

$$p_2(x) = Ax^2 + Bx + C$$

$$\int_{-h}^h p_2(x) = \left. \frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx \right|_{-h}^h \rightarrow I$$

$$= \left(\frac{Ah^3}{3} + \frac{Bh^2}{2} + Ch \right) - \left(-\frac{Ah^3}{3} + \frac{Bh^2}{2} - Ch \right) = \frac{2Ah^3}{3} + 2Ch = \frac{h}{3} (2Ah^2 + 6C)$$

$$I = 2A \frac{h^3}{3} + 2Ch$$

donde :

$$\begin{matrix} \textcircled{i} & \textcircled{ii} & \textcircled{iii} \\ (-h, f_0), (0, f_1) & \text{y} & (h, f_2) \end{matrix} \left. \vphantom{\begin{matrix} \textcircled{i} \\ \textcircled{ii} \\ \textcircled{iii} \end{matrix}} \right\} \text{evaluados en } p_2(x)$$

$$\begin{aligned} \textcircled{i} \quad Ax^2 + Bx + C &= A(-x)^2 + B(-x) + C \\ &= Ax^2 - Bx + C \\ &= Ah^2 - Bh + C \end{aligned}$$

$$\begin{aligned} \textcircled{iii} \quad Ax^2 + Bx + C &= Ax^2 + Bx + C \\ &= Ah^2 + Bh + C \end{aligned}$$

$$\begin{aligned} \textcircled{ii} \quad Ax^2 + Bx + C &= A(0)^2 + B(0) + C \\ &= C \end{aligned} \quad \left. \vphantom{\begin{aligned} \textcircled{ii} \\ \textcircled{ii} \end{aligned}} \right\} y_1 \text{ (o } f_1)$$

$$\begin{aligned} y_0 &= Ah^2 - Bh + C \\ y_2 &= Ah^2 + Bh + C \\ \hline &2Ah^2 + 2C \end{aligned}$$

$$\begin{aligned} 2Ah^2 + 2C &= y_0 + y_2 \\ 2Ah^2 &= (y_0 + y_2) - 2C \end{aligned} \quad \textcircled{IV}$$

reemplazando \textcircled{IV} en I ...

$$I = \frac{h}{3} [(y_0 + y_2) - 2C + 6C] = \frac{h}{3} [(y_0 + y_2) + 4C] = \frac{h}{3} [(y_0 + y_2) + 4y_1] = \frac{h}{3} [y_0 + 4y_1 + y_2]$$