PORTFOLIO OPTIMIZATION WITH FEEDBACK STRATEGIES BASED ON ARTIFICIAL NEURAL NETWORKS

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Supplementary Comparisons

S1 Geometric Brownian Motion

Assuming constant volatility dynamics $Y_t \equiv \sigma^2$, the full Heston model reduced to geometric Brownian motion (GBM)

$$\mathrm{d}P_t = rP_t, \qquad P_0 = 1, \tag{S1.1}$$

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t, \qquad S_0 = S^0. \tag{S1.2}$$

Equation S1.2 is calibrated on three years' worth of adjusted daily GSPC closing prices (1/1/2021–12/31/2023) with usual maximum likelihood estimation (Anum and Pokojovy, 2023). Table S(1) lists the parameter values used in simulations.

S^0	W^0	r	μ	σ
\$4770	\$1.0	0.050	0.085	0.176

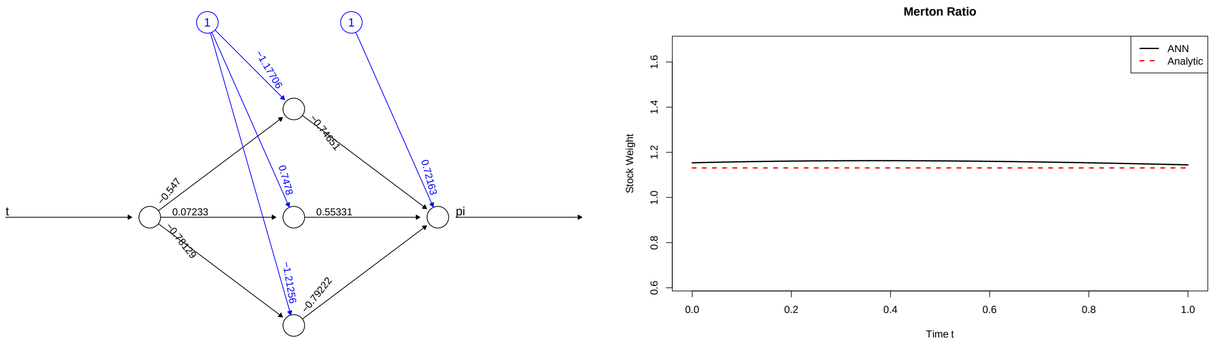
Table S(1): Initial GSPC price, initial wealth and estimated annualized parameter values.

Under general isoelastic utility, the optimal stock weight, known as classical Merton ratio, is given by

$$\pi^*_{\mathrm{GBM}} = \frac{\mu - r}{\eta\sigma^2}. \tag{S1.3}$$

For comparison purposes, we chose a grid of seven equispaced $1/\eta$ values on $[0.25, 1]$. Selecting ANN with a single three-neuron hidden layer (cf. Figure S(1)(a)), the Adam method was used to maximize the empirical power utility for each η value using the following training schedule: 100 steps with minibatch size $B = 10$ and $\varepsilon = 0.1$, 100 steps with $B = 10$, $\varepsilon = 0.05$ and 500 steps with $B = 50$, $\varepsilon = 0.01$. The training time did not exceed 3.5 hrs for each η.

Figure S(1) displays the trained ANN (left panel) and the resulting weight profile $\pi(t)$ (right panel) for $\eta = 1$. The profile appears to be nearly flat closely following analytic



(a) Trained ANN model

(b) Estimated Merton ratio vs time t

Figure S(1): Empirical results for logarithmic utility.

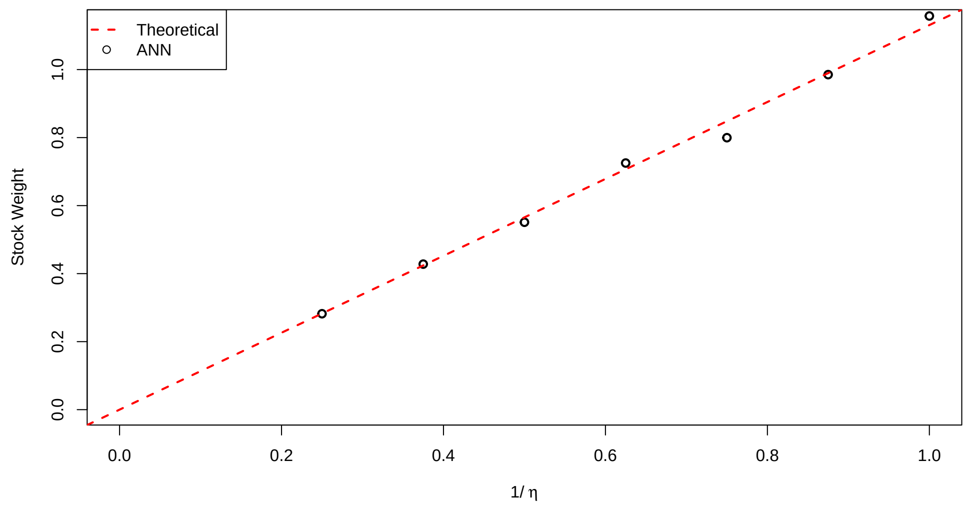


Figure S(2): Merton ratio vs $1/\eta$.

	$1/\eta$	0.250	0.375	0.500	0.625	0.750	0.875	1.000
ANN	Mean	0.05055	0.05563	0.05897	0.06050	0.06479	0.06643	0.07200
	Std error	$1.83{\cdot}10^{-5}$	$4.86{\cdot}10^{-5}$	$8.61{\cdot}10^{-5}$	0.00015	0.00019	0.00029	0.00041
Analyt.	Mean	0.05027	0.05555	0.05765	0.06408	0.06387	0.06753	0.07000
	Std error	$1.77{\cdot}10^{-5}$	$4.65{\cdot}10^{-5}$	$9.01{\cdot}10^{-5}$	0.00014	0.00021	0.00030	0.00040

Table S(2): Empirically estimated (based on 10,000 Monte Carlo replications) expected terminal utility and standard error thereof under analytic and ANN weights for various $1/\eta$ values.

one. Similar results were obtained for the remaining six η values. Figure S(2) plots the stock weight (averaged over 500 equispaced time points) vs $1/\eta$. Good agreement with the analytic optimum can be observed. The expected utility under both ANN and analytic optimal weights was simulated based on 10,000 Monte Carlo replications as reported in Table S(2). Again, the expected utility under ANN was on par with that under the analytic weight from Equation (S1.3) over the range of η's considered.

S2 Heston Model for S&P 500 and VIX

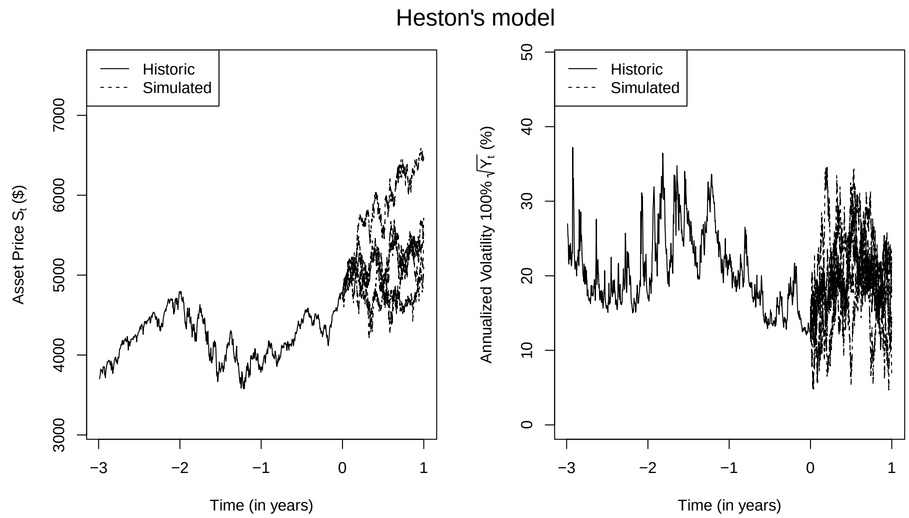
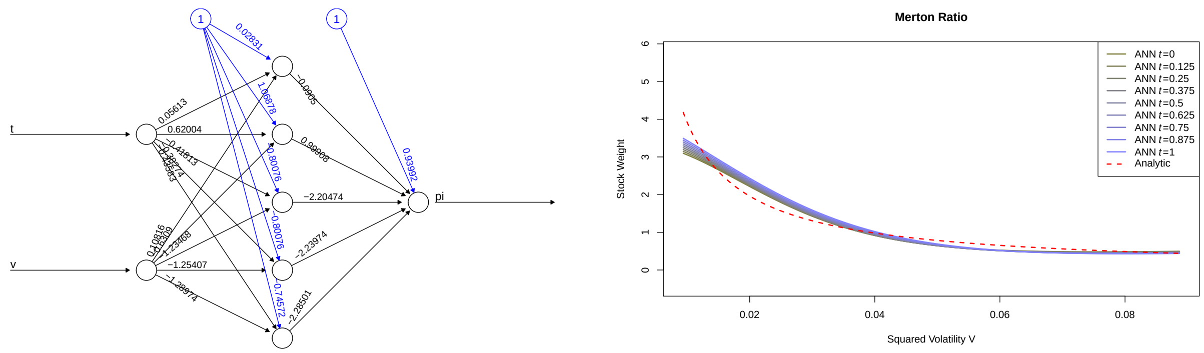


Figure S(3): Three years of historical data and five simulated one-year Heston market forecast paths for GSPC (left) and VIX (right).

S2.1 Isoelastic Utility with $\eta = 1$: Logarithmic Utility

	ANN	Analytic
Mean	0.07830822	0.07874819
Standard error	0.00060016	0.00060721

Table S(3): Empirically estimated (based on 10,000 Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and analytic strategies for Heston model applied to GSPC and VIX under logarithmic utility ($\eta = 1$).



(a) Trained ANN model

(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(4): Trained ANN model (left) and volatility-specific ANN-based strategy compared to analytic optimal strategy (right) for Heston model applied to GSPC and VIX under logarithmic utility ($\eta = 1$).

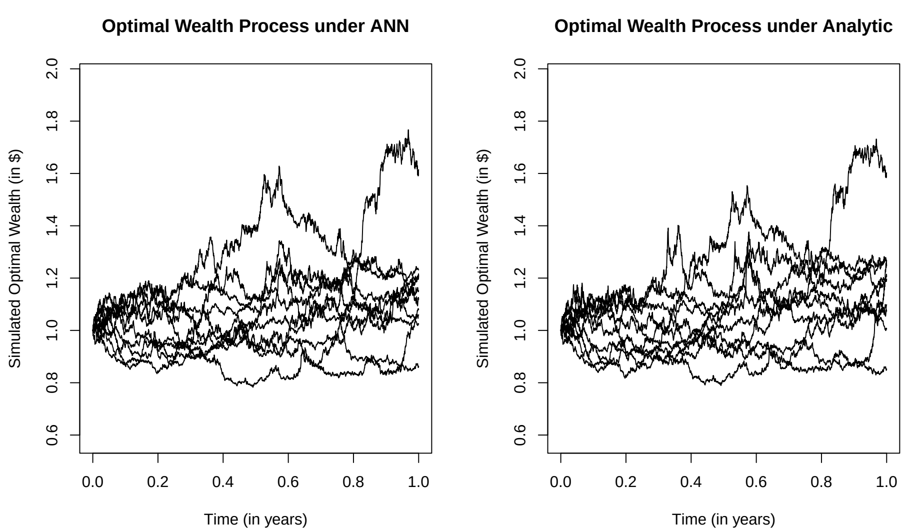
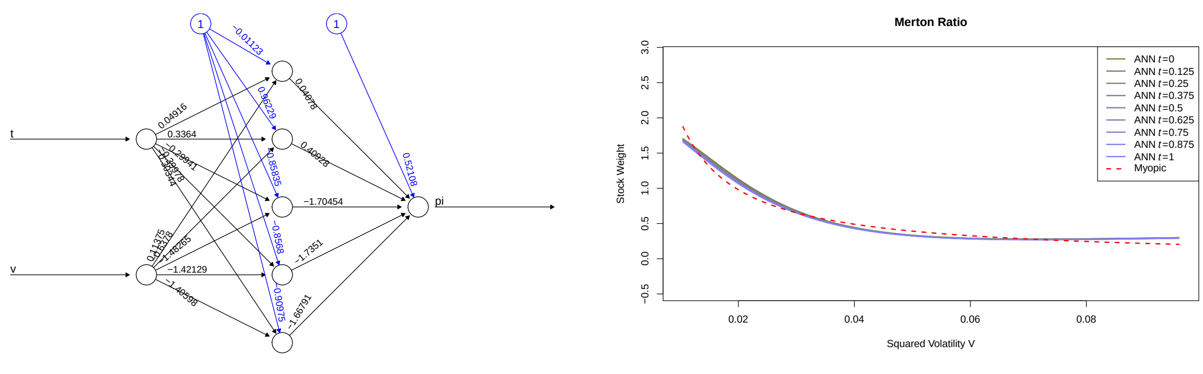


Figure S(5): Five simulated wealth paths under ANN-based (left) and analytic optimal strategy (right) for Heston model applied to GSPC and VIX under logarithmic utility ($\eta = 1$).

S2.2 Isoelastic Utility with $\eta = 2$

	ANN	Analytic
Mean	0.06226655	0.06223521
Standard error	0.00011913	0.00012930

Table S(4): Empirically estimated (based on 10,000 Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and myopic strategies for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 2$.



(a) Trained ANN model

(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(6): Trained ANN model (left) and volatility-specific ANN-based strategy compared to myopic strategy (right) for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 2$.

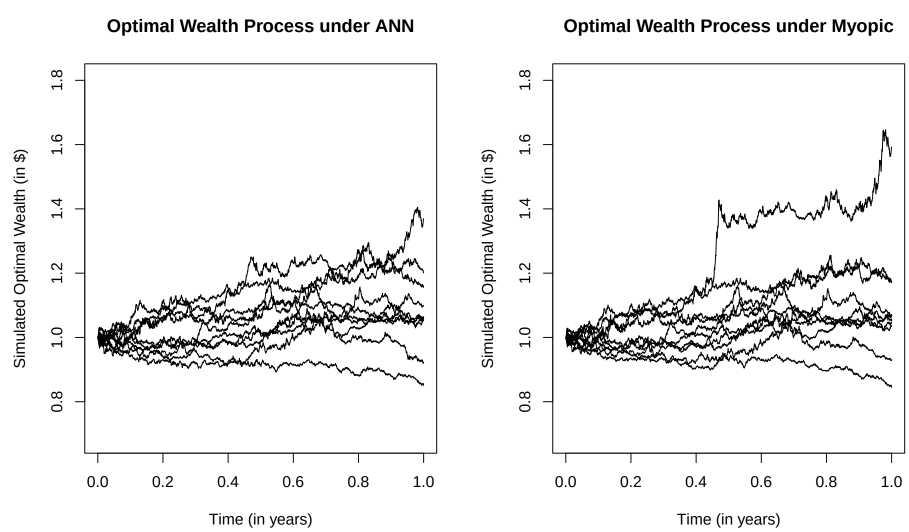
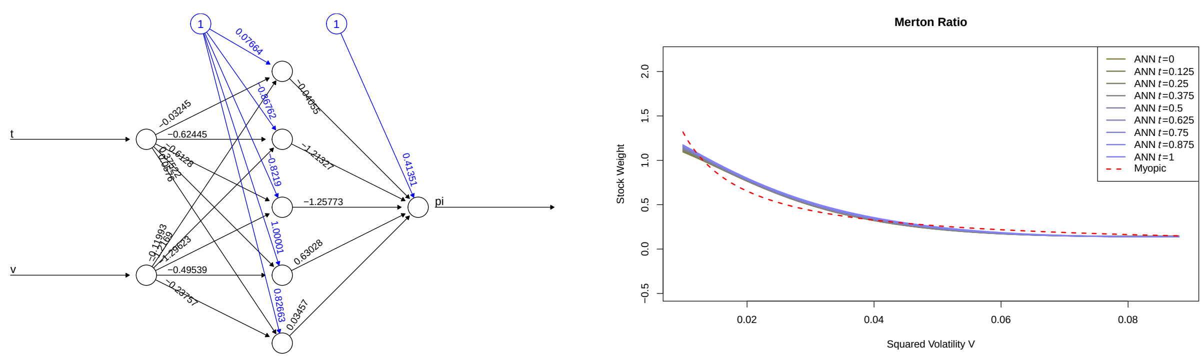


Figure S(7): Five simulated wealth paths under ANN-based (left) and myopic strategy (right) for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 2$.

S2.3 Isoelastic Utility with $\eta = 3$

	ANN	Analytic
Mean	0.05553235	0.05612797
Standard error	$5.228593 \cdot 10^{-5}$	$5.059058 \cdot 10^{-5}$

Table S(5): Empirically estimated (based on 10,000 Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and myopic strategies for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 3$.



(a) Trained ANN model

(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(8): Trained ANN model (left) and volatility-specific ANN-based strategy compared to myopic strategy (right) for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 3$.

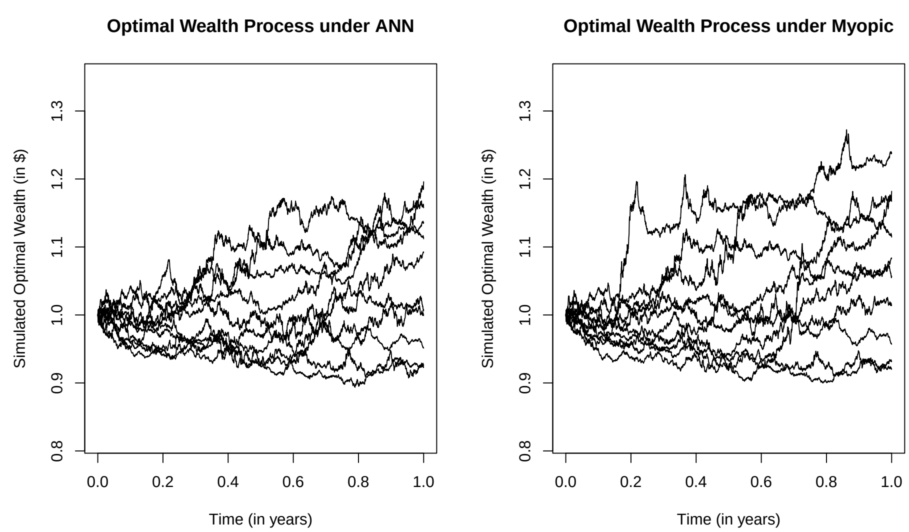
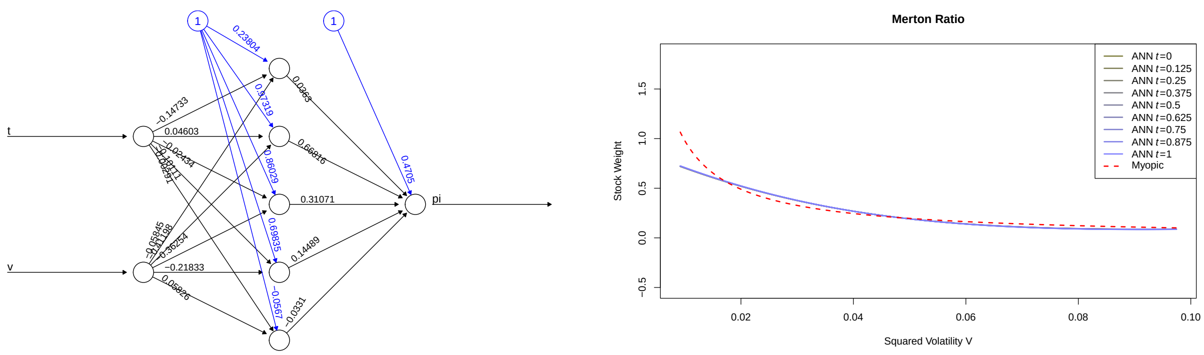


Figure S(9): Five simulated wealth paths under ANN-based (left) and myopic strategy (right) for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 3$.

S2.4 Isoelastic Utility with $\eta = 4$

	ANN	Analytic
Mean	0.05274725	0.0515179
Standard error	$2.407664 \cdot 10^{-5}$	$2.536713 \cdot 10^{-5}$

Table S(6): Empirically estimated (based on 10,000 Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and myopic strategies for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 4$.



(a) Trained ANN model

(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(10): Trained ANN model (left) and volatility-specific ANN-based strategy compared to myopic strategy (right) for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 4$.

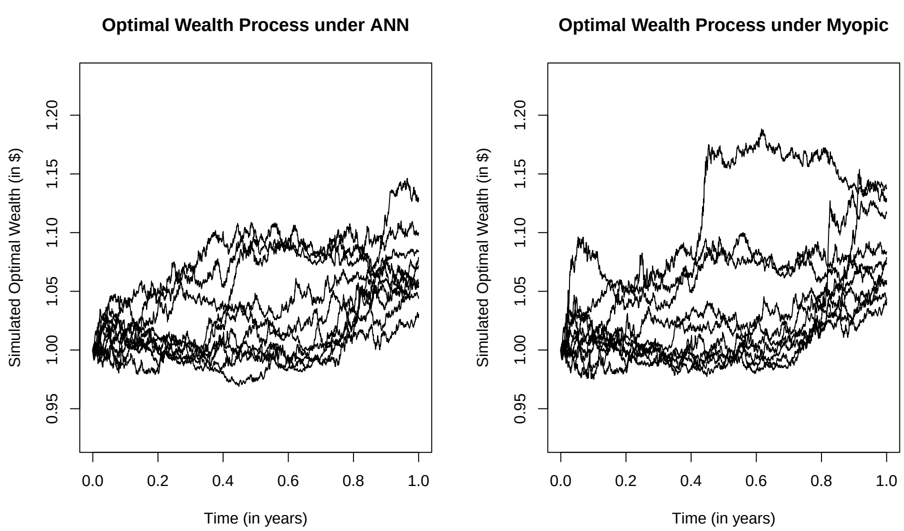


Figure S(11): Five simulated wealth paths under ANN-based (left) and myopic strategy (right) for Heston model applied to GSPC and VIX under isoelastic utility with $\eta = 4$.

S3 Heston Model for USO and OVX

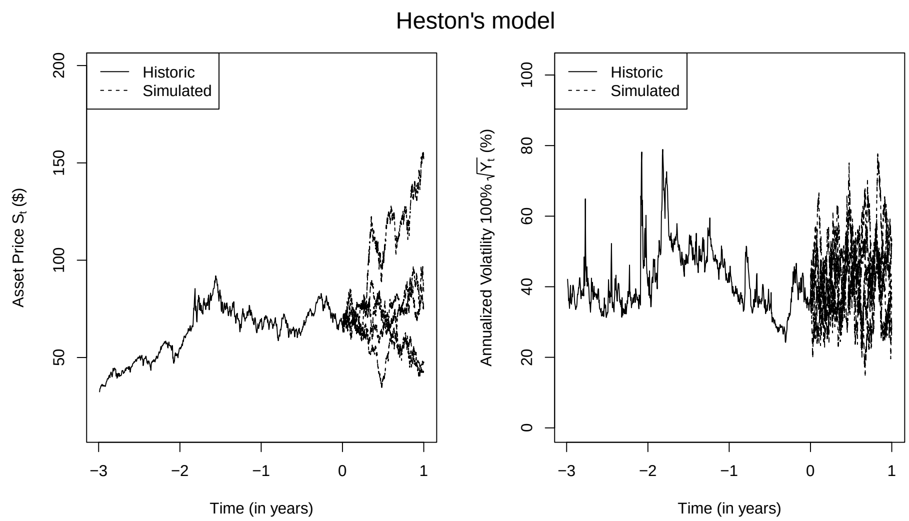
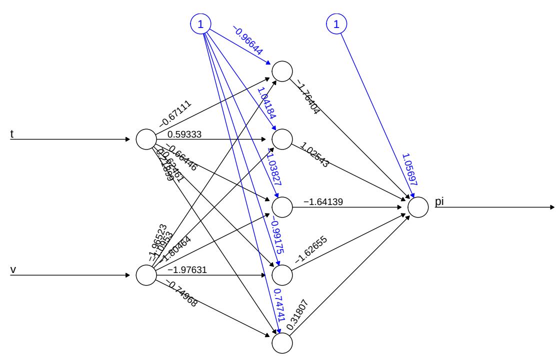


Figure S(12): Three years of historical data and five simulated one-year Heston market forecast paths for USO (left) and OVX (right).

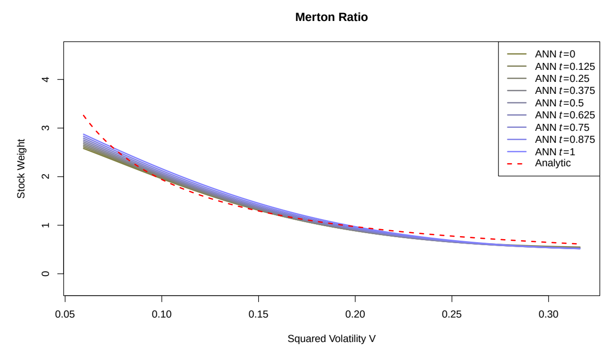
S3.1 Isoelastic Utility with $\eta = 1$: Logarithmic Utility

	ANN	Analytic
Mean	0.1823288	0.2039854
Standard error	0.0025774	0.0028946

Table S(7): Empirically estimated (based on $10,000$ Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and analytic strategies for Heston model applied to USO and OVX under logarithmic utility ($\eta = 1$).



(a) Trained ANN model



(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(13): Trained ANN model (left) and volatility-specific ANN-based strategy compared to analytic optimal strategy (right) for Heston model applied to USO and OVX under logarithmic utility ($\eta = 1$).

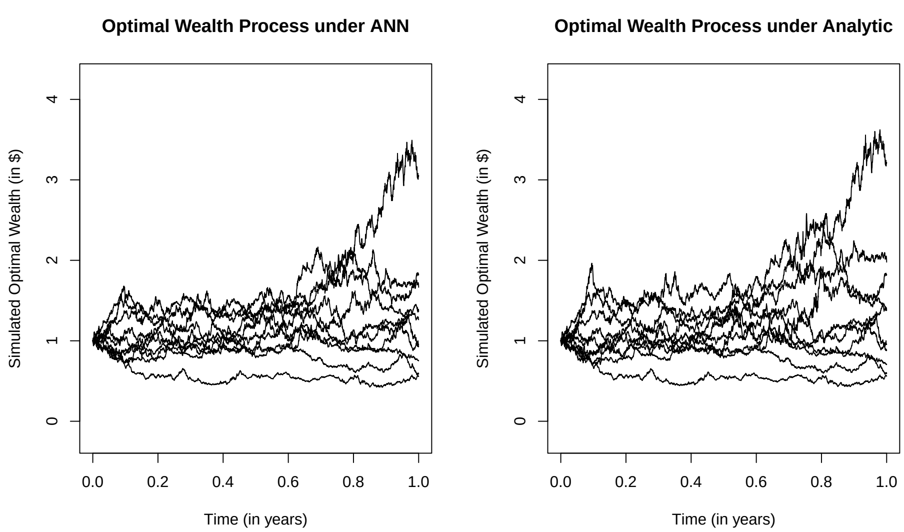
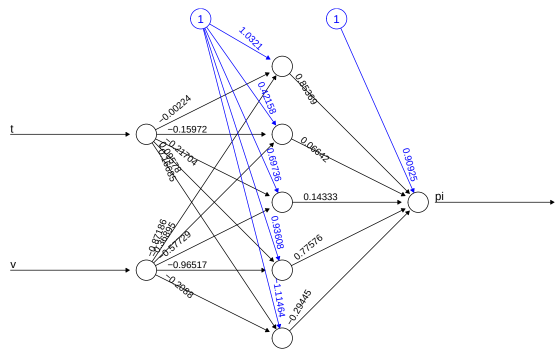


Figure S(14): Five simulated wealth paths under ANN-based (left) and analytic optimal strategy (right) for Heston model applied to USO and OVX under logarithmic utility ($\eta = 1$).

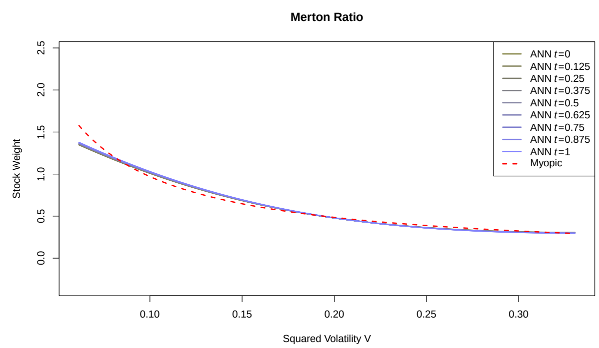
S3.2 Isoelastic Utility with $\eta = 2$

	ANN	Analytic
Mean	0.1127781	0.1132771
Standard error	0.0005482	0.0005552

Table S(8): Empirically estimated (based on 10,000 Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and myopic strategies for Heston model applied to USO and OVX under isoelastic utility with $\eta = 2$.



(a) Trained ANN model



(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(15): Trained ANN model (left) and volatility-specific ANN-based strategy compared to myopic strategy (right) for Heston model applied to USO and OVX under isoelastic utility with $\eta = 2$.

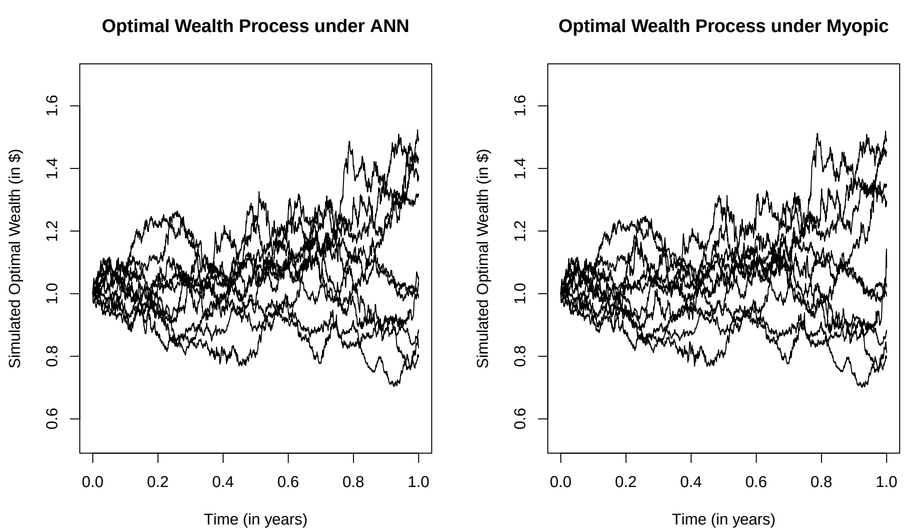
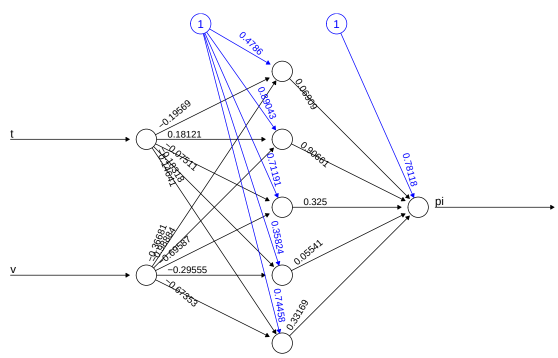


Figure S(16): Five simulated wealth paths under ANN-based (left) and myopic strategy (right) for Heston model applied to USO and OVX under isoelastic utility with $\eta = 2$.

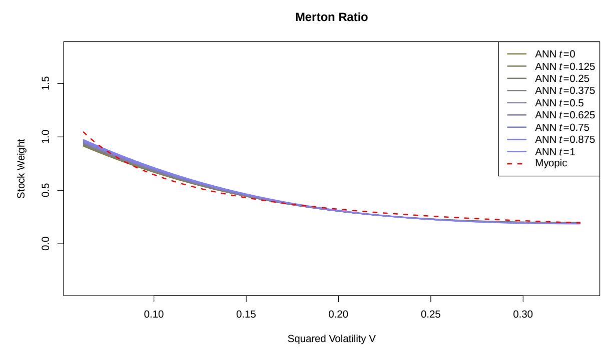
S3.3 Isoelastic Utility with $\eta = 3$

	ANN	Analytic
Mean	0.08527496	0.08565635
Standard error	0.00022383	0.00022056

Table S(9): Empirically estimated (based on $10,000$ Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and myopic strategies for Heston model applied to USO and OVX under isoelastic utility with $\eta = 3$.



(a) Trained ANN model



(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(17): Trained ANN model (left) and volatility-specific ANN-based strategy compared to myopic strategy (right) for Heston model applied to USO and OVX under isoelastic utility with $\eta = 3$.

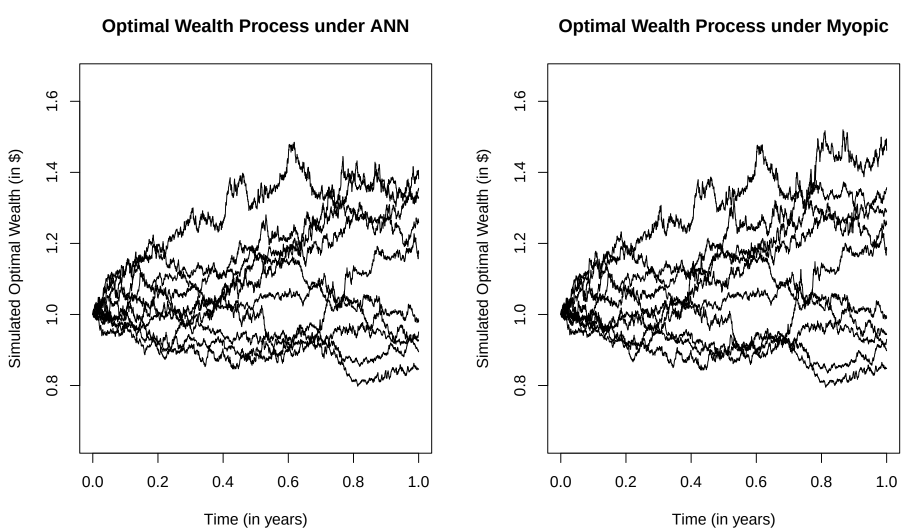
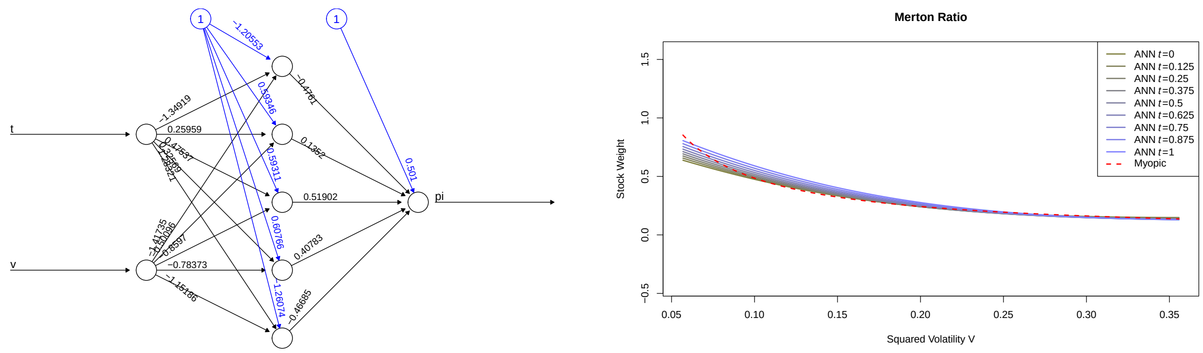


Figure S(18): Five simulated wealth paths under ANN-based (left) and myopic strategy (right) for Heston model applied to USO and OVX under isoelastic utility with $\eta = 3$.

S3.4 Isoelastic Utility with $\eta = 4$

	ANN	Analytic
Mean	0.07270631	0.07375263
Standard error	0.00011766	0.00011054

Table S(10): Empirically estimated (based on 10,000 Monte Carlo replications) expected terminal utility and standard error thereof under ANN-based and myopic strategies for Heston model applied to USO and OVX under isoelastic utility with $\eta = 4$.



(a) Trained ANN model

(b) Estimated Merton ratio vs sqr'd volatility y

Figure S(19): Trained ANN model (left) and volatility-specific ANN-based strategy compared to myopic strategy (right) for Heston model applied to USO and OVX under isoelastic utility with $\eta = 4$.

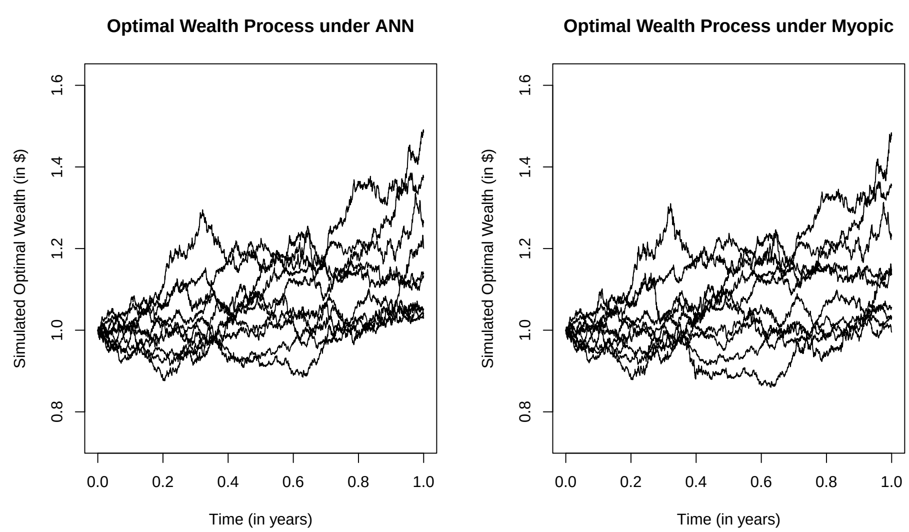


Figure S(20): Five simulated wealth paths under ANN-based (left) and myopic strategy (right) for Heston model applied to USO and OVX under isoelastic utility with $\eta = 4$.

Bibliography

Anum, A.T., Pokojovy, M., 2023. A hybrid method for density power divergence minimization with application to robust univariate location and scale estimation. Communications in Statistics – Theory and Methods 53, 5186–5209.