Definitions of fluence with Gaussian beam profiles

A Gaussian beam has an intensity profile

$$I(r) = I_0 \exp\left(-2\frac{r^2}{w^2}\right) \tag{1}$$

where r is the radial coordinate and w is the $1/e^2$ radius sometime called the waist of the beam. The total energy in such a pulse within radius r can be computed via an integral

$$E(r) = I_0 \int_0^r \exp\left(-2\frac{{r'}^2}{w^2}\right) \mathrm{d}r' = \frac{I_0 \pi w^2}{2}\left\{1 - \exp\left(-2\frac{r^2}{w^2}\right)\right\}. \tag{2}$$

The large r limit shows that the total energy is

$$E_0 = \frac{I_0 \pi w^2}{2}. \tag{3}$$

The average fluence with radius r is then

$$F(r) = \frac{E(r)}{\pi r^2} = \frac{E_0}{\pi}\frac{1}{r^2}\left\{1 - \exp\left(-2\frac{r^2}{w^2}\right)\right\}. \tag{4}$$

The peak fluence can be derived by expanding the exponential in a Taylor series giving

$$F^{\max} = F(r \to 0) = \frac{2E_0}{\pi w^2} \tag{5}$$

Definition 1

A common defintion uses the waist of the beam to define the radius of a circle. Definition 1 of fluence assumes that all the energy lies within the circle giving

$$F_1 = \frac{E_0}{\pi w^2}. \tag{6}$$

The average fluence within this definition would be

$$F_1^{\text{average}} = \frac{E_0}{\pi}\frac{1}{r^2}\left(1 - e^{-2}\right) \approx \frac{0.865 E_0}{\pi w^2} \tag{7}$$

and the max fluence is

$$F_1^{\max} = F(r \to 0) = \frac{2E_0}{\pi w^2} \tag{8}$$

Definition 2

Other size metrics include the full-width at half-maximum (FWHM) X_{FWHM} or the half-width at half-maximum (HWHM) X_{HWHM}. Expressing these in terms of the beam waist w gives

$$X_{\text{FWHM}} = 2X_{\text{HWHM}} = \sqrt{2\ln(2)}w. \tag{9}$$

If one considers X_{HWHM} the radius of a circle this gives our second definition

$$F_2 = \frac{E_0}{\pi X_{\mathrm{HWHM}}^2}, \tag{10}$$

a somewhat higher fluence than that in obtained by definition 1 (Equation 6). The average power within a circle of radius X_{HWHM} is

$$F_2^{\mathrm{average}} = \frac{E_0}{2\pi X_{\mathrm{HWHM}}^2} \tag{11}$$

Definition 3

Equation 11 motivates a third definition

$$F_3 = \frac{E_0}{2\pi X_{\mathrm{HWHM}}^2} = \frac{2E_0}{\pi X_{\mathrm{FWHM}}^2}. \tag{12}$$

which is appealing as the fluence definition matches the average energy per unit area.