A dynamic network analysis of ancient Roman social complexity

· *work in progress* ·

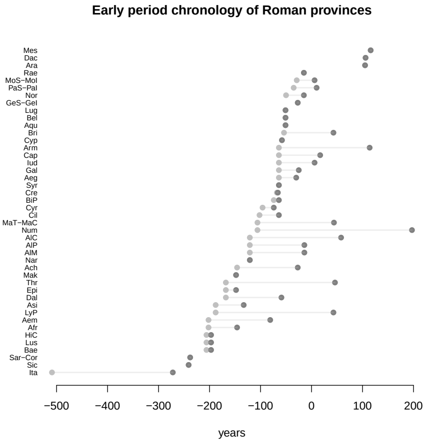
Antonio Rivero Ostoic
School of Culture and Society

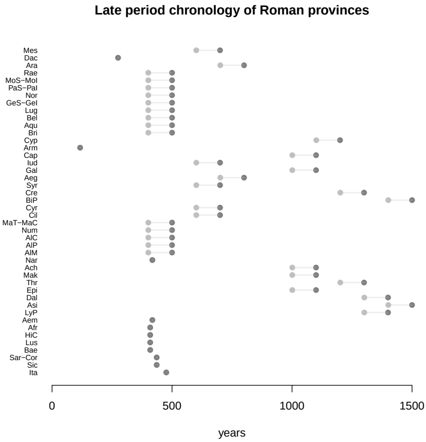
Networks 2021 Conference ☆ 9 July 2021

AARHUS UNIVERSITY

Ancient Roman provinces historical periods

white time intervals



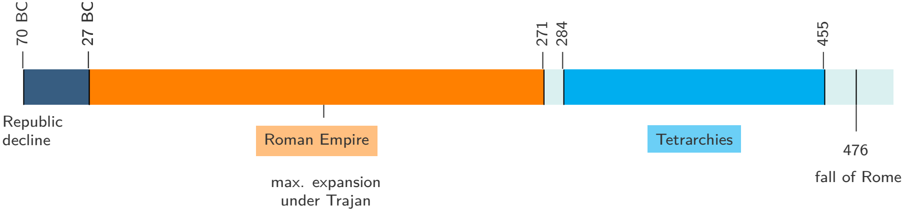



```
sdam::plot.dates(x, taq, tpq, ...)
```

based on www.unrv.com/provinces/

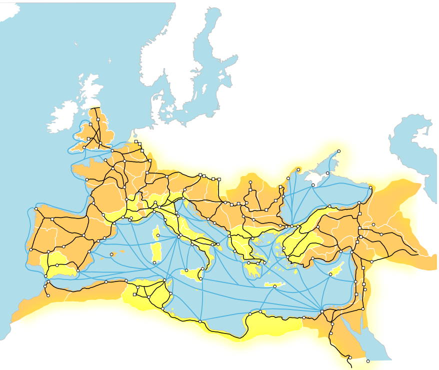
Chronological periods of ancient Rome

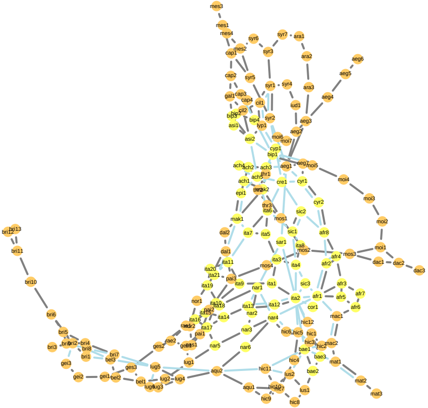
example with a 546 years range of time



Roman Empire transport network (maximum extent ca. 117 AD)

main roads & maritime routes and province types





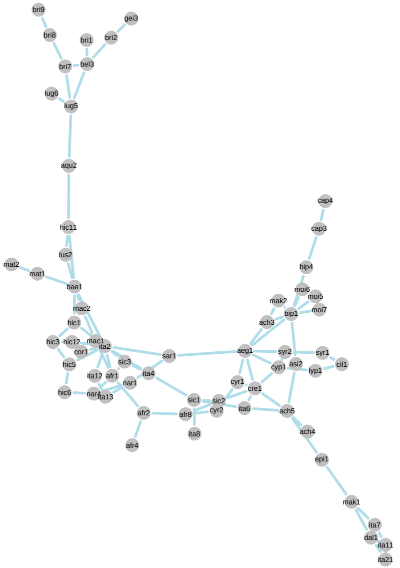
routes based on Rodrigue (2013)

```
sdam::plot.map(type = "si")
```

senatorial , imperial · maritime , roads

Main maritime routes

Roman Empire transport network



Spatial networks

transportation systems

The spatial network of the ancient Roman transport system constitutes one domain of a multilevel configuration

In a graph representation of a transportation network

- settlements, crossing roads, and terminals are vertices
- roads and shipping routes are edges (two "modes")
  - ➟ where distance is a deterrence function of journey cost

☛ unlike social networks, the design and evolution of spatial networks are physically constrained

Spatial networks construction techniques

Assumed agency to create connection

- Maximum distance expressed in units of time
  ➟ convert journey costs into a likehood
- Proximal point
- Efficiency criterion

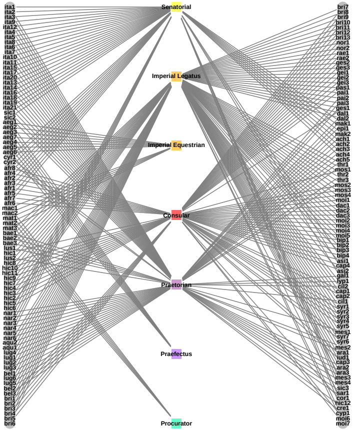
Mathematical

- Delaunay triangulation
  ➟ geodesic distances
- Gabriel graph
  ➟ allows temporal distances between places

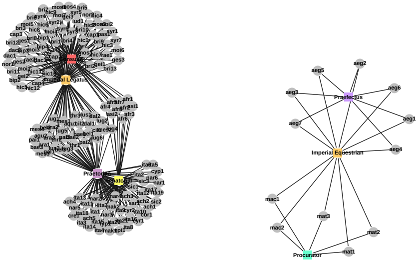
Nodes with province and government types

"multimodal" network graphs

Bipartite



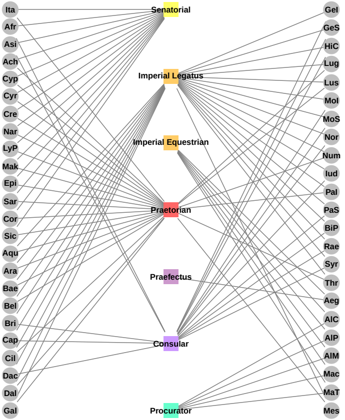
Binomial projection



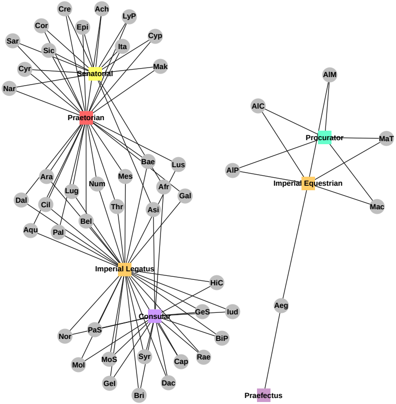
Roman provinces and government types

multimodal network graphs

Bipartite



Binomial projection



Multilevel systems

addressing complexity

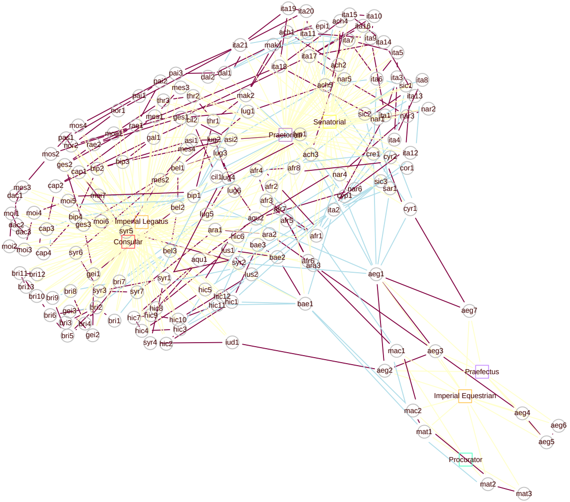
A *multilevel network* X^M for vertex sets N (domain), M (codomain), and edge sets E

$$X^M \; = \; \langle \; N, \; M, \; E_N, \; E_M, \; E_{N \times M} \; \rangle$$

- An *affiliation* network is $X^B = \langle N, M, E_{N \times M} \rangle$, where $E_N, E_M = \varnothing$
- A *valued* network $X^V = \langle N, E, V \rangle$ with V for weights
- A *multiplex* system adds R for types of E
- A *dynamic* system has $t > 1$ time stamps, $X^+ = X_1 \ldots X_t$

Multilevel structures

a projection of Roman Empire transport network with political affiliations

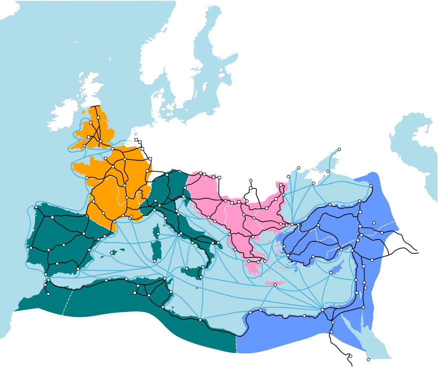


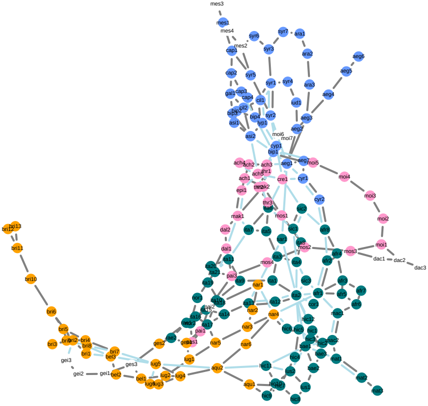
circles: *actors* in N

squares: *events* in M

Roman Empire transport network

main roads & maritime routes under a tetrarchy (ca. 284 AD)





```
sdam::plot.map(type = "tetra")
```

MAXIMIAM , DIOCLECIANO · GALERIO , CONSTANCIO

a dynamic spatial network analysis

Oriens diocese in Asian continent example

- both network size and density are reduced at t_2

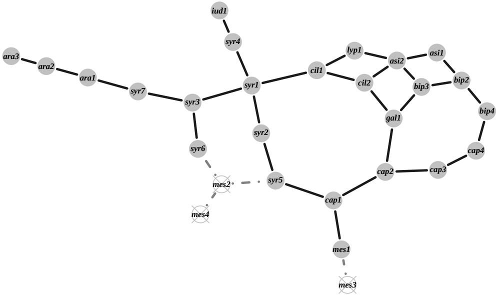
  ➟ affect centrality measures

– there is one less cycle

  ➟ double transportation cost from `syr6` to `syr5` and viceversa

– there are changing pendants

  ➟ `mes1` and `syr6` become structurally equivalent



Path-finding problems in spatial networks

reachability and betweenness centrality

Reachability is an indicator of potential flow diffusion paths in networks

- *static*: vertex adjacency
- *dynamic*: plus concurrence or precedence of relational "events"
  ➟ time-ordered events

☛ combine with changing affiliations over time with another path problem of temporal *betweenness centrality*

Algebra for dynamic networks

Use algebraic structures with the composition of interval sets for analysis of temporal or dynamic (simple) networks

A temporal interval $[u, v]$ from a base set $\mathbb{E}$ where each element

$$E = \cup_i \in \{1, 2, \ldots, t\}[u_i, v_j] \in \mathbb{E}$$

is finite, disjoint, and bounded.

- ➟ u and v are *onset* and *offset* resp.
- ➟ $(u - v)$ is the duration of the interval
- ➟ a discrete time point is when $u = v$
- ➟ the set of endomorphisms over $\mathbb{E}$ is denoted as $\mathbb{H}$

Relational compositions

for time-ordered paths and decay

- The composition of interval sets $E_1 = \bigcup_{p\in\{1\ldots r\}}[u_p, v_p]$ and $E_2 = \bigcup_{q\in\{1\ldots s\}}[t_q, w_q]$ in $\mathbb{E}$ is

$$E_1 \otimes E_2 = \bigcup_{\substack{p=1\ldots r\\ q=1\ldots s}} [u_p, v_p] \otimes [t_q, w_q]$$

  being w the latest time point.

- Another operation on $\mathbb{E}$ is the pairwise interval union over all the intervals in $E_1, E_2 \in \mathbb{E}$

$$E1 \oplus E2 = \bigcup_{i,j}\Big([u_i, v_i] \cup [t_j, w_j]\Big)$$

☛ ordered semirings with composition and union operations over time help us to compute solutions to path-finding problems

cf. Kontoleon, Falzon, Pattison (2013); Moody (2002)

Reachability in dynamic networks

a path-finding problem for transport systems

Determine H^*, the quasi-inverse of $H \in \mathbb{H}$, to compute solutions to path-finding problems

$$H^* = \lim_{k\to\infty} H^{(k)}$$

➟ find conditions for convergence under which $H^* = H^{(k)}$ for a finite k

going further ...

I. Complex dynamic systems: develop visualization tools and perform algebraic analyses

➟ implement algorithms for the computation of multilevel graphs, semirings and other algebraic structures

➟ describe the evolution of social complexity–case periods of the Roman Empire–through dynamic networks analysis

➟ geography and time support in R packages `sdam`, `multiplex` and `multigraph`

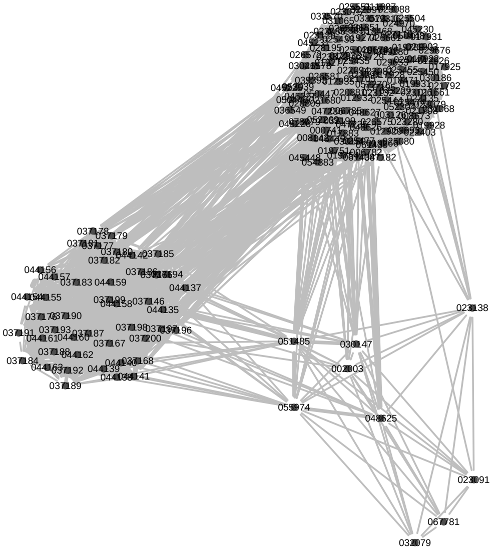
II. Temporal uncertainty with network imputation methods

➟ apply to Roman inscriptions in Latin and Greek from the EDH dataset

Epigraphic networks

addressing social complexity

Similarity among Egyptian epigraphs



EDH dataset with attribute simple matching

(cf. SDAM project)

Thanks!

Social Dynamics

in the Ancient Mediterranean

Social Dynamics in the Ancient Mediterranean Sea – CEDHAR

Research programme activities at the Department of History and Classical Studies, AU

`github.com/mplex`

`jaro@cas.au.dk`