Certamen 3 Jorge Hormazabal

P1)

A) $\lim\limits_{n\to\infty} \sqrt{n^2+n}-n \qquad \lim\limits_{n\to\infty} \left(\sqrt{n^2+n}-n\right)\cdot\frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n}$

$\lim\limits_{n\to\infty} \frac{n^2+n-n^2}{\sqrt{n^2+n}+n} \qquad \lim\limits_{n\to\infty} \frac{n}{\cancel{n}\left(\sqrt{1+\frac{1}{n}}+1\right)} \qquad \lim\limits_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \boxed{\frac{1}{2}}$

P_3) A) $\lim_{N\to\infty} \left| \frac{X^{N+1}}{(N+1)^5 \cdot 5^{(N+1)}} \cdot \frac{5^N \cdot N^5}{X^N} \right|$

$\lim_{N\to\infty} \left| \frac{\cancel{X^N} \cdot X \cdot \cancel{5^N} \cdot N^5}{(N+1)^5 \cdot \cancel{5^N} \cdot 5 \cdot \cancel{X^N}} \right|$

$\lim_{N\to\infty} \frac{X \cdot N^5}{(N+1)^5 \cdot 5} = \left|\frac{X}{5}\right| < 1$

$= \frac{1}{5} |x| < 1 \Rightarrow R = 5$

B) $\lim_{N\to\infty} \left| \frac{(-2)^{N+1} \cdot x^{N+1}}{\sqrt[4]{N+1}} \cdot \frac{\sqrt[4]{N}}{(-2)^N \cdot x^N} \right|$

$\lim_{N\to\infty} \left| \frac{\cancel{-2^N} \cdot -2 \cdot \cancel{x^N} \cdot x \cdot \sqrt[4]{N}}{\sqrt[4]{N+1} \cdot \cancel{(-2)^N} \cdot \cancel{x^N}} \right|$

$\lim_{N\to\infty} \cancel{\frac{\sqrt[4]{N}}{\sqrt[4]{N+1}}}^{1} \cdot 2|x| < 1 \Rightarrow R = 1/2$

4)

A) $f(x) = x e^{x}$

$$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$$

$f(x) = xe^{x}$

$f'(x) = e^{x} + xe^{x}$

$f''(x) = 2e^{x} + xe^{x}$

$f'''(x) = 3e^{x} + xe^{x}$

$f''''(x) = 4e^{x} + xe^{x}$

$$\sum \frac{f^{(n)}(0)\, x^{n}}{n!}$$

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$$

$f(0) = 0$

$f'(0) = 1$

$f''(0) = 2$

$f'''(0) = 3$

$f''''(0) = 4$

$$0 + x + \frac{2}{2!}x^2 + \frac{3x^3}{3!} + \frac{4x^4}{4!}$$

$$x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$$

$$x \cdot \sum_{n=0}^{\infty} \frac{x^{n}}{n!} = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

B) $f(x) = \frac{e^{x} - e^{-x}}{2}$

$f'(x) = \frac{e^{x} + e^{-x}}{2}$

$f''(x) = \frac{e^{x} + e^{-x}}{2}$

$f'''(x) = \frac{e^{x} + e^{-x}}{2}$

$f(x) = 0$

$f'(x) = 1$

$f''(x) = 0$

$f'''(x) = 1$

$f(0) = f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$

$\sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!} x^{n}$

$\overset{0}{x} + \overset{1}{\frac{x}{3!}} + \overset{2}{\frac{x^3}{5!}}$

$\sum_{n=0}^{\infty} \frac{x^{(2n+1)}}{(2n+1)!}$

$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{x^{2(n+1)+1}}{(2(n+n)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right|$

$= \lim_{n \to \infty} \left| \frac{x^{2n+3}}{x^{2n+1}} \cdot \frac{(2n+1)!}{(2n+3)!} \right|$

$= \lim_{n \to \infty} \frac{x^3}{(2n+3)(2n+2)}$

$x^2 \cdot \lim_{n \to \infty} \frac{1}{(2n \cdot 3)(1n+2)}$

$x^2 \cdot 0$

0 // $h = \infty$