Section 3.2 - Derivatives of exponential and logarithmic functions

For $b > 0$, $b \neq 1$

$$\frac{d}{dx}e^x = e^x$$

$x^2 \mid 2x$

$$\frac{d}{dx}b^x = b^x \ln b$$

Be careful! The power rule *only applies* for functions of the form $f(x) = x^n$ for any real number n.

For $b > 0$, $b \neq 1$, and $x > 0$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\log_b x = \frac{1}{\ln b}\left(\frac{1}{x}\right)$$

Find $f'(x)$

(a) $f(x) = 5e^x$ Same answer

$f'(x) = 5e^x$

(b) $f(x) = 5e^x + 3x + 1$

$5e^x + 3 + 0$

$f'(x) = 5e^x + 3$

(c) $f(x) = x^3 - 3e^x$

$f'(x) = 3x^2 - 3e^x$

$6 \ln x = \frac{6}{x}$

(d) $f(x) = 6 \ln x - x^2 + 1$

$f'(x) = \frac{6}{x} - 2x$

(e) $f(x) = x^e$ ← power function

$f'(x) = ex^{e-1}$

(f) $f(x) = \log x \longrightarrow \log_{10} x$

$f'(x) = \frac{1}{\ln(10)}\left(\frac{1}{x}\right)$

Refer to the new rules on the previous page

(g) $f(x) = \log_3 x + 2x - 1$

$f'(x) = \frac{1}{\ln(3)}\left(\frac{1}{x}\right) + 2$

Example: Find the equation of the line tangent to the graph of $f(x) = 3 + \ln x$ when $x = 1$

tangent line intersects with $(1,3)$

$f(1) = 3 + \ln(1) = 3$

$f'(x) = \frac{1}{x}$

$f'(1) = \frac{1}{1} = 1$

$m = 1$

$m = 1 \quad (1,3)$

$y = mx + b$

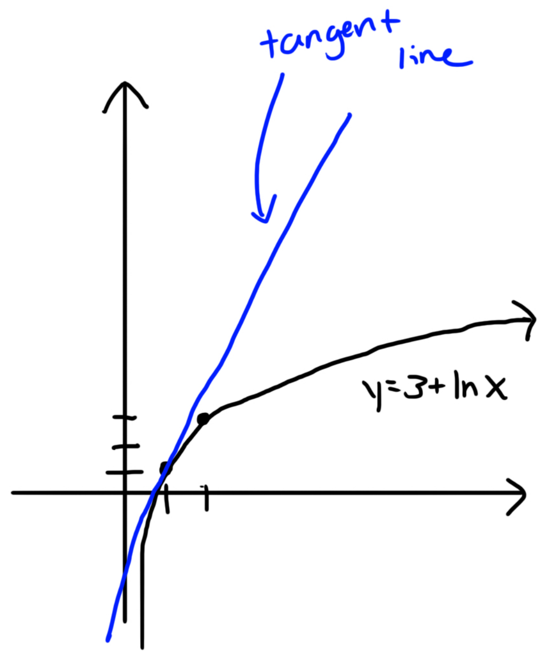
$y = x + b$

$3 = 1 + b$

$b = 2$

equation of the tangent line:

$y = x + 2$



Recall...

<u>Sum rule:</u> The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

or

$$(f(x)+g(x))'=f'(x)+g'(x)$$

<u>Difference rule:</u> The derivative of a difference is the difference of the derivatives.

$$\frac{d}{dx}\left(f(x)-g(x)\right)=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

or

$$(f(x)-g(x))'=f'(x)+g'(x)$$

Example: If $f(x)=x^3-x^2+x-1$, find $f'(x)$.

$3x^2 \quad 2x \quad 1 \quad 0$

$f'(x)=3x^2-2x+1$

3.3 - Derivatives of products and quotients

The product rule: If

$$y = f(x) = F(x)S(x)$$

and if $F'(x)$ and $S'(x)$ exist, then

$$f'(x) = F(x)S'(x) + F'(x)S(x)$$

In words, *the derivative of a product is the first times the derivative of the second plus the derivative of the first times the second.*

Example: If $f(x) = (x-1)(x^2+3x+1)$, find $f'(x)$.

$f'(x) = F(x)S'(x) + F'(x)S(x)$

$F'(x) = 1$

$S'(x) = 2x+3$

$f'(x) = (x-1)(2x+3) + (1)(x^2+3x+1)$

$(2x^2+3x-2x-3) + (x^2+3x+1)$

$(2x^2+x-3) + (x^2+3x+1)$

$f'(x) = 3x^2+4x-2$

Problem: Find $f'(x)$

$f'(x) = F(x)S'(x) + F'(x)S(x)$

(a) $f(x) = (x-1)(x+4)$

$f'(x) = (x-1)(1) + (1)(x+4)$

$= x-1 + x+4$

$f'(x) = 2x+3$

(b) $f(x) = 2x^3(x^2-2)$

$f'(x) = (2x^3)(2x) + (6x^2)(x^2-2)$

$f'(x) = 2x^4 + 6x^4 - 12x^2$

$f'(x) = 8x^4 - 12x^2$ $f'(x) = 4(2x^4 - 3x^2)$

$(6x^2)(x^2-2)$

$6x^4 - 12x^2$

(c) $f(x) = (2x^3+x)(x^2-2)$

$f'(x) = (6x^2+1)(x^2-2) + (2x)(2x^3+x)$

$f'(x) = 6x^4 - 12x^2 + x^2 - 2 + 2x^4 + 2x^2$

$f'(x) = 8x^4 - 9x^2 - 2$

$(6x^2+1)(x^2-2)$

$6x^4 - 12x^2 + x^2 - 2$

$(2x)(2x^3+x)$

$2x^4 + 2x^2$

(d) $f(x) = x^2 \ln x$

$f'(x) = x^2\left(\frac{1}{x}\right) + 2x \ln x$

(e) $f(x) = 4x^2e^x$

$f'(x) = 4x^2e^x + 8x^2e^x$

The quotient rule: If

$$y = f(x) = \frac{T(x)}{B(x)}$$

and if $T'(x)$ and $B'(x)$ exist, then

$$f'(x) = \frac{B(x)T'(x) - T(x)B'(x)}{[B(x)]^2}$$

In words, *the derivative of a quotient is the bottom times the derivative of the top minus the top times derivative of the bottom divided by the bottom squared.*

Example: If $f(x) = \dfrac{2x-1}{5x+3}$, find $f'(x)$.

$T(x) = 2x-1 \qquad T'(x) = 2$

$B(x) = 5x+3 \qquad B'(x) = 5$

$$\frac{(5x+3)(2) - (2x-1)(5)}{(5x+3)^2}$$

$(5x+3)(5x+3)$

$10x^2 + 15x + 15x + 9$

$10x^2 + 30x + 9$

$$\frac{\cancel{10x} + 6 - \cancel{10x} - 5}{10x^2+30x+9}$$

$$\frac{1}{10x^2+30x+9}$$

Problem: Find $f'(x)$

(a) $f(x) = \dfrac{x-1}{x+4}$

$$f'(x) = \frac{(x+4)(1)-(1)(x-1)}{(x+4)^2}$$

(b) $f(x) = \dfrac{x^2-1}{3x-1}$

$$f'(x) = \frac{(3x-1)(2x)-(x^2-1)(3)}{(3x-1)^2}$$

(c) $f(x) = \dfrac{\ln x}{x+1}$

(d) $f(x) = \dfrac{1+e^x}{1-e^x}$

(e) $f(x) = \dfrac{2x}{1+\ln x}$

Example: What about $f(x) = \frac{1}{x}$? Should I use the quotient rule? Or?