

Definitions of fluence with Gaussian beam profiles

A Gaussian beam has an intensity profile

$$I(r) = I_0 \exp\left(-2\frac{r^2}{w^2}\right) \quad (1)$$

where r is the radial coordinate and w is the $1/e^2$ radius sometime called the waist of the beam. The total energy in such a pulse within radius r can be computed via an integral

$$E(r) = I_0 \int_0^r \exp\left(-2\frac{r'^2}{w^2}\right) 2\pi r' dr' = \frac{I_0 \pi w^2}{2} \left\{ 1 - \exp\left(-2\frac{r^2}{w^2}\right) \right\}. \quad (2)$$

The large r limit shows that the total energy is

$$E_0 = \frac{I_0 \pi w^2}{2}. \quad (3)$$

The average fluence with radius r is then

$$F(r) = \frac{E(r)}{\pi r^2} = \frac{E_0}{\pi} \frac{1}{r^2} \left\{ 1 - \exp\left(-2\frac{r^2}{w^2}\right) \right\}. \quad (4)$$

The peak fluence can be derived by expanding the exponential in a Taylor series giving

$$F^{\max} = F(r \rightarrow 0) = \frac{2E_0}{\pi w^2} \quad (5)$$

Definition 1

A common definition uses the waist of the beam to define the radius of a circle. Definition 1 of fluence assumes that all the energy lies within the circle giving

$$F_1 = \frac{E_0}{\pi w^2}. \quad (6)$$

The average fluence within this definition would be

$$F_1^{\text{average}} = \frac{E_0}{\pi} \frac{1}{r^2} (1 - e^{-2}) \approx \frac{0.865 E_0}{\pi w^2} \quad (7)$$

and the max fluence is

$$F_1^{\max} = F(r \rightarrow 0) = \frac{2E_0}{\pi w^2} \quad (8)$$

Definition 2

Other size metrics include the full-width at half-maximum (FWHM) X_{FWHM} or the half-width at half-maximum (HWHM) X_{HWHM} . Expressing these in terms of the beam waist w gives

$$X_{\text{FWHM}} = 2X_{\text{HWHM}} = 2\sqrt{2\ln(2)}w. \quad (9)$$

If one considers X_{HWHM} the radius of a circle this gives our second definition

$$F_2 = \frac{E_0}{\pi X_{\text{HWHM}}^2}, \quad (10)$$

a somewhat higher fluence than that obtained by definition 1 (Equation 6). The average power within a circle of radius X_{HWHM} is

$$F_2^{\text{average}} = \frac{E_0}{2\pi X_{\text{HWHM}}^2}. \quad (11)$$

Definition 3

Equation 11 motivates a third definition

$$F_3 = \frac{E_0}{2\pi X_{\text{HWHM}}^2} = \frac{2E_0}{\pi X_{\text{FWHM}}^2}. \quad (12)$$

which is appealing as the fluence definition matches the average energy per unit area.