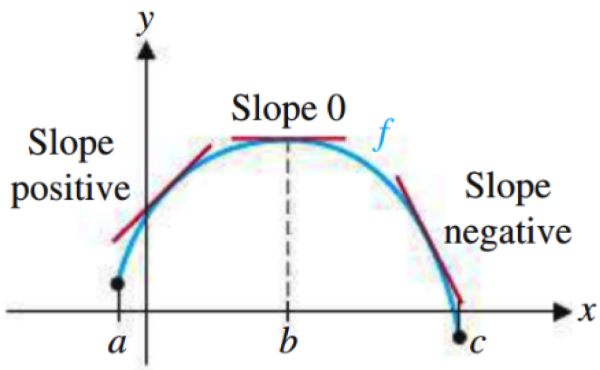
Finding intervals of increase or decrease



Theorem 1: For the interval (a, b), if $f' > 0$, then f is increasing (i.e. rises), and if $f' < 0$, then f is decreasing (i.e. falls).

Conclusion and key idea: We can determine the intervals of increase and decrease for our function f by creating a sign chart for its derivative, f'.

Example: Let $f(x) = 8x - x^2$

$f'(x) = 8 - 2x$

$-8 = -2x$

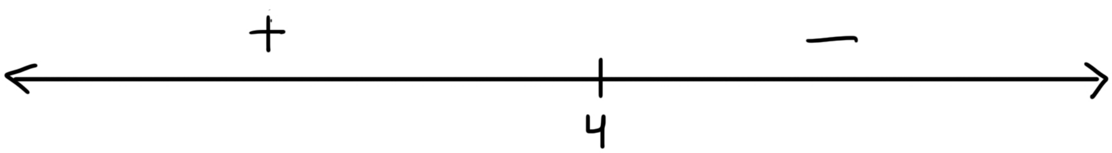
$4 = x$

(a) For what interval(s) is f increasing? Decreasing?

$f'(3) = 8 - 2(3)$
$= 8 - 6$
$= 2$

$f'(5) = 8 - 2(5)$
$= -10$

$f(x)$ is increasing on the interval $(-\infty, 4)$ and decreasing on the interval $(4, \infty)$



(b) For what values of x does the graph of f have a horizontal tangent line? What does that tell us about the graph?

$x = 4$

$f'(4) = 0$

$f(4) = 8(4) - 4^2$
$= 32 - 16$
$= 16$

$f(0) = 8(0) - 0^2$
$= 0$

$0 = 8x - x^2$

$x(x-8) = 0$

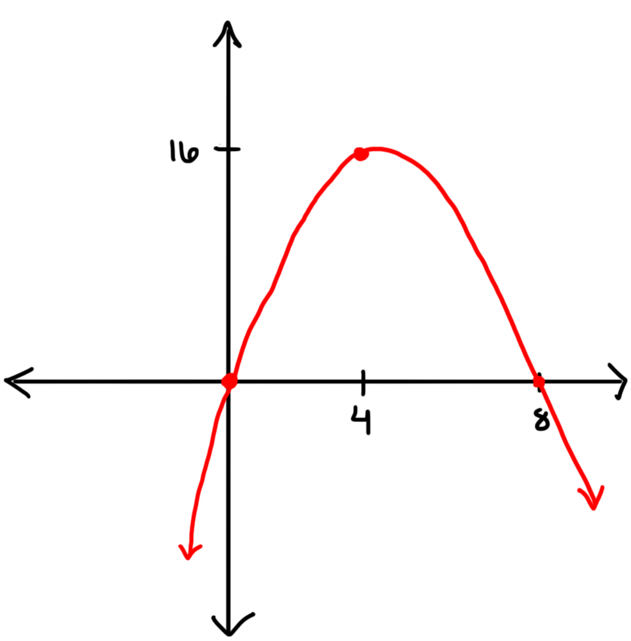
$x = 0 \quad x = 8$

$f(4) = 16 \longrightarrow (4,16)$ point of vertex

$(0,0)$ y-intercept

$(0,8)$ x-intercept

(c) Sketch the graph.



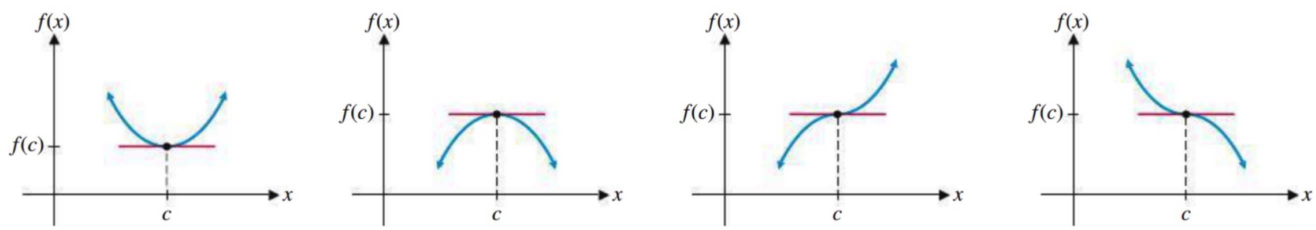
Critical numbers

As we saw in the last example, the sign chart for f' is a useful tool for analyzing the graph of f. This means that the *partition numbers* for f' (used to create the sign chart for f') are key to understanding the behavior of the function f.

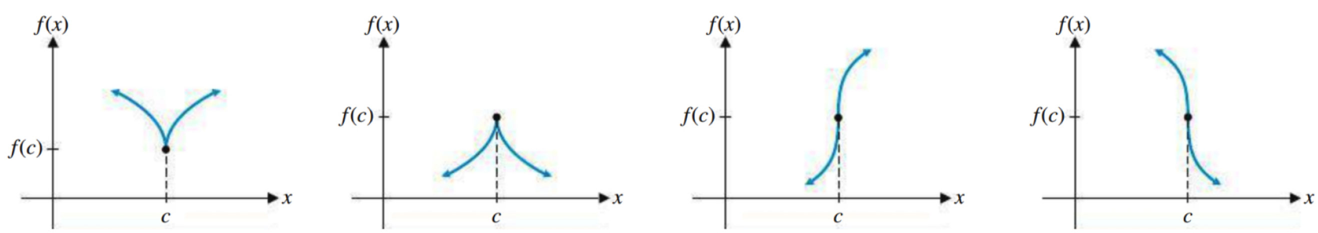
We call the partition numbers of f' the *critical numbers* of f (as long as they are in the domain of f).

Critical numbers: A real number x in the domain of f such that $f'(x) = 0$ or $f'(x)$ doesn't exist is called a **critical number** of f.

$f'(c) = 0$**: Horizontal tangent**



$f'(c)$ **is not defined but** $f(c)$ **is defined**



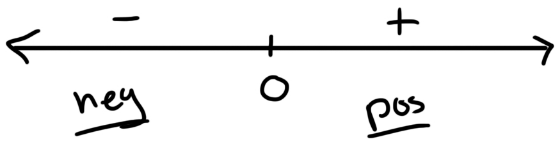
Example: Find the critical numbers for $f(x) = x^2$, the intervals on which f is increasing, and those on which f is decreasing.

$f'(x) = 2x$

$0 = x$

$x = 0$

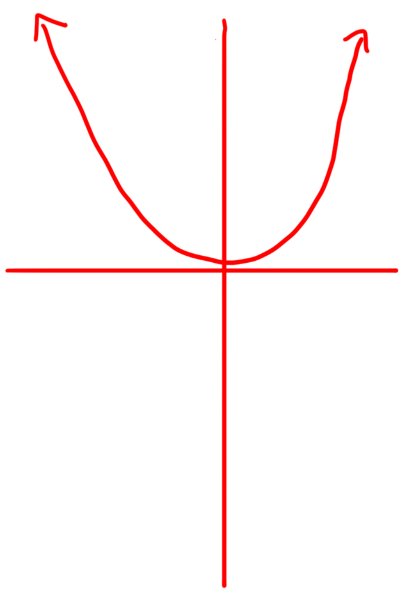
one partition number and it is a criticle number



increasing $(0, \infty)$

decreasing $(-\infty, 0)$

Desmos.com



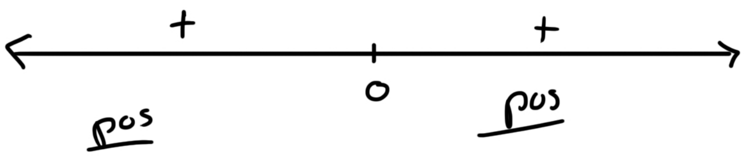
$f(x) = x^2$

Example: Find the critical numbers for $f(x) = x^3$, the intervals on which f is increasing, and those on which f is decreasing.

$f'(x) = 3x^2$

$0 = x$

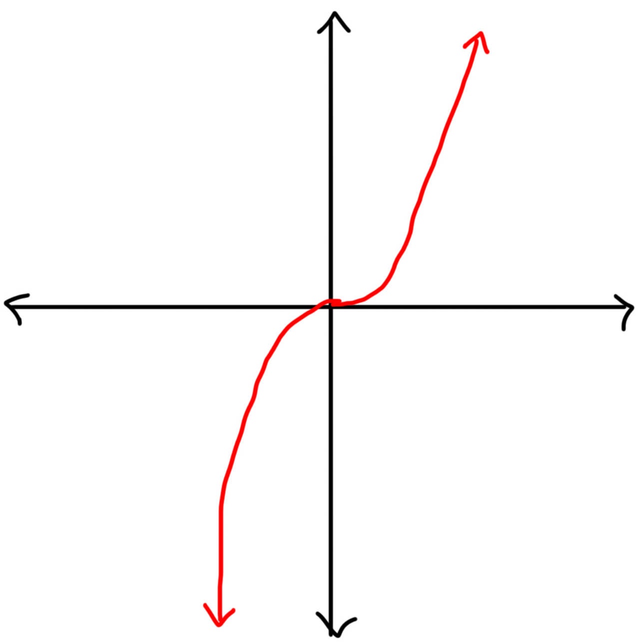
$x=0$ partition number is also a criticle number



recommended to graph

increasing from $(-\infty, \infty)$

never decreasing



Example: Find the critical numbers for $f(x) = \sqrt[3]{x}$, the intervals on which f is increasing, and those on which f is decreasing.

$$f(x) = x^{1/3}$$

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$f'(x) = \frac{1}{3x^{2/3}}$$

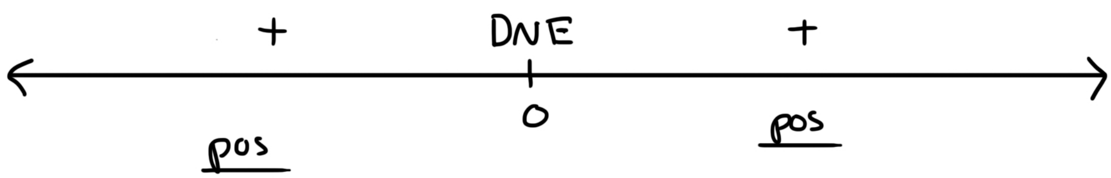
$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

<u>partition numbers</u>

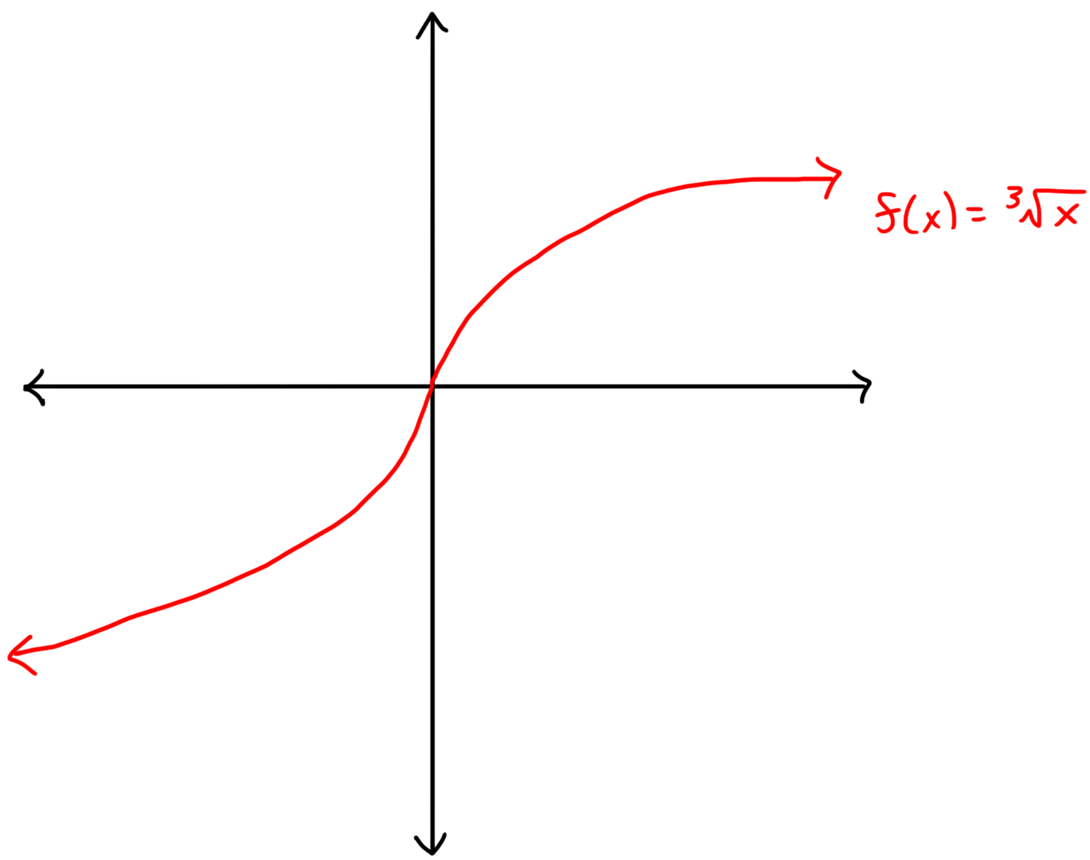
$\frac{1}{3\sqrt[3]{x^2}} = 0$ <u>No Solution</u>

$\boxed{x=0 \text{ partition number because the derivative DNE}}$

$\boxed{x=0 \text{ is also a critical number}}$



increases on $(-\infty, \infty)$, never decreases

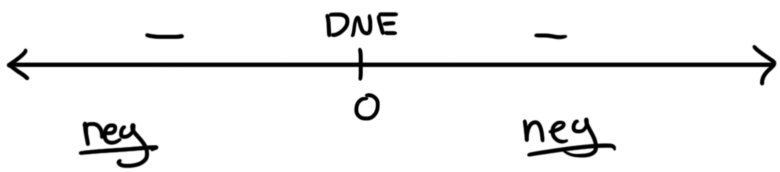


Example: Find the critical numbers for $f(x) = \dfrac{1}{x}$, the intervals on which f is increasing, and those on which f is decreasing.

$f(x) = x^{-1}$

$f'(x) = -1x^{-2}$

$f'(x) \; - \frac{1}{x^2}$

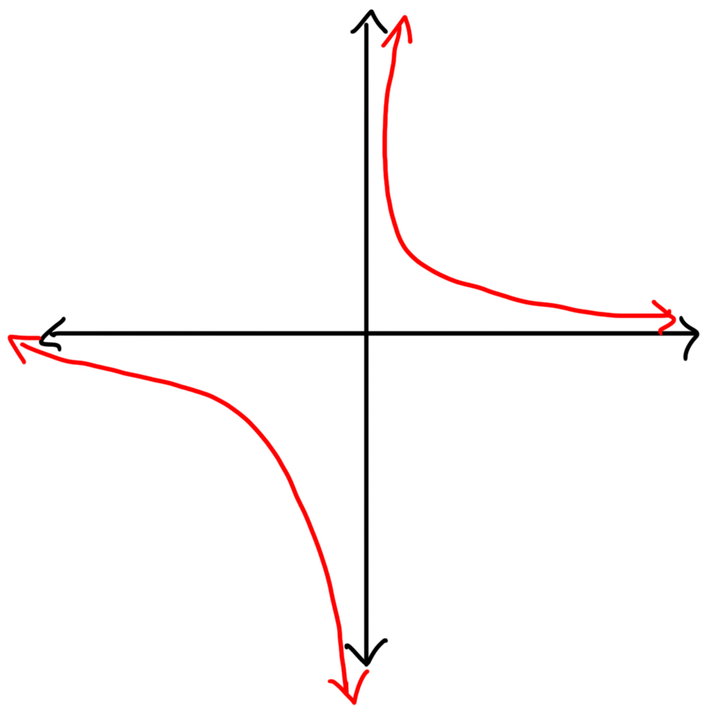


Decreasing on $(-\infty, \infty)$, never increasing

Partition numbers

$\boxed{x = 0}$

only partition number but NOT a critical number



Example: Find the critical numbers for $f(x) = 8\ln x - x^2$, the intervals on which f is increasing, and those on which f is decreasing.

Local extrema

We say that $f(c)$ is a **local maximum** if there is an interval (a, b) containing c such that

$$f(x) \leq f(c)$$

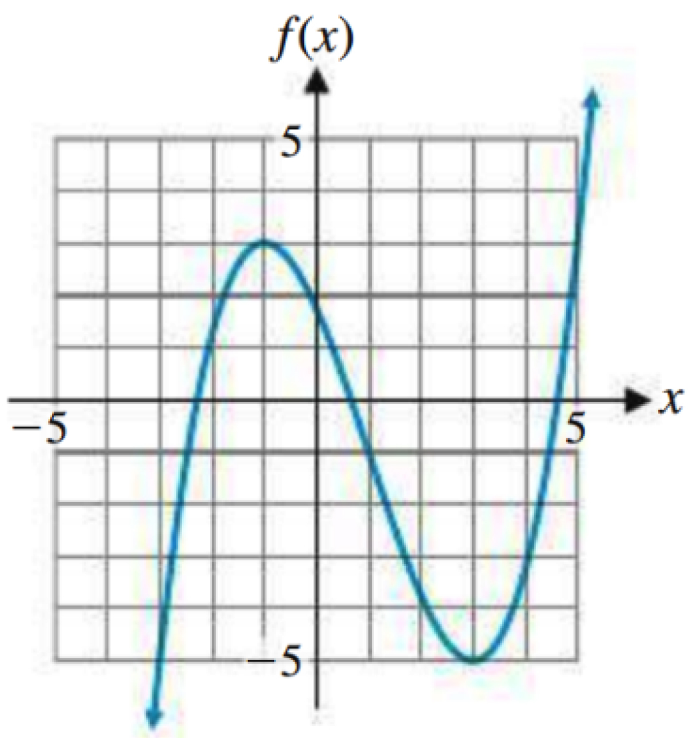
for all x in (a, b).

We say that $f(c)$ is a **local minimum** if there is an interval (a, b) containing c such that

$$f(x) \geq f(c)$$

for all x in (a, b).

Let's look at the following graph.



Question: What do you notice about the graph of f "near" the local extrema? *Think about the tangent lines!*

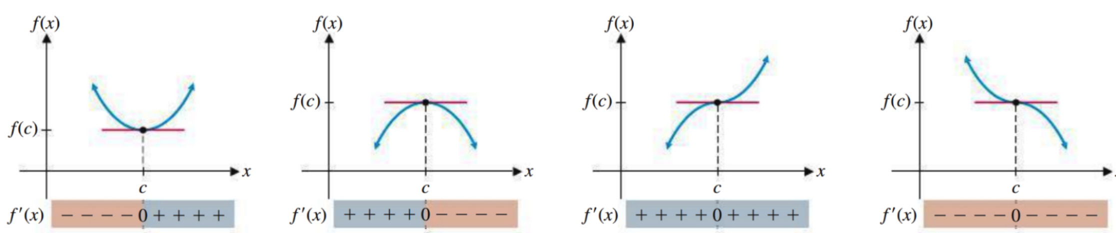
The first derivative test for local extrema

Theorem 2: If $f(c)$ is a local extremum of f, then c is a critical number of f.

First derivative test for local extrema: Let c be a critical number of f and construct a sign chart for f' around c.

- If the sign of f' changes from $-$ to $+$ at c, $f(c)$ is a *local minimum.*
- If the sign of f' changes from $+$ to $-$ at c, $f(c)$ is a *local maximum.*
- If the sign of f' does not change at c, then $f(c)$ is neither a local maximum nor local minimum.

$f'(c) = 0$: Horizontal tangent



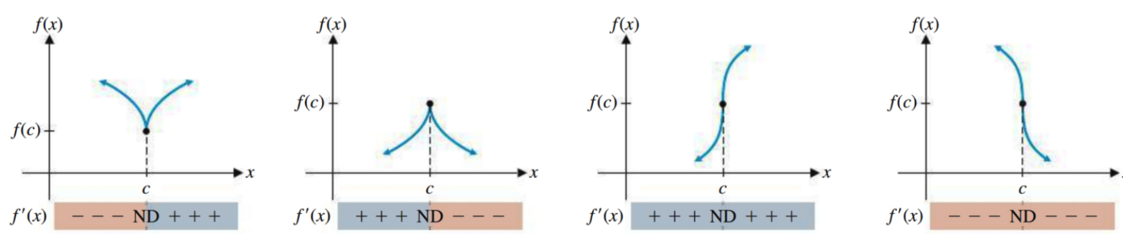
(A) $f(c)$ is a local minimum

(B) $f(c)$ is a local maximum

(C) $f(c)$ is neither a local maximum nor a local minimum

(D) $f(c)$ is neither a local maximum nor a local minimum

$f'(c)$ is not defined but $f(c)$ is defined



(E) $f(c)$ is a local minimum

(F) $f(c)$ is a local maximum

(G) $f(c)$ is neither a local maximum nor a local minimum

(H) $f(c)$ is neither a local maximum nor a local minimum

Example: Let $f(x) = x^3 - 6x^2 + 9x + 1$

(a) Find the critical numbers of f.

(b) Find the local extrema for f.

(c) Sketch the graph of f.

Example: Let $f(x) = x^3 - 9x^2 + 24x - 10$

(a) Find the critical numbers of f.

(b) Find the local extrema for f.

(c) Sketch the graph of f.