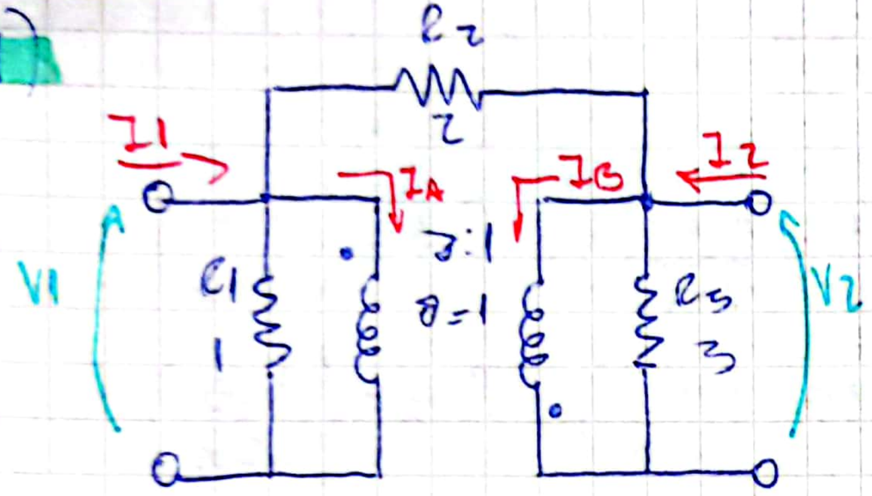
Tarea Semanal 7:

1)



$$V_2 = -\frac{V_1}{a} \quad ; \quad I_B = I_A \cdot a$$

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + I_A \quad (1)$$

$$I_2 = \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} + I_B \quad (2)$$

• Analizando con $I_2 = 0$:

$$-I_B = \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} = -\frac{V_1}{a . R_3} + \frac{-\frac{V_1}{a} - V_1}{R_2}$$

$$-I_B = -I_A . a \Rightarrow -I_A . a = -\frac{V_1}{a . R_3} - \frac{V_1 (1 + 1/a)}{R_2}$$

Trafo. Real:

$$|Z| = \begin{pmatrix} sL_1 & -sM \\ -sM & sL_2 \end{pmatrix} \qquad Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

$$I_A = \frac{V_1}{a^2 R_3} + \frac{V_1(1+1/a)}{a \cdot R_2} \rightarrow V_1 + \frac{V_1}{a} = V_1\left(1+1/a\right)$$

En (1): $$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1}{a^2 R_3} + \frac{V_1(1+1/a)}{a \cdot R_2}$$

$$a = 1;\ R_1 = 1;\ R_2 = 2;\ R_3 = 3$$

$$I_1 = \frac{V_1}{1} + \frac{V_1 \cdot 2}{2} + \frac{V_1}{3} + \frac{V_1 \cdot 2}{2} = \frac{10}{3} V_1$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{3}{10} = Z_{22}$$

- Analizando con $I_1 = 0$:

$$-I_A = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}\ ;\ -\frac{I_B}{a} = \frac{V_1}{R_1} + \frac{V_1(1+1/a)}{R_2}$$

$$I_B = -\frac{V_1 \cdot a}{R_1} - \frac{V_1(1+1/a) \cdot a}{R_2} = \frac{V_2 \cdot a^2}{R_1} + \frac{V_2 \cdot a^2(1+1/a)}{R_2}$$

En (2)

$$I_2 = \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} + \frac{V_2 \cdot a^2}{R_1} + \frac{V_2 \cdot a^2(1+1/a)}{R_2}$$

$$I_2 = \frac{V_2}{R_3} + \frac{V_2(1+\alpha)}{R_2} + \frac{V_2 \cdot \alpha^2}{R_1} + \frac{V_2 \cdot \alpha(1+\alpha)}{R_2}$$

$\alpha = 1 ; R_1 = 1 ; R_2 = 2 ; R_3 = 3$

$$I_2 = \frac{V_2}{3} + \frac{V_2 \cdot 2}{2} + \frac{V_2}{1} + \frac{V_2 \cdot 2}{2} = \frac{10}{3} \cdot V_2$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{3}{10}$$

- Z_{12} y Z_{21}:

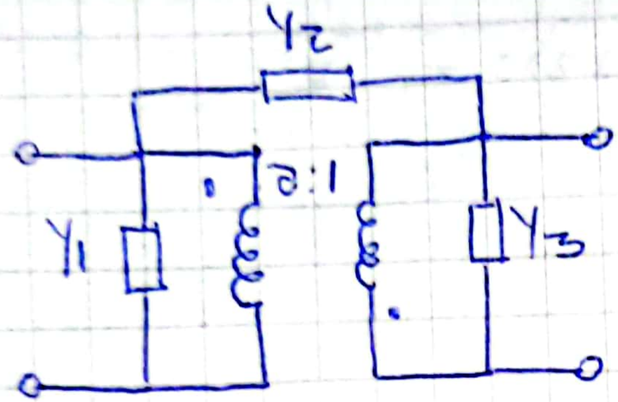
$$Z_{12} = \frac{V_1}{I_2} = -\frac{V_2 \cdot \alpha}{I_2} = -Z_{22} \cdot \alpha = -0{,}3$$

$$Z_{21} = \frac{V_2}{I_1} = -\frac{V_1}{I_1} \frac{1}{\alpha} = -Z_{11} \cdot \frac{1}{\alpha} = -0{,}3$$

$$Z = \begin{pmatrix} 0{,}3 & -0{,}3 \\ -0{,}3 & 0{,}3 \end{pmatrix}$$

Escaneado con CamScanner

2)



$$Z_1 = \frac{1}{Y_1} \; ; \; Z_2 = \frac{1}{Y_2} \; ; \; Z_3 = \frac{1}{Y_3}$$

Del punto 1):

$$I_1 = \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} + \frac{V_1}{a^2 . Z_3} + \frac{V_1(1+1/a)}{a . Z_2}$$

$$I_1 = V_1 . Y_1 + (V_1 - V_2) . Y_2 + \frac{V_1 . Y_3}{a^2} + \frac{V_1(1+1/a)}{a} . Y_2$$

$$I_1 = V_1 . (Y_1) + V_1(1+1/a) Y_2 + \frac{V_1 . Y_3}{a^2} + \frac{V_1(1+1/a)}{a} . Y_2$$

$$I_1 = V_1 \left[Y_1 + (1+1/a) Y_2 + \frac{Y_3}{a^2} + \frac{(1+1/a)}{a} . Y_2 \right]$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{1}{Y_1 + Y_2 + \frac{Y_2}{a} + \frac{Y_3}{a^2} + \frac{Y_2}{a} + \frac{Y_2}{a^2}}$$

$$Z_{11} = \frac{\partial^2}{Y_1 \cdot \partial^2 + Y_2 \cdot \partial^2 + Y_2 \cdot \partial + Y_3 + Y_2 \cdot \partial + Y_2}$$

$$Z_{11} = \frac{\partial^2}{Y_1 \cdot \partial^2 + Y_2 \cdot \partial^2 + Y_2 + Y_3 + 2 \cdot Y_2 \cdot \partial}$$

Del punto 1):

$$I_2 = \frac{V_2}{Z_3} + \frac{V_2(1+\partial)}{Z_2} + \frac{V_2\, \partial^2}{Z_1} + \frac{V_2 \cdot \partial \cdot (1+\partial)}{Z_2}$$

$$I_2 = V_2 \cdot Y_3 + V_2(1+\partial) \cdot Y_2 + V_2 \cdot \partial^2 \cdot Y_1 + V_2 \cdot \partial(1+\partial) \cdot Y_2$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{1}{Y_3 + (1+\partial)Y_2 + \partial^2 Y_1 + \partial(1+\partial)Y_2}$$

$$Z_{22} = \frac{1}{Y_3 + Y_2 + Y_2 \cdot \partial + \partial^2 \cdot Y_1 + \partial \cdot Y_2 + Y_2\, \partial^2}$$

$$Z_{22} = \frac{1}{Y_1 \cdot \partial^2 + Y_2 \cdot \partial^2 + Y_2 + Y_3 + 2 \cdot Y_2 \cdot \partial}$$

Del punto 1):

$$Z_{12} = -Z_{22}\cdot a = \frac{-a}{Y_1\cdot a^2 + Y_2\cdot a^2 + Y_2 + Y_3 + 2Y_2\cdot a}$$

$$Z_{21} = -Z_{11}\cdot\frac{1}{a} = \frac{-a^{\cancel{2}}}{Y_1\cdot a^2 + Y_2\cdot a^2 + Y_2 + Y_3 + 2Y_2\cdot a}\cdot\frac{1}{\cancel{a}}$$

$Z_{12} = Z_{21}$ $\longrightarrow$ Red reciproca

Para simetria:

$$Z_{11} = Z_{22} \Rightarrow \frac{a^2}{\cancel{Y_1 a^2 + Y_2 a^2 + Y_2 + Y_3 + 2Y_2 a}} = \frac{1}{\cancel{Y_1 a^2 + Y_2 a^2 + Y_2 + Y_3 + 2Y_2 a}}$$

$a^2 = 1 \Rightarrow a = 1$ para que la red sea simétrica, no importan los valores de Y_1, Y_2, Y_3

Escaneado con CamScanner