

checking_error_function

December 12, 2019

```
[1]: import numpy as np
      from scipy.special import erf
      from scipy.integrate import quad
      import matplotlib.pyplot as plt
      import lmfit

      from IPython.display import Latex
```

Scipy defines erf as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-z^2) dz$$

which goes from $-1 \rightarrow 1$ as x goes from $-\infty \rightarrow \infty$. Note that the dummy variable has been relabeled to z (rather than t) as the latter is confusing if we want to use t as time. By inspection we can see that is equivalent to integrating a normal Gaussian distribution with

$$\sigma = \frac{1}{\sqrt{2}}$$

$$\text{where FWHM} = 2\sigma\sqrt{2\log 2}$$

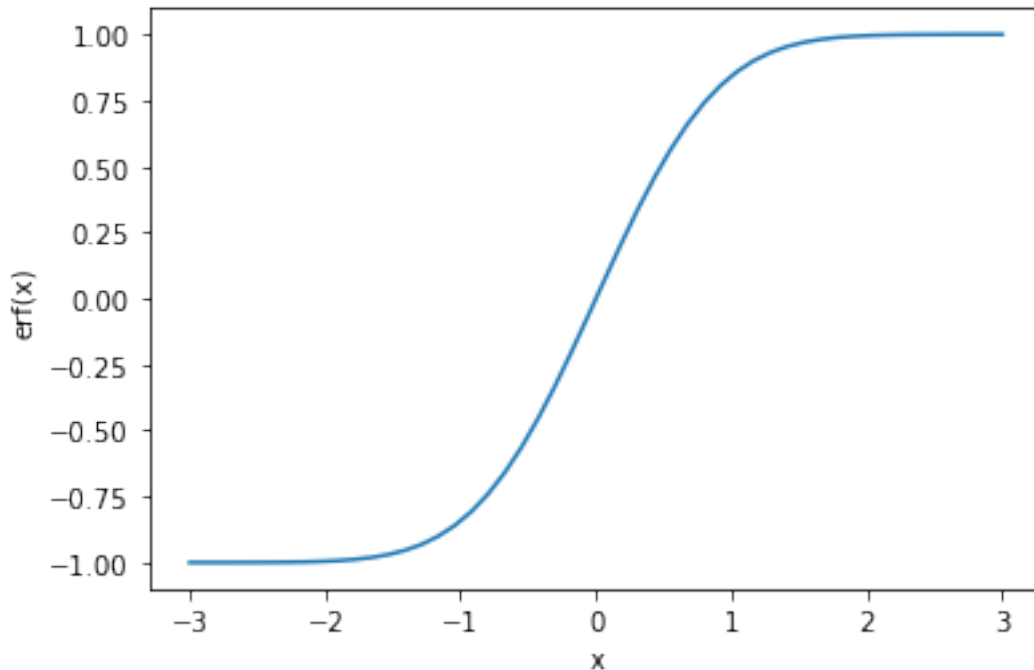
Let's plot the error function

```
[2]: x = np.linspace(-3, 3)

      fig, ax = plt.subplots()

      ax.plot(x, erf(x))
      ax.set_xlabel("x")
      ax.set_ylabel("erf(x)")
```

```
[2]: Text(0, 0.5, 'erf(x)')
```



The function required to go from 1 to 1-A is

$$1 - \frac{A}{2}[1 - \operatorname{erf}(x)]$$

To to be equivant to convolving with a Guassian of with FWHM=1 we need a variable substitution.

Multiplying x by $1/\sqrt{2}$ converts the function to have a effective crossover width of $\sigma = 1$ so that a valid variable substitution is

$$x \rightarrow \frac{x}{\sqrt{2}\sigma}$$

We can then insert $\sigma = FWHM/(2\sqrt{2\log 2})$ and cancel the $\sqrt{2}$ s.

The whole function is

$$1 - \frac{A}{2}[1 - \operatorname{erf}(2x\sqrt{\log 2})]$$

Let's check this numerically

```
[3]: def delay_norm(x, A, FWHM):
      return 1- A/2*(1-erf(-x*2*np.sqrt(np.log(2))))

x = np.linspace(-3, 3, 5000)

fig, (ax, axr) = plt.subplots(1, 2, figsize=(10, 4))

y = delay_norm(x, A=0.2, FWHM=1)
ax.plot(x, y)
```

```

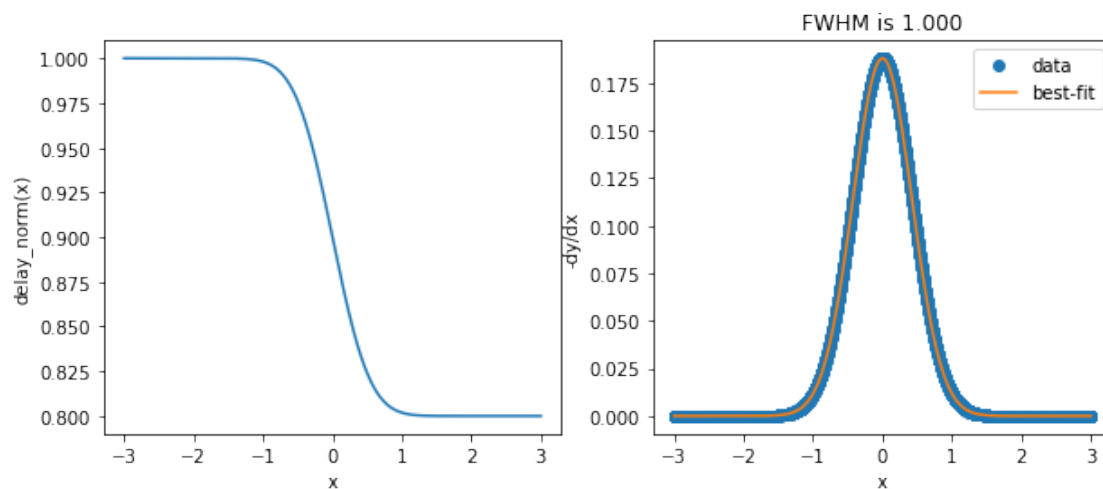
ax.set_xlabel("x")
ax.set_ylabel("delay_norm(x)")

dx = x[1] - x[0]

dy_dx = np.diff(y)/dx
model = lmfit.models.GaussianModel()
result = model.fit(-dy_dx, x=x[:-1])
result.plot_fit(ax=axr)
axr.set_xlabel("x")
axr.set_ylabel("-dy/dx")
axr.set_title("FWHM is {:.3f}".format(result.params['fwhm'].value))

```

[3]: Text(0.5, 1.0, 'FWHM is 1.000')



Since this worked, we just need to substitute the form for erf . The function we want is

$$1 - \frac{A}{2} - \frac{A}{\pi} \int_0^{x^2 \sqrt{\log 2}} \exp(-z^2) dz$$

[]: