

Definitions

$$-\lim_{x \rightarrow a^-} f(x), \lim_{x \rightarrow a^+} f(x), \lim_{x \rightarrow a} f(x), \lim_{x \rightarrow \pm \infty} f(x)$$

- Algebraically, Graphically

$$f(x) = \begin{cases} 8+x^2 & \text{if } x < 0 \\ 8-x^2 & \text{if } x \geq 0 \end{cases}$$

$\lim_{x \rightarrow 0} f(x)$: compare both. If they are dif, the limit DNE, otherwise, it exists.

$$\lim_{x \rightarrow 0} f(x) = 2$$

$$\lim_{x \rightarrow 1} 8 - x^2 = 7$$

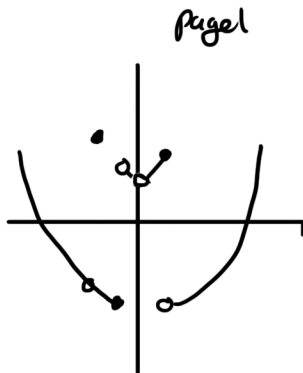
$$\left. \begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= 8+0 = 8 \\ \lim_{x \rightarrow 0^+} f(x) &= 8-0 = 8 \end{aligned} \right\} \text{Same so the limit exists}$$

Asymptotes

2.2.1

$$\lim_{x \rightarrow \infty} \frac{\cancel{3+2x} + 2x^2}{1x^2} = \frac{2}{1} = 2$$

If degrees are same, divide coefficient
if bottom degree is higher, limit = 0
if top degree is higher, limit = ∞



2.3: Continuity

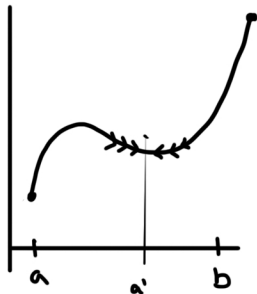
- Sign Chart, partition numbers
- Continuous on a point, $()$, $[]$
- Types of discont. Jump, Removable (hole), Infinite

$f(x)$ is continuous on $x = a$

$$\lim_{x \rightarrow a} f(x) = f(a)$$

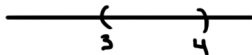
$$\lim_{x \rightarrow b^-} f(x) = f(b)$$

$$\lim_{x \rightarrow a^+} f(x) = f(a)$$



2.3 problems 9 and 11
include these

Does not include $(3, 4) \neq [3, 4]$ Includes



- Sign Chart and partitions

Determine whether the function is continuous

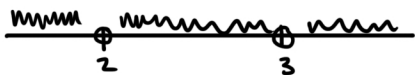
$$g(x) = \frac{(x+3)}{(x-3)(x-2)} \quad \text{at } x=3, 2$$

$$\lim_{x \rightarrow a} g(x) = \frac{-3+3}{(-3-3)(-3-2)} = \frac{0}{(-6)(-5)} = \frac{0}{30} = 0$$

$$\lim_{x \rightarrow a} g(x) = \frac{3+3}{(3-3)(3-2)} = \frac{6}{(0)(1)} = \frac{6}{0}$$

$$\lim_{x \rightarrow a} g(x) = \frac{2+3}{(2-3)(2-2)} = \frac{5}{(-1)(0)} = \frac{5}{0}$$

} Discontinuous



$$(-\infty, 2) \cup (2, 3) \cup (3, \infty)$$

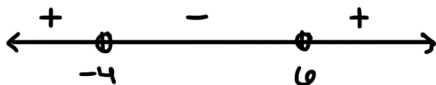
2.3-11: Sign Chart

$$x^2 - 2x - 24 < 0$$

$$(x-6)(x+4) < 0$$

$$x-6=0 \quad x+4=0$$

$$x=6 \quad x=-4$$



$(-9-6)(-9+4)$	$(0-6)(0+4)$	$(7-6)(7+4)$
-11 -1	-6 4	1 11
positive	negative	positive

ALWAYS make equal to 0.

If not equal to, open circles

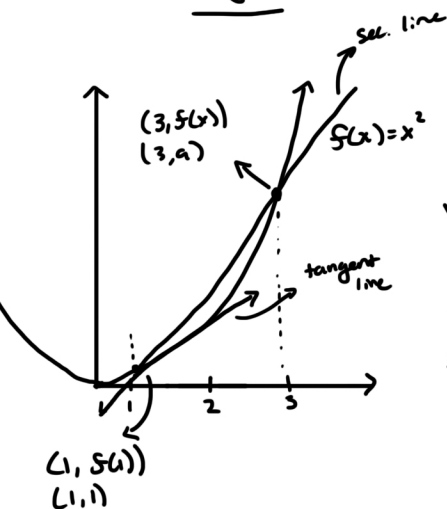
If equal to, closed circles

asking for negative only so...

Ans: $(-4, 6)$

Derivatives

Sec. line passes through multiple points
Tangent line passes through one line



$$h(x) = mx + b$$

$$m = \frac{9-1}{3-1} = \frac{8}{2}$$

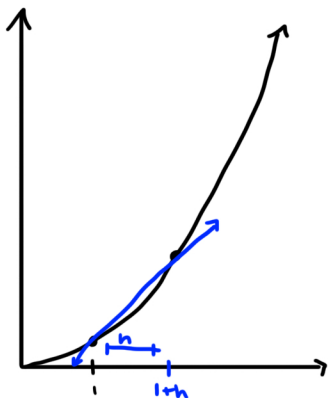
$$h = 4x + b$$

$$\begin{array}{l|l} 4-4 = 4(x-x_1) & 1 = 4(1) \\ & 1 = 4+b \\ & -3 = b \end{array}$$

For a tangent line, you don't have two points.

Slope and derivative
are similar concepts.

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$
$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{1+h-1} = \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1^2}{h}$$
$$= \frac{1^2 + 2h + h^2 - 1}{h}$$

$$= \lim_{h \rightarrow 0} 2h + 1 \rightarrow \lim_{h \rightarrow 0} = 2$$