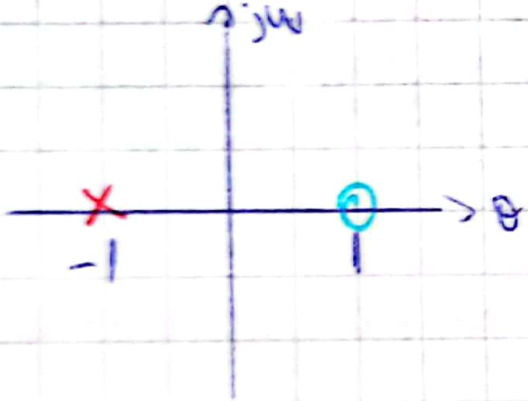
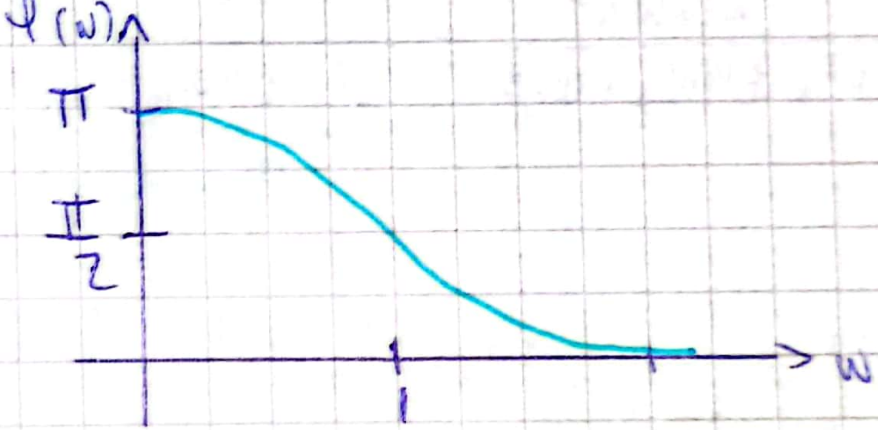
1) a) Rotador de fase de 1er orden:

$$T(s)=\frac{s-1}{s+1} \longrightarrow \text{Normalizada}$$





$$T(jw)=\frac{jw-1}{jw+1}\cdot\frac{jw-1}{jw-1}=\frac{-w^2+2jw+1}{-w^2+1} \; ; \; \varphi(jw)=\operatorname{atan}\left(\frac{2w}{-w^2+1}\right)-\operatorname{atan}\left(\frac{1}{-w^2+1}\right)$$

$$\tau(w)=\frac{1}{1+\frac{2w}{-w^2+1}}\cdot\frac{2.(-w^2+1)-2w(-2w)}{(-w^2+1)^2}-\frac{1}{1+\frac{1}{-w^2+1}}\cdot\frac{-(-2w)}{(-w^2+1)^2}$$

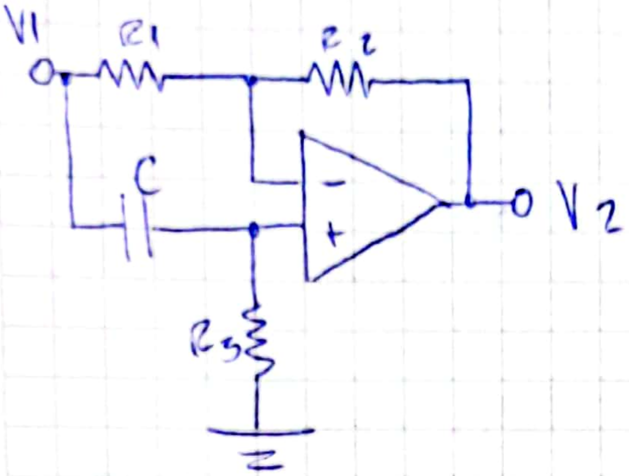
$$\tau(w) = \frac{-w^2+1}{-w^2+1+2w} \cdot \frac{-2w^2+1+4w^2}{(-w^2+1)^2} - \frac{-w^2+1}{-w^2+2} \cdot \frac{2w}{(-w^2+1)^2}$$

$$\tau(w) = \frac{\cancel{-w^2+1}}{-w^2+2w+1} \cdot \frac{2w^2+1}{(-w^2+1)^{\cancel{2}}} - \frac{\cancel{-w^2+1}}{-w^2+2} \cdot \frac{2w}{(-w^2+1)^{\cancel{2}}}$$

$$\tau(w) = \frac{2w^2+1}{(-w^2+2w+1)(-w^2+1)} - \frac{2w}{(-w^2+2)(-w^2+1)}$$

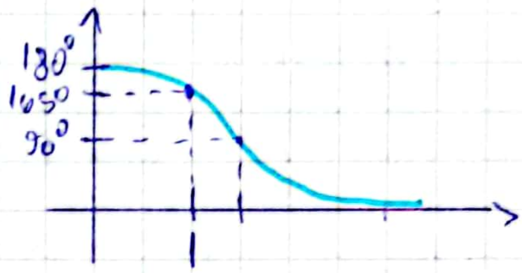
b) Implementación con topología activa y pasiva:

- Activa:



$$\frac{V_2(s)}{V_1(s)} = T(s) = \frac{s - {}^{R_2}/_{C R_3 R_1}}{s + \frac{1}{C R_3}}$$

$$\varphi = 15^\circ \Rightarrow \varphi = 180^\circ - 15^\circ = 165^\circ$$



$$\Omega_z = R; \quad R_1 = R_2 = R_3 = R \Rightarrow T(s) = \frac{s - 1/CR}{s + 1/CR}$$

- Busco valor θ para $\varphi = 165^\circ$:

$$\begin{cases} \measuredangle Z - \measuredangle P = 165^\circ \longrightarrow \\ \measuredangle Z + \measuredangle P = 180^\circ \end{cases} \quad \measuredangle Z = 165^\circ + \measuredangle P$$

$$165^\circ + 2 \cdot \measuredangle P = 180^\circ$$

$$\measuredangle P = 7{,}5^\circ \; ; \; \measuredangle Z = 172{,}5^\circ$$

$$tg(\measuredangle P) = \frac{\overset{\omega = 1}{1}}{\theta} \Rightarrow \theta = \frac{1}{tg(\measuredangle P)} = 7{,}59$$

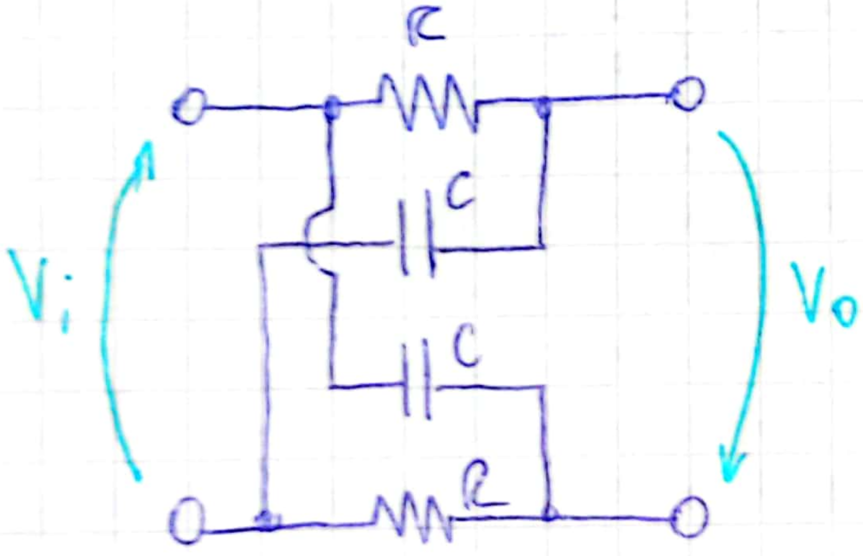
- Valores de R y C:

$$T(s) = \frac{s - 7{,}59}{s + 7{,}59} \; , \; \frac{1}{RC} = 7{,}59$$

$$\text{Si: } R = 1 \Rightarrow C = 0{,}131$$

- Pasiva:

Lattice.



$$T(s) = \frac{-Z_2 + Z_1}{Z_2 + Z_1}$$

$$Z_1 = R \; ; \; Z_2 = 1/sC$$

$$T(s) = \frac{s - 1/RC}{s + 1/RC}$$

- Mismos valores de R y C que la red activa

$R = 1 \; ; \; C = 0{,}131$

2) Low-Pass Notch:

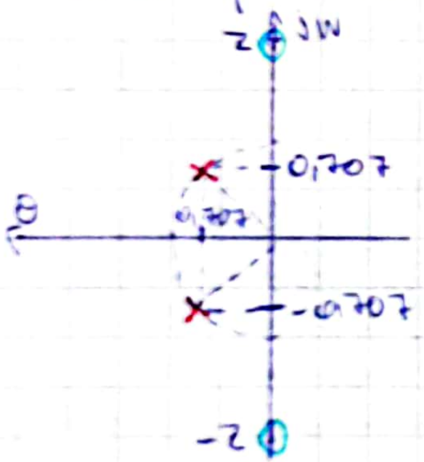
$K = -12\ dB = 0{,}251$

Denominador Butter: $s^2 + \sqrt{2}\, s + 1$

$W_z = 2 \rightarrow$ Elimina esa franja

$$T(s) = 0{,}251 \cdot \frac{s^2 + 2^2}{s^2 + \sqrt{2}\, s + 1^2}$$

Polos y ceros:



Circuito pasivo:

$$T(s) = \frac{s^2 + s\, b/a \cdot 1/RC + d/a \cdot 1/LC}{s^2 + s\,\frac{1}{RC} + \frac{1}{LC}}$$

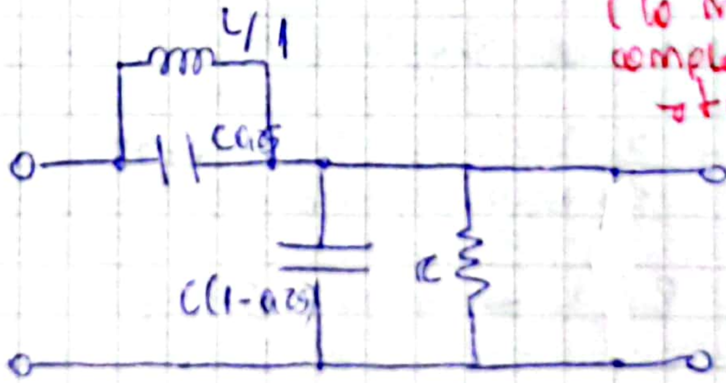
$b = 0;\ W_z^2 = \frac{d}{a} > 1 \Rightarrow W_z^2 = 4 = \frac{d}{a}$

$d = 1\ ;\ a = 0{,}25$ → L levantada de tierra (lo mejor sería completamente aterrizada pero la condición $\frac{d}{a} = 4$ no lo permite)

$R = 1$

$L = Q = \sqrt{2}$

$C = \frac{1}{Q} = \frac{1}{\sqrt{2}}$



Escaneado con CamScanner

b) $\omega_0 = 1 = \omega_z$

$$T(j\omega) = T(s)\Big|_{s=j\omega} = \frac{-\omega^2 - j\omega\frac{\omega_0}{Q_z} + \omega_0^2}{-\omega^2 + j\omega\frac{\omega_0}{Q_p} + \omega_0^2}$$

$$|T(j\omega)| = \frac{\sqrt{(-\omega^2+\omega_0^2)^2 + \left(\omega\cdot\frac{\omega_0}{Q_z}\right)^2}}{\sqrt{(-\omega^2+\omega_0^2)^2 + \left(\omega\,\frac{\omega_0}{Q_p}\right)^2}}$$

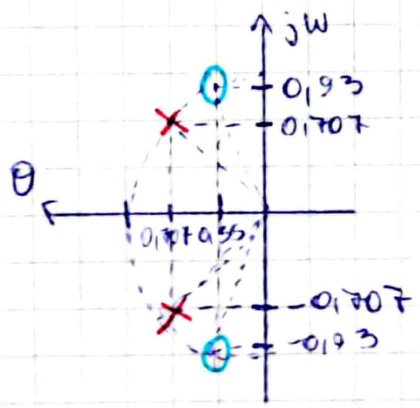
$$|T(j\omega)|\Big|_{\omega=\omega_0} = \frac{\omega_0\,\omega_0/Q_z}{\omega_0\,\omega_0/Q_p} = \frac{Q_p}{Q_z} = -6\,dB = 0{,}5$$

$$Q_z > Q_p$$

$$Q_p = \frac{1}{\sqrt{2}} \Rightarrow Q_z = 2Q_p = \sqrt{2} = \frac{1}{2\cdot\cos\psi} \Rightarrow \psi = 69{,}29^\circ$$

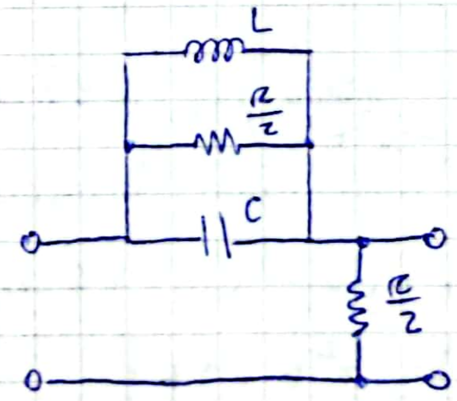
$\hookrightarrow \theta = 0{,}35$

$\hookrightarrow j\omega = 0{,}93$



$$T(s) = \frac{s^2 + s\,{}^{b}/_{a}\cdot{}^{1}/_{RC} + {}^{d}/_{a}\cdot{}^{1}/_{LC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

$$\frac{d}{a} = 1\ ;\ \frac{b}{a} = \frac{1}{2}\ ;\ d = a = 1\ ;\ b = 0{,}5$$



$R = 1$

$L = Q_z = \sqrt{2}$

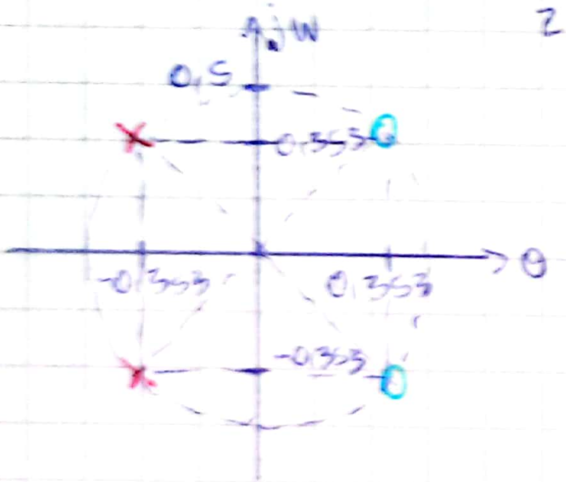
$$C = \frac{1}{Q_z} = \frac{1}{\sqrt{2}}$$

c) En $-\pi$ tendré $w_0 \Rightarrow w_0 = 0{,}5$

$$T(s) = \frac{s^2 - \frac{\sqrt{2}}{2} s + 0{,}5^2}{s^2 + \frac{\sqrt{2}}{2} s + 0{,}5^2}$$

Lattice pasiva:

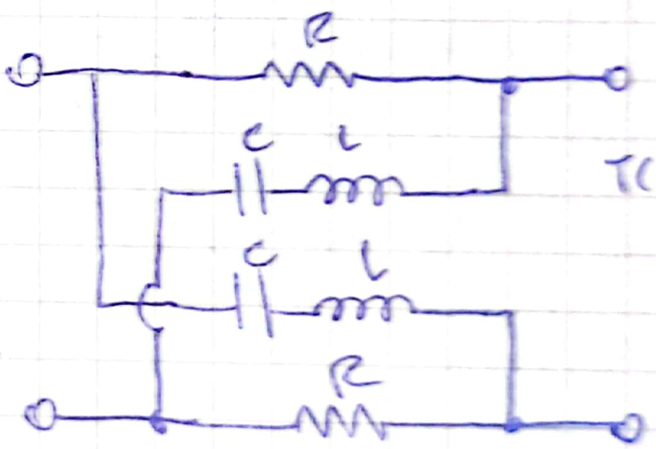
$$Z_1 = R \quad ; \quad Z_2 = \frac{1}{sC} + sL = \frac{s^2 + 1/LC}{s\,1/L}$$



$$L = \frac{Q}{w_0} = \frac{1}{\sqrt{2}.2}$$

$$C = \frac{1}{Qw_0} = \frac{2}{1/\sqrt{2}}$$

$$R = 1$$



$$T(s) = \frac{s^2 - s\,R/L + 1/LC}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

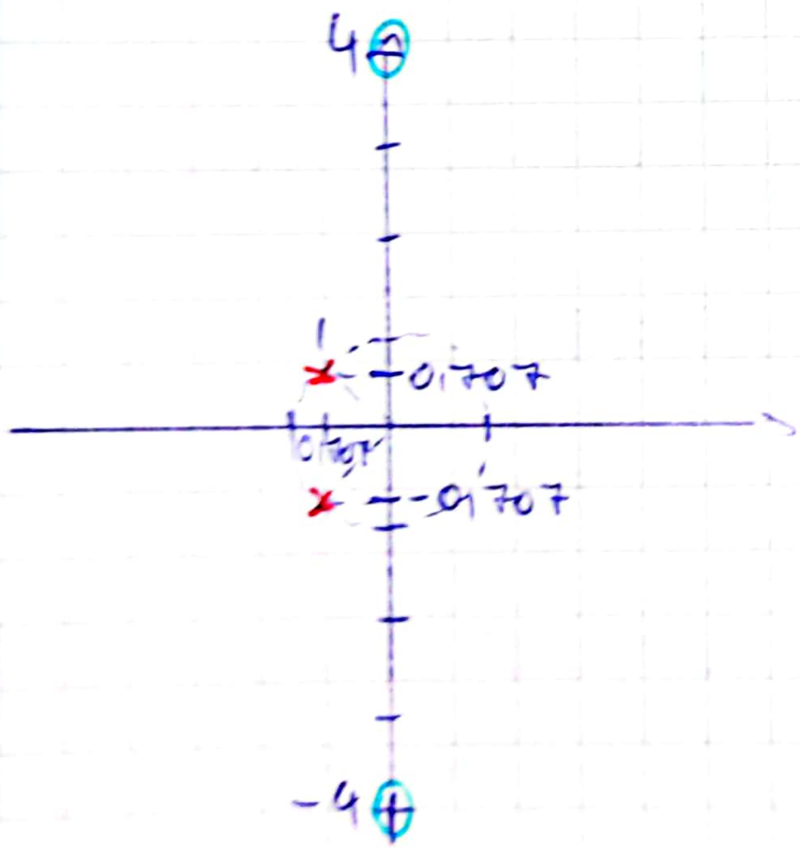
d) Fase $-\frac{\pi}{2} \Rightarrow \omega_p = 1$

Salto Fase $\omega_z \Rightarrow \omega_z = 4$

Notch $\omega_p \neq \omega_z$

$\omega_z > \omega_p \Rightarrow$ Low-Pass Notch

$$T(s) = \frac{s^2 + 4^2}{s^2 + \sqrt{2}s + 1}$$



Implementación pasiva:

Igual que a) solo que

$\omega_z^2 = 16 = \frac{d}{a}$

$d = 1$; $a = 0{,}0625$

3) $\phi(\omega) = \underbrace{\frac{\pi}{2}}_{\text{Fase ceros}} - \underbrace{\operatorname{arctg}\left(\frac{6\omega}{-\omega^2+4}\right)}_{\text{Fase polos}}$

Fase ceros $= \frac{\pi}{2} \Rightarrow$ \$ aporta $\frac{\pi}{2}$ $\forall$ ω

Fase polos $= \operatorname{arctg}\left(\frac{6\omega}{-\omega^2+4}\right)$; $\mathrm{Re}\{Denp(s)\} = -\omega^2+4$

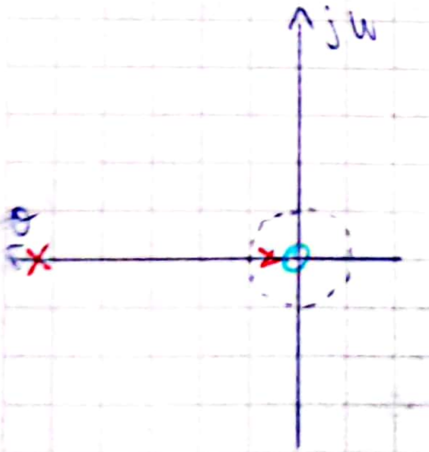
$\mathrm{Imag}\{Denp(s)\} = 6\omega$

$Denp(j\omega) = -\omega^2+4+6j\omega = (j\omega)^2+6j\omega+4$

$s = j\omega \Rightarrow Denp(s) = s^2+6s+4$

- Propongo:

$$T(s) = \frac{s}{s^2+6s+4} = \frac{s}{(s+0{,}764)(s+5{,}236)}$$



$\omega = 0 \Rightarrow \varphi = \frac{\pi}{2} - 0$

$\omega \to \infty \Rightarrow \varphi = \frac{\pi}{2} - 2\cdot\frac{\pi}{2} = -\frac{\pi}{2}$