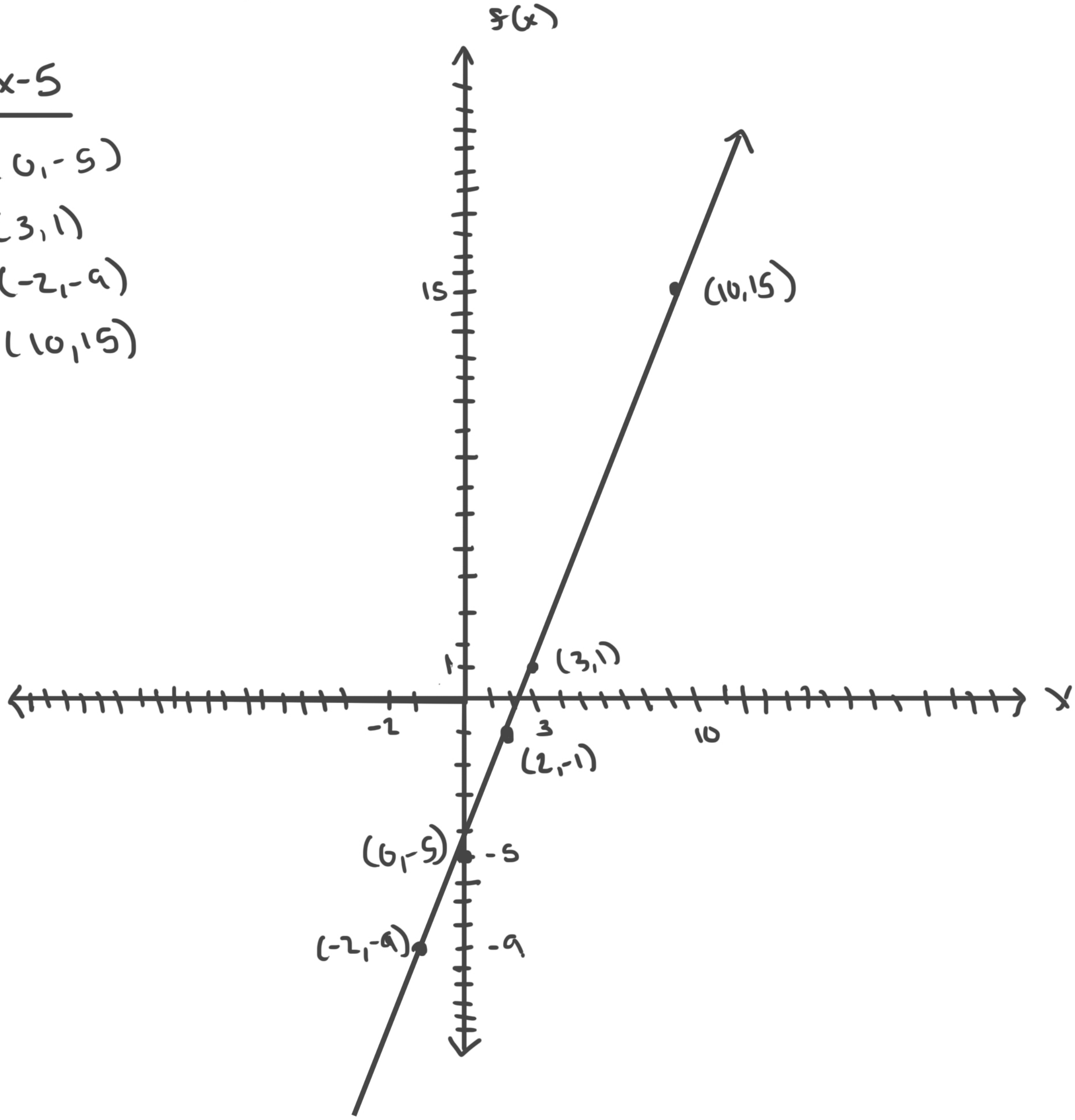
Point-by-point Plotting

To sketch the graph of a function in two variables, we plot sufficiently many points of the form $(x, f(x))$ so that the shape of the graph is apparent, and then we connect those points with a smooth curve. This process is called point-by-point plotting.

$(x, f(x))$

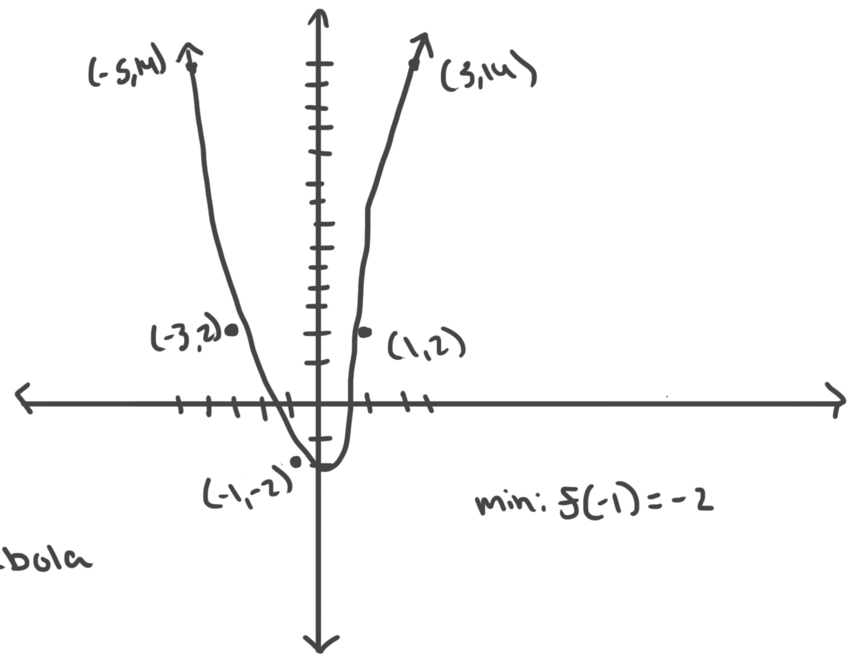
Example: Sketch a graph of $f(x) = 2x - 5$

x	$f(x)=2x-5$	
0	-5	(0,-5)
3	1	(3,1)
-2	-9	(-2,-9)
10	15	(10,15)



Example: Sketch a graph of $f(x) = (x+1)^2 - 2$

x	$f(x) = (x+1)^2 - 2$
-5	14
-3	2
-1	-2
1	2
3	14



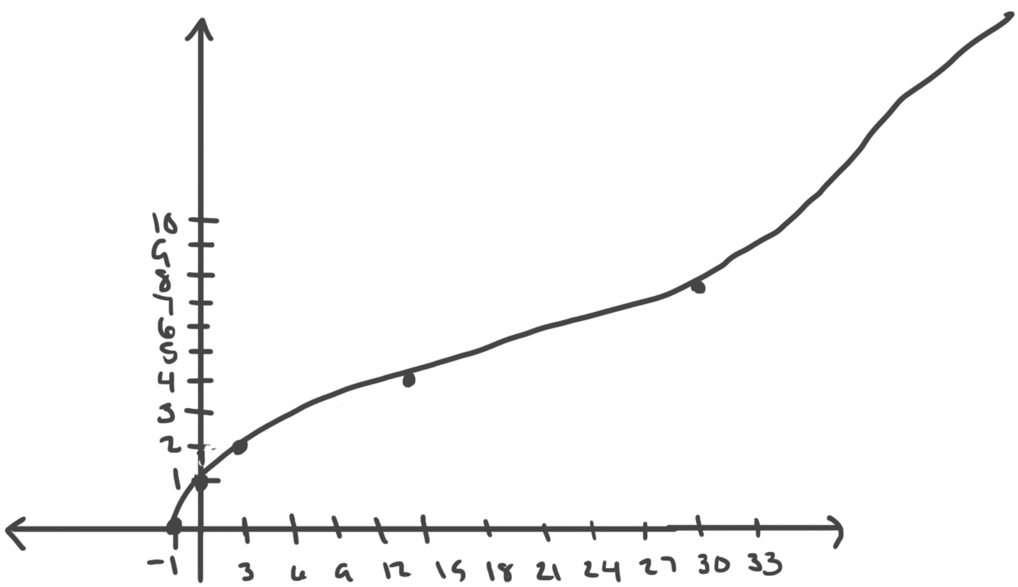
min: $f(-1) = -2$

resembles a positive parabola

Example: Sketch a graph of $f(x) = \sqrt{x+1}$

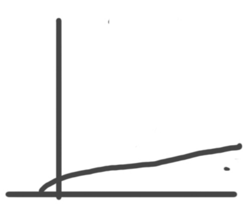
cannot have a negative domain

x	$f(x) = \sqrt{x+1}$
3	2
15	4
31	8
-1	0
0	1



resembles half a parabola

Zoomed out:



domain: $[-1, \infty)$

range: $[0, \infty)$

Application

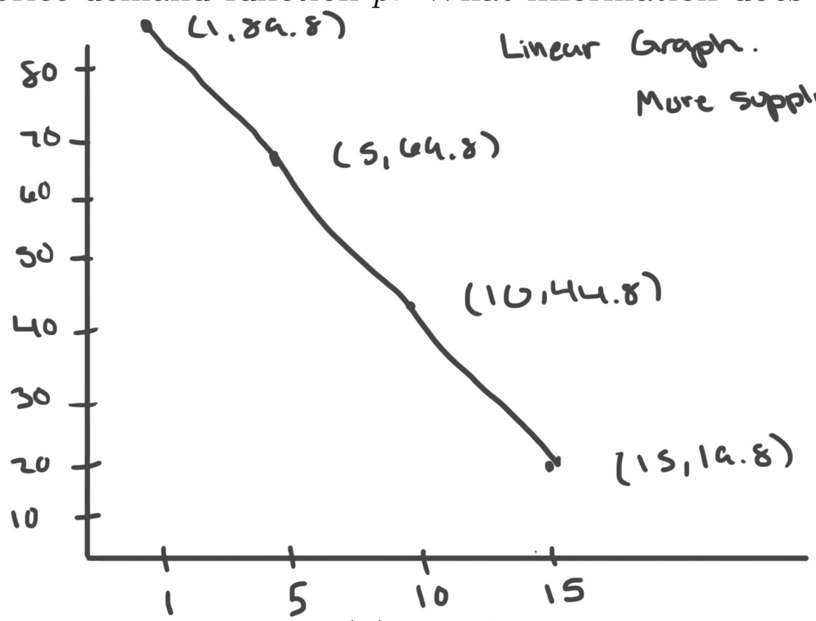
A manufacturer of a popular digital camera wholesales the camera to retail outlets throughout the United States. Using statistical methods, the financial department in the company produced price–demand data and an analyst obtained the following price–demand function that modeled that data

$$p(x) = 94.8 - 5x, \quad 1 \leq x \leq 15$$

where p is the wholesale price per camera at which x million cameras are sold.

(A) Plot the graph of the price-demand function p. What information does this graph give?

X	P(x) = 94.8 - 5x
1	89.8
5	69.8
10	44.8
15	19.8



Linear Graph.

More supply = lower price

(B) What is the company's revenue function, $R(x)$, for this camera, and what is its domain?

revenue = price × number of sales

$R(x) = x\,p(x)$

$R(x) = x(94.8 - 5x) = 94.8x - 5x^2$

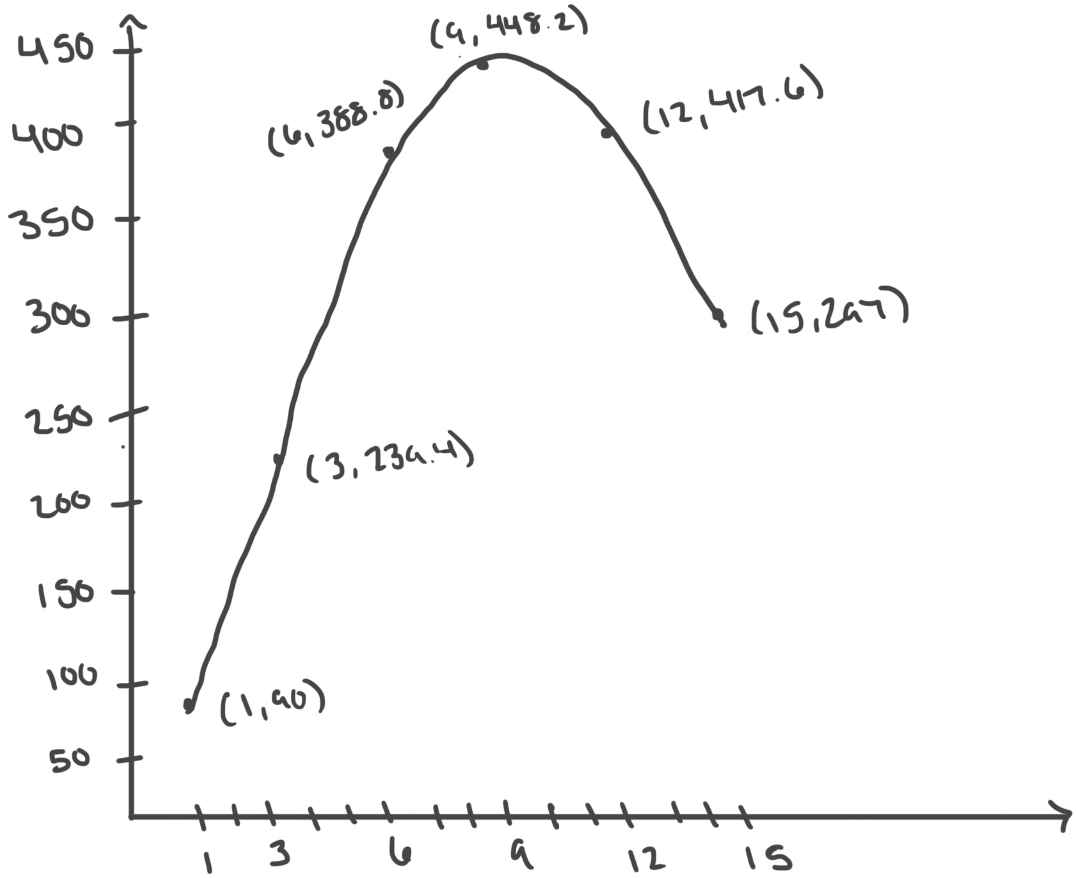
Domain: $[1, 15]$

(C) Complete the table, computing revenues to the nearest million dollars.

$R(x) = 94.8x - 5x^2$
$[1, 15]$

x (millions)	$R(x)$ (million \$)
1	90
3	239.4
6	388.8
9	448.2
12	417.6
15	297

(D) Plot the data in the table. Then sketch a graph of the revenue function using these points. What information does this graph give?



max: 448.2

The graph resembles a negative parabola

The graph tells us that their revenue is highest when they have sold 9 million products for a total revenue of \$448.2 million.

Linear equations and linear functions

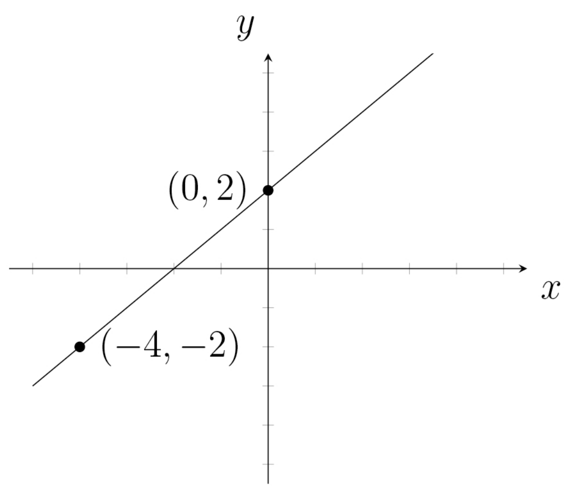
A **linear equation in two variables** is an equation that can be written in the standard form

$$Ax + By = C$$

where A, B, and C are constants (A and B not both 0), and x and y are variables.

The graph of a linear equation is a straight line and the equation for a straight line is a linear equation.

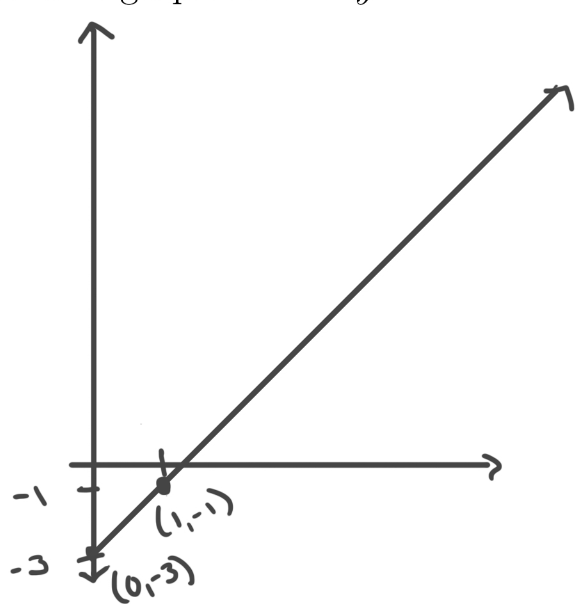
Example: Here's the graph of $-x + y = 2$.



Fact: To graph a line you need *only* two (distinct) points on that line.

Example: Sketch a graph of $2x - y = 3$.

$2x - y = 3$
$y = 2x - 3$



x	f(x)	
0	-3	(0,-3)
1	-1	(1,-1)
2	1	(2,1)

Slope of a line

All non-vertical lines have a slope.

If (x_1, y_1) and (x_2, y_2) are two points on a line with $x_1 \neq x_2$, then the **slope** of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: *The slope of a line, m, is the same for any two points on that line.*

The slope of a line gives lots of information about that line (more in a bit) and–in some sense–the usefulness of that information is why we have Calculus!

Useful fact: We can write an equation for a given line if we know…

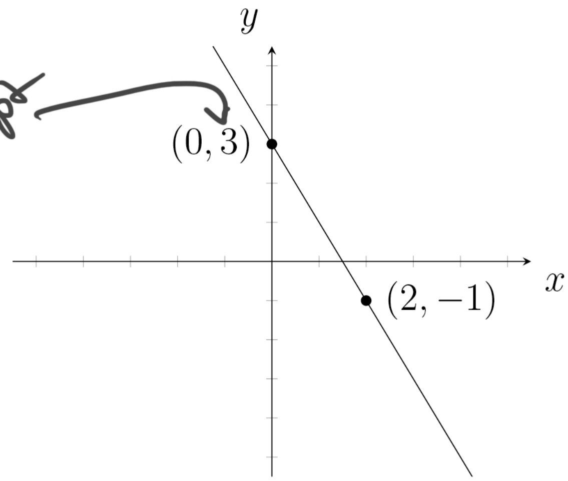
- any two points that lie on the line, or
- the slope and any point that lies on the line

It's helpful to know the common forms for the equation of a line:

- **Slope-intercept form:**
$$y = mx + b$$
where m is the slope and b is the $y-$intercept
- **Point-slope form:**
$$y - y_1 = m(x - x_1)$$
where m is the slope and (x_1, y_1) is a point on the line
- **Standard form:**
$$Ax + By = C$$
where A, B, C are constants and not both A and B are 0

Example: Find an equation for the line.

y-intercept



$$y = mx + b$$

slope

y-intercept

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$$

$$y = -2x + 3$$

x	$f(x) = -2x+3$
0	$-2(0)+3$ $= 3$
2	$-2(2)+3$ $= -1$

Example: Find an equation for the line with slope $m = -3$ that passes through the point $(0, 4)$.

y-intercept

slope

$y = -3x + 4$

Example: Find an equation for the line with slope $m = 2$ that passes through the point $(-1, 2)$.

$x_1 \quad y_1$

slope

point slope

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - (-1))$

$y = 2(x+1) + 2$

$y = 2x + 2 + 2$

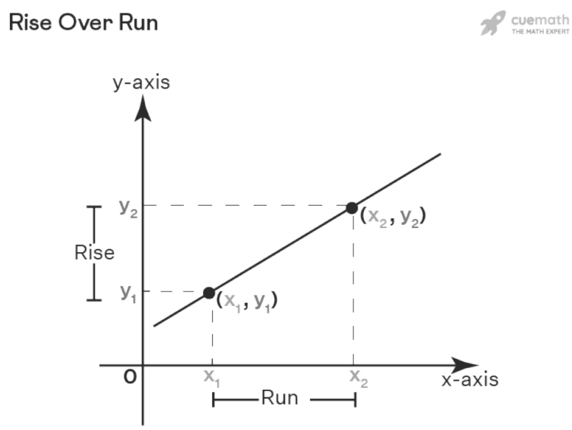
$y = 2x + 4$

Interpreting slope

Slope is a measure of the *steepness* of a line.

Slope is often described as

$$m = \frac{\text{rise}}{\text{run}}$$



If the slope < 0, then the line drops as we go from left to right.

If the slope > 0, then the line rises as we go from left to right.

Example: Sketch the graph of a line with slope $\frac{1}{3}$ that passes through $(0, 2)$.

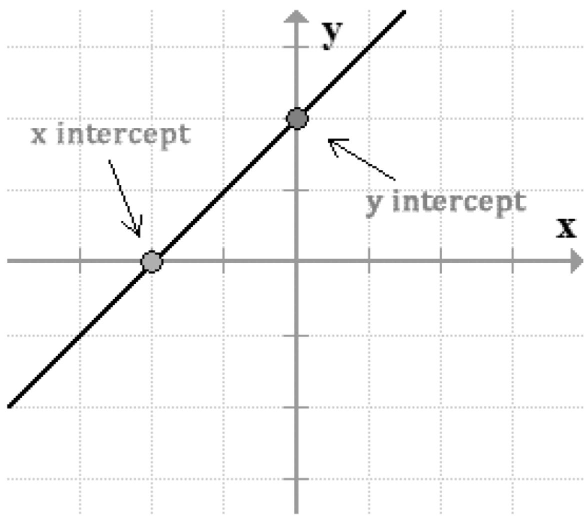
Example: Sketch the graph of a line with slope -2 that passes through $(-1, 2)$.

Question: How would you describe a line with slope $m = 0$?

Finding intercepts

A point where a graph crosses the $x-$axis is called an $x-$**intercept** and has a $y-$coordinate of $y = 0$.

The point where a graph crosses the $y-$axis is called the $y-$**intercept** and has an $x-$coordinate of $x = 0$.



Example: Find all intercepts for $f(x) = 3x - 9$.

Example: Find all intercepts for $f(x) = x^2 - 2x - 3$.