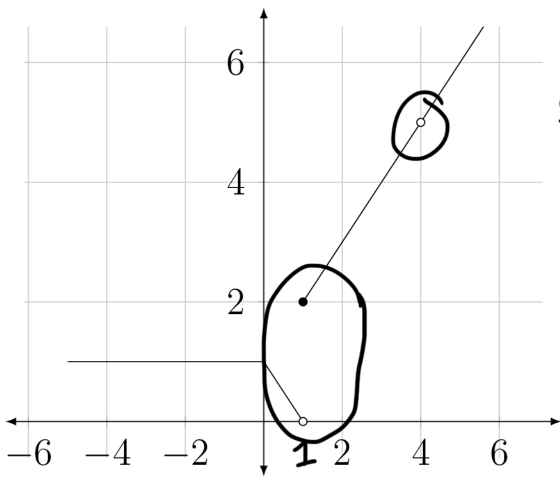
Survey

Last time...

Definition of Limit:
We write
$$\lim_{x\to c} f(x) = L$$
or
$$f(x) \to L \text{ as } x \to c$$
if the functional value $f(x)$ is close to the *single* real number L whenever x is close, but not equal, to c (on either side of c).

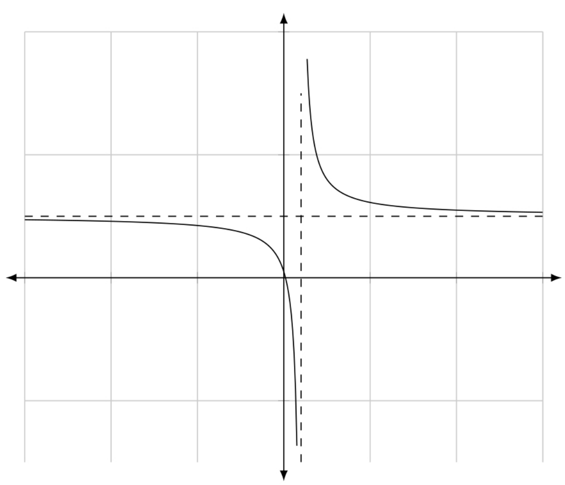
$\lim_{x\to 1^+} f(x)=2$

$\lim_{x\to 1^-} f(x)=0$



$f(4)=$ DNE or undefined

$\lim_{x\to 4} f(x)=5$



horizontal and verticle asymptote

Most of the time
$$\lim_{x\to c} f(x) = f(c)$$
But – as we've seen – this isn't true **all** of the time ...and that's why we have limits!

Indeterminate form

Recall: $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$ for any rational function $\frac{f(x)}{g(x)}$ with $g(c) \neq 0$.

Question: What if $g(c) = 0$?

Simplify!

Example: Find

$$\lim_{x\to 1} \frac{x^2-1}{x-1}$$

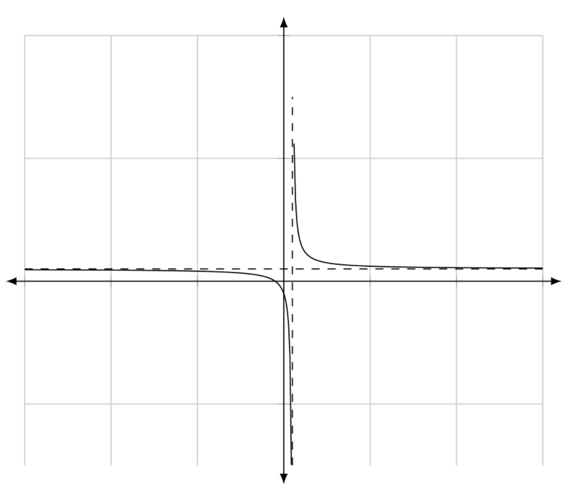
$$\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} \qquad x+1 \qquad 1+1=2$$

$$\lim_{x\to 1} f(x) = \frac{x^2-1}{x-1} = 2$$

Example: Find

$$\lim_{x\to 1} \frac{x^2-1}{(x-1)^2}$$

$$\frac{(x+1)\cancel{(x-1)}}{(x-1)\cancel{(x-1)}} \qquad \frac{(x+1)}{(x-1)}$$

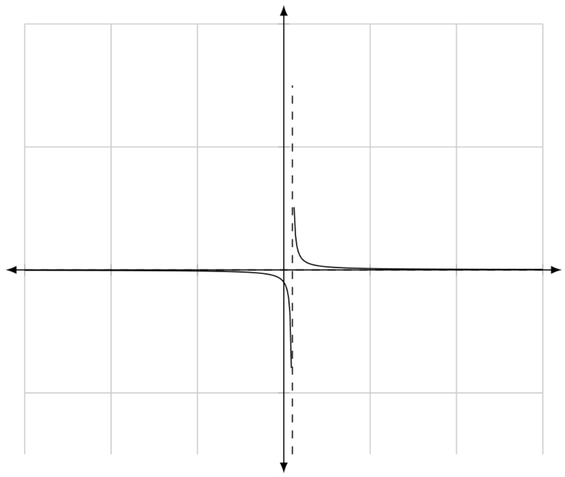


$$\frac{1+1}{1-1} \qquad \frac{2}{0} = \text{Undefined}$$

Example: Find

$$\lim_{x \to 1} \frac{1}{x-1}$$

$\frac{1}{x-1}$ $\frac{1}{1-1}$ $\frac{1}{0}$ = undefined



If $\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

is said to be **indeterminate**.

If $\lim_{x \to c} f(x) \neq 0$ and $\lim_{x \to c} g(x) = 0$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

does not exist.

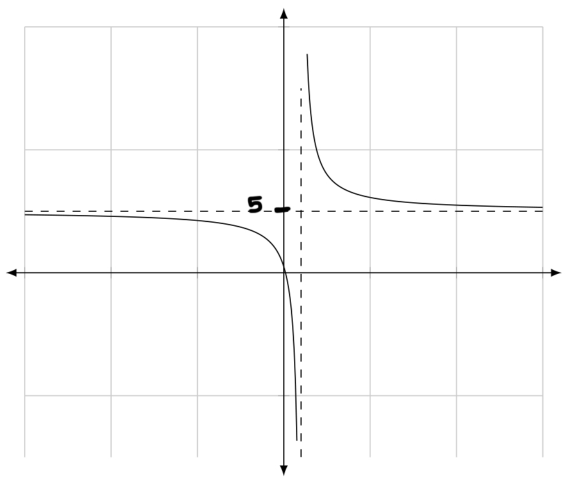
Infinite limits and limits at infinity

Reminder:

$$\lim_{x \to c} f(x) = L \text{ means } f(x) \to L \text{ as } x \to c$$

Infinite limits and **limits at infinity** are two *special cases* of limits.

- An *infinite limit* is the idea that sometimes a limit doesn't exist because the value of the function *grows without bound* as $x \to c$.
- A *limit at infinity* refers to the idea that the value of a function might *approach* a single value as the input "grows without bound;" i.e. as $x \to \infty$.



What does $f(x)$ approach as $x \to 2^-$?

$\lim_{x \to 2^-} f(x) = -\infty$ (going down infinitely)

What does $f(x)$ approach as $x \to 2^+$?

(going upward infinitely)

$\lim_{x \to 2^+} f(x) = \infty$

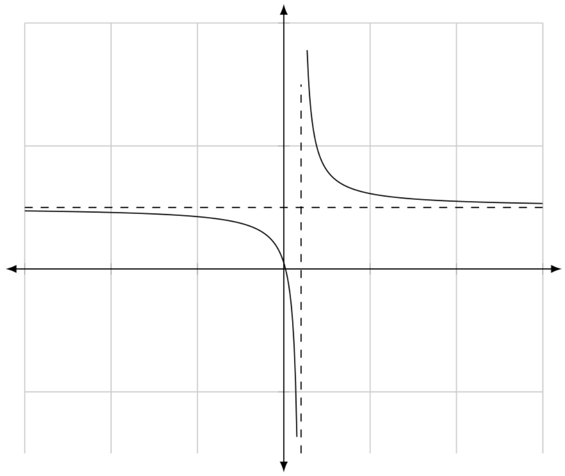
What does $f(x)$ approach as $x \to \infty$?

$\lim_{x \to \infty} f(x) = 5$

What does $f(x)$ approach as $x \to -\infty$?

$\lim_{x \to -\infty} f(x) = 5$

A graphical approach to infinite limits and limits at infinity



Infinite limit $\Leftrightarrow$ vertical asymptote

The vertical line $x = a$ is a **vertical asymptote** for the graph of $y = f(x)$ if

$$\lim_{x \to a^+} f(x) = \infty \text{ (or } -\infty)$$

$$\text{or } \lim_{x \to a^-} f(x) = \infty \text{ (or } -\infty)$$

Limit at infinity $\Leftrightarrow$ horizontal asymptote

The horizontal line $y = b$ is a **horizontal asymptote** for the graph of $y = f(x)$ if

$$\lim_{x \to \infty} f(x) = b \text{ or } \lim_{x \to -\infty} f(x) = b$$

Finding asymptotes for rational functions

If

$$f(x) = \frac{n(x)}{d(x)}$$

is a rational function and $n(c) \neq 0$ but $d(c) = 0$, then $x = c$ is a vertical asymptote.

Example: Does $f(x) = \dfrac{x+2}{x-2}$ have a vertical asymptote?

yes, X=2 is the verticle asymptote.

The degree is the value of the largest exponent.

Consider the rational function

$$f(x) = \frac{n(x)}{d(x)}$$

where $n(x)$ and $d(x)$ are polynomials.

numerator denominator

Case 1. If degree $n(x) <$ degree $d(x)$, then $y = 0$ is the horizontal asymptote.

numerator denominator

Case 2. If degree $n(x) =$ degree $d(x)$, then $y = \dfrac{a}{b}$ is the horizontal asymptote where a is the leading coefficient of $n(x)$ and b is the leading coefficient of $d(x)$.

numerator denominator

Case 3. If degree $n(x) >$ degree $d(x)$, there is no horizontal asymptote.

Example: Does $f(x) = \dfrac{x+2}{x-2}$ have a horizontal asymptote?

degrees are the same

yes, y= 1 is the horizontal asymptote.

Find all vertical and horizontal asymptotes

Problem 1: $f(x) = \dfrac{1}{x-2}$

horizontal asymptote: $y=0$ ✓

verticle asymptote: $x=2$ ✓

Problem 2: $g(x) = \dfrac{3x-1}{x-2}$

horizontal asymptote, $y=3$ ✓

verticle asymptote: $x=2$ ✓

Problem 3: $h(x) = \dfrac{x^2}{x-2}$

horizontal asymptote: DNE

verticle asymptote: $x=2$

Problem 4: $j(x) = \dfrac{x-3}{x^2-4x+3}$ $\dfrac{\cancel{x-3}}{\cancel{(x-3)}(x-1)}$ $\dfrac{1}{(x-1)}$

horizontal asymptote: $y=0$

verticle asymptote: $x=1$

Problem 5: $k(x) = \dfrac{2x^2}{x^2-x-6}$ $\dfrac{2x^2}{(x-3)(x+2)}$

horizontal asymptote: $y=2$

verticle asymptote: $x=-2$

$(-2)^2-(-2)-6$

$4+2-6$

$6-6$

0

Limits at infinity

Example: For $p(x) = 5x^3 - 2x^2 + x - 7$. Find $\lim_{x\to\infty} p(x)$ and $\lim_{x\to-\infty} p(x)$.

For a polynomial function

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

$$\lim_{x\to\infty} p(x) = \lim_{x\to\infty} a_n x^n$$

and

$$\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} a_n x^n$$