San Diego State University

Department of Mathematics and Statistics
Math 638
Continuous Dynamical Systems

Project Proposal:

Equation-free modeling vs. Data Assimilation: Accuracy Comparison for the Lorentz's System

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1 Introduction

In this project we will look at two methods of analyzing and interpreting data dynamics. We will focus on using *dynamic mode decomposition* (DMD) which is an equation-free method and the *extended kalman filter* (EKF) which is a data-assimilation method. The goal is to apply these to the Lorentz's system and measure how effective each method is at generating future data from a Lorentz's system with added noise.

2 Background and Motivation

2.1 Dynamic Mode Decomposition (DMD)

The technique of *dynamic mode decomposition* has important advantages for modeling high-dimensional complex systems. The algorithm for the dynamic mode decomposition is data-driven, equation-free, and reconstructs the underlying dynamics of the system from snapshot measurements. DMD has become popular as a method for systems with nonlinear dynamics. In addition, DMD can be modified to take advantage of limited measurements of a complex system.

The motivation of studying how dynamic mode decomposition works is that it has applications in physical, biological, and engineering systems.

2.2 Data Assimilation

Data assimilation is arguably one of the most useful techniques available to perform data-driven modeling. This method makes use of both data measurements collected in time about the system and a set of governing equations.

A major issue of modeling in general is the fact that both simulations and measurements are substantially influenced by noise and uncertainty, which corrupts their trustworthiness, at least to a degree. However, combining the two so that experimental data and the model help informing each other will improve the predictive capabilities of the model itself. In this project, we will outline the method of incorporating *innovation* to the model predictions, in particular by studying the role of the *Kalman filter* in the modeling of realistic systems, such as the Lorentz's Equation.

3 Presented Work

We will present and analyze the techniques of *dynamic mode decomposition* and *data assimilation*. By randomly generating data, we will use both of the aforementioned approaches to model the Lorentz's system with perturbed initial conditions and randomly-added noise. Finally, we will compare their results and derive conclusions with regards of the accuracy and efficiency of both data-driven and equation-free models.

At this stage, we are not ready to discuss or present the analysis of other dynamical systems or modeling techniques, but, if time and resources allow, we will look into it.

4 Bibliography

References

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