Announcements

question 2: $dy = f'(x)dx$

Test 2 will be given in your recitation meeting, THIS WEEK (Wednesday, 10/16). This test will cover sections 2.5 – 3.4.

- 2.5 - Basic differentiation properties
  - The first set of derivative rules
  - The notations used for derivatives; e.g. $f'(x)$, y', $\frac{dy}{dx}$, $\frac{d}{dx}$, etc.
- 2.6 - Differentials
  - Increments and differentials are used to analyze the change in y for a given change in x.
  - The increment Δy is the *actual* change in y for a given change in x
  - The differential $dy = f'(x)dx$ is the *estimated* change in y for a *small* change in x.
- 2.7 - Marginal analysis
  - Derivatives are used to analyze change in functions; e.g the derivative of the cost function (marginal cost) is the instantaneous rate of change of cost relative to production at a given production level.
- 3.1 - This section covers a quick review of exponentials and logarithms
  - Exponential functions, $f(x) = b^x$, $b > 0$
  - The constant e and the exponential function $f(x) = e^x$
  - Continuously compounded interest formula, $A = Pe^{rt}$
  - Logarithms; i.e $y = \log_b x$ means that $b^y = x$ (in words, $\log_b x$ is the power of b that gives x).
  - Solving exponential equations (using logarithms); i.e. using the properties of logarithms.
  - For more detailed information see 1.5, 1.6
- 3.2 - Derivatives of Exponential and Logarithmic Functions
- 3.3 - Derivatives of Products and Quotients
- 3.4 - The Chain Rule

Problems from the review

$$\frac{BT' - BT}{B^2}$$

1(g): y' if $y = \dfrac{x-1}{x+1}$

$$\boxed{\cancel{y'}\ \frac{(x-1)(1)-(1)(x+1)}{(x+1)^2}}$$

$$\boxed{y'\ \frac{(x+1)(1)-(1)(x-1)}{(x+1)^2}}$$

quotient rule

1(h): $f'(x)$ if $f(x) = \dfrac{1}{x^2-1}$

$$\frac{(1)(2x)-(0)(x^2-1)}{(x^2-1)^2} \rightarrow \frac{(x^2-1)(0)-1(2x)}{(x^2-1)^2}$$

quotient rule

$$\boxed{\cancel{f'(x)} = \frac{2x}{(2x^2-1)^2}}$$

$$\boxed{\frac{-2x}{(x^2-1)^2}}$$

1(i): $p'(x)$ if $p(x) = (x^2-2x-1)(x^3+5x^2-x-4)$

product rule

$$\boxed{p'(x) = (2x-2)(x^3+5x^2-x-4) + (x^2-2x-1)(3x^2+10x-1)}$$

1(j): $f'(x)$ if $f(x) = (5-2x)^4$

chain rule

$f(x) = x^4$ $\quad g(x) = 5-2x$

$f'(x) = 4x^3$ $\quad g'(x) = -2$

$f'(g(x)) = 4(5-2x)^3$

$f'(x) = 4(5-2x)^3 \cdot (-2)$

$$\boxed{f'(x) = -8(5-2x)^3}$$

1(k): $f'(x)$ if $f(x) = e^{5x}$

chain rule

$f(x) = e^x$ $\quad g(x) = 5x$

$f'(x) = e^x$ $\quad g'(x) = 5$

$f'(g(x)) = e^{5x}$

$f'(x) = (e^{5x})(5)$

$$\boxed{f'(x) = 5e^{5x}}$$

1(l): $f'(x)$ if $f(x) = \ln(5x+1)$

chain rule

$f(x) = \ln x$ $\quad g(x) = 5x+1$

$f'(x) = \frac{1}{x}$ $\quad g'(x) = 5$

$f'(g(x)) = \frac{1}{5x+1}$

$f'(x) = \left(\frac{1}{5x+1}\right)(5)$

$$\boxed{f'(x) = \frac{5}{5x+1}}$$

1(m): $f'(x)$ if $f(x) = \dfrac{1}{(1-x)^4}$

chain rule

$f(x) = \frac{1}{x^4}$ $\quad g(x) = 1-x$

$f'(x) = \frac{-4}{x^5}$ $\quad g'(x) = -1$

$f'(x) = \frac{-4}{(1-x)^5}(-1)$

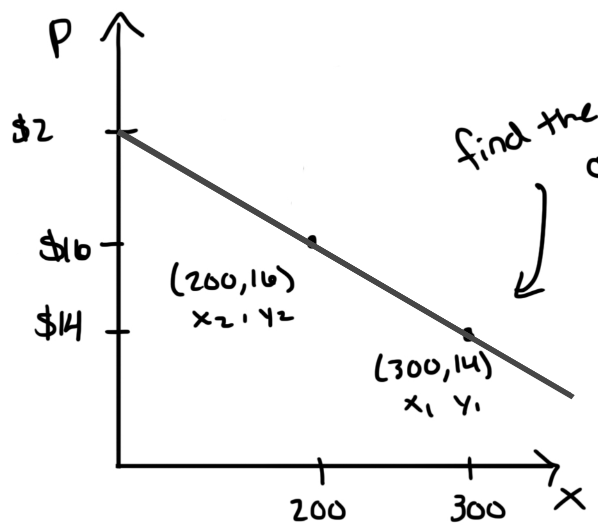
$f'(g(x)) = \frac{-4}{(1-x)^5}$

$$\boxed{f'(x) = \frac{4}{(1-x)^5}}$$

Question 8: A company is planning to manufacture and market a new two-slice electric toaster. After conducting extensive market surveys, the research department provides the following estimates: a weekly demand of 200 toasters at a price of \$16 per toaster and a weekly demand of 300 toasters at a price of \$14 per toaster. The financial department estimates that weekly fixed costs will be \$1,400 and variable costs (cost per unit) will be \$4.

(a) Assume that the relationship between price p and demand x is linear. Use the research department's estimates to express p as a function of x and find the domain of this function.

$y = mx + b$



find the equation of the line

$$m = \frac{\Delta p}{\Delta x} = \frac{p_2 - p_1}{x_2 - x_1}$$

$$\frac{16-14}{200-300} = \frac{2}{-100} = -\frac{1}{50}$$

$m = -\frac{1}{50}$

$x = 200$
$p = 16$

$p = -\frac{1}{50}x + b$

$16 = -\frac{1}{50}(200) + b$

$16 = -4 + b$

$20 = b$

$$\boxed{p = -\frac{1}{50}x + 20}$$

$$\boxed{0 \le x \le 1000}$$

$0 = -\frac{1}{50}x + 20$

$-20 = -\frac{1}{50}x$

$1000 = x$

(b) Find the revenue function in terms of x and state its domain.

Revenue = (demand) × (price)

$f(x) = x\left(-\frac{1}{50}x + 20\right) \qquad 0 \le x \le 1000$

$\boxed{R(x) = -\frac{1}{50}x^2 + 20x}$

(c) Assume that the cost function is linear. Use the financial department's estimates to express the cost function in terms of x.

Cost = fixed cost + variable costs

$\boxed{C(x) = 1400 + 4x}$

(d) Find the profit function in terms of x.

Profit = Revenue - Cost

$P(x) = \left(-\frac{1}{50}x^2+20x\right) - (1400+4x)$

$= \left(-\frac{1}{50}x^2+20x\right) - 1400 - 4x$

$\boxed{P(x) = -\frac{1}{50}x^2+16x-1400}$

(e) Evaluate the marginal profit at $x = 250$ and $x = 475$ and interpret the results.

$p(x) = -\frac{1}{50}x^2+16x-1500$

$p'(x) = -\frac{2}{50}x+16$

$p'(x) = -\frac{1}{25}x+16$

$p'(250) = -\frac{1}{25}(250)+16$

$\boxed{p'(250) = 6}$

When our demand is 250, we are making \$6 per toaster

$p'(475) = -\frac{1}{25}(475)+16$

$\boxed{p'(475) = -3}$

When our demand is 475, our profit is decreasing at a rate of \$3 per toaster

3.5 - Implicit differentiation (an application of the chain rule)

Recall that a **function** is a rule that assigns a unique output to each input.

$3x^2+y-2=0$

$3x^2-2=-y$

$y=2-3x^2$

Consider the following:

$$3x^2 + y - 2 = 0$$

and

$$y = 2 - 3x^2$$

Notice anything? Are the equations related?

They're the same function

Sometimes the rule is *direct* or **explicit**

$$y = 2 - 3x^2$$

Sometimes the rule is *indirect* or **implicit**:

$$3x^2 + y - 2 = 0$$

Implicit differentiation

When the rule is **explicit**, we take the derivative in the *usual* way...

$$y = 2 - 3x^2$$

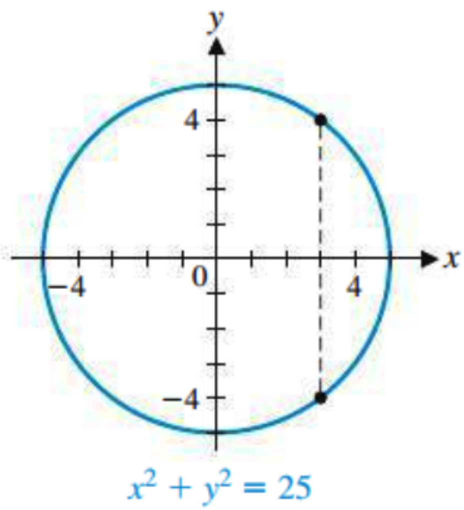
$$y' = -6x$$

When the rule is **implicit**, keeping in mind that the equation does define y as a function of x, we can take the derivative of both sides...

$$\frac{d}{dx}\left(3x^2 + y(x) - 2\right) = \frac{d}{dx}(0)$$

This process is called **implicit differentiation** and it is useful in situations where we have an equation that defines y as a function of x but it's not "convenient" or *sometimes* even *possible* to solve for y in terms of x.

Example: This equation
$$x^2 + y^2 = 25$$
describes the circle of radius 5 centered at the origin $(0, 0)$.



Goal: Find the slope of the graph when $x = 3$.

First, there are actually two such points and for one the slope will be negative and for the other it will be positive!

To find the slope when $x = 3$, we could go the **explicit route**...

Or we could go the **implicit route**...

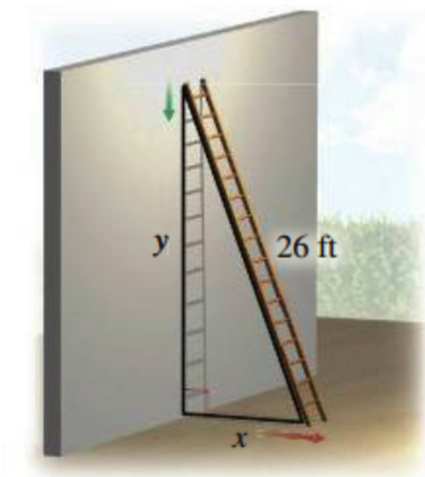
Example: Find the slope of the graph of

$$y - xy^2 + x^2 + 1 = 0$$

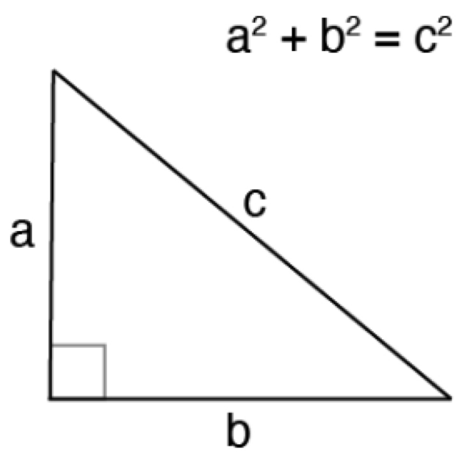
when $x = 1$.

An application of implicit differentiation: Related rates (3.6)

Example: A 26-foot ladder is placed against a wall. If the top of the ladder is sliding down the wall at 2 feet per second, at what rate is the bottom of the ladder moving away from the wall when the bottom of the ladder is 10 feet away from the wall?



Recall: The figure forms a right triangle so we can use the Pythagorean theorem!



So $x^2 + y^2 = 26^2$.

Key point: Think carefully about the problem description. What's really behind the changes described? Also, think about the rate given, "2 feet *per second*."

So we're thinking of x and y in terms of time t and we want to find dx/dt when $x = 10$ and $dy/dt = -2$ (Why -2?)

$$x^2 + y^2 = 26^2$$

Example: Suppose that two motorboats leave from the same point at the same time. If one travels north at 15 miles per hour and the other travels east at 20 miles per hour, how fast will the distance between them be changing after 2 hours

Example: Suppose that for a company manufacturing flash drives, the cost, revenue, and profit equations are given by

$$C = 5000 + 2x$$

$$R = 10000x - 0.001x^2$$

where the production output in 1 week is x flash drives. If production is increasing at the rate of 500 flash drives per week when production is 2000 flash drives, find the rate of increase in cost and the rate of increase in revenue.