Review of exponential functions

Note: For a more detailed review, see sections 1.5 and 1.6 in the textbook.

Exponential functions are often used for modeling and solving real-world situations involving *growth* or *decay*; e.g. financial growth, population growth, radioactive decay, etc.

variable is the exponent

The equation

$$f(x) = b^x, b > 0, b \neq 1$$

defines an **exponential function**. We call the constant b the **base**. The domain is all real numbers and the range is all positive numbers.

$$\left.\begin{matrix} x^2 \\ x^3 \\ x^4 \end{matrix}\right\} \text{Power Functions}$$

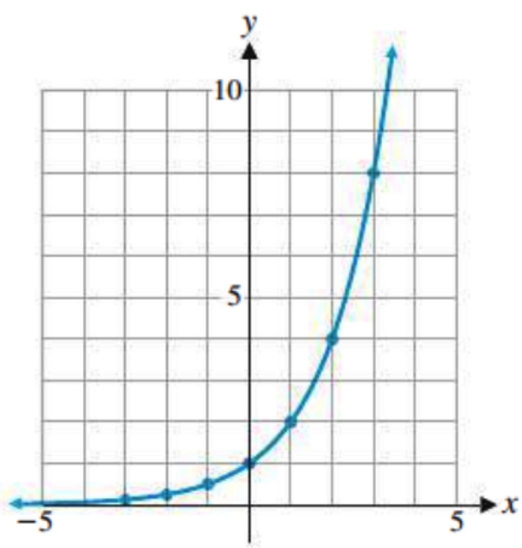


Figure 1 $y = 2^x$

Exponential Growth

The constant e

e is a constant that occurs frequently in problems involving population or financial growth.

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2.718281828459045\ldots$$

Kind of a "cousin" to pi (π)

Examples involving exponential functions:

(a) $f(x) = 2^x$

(b) $g(x) = e^x$

(c) $h(x) = 3e^x - x^2 + 5x - 1$

If a principal P is invested at an annual rate r (written as a decimal) compounded continuously, then the amount A in the account at the end of t years is given by

$$A = Pe^{rt}$$

Example: If \$100 is invested at 6% compounded continuously, what amount will be in the account in 2 years?

P = principle = 100

r = interest rate (%) = 0.06

t = time = 2

$A = 100e^{0.06(2)}$

$A = 100e^{0.12}$

$A = \$112.75$

In 2 years, there will be \$112.75 in the account

Review of logarithmic functions

Logarithms provide a tool that we can use to manipulate exponential expressions; i.e. a tool we can use to solve exponential equations.

For $b > 0$ and $b \neq 1$

$$x = b^y \qquad \Leftrightarrow \qquad y = \log_b x$$

Exponential form **Logarithmic form**

In words,

$$y = \log_b x$$

means:

y is the exponent of b that gives us x

Example: Solve $52 = 10^x$

$x = b^y \rightarrow y = \log_b x$

$52 = 10^x \; = \; x = \log_{10} 52$

$x = 1.716$

since log has a base of 10 by default, it can be written as:

$x = \log 52$

x is the power of 10 that gives us 52

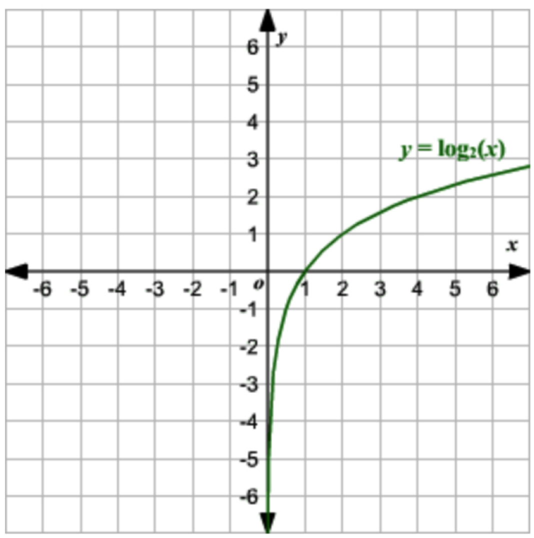
$x = \log_{10} 52 = \log 52$

$x = \log 52$

$x = 1.716$

$10^{1.716} \approx 51.99$

The **domain** of the logarithmic function is $(0, \infty)$ and the **range** is $(-\infty, \infty)$.



If b, M, and N are positive real numbers, $b \neq 1$, and p and x are real numbers, then

Product rule: $\log_b MN = \log_b M + \log_b N$

Quotient rule: $\log_b \frac{M}{N} = \log_b M - \log_b N$

Power rule: $\log_b M^p = p \log_b M$

Example: If \$100 is invested at 6% <u>compounded continuously</u>, how long until the amount doubles?

$$A = Pe^{rt}$$

$P = \$100$

$r = 0.06$

$t = ?$

$A = 2P = \$200$

$$200 = 100e^{0.06t}$$ ← Exponential Equation

$$\frac{200}{100} = \frac{100e^{0.06t}}{100}$$

$$2 = e^{0.06t}$$ First Isolate and then do logs

$$\ln 2 = \ln(e^{0.06t})$$ ← take ln of both sides

$$\ln 2 = 0.06t \ln(e)$$

$$\ln 2 = 0.06t\,(1)$$

$$\frac{\ln}{0.06} = \frac{0.06t}{0.06}$$

$$t = \frac{\ln}{0.06} \approx 11.55$$

It will take 11.55 years to double our money

Section 3.2 - Derivatives of exponential and logarithmic functions

For $b > 0$, $b \neq 1$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}b^x = b^x \ln b$$

Be careful! The power rule *only applies* for functions of the form $f(x) = x^n$ for any real number n.

For $b > 0$, $b \neq 1$, and $x > 0$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\log_b x = \frac{1}{\ln b}\left(\frac{1}{x}\right)$$

Find $f'(x)$

(a) $f(x) = 5e^x$

(b) $f(x) = 5e^x + 3x + 1$

(c) $f(x) = x^3 - 3e^x$

(d) $f(x) = 6\ln x - x^2 + 1$

(e) $f(x) = x^e$

(f) $f(x) = \log x$

(g) $f(x) = \log_3 x + 2x - 1$

Example: Find the equation of the line tangent to the graph of $f(x) = 3 + \ln x$ when $x = 1$