P1.

a) $\lim_{n\to\infty} \sqrt{n^2+n} - n$

$\lim_{n\to\infty} \sqrt{n^2+n} = n^2+n \Rightarrow$ polinomio de grado positivo $\Rightarrow +\infty$

$\lim_{n\to\infty} n = +\infty$

Transformar el límite para transformar la indeterminación

$$\lim_{n\to\infty} \left(\sqrt{n^2+n} - n\right) \Big/ \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n}$$

$$\frac{\left(\sqrt{n^2+n}-n\right)\left(\sqrt{n^2+n}+n\right)}{\sqrt{n^2+n}+n}$$

$$\frac{\cancel{n^2}+n-\cancel{n^2}}{\sqrt{n^2+n}+n} =$$

$$\sqrt{n^2\cdot\left(1+\frac{1}{n}\right)}+n$$

$$\sqrt{n^2}\cdot\sqrt{\left(1+\frac{1}{n}\right)}+n$$

$$n\left(\sqrt{1+\frac{1}{n}}+1\right)$$

$$\lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\left(\sqrt{1+\frac{1}{n}}+1\right)} = \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty}\sqrt{1+\frac{1}{n}}+1}$$

$$\lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0$$

$$\frac{\lim_{n \to \infty} 1}{\lim_{n \to \infty} \left(\sqrt{1 + \frac{1}{n}} + 1\right)} = \frac{1}{\sqrt{1+0} \quad 1} = \frac{1}{2}$$

//

b) $\lim_{n \to \infty} \frac{n!}{n^n} = \frac{\lim_{n \to \infty} n!}{\lim_{n \to \infty} n^n} = \frac{\lim_{n \to \infty} (n)! = +\infty}{\lim_{n \to \infty} n^n = +\infty}$

$$\lim_{n \to \infty} \frac{n!}{n^n} = 0$$

P2

1)

$$a_0 = 1 \quad , \quad a_{n+1} = 3 - \frac{1}{a_n}$$

es creciente si $\forall n$ se cumple $x_{n+1} \geq x_n$

Desarrollamos la desigualdad

$$3 - \frac{1}{a_n} \geq a_n$$

$$\frac{2}{a_n} \geq a_n$$

$$2 \geq a_n^2$$

$$\sqrt{2} \geq a_n$$

b) $a_0 = 2$ $\qquad a_{n+1} = \dfrac{1}{3 - a_n}$

es Decresiente si $a_{n+1} \leq a_n$

$$\frac{1}{3-a_n} \leq a_n$$

$a_0 = 2$

$a_1 = \dfrac{1}{3-2} = 1$

$a_2 = \dfrac{1}{3-1} = \dfrac{1}{2}$

Como $2 \geq 1 > \frac{1}{2} \geq \dots \geq \dfrac{1}{3-a_n} \Rightarrow a_n$ es Decresiente

Demostrar que $0 < a_n \leq 2$

Si $a_n > 0$, entonces $a_{n+1} > 0$

$0 < a_{n+1} \Rightarrow 0 < \dfrac{1}{3-a_n}$ → como es decreciente y acotado superiormente

P3

a) $\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5} =$

$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\lim_{n \to \infty} \left| \frac{x^{n+1}}{(n+1)^5 \cdot 5^{(n+1)}} \cdot \frac{5^n \cdot n^5}{x^n} \right|$

$\lim_{n \to \infty} \left| \frac{\cancel{x^n} \cdot x \cdot \cancel{5^n} \cdot n^5}{(n+1)^5 \cdot \cancel{5^n} \cdot 5 \cdot \cancel{x^n}} \right|$

$\lim_{n \to \infty} \frac{x \cdot n^5}{(n+1)^5 \cdot 5} = \left| \frac{x}{5} \right| < 1 = \frac{1}{5} |x| < 5 \Rightarrow$ $\boxed{R = 5}$

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b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$

$$\lim_{n\to\infty} \left| \frac{(-2)^{n+1} \cdot x^{n+1}}{\sqrt[4]{n+1}} \cdot \frac{\sqrt[4]{n}}{(-2)^n \cdot x^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{-2^n} \cdot -2 \cdot \cancel{x^n} \cdot x \cdot \sqrt[4]{n}}{\sqrt[4]{n+1} \cdot \cancel{(-2)^n} \cdot \cancel{x}} \right|$$

$$\lim_{n\to\infty} \cancel{\frac{\sqrt[4]{n}}{\sqrt[4]{n+1}}} \cdot 2|x| < 1 \Rightarrow$$

$$1 \cdot 2|x| < 1 \Rightarrow \frac{1}{2}$$

$$|x| < \frac{1}{2}$$

P4

a) $f(x) = xe^x$

$f(0) + f'(0)x + \frac{f''(0)}{2!} \cdot x^2 + \frac{f^{n+1}(0)}{3!} x^3 + \dots$

$\sum_{n=0}^{\infty} \frac{f^n(0)}{n!} \cdot x^n$

$f(x) = xe^x$

$f'(x) = e^x + xe^x$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$f''(x) = 2e^x + xe^x$

$f'''(x) = 3e^x + xe^x$

$f''''(x) = 4e^x + xe^x$

$f(0) = 0$

$0 + x + \frac{2}{2!} x^2 + \frac{3x^3}{3!} + \frac{4x^4}{4!}$

$f'(0) = 1$

$x + x^2 + \frac{1}{2} x^3 + \frac{1}{6} x^4$

$f''(0) = 2$

$f'''(0) = 3$

$x \cdot \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$ //

$f''''(0) = 4$

(b)

$f(x) = \sinh x = \frac{1}{2}(e^{x} - e^{-x})$.

$f(x) = \frac{e^{x} - e^{-x}}{2}$ $\qquad f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$

$f'(x) = \frac{e^{x} + e^{-x}}{2}$ $\qquad \sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!} x^{n}$

$f''(x) = \frac{e^{x} - e^{-x}}{2}$

$f'''(x) = \frac{e^{x} + e^{-x}}{2}$

$f(0) = 0$ $\qquad \overset{0}{x} + \overset{1}{\frac{x^3}{3!}} + \overset{2}{\frac{x^3}{5!}}$

$f'(0) = 1$

$f''(0) = 0$ $\qquad \sum_{n=0}^{\infty} \frac{x^{(2n+1)}}{(2n+1)!}$ //

$f'''(0) = 1$

Radio de convergencia

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right|$$

$$\lim_{n\to\infty} \left| \frac{x^{2n+3}}{x^{2n+1}} \cdot \frac{(2n+1)!}{2n+3!} \right|$$

$$\lim_{n\to\infty} \frac{x^2}{(2n+3)(2n+2)}$$

$$x^2 \cdot \lim_{n\to\infty} \frac{1}{(2n+3)(2n+2)} = x^2 \cdot 0 = 0 //$$

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