuable to constrained agents). The planner cuts rates, stimulating output (equal to aggregate labor) and consumption. For the look through policy, output (labor) and consumption both fall. Home goods inflation is mechanically set to 0. Reflecting the fiscal externality and a desire to give additional transfers to the constrained, foreign consumption falls by less in the optimal policy than the look through policy, leading to a much larger increase in transfers. Relative to the look through policy, the planner stimulates output by about 1 percentage point and consumption by slightly more than 1 percentage point.

Figure 3 plots distributional outcomes for the optimal and look through policies.

Figure 3: Optimal vs Look Through Policy, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to distributional variables. I define the fraction of constrained agents as the share with $b = \phi$ and plot its deviation from the steady state. The optimal policy has a much larger drop in constrained agents, stemming from the larger increase in transfers. The (total) income Gini coefficient falls by slightly more than 1 percentage point relative to its steady state value in the optimal policy; it falls by around half of that in the look through policy. In the optimal policy, constrained consumption increases by 5% and constrained labor falls by slightly less than 5%. For the look through policy, the effects are dampened. Constrained consumption slightly increases and constrained labor slightly decreases.

The fraction of constrained agents (with $b = \phi$) falls under both policy responses, but by more in the optimal response. This largely stems from the increase in transfers; agents can afford to save since their transfers increase dramatically as a result of the increased tariff revenue. Similarly, since rates fall by more (flattening the distribution of capital income) and transfers increase by more, the total income Gini coefficient falls by more in the optimal policy. The constrained's consumption increases by 5% on impact in the optimal policy response, it increases by less than 1% in the look through response. The labor supply response in the optimal policy for the constrained is similarly amplified. There is an on-impact decrease of over 4% in the optimal response, the decrease is by less than half that factor in the look through policy. The optimal policy dramatically improves welfare relevant outcomes for the poorest agents and generally leads to a less unequal distribution of income and wealth.

5.3 Fiscal Adjustment

In incomplete markets models, Ricardian equivalence fails. Thus, which variable adjusts to clear the government budget constraint affects equilibrium outcomes. Carroll and Hur (2023) highlights how the redistribution of tariff revenue affects *positively* the welfare gains and losses from tariff shocks in a heterogeneous agent neoclassical model. I build on these insights and show how the failure of Ricardian equivalence affects equilibirum outcomes in a *normative* sense. In this model, the government has 3 non-tariff fiscal tools they can adjust: labor income taxes, government debt, and transfers. Interestingly, the Trump administration has proposed adjusting each of these 3 instruments in response to the revenues their tariffs bring in,[14] highlighting the direct policy relevance of understanding how the failures of Ricardian Equivalance affect both equilibrium outcomes and optimal policy.

I solve for the same fully nonlinear, perfect foresight transition path for optimal monetary policy in response to the tariff shock in figure 1 under the assumption that each of the 3 aforementioned instruments adjust to satisfy the government budget constraint[15]. I calibrate the other two instruments in each of the 3 scenarios as follows

1. Transfers Adjust (Baseline): Bonds and labor taxes are constant as calibrated in table 2

2. Taxes Adjust: Bonds are at the same constant level as in table 2, transfers are set to be 1/6th of GDP, in line with the data and the transfer to GDP ratio in the baseline model. Unlike the debt to GDP ratio, this value *is* internalized by the planner.

3. Bonds Adjust: Labor taxes are at the same constant rate as in table 2, transfers are set to be 1/6th of GDP, in line with the data and the transfer to GDP ratio in the baseline model. Again, this value *is* internalized by the planner.

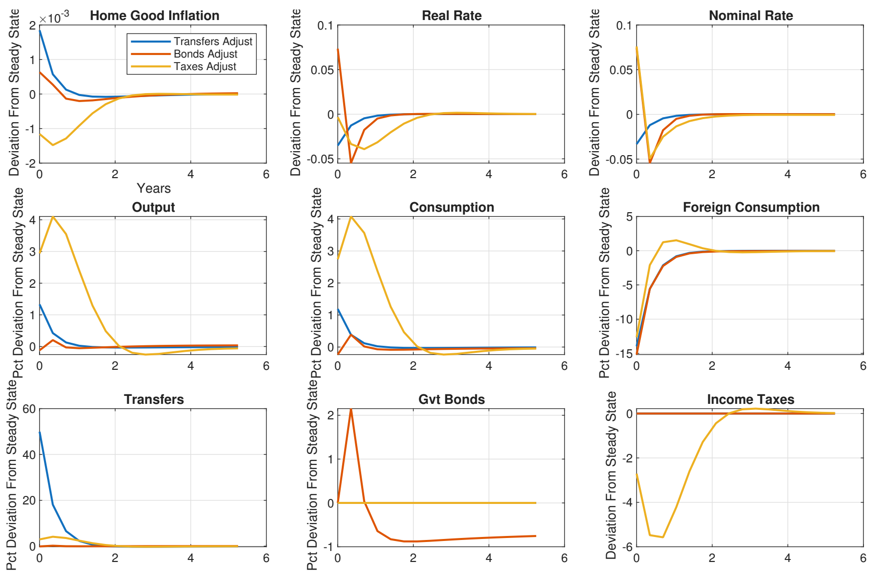
[14] Proposal to adjust transfers, proposal to adjust debt, proposal to adjust income taxes

[15] Planner first order conditions are different in each of these environments, I outline the differences in appendix D

This calibration ensures the steady state outcomes are identical in the 3 alternatives. However, the optimal policy response need not be identical. I do not consider the production subsidy adjusting, as its point is to exactly counteract the monopoly distortion in the steady state.

Figure 4 plots aggregate outcomes for optimal monetary policy under each of the 3 fiscal instruments adjusting.

Figure 4: Optimal Policy, Varying Fiscal Adjustment, Aggregate Variables



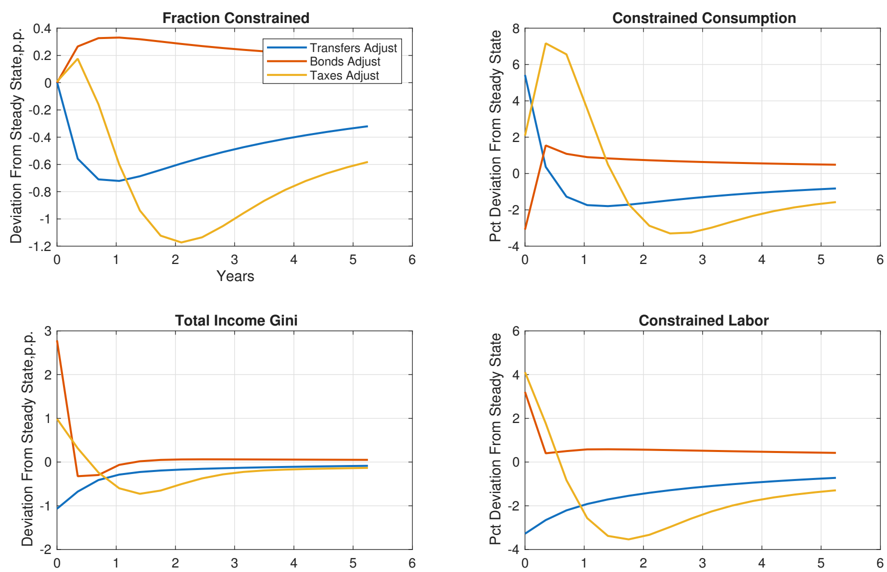
Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables when transfers, taxes, or bonds adjust under optimal monetary policy. Unlike when transfers adjust, optimal monetary policy when bonds adjust features deflation, a decrease in output (labor) and consumption, and the largest decrease in foreign consumption. Adjusting income taxes operates generally similarly to adjusting transfers. Rates are cut, labor and consumption increase, and the decrease in foreign consumption is not as severe.

The variable that adjusts affects not only the magnitude of the optimal policy response, but also the sign. When bonds adjust, the central bank instead chooses to raise rates on impact. Output and consumption or remain constant on impact. The decrease in foreign consumption is larger than in the two alternative scenarios. Changing government debt doesn't directly impact households in the way that decreasing taxes or increasing transfers do, so the central bank does not attempt to partially correct the decrease in foreign consumption; in other words the fiscal externality is weakest when bonds adjust.

When income taxes adjust, the monetary policy stance is similarly expansionary to transfer adjustment. However, instead of transfers increasing, income taxes decrease by 5 percentage points. Somewhat mechanically, allowing income taxes to adjust increases labor supply, output, and consumption by the largest amount, since the labor supply distortion is weakened. Similar to the logic of Alessandria et al. (2025), using tariff revenue to correct the distortionary labor income tax is particularly powerful.

Figure 5 plots distributional outcomes for optimal monetary policy under each of the 3 fiscal instruments adjusting

Figure 5: Optimal Policy, Varying Fiscal Adjustment, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to distributional variables when transfers, taxes, or bonds adjust under optimal monetary policy. The fraction of constrained agents falls when transfers or taxes adjust, it is effectively unchanged when bonds adjust. There are noticeable increases in constrained consumption and decreases in constrained labor when transfers or taxes adjust. There are weak decreases in consumption and labor when bonds adjust. The total income Gini falls much more modestly when bonds or taxes adjust compared to when transfers adjust.

Similar to the aggregate outcomes, the distributional outcomes are also greatly affected by which variable adjusts. When bonds adjust, the fraction of constrained agents increases. Their consumption decreases, income inequality rises, and their labor supply increases. Since the adjusting fiscal variable does not directly affect distributional outcomes, the poor tend to suffer under this regime. This further explains why the planner does not cut rates or induce inflation to the same extent he did when transfers adjusted: he cannot use a rate cut to improve the welfare of the high marginal value, constrained agents. The fiscal externality is weak here. A change in the debt does not greatly affect agents on average, nor does it have particularly positive outcomes for the poor.

When taxes adjust, there is a large decrease in the fraction of constrained agents. Since the wealth poor supply the most labor, decreasing taxes greatly affects their ability to save their way away from the constraint. This similarly explains why the constrained consumption response is large and positive. This economy features the largest and most persistent interest rate cut because a tax reduction is both (somewhat) progressive and has large aggregate effects. The fiscal externality has the strongest average *and* distributional effects. While transfers may be more progressive, they do not affect the unconstrained agents in the same way an income tax cut will.

6 Efficient Steady State

Even with the use of the timeless penalty, the presence of market incompleteness, borrowing constraints, and potentially suboptimal fiscal policy implies the initial steady state in section 5 is inefficient. The monetary policy response to the tariff shock may then be responding to these pre-existing inefficiencies or the new ones induced by the higher tariff. The planner may be unhappy with the ex-ante amount of redistribution in the economy and uses the tariff shock as an opportunity to engage in more redistribution. Since directly solving for optimal steady state fiscal instruments is infeasible (see Auclert et al. (2024)), I follow an inverse optimal approach similar to that of McKay and Wolf (2022).

In particular, I find welfare weights for each household type $\varphi(b, z)$ such that the initial steady state from section 5 is first-best (equalizing marginal utility of consumption). In the steady state, for given parameters and fiscal policy, all allocations are invariant to the welfare weights used (since steady state inflation is 0 and all other economic variables are pinned down by implementability conditions). Thus, the welfare weights are given by

$$\varphi(b,z) = \frac{u_h(C^h_{ss}, C^f_{ss})}{u_h(c^h_{ss}, c^f_{ss})} \tag{26}$$

where C^h and C^f are aggregate home and foreign consumption.

Proposition 7 (Weighted Primal Timeless Ramsey Problem) *Given welfare weights $\varphi(b,z)$, the timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{9}} \max_{r_t,\pi_t,V_t(.),f_t(.),c^f_t(.),c^h_t(.),n_t(.),w_t,N_t,C^f_t} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0}, \eta_{2,0}) \tag{27}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ defined as

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_\phi^\infty \varphi(b,z)\left[u(c^h_t, c^f_t) - g(n_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$

$$\eta_{2,t}(b,z)\left[u(c^f_t, c^h_t) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r^h_t - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right] +$$

$$\eta_{4,t}[b^h_t - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c^f_t - C^f_t]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p^h_t} u_h(c^h_t, c^f_t)\right]$$

$$\eta_{8,t}(b,z)\left[u_h(c^h_t, c^f_t) - \frac{\partial V}{\partial b^h_t} p^h_t\right] + \eta_{9,t}(b,z)\left[u_f(c^h_t, c^f_t) - \frac{\partial V}{\partial b^h_t} p^h_t(1+\tau)\right] \mathrm{d}b\, \mathrm{d}z\} \,\mathrm{d}t$$

where $\eta_{1,t}, \eta_{2,t}, \eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ are functional Lagrange multipliers and $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, and $\eta_{6,t}$ are scalar Lagrange multipliers and z_1 and z_J are the lower and upper bounds, respectively, on the idiosyncratic shock z. Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in $\mathcal{A}$ by substituting for transfers.

The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ is defined as in Dávila and Schaab (2023) and equals:

$$\int_{z_1}^{z_J}\int_\phi^\infty \eta_{2,0} V_0(b,z)\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{9}$ and choice variables $r_t, \pi_t, V_t(.), f_t(.), c^f_t(.), c^h_t(.), n_t(.), w_t, N_t, C^f_t$ that solve 27

The planner first order conditions for this version of the model are described in detail in appendix E. The major difference is that the relationship between marginal social and private values differs. Now,

$$\mathcal{M}\eta_b = \varphi\mathcal{M}V_b + \varphi_b u - \varphi_b g + \eta_4 + \eta_5 n_b + \eta_6 c^f_b$$

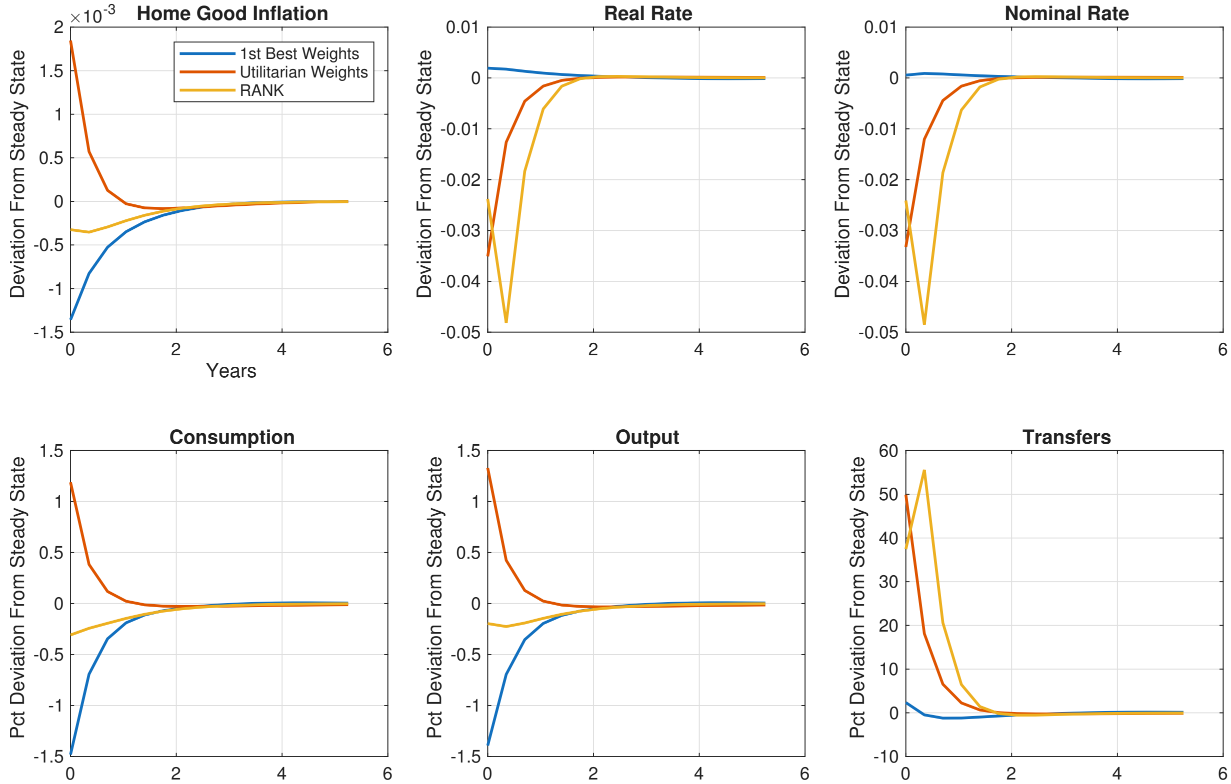
where $\mathcal{M}$ is defined as in equation 24 and φ_b is the partial derivative of the weighting function with respect to b. Now, since welfare weights are heterogeneous across wealth and productivity levels, the planner internalizes how a change in wealth will affect their future value of the agent's welfare. The presence of these weights will also affect the planner's attitude towards redistribution. In the empirically relevant case where welfare weights are increasing in b (since marginal utility is decreasing in b), this will mean the planner explicitly *dislikes* redistribution from the rich to the poor, as the rich have much higher welfare weights.

6.1 Numerical Results

With these welfare weights, the steady state allocation is efficient. The planner does not view the amount of redistribution in the economy as suboptimal, nor do they view fiscal variables as suboptimally chosen. Any policy response is a response to the new inefficiencies induced by the tariff. As before, I hit the economy with the MIT tariff shock given by 1 and calculate the fully nonlinear, perfect foresight transition paths.

First, I compare impulse responses for aggregate (figure 6) and distributional (figure 7) variables for the planner with utilitarian welfare weights, the welfare weights given by equation 26, and RANK.

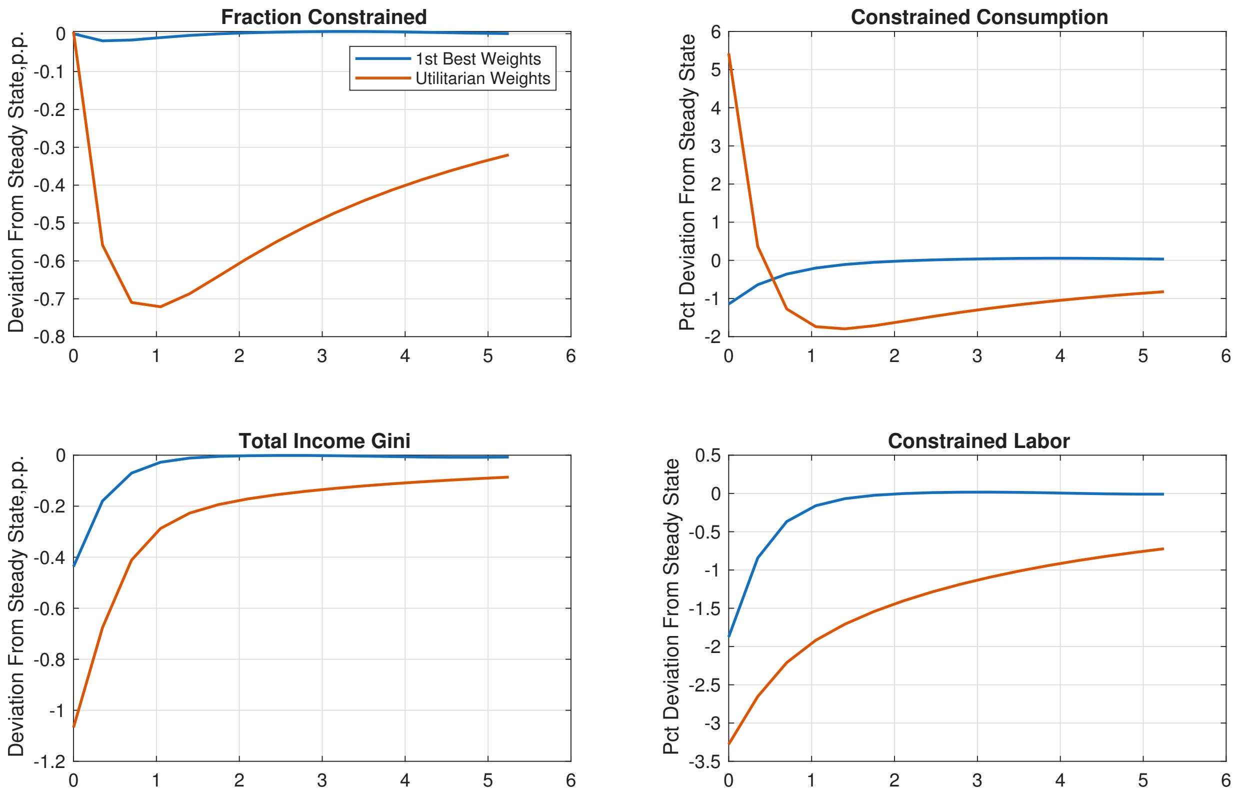
Figure 6: First Best vs Utilitarian Weights, Aggregate Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables for a planner whose has welfare weights given by equation 57 versus a planner who has a utilitarian social welfare function. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

In the economy with first best welfare weights, there is now deflation and a very small rate hike. The optimal change in the nominal rate is close to 0; a look through policy is close to optimal. Consumption and output both fall, while transfers remain nearly flat. This is a stark contrast from the policy with utilitarian weights, which featured inflation, sizeable rate cuts, increases in consumption, output, and transfers.

Figure 7: First Best vs Utilitarian Weights, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to for a planner whose has welfare weights given by equation 57 versus a planner who has a utilitarian social welfare function. . Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

In the economy with first best welfare weights, there is nearly no change in the fraction of constrained agents, rather than the large decrease present with utilitarian weights. Their consumption falls (rises with utilitarian weights) and their labor supply contracts by less than in the utilitarian weight case.

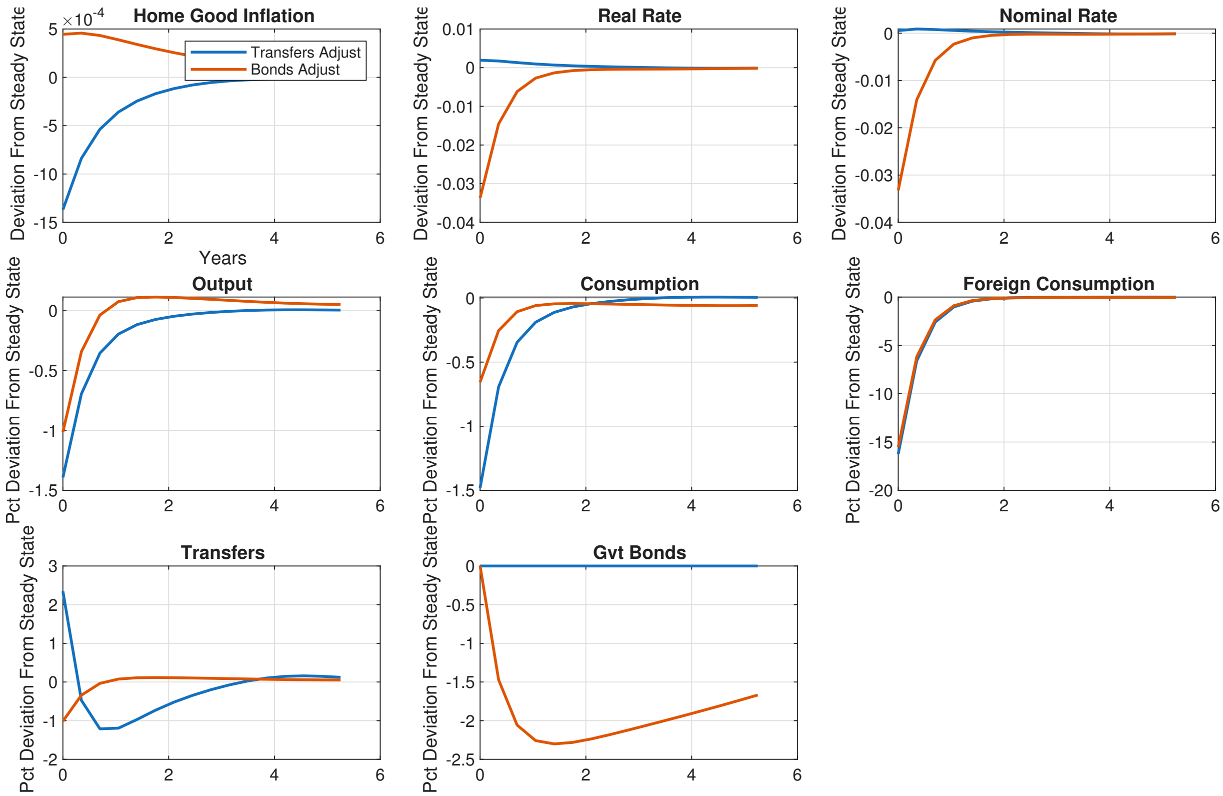
Why are there such stark differences between the optimal policy with utilitarian weights versus first best weights? The differences stem from the planner's attitude towards the fiscal externality in the two models. The expression for the magnitude of the fiscal externality is given by:

$$\eta_{6,t} = \underbrace{\mathrm{COV}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right), \frac{\partial s_t}{\partial T}\frac{\partial T}{\partial C^f}\right]}_{\text{Distributional Effect}} + \underbrace{\mathbb{E}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right)\right]\mathbb{E}_{f_t}\left[\frac{\partial s_t}{\partial T}\frac{\partial T}{\partial C^f}\right]}_{\text{Average Effect}}$$

In the utilitarian welfare weight case, $\eta_{6,t} > 0$. Since the fiscal externality redistributed from the rich to the poor and made agents on average better off, the existence of this motive pushed the planner to inflate. In the first best case, however, $\eta_{6,t} < 0$. Using transfers as the "marginal" instrument means additional foreign consumption is redistributing from the rich to the poor. Since the planner puts higher weight on the welfare of the rich, this redistribution is now an explicit bad. The existence of this fiscal externality pushes the planner towards deflating. They do not want additional foreign consumption, as they view it, on the margin, as a social bad.

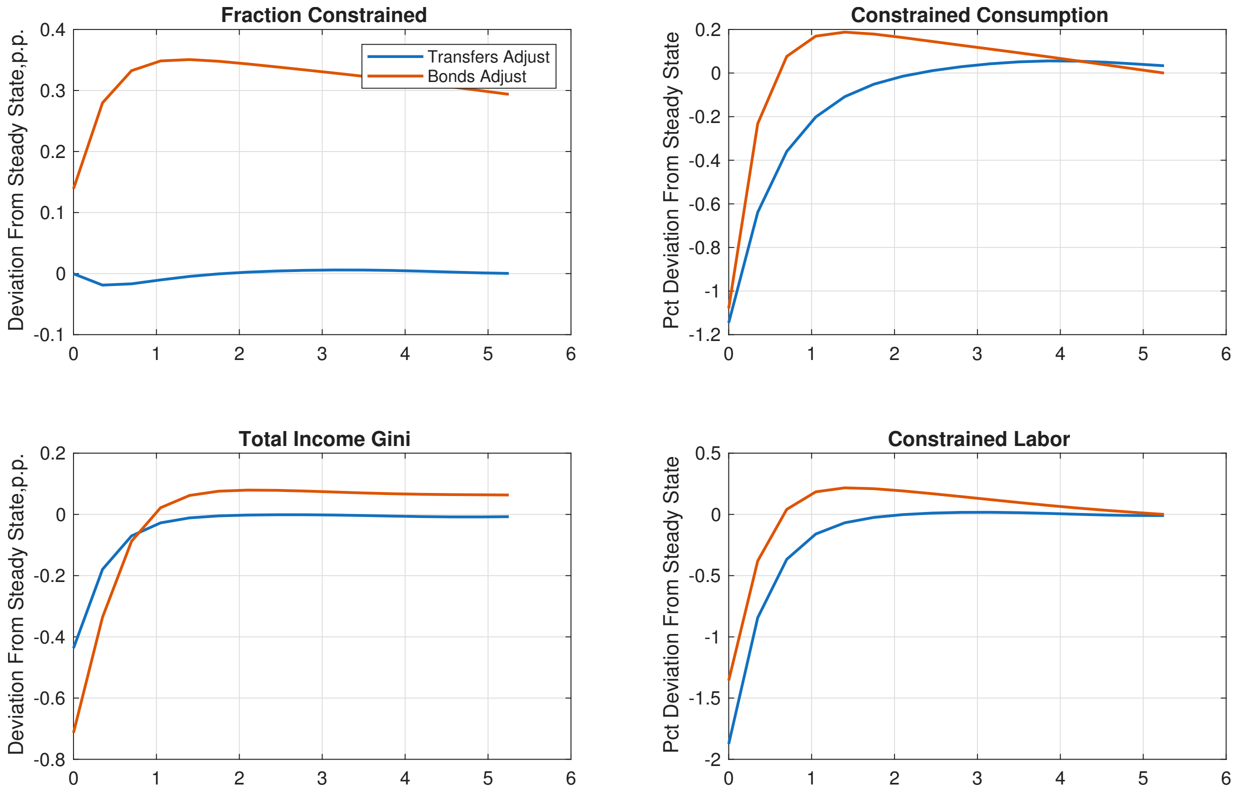
To further emphasize how the "marginal" fiscal instrument affects the planner's optimal policy, I compare two economies with first best weights, one with transfers adjusting to clear the government's budget constraint, one with debt adjusting. Recall that transfers adjusting is a redistribution from the rich to the poor: the rich pay a disproportionate share of the tariff revenue. Debt adjusting is (roughly) distributionally neutral, it does not directly affect the budget of households, only indirectly through general equilibrium interest rate effects. Figures 8 and 9 plot the aggregate and distributional outcomes for these two economies.

Figure 8: First Best, Varying Fiscal Adjustment, Aggregate Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables for a planner whose has welfare weights given by equation 57 versus a planner who has a utilitarian social welfare function. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

Figure 9: First Best, Varying Fiscal Adjustment, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to for a planner whose has welfare weights given by equation 57 versus a planner who has a utilitarian social welfare function. . Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

The outcomes when bonds adjust look more similar to the outcomes when bonds adjusts in the utilitarian case (figure 5). There is now inflation and a more modest reduction in output and consumption on impact. There is a slight increase in the fraction of constrained agents and modest decreases in their consumption and labor supply.

The minimal difference between the optimal policy with first best and utilitarian weights when bonds adjusts compared to the large difference between optimal policy with first best and utilitarian weights when transfers adjust stems from the distributional effects of the "marginal" fiscal variable. Bonds adjusting are roughly distributionally neutral. The fact that the planner puts more weight on the rich relative to the poor does not change how they view the fiscal externality: it is neutral in both cases and does not substantially push them towards inflation or deflation. However, transfers adjusting have a clear distributional impact. They redistribute from the rich to the poor. The planner views this as an explicit bad when they have first best welfare weights and an explicit good when they have utilitarian welfare weights. This leads to deflation and contraction in the first best case and inflation and expansion in the utilitarian case.

In summary, making the initial steady state efficient by adding increasing welfare weights can have stark consequences for optimal monetary policy. The planner's attitude towards the fiscal externality can be drastically altered. When the marginal fiscal variable is progressive (in the traditional sense of the word), the planner explicitly dislikes the presence of the fiscal externality and pushes against it. They raise rates, decreasing foreign consumption, tariff revenue, and the redistribution the fiscal system induces. When the marginal fiscal variable is more neutral, the planner's attitude towards the fiscal externality is unchanged. Optimal policy with and without these welfare weights is similarly unchanged.

7 Model Extensions

I now consider some additional extensions to the model in the vein of the extensions studies by Bianchi and Coulibaly (2025). Each extension has an appendix that provides more details; I write out the new planner's problem and discuss particularly important theoretical differences in the main body of the text before describing numerical results.

7.1 Endogenous Relative Prices

In the baseline model, I assume the relative price of home and foreign goods, p, is exogenous. This implies the central bank will have no incentive or ability to use its policy instrument to affect terms of trade. Additionally, the optimal (first best) tariff in this economy is 0. Now, I extend the model to have an endogenous relative price. Following Bianchi and Coulibaly (2025), I assume the relative price takes the following functional form

$$p_t = A(N_t - C_t^h)^{\frac{1}{\vartheta}} \tag{28}$$

where A is an exogenous foreign demand shifter, N_t is aggregate output at home, C_t^h is aggregate home consumption of the home-produced good, and ϑ is the export demand elasticity.

This functional form is implied by assuming the rest of the world is comprised of a continuum of countries each with a representative household that produces one foreign variety. The foreign block is written and solved explicitly in appendix F.

Even though home households are heterogeneous, a usual static optimal tariff formula applies since foreign households are homogeneous, and the foreign export supply function is only a function of the contemporaneous price[16]. The optimal tariff then satisfies

$$\tau_{\text{opt}} = \frac{1}{\vartheta - 1} \tag{29}$$

Denoting the relative price as $p(N, C^h)$, the new planner problem is

Proposition 8 (Primal Timeless Ramsey Problem with Endogenous Prices) *The timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{1}0} \max_{r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, C_t^h} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0}, \eta_{2,0}) \tag{30}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ defined as

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t} \{ \int_{z_1}^{z_J} \int_\phi^\infty \varphi(b, z) \left[u(c_t^h, c_t^f) - g(n_t) \right] f_t + \eta_{1,t}(b, z) \left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t} \right] +$$
$$\eta_{2,t}(b, z) \left[u(c_t^f, c_t^h) - g(n_t) + \mathcal{A} V_t + \frac{\partial V_t}{\partial t} - \rho V_t \right] + \eta_{3,t} \left[\pi_t (r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi} \right] +$$
$$\eta_{4,t}[b_t^h - B_t] f_t + \eta_{5,t}[n_t z_t - N_t] f_t + \eta_{6,t}[c_t^f - C_t^f] f_t + \eta_{7,t}(b, z) \left[g'(n_t) - \frac{w_t z_{j,t}(1 - \tau^n)}{p_t^h} u_h(c_t^h, c_t^f) \right]$$
$$\eta_{8,t}(b, z) \left[u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} \right] + \eta_{9,t}(b, z) \left[u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(N_t, C^h) t (1 + \tau) \right] + \eta_{10,t}[c_t^h - C_t^f] f_t \, \mathrm{d}b \, \mathrm{d}z \} \, \mathrm{d}t$$

where $\eta_{1,t}, \eta_{2,t}, \eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ are functional Lagrange multipliers and $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, and $\eta_{10,t}$ are scalar Lagrange multipliers and z_1 and z_J are the lower and upper bounds, respectively, on the idiosyncratic shock z. Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in $\mathcal{A}$ by substituting for transfers.

The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ is defined as in Dávila and Schaab (2023) and equals:

$$\int_{z_1}^{z_J} \int_\phi^\infty \eta_{2,0} V_0(b, z) \, \mathrm{d}b \, \mathrm{d}z + \eta_{3,0} \pi_0$$

[16] see section 2 of Dávila et al. (2025) for details

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{1}0$ *and choice variables* $r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, C_t^h$ *that solve* 30

The new multiplier $\eta_{10,t}$ reflects a pecuniary externality on the relative price caused by marginal changes in home consumption.

I can derive a new planner targeting rule that illustrates the terms of trade motive

Proposition 9 (An Endogenous Price Targeting Rule) *Optimal interest rate policy is determined by the following equation*

$$0 = \underbrace{\mathbb{E}_{f_t}\left[(1-m_t)\frac{u_h}{p}\right]}_{\textit{Output Gap}} - \underbrace{\mathbb{E}_{f_t}\left[\frac{\theta}{2}\pi_t^2\frac{u_h}{p}\right]}_{\textit{Inflation Costs}} - \frac{\Omega_2}{\Omega_1}\underbrace{\mathbb{E}_{f_t}\left[\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{u_h}{p}\right]}_{\textit{Tariff Fiscal Externality}} +$$

$$\underbrace{\mathbb{E}_{f_t}[\frac{\partial s}{\partial T}\frac{\partial IncTax}{\partial N}\frac{u_h}{p}]}_{\textit{Income Tax Fiscal Externality}} + \frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\underbrace{\mathbb{E}_{f_t}\left[(B-b)\frac{u_h}{p}\right]}_{\textit{Redistribution}} + \underbrace{\mathbb{E}_{f_t}[\frac{\partial p}{\partial N}\left(\tau C^f - (1+\tau)c^f\right)u_h]}_{\textit{Terms of Trade Manipulation}}$$

$$\underbrace{\int\int \frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)[B-b]\frac{u_h}{p}\eta_{2,t} + \frac{\partial s}{\partial N}\frac{u_h}{p}\eta_{2,t} - \frac{\Omega_2}{\Omega_1}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{u_h}{p}\eta_{2,t} + \frac{\partial p}{\partial N}\left(\tau C^f - (1+\tau)c^f\right)u_h\eta_{2,t}\,\mathrm{d}b}_{\textit{Commitment Penalties}}$$

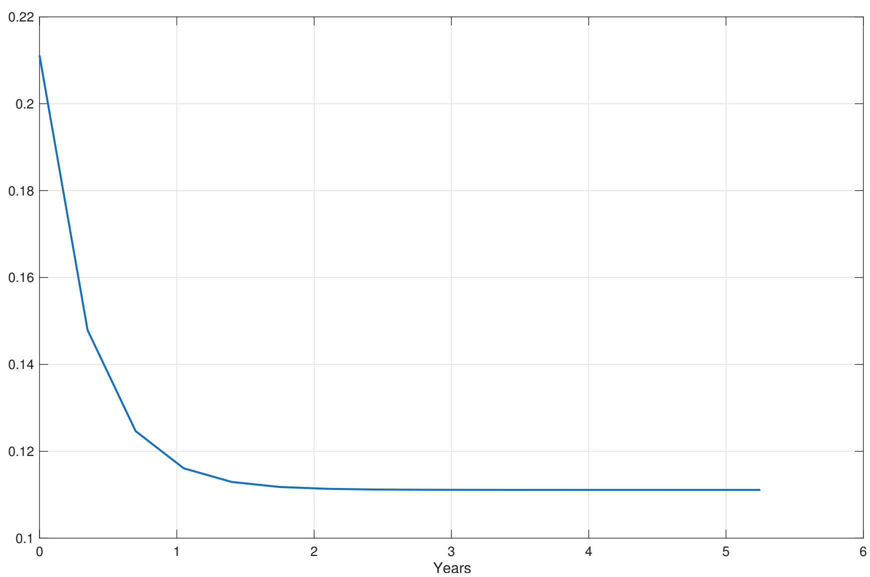
where Ω_1 *and* Ω_2 *are (endogenous and generally positive) constants defined in this proposition's proof in Appendix* F

This targeting rule features a new term that captures how the planner values terms of trade manipulation: $\mathbb{E}_{f_t}[\frac{\partial p}{\partial N}\left(\tau C^f - (1+\tau)c^f\right)u_h]$. Expansionary policy will increase the relative price p, since output will increase and home consumption will increase by less than one-to-one. Increasing the price will give all agents additional transfers equal to $\frac{\partial p}{\partial N}\tau C^f$, but make their consumption of the foreign good more expensive by a factor of $\frac{\partial p}{\partial N}(1+\tau)c^f$. In the representative agent case $c^f = C^f$, so an increase in the relative price unambiguously costs the household resources, dampening the planner's incentives to expand.

However, in the heterogeneous agent case, if c^f is very low relative to C^f it is possible some households will see a net resource increase. For example, if foreign goods are a luxury, low wealth households will have $c^f \approx 0$ and will gain resources from an increase in the relative price. Thus, it is ex-ante unclear how the terms of trade motive affects optimal policy. If there are households who consume very few imports and those households have high marginal utility, the planner may want to have an even greater expansion than the exogenous price case. However, under most parameterizations (including my baseline parameterization), this will not be the case. The planner will likely prefer less expansionary policy since expansion raises the relative price, which is welfare-reducing

Numerical Results Similar to my previous numerical results, I consider the planner's optimal response to a one-time, temporary, MIT shock to tariffs. However, unlike before, I assume the tariff at time 0 is the optimal tariff from equation 29. There is a 10 percentage point increase to the tariff rate, which again reverts exponentially back to 0 after approximately 2 years. Figure 10 plots the tariff shock I use for the numerical experiments in this section.

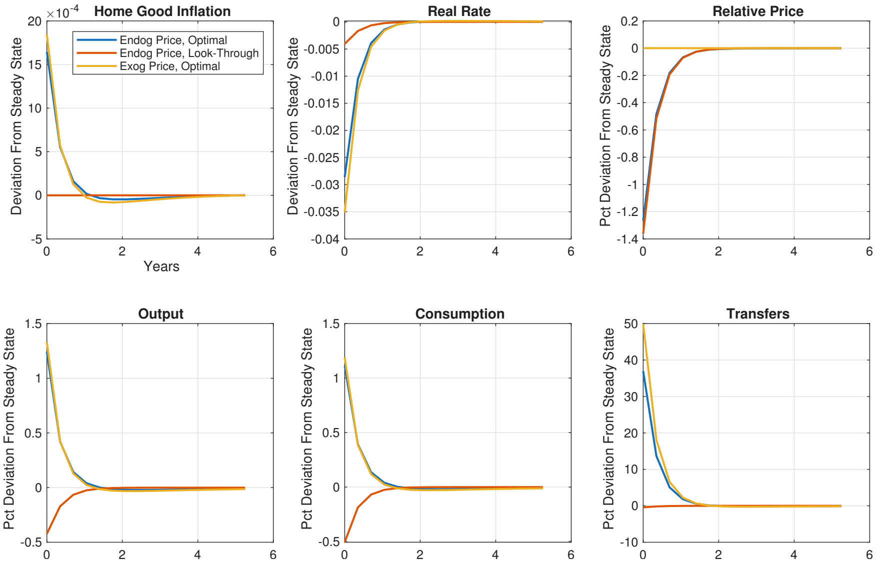
Figure 10: Path of Exogenous Tariff Shock



Notes: The path of tariffs satisfies $\tau_t = .1 \cdot e^{-\xi t}$ with $\xi = .35$.

I compare aggregate and distributional outcomes for the Ramsey and "look-through" policy and include the Ramsey policy with exogenous relative prices for reference

Figure 11: Optimal vs Look Through Policy, Aggregate Variables

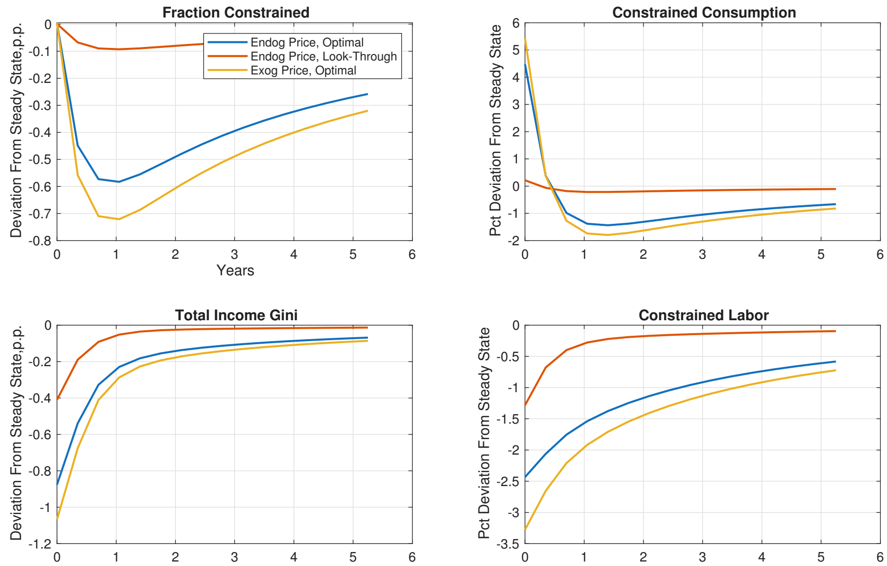


Notes: For the path of tariffs from figure 10, I plot impulse responses to aggregate variables. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

When the relative price is endogenous, the tariff shock will reduce the relative price p of the foreign good. Expansionary policy will counteract this reduction in the relative price, since it will increase N, but C^h will increase less than one for one. Since an increase in the relative price is welfare reducing for all agents in my calibration (foreign consumption is not enough of a luxury good to make $\tau C^f - (1+\tau)c^f$ positive for any agent at any time), the planner's incentive to engage in expansionary policy is dampened.

Relative to optimal policy when prices are exogenous, there is a smaller interest rate cut, inflation rises by less, consumption, output, and transfers increase by less. However, the signs of the responses to each variable are unchanged and are quantitatively not too different. This reflects the fact that the fiscal externality is still present and very powerful quantitatively. The marginal welfare costs of higher prices are weak compared to the gains from fiscal redistribution.

Figure 12: Optimal vs Look Through Policy, Distributional Variables



Notes: For the path of tariffs from figure 10, I plot impulse responses to aggregate variables. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

Since there is less expansion and less redistribution through tariff revenue, more agents remain constrained, the consumption of the constrained rises by less, and their labor supply falls by less. Relative to the look-through policy, most of the insights from the previous sections remain; there is a much stronger rate cut, labor, consumption, and transfers all rise instead of remaining flat or falling, and there are larger improvements in distributional outcomes. Notably, the relative price falls by more in the look-through policy, since the optimal policy entails an expansion, which raises the relative price.

7.2 Progressive Fiscal Policy

I now consider a transfer specification that is not flat. I use a transfer function of the following form, similar to the one of Ferriere et al. (2023):

$$\mathcal{T}(z_t) = \xi_t \frac{2e^{-\gamma_1(z_t-\gamma_0)}}{1+e^{-\gamma_1(z_t-\gamma_0)}}$$

and is parameterized by $\gamma_1 \in \mathbb{R}^+, \gamma_0 \in \mathbb{R}$ This sigmoid is a continuous approximation of a step function with a cutoff at γ_0, meaning households with productivity above γ_0 receive (approximately) 0 transfers, while households with productivity below γ_0 receive (approximately) $2\xi_t$ transfers. This captures the fact that many government transfer programs have eligibility thresholds at some level of income[17]. γ_1 represents the steepness of the transfer function, if $\gamma_1 = 0$, transfers are flat, while the limiting case $\gamma_1 \to \infty$ recovers the aforementioned step function. ξ is an equilibrium object, adjusting to satisfy the government budget constraint (more generally it is set so $\int \mathcal{T}(b) f(b,z)\,\mathrm{d}b\,\mathrm{d}z = T$ where T are aggregate transfers set according to an ad hoc rule or to satisfy the government budget).

Model I now detail the different features of the model. In this section, I assume a general transfer function $\mathcal{T}(b,z)$. The new government budget constraint is

$$\dot{B}_t = rB_t + sp^h_t Y_t + \int \mathcal{T}(b_t, z_t)\,\mathrm{d}f(b,z) - \tau p^f \int c_f\,\mathrm{d}f(b,z) - \tau_n w_t \int nz\,\mathrm{d}f(b,z)$$

The new savings function is

$$s^h = (r+\eta)b + (1-\tau^n)zwn + \mathcal{T}(b_t, z_t) + (1+s)N_t - m_t N_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f$$

Proposition 10 (Primal Timeless Ramsey Problem with Progressive Transfers) *The timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{1}0} \; \max_{r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f,\mathcal{T}(.)} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0}, \eta_{2,0}) \tag{31}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ defined as

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_\phi^\infty \left[u(c_t^h, c_t^f) - g(n_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$

$$\eta_{2,t}(b,z)\left[u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right] +$$

$$\eta_{4,t}[b_t^h - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c_t^f - C_t^f]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_h(c_t^h, c_t^f)\right]$$

$$\eta_{8,t}(b,z)\left[u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}\right] + \eta_{9,t}(b,z)\left[u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(N_t, C^h)t(1+\tau)\right] + \eta_{10,t}[\mathcal{T}(b,z) - \dot{B}_t + r_t B_t + sN$$

[17] Indexing the same sigmoid off of wealth provides too strong of incentives to remain borrowing constrained, while indexing to total income complicates the labor supply decision. Productivity is a reasonable proxy for total labor income, while remaining numerically tractable

where $\eta_{1,t}, \eta_{2,t}, \eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ are functional Lagrange multipliers and $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, and $\eta_{10,t}$ are scalar Lagrange multipliers and z_1 and z_J are the lower and upper bounds, respectively, on the idiosyncratic shock z. Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in $\mathcal{A}$ by substituting for transfers.

The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ is defined as in Dávila and Schaab (2023) and equals:

$$\int_{z_1}^{z_J} \int_{\phi}^{\infty} \eta_{2,0} V_0(b, z) \, \mathrm{d}b \, \mathrm{d}z + \eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{1}0$ and choice variables $r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, \mathcal{T}(.)$ that solve 31

I explicitly describe the planner first order conditions in appendix G.[18]

To highlight how the fiscal externality is quantitatively different in this environment, I obtain the following expression derived in appendix G

$$\eta_{6,t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \underbrace{\left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b} \eta_{2,t} \right)}_{\text{Net Planner Marginal Value}} \underbrace{\frac{\partial s^h}{\partial \mathcal{T}(b)} \frac{\partial \mathcal{T}(b)}{\partial T}}_{\text{Individual Transfers}} \underbrace{\frac{\partial T}{\partial C^f}}_{\text{Change in Aggregate Fiscal Revenue}} \mathrm{d}b \, \mathrm{d}z$$

$$\eta_{6,t} = \underbrace{\mathrm{COV}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right), \frac{\partial s^h}{\partial \mathcal{T}(b)} \frac{\partial \mathcal{T}(b)}{\partial T} \frac{\partial T}{\partial C^f} \right]}_{\text{Distributional Effect}} + \underbrace{\mathbb{E}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right) \right] \mathbb{E}_{f_t}\left[\frac{\partial s^h}{\partial \mathcal{T}(b)} \frac{\partial \mathcal{T}(b)}{\partial T} \frac{\partial T}{\partial C^f} \right]}_{\text{Average Effect}}$$

which shows the strength of the fiscal externality $\eta_{6,t}$ depends on how individual savings respond to individual transfers (one-to-one by construction), how individual transfers respond to aggregate transfers (a function of progressivity), and how aggregate transfers respond to aggregate foreign consumption (if transfers adjust in the government budget, aggregate transfers change by τp). As before, this is comprised of an average and distributional

[18] Note that even though the problem has an additional choice variable and transfers are not flat, the only first order condition that is explicitly different is the planner HJB; each other first order condition, including the fiscal externality, is expressed exactly as in the baseline model. The planner HJB reflects a new pecuniary externality on transfers, as changes in wealth will now affect idiosyncratic and therefore aggregate transfers:

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{9,t}(c_t^f - C_t^f) + \eta_{10,t}(\mathcal{T}(b_t, z_t) - T_t)$$

effect. When transfers are progressive, the distributional effect will be particularly strong. There will be a large, positive covariance between planner marginal values and the effects of transfers on household budgets. This will generally strengthen the fiscal externality and push the planner towards more inflation

Calibration I calibrate the parameters γ_1 and γ_0 to match the empirical transfer distribution. I use CBO data on non-Medicare and non-SSI transfers and assume UI payments are transfers to generate the empirical transfer distribution by income decile. I follow directly the methods of Ferriere et al. (2023): using the methods and code of Habib (2018) to impute transfer levels on 2013 CPS-ASEC data, merging into IPUMS CPS-ASEC data (to obtain income), and then calculating transfer rates for each income decile. As they do, I only incorporate transfers designed to provide income insurance to the non-elderly: SNAP (food stamps), Temporary Assistance for Needy Families, housing assistance, UI benefits, and other small means-tested programs. Since the model does not consider disability, old age, or other adverse health shocks, I exclude Medicaid and Supplemental Security Income.

Table 3: Transfer Rates

Income Decile	1	2	3	4	5	6	7	8	9	10
Model										
Data	.528	.167	.074	.032	.019	.011	.007	.005	.002	.0009

Notes: This table reports, by decile, household transfer rates in the model and in CBO data. I define a transfer rate as the fraction of total income attributed to transfers.

To do the calibration, I solve the model with flat transfers ($\gamma_1 = 0$) and obtain total income and wealth levels by decile. I then obtain transfer levels consistent with these income levels and the data. Finally, I obtain γ_1 and γ_0 by doing NLS on the corresponding levels of wealth and transfer levels (letting ξ be aggregate transfers in the model with flat transfers). I then solve the model with these new γ_1 and γ_0 coefficients, repeating the process until model transfer rates and data transfer rates are approximately equal.

Numerical Results WORK IN PROGRESS

7.3 Intermediate Inputs

In the baseline model, tariffs only distort consumption and function similarly to a negative demand shock combined with a fiscal shock. They do not enter into the problem of the firm and do not distort production decisions. Now I assume that there are two types of goods that are imported (and subject to tariffs), a foreign produced consumption good as before and foreign-produced intermediate inputs that must be combined with labor to produce the home consumption good. This allows the tariff to function as a supply shock.

Model I now detail the different features of the model. Each intermediate firm $j \in (0,1)$ now produces according to $y_{jt} = x_{jt}^{\alpha} n_{jt}^{1-\alpha}$ where x_{jt} is the quantity of intermediate inputs used by firm j at time t. The leads to a new expression for real marginal costs and dividends

$$
\begin{aligned}
m_t &= \left(\frac{w_t}{(1-\alpha)}\right)^{1-\alpha} \left(\frac{p_t^x(1+\tau^x)}{\alpha}\right)^{\alpha} \\
D_t &= (1+s)Y_t - w_t N_t - p(1+\tau_t^x)X_t - \Theta(\pi)
\end{aligned}
$$

where p_t^x is the relative price of the intermediate input and τ_t^x is the tariff on the intermediate input. Going forward, I follow Bianchi and Coulibaly (2025) and Werning et al. (2025) and assume that $p_t^x = p_t = p$ (i.e. the intermediate input and foreign consumption good are perfect substitutes and both relative prices are constant). In my numerical exercises, I will additionally assume that $\tau_t = \tau_t^x$ (the foreign consumption good and intermediate input are tariffed at the same rate). The government budget is given by

$$
\dot{B}_t = rB_t + T_t + sp_t^h Y_t - \tau p^f \int c_f \, \mathrm{d}f(b^h, b^f, z) - \tau_n \int nz \, \mathrm{d}f(b^h, b^f, z) - \tau_t p X_t
$$

Finally, the goods market clearing is

$$
Y_t = \frac{\theta}{2}\pi_t^2 Y_t + C_t^h + pC_t^f + pX_t
$$

Proposition 11 (Primal Timeless Ramsey Problem with Intermediate Inputs) *The timeless Ramsey planner's problem solves*

$$
\min_{\{\eta_{k,t}\}_{k=1}^{1}0} \max_{r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f,X_t} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0},\eta_{2,0}) \tag{32}
$$

with the Lagrangian functional $\mathcal{L}(f_0)$ defined as

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_{\phi}^{\infty} \varphi(b,z)\left[u(c_t^h, c_t^f) - g(n_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$
$$\eta_{2,t}(b,z)\left[u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r_t^h - \alpha\frac{\dot{X}_t}{X_t} - (1-\alpha)\frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right] +$$
$$\eta_{4,t}[b_t^h - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c_t^f - C_t^f]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h, c_t^f)\right]$$
$$\eta_{8,t}(b,z)\left[u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}\right] + \eta_{9,t}(b,z)\left[u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}p(1+\tau)\right] + \eta_{10,t}[p(1+\tau) - m_t X_t^{\alpha-1}N_t^{1-\alpha}\alpha]\,\mathrm{d}b\,\mathrm{d}z\}$$

where $\eta_{1,t}, \eta_{2,t}, \eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ are functional Lagrange multipliers and $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, and $\eta_{10,t}$ are scalar Lagrange multipliers and z_1 and z_J are the lower and upper bounds, respectively, on the idiosyncratic shock z. Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in $\mathcal{A}$ by substituting for transfers.

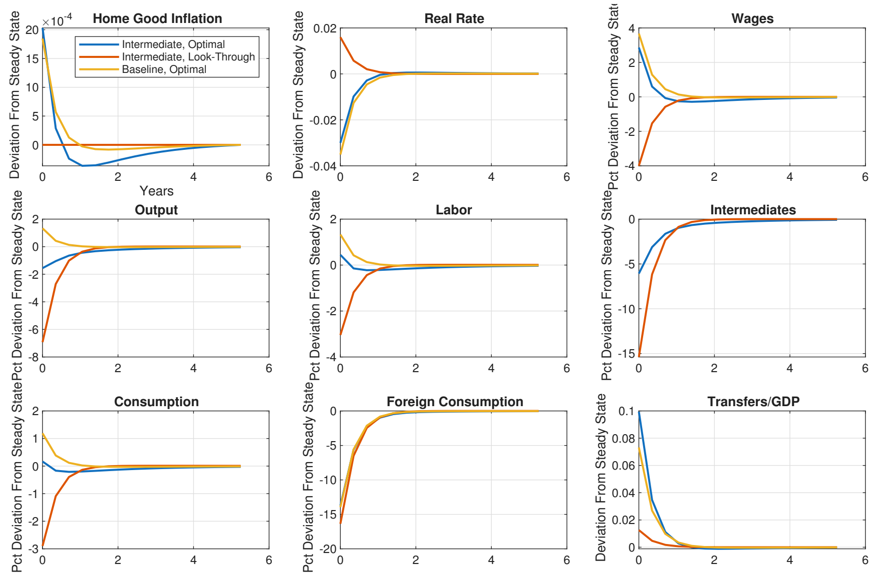
The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ is defined as in Dávila and Schaab (2023) and equals:

$$\int_{z_1}^{z_J}\int_{\phi}^{\infty} \eta_{2,0}V_0(b,z)\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{1}0$ and choice variables $r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, X_t$ that solve 32

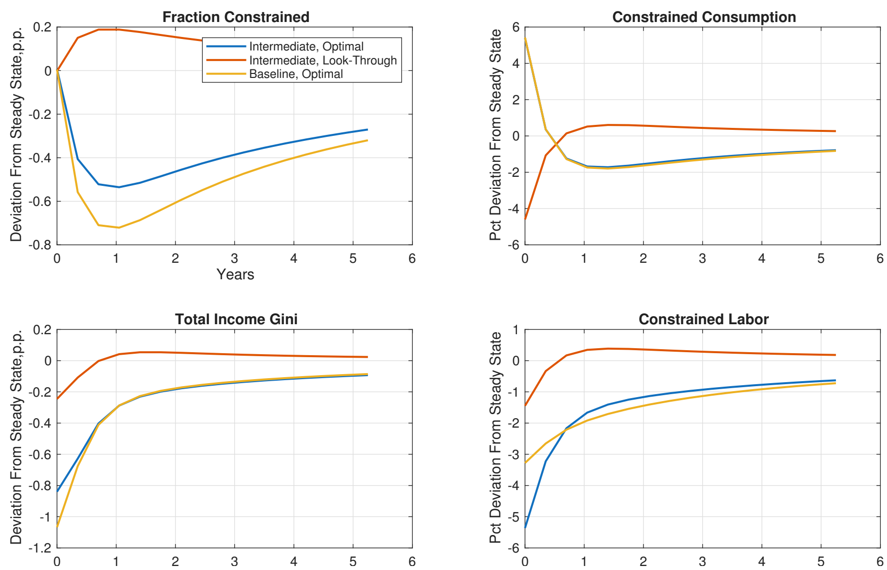
Numerical Results Figures 13 and 14 plot results for the tariff shock in figure 1 to τ and τ^x.

Figure 13: Tariff Shock with Imported Inputs, Aggregate Variables



Notes: For the path of tariffs to both imported intermediate goods and final goods from figure 1, I plot impulse responses to aggregate variables. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

Figure 14: Tariff Shock with Imported Inputs, Distributional Variables



Notes: For the path of tariffs to both imported intermediate goods and final goods from figure 1, I plot impulse responses for distributional variables.

When there are tariffs on both intermediate inputs and final consumption goods, the optimal interest rate policy remains generally similar compared to the baseline model with no intermediate inputs. There is still an approximately 3 percent rate cut and a modest increase in inflation on impact. The rate cut induces less of a decrease in foreign consumption and purchases of intermediate inputs. The decrease in foreign consumption is nearly identical to the decrease in the economy without intermediate inputs. Since production is now more expensive, output and consumption fall (rather than rise), though by less than they do in the look-through policy. Firms substitute towards labor and wages and transfers both rise. The distributional effects of the policy are broadly similar to the model without intermediates. The fraction of constrained agents falls, the consumption and labor supply of constrained agents falls, as does income inequality

Qualitatively, the expansionary stance reflects the fact that the fiscal motive induced by the tariffs on consumption goods remains. It may appear counterintuitive that the rate cut is not stronger in this environment, as there is a new source of fiscal revenue that the central bank should like to increase. However, this revenue motive is neutral by construction and cannot affect optimal policy under the assumption that dividends and transfers are flat. Under a more empirically realstic regressive distribution of profits and progressive distribution of transfers, this should strength the incentive to cut rates, as doing so would redistribute from the rich to the poor.

7.4 Heterogeneous Consumption Baskets

In the baseline version of my model, I assume the preference weight on home goods ω_i is constant for all households. Now, I loosen that assumption and allow ω_i to be a function of household states b, z. This induces preferences that are as-if non-homothetic, as consumption shares of each good type will vary with wealth. The model and planner FOC are the same as the baseline. The key theoretical difference comes from the fact that household marginal utilities and therefore planner marginal values now depend on relative exposure to the shock, as discussed in section 4.

Data To calibrate the expenditure weights ω_i in my model, I use the Consumer Expenditure Survey (CEX) from 2004 to 2013 to obtain the share of consumption spent on imports or tradable goods across the income distribution. The CEX contains data on expenditures at the household level for a wide set of goods and services categories. For each of those categories, to calculate the share of imports, I use data from Johnson (2017), which uses detailed 2007 I/O tables to calculate a "tradability index" (defined as the ratio of the maximum of imports or exports to domestic production) and defines a tradable good as one with a tradability index ≥ 11. For each post-tax income decile, I calculate the average share of expenditure on tradable goods. As validation, I repeat this exercise with import shares for very broad categories provided by the US Trade Representative and used by Furman et al. (2017), calculating the average import share for each post-tax income decile.

Figure 7.4 plots expenditure shares on tradable goods and imported goods for each income decile. Like Borusyak and Jaravel (2021), I find that import shares/exposure are not decreasing in wealth, but are instead (modestly) increasing in wealth. This implies the the expenditure channel of tariffs does not disproportionately affect the poor, but instead the rich. Furthermore, properly calibrated home expenditure weights ω_i will be slightly *decreasing* in income/wealth. Since import shares are log-linear in income, I use a logarthmically spaced grid ranging from I use a linearly spaced grid for ω_i ranging from .7044 (for the constrained) to .6844 (for the richest). I do not vary ω over values of z, only over b. This is sufficient to generate the magnitude of variation in import shares from top of the distribution to the bottom seen in figure 7.4. Additionally, this calibration replicates the average expenditure share on imports from the homogeneous ω model, where $\omega = .7$. Note that even though tradable shares are increasing in income, this does not imply that tariffs are not regressive: the lower end of the income distribution has a higher consumption share of income and higher marginal utility, thus tariffs still disproportionately harm them.

Figure 15: Tradable Shares by Income

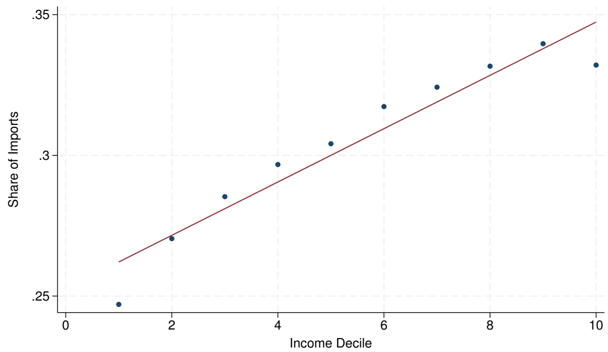
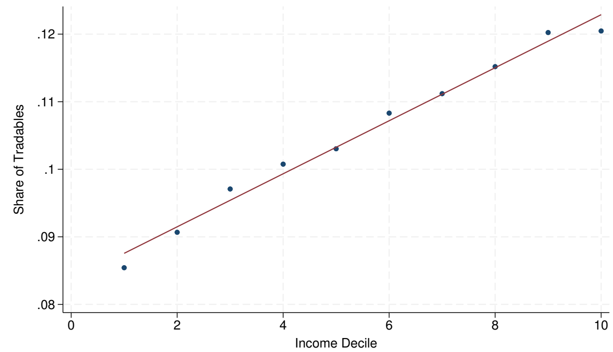
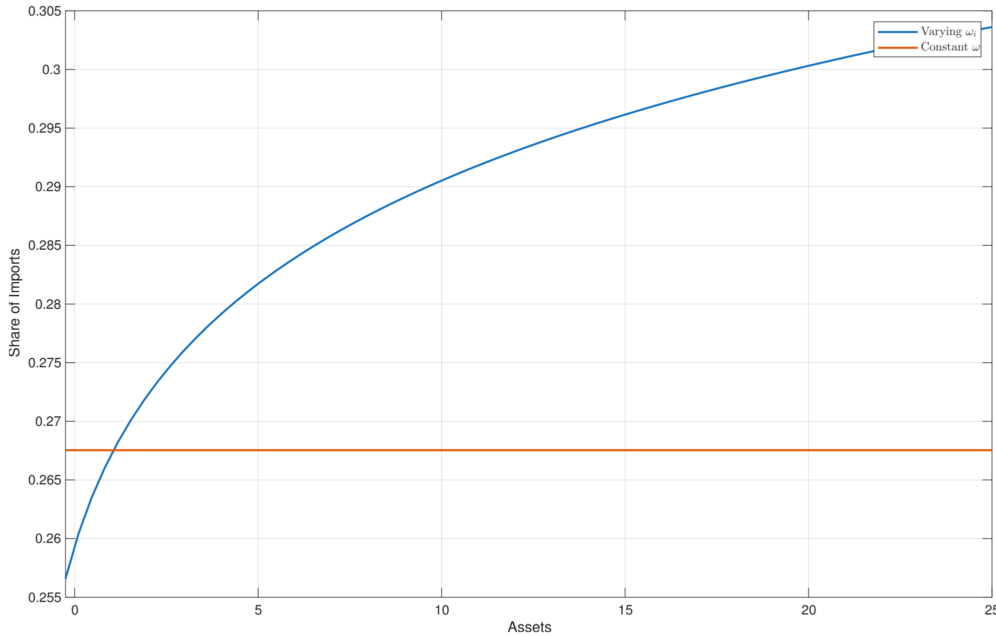


Figure 16: Import Shares by Income



Notes: For each income decile, I plot the average share of expenditure on tradable goods (left) and imports (right) using 2004-2013 public microdata from the CEX's Diary and Interview surveys. Tradables are defined using the approach of Johnson (2017) which uses detailed I/O tables to define tradability, while import share data are from Furman et al. (2017). To deal with housing consumption, I replace expenditures on mortgage interest, homeowner's insurance, and property taxes with self-reported owner equivalent rent following Carroll and Hur (2020)

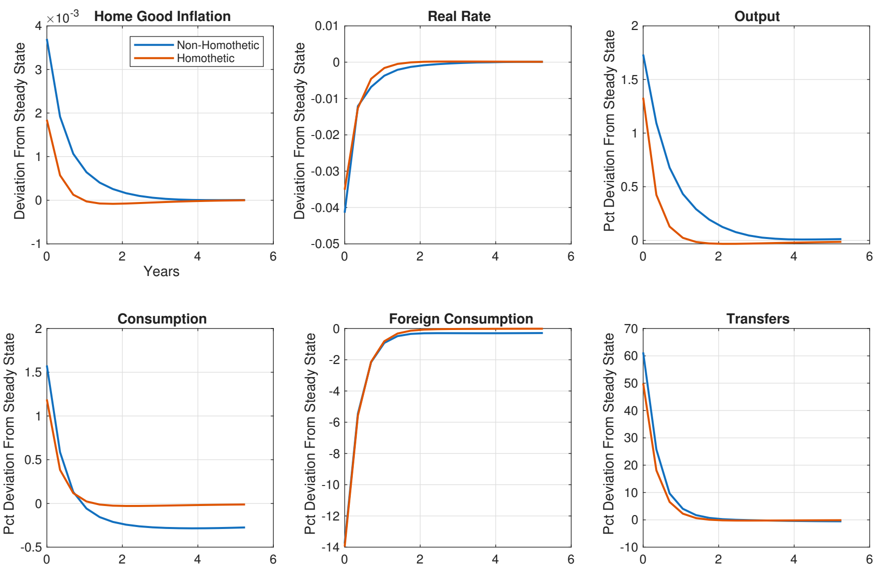
Figure 17: Steady State Import Shares



Notes: Using a logarithmically spaced vector of preference weights over my asset grid, ranging from $\omega(b_{\min}) = .702$ to $\omega(b_{\max}) = .682$, I plot the steady state import shares in the baseline economy with no tariffs. I generate approximately the variation in import shares found in the data from the top of the distribution to the bottom, as well as the shape of the import share curve. Additionally, the average import share in the economy with heterogeneous ω (a share of .267) matches the import share of the economy with $\omega = .7$ for all households.

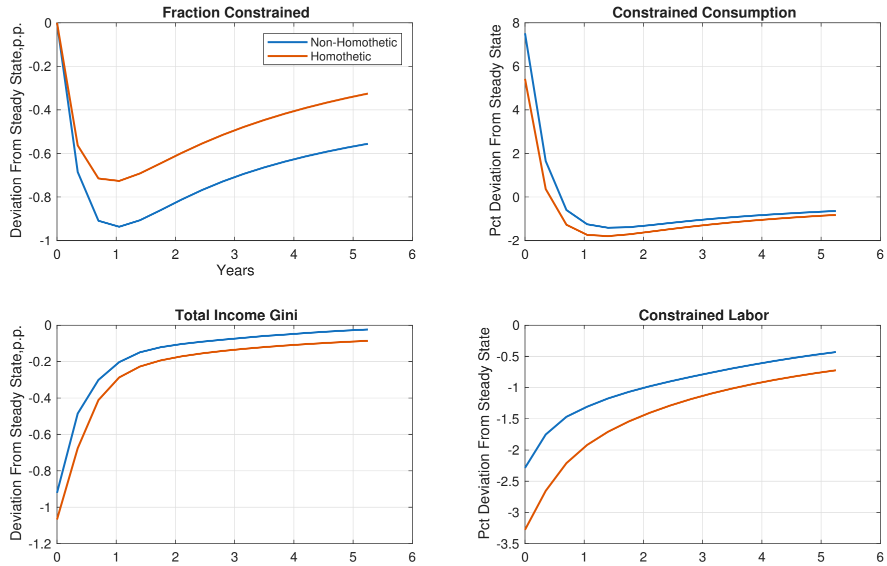
Numerical Results Given the calibration of ω_i from the previous section, I now calculate the same perfect foresight impulse response to the shock in figure 1. Figures 18 and 19 compare the optimal policy with heterogeneous preference weights on home consumption ω. For the distribution of ω_i weights described in figure 17, I compare the optimal policy under this preference structure to the baseline with homogeneous ω. Both economies feature a rate cut and inflation, though the economy with heterogeneous ω features a larger rate cut and more inflation. As such, consumption, output, and transfers increase by more in the heterogeneous ω economy. When the tariff is active, it is a relatively "good" time to buy the home good, which the poor buy a higher share of. Thus, the planner cuts rates more. For the same reason, the consumption of the constrained increases by more: they consume more of the relatively cheaper home good, so the planner is more willing to intertemporally redistribute when the home good is relatively cheaper.

Figure 18: Optimal Policy, Heterogeneous ω, Aggregate Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables when transfers, taxes, or bonds adjust under optimal monetary policy. Unlike when transfers adjust, optimal monetary policy when bonds adjust features deflation, a decrease in output (labor) and consumption, and the largest decrease in foreign consumption. Adjusting income taxes operates generally similarly to adjusting transfers. Rates are cut, labor and consumption increase, and the decrease in foreign consumption is not as severe.

Figure 19: Optimal Policy, Heterogeneous ω, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to distributional variables when transfers, taxes, or bonds adjust under optimal monetary policy. The fraction of constrained agents falls when transfers or taxes adjust, it is effectively unchanged when bonds adjust. There are noticeable increases in constrained consumption and decreases in constrained labor when transfers or taxes adjust. There are weak decreases in consumption and labor when bonds adjust. The total income Gini falls much more modestly when bonds or taxes adjust compared to when transfers adjust.

7.5 Dual Mandate Planner

The key difference between a tariff shock and a conventional supply/demand shock in the lens of my model is the fiscal externality. Agents view the private cost of foreign consumption as greater than its social cost, due to the tariff revenue that can be used as transfers, to reduce the debt, or to reduce taxes. This externality is present in both RANK (as shown in Bianchi and Coulibaly (2025)) and in my HANK model and in both cases means the planner would prefer higher levels of foreign consumption than private agents. However, this fiscal externality is particularly powerful in HANK. For exposition, consider the case where fiscal revenues are redistributed as transfers. The planner realizes they can use these fiscal revenues to effectively redistribute from rich agents to poor agents, rather than simply "refund" the tariff revenues as in RANK. This redistributive desire is necessarily subjective.

To abstract from this, I change the planner objective function to

$$\int_0^\infty e^{-\tilde{\rho}t}\mathbb{E}_{f_0(b,z)}[u(C_t^f, C_t^h) - g(N_t)]\,\mathrm{d}t \tag{33}$$

where C_t^f, C_t^h, and N_t are respectively aggregate consumption of the foreign and home goods and labor. The planner is still constrained by the same constraints as before: the KFE, HJB (and all household optimality conditions), the NKPC, the government budget, and market clearing. It is as if the planner is maximizing the utility of a fictitious representative agent, or in the logic of the approximation of Benigno and Woodford (2005) is as if the central bank has a standard dual mandate to maintain maximum employment and price stability and has no explicit preference for redistribution.

Proposition 12 (Dual Mandate Primal Timeless Ramsey Problem) *The timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{1}0}\ \max_{r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f,\mathcal{T}(.)} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0}, \eta_{2,0}) \tag{34}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ defined as

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_{\phi}^{\infty}\left[u(C_t^h, C_t^f) - g(N_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$
$$\eta_{2,t}(b,z)\left[u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right] +$$
$$\eta_{4,t}[b_t^h - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c_t^f - C_t^f]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h, c_t^f)\right]$$
$$\eta_{8,t}(b,z)\left[u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}\right] + \eta_{9,t}(b,z)\left[u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}p(N_t, C^h)t(1+\tau)\right]\mathrm{d}b\,\mathrm{d}z\}\,\mathrm{d}t$$

where $\eta_{1,t}, \eta_{2,t}, \eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ are functional Lagrange multipliers and $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, and $\eta_{6,t}$ are scalar Lagrange multipliers and z_1 and z_J are the lower and upper bounds, respectively, on the idiosyncratic shock z. Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in $\mathcal{A}$ by substituting for transfers.

The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ is defined as in Dávila and Schaab (2023) and equals:

$$\int_{z_1}^{z_J}\int_{\phi}^{\infty}\eta_{2,0}V_0(b,z)\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{1}0$ *and choice variables* $r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, \mathcal{T}(.)$ *that solve 34*

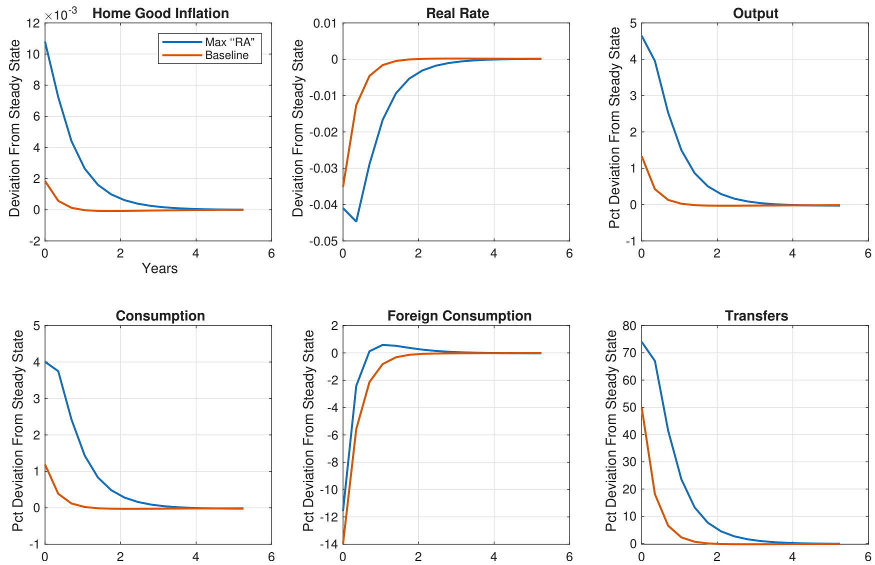
I include the new first order conditions in appendix I. The key different mechanism can be understood by examining the planner's marginal private value of wealth

$$\eta_b = \mathcal{M}^{-1}\eta_4 + \mathcal{M}^{-1}\eta_5 n_b + \mathcal{M}^{-1}\eta_6 c_b^f + \mathcal{M}^{-1}\eta_{10} c_b^h$$

with $\mathcal{M}$ defined identically to before. This marginal value of wealth can be interpreted (only) as the present discounted value of the contribution of future consumption, savings, and labor to aggregate excess consumption, savings, and labor caused by an increase in current wealth. Crucially, it does not contain the household's private value of wealth and will be much more flat over the wealth distribution. This reflects the planner's lack of a *direct* redistributive motive. Due to, for example, Keynesian-style multipliers, these marginal values will however not be completely flat. The planner may still want to redistribute towards the poor, as increasing the income of the poor will have a stronger multiplier on aggregate demand due to their higher MPC's, which *is* in the planner's objective function.

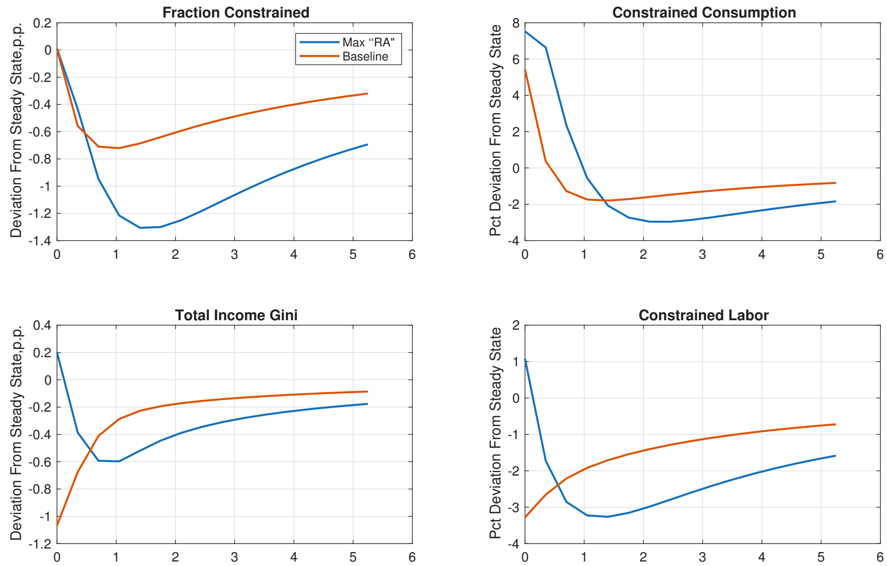
Numerical Results I now plot the optimal response to the shock in figure 1 under the assumption the planner has the objective function in equation 33. For comparison, I also include the optimal response in the baseline model. Figure 20 presents results for aggregate variables, while figure 21 presents results for distributional outcomes.

Figure 20: Optimal vs Look Through Policy, Aggregate Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables for a planner whose objective is to maximize the welfare of an "as-if" representative agent compared to a planner whose objective is to maximize utilitarian welfare. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

Figure 21: Optimal vs Look Through Policy, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to distributional variables for a planner whose objective is to maximize the welfare of an "as-if" representative agent compared to a planner whose objective is to maximize utilitarian welfare. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

The results are quite counterintuitive. I would expect there to be less inflation, a lower rate cut, and less expansion, as the planner prefers redistribution less in this environment than the baseline model, dampening the incentive to inflate. However, the opposite is true. I think the intuition is that the planner is more willing to force the poor to work in this model. The labor of the constrained increases rather than decreases.

7.6 Refunds

In Bianchi and Coulibaly (2025) the representative agent is refunded *exactly* the revenue they spend on tariffs. In HANK, this need not be the case, as agents that have below average foreign consumption are refunded more than what they "put in". To better mimic the spirit of the original model and obtain a fiscal system that is truly distributionally neutral, in this section I assume the following structure of transfers:

$$\mathcal{T}(b,z) = T^{\text{base}} + T^{\text{refund}}$$
$$T^{\text{refund}} = \tau p c^f$$

i.e. each household receives *exactly* the revenue they spend on tariffs plus a flat, baseline level of transfers (calibrated ex-ante). However, they take these transfers as given and do not internalize that additional expenditure on c^f increases T^{refund}. Then, I need to specify how the budget constraint adjusts if the government receives increased or decreased non-tariff revenue (for example, from GE effects on labor supply and labor income tax revenue). I assume this non-tariff revenue simply adjusts the debt B. The government budget is then

$$\dot{B}_t^h = rB_t^h + T^{\text{base}} + \int T_t^{\text{refund}}(b,z)\,\mathrm{d}f(b^h,b^f,z) + sp_t^h Y_t - \tau p^f \int c^f\,\mathrm{d}f(b^h,b^f,z) - \tau_n w \int nz\,\mathrm{d}f(b^h,b^f,z)$$
$$\dot{B}_t^h = rB_t^h + T^{\text{base}} + sp_t^h Y_t - \tau_n w \int nz\,\mathrm{d}f(b^h,b^f,z)$$

I define T^{base} to be a fraction of GDP (calibrated to be the same as in the baseline model). Then, bonds adjust to clear the gvt budget constraint.

Proposition 13 (Primal Timeless Ramsey Problem with Refunds) *The timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{1}0}\ \max_{r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f,B_t} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0},\eta_{2,0}) \tag{35}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ defined as

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_\phi^\infty \left[u(c_t^h,c_t^f) - g(n_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$
$$\eta_{2,t}(b,z)\left[u(c_t^f,c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t-1) - \dot{\pi}\right] +$$
$$\eta_{4,t}[b_t^h - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c_t^f - C_t^f]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h,c_t^f)\right]$$
$$\eta_{8,t}(b,z)\left[u_h(c_t^h,c_t^f) - \frac{\partial V}{\partial b_t^h}\right] + \eta_{9,t}(b,z)\left[u_f(c_t^h,c_t^f) - \frac{\partial V}{\partial b_t^h}p(N_t,C^h)t(1+\tau)\right] + \eta_{10,t}[\mathcal{T}(b,z) - \dot{B}_t + r_t B_t + sN$$

where $\eta_{1,t},\eta_{2,t},\eta_{7,t},\eta_{8,t},\eta_{9,t}$ are functional Lagrange multipliers and $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, and $\eta_{10,t}$ are scalar Lagrange multipliers and z_1 and z_J are the lower and upper bounds, respectively, on *the idiosyncratic shock z. Additionally, I assume that bonds adjust to clear the government budget constraint. The constraint associated with $\eta_{10,t}$ is the government budget without tariff revenue (which is refunded to households)*

The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ is defined as in Dávila and Schaab (2023) and equals:

$$\int_{z_1}^{z_J} \int_{\phi}^{\infty} \eta_{2,0} V_0(b,z) \, \mathrm{d}b \, \mathrm{d}z + \eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{10}$ and choice variables $r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, \mathcal{T}(.)$ that solve 35

Numerical Results WORK IN PROGRESS[19]

8 Conclusion

The presence of household heterogeneity is a key determinant of the optimal monetary response to tariff shocks. Unlike a conventional supply or demand a shock, tariff shocks are differentiated by their strong effects on fiscal revenue. When households make private consumption decisions, they do not internalize that the tariffs they pay increase fiscal revenue and allow the government to provide additional transfers, reduce the debt, or reduce taxes. A planner does internalize this fiscal externality. Like in Bianchi and Coulibaly (2025), I show that the presence of this fiscal externality generally pushes the planner towards expansionary monetary policy. The planner perceives aggregate foreign consumption as being inefficiently low, and the only margin available to correct this is by cutting rates and stimulating aggregate demand. I extend the model and targeting rule of Dávila and Schaab (2023) and show that this fiscal motive combines with standard output gap closing, inflation cost, and redistributive motives to shape optimal policy.

[19] The computation here is quite non-trivial, for both the household block and the planner block. The household block is tricky because I need to find a sort of fixed point over refunds: a level of transfers that induces the household to spend exactly $\tau p c^f$ on foreign consumption. This is not too hard, it ends up just being a very ugly quadratic equation. The planner block is tricky because one of the first order conditions for individual choices contains $\eta_{2,t}$ (normally it would cancel). This means that $\eta_{2,t}$ is codetermined with the penalties $\eta_{7,t}, \eta_{8,t}, \eta_{9,t}$, which makes it much harder to solve the planner KFE. See appendix J for details.

However, unlike in the representative agent case, the fiscal externality in HANK does not simply reflect the fact that transfers refund households the tariff costs they pay. Moreover, the failure of Ricardian Equivalence places additional emphasis on how the government uses its tariff revenue. Even when transfers are lump sum, the fact that the rich consume more of the foreign good implies redistribution from the rich to the poor. Since the planner places greater weight on the welfare of the poor, this further pushes towards expansionary policy.

More generally, the extent to which this fiscal externality incentivizes the planner to inflate is determined by aggregate and distributional effects. First, when the aggregate effects of additional fiscal revenue are positive, the planner will want to inflate. In HANK, there are additional distributional effects that impact the size of the fiscal externality. These distributional effects are equal to the covariance between the income effects of fiscal revenue on households and the planner marginal values on households. The incentive to inflate is stronger and rate cuts are larger, as I show numerically, when progressive instruments like transfers or income taxes adjust. However, when a distributionally neutral instrument adjusts, like debt, optimal policy features weaker cuts or rate hikes. Additional features of the model, such as endogenous terms of trade, non-homothetic consumption behavior, or tariffed intermediate inputs do not meaningfully change this fiscal mechanism.

However, unlike in Bianchi and Coulibaly (2025), there are realistic circumstances where the fiscal externality would explicitly induce deflation. I show this is the case numerically when the planner uses welfare weights that place more value on higher wealth households. If the government spends tariff revenue on lump sum transfers, the distributional effects of the fiscal externality become explicitly negative and induce the planner to raise rates, attempting to decrease fiscal revenue, preventing redistribution from the rich to the poor (which this planner dislikes).

Future work studying optimal monetary responses to tariffs with heterogeneous agents should consider heterogeneity in exposure to tariff shocks through sectoral employment. Past work has shown the importance of this mechanism for the positive effect of shocks. It should be similarly important normatively. Work studying optimal policy with monetary and fiscal interactions should take more seriously the active role of fiscal policy. Here, I assume the fiscal authority makes no strategic or optimizing decisions. In practice, how fiscal revenue is spent is an active choice and can create a game between monetary and fiscal authorities.

A Mathematical Preliminaries

Following the notation of Nuño and Thomas (2022), define the following pieces of notation, letting $\Phi \equiv \{z_1, ..., z_J\} \times \mathbb{R}$, $\hat{\Phi} = [0, \infty] \times \Phi$, and $L^2(\Phi), L^2([0, \infty])$, $L^2(\hat{\Phi})$ be Lebesgue-integrable function spaces:

$$\langle f, g \rangle_\Phi = \int \int_\Phi f_i g_i \, \mathrm{d}b \, \mathrm{d}z = \int_\Phi f^T g \quad \forall f, g \in L^2(\Phi) \tag{36}$$

$$(f, g)_{\mathbb{R}^+} = \int_0^\infty e^{-\tilde{\rho}t} f g \, \mathrm{d}t \quad \forall f, g \in L^2[0, \infty] \tag{37}$$

$$(f, g)_\Phi = \int_0^\infty e^{-\tilde{\rho}t} \langle f, g \rangle_\Phi = \langle e^{-\tilde{\rho}t} f, g \rangle_{\hat{\Phi}} \quad \forall f, g \in L^2(\hat{\Phi}) \tag{38}$$

Then $L^2(\Phi), L^2([0, \infty])$, and $L^2(\hat{\Phi})$ are Hilbert spaces with inner products 36, 37, and 38 respectively.

To solve the central bank's problem (eq 12), I need a generalization of the derivative for function spaces. The Gâteaux derivative is this generalization. See Nuño and Moll (2018) and Nuño and Thomas (2022) for a more detailed discussion of the mathematics; here I state the necessary results for my application without proof.

Definition 2 (Gâteaux Derivative) *Let $W[f]$ be a functional in an arbitrary function space and h be any function in that function space. Then, the Gâteaux derivative of W at f with increment h is*

$$\delta W[f; h] = \lim_{\alpha \to 0} \frac{W[f + \alpha h] - W[f]}{\alpha} = \frac{\mathrm{d}}{\mathrm{d}\alpha} W[f + \alpha h]|_{\alpha=0}$$

The Gâteaux derivative can be used to generalize usual constrained optimization results to function spaces. First, I define a Lagrangian Functional

Definition 3 (Lagrangian Functional) *Suppose $W[f]$ is a functional in function space $L^2(\Phi)$ and $G[f]$ is a mapping $L^2(\Phi) \to \mathbb{R}^n$. Then the Lagrangian functional for the opti-*

mization problem

$$\max W[f]$$
$$s.t. \quad G[f] = 0$$

is defined as $\mathcal{L}[f] = W[f] + \langle \eta, G[f] \rangle_\Phi$ *where* $\eta \in L^2(\Phi)$ *is the Lagrange multiplier*

Theorem 14 (Constrained Optimality) *Suppose* $W[f]$ *is a functional in function space* $L^2(\Phi)$ *and* $G[f]$ *is a mapping* $L^2(\Phi) \to \mathbb{R}^n$*. Then a necessary condition for* W *to have a maximum at* f *subject to the constraint* $G[f] = 0$ *is that there exists a Lagrange multiplier* $\eta \in L^2(\Phi)$ *such that the Gâteaux derivative of the Lagrangian Functional with multiplier* η *is 0:*

$$\delta\mathcal{L}[f; h] = 0, \quad \forall h \in L^2(\Phi)$$

B Solution to Optimal Policy Problem

B.1 Lagrangian

Using the notation from equations 37-38, the Lagrangian for the central bank's problem (eq 12) is defined in the space $L^2(\hat{\Phi})$ as

$$\begin{aligned}\mathcal{L} = &\Big(u(c_t^h, c_t^f) - g(n_t), f_t\Big)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho} V_t\right)_\Phi \\ &\left(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \big(\eta_{4,t}, f_t(b_t^h - B_t)\big)_\Phi + \left(\eta_{5,t}, f_t(n_t - N_t)\right)_\Phi + \Big(\eta_{6,t}, f_t(c_t^f - C_t^f)\Big)_\Phi \\ &+ \left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n) w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\right.\end{aligned} \tag{39}$$

Re-defining the generator and its adjoint:

$$\mathcal{A}^* f = \begin{bmatrix} -\frac{\partial(s_1^f f_1)}{\partial b^f} - \frac{\partial(s_1^h f_1)}{\partial b^h} - f_1 \sum_{j' \neq 1} \lambda_{1j'} + \sum_{j' \neq 1} \lambda_{j'1} f_1 \\ \vdots \\ -\frac{\partial(s_J^f f_J)}{\partial b^f} - \frac{\partial(s_J^h f_J)}{\partial b^h} - f_J \sum_{j' \neq J} \lambda_{Jj'} + \sum_{j' \neq J} \lambda_{j'J} f_J \end{bmatrix}$$

and

$$\mathcal{A}V = \begin{bmatrix} \frac{\partial V_1}{\partial b^h} s_1^h + \frac{\partial V_1}{\partial b^f} s_1^f + \sum_{j' \neq 1} \lambda_{1,j'} \left[V_{j'} - V_1 \right] \\ \vdots \\ \frac{\partial V_J}{\partial b^h} s_J^h + \frac{\partial V_J}{\partial b^f} s_J^f + \sum_{j' \neq J} \lambda_{J,j'} \left[V_{j'} - V_J \right] \end{bmatrix}.$$

where after substituting in for lump sum dividends and the government budget, the savings function is $s^h = rb^h + (1-\tau^n)zwn + \dot{B} - rB - sN + \tau p C_t^f + \tau^n w N_t + (1+s)N_t - m_t N_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f$. I define $C_t^f = \langle c_t^f, f_t \rangle_\Phi$ and add C_t^f as an additional choice of the planner. This avoids having f appear directly in the generator $\mathcal{A}$, simplifying the derivations going forward, and facilitating a more intuitive interpretation of the equilibrium conditions.

The generator and its adjoint satisfy $\langle f, \mathcal{A}v \rangle_\Phi = \langle \mathcal{A}^* f, v \rangle_\Phi$ for any $f, v \in L^2(\Phi)$. Using this result and integration by parts, it will be useful at times to rewrite the $\eta_{1,t}$ and $\eta_{2,t}$ terms of the Lagrangian as

$$\begin{aligned}
&\int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t} \rangle_\Phi \, \mathrm{d}t \\
&= \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}\eta_{1,t}, f_t \rangle_\Phi + e^{-\tilde{\rho}t} \langle \eta_{1,t}, -\frac{\partial f_t}{\partial t} \rangle_\Phi \\
&= \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}\eta_{1,t}, f_t \rangle_\Phi + \langle \eta_{1,0}, f_0 \rangle_\Phi - \lim_{T\to\infty} \langle e^{-\tilde{\rho}T} \eta_{1,T}, f_T \rangle + e^{-\tilde{\rho}t} \langle \frac{\partial \eta_{1,t}}{\partial t} - \tilde{\rho}\eta_{1,t}, f_t \rangle_\Phi
\end{aligned}$$

and

$$\begin{aligned}
&\int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t \rangle_\Phi \, \mathrm{d}t \\
&= \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) \rangle_\Phi \, \mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t \rangle_\Phi \, \mathrm{d}t \\
&= \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) \rangle_\Phi \, \mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}^* \eta_{2,t}, V_t \rangle_\Phi \, \mathrm{d}t + \lim_{T\to\infty} \langle e^{-\tilde{\rho}T} \eta_{2,T}, V_T \rangle_\Phi - \langle \eta_{2,0}, v_0 \rangle_\Phi + \\
&\int_0^\infty e^{-\tilde{\rho}t} \langle V_t, -\frac{\partial \eta_{2,t}}{\partial t} \rangle_\Phi \, \mathrm{d}t + \sum_{j=1}^J v_{j,t}(b) s_{j,t}(b) \theta_{j,t}(b) |_{-\infty}^{\infty}
\end{aligned}$$

Note that the rewritten $\eta_{1,t}$ term can be combined with the first term of the original Lagrangian to construct a "social HJB Equation"

$$\int_0^\infty e^{-\tilde{\rho}t} \langle u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\partial \eta_{1,t}}{\partial t} - \tilde{\rho}\eta_{1,t}, f_t \rangle_\Phi \, \mathrm{d}t + \langle \eta_{1,0}, f_0 \rangle_\Phi - \lim_{T\to\infty} \langle e^{-\tilde{\rho}T} \eta_{1,T}, f_T \rangle + \ldots$$

where $\eta_{1,t}$ is the social value function with the latter two terms later being used to define boundary conditions.

Similarly, the $\eta_{2,t}$ term can be rewritten as a "social KFE Equation" plus a social welfare term

$$\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t)\rangle_\Phi \,\mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t}\langle \mathcal{A}^*\eta_{2,t} - \frac{\partial \eta_{2,t}}{\partial t}, V_t\rangle_\Phi \,\mathrm{d}t + \lim_{T\to\infty}\langle e^{-\tilde{\rho}T}\eta_{2,T}, V_T\rangle_\Phi - \langle \eta_{2,0}, v_0\rangle_\Phi + \sum_{j=1}^{J} v_{j,t}(b)s_{j,t}(b)\theta_{j,t}(b)|_{-\infty}^{\infty}$$

where $\eta_{2,t}$ is the planner's density (the latter three terms will be used to create usual boundary conditions).

Additionally, it will be useful to rewrite the FOC on home and foreign consumption according to

$$\begin{aligned} &-\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{8,t}, \frac{\partial V}{\partial b^h}\rangle_\Phi \\ &= \int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial \eta_{8,t}}{\partial b^h}, V_t\rangle_\Phi \,\mathrm{d}t \end{aligned}$$

by integrating by parts.

B.2 Proof of Proposition 2

Characterizing the solution to the planner's problem and completing the proof of proposition 2 requires taking Gâteaux derivatives of 39 with respect to $\pi_t, r_t^h, V_t(.), c_t^f(.), c_t^h(.), n_t(.), f_t(.), w_t, B_t, N_t, C_t^f$.

KFE and HJB The derivative wrt f in the direction h is

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle u(c_t^h, c_t^f) - g(n_t), f_t + \alpha h\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle \frac{\partial \eta_{1,t}}{\partial t} - \tilde{\rho}\eta_{1,t} + \mathcal{A}\eta_{1,t}, f_t + \alpha h\rangle_\Phi|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle \eta_{1,0}, f_0 + \alpha h_0\rangle|_{\alpha=0} - \frac{\mathrm{d}}{\mathrm{d}\alpha}\lim_{T\to\infty}\langle e^{-\tilde{\rho}T}\eta_{1,T}, f_T + \alpha h_T\rangle_\Phi + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle e^{-\tilde{\rho}t}\eta_{4,t}, (b_t^h - B_t)(f+\alpha h)\rangle_\Phi|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle e^{-\tilde{\rho}t}\eta_{5,t}, (z_t n_t - N_t)(f+\alpha h)\rangle_\Phi|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle e^{-\tilde{\rho}t}\eta_{6,t}, (c_t^f - C_t^f)(f+\alpha h)\rangle_\Phi|_{\alpha=0}$$

Which simplifies to

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{9,t}(c_t^f - C_t^f) \tag{40}$$

$$\lim_{T\to\infty} e^{-\tilde{\rho}T}\eta_{1,T} = 0$$

This can be interpreted as a planner's HJB equation and usual transversality condition, where $\eta_{1,t}$ is the planner's value function.

The derivative wrt v in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \mathcal{A}^*\eta_{2,t} - \frac{\partial \eta_{2,t}}{\partial t}, V_t + \alpha h\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}(\lim_{T\to\infty}\langle e^{-\tilde{\rho}T}\eta_{2,T}, V_T + \alpha h_T\rangle_\Phi - \langle \eta_{2,0}, v_0 + \alpha h_0\rangle_\Phi)|_{\alpha=0}$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\left(\sum_{j=1}^{J}(v_{j,t} + \alpha h)(b)s_t^h(b,j)\eta_{2,t}(b,j)|_{b=-\infty}^{b=\infty}\right)|_{\alpha=0} + \int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial \eta_{8,t}}{\partial b}p_t^h + \frac{\partial \eta_{9,t}}{\partial b}(1+\tau)p_t^f\rangle_\Phi$$

Which simplifies to (for $b \geq \phi$ and $t > 0$)

$$\frac{\partial \eta_{2,t}}{\partial t} = \mathcal{A}^*\eta_{2,t} + \frac{\partial \eta_{8,t}}{\partial b}p_t^h + \frac{\partial \eta_{9,t}}{\partial b}(1+\tau)p_t^f \tag{41}$$

This is a "planner's KFE" with density $\eta_{2,t}$. The boundary conditions are

$$\lim_{b\to\infty} s_t^h(b,j)\eta_{2,t}(b,j) = \lim_{b\to-\infty} s_t^h(b,j)\eta_{2,t}(b,j) = 0 \quad j = 1,...J$$
$$\eta_{2,0} = \lim_{T\to\infty} e^{-\tilde{\rho}T}\eta_{2,T} = 0$$

Prices The derivative wrt π in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial \eta_{1,t}}{\partial b}(-N_t\frac{\theta}{2}(\pi + \alpha h)^2), f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial v}{\partial b}(-N_t\frac{\theta}{2}(\pi + \alpha h)^2), \eta_{2,t}\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left((r - \frac{\dot{N}_t}{N_t})(\pi + \alpha h) - \dot{(\pi + \alpha h)}\right)\mathrm{d}t$$

After taking derivatives wrt α, the third term requires evaluating $-\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\dot{h}\,\mathrm{d}t$. Using integration by parts and $\langle 1, f_t\rangle_\Phi = 1$

$$\begin{aligned}
&-\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\dot{h}\,\mathrm{d}t \\
&= \eta_{3,0}h_0 + \int_0^\infty e^{-\tilde{\rho}t}\langle(\dot{\eta}_{3,t} - \tilde{\rho}\eta_{3,t})h_t, f_t\rangle_\Phi
\end{aligned}$$

Thus, this FOC simplifies to

$$0 = \langle -\frac{\partial \eta_{1,t}}{\partial b}(N_t\theta\pi_t), f_t\rangle_\Phi + \langle -\frac{\partial V_{1,t}}{\partial b}(N_t\theta\pi_t), \eta_{2,t}\rangle_\Phi + \eta_{3,t}(r - \frac{\dot{N}_t}{N_t} - \tilde{\rho}) + \dot{\eta}_{3,t} \tag{42}$$

$$\eta_{3,0} = 0$$

The derivative wrt r in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle (r+\alpha h)(b-B)\frac{\partial \eta_{1,t}}{\partial b}, f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle (r+\alpha h)b\frac{\partial V}{\partial b}, \eta_{2,t}\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left((r_t+\alpha h) - \frac{\dot{N}_t}{N_t}\right)\pi_t$$

which simplifies to

$$\eta_{3,t}\pi_t = \langle (B_t - b_t^h)\frac{\partial \eta_{1,t}}{\partial b}, f_t\rangle_\Phi + \langle (B_t - b_t^h)\frac{\partial V_{1,t}}{\partial b}, \eta_{2,t}\rangle_\Phi \tag{43}$$

The derivative wrt w in direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle\left(\frac{(w+\alpha h)}{p^h}(1-\tau^n)zn_t + \tau^n N_t\frac{(w+\alpha h)}{p^h}\right)\frac{\partial \eta_{1,t}}{\partial b}, f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle\frac{(w+\alpha h)}{p^h}(1-$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left(-\frac{\epsilon}{\theta}(\frac{w+\alpha h}{p^h}-1)\right)\mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{7,t}, -\frac{(w_t+\alpha h)(1-\tau^n)z_{j,t}}{p_t^h}u_h(c_t^h, c_t^f)\rangle \,\mathrm{d}t$$

which simplifies to

$$0 = \langle (1-\tau^n)(z_t n_t - N_t)\frac{\partial \eta_{1,t}}{\partial b}, f\rangle_\Phi + \langle (1-\tau^n)(z_t n_t - N_t)\frac{\partial V_t}{\partial b}, \eta_{2,t}\rangle_\Phi - \eta_{3,t}\frac{\epsilon}{\theta} - \langle (1-\tau^n)zu_h(c^f, c^h), \eta_{7,t}\rangle_\Phi \tag{44}$$

Household Choices The derivative wrt c^f in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle u(c_t^h, c_t^f+\alpha h) - (c^f+\alpha h)p(1+\tau)\frac{\partial \eta_{1,t}}{\partial b}, f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle u(c_t^h, c_t^f+\alpha h) - (c^f+\alpha h)p(1+\tau)\frac{\partial V_t}{\partial b}, \eta_{2,t}\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{9,t}, u_f(c^h, c^f+\alpha h)\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{7,t}, -\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h, c_t^f+\alpha h)\rangle \,\mathrm{d}t +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{8,t}, u_h(c_t^h, c_t^f+\alpha h)\rangle_\Phi \,\mathrm{d}t + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle e^{-\tilde{\rho}t}\eta_{6,t}, (c_t^f+\alpha h - C_t^f)f_t\rangle_\Phi|_{\alpha=0}$$

which, after noting that the private FOC hold, simplifies to

$$0 = u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) + \frac{\eta_{9,t}}{f_t}u_{ff}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hf}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{fh}(c_t^h, c_t^f) + \eta_{6,t} \tag{45}$$

The derivative wrt c^h in the direction h is similar

$$
\begin{aligned}
0 = &\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle u(c_t^h+\alpha h, c_t^f) - (c^h+\alpha h)\frac{\partial \eta_{1,t}}{\partial b}, f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \\
&\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle u(c_t^h+\alpha h, c_t^f) - (c^h+\alpha h)\frac{\partial V_t}{\partial b}, \eta_{2,t}\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \\
&\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{9,t}, u_f(c^h+\alpha h, c^f)\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{7,t}, -\frac{(1-\tau^n)w_t z_{j,t}}{p_t^h}u_h(c_t^h, c_t^f+\alpha h)\rangle \,\mathrm{d}t|_{\alpha=0} + \\
&\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{8,t}, u_h(c_t^h+\alpha h, c_t^f)\rangle_\Phi \,\mathrm{d}t|_{\alpha=0}
\end{aligned}
$$

and after taking into account the fact that the household FOCs must hold simplifies to

$$
0 = u_h(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b} + \frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{(1-\tau^n)w_t z_{j,t}}{p_t^h}u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) \tag{46}
$$

Note that unlike in Nuño and Moll (2018), I explicitly set the individual FOC as a constraint. Without this, the planner's FOC would simply be the partials of the planner's HJB (eqn (40)) with respect to c^h, c^f, and n respectively. However, enforcing the private FOC hold directly adds η_7, η_8, and η_9 terms (with non-zero multipliers), capturing the fact that the planner would prefer levels of individual consumption and labor not consistent with households' optimization. The values of $\eta_{7,t}(b,z)$, $\eta_{8,t}(b,z)$, $\eta_{9,t}(b,z)$ penalize the planner enough to make them choose consumption and labor consistent with household choices rather than allocation them optimally subject only to feasibility

The derivative wrt n in direction h

$$
\begin{aligned}
0 = &\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial \eta_{1,t}}{\partial b} zw(n+\alpha h) - g(n_t+\alpha h), f_t\rangle_\Phi \,\mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial V_t}{\partial b} zw(n+\alpha h) - g(n_t+\alpha h), \eta_{2,t}\rangle_\Phi \,\mathrm{d}t|_{\alpha} \\
&+ \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{5,t}, f_t(n_t+\alpha h - N_t)\rangle_\Phi \,\mathrm{d}t \int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{7,t}, g'(n_t+\alpha h)\rangle \,\mathrm{d}t|_{\alpha=0}
\end{aligned}
$$

This simplifies to

$$
0 = -g'(n_t) + \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h}\frac{\partial \eta_{1,t}}{\partial b} + \eta_{7,t}\frac{g''(n_t)}{f_t} + z\eta_{5,t} \tag{47}
$$

Aggregate Choices The derivative wrt B in direction h

$$
\begin{aligned}
0 = &\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{1,t}, \mathcal{A}^*_{B+\alpha h} f_t\rangle_\Phi \,\mathrm{d}t + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{4,t}, f_t(b_t^h - (B+\alpha h))\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} \\
&+ \int_0^\infty e^{-\tilde{\rho}t}\langle V_t, \mathcal{A}^*_{B+\alpha h}\eta_{2,t}\rangle_\Phi \,\mathrm{d}t
\end{aligned}
$$

which simplifies to

$$\eta_{4,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\frac{\partial s^h}{\partial B}\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\frac{\partial s^h}{\partial B}\rangle_\Phi \tag{48}$$

Under the assumption of flat transfers, this evaluates to (using integration by parts to evaluate the derivative of $\dot{B}$ terms)

$$\eta_{4,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left((\rho - r) - \frac{\dot{f}_t}{f_t} - \frac{\frac{\partial \dot{\eta}_{1,t}}{\partial b}}{\frac{\partial \eta_{1,t}}{\partial b}}\right)\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\left((\rho - r) - \frac{\dot{\eta}_{2,t}}{\eta_{2,t}} - \frac{\frac{\partial \dot{V}_t}{\partial b}}{\frac{\partial V_t}{\partial b}}\right)\rangle_\Phi$$

The derivative wrt N in direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left((1+s-s)(N_t+\alpha h) - m_t(N_t+\alpha h) - \frac{\theta}{2}\pi_t^2(N_t+\alpha h) + \tau w(N+\alpha h)\right)\rangle_\Phi \,\mathrm{d}t_{\alpha=0}+$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{2,t}, \frac{\partial V}{\partial b}\left((1+s-s)(N_t+\alpha h) - m_t(N_t+\alpha h) - \frac{\theta}{2}\pi_t^2(N_t+\alpha h) + \tau w(N+\alpha h)\right)\rangle_\Phi \,\mathrm{d}t_{\alpha=0}+$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left(-\frac{\dot{\overline{N_t+\alpha h}}}{N_t+\alpha h}\right)\pi_t \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{5,t}, f_t(n_t - (N+\alpha h))\rangle_\Phi \,\mathrm{d}t|_{\alpha=0}$$

which simplifies to

$$\eta_{5,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left(-\frac{\dot{\overline{N_t+\alpha h}}}{N_t+\alpha h}\right)\pi_t \,\mathrm{d}t|_{\alpha=0}$$

The final term evaluates to

$$\int_0^\infty e^{-\tilde{\rho}t}(-\eta_{3,t})\frac{\dot{h}N_t - h\dot{N}_t}{N_t^2}\pi_t \,\mathrm{d}t$$

Using integration by parts

$$\int_0^\infty e^{-\tilde{\rho}t}(-\eta_{3,t})\frac{\dot{h}}{N_t}\,\mathrm{d}t$$
$$= -\left(\frac{e^{-\tilde{\rho}t}h_t\eta_{3,t}\pi_t}{N_t}|_0^\infty\right) + \int_0^\infty h_t\left(\frac{\pi_t\eta_{3,t}(-\tilde{\rho}e^{-\tilde{\rho}t})}{N_t} + \frac{\pi_t\dot{\eta}_{3,t}e^{-\tilde{\rho}t}}{N_t} + \frac{\dot{\pi}\eta_{3,t}e^{-\tilde{\rho}t}}{N_t} - \frac{\pi_t\eta_{3,t}e^{-\tilde{\rho}t}\dot{N}_t}{N_t^2}\right)\mathrm{d}t + \int_0^\infty h_t\frac{\pi_t\eta_{3,}}{}$$
$$= \frac{h_0\eta_{3,0}}{N_0} + \int_0^\infty \frac{h_t e^{-\tilde{\rho}t}}{N_t}\left(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t}\right)\mathrm{d}t$$

So finally,

$$\eta_{5,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} \tag{49}$$

The derivative wrt C^f in direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\tau p(C_t^f + \alpha h)\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\tau p(C_t^f + \alpha h)\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{9,t}, (c_t^f - C_t^f - \alpha h) f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0}$$

which simplifies to

$$\eta_{9,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\tau p\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\tau p\rangle_\Phi \tag{50}$$

Combining the simplified first order conditions completes the proof of Proposition 2.

B.3 Proof of Proposition 3

First, I define some notation. Taking the derivative of the household's HJB, equation 1, and the planner's HJB, equation 40, with respect to current bonds b yields

$$\tilde{\rho}V_b = u_h \cdot c_b^h + u_f \cdot c_b^f - g' \cdot n_b + V_b s_b^h + \frac{\partial^2 V}{\partial b^2}s^h + \sum_{j' \neq j}\lambda_{j,j'}[V_b(j') - V_b(j)] + \frac{\mathrm{d}}{\mathrm{d}t}V_b$$
$$\tilde{\rho}\eta_b = \cdot c_b - g' \cdot n_b + \eta_b s_b^h + \frac{\partial^2 \eta}{\partial b^2}s^h + \sum_{j' \neq j}\lambda_{j,j'}[\eta_b(j') - \eta_b(j)] + \frac{\mathrm{d}}{\mathrm{d}t}\eta_b + \eta_4 + \eta_5 n_b + \eta_6 c_b^f$$

where variables with a subscript b denote partial derivatives with respect to bonds and u and g are the instantaneous utilities of consumption and labor. Now define the operator $\mathcal{M} = \tilde{\rho} - s_b^h - \mathrm{d}t - \mathcal{A}$. Then, noting that $\frac{\partial^2 V}{\partial b^2}s^h + \sum_{j' \neq j}\lambda_{j,j'}[V_b(j') - V_b(j)] = \mathcal{A}V_b$, the envelope conditions can be rewritten according to

$$\mathcal{M}V_b = u' \cdot c_b - g' \cdot n_b$$
$$\mathcal{M}\eta_b = u' \cdot c_b - g' \cdot n_b + \eta_4 + \eta_5 n_b + \eta_6 c_b^f$$

and following Dávila and Schaab (2023), the difference between the marginal social and public value of wealth can be written as

$$\eta_b = V_b + \eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f \tag{51}$$

where the term $\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f$ captures the present discounted value of the future savings, labor supply, and foreign consumption to aggregate excess bond demand, aggregate excess labor, and aggregate excess foreign consumption (respectively) caused by an increase in time t assets.

Next, define "promise keeping values"

$$\begin{aligned}
\mathcal{H}_t^f &\equiv -\frac{\eta_{9,t}}{f_t}u_{ff}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hf}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{fh}(c_t^h, c_t^f) \\
\mathcal{H}_t^h &\equiv -\frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hh}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) \\
\mathcal{H}_t^n &\equiv -\eta_{7,t}\frac{g''(n_t)}{f_t}
\end{aligned}$$

i.e. the η_7, η_8, and η_9 terms in the planner's FOC's for individual choices (equations 45-47) which represent value/cost of the planner being forced to adhere to household optimality conditions for foreign consumption, domestic consumption, and labor rather than allocating consumption and labor only subject to feasibility.

Additionally, note that the "promise keeping values" are related to the wedge between the private and social values of wealth according to

$$\begin{aligned}
\mathcal{H}_t^f &= \eta_{6,t} - p(1+\tau)(\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f) \\
\mathcal{H}_t^h &= -(\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f) \\
\mathcal{H}_t^n &= z\eta_{5,t} - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}(\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f)
\end{aligned}$$

Now with the requisite notation defined, I am ready to prove proposition 3. The general strategy will be to substitute the planner's first order conditions for the individual choices (equations 45-47) into the first order conditions for interest rates (equation 43) and aggregates (equations 49-50)

Rewrite the first order condition for individual foreign consumption

$$\begin{aligned}
0 &= u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t} \\
0 &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}(B-b)\left[u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z \\
\int_{z_1}^{z_J}\int_{\phi}^{\infty}(B-b)\frac{\partial \eta_{1,t}}{\partial b} f_t \,\mathrm{d}b\,\mathrm{d}z &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}(B-b)\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)}\right] f_t \,\mathrm{d}b\,\mathrm{d}z
\end{aligned}$$

Substituting into the first order condition for interest rates and using the private consumption

FOC to substitute for $\frac{\partial V_t}{\partial b}$

$$\eta_{3,t}\pi_t = \int_{z_1}^{z_J}\int_{\phi}^{\infty}(B_t - b_t^h)\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)} + \frac{u_f}{p(1+\tau)}\frac{\eta_{2,t}}{f_t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z \tag{52}$$

Next, the first order condition for aggregate foreign consumption is rewritten as

$$0 = u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t}$$

$$0 = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z$$

$$\int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{\partial \eta_{1,t}}{\partial b} f_t\,\mathrm{d}b\,\mathrm{d}z = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)}\right] f_t\,\mathrm{d}b\,\mathrm{d}z + p\tau\frac{\eta_{6,t}}{p(1+\tau)}$$

paired with the first order condition for individual foreign consumption yields

$$\eta_{6,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[\frac{u_f}{p} - \frac{\mathcal{H}_t^f}{p} + \frac{u_f}{p}\frac{\eta_{2,t}}{f_t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z \tag{53}$$

Rewriting the first order condition for individual labor choice yields

$$\int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\frac{\partial \eta_{1,t}}{\partial b} f_t\,\mathrm{d}b\,\mathrm{d}z = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\left[\frac{g'(n)}{w_t z_t(1-\tau^n)} + \frac{\mathcal{H}_t^n}{(1-\tau^n)w_t z_t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z - \frac{\partial s^h}{\partial N}\frac{\eta_{5,t}}{w_t(1-\tau^n)}$$

Then substituting into the first order condition for aggregate labor

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\frac{1}{(1-\tau^n)wz}\left[g'(n) + \mathcal{H}_t^n + g'(n_t)\frac{\eta_{2,t}}{f_t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z - \frac{\partial s^h}{\partial N}\frac{\eta_{5,t}}{w_t(1-\tau^n)} + \frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\pi \tag{54}$$

Next, I combine equation 52 with equation 54

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\frac{1}{w_t z_t(1-\tau^n)}\left[g'(n) + \mathcal{H}_t^n + g'(n_t)\frac{\eta_{2,t}}{f_t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z - \frac{\partial s^h}{\partial N}\frac{\eta_{5,t}}{w_t(1-\tau^n)} +$$
$$\frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\int_{z_1}^{z_J}\int_{\phi}^{\infty}(B_t - b_t^h)\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)} + \frac{u_f}{p(1+\tau)}\frac{\eta_{2,t}}{f_t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z$$

$$\eta_{6,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[\frac{u_f}{p} - \frac{\mathcal{H}_t^f}{p} + \frac{u_f}{p}\frac{\eta_{2,t}}{f_t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z$$

I then define Ω_1 and Ω_2 according to

$$\Omega_1 = \eta_{6,t} + \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{\mathcal{H}_t^f}{p} f_t\,\mathrm{d}b\,\mathrm{d}z$$

$$\Omega_2 = (1 + \frac{\partial s^h}{\partial N})\eta_{5,t} + \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left[\frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)(B - b^h)\frac{\mathcal{H}_t^f}{p(1+\tau)} - \frac{\partial s^h}{\partial N}\frac{\mathcal{H}_t^n}{w_t z_t}\right]$$

Given usual assumptions about concavity of the utility functions and standard parameter values, $\Omega_1, \Omega_2 > 0$.

So

$$1 = \frac{1}{\Omega_2} \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s^h}{\partial N} \left[\frac{u_h}{p} + \frac{u_h}{p} \frac{\eta_{2,t}}{f_t} \right] f_t \, \mathrm{d}b \, \mathrm{d}z + \frac{1}{\Omega_2 N_t} \left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} \right) \int_{z_1}^{z_J} \int_{\phi}^{\infty} (B_t - b_t^h) \left[\frac{u_h}{p} + \frac{u_h}{p} \frac{\eta_{2,t}}{f_t} \right] f_t \, \mathrm{d} \tag{55}$$

$$1 = \frac{1}{\Omega_1} \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s}{\partial T} \frac{\partial T}{\partial C^f} \left[\frac{u_h}{p} + \frac{u_h}{p} \frac{\eta_{2,t}}{f_t} \right] f_t \, \mathrm{d}b \, \mathrm{d}z \tag{56}$$

Combining equations 55 and 56 and rearranging completes the proof.

Note that manipulating equation 42 can allow one to obtain a more intuitive formulation of term that rescales the redistributive motive, $\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} \right)$.

$$\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{N_t \theta \pi_t}{\eta_{3,t}} \left(\frac{u_h}{p} - \frac{\mathcal{H}^h}{p} + \frac{u_h}{p} \frac{\eta_{2,t}}{f_t} \right) f_t \, \mathrm{d}b \, \mathrm{d}z + \tilde{\rho} - r + \frac{\dot{N}_t}{N_t}$$

$$\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{N_t \theta \pi_t}{\eta_{3,t}} \left(\frac{u_h}{p} - \frac{\mathcal{H}^h}{p} + \frac{u_h}{p} \frac{\eta_{2,t}}{f_t} \right) f_t \, \mathrm{d}b \, \mathrm{d}z + \frac{\dot{N}_t}{N_t} - r + \frac{\dot{\pi}_t}{\pi_t}$$

Finally from the NKPC

$$\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{N_t \theta \pi_t}{\eta_{3,t}} \left(\frac{u_h}{p} - \frac{\mathcal{H}^h}{p} + \frac{u_h}{p} \frac{\eta_{2,t}}{f_t} \right) f_t \, \mathrm{d}b \, \mathrm{d}z - \frac{\epsilon}{\theta} \frac{m_t - 1}{\pi_t}$$

This term thus implies that the redistributive motive is scaled by how the marginal costs of inflation accrue to households.

C Computational Algorithm

The algorithm to calculate the optimal stationary equilibrium and optimal transitional dynamics combines insights from the algorithms of Nuño and Moll (2018) and Nuño and Thomas (2022). In particular, it is a double fixed point algorithm. In the inner loop, I solve the usual household HJB, given guesses of Lagrange multipliers η_4, η_5, η_9 and optimal inflation π_t, a standard problem with algorithms outlined in, for example, Achdou et al. (2021). In the outer loop, I solve algebraically for the planner's value function, density, and multipliers on individual choices. Then I use the planner's first order conditions on aggregates (C^f, N, B) and inflation/interest rates to verify my guesses for the unknown multipliers and inflation.

Stationary Equilibrium Algorithm First, I specify the inner loop. Entering the inner loop, I take as given values of η_4, η_5, η_9, and π. I am implicitly treating T as a residual in the baseline version of the model. So since s and τ are ex-ante specified and T is the residual, I need a specification for B. Here, I assume that it is flat, but I *calibrate* it to be equal to some constant fraction of steady state GDP: $B_t = B_{\text{perc}} Y_{ss}$, which greatly facilitates numerical implementation. Note that I am assuming bonds are flat; the monetary authority does not internalize how their choices of policy affect equilibrium supply of bonds by the fiscal authority.

So in this inner loop, I am effectively seeking the equilibrium consistent with a given level of inflation

1. Guess aggregate labor N_t, the interest rate r_t, and aggregate transfers T
2. Solve the firm's problem (eqs 6 and 7) for the wage and dividends.
3. Using standard methods, solve the household's HJB equation (eq 1)
4. Solve the KFE (eq 5) for the stationary distribution
5. Using implied aggregate bonds, use the government budget constraint to calculate the implied transfer T (eq 8)
6. Check market clearing, for bonds comparing the guessed B and the model-implied B (eqs 9 10)
7. If markets clear and the gvt budget constraint holds, stop. Otherwise, return to step 1 and update guesses until convergence

Now, I specify the outer loop, which verifies the planner's conditions holds.

1. Guess values for $\eta_4, \eta_5, \eta_6,$, and π
2. Do the inner loop, described above. Store all equilibrium policy functions, value functions, and aggregates.
3. Solve the planner's HJB (eq 40) for $\eta_{1,t}$.

4. Use equation 51 to calculate the derivative of $\eta_{1,t}$ with respect to wealth (the usual upwinding scheme is not effective since we cannot use backwinding to calculate the derivative for constrained agents. The planner's FOC for individual consumption and labor that would be used to deal with constrained agents include unknown Lagrange multiplier terms. It is less efficient to jointly solve for $\frac{\partial \eta_{1,t}}{\partial b}$ and the multipliers than it is to back out the derivative using equation 51)

5. Jointly solve the planner's first order conditions for individual choices and obtain η_7, η_8, η_9 (eqns 45-47)

6. Solve the planner's "KFE" (eq 41) for $\eta_{2,t}$

7. Using the planner's first order condition for π_t, the conjectured value of π_t, and the equilibrium objects, solve equation 42 for η_3

8. Use the planner's first order condition for r_t and w_t (equation 43 and 44) and aggregates (equations 48-50) to calculate 4 residuals

9. If the 4 residuals are 0, stop. Otherwise, return to step 1 of the outer loop and update guesses for η_4, η_5, η_9, and π.

For both loops, I use a numerical solver in matlab (the default fsolve algorithm is fine).

C.1 Transitional Dynamics Algorithm

I follow the timeless approach of, for example, Dávila and Schaab (2023). In principle, one can solve the transitional dynamics in response to an MIT shock in the same way as the stationary equilibrium, using an inner loop to guess and verify paths of N, r, and T and an outer loop to guess and verify paths of multipliers η_4, η_5, η_6, and π. However, given that most shocks I consider are relatively small (meaning initial guesses should be reasonable) and there are a large number of equations and unknowns, it is more efficient to simultaneously guess paths of N, r, T, η_4, η_5, η_6, and π and use one solver. Like in the stationary equilibrium, I assume that bond are constant ($\dot{B}_t = 0$) and calibrated to be a fixed fraction of steady state

GDP.[20]

1. Solve for the initial stationary equilibria. For a temporary shock, the initial and ending equilibria are identical. For a permanent shock, the equilibria will be different

2. Discretize time. Set the ending time to be some large number $\bar{T}$ so the model converges to the steady state after $\bar{T}$. Additionally choose a number of steps ns, which will be the number of discrete time steps the model is solved on. Thus, I define $\mathrm{d}t = \frac{\bar{T}}{ns}$.

3. Specify a path for the tariff shock $\{\tau_t\}_{t=0}^{ns}$. For the temporary shock, I assume an exponentially decaying process at some rate ξ.

4. Construct initial guess of paths for $\{N\}_{t=0}^{ns}$, $\{r\}_{t=0}^{ns}$, $\{T\}_{t=0}^{ns}$, $\{\eta_4\}_{t=0}^{ns}$, $\{\eta_5\}_{t=0}^{ns}$, $\{\eta_6\}_{t=0}^{ns}$, and $\{\pi\}_{t=0}^{ns}$. I simply linearly interpolate between the initial and terminal steady state values.

5. Generate and provide initial or terminal conditions for all other endogenous equilibrium variables' paths (aggregates, household policies, the density, value function, its derivative, the generator, and all of the planner's multipliers). $\eta_{2,t}$ and f_t should have initial conditions, as they are determined by forward looking equations. All other endogenous variables should have terminal conditions.

6. Solve the firm's problem for paths of m (from the NKPC), w (from firm labor FOC), Y, D, and B (from the assumption that B is a constant fraction of GDP). Since I guess π and N, iteration over time on the NKPC is unnecessary, I can simply calculate $\dot{\pi}$ and $\dot{N}$ directly.

7. Starting from the terminal steady state, solve the HJB (eq 1). Discretize time as $\dot{V} = \frac{V_{t+1} - V_t}{\mathrm{d}t}$ and solve the HJB with the implicit method of Achdou et al. (2021).

[20] I found that allowing bonds to be a fraction of GDP, adjusting during the transition path, and having the monetary authority internalize this made market clearing very difficult numerically. In particular, it implied that output must remain constant on impact. Household bond demand must remain constant (b is a state variable, f cannot adjust on impact due to the initial condition on the density). Thus, bond supply also needs to remain constant, so if bond supply is a fixed fraction of GDP, GDP cannot change on impact for the bond market to clear.

8. Solve the KFE (equation 5) forwards. Implicit updating implies $\frac{f_{t+1}-f_t}{\mathrm{d}t} = \mathcal{A}_t' f_{t+1}$ and requires solving the aforementioned for f_{t+1}.

9. Calculate aggregate consumption, labor, and bonds implied by the path of distributions and household policies. Then solve the government budget constraint for implied transfers, discretizing $\dot{B}$ in the gvt budget as $\frac{B_{t+1}-B_t}{\mathrm{d}t}$

10. Generate 3 vectors of residuals by verifying whether the implied paths of labor, bonds, and transfers from the previous step equal the guessed paths.

11. Now I solve the social planner's FOC's. First, iterate backwards in the same way as the household problem to solve for $\eta_{1,t}$ (equation 40). Use equation 51 to find the corresponding derivative.

12. Jointly solve the planner's first order conditions for individual choices and obtain η_7, η_8, η_9 (eqns 45-47). These are linear equations in the unknown multipliers that can be solved efficiently using sparse matrices.

13. Solve the planner's KFE (eq 41) forwards in the same way as the KFE to obtain $\eta_{2,t}$.

14. Solve the π FOC backwards for a path of $\eta_{3,t}$, setting $\dot{\eta}_{3,t} = \frac{\eta_{3,t+1}-\eta_{3,t}}{\mathrm{d}t}$.

15. Use the planner's first order condition for r_t and w_t (equation 43 and 44) and aggregates (equations 48-50) to calculate 4 vectors of residuals

16. If the 7 vectors of residuals (4 from verifying social planner FOCs hold, 3 from verifying competitive equilibrium conditions hold), stop. Otherwise update the guesses and repeat until convergence

I use a numerical solver in matlab to solve this system of $7 \cdot ns$ equations in $7 \cdot ns$ unknowns. The default algorithm converges quickly and reliably. All matrix inversions should be done with sparse matrices to speed up computation; otherwise computation time will be order(s) of magnitude slower.

Boundary Conditions At the borrowing constraint,

$$
\begin{aligned}
&0 \leq \dot{b} = rb + zwn + D + T - c^h - p(1+\tau)c^f \\
&c^h + p(1+\tau)c^f \leq rb + zwn + D + T \\
&c^h \left(1 + p(1+\tau)\Theta_{i,\tau}\right) \leq rb + zwn + D + T \\
&(c^h)^{-\sigma}\mathcal{C}_{i,\tau} \geq \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)^{-\sigma} \mathcal{C}_{i,\tau} \\
&u_h \geq \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)^{-\sigma} \mathcal{C}_{i,\tau} \\
&V_b \geq \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)^{-\sigma} \mathcal{C}_{i,\tau} \\
&V_b \geq u_h \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)
\end{aligned}
$$

where $u_h(c_h) \equiv c_h^{-\sigma}\mathcal{M}_{i,\tau}$

To evaluate the above formula when the inequality binds, need a solution for n. Labor supply at the *binding* boundary constraint $b = \phi$ is determined by

$$
\begin{aligned}
n^\kappa &= \frac{wz}{p^h}(c^h)^{-\sigma}\mathcal{C}_{i,\tau} \\
c^h &= \frac{r\phi + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}
\end{aligned}
$$

which is a quadratic equation under the assumption that $\kappa = \sigma = 1$

D Adjustment of Gvt Budget, Details

The variable that adjusts in the gvt's budget constraint affects equilibrium outcomes and (some of) the conditions to determine equilibrium, as Ricardian Equivalence does not hold. As a baseline, I assume T adjusts while B is a fixed fraction of GDP and τ_n is some fixed value (i.e. tariff revenue is redistributed to households as transfers). Below, I rewrite the different equilibrium conditions if instead τ_n or B adjust (tariff revenue is used to reduce the labor income tax or reduce the deficit while other variables are fixed at a certain level or fraction of GDP) and briefly describe how the computational algorithm changes. As the production subsidy s is designed to eliminate steady state markup distortions, I will not consider a case where it adjusts and instead always set it to $s = \frac{1}{1-\epsilon}$.

D.1 Adjusting Bonds

Here I assume that τ_n is a fixed number, T is some fraction of GDP ($T = T_{\text{perc}}Y$), and B adjusts to clear the government budget constraint. Since I will be able to eliminate one guess variable in the competitive equilibrium loop, I add the government budget constraint directly as an additional planner constraint with multiplier $\eta_{10,t}$ without affecting computation speed

$$0 = rB_t + T_{\text{perc}}N_t + sN_t - \tau pC_t^f - \tau^n wN_t - \dot{B}_t$$

The new savings function is

$$s^h = (r+\eta)b^h + (1-\tau^n)zwn + T_{\text{perc}}N_t + (1+s)N_t - m_tN_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f$$

The first order conditions for N_t, w_t, r_t, and C_t^f are affected and there is now an FOC for B_t; all others are unchanged.

The new FOC for N_t is

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial\eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\left(T_{\text{perc}} + (1+s) - m - \frac{\theta}{2}\pi_t^2\right)\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} + \eta_{10,t}(T_{\text{perc}} + s - \tau_n w)$$

The new FOC for C_t^f is

$$-\eta_{10,t}\tau p - \eta_{6,t} = 0$$

There is no integral containing marginal social and private values since $\frac{\partial s^h}{\partial C^f} = 0$

The new FOC for r_t is

$$0 = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial\eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)b\,\mathrm{d}b\,\mathrm{d}z + \pi_t\eta_{3,t} + B_t\eta_{10,t}$$

The new first order condition for w_t is

$$\eta_{3,t}\frac{\epsilon}{\theta} + \eta_{10,t}\tau_n N_t = \int_{z_1}^{z_J}\int_{\phi}^{\infty}((z_tn_t - N_t) - \tau z_t n_t)\left(\frac{\partial\eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right) - (1-\tau^n)zu_h(c^f, c^h)\eta_{7,t}\,\mathrm{d}b\,\mathrm{d}z$$

The first order condition for B_t is

$$0 = -\eta_{4,t} + \eta_{10,t}r - (\tilde{\rho}\eta_{10,t} - \dot{\eta}_{10,t})$$

where the final term is derived by integrating by parts over the time dimension and has initial condition $\eta_{10,0} = 0$

Now, instead of guessing N, r, and T to solve the competitive equilibrium and $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, and π_t to verify the planner FOC's, I only need to guess N and r to solve the competitive equilibrium but need to also guess $\eta_{10,t}$ to verify the planner FOC. Similarly, in the transitional dynamics, I guess a path for $\eta_{10,t}$ instead of T.

D.2 Adjusting Taxes

Here I assume that T and B are some constant fraction of GDP and τ_n adjusts to clear the government budget constraint. τ_n is then defined as

$$\tau_n(r, N, w, C^f) = \frac{rB_{\text{perc}}N + T_{\text{perc}}N + sN - \tau pC^f - B_{\text{perc}}\dot{N}}{wN}$$

The new savings function is

$$s^h = (r+\eta)b^h + (1-\tau_n(r, N, w, C^f))zwn + T_{\text{perc}}N_t + (1+s)N_t - m_tN_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f$$

The first order conditions for N_t, w_t, r_t, and C_t^f are affected; all others are unchanged. Since τ_n only appears in the savings rule, the general structure of the FOC are unchanged from the basic case, though the derivative of the savings function with respect to labor, foreign consumption, interest rate, and wages is different:

$$\begin{aligned}
\eta_{5,t} &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V}{\partial b}\eta_{2,t}\right)\frac{\partial s^h}{\partial N}\,\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} \\
\eta_{6,t} &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V}{\partial b}\eta_{2,t}\right)\frac{\partial s^h}{\partial C^f}\,\mathrm{d}b\,\mathrm{d}z \\
-\eta_{3,t}\pi_t &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V}{\partial b}\eta_{2,t}\right)\frac{\partial s^h}{\partial r}\,\mathrm{d}b\,\mathrm{d}z \\
\eta_{3,t}\frac{\epsilon}{\theta} &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial s^h}{\partial w} - zu_h(c^f, c^h)\eta_{7,t}\,\mathrm{d}b\,\mathrm{d}z
\end{aligned}$$

Where the savings derivatives are

$$
\begin{aligned}
\frac{\partial s^h}{\partial N} &= -\frac{\partial \tau_n}{\partial N} zwn + T_{\text{perc}} + (1+s) - w - \frac{\theta}{2}\pi_t^2 \\
\frac{\partial \tau_n}{\partial N} &= \frac{\tau p C^f}{wN^2} - \frac{B_{\text{perc}}}{wN}\left(\tilde{\rho} - \frac{\dot{f}}{f} - \frac{\frac{\partial \dot{\eta}_{1,t}}{\partial b}}{\frac{\partial \eta_{1,t}}{\partial b}}\right) \\
\frac{\partial s^h}{\partial C^f} &= -(\frac{-\tau p}{wN}) \\
\frac{\partial s^h}{\partial r} &= b - (\frac{B_{\text{perc}}}{w}) \\
\frac{\partial s^h}{\partial w} &= zn
\end{aligned}
$$

The computational algorithm for the steady state requires guessing τ_n, r, and N to solve for a competitive equilibrium. I guess the same multipliers and inflation to verify the planner's FOCs hold. Similarly in the transitional dynamics, I guess a path of τ_n instead of a path of T.

E Inverse Optimal Approach, Details

This section provides details on the inverse optimal approach I take to obtain an efficient steady state.

E.1 Model

The new Lagrangian is

$$
\begin{aligned}
\mathcal{L} = &\Big(\varphi(b,z)(u(c_t^h, c_t^f) - g(n_t)), f_t\Big)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t\right)_\Phi \\
&\left(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \big(\eta_{4,t}, f_t(b_t^h - B_t)\big)_\Phi + (\eta_{5,t}, f_t(n_t - N_t))_\Phi + \Big(\eta_{6,t}, f_t(c_t^f - C_t^f)\Big)_\Phi \\
&+ \left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\right.
\end{aligned}
$$

The FOC for c^h, c^f, n, and f differ. They are

$$\tilde{\rho}\eta_{1,t} = \varphi u(c_t^h, c_t^f) - \varphi g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(n_t - N_t) + \eta_{6,t}(c_t^f - C_t^f)$$

$$0 = \varphi u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) + \frac{\eta_{9,t}}{f_t} u_{ff}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hf}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{fh}(c_t^h, c_t^f) + \eta_{6,t}$$

$$0 = \varphi u_h(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b} p + \frac{\eta_{9,t}}{f_t} u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{hh}(c_t^h, c_t^f)$$

$$0 = -\varphi g'(n_t) + \frac{(1-\tau^n) w_t z_{j,t}}{p_t^h}\frac{\partial \eta_{1,t}}{\partial b} + \eta_{7,t}\frac{g''(n_t)}{f_t} + z\eta_{5,t}$$

Computation Recall that I define the weights as

$$\varphi(b,z) = \frac{u_h(C^h, C^f)}{u_h(c_i^h, c_i^f)} \tag{57}$$

where C^h, C^f are aggregate home and foreign consumption. First, I solve for the utilitarian planner steady state and obtain the welfare weights. I solve for the stationary equilibrium again, using these welfare weights. The allocations will be identical to the baseline, as inflation will remain 0 in the steady state, but planner multipliers will be different. I use these results to initialize and solve the transitional dynamics to the same shock as before. Crucially, planner welfare weights are constant and equal to those defined in equation 57.

F Endogenous Terms of Trade, Details

This section provides details on the model with endogenous terms of trade.

Derivation of Relative Price and Optimal Tariff The rest of the world is comprised of a continuum of countries $k \in (0,1)$ each with a representative household. Each country produces a single variety a la Armington aggregated into the (composite) foreign good with elasticity of substitution ϑ, i.e.

$$C_{k,t}^f = \left(\int (C_{j,k,t}^f)^{\frac{\vartheta-1}{\vartheta}}\,\mathrm{d}j\right)^{\frac{\vartheta}{\vartheta-1}}$$

where $C_{j,k,t}$ is the consumption of variety j by country k at time t. The representative foreign household in country k has a value function V_k given by

$$\rho V_{k,t}(B) = \max_{C_{k,t}^h, C_{j,k,t}, N_{k,t}} u(C_{k,t}^h, C_{k,t}^f) - g(N_{k,t}) + \frac{\partial V_{k,t}}{\partial B}\dot{B}_t + \frac{\partial V_{k,t}}{\partial t}$$

$$\dot{B}_t = W_{k,t}N_{k,t} + D_{k,t} + r_t B_{k,t} - P_{k,t}^h - \int P_{j,k,t}C_{j,k,t}$$

where home and foreign consumption are aggregated according to the CES aggregator

$$(C_{k,t})^{\frac{\gamma-1}{\gamma}} = \omega(C_{k,t}^h)^{\frac{\gamma-1}{\gamma}} + (1-\omega)\left(C_{k,t}^f\right)^{\frac{\gamma-1}{\gamma}}$$

with trade elasticity γ. Solving the household's problem, invoking the market clearing condition for the home produced good $N_t = C_t^h + C_t^{h*}$ (where C_t^{h*} is aggregated foreign consumption of the home produced good) yields equation 28 with the exogenous demand shifter $A = ((1-\omega)C_t^*)^{-\frac{1}{\vartheta}}$.

In this model, the usual static optimal tariff formula applies as foreign households are homogeneous, implying the foreign export supply function is only a function of the contemporaneous price (see section 2 of Dávila et al. (2025) for details). The first best tariff for the home country is then

$$\tau_{\text{opt}} = \frac{1}{\vartheta - 1}$$

In my numerical simulations, I will consider tariff shocks that deviate from this optimal benchmark.

F.1 Model

I express the price as $p_t = p(N_t, C_t^h)$. I now need to add an additional planner choice of C_t^h. The planner Lagrangian is

$$\mathcal{L} = \left(u(c_t^h, c_t^f) - g(n_t), f_t\right)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t\right)_\Phi$$

$$\left(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \left(\eta_{4,t}, f_t(b_t^h - B_t)\right)_\Phi + (\eta_{5,t}, f_t(n_t - N_t))_\Phi + \left(\eta_{6,t}, f_t(c_t^f - C_t^f)\right)_\Phi$$

$$+ \left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(N_t, C_t^h)(\right.$$

$$+ \left(\eta_{10,t}, f_t(c_t^h - C_t^h)\right)_\Phi$$

where $s^h = rb^h + (1-\tau^n)zwn + \dot{B} - rB - sN + \tau p(N_t, C_t^h)C_t^f + \tau^n wN_t + (1+s)N_t - m_tN_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(N_t, C_t^h)(1+\tau)c^f$.

There are different FOC for f_t, c_t^h, N_t, and a new FOC for C_t^h.

The FOC for f_t is

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_tn_t - N_t) + \eta_{6,t}(c_t^f - C_t^f) + \eta_{10,t}(c_t^h - C_t^h)$$

The FOC for c_t^h is

$$0 = u_h(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b} + \frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{(1-\tau^n)w_tz_{j,t}}{p_t^h}u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) + \eta_{10,t}$$

The FOC for N_t is

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\left(1 - m_t - \frac{\theta}{2}\pi_t^2 + \tau w + \frac{\partial p}{\partial N_t}\left(\tau C_t^f - (1+\tau)c^f\right)\right)\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} - \int_{z_1}^{z_J}\int_{\phi}^{\infty}\eta_{9,t}(\frac{\partial V}{\partial b_t^h}\frac{\partial p}{\partial N}(1+\tau))\,\mathrm{d}b\,\mathrm{d}z$$

The FOC for C_t^h is

$$\eta_{10,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial p}{\partial C_t^h}\left(\tau C_t^f - (1+\tau)c^f\right) - \eta_{9,t}(\frac{\partial V}{\partial b_t^h}\frac{\partial p}{\partial C^h}(1+\tau))\,\mathrm{d}b\,\mathrm{d}z$$

The new multiplier $\eta_{10,t}$ reflects how marginal changes in home consumption affect the relative price. The first order condition for aggregate labor and home consumption additionally reflect how the planner optimally manipulates terms of trade, accounting for how the consumption bundle of the household changes ($\frac{\partial p}{\partial C_t^h}(1+\tau)c^f$), how the change in the relative price affects the transfer the household receives ($\frac{\partial p}{\partial C_t^h}\tau C^f$), and how the change in the relative price directly affects the household's FOC for foreign consumption ($\eta_{9,t}\frac{\partial V}{\partial b_t^h}\frac{\partial p}{\partial C^h}(1+\tau)$)

F.2 New Planner Rule

I now derive a new planner targeting rule that illustrates the terms of trade motive. Promise keeping values become

$$\begin{aligned}
\mathcal{H}_t^f &\equiv -\frac{\eta_{9,t}}{f_t}u_{ff}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_tz_{j,t}(1-\tau^n)}{p_t^h}u_{hf}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{fh}(c_t^h, c_t^f) \\
\mathcal{H}_t^h &\equiv -\frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_tz_{j,t}(1-\tau^n)}{p_t^h}u_{hh}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) \\
\mathcal{H}_t^n &\equiv -\eta_{7,t}\frac{g''(n_t)}{f_t}
\end{aligned}$$

In the following derivations, I follow the proof of Proposition 3 in appendix B. More details can be found there. I obtain two equations, combining planner first order conditions exactly as in the baseline model

$$\eta_{6,t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s}{\partial T} \frac{\partial T}{\partial C^f} \left[\frac{u_f}{p} - \frac{\mathcal{H}_t^f}{p} + \frac{u_f}{p} \frac{\eta_{2,t}}{f_t} \right] f_t \, \mathrm{d}b \, \mathrm{d}z$$

$$\begin{aligned}
\eta_{5,t} = & \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s^h}{\partial N} \left[u_h + \frac{\mathcal{H}_t^n}{(1-\tau^n) w z} + u_h \frac{\eta_{2,t}}{f_t} \right] f_t \, \mathrm{d}b \, \mathrm{d}z - \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s^h}{\partial N} \frac{\eta_{5,t}}{z w_t (1-\tau^n)} \, \mathrm{d}b \, \mathrm{d}z \\
& - \frac{\partial p}{\partial N} \int_{z_1}^{z_J} \int_{\phi}^{\infty} \eta_{9,t} \left(\frac{\partial V}{\partial b_t^h} (1+\tau) \right) \mathrm{d}b \, \mathrm{d}z + \frac{1}{N_t} \left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} \right) \int_{z_1}^{z_J} \int_{\phi}^{\infty} (B_t - b_t^h) \left[u_h - \frac{\mathcal{H}_t^f}{p(1+\tau)} + u_h \frac{\eta_{2,t}}{f_t} \right] f_t
\end{aligned}$$

I then define Ω_1 and Ω_2 in the same way as before.

$$\begin{aligned}
\Omega_1 &= \eta_{6,t} + \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s}{\partial T} \frac{\partial T}{\partial C^f} \frac{\mathcal{H}_t^f}{p} f_t \, \mathrm{d}b \, \mathrm{d}z \\
\Omega_2 &= \eta_{5,t} \left(1 + \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s^h}{\partial N} \frac{1}{z w_t (1-\tau^n)} f_t \, \mathrm{d}b \, \mathrm{d}z \right) + \frac{\partial p}{\partial N} \int_{z_1}^{z_J} \int_{\phi}^{\infty} \eta_{9,t} \left(\frac{\partial V}{\partial b_t^h} (1+\tau) \right) \mathrm{d}b \, \mathrm{d}z \\
&+ \int_{z_1}^{z_J} \int_{\phi}^{\infty} \left[\frac{1}{N_t} \left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} \right) (B - b^h) \frac{\mathcal{H}_t^f}{p(1+\tau)} - \frac{\partial s^h}{\partial N} \frac{\mathcal{H}_t^n}{w_t z_t (1-\tau^n)} \right] \mathrm{d}b \, \mathrm{d}z
\end{aligned}$$

After some algebra and simplifications, I obtain the targeting rule in the main body.

G Progressive Transfers, Details

This section provides details on the model with progressive transfers.

G.1 Model

The new government budget constraint is

$$\dot{B}_t = r B_t + s p_t^h Y_t + \int \mathcal{T}(b_t) \, \mathrm{d}f(b,z) - \tau p^f \int c_f \, \mathrm{d}f(b,z) - \tau_n w_t \int n z \, \mathrm{d}f(b,z)$$

The new savings function is

$$s^h = (r+\eta) b + (1-\tau^n) z w n + \mathcal{T}(b) + (1+s) N_t - m_t N_t - \frac{\theta}{2} \pi_t^2 N_t - c^h - p(1+\tau) c^f$$

I then need to add the government budget/transfer clearing as a new planner condition with multiplier $\eta_{10,t}$, so the planner Lagrangian is

$$\mathcal{L} = \left(u(c_t^h, c_t^f) - g(n_t), f_t\right)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t\right)_\Phi$$
$$\left(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \left(\eta_{4,t}, f_t(b_t^h - B_t)\right)_\Phi + \left(\eta_{5,t}, f_t(n_t - N_t)\right)_\Phi + \left(\eta_{6,t}, f_t(c_t^f - C_t^f)\right)_\Phi$$
$$+ \left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\right)$$
$$+ \left(\eta_{10,t}, f_t(\mathcal{T}(b_t) - \dot{B}_t + rB_t + sp_t^h N_t - \tau p^f C^f - \tau_n w_t N_t)\right)_\Phi$$

adding $\mathcal{T}(b)$ as a planner choice (since it cannot be substituted for anymore in the household budget). Planner choices of r, N, C^f, w, and f are different than in the baseline model; other choices are unaffected.

The planner FOC for f is

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{9,t}(c_t^f - C_t^f) + \eta_{10,t}(\mathcal{T}(b_t) - T_t)$$

where $T_t = \dot{B}_t - rB_t - sp_t^h N_t + \tau p^f C^f + \tau_n w_t N_t$

The planner FOC for N is

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial s^h}{\partial N}\,\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} + \eta_{10,t}(s - \tau_n w)$$

The planner FOC for C^f is

$$\eta_{6,t} = -\eta_{10,t}\tau p$$

The planner FOC for r_t is

$$0 = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right) b\,\mathrm{d}b\,\mathrm{d}z + \pi_t\eta_{3,t} + B_t\eta_{10,t}$$

The planner FOC for w_t is

$$\eta_{3,t}\frac{\epsilon}{\theta} + \eta_{10,t}\tau_n N_t = \int_{z_1}^{z_J}\int_{\phi}^{\infty}((z_t n_t - N_t) - \tau z_t n_t)\left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right) - (1-\tau^n) z u_h(c^f, c^h)\eta_{7,t}\,\mathrm{d}b\,\mathrm{d}z$$

The planner FOC for $\mathcal{T}(b)$ is

$$\left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial s^h}{\partial \mathcal{T}(b)} = -f_t\eta_{10,t} \tag{58}$$

with $\frac{\partial s^h}{\partial \mathcal{T}(b)} = 1$.

The FOC for r, N, C^f, w in this formulation can be re-expressed as exactly the FOC in the baseline formulation of the model with non-progressive transfers. I integrate eq 58 over b and z and substitute for $\eta_{10,t}$ in those 4 FOC. The planner HJB is the only FOC that has a different expression as long as transfers are not flat, i.e. $\mathcal{T}(b) = T \quad \forall b$ does not hold. Then, the social value function, $\eta_{1,t}$, and its derivative, $\frac{\partial \eta_{1,t}}{\partial b}$, are different.

H Intermediate Inputs, Details

In the baseline model, tariffs only distort consumption. They do not enter into the problem of the firm and do not distort production decisions. Now I assume that there are two types of goods that are imported (and subject to tariffs), a foreign produced consumption good as before and foreign-produced intermediate inputs that must be combined with labor to produce the home consumption good.

H.1 Model

Here, I discuss the changes in the economic environment. The problem of the household is the same as in the baseline model. The problem of the final good producer is the same as in the baseline model. The problem of the intermediate firms differs. Each intermediate firm $j \in (0,1)$ produces according to $y_{jt} = x_{jt}^{\alpha} n_{jt}^{1-\alpha}$ where x_{jt} is the quantity of intermediate inputs used by firm j at time t. To determine the optimal mix of inputs, the firm solves the following static problem

$$\min_{x_{jt}, n_{jt}} w_t n_{jt} + p_{xt}(1+\tau^x) x_{jt}$$
$$\text{s.t.} y_{jt} = x_{jt}^{\alpha} n_{jt}^{1-\alpha}$$

Real marginal costs are then

$$m_t = \left(\frac{w_t}{(1-\alpha)}\right)^{1-\alpha} \left(\frac{p_t^x (1+\tau^x)}{\alpha}\right)^{\alpha}$$

where p_t^x is the relative price of the intermediate input and τ_t^x is the tariff on the intermediate input. Going forward, I follow Bianchi and Coulibaly (2025) and Werning et al. (2025) and assume that $p_t^x = p_t = p$ (i.e. the intermediate input and foreign consumption good are perfect substitutes and both relative prices are constant). In my numerical exercises, I will additionally assume that $\tau_t = \tau_t^x$ (the foreign consumption good and intermediate input are tariffed at the same rate). The Phillips curve is the same as in the baseline model. Real dividends are then given by

$$D_t = (1+s)Y_t - w_t N_t - p(1+\tau_t^x)X_t - \Theta(\pi)$$

The government budget is given by

$$\dot{B}_t = rB_t + T_t + sp_t^h Y_t - \tau p^f \int c_f \,\mathrm{d}f(b^h, b^f, z) - \tau_n \int nz \,\mathrm{d}f(b^h, b^f, z) - \tau_t p X_t$$

Finally, the goods market clearing is

$$Y_t = \frac{\theta}{2}\pi_t^2 Y_t + C_t^h + pC_t^f + pX_t$$

H.2 Policy Problem

The solution to the optimal policy problem can be obtained by maximizing the following Lagrangian functional with respect to the functions $\pi_t, r_t^h, V_t(.), c_t^f(.), c_t^h(.), n_t(.), f_t(.), w_t,$ N_t, C_t^f, X_t, Y_t.

$$\mathcal{L} \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_\phi^\infty \left[u(c_t^h, c_t^f) - g(n_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$
$$\eta_{2,t}(b,z)\left[u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r_t^h - \alpha\frac{\dot{X}_t}{X_t} - (1-\alpha)\frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right] +$$
$$\eta_{4,t}[b_t^h - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c_t^f - C_t^f]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h, c_t^f)\right]$$
$$\eta_{8,t}(b,z)\left[u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}p_t^h\right] + \eta_{9,t}(b,z)\left[u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}p_t^h(1+\tau)\right] + \eta_{10,t}[p(1+\tau) - m_t X_t^{\alpha-1}N_t^{1-\alpha}\alpha]\,\mathrm{d}b\,\mathrm{d}$$

where after substituting in for lump sum dividends and the government budget, the savings function is $s^h = rb^h + (1-\tau^n)zwn + \dot{B} - rB - sX_t^\alpha N_t^{1-\alpha} + \tau p(C_t^f + X_t) + \tau^n w N_t + (1+s)X_t^\alpha N_t^{1-\alpha} - w_t N_t - p(1+\tau)X_t - \frac{\theta}{2}\pi_t^2 X_t^\alpha N_t^{1-\alpha} - c^h - p(1+\tau)c^f$. Relative to the baseline model, there is now a choice of X_t, pinned down by the first order condition for X_t. In the

Phillips curve, the term $\frac{\dot{Y}_t}{Y_t}$ can be re-expressed as

$$\begin{aligned}\frac{\dot{Y}}{Y} &= \frac{\frac{\mathrm{d}}{\mathrm{d}t}X_t^{\alpha}N_t^{1-\alpha}}{Y_t} \\ &= \frac{\alpha X_t^{\alpha-1}N_t^{1-\alpha}\dot{X}_t + (1-\alpha)X_t^{\alpha}N_t^{-\alpha}\dot{N}_t}{Y_t} \\ &= \alpha\frac{\dot{X}_t}{X_t} + (1-\alpha)\frac{\dot{N}_t}{N_t}\end{aligned}$$

The first order conditions for V, f, c^h, c^f, n, C^f, and r are identical to the baseline model.

The first order condition with respect to π is now

$$\dot{\eta}_{3,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)Y_t\theta\pi_t\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,t}(\tilde{\rho} - r + \frac{\dot{Y}_t}{Y_t})$$

The first order condition with respect to w is now

$$\eta_{3,t}\frac{\epsilon}{\theta}\frac{\partial m}{\partial w} + \eta_{10,t}\frac{\partial m}{\partial w}\frac{\partial Y}{\partial X} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}(1-\tau^n)(z_t n_t - N_t)\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right) - (1-\tau^n)zu_h(c^f, c^h)\eta_{7,t}\,\mathrm{d}b\,\mathrm{d}z$$

The first order condition with respect to N is now

$$\begin{aligned}\eta_{5,t} = &\int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\left(\frac{\partial Y}{\partial N}\left(1 - \frac{\theta}{2}\pi_t^2 - m\right) + \tau^n w\right)\mathrm{d}b\,\mathrm{d}z + \\ &(1-\alpha)\frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} - \eta_{10,t}m_t\frac{\partial^2 Y}{\partial X \partial N}\end{aligned}$$

Finally, the first order condition with respect to X is

$$\eta_{10,t}m_t\frac{\partial^2 Y}{\partial X^2} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\left(\frac{\partial Y}{\partial X}\left(1 - \frac{\theta}{2}\pi_t^2 - m\right) + \tau p\right) + \alpha\frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{X_t}$$

The computational approach is mostly the same. I explain the algorithm's differences for the dynamics. Now, I guess paths of Y, r, η_4, η_5, η_6, and π. To solve the firm's problem, I solve the Phillips curve for paths of m, the marginal cost expression for w, the X first order condition for X, then use the definition of Y to obtain N. Solving the household's problem, government budget, KFE, and market clearing proceeds as standard. Verifying the planner FOCs hold proceeds as in the baseline model, solving for η_{10} after solving for η_3, the proceeding to check the remaining residuals.

I Abstracting From Redistribution, Details

I.1 Model

The planner's new Lagrangian is

$$\mathcal{L} = \Big(u(C_t^h, C_t^f) - g(N_t), f_t\Big)_\Phi + \Big(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\Big)_\Phi + \Big(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A} V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho} V_t\Big)_\Phi$$

$$\Big(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\Big)_{\mathbb{R}^+} + \big(\eta_{4,t}, f_t(b_t^h - B_t)\big)_\Phi + (\eta_{5,t}, f_t(n_t - N_t))_\Phi + \Big(\eta_{6,t}, f_t(c_t^f - C_t^f)\Big)_\Phi$$

$$+ \Big(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n) w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\Big)_\Phi + \Big(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\Big)_\Phi + \Big(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\Big)$$

$$+ \big(\eta_{10,t}, f_t(c_t^h - C_t^h)\big)_\Phi$$

and will have modified first order conditions for C^h, C^f, N, c^h, c^f, n, and f

The first order condition with respect to f_t yields the planner's HJB

$$\tilde{\rho}\eta_{1,t} = u(C_t^h, C_t^f) - g(N_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{6,t}(c_t^f - C_t^f) + \eta_{10,t}(c_t^h - C_t^h)$$

The first order condition with respect to C^h is

$$u_h(C_t^h, C_t^f) = \eta_{10,t}$$

(C^h enters nowhere in the generator or savings rule s^h)

The first order condition with respect to C^f is

$$\eta_{6,t} = u_f(C_t^h, C_t^f) + \int_{z_1}^{z_J} \int_\phi^\infty \left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right) \frac{\partial s^h}{\partial C_t^f} \,\mathrm{d}b\,\mathrm{d}z$$

The first order condition with respect to N is

$$\eta_{5,t} = -g'(N_t) + \int_{z_1}^{z_J} \int_\phi^\infty \left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right) \frac{\partial s^h}{\partial N_t} \,\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t}$$

The first order condition with respect to individual choices c^h, c^f, and n remain the partial derivatives of the planner's HJB, augmented with terms corresponding to "promise-keeping" constraints $\eta_{7,t}, \eta_{8,t}, \eta_{9,t}$:

$$0 = \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) + \frac{\eta_{9,t}}{f_t} u_{ff}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hf}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{fh}(c_t^h, c_t^f) + \eta_{6,t}$$

$$0 = -\frac{\partial \eta_{1,t}}{\partial b} p + \frac{\eta_{9,t}}{f_t} u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{hh}(c_t^h, c_t^f) + \eta_{10,t}$$

$$0 = \frac{(1-\tau^n) w_t z_{j,t}}{p_t^h}\frac{\partial \eta_{1,t}}{\partial b} + \eta_{7,t}\frac{g''(n_t)}{f_t} + z\eta_{5,t}$$

General issues to fix: endogenous relative price, new formula; aggregate foreign supply of bonds (auclert paper or intermediary); welfare decomp; pure monetary and pure fiscal shock; 0 welfare cost of inflation a la Bianchi; redistribute *just* what you spent (hard computationally, but would be cool)

J Refunds, Details

The planner Lagrangian is

$$
\begin{aligned}
\mathcal{L} = & \left(u(c_t^h, c_t^f) - g(n_t), f_t\right)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho} V_t\right)_\Phi \\
& \left(\eta_{3,t}, \pi_t\left(r_t^h - \frac{\dot{N}_t}{N_t}\right) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \left(\eta_{4,t}, f_t(b_t^h - B_t)\right)_\Phi + \left(\eta_{5,t}, f_t(n_t - N_t)\right)_\Phi + \left(\eta_{6,t}, f_t(c_t^f - C_t^f)\right)_\Phi \\
& + \left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n) w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\right) \\
& + \left(\eta_{10,t}, (-\dot{B}_t + rB_t + T^{\text{base}} s p_t^h N_t - \tau_n w_t N_t)\right)_\Phi
\end{aligned}
$$

Planner choices of r, N, C^f, c^f, and w are different than in the baseline model; other choices are unaffected. The new savings function is

$$
s^h = (r+\eta)b + (1-\tau^n)zwn + T^{\text{perc}}N + T^{\text{refund}}(b,z) + (1+s)N_t - m_t N_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f
$$

Relative to the model where B adjusts, only 2 FOC are meaningfully different. First for C^f

$$
\eta_{6,t} = 0
$$

now for c^f

$$
\begin{aligned}
0 = & \left(u_f(c^h, c^f) - p\frac{\partial \eta_1}{\partial b} + \eta_6\right) f_t + \left(\tau \frac{\partial V}{\partial b}\right)\eta_{2,t} + \\
& \eta_{9,t} u_{ff}(c_t^h, c_t^f) - \eta_{7,t} \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hf}(c_t^h, c_t^f) + \eta_{8,t} u_{fh}(c_t^h, c_t^f)
\end{aligned}
$$

There are two noteworthy differences. The marginal social value of wealth is only multiplied by p rather than $p(1+\tau)$ and there is an additional penalty $\left(\tau p \frac{\partial V}{\partial b}\right)\eta_{2,t}$. Both reflect that the planner and household perceive the cost of foreign consumption differently. The planner internalizes that tariff spending is refunded, while the household does not. The remaining FOC are written out in appendix D where I solve the model with bonds adjusting

J.1 Computation

The computation of the household's problem is now somewhat more difficult. I need to find the value of T^r (that households take as given) that satisfies $T^r = \tau p c^f$ where c^f is itself a function of T^r. Given the functional forms I use, this problem has a closed form solution. The household's labor supply is given by $\hat{a}n^2 + \hat{b}(T^r)n + \hat{c} = 0$ where only $\hat{b}(T^r)$ depends on T^r. Then,

$$
\begin{aligned}
T^r &= \tau p c^f \\
T^r &= \tau p \Theta_h c^h \\
T^r &= \tau p \Theta_h \left(\frac{rb + (1-\tau^n) z w \left(\frac{-\hat{b}(T^r) + \sqrt{\hat{b}(T^r)^2 - r\hat{a}\hat{c}}}{2\hat{a}} \right) + D + T^{\text{base}} + T^r}{1 + p(1+\tau)\Theta_h} \right)
\end{aligned}
$$

After some algebra, this radical equation can be re-expressed as a quadratic, which can them be solved in closed form (taking the positive root). I can then proceed with the solving of the households' problem as usual.

Solving the planner's first order conditions is also more complicated, due to the addition of $\eta_{2,t}$ in the first order condition for c^f. Usually, I solve the 3 planner individual choice FOC for the multipliers on the household private FOC, then solve for the KFE for $\eta_{2,t}$. This, however doesn't work since the private FOC depend on $\eta_{2,t}$ now. This equation is non-linear (due to derivatives of $\eta_{8,t}$ appearing). It turns out, rather than using a solver (which is prohibitively slow), starting w/ an initial guess of η_2 and naively iterating on the planner foc for private households and the planner KFE has good convergence properties in practice (i.e. guess η_2, solve for η_7, η_8, η_9, solve the KFE for η_2, then update the guess of η_2 with the solution of the KFE until convergence). Note that this technique is only necessary in the stationary equilibrium. In the transition dynamics, $\eta_{2,t}$ has an initial condition, which allows me to find $\eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ since they depend on the (known) contemporaneous $\eta_{2,t}$. Then, the KFE gives $\eta_{2,t+1}$. Iterating forwards solve the system.

K RANK, Details

Here, I explicitly characterize the first order conditions for optimal policy in the RANK limit of my economy. In particular, the RANK limit I consider is one with the death rate $\eta = 0$, the borrowing limit set to the natural borrowing limit, and the standard deviation of the idiosyncratic shock equal to zero.[21] This implies the density is a point mass, individual labor supply is equal to aggregate labor supply, individual foreign consumption is equal to aggregate foreign consumption, and individual bond holdings are equal to aggregate bond holdings. I rewrite the Lagrangian of the planner with multipliers indexed by λ and time subscripts omitted for notational simplicity:

$$\mathcal{L} = \int_0^\infty e^{-\rho t}\{u(C^h, C^f) - g(N) + \lambda_1 \left[C^h \frac{1}{\sigma}(r-\rho) - \dot{C}^h\right] + \lambda_3 \left[g'(N) - u_h(C^h, C^f)(1-\tau_n)w\right]$$
$$+ \lambda_2 \left[C^f - C^h \Theta_\tau\right] + \lambda_4 \left[\pi_t(r - \frac{\dot{N}}{N}) - \frac{\epsilon}{\theta}(w-1) - \dot{\pi}\right] + \lambda_5 \left[N - \frac{\theta}{2}\pi^2 N - C^h - p(1+\tau)C^f + T(C^f)\right]\}$$

The first three planner constraints (λ_1, λ_2, λ_3) are household optimality conditions: the Euler equation w/ risk aversion coefficient σ, the distribution of home and foreign consumption with $\Theta_\tau = \left(\frac{1-\omega}{\omega(1+\tau)p}\right)^\gamma$, and the intratemporal labor/leisure choice. The final two conditions (λ_4 and λ_5) are the Phillips Curve and goods market clearing. To make the fiscal externality more explicit in the derivation that follows, I follow Bianchi and Coulibaly (2025) and define the (rebated) fiscal revenue as $T(C^f) = p\tau C^f$.

Again, this Lagrangian is a functional, so I take Gâteaux derivatives with respect to π,

[21] Since the generator $\mathcal{A}$ would be non-invertible and f would be a point mass, it is easier computationally to solve an explicit RANK representation of the model rather than the implicit representation with the idiosyncratic shock, death rate, and borrowing limit removed.

r, w, N, C^h, and C^f to generate planner first order conditions. The FOC are

$$\lambda_4\left(r-\frac{\dot{N}}{N}-\rho\right)+\dot{\lambda}_4=\lambda_5\theta\pi N$$

$$\frac{\lambda_1 C^h}{\sigma}=-\lambda_4\pi$$

$$\lambda_4\frac{\epsilon}{\theta}=-\lambda_3\left(u_h(C^h,C^f)(1-\tau_n)\right)$$

$$g'(N)=\lambda_3 g''(N)+\lambda_5(1-\frac{\theta}{2}\pi^2)+\frac{\dot{\lambda}_4\pi-\lambda_4\pi\rho+\dot{\pi}\lambda_4}{N}$$

$$0=u_h(C^h,C^f)+\lambda_1\left[\frac{1}{\sigma}(r-\rho)\right]-\rho\lambda_1+\dot{\lambda}_1+\lambda_2\left[-\Theta_\tau\right]+\lambda_3\left[-u_{hh}(C^h,C^f)(1-\tau_n)w\right]-\lambda_5$$

$$0=u_f(C^h,C^f)+\lambda_2+\lambda_3\left[-u_{hf}(C^h,C^f)(1-\tau_n)w\right]-\lambda_5 p(1+\tau)+\lambda_5\frac{\partial T}{\partial C^f}$$

Like in HANK, multipliers λ_1, λ_2, λ_3 are promise-keeping values. The planner may prefer to not obey private first order conditions; these multipliers/penalties force him to do so. Consider the first order condition on foreign consumption. When choosing the optimal level of foreign consumption, the planner trades off the marginal utility gain u_f, the marginal cost to the household of foreign consumption $p(1+\tau)$, the marginal effect on the slackness of household first order conditions (λ_2 and λ_3 terms), and the fiscal revenue gained $\frac{\partial T}{\partial C^f}$. Since λ_5 is the multiplier on the resource constraint, I interpret λ_5 as the *social* marginal value of wealth. For the private household, the first order condition for foreign consumption is $u_f(C^h,C^f)=\tilde{\lambda}p(1+\tau)$ where $\tilde{\lambda}$ is the multiplier on the household budget constraint or the *private* marginal value of wealth. Unlike the household, the planner internalizes how changes in foreign consumption affect transfers, valuing them according to $\lambda_5\frac{\partial T}{\partial C^f}>0$. This is the fiscal externality. Since this term is positive, the planner will want higher foreign consumption.

This intuition extends directly to HANK. First, I will rewrite the planner's FOC for foreign consumption in RANK by condensing the "promise-keeping" terms

$$0=u_f(C^h_t,C^f_t)-\mathcal{H}^{f,RA}_t-\lambda_{5,t}p(1+\tau_t)+\lambda_{5,t}\frac{\partial T}{\partial C^f}$$

Then, in HANK, I combine the first order conditions for individual and aggregate foreign consumption, defining $\mathcal{H}^f_t$ as in appendix B.

$$0=\int_{z_1}^{z_J}\int_{\phi}^{\infty}\left[u_f(c^h_t,c^f_t)-\mathcal{H}^f_t-\frac{\partial\eta_{1,t}}{\partial b}p(1+\tau)+\frac{\partial s^h}{\partial T}\frac{\partial T}{\partial C^f}\left(\frac{\partial\eta_{1,t}}{\partial b}+\frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right)\right]f_t\,\mathrm{d}b\,\mathrm{d}z$$

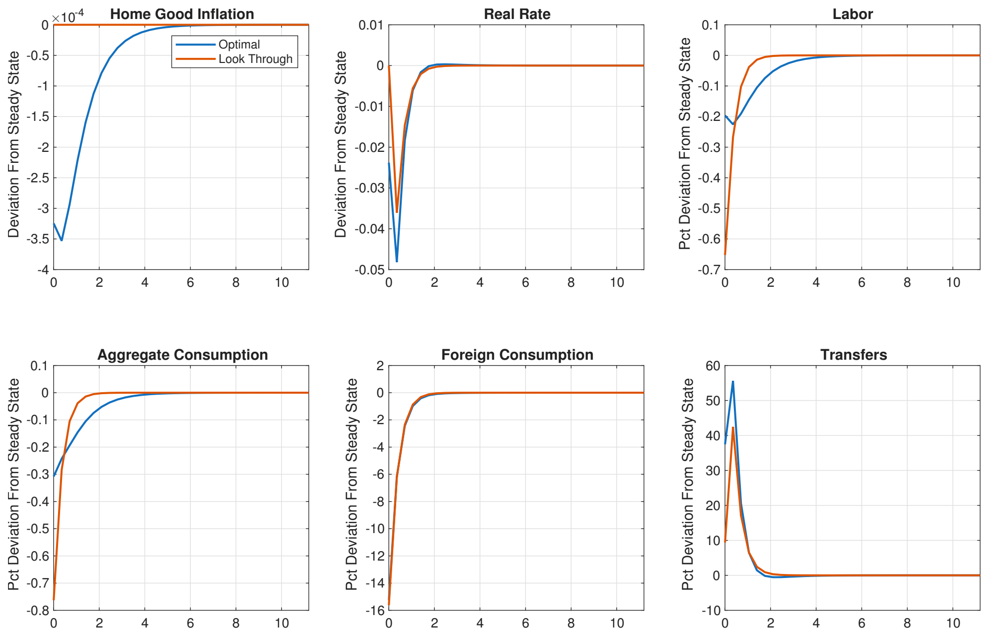
One can show these equations are identical when the HANK economy sets the death rate to 0, the variance of the idiosyncratic shock to 0, and the borrowing limit to its natural value (the RANK limit). When choosing the optimal level of foreign consumption in HANK, the planner still trades off the same benefits and costs as in RANK, in HANK he simply values these benefits differently. In RANK $\frac{\partial s^h}{\partial T} = 1$; in HANK this need not be the case. Similarly, $\frac{\partial \eta_{1,t}}{\partial b}$ is heterogeneous across HANK households (and the planner must account for the distributional penalties), the analogous marginal social value of wealth $\lambda_{5,t}$ is homogeneous in RANK.

K.1 Numerical Results

The 6 planner FOC, the 3 household FOC, the Phillips curve, and market clearing jointly determine the values of all planner multipliers and the 6 economic variables. For computation, it is sufficient to guess (paths) for r, π, and λ_1. Given these guesses, it is straightforward to back out all other unknowns and then use the remaining 3 equations as residuals.

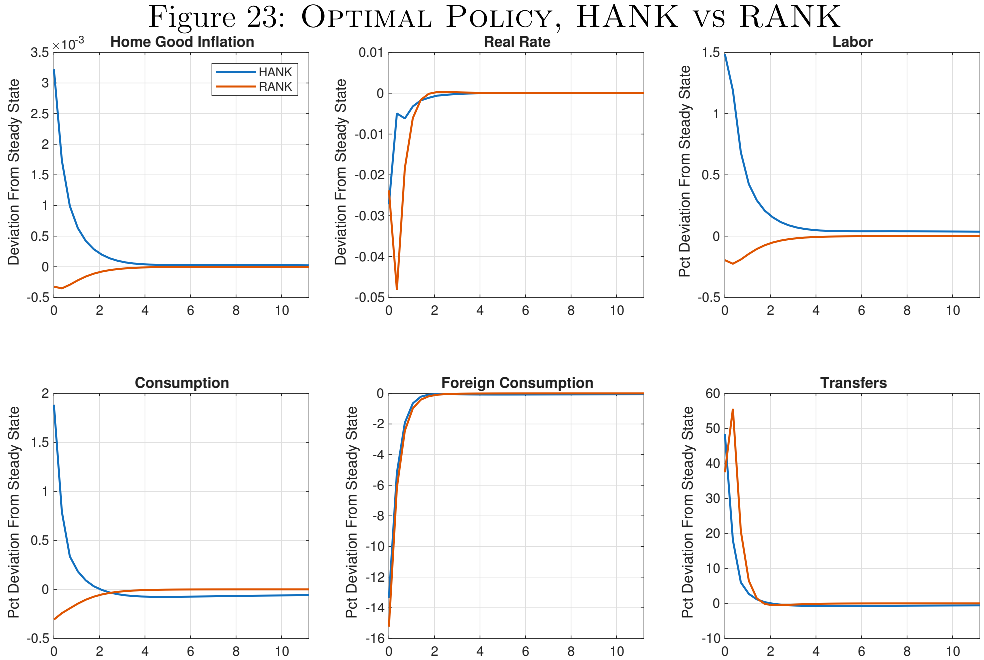
Figure 22 plots the impulse response for interest rates, inflation, aggregate and foreign consumption, labor, and transfers for the same shock as in figure 1 for the RANK model under the optimal policy response and the look-through response. Though I still see (slightly) less of a decrease in foreign consumption and an increase in transfers in line with the logic of Bianchi and Coulibaly (2025), I do not see the optimally inflationary response that they do since I assume the interest rate is endogenous, rather than set to $r^* = \rho$ as they do. However, the general intuition that the fiscal externality incentivizes the planner to have higher output and a stronger rate cut to induce higher consumption to counteract the effects of the tariff remains in my model.

Figure 22: Optimal vs Look Through Policy, RANK



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables in RANK. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state. 3

Figure 23: Optimal Policy, HANK vs RANK



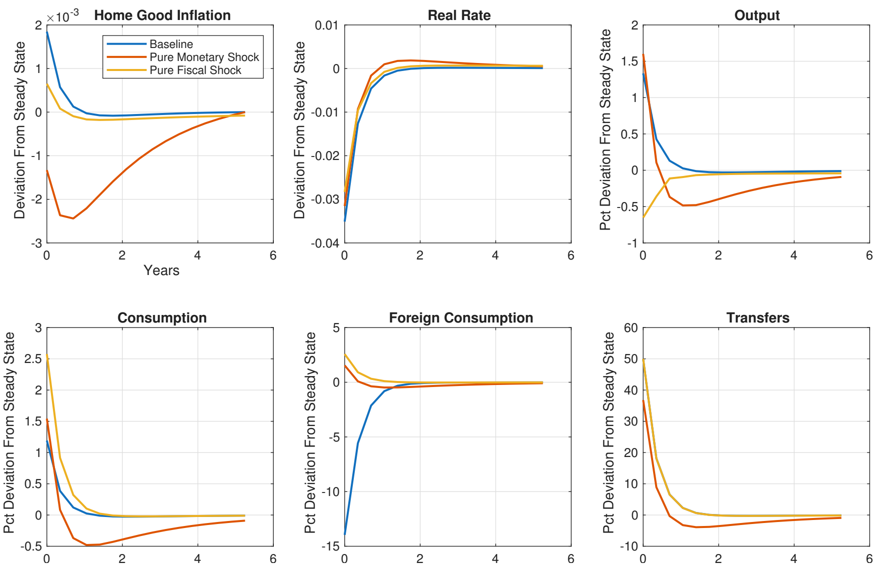
L Decomposition

The overall response to a tariff shock can thought of as a combination of 3 separate parts: the direct effect of the tariffs, the indirect effect of the monetary response, and the indirect effect of the fiscal response. To understand separately the strength of both the monetary and fiscal mechanisms in my model, I do 2 sets of experiments, in each keeping the tariff level to 0:

1. I take as given the optimal path of *nominal* rates from the solution of the baseline model to the tariff shock in figure 1. I then guess paths of r_t, π_t, and N_t that clear the bond and labor markets and satisfy the Fisher equation.

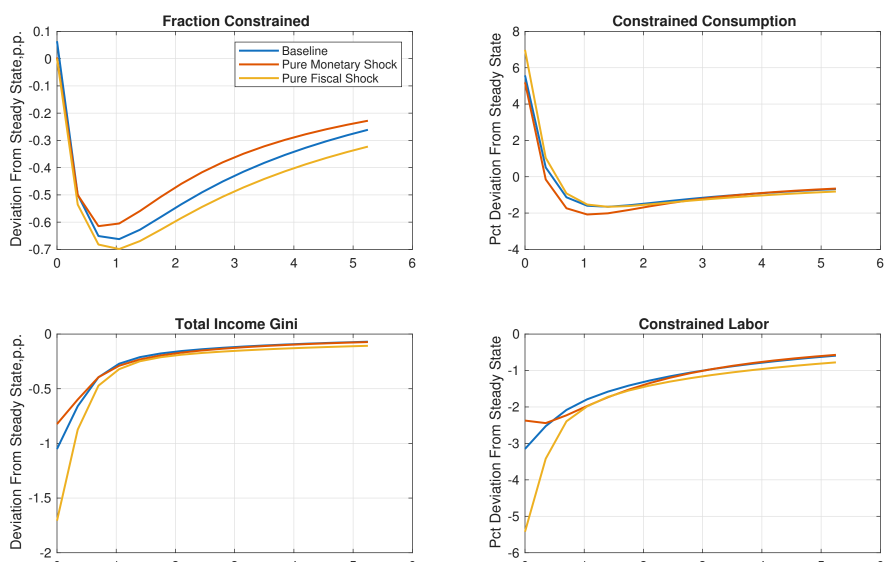
2. I take as given the optimal path of transfers from the solution of the baseline model. I then solve for the optimal monetary response to this "fiscal shock", allowing the debt to adjust to satisfy the government budget (the planner FOC are almost identical to the ones where B adjusts, save for the transfer specification). In order to keep the supply of debt (that the households must purchase) consistent with the baseline version of the model, I assume that the foreign debt is purchased by the rest of the world.

Figure 24: Tariff Shock with Imported Inputs, Aggregate Variables



Notes: I plot impulse responses to aggregate variables for the monetary and fiscal shock outlined above. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

Figure 25: Tariff Shock with Imported Inputs, Distributional Variables



Notes: I plot impulse responses for distributional variables for the monetary and fiscal shock outlined above.

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