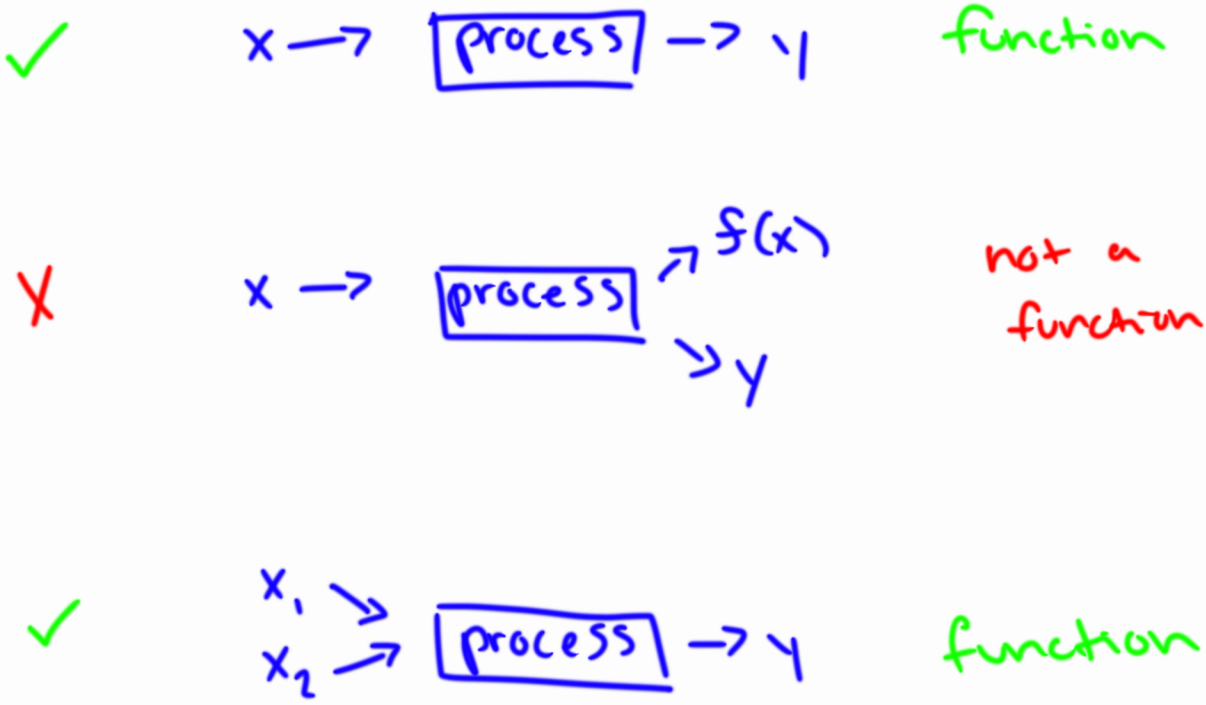
MAT 180 August 19th, 2024 Michael Porter

Chapter 1 - Review of key concepts in Algebra

A **function** is a correspondence (or rule) between two sets of elements (usually both are sets of real numbers) such that to each element in the first set, there is a unique corresponding element in the second set.

Another way: A **function** assigns a unique output to each input.



Is this a function?

Rule: Take a number, square it, then add 7.

yes, unique output

Rule: Multiply a number by 5, then subtract 3.

yes, unique output

Rule: Take the square root of a number.

No, if you input a negative, you get 2 outputs

$\sqrt{-1} = \pm 1 =$ -1 and 1

2 outputs

Understanding square roots

Definition: The **square root** of a number x is a number whose square is x.

Question: What are the *square roots* of 4?

$(2)^2 = 4$

$(-2)^2 = 4$

So 2 and -2 are both square roots of 4.

By convention, the symbol $\sqrt{x}$ means the **positive** square root of x.

Question: What is $\sqrt{4}$?

$\sqrt{4} = 2$

More later...but this is why we use $\pm$ when solving equations by taking the square root of both sides.

Function notation

Rule: Square a number and then add 2.

Equation: $y = x^2 + 2$

Function notation: $f(x) = x^2 + 2$

Key terms: Input, output, independent variable, dependent variable

Example: Evaluate $f(1)$

$f(1) = 1^2 + 2 = 3$

Example: Evaluate $f(-3)$

$f(-3) = -3^2 + 2 = 11$

Point-by-point Plotting

To sketch the graph of a function in two variables, we plot sufficiently many points of the form $(x, f(x))$ so that the shape of the graph is apparent, and then we connect those points with a smooth curve. This process is called point-by-point plotting.

Example: Sketch a graph of $f(x) = 2x - 5$

Example: Sketch a graph of $f(x) = (x+1)^2 - 2$

Example: Sketch a graph of $f(x) = \sqrt{x+1}$

Domain and Range

You might have noticed in the last example that sometimes inputs don't "work!"

Example: Let $f(x) = \dfrac{1}{x-2}$. What is $f(2)$?

The **domain** of a function is the set of all possible inputs. The **range** of a function is the set of all possible outputs.

Example: What is the domain of $f(x) = \dfrac{1}{x-2}$?

$\mathbb{R}$ = All Real numbers

$\mathbb{R}(-\infty, 2) \cup (2, \infty)$

Example: What is the domain of $f(x) = \sqrt{x}$?

$\mathbb{R}\ [0, \infty)$

Example: What is the domain of $f(x) = 2x - 7$?

$\mathbb{R}(-\infty, \infty)$ or $\mathbb{R}$

Rules for finding the domain

If a domain isn't already stated, there are three scenarios to consider:

- If the function has a denominator, the denominator can **never** be 0.
- If the function has a square root, the argument of the square root must always be ≥ 0.
- If the function has a logarithm, the argument of the logarithm must always be > 0.

Example: What is the domain of $f(x) = \dfrac{1}{x-1}$?

$\mathbb{R}(-\infty, 1) \cup (1, \infty)$

Example: What is the domain of $f(x) = \sqrt{x-2}$?

$\mathbb{R}[2, \infty)$

Example: What is the domain of $f(x) = \dfrac{1}{x^2+1}$?

$\mathbb{R}$

Example: What is the domain of $f(x) = x^2 + 1$?

$\mathbb{R}$

Application

A manufacturer of a popular digital camera wholesales the camera to retail outlets throughout the United States. Using statistical methods, the financial department in the company produced price−demand data and an analyst obtained the following price−demand function that modeled that data

$$p(x) = 94.8 - 5x, \quad 1 \leq x \leq 15$$

where p is the wholesale price per camera at which x million cameras are sold.

(A) Plot the graph of the price-demand function p. What information does this graph give?

(B) What is the company's revenue function, $R(x)$, for this camera, and what is its domain?

(C) Complete the table, computing revenues to the nearest million dollars.

x (millions)	$R(x)$ (million \$)
1	90
3	
6	
9	
12	
15	

(D) Plot the data in the table. Then sketch a graph of the revenue function using these points. What information does this graph give?