Instructions. Identify the group of 2 classmates with whom you will complete this assignment. Then, for each of the homework problems below,

1. Please have the group write up one complete solution in LaTeX.
2. Document the contributions of each member of the group in solving this problem. Be sure to include any references you have used to arrive at this solution, *e.g.* web pages, texts, etc.
3. Include sufficient written discussion to demonstrate your understanding of the problem.

Please email a PDF file of your homework solutions by **February 13** before 11:59 pm.

Homework Problems.

1. *This problem allows you to extend our discussion on Green's functions to a problem with a Neumann boundary condition.*

   Consider the boundary value problem

   $$u'' = f \quad \text{on } (0,1),$$
   $$u'(0) = \alpha, \quad u(1) = \beta.$$

   (a) Determine the boundary value problem for Green's function and solve it.

   (b) Use Green's function to determine the solution of the boundary value problem above.

   (c) Using your results from above as guidance, find the general formulas for the elements of the inverse of the matrix in equation (2.54) in the text. Write out the 5×5 matrices A and A^{-1} for the case in which $h = 0.25$.

2. *This problem allows you to investigate the solvability condition for Neumann problems.*

   Determine the null space of the matrix A^T, where A is given in equation (2.58) in the text. Verify that condition (2.62) in the text must hold for the linear system to have solutions.

3. *Solving a nonlinear boundary value problem.*

   (a) Write a program to solve the boundary value problem for the nonlinear pendulum discussed in the text. See if you can find yet another solution for the boundary conditions illustrated in Figures 2.4 and 2.5 in the text.

   (b) Find a numerical solution to this boundary value problem with the same general behavior as see in Figure 2.5 for the case of a longer time interval, say $T = 20$, again with $\alpha = \beta = 0.7$. Try larger values of T. What does $\max_i \theta_i$ approach as T is increased? Note that for large T this solution exhibits "boundary layers."