1.1)

a) $f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f'''(x_0) + \frac{h^4}{4!} f''''(x_0) + O(h^5)$

b) $f(x_0-h) = f(x_0) - hf'(x_0) + \frac{h^2}{2!} f''(x_0) - \frac{h^3}{3!} f'''(x_0) + \frac{h^4}{4!} f^{IV}(x_0) + O(h^5)$

$a+b = 2f(x_0) + f''(x_0) \cdot h^2 + \frac{h^4}{12} f^{IV}(x_0) + O(h^5)$

$$f''(x_0) = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}$$

$- \frac{h^2}{12} f^{IV}(x_0) + O(h^3)$

$$f''(x_0) = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}$$

1.5) Usando 1.8 de los notas y su conjugada:

$$f(x+h)+f(x-h) = -2f(x_0) - h^2 f''(x_0) + \frac{h^4}{12} f^{IV}(x_0)$$

$$f''(x_0) = \frac{12}{h^4}\left(f(x-h)+2f(x_0) - h^2 f''(x_0) + f(x+h)\right)$$

$$h^2 f''(x_0) = -f(x_0-h)+2f(x_0)-f(x_0+h)$$

$$\cancel{f^{IV}(x_0) = \frac{3}{4h^4}\left(f(x-2h) - f(x_0-h) + 4 f(x_0)\right.}$$

$$-f(x+h)+f(x+2h)$$

$$h^2 f''(x_0) = \frac{-f(x_0+2h)+2f(x_0) - f(x_0+2h)}{4}$$

$$f^{V}(x_0) = \left(12 f(x-h) - 24 f(x_0) - 3 f(x_0+2h) + 6 f(x_0)\right.$$

$$\left.3f(x_0-2h)+12f(x-h)\right)/h^4$$

$$f^{V}(x_0) \frac{-3}{h^4}\left(-4f(x-h)+\cancel{8}^{6} f(x_0)+f(x_0+2h) \cancel{-2f(x_0)}\right.$$

$$\left.f(x_0-2h)-4f(x-h)\right)$$

$$f^{IV}(x_0) = \frac{-3}{h^4}\big(f(x_0+2h) - 4 f(x_0+h) + 6 f(x_0) - 4 f(x_0+h) + f(x_0-2h) \big)$$

El orden de la aproximación vendría dado por:

$$-\frac{h^2}{30} f^{VI}(x_0) + O(h^3)$$ (originalmente sería $O(h^5)$, pero $\frac{h^5}{h^2} = h^3$)

1.4 a)

$$f''(x_n) = \frac{1}{4h^2} \sum_{m=-\infty}^{\infty} M[m+2]\, f(x_n - m)$$

donde $M = [1, -2, -1]$