Example: Find two numbers whose difference is 19 and whose product is a minimum.

Define Variables: Let x, y be the two numbers

$$x - y = 19$$ ← use this to write y in terms of x.

Product: xy ← minimize

$$y = x - 19$$

$$P(x) = x(x-19)$$

$$P(x) = x^2 - 19x$$

$$P'(x) = 2x - 19$$

$$0 = 2x - 19$$

$$19 = 2x$$

$$\frac{19}{2} = x$$ ← only critical number

$$y = -\frac{19}{2}$$

$$x = \frac{19}{2}$$

$$P''(x) = 2 > 0$$

$$P''\left(\frac{19}{2}\right) = 2 > 0$$

We get an absolute minimum when $x = \frac{19}{2}$

We need to find two numbers so solve for y now

$$y = x - 19$$

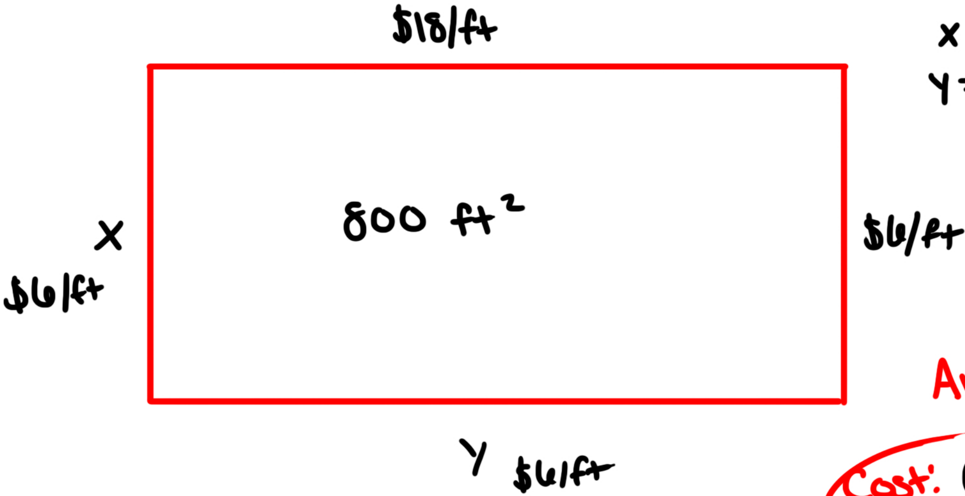
$$y = \frac{19}{2} - 19 = -\frac{19}{2}$$

The minimum product $= \frac{19}{2}\left(-\frac{19}{2}\right) = -\left(\frac{19}{2}\right)^2$

4.6 - Optimization

Example: A fence is to be built to enclose a rectangular area of 800 square feet. The fence along three sides is to be made of material that costs \$6 per foot. The material for the fourth side costs \$18 per foot. Find the dimensions of the rectangle that will allow for the most economical fence tobe built.

minimize cost



x = width (ft)

y = length (ft)

both x and y HAVE to be positive

Area: $xy = 800$

Cost: $6x + 6y + 6x + 18y$
$= 12x + 24y$

minimize

$xy = 800 \rightarrow y = \frac{800}{x}$

$C(x) = 12x + 24\left(\frac{800}{x}\right)$

$C(x) = 12x + \frac{19200}{x} = 12x + 19200x^{-1}$

$C'(x) = 12 - \frac{19200}{x^2}$

$0 = 12 - \frac{19200}{x^2}$

$0 = 12x^2 - 19200$

$0 = x^2 - 1600$

$0 = (x-40)(x+40)$

only critical number

$x = 40$ $\quad$ $\cancel{x = -40}$

$C''(x) = \frac{38400}{x^3}$

$C''(40) = \frac{38400}{40^3}$ positive

Absolute minimum when $x = 40$

$y = \frac{800}{40}$

$y = 20$

The dimensions of the rectangle with minimum cost are $40\text{ft} \times 20\text{ft}$

x= number of smart phones sold per week

Example: A company manufactures and sells x smartphones per week. The weekly price-demand and cost equations are, respectively,

$0 \le x \le 1250$

price-demand $$p = 500 - 0.4x$$

and

cost $$C(x) = 20,000 + 20x$$

(A) What price should the company charge for the phones, and how many phones should be produced to maximize the weekly revenue? What is the maximum weekly revenue?

(B) What is the maximum weekly profit? How much should the company charge for the phones, and how many phones should be produced to realize the maximum weekly profit?

Revenue = (number sold) x (price/sale)

$R(x) = x(500-0.4x)$

$R(x) = 500x - 0.4x^2$

$R'(x) = 500 - 0.8x$

$0 = 500 - 0.8x$

$0.8x = 500$

$x = 625$

R(x) is defined on [0, 1250]

x	$R(x) = 500x - 0.4x^2$
0	0
1250	0
625	\$156,250

The max revenue occurs when demand is 625/week & the price \$250 & the max revenue is \$156,250

$P(x) = 500x - 0.4x^2 - (20000 + 20x) = 480x - 0.4x^2 - 20000$

$P'(x) = 480 - 0.8x$ on [0, 1250]

$0 = 480 - 0.8x$

$0.8x = 480$

$x = 600$

x	$P(x) = 480x - 0.4x^2 - 20000$
0	-20000
1250	-45000
600	\$267,760 ← max profit

The max weekly profit is \$267,760. The price should be \$260 per phone, we need to make 600 phones per week

Example: A lake used for recreational swimming is treated periodically to control harmful bacteria growth. Suppose that t days after a treatment, the concentration of bacteria per cubic centimeter is given by

$$C(t) = 30t^2 - 240t + 500, \quad 0 \le t \le 8$$

How many days after a treatment will the concentration be minimal? What is the minimum concentration?

Let t = number of days since treatment

$C(t) = 30t^2 - 240t + 500, \ 0 \le t \le 8$ $[0,8]$

$C'(t) = 60t - 240$

$0 = 60t - 240$

$240 = 60t$

$4 = t$

x	$C(t) = 30t^2 - 240t + 500$
0	500
8	500
4	20 ← Absolute minimum

After 4 days, the treatment be at a minimum of 20 bacteria per cubic centimeter