

Note on the definition of fluence in optical experiments

A Gaussian beam has an intensity profile

$$I(r) = I_0 \exp\left(-2\frac{r^2}{w^2}\right) \quad (1)$$

where r is the radial coordinate and w is the $1/e^2$ radius sometime called the waist of the beam. The total energy in such a pulse within radius r can be computed via an integral

$$E(r) = I_0 \int_0^r \exp\left(-2\frac{r'^2}{w^2}\right) dr' = I_0 \pi \frac{w^2}{2} \left\{ 1 - \exp\left(-2\frac{r^2}{w^2}\right) \right\}. \quad (2)$$

The large r limit shows that the total energy is

$$E_0 = I_0 \pi \frac{w^2}{2}. \quad (3)$$

The average fluence with radius r is then

$$F(r) = \frac{E(r)}{\pi r^2} = \frac{E_0}{\pi} \frac{1}{r^2} \left\{ 1 - \exp\left(-2\frac{r^2}{w^2}\right) \right\}. \quad (4)$$

We want to work in terms of the more common peak size metric the full-width at half-maximum (FWHM), which is $X = \sqrt{2 \ln(2)} w$ giving

$$F(r) = \frac{E_0}{\pi} \frac{1}{r^2} \left\{ 1 - \exp\left(-4 \ln(2) \frac{r^2}{X^2}\right) \right\}, \quad (5)$$

as shown in Figure 1.

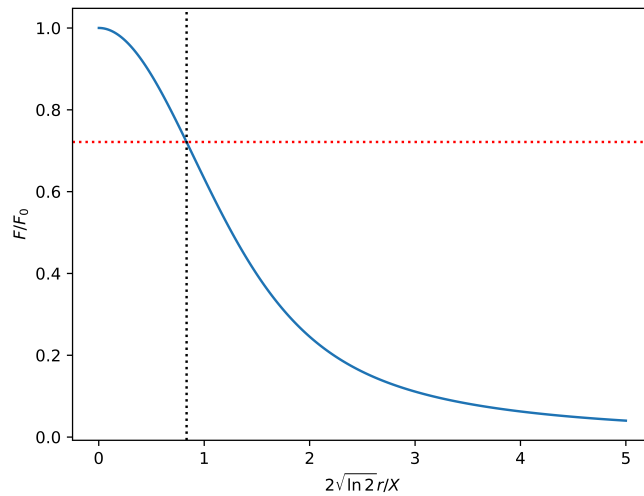


Figure 1: Average fluence, F of Gaussian pulse within radius r in units of the FWHM. The vertical black dotted line corresponds to $r = \frac{X}{2}$ and the horizontal red line is $F = \frac{F_0}{2 \ln(2)}$.

At the center of the beam, the peak fluence can be derived by expanding the exponential in a Taylor series

$$F_0 = \frac{4 \ln(2) E_0}{\pi X^2}. \quad (6)$$

The average fluence within the FWHM is

$$F(r = X/2) = \frac{2E_0}{\pi X^2} \quad (7)$$

which is the standard form for the fluence equation and which corresponds to $\frac{1}{2 \ln(2)} \approx 72\%$ of the peak fluence.