checking_error_function

December 16, 2019

[1]:
```
import numpy as np
from scipy.special import erf
from scipy.integrate import quad
import matplotlib.pyplot as plt
import lmfit

from IPython.display import Latex
```

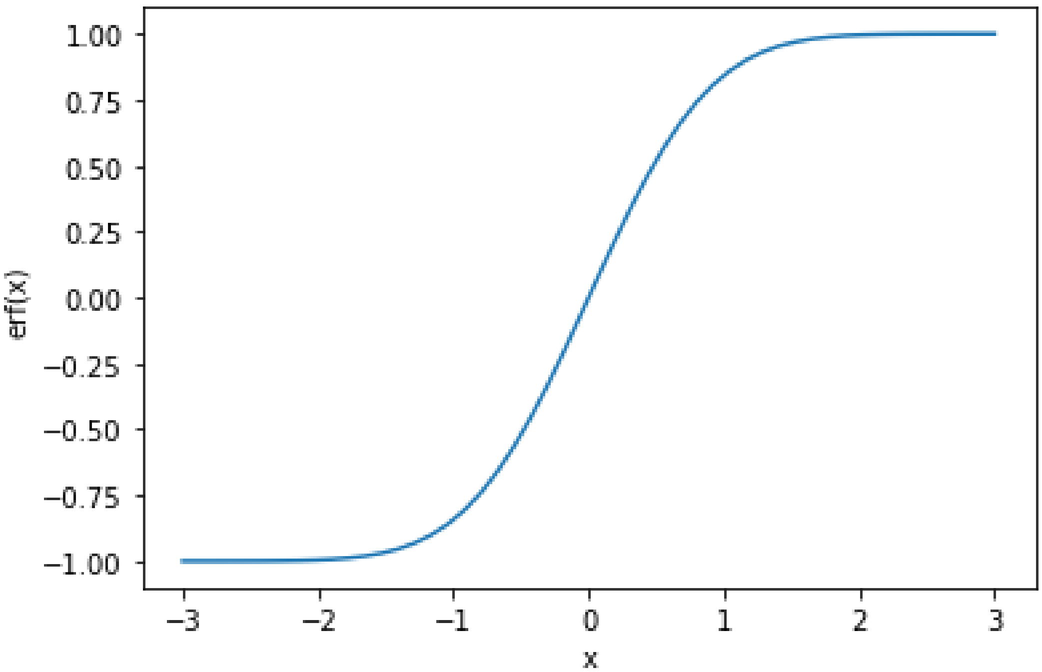
Scipy defines erf as

$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-z^2) dz$
which goes from $-1 \rightarrow 1$ as x goes from $-\inf \rightarrow \inf$. Note that the dummy variable has been relabeled to z (rather than t) as the latter is confusing if we want to use t as time. By inspection we can see that is equivalent to integrating a normal Guassian distribution with
$\sigma = \frac{1}{\sqrt{2}}$
where FHWM $= 2\sigma\sqrt{2 \log 2}$

Let's plot the error function

[2]:
```
x = np.linspace(-3, 3)

fig, ax = plt.subplots()

ax.plot(x, erf(x))
ax.set_xlabel("x")
ax.set_ylabel("erf(x)")
```

[2]: `Text(0, 0.5, 'erf(x)')`



The function required to go from 1 to 1-A is
$1 - \frac{A}{2}[1 - \text{erf}(x)]$

To to be equilvant to convolving with a Guassian of with FWHM=1 we need a variable substitution.

Multiplying x by $1/\sqrt{2}$ converts the function to have a effective crossover width of $\sigma = 1$ so that a valid variable substituion is
$x \rightarrow \frac{x}{\sqrt{2}\sigma}$
We can then insert $\sigma = FWHM/(2\sqrt{2\log 2})$ and cancel the $\sqrt{2}$s.

The whole function is

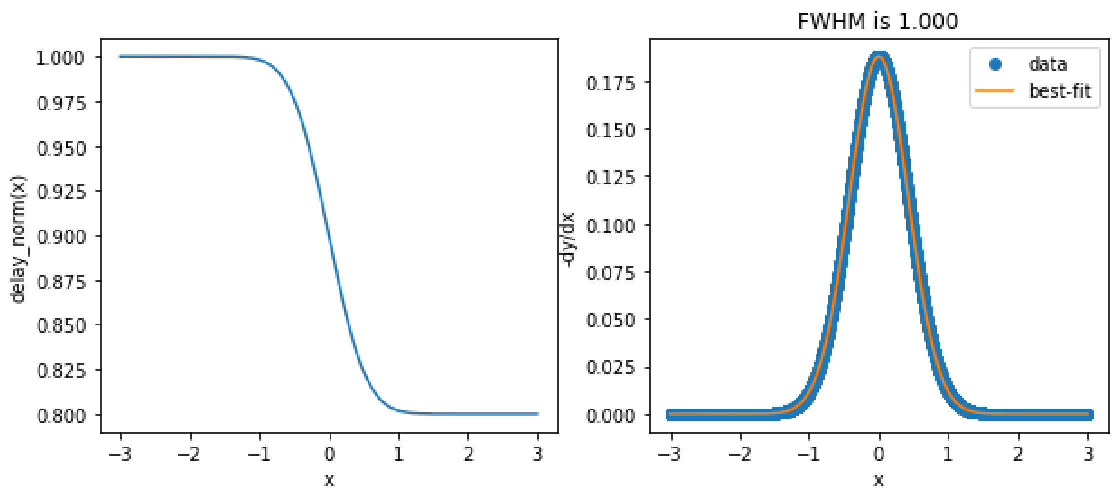
$1 - \frac{A}{2}[1 - \text{erf}(2x\sqrt{\log 2}]$

Let's check this numerically

[3]:
```
def delay_norm(x, A, FWHM):
    return 1- A/2*(1-erf(-x*2*np.sqrt(np.log(2))))

x = np.linspace(-3, 3, 5000)

fig, (ax, axr) = plt.subplots(1, 2, figsize=(10, 4))

y = delay_norm(x, A=0.2, FWHM=1)
ax.plot(x, y)
ax.set_xlabel("x")
```

```python
ax.set_ylabel("delay_norm(x)")

dx = x[1] - x[0]

dy_dx = np.diff(y)/dx
model = lmfit.models.GaussianModel()
result = model.fit(-dy_dx, x=x[:-1])
result.plot_fit(ax=axr)
axr.set_xlabel("x")
axr.set_ylabel("-dy/dx")
axr.set_title("FWHM is {:.3f}".format(result.params['fwhm'].value))
```

[3]: Text(0.5, 1.0, 'FWHM is 1.000')



Since this worked, we just need to substitute the form for erf. The function we want is $1 - \frac{A}{2} - \frac{A}{\pi}\int_0^{x2\sqrt{\log 2}} \exp(-z^2)dz$

[ ]: