

$$\int_a^b \frac{f^3(\xi)}{4} (x-a)(x-b) \left(x - \frac{(a+b)}{2}\right) dx$$

$$\frac{f^3(\xi)}{4} \cdot \underbrace{\int_a^b (x-a)(x-b) \left(x - \frac{(a+b)}{2}\right) dx}_{①} = 0$$

$$① = \int_a^b x^3 + 2abx + \frac{a^2x}{2} - \frac{a^2b}{2} - \frac{ab^2}{2} - \frac{3ax^2}{2} + \frac{b^2x}{2} - \frac{3bx^2}{2}$$

$$= \int_a^b x^3 + \left(-\frac{3a}{2} - \frac{3b}{2}\right) \int_a^b x^2 + \left(\frac{a^2}{2} + 2ab + \frac{b^2}{2}\right) \int_a^b x + \left(-\frac{a^2b}{2} - \frac{ab^2}{2}\right) \int_a^b 1$$

$$= \frac{1}{4} \left(x(a^2(x-2b) - 2a(b-x)^2 + x(b-x)^2) \right) \Big|_a^b = 0$$