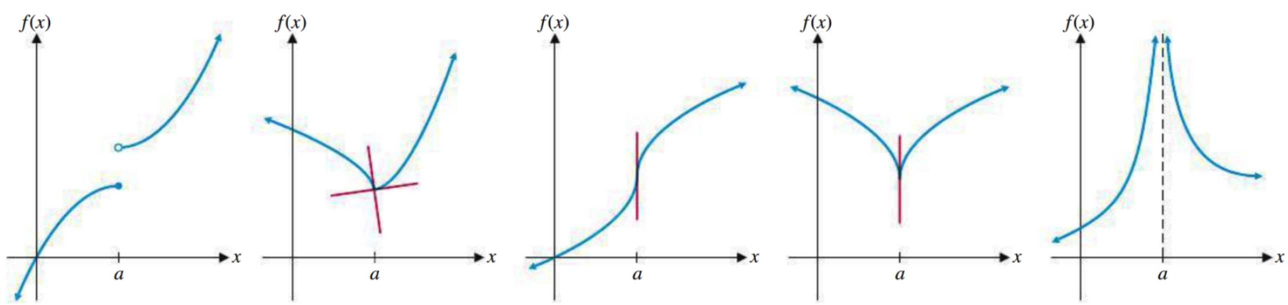
Could a function be nondifferentiable?

Recall the definition...

Given $y = f(x)$, we define the **derivative** of f at x, denoted $f'(x)$, by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

<u>if the limit exists</u>.



(A) Not continuous at $x = a$ (B) Graph has sharp corner at $x = a$ (C) Vertical tangent at $x = a$ (D) Vertical tangent at $x = a$ (E) Vertical asymptote at $x = a$

If $f'(x)$ exists for each x in the open interval (a, b), then f is said to be **differentiable** over (a, b).

Notation

If $y = f(x)$, then

$$f'(x), y', \text{ and } \frac{dy}{dx}$$

all denote the derivative of f at x.

$f(x) = f'(x)$

$y = f(x)$

$y = y'$

$\frac{dy}{dx}$

derivative of y

derivative of x

$\frac{d}{dx} f(x)$

Circled ones, we will see occasionally

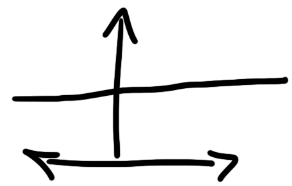
2.5 - Basic differentiation properties

The derivative rules

Constant function rule: If $y = f(x) = C$, then

no slope
when constant

$$f'(x) = 0$$



$f'(x)=C$

Power rule: If $y = f(x) = x^n$ for any real number n, then

$f(x) = x^4$
$f'(x) = 4x^3$

$$f'(x) = nx^{n-1}$$

$f(x) = x^2$
$f'(x) = 2x$

Constant multiple property: If c is a real number and $f(x) = cg(x)$,

$$f'(x) = cg'(x)$$

Sum/difference property:

$$\text{If } f(x) = g(x) + h(x), \text{ then } f'(x) = g'(x) + h'(x)$$

$$\text{If } f(x) = g(x) - h(x), \text{ then } f'(x) = g'(x) - h'(x)$$

Find the indicated derivatives

Example: $\frac{d}{dx}x^3$ $\boxed{\frac{d}{dx}3x^2}$

$f(x) = x^n$
$f'(x) = nx^{n-1}$

Example: $\frac{d}{dx}x^{\frac{3}{2}}$ $\frac{d}{dx}\frac{3}{2}x^{\frac{3}{2}-1}$ $\boxed{= \frac{3}{2}x^{\frac{1}{2}}}$

Example: $\frac{d}{dx}x^{-2}$ $\frac{d}{dx}\boxed{-2x^{-3}}$

Example: $\frac{d}{dx}\frac{1}{x^2}$ $\frac{d}{dx}x^{-2}$ $\boxed{= -2x^{-3}}$

$\frac{1}{x^n} = x^{-n}$

Example: $\frac{d}{dx}\sqrt{x}$ $= x^{\frac{1}{2}}$ $\frac{1}{2}x^{\frac{1}{2}-1}$ $\boxed{= \frac{1}{2}x^{-\frac{1}{2}}}$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$\frac{1}{x^n} = x^{-n}$

Example: $\frac{d}{dx}\frac{1+x}{x^2}$ $\frac{d}{dx}\frac{1}{x^2} + \frac{x}{x^2}$

$\frac{1}{x^2} + \frac{1}{x} =$ $x^{-2} + x^{-1}$

$\frac{d}{dx} -2x^{-3} + (-1)x^{-2} = \boxed{\frac{d}{dx} -2x^{-3} - x^{-2}}$

Find the indicated derivatives

Example: $\frac{d}{dx}2x = 2\frac{d}{dx}x = 2\frac{d}{dx}x^1 = 2(1)x^0$

$= 2 \qquad 0$

$$\boxed{\frac{d}{dx} = 2}$$

Example: $\frac{d}{dx}(5x-7) = 5 \;/\; \frac{d}{dx}5x - \frac{d}{dx}7 = 5 = 5$

$$\boxed{\frac{d}{dx} = 5}$$

Example: $\frac{d}{dx}(x^2-x+1)$

$\frac{d}{dx}x^2 - \frac{d}{dx}x + \frac{d}{dx}1$

$= 2x-1+0$

$$\frac{d}{dx} = 2x-1+0 = \boxed{\frac{d}{dx}2x-1}$$

Example: $\frac{d}{dx}(4x^3-2x^2+x-5)$

$\frac{d}{dx}4x^3 - \frac{d}{dx}2x^2 + \frac{d}{dx}x - \frac{d}{dx}5$

$3(4)x^2 - 2(2)x + 1 - 0$

$12x^2 - 4x + 1 - 0$

$$\boxed{\frac{d}{dx}12x^2-4x+1}$$

Example: $\frac{d}{dx}(x^{10}-5x^7+x^3-x-1)$

$$\boxed{\frac{d}{dx}10x^9-35x^6+3x^2-1}$$

$-5(7)x^6$

True/False: You can take the derivative of any polynomial function.

$\boxed{\text{True,}}$ Just solve term-by-term using the rules

Example: For $f(x) = 6x - x^2$.

(a) Find $f'(x)$

$\boxed{f'(x) = 6 - 2x}$

(b) Find the slope of the graph at $x = 2$

$6 - 2(2) = 6 - 4$ $\boxed{m = 2}$

(c) Find the slope of the graph at $x = 4$

$6 - 2(4) = 6 - 8$ $\boxed{m = -2}$

(d) Find the equation of the tangent line at $(2, 8)$

$y - y_1 = m(x - x_1)$

$y - 8 = 2(x - 2)$

$y = 2x - 4 + 8$

$\boxed{y = 2x + 4}$

(e) Find the value(s) of x for which the tangent is horizontal

Example: A company's total sales (in millions of dollars) t months from now are given by

$$S(t) = 0.03t^3 + 0.5t^2 + 2t + 3$$

(a) Find $S'(t)$

$S'(t) = 0.03(3)t^2 + 0.5(2)t + 2 + 0$

$S'(t) = 0.09t^2 + 1t + 2$

(b) Find $S(5)$ and $S'(5)$ (round to 2 decimal places). What does each value represent?

$S(5) = 0.03(5)^3 + 0.5(5)^2 + 2(5) + 3$
$= 3.75 + 12.5 + 10 + 3$
$= 29.5$

\$29.5 million over 5 months

$S'(5) = 0.09(5)^2 + 1(5) + 2$
$= 2.25 + 5 + 2$
$= 9.25$

rate of change is +\$9.25 million per month

(c) Does $S'(t) = 0$ for any t? What would that mean?

total sales has leveled out

no increase or decrease