Slope of a line

Mon, August 26, 2024
Professor Scobee absent
Dr. Luke is teaching with help of SLAs

From last class...

All non-vertical lines have a slope.

If (x_1, y_1) and (x_2, y_2) are two points on a line with $x_1 \neq x_2$, then the **slope** of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Useful fact: We can write an equation for a given line if we know...

- any two points that lie on the line, or
- the slope and any point that lies on the line

It's helpful to know the common forms for the equation of a line:

- **Slope-intercept form:**
$$y = mx + b$$
where m is the slope and b is the $y-$intercept
- **Point-slope form:**
$$y - y_1 = m(x - x_1)$$
where m is the slope and (x_1, y_1) is a point on the line
- **Standard form:**
$$Ax + By = C$$
where A, B, C are constants and not both A and B are 0

Example: Find an equation for the line with slope $m = -3$ that passes through the point $(0, 4)$.

$x_1 \ y_1$

$y - y_1 = m(x - x_1)$

$y - 4 = -3(x - 0)$

$y = -3x + 4 \rightarrow$ perpendicular: $y = \frac{1}{3}x + 4$

Example: Find an equation for the line with slope $m = 2$ that passes through the point $(-1, 2)$.

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - (-1))$

$y = 2(x+1) + 2$

$y = 2x + 2 + 2$

$y = 2x + 4$

perpendicular:

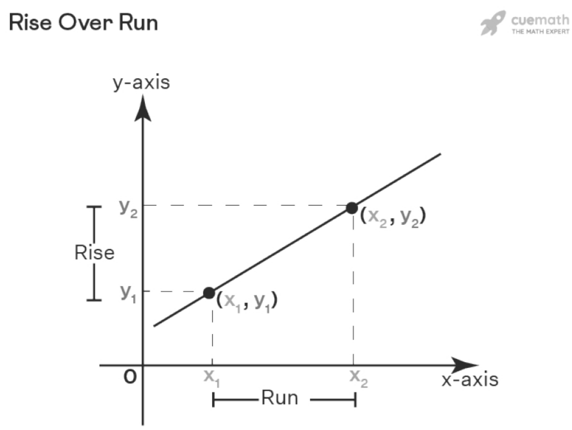
$y = -\frac{1}{2}x + 4$

Interpreting slope

Slope is a measure of the *steepness* of a line.

Slope is often described as

$$m = \frac{\text{rise}}{\text{run}}$$



If the slope < 0, then the line drops as we go from left to right.

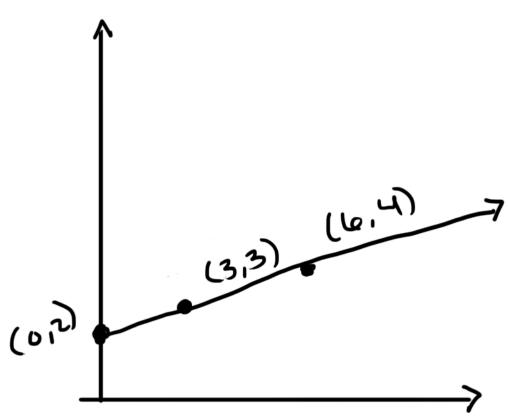
If the slope > 0, then the line rises as we go from left to right.

Example: Sketch the graph of a line with slope $\frac{1}{3}$ that passes through $(0, 2)$.

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{3}(x - 0)$

$y = \frac{1}{3}x + 2$



Example: Sketch the graph of a line with slope -2 that passes through $(-1, 2)$.

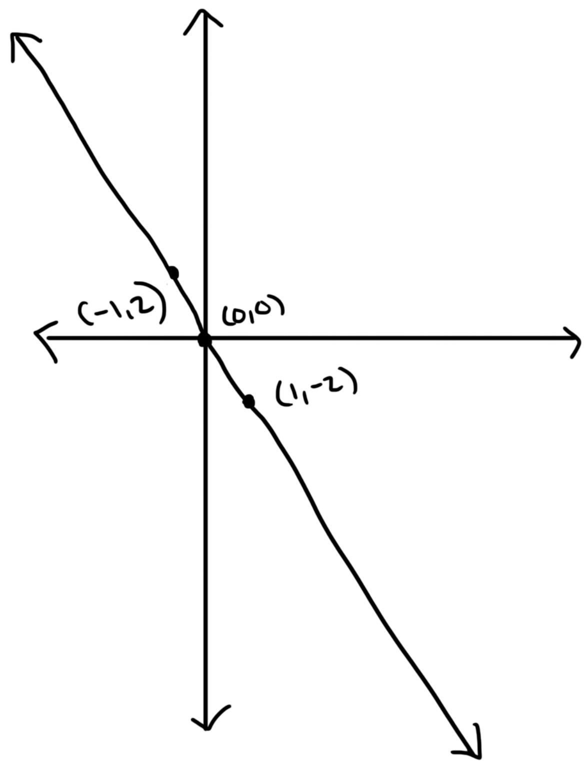
$y - y_1 = m(x - x_1)$

$y - 2 = -2(x - (-2))$

$y = -2(x + 2) + 2$

$y = -2x - 4 + 2$

$y = -2x - 2$



Question: How would you describe a line with slope $m = 0$?

undefined

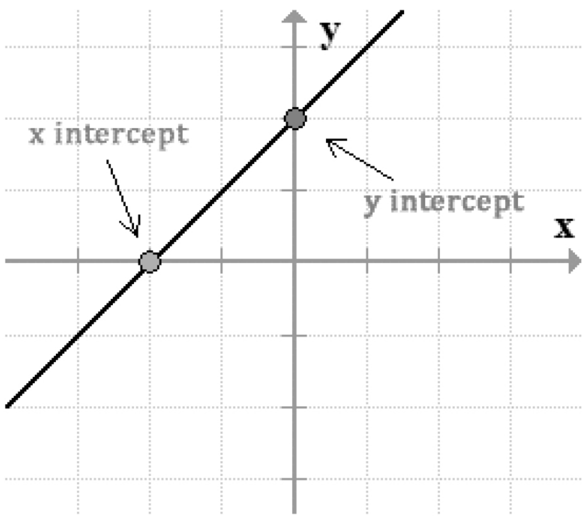
$m = 0$

Finding intercepts

A point where a graph crosses the x−axis is called an x−**intercept** and has a y−coordinate of $y = 0$.

The point where a graph crosses the y−axis is called the y−**intercept** and has an x−coordinate of $x = 0$.

They do not have to actually cross but can just touch.



Example: Find all intercepts for $f(x) = 3x - 9$.

$f(0) = 3(0) - 9$
$= 0 - 9$
$= -9$
$(0, -9)$

$0 = 3x - 9$
$-3x = -9$
$x = 3$
$(3, 0)$

y-intercept: $(0, -9)$
x-intercept: $(3, 0)$

Example: Find all intercepts for $f(x) = x^2 - 2x - 3$.

$f(0) = 0^2 - 2(0) - 3$
$= 0 - 0 - 3$
$= -3$
$(0, -3)$

$0 = x^2 - 2x - 3$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$2 \pm \sqrt{-2^2 - 4(1)(-3)}$

$\frac{2 \pm 4}{2}$

$= 3 \qquad = -1$

y-intercepts: $(0, -3)$
x-intercepts: $(3, 0)$
$(-1, 0)$

opens upward due to positive square (x^2).

2.1: Introduction to limits

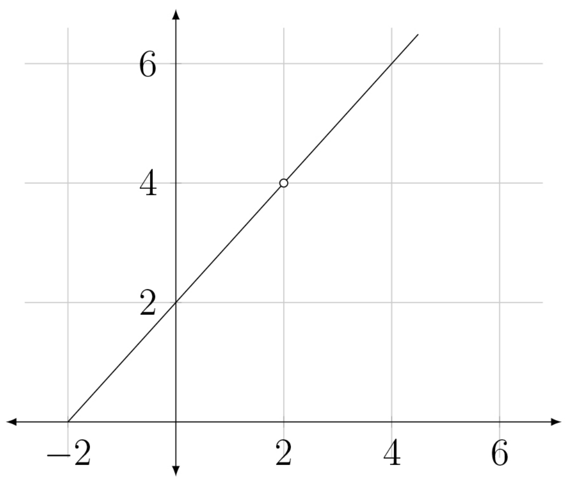
Why limits?

Limits are the cornerstone of Calculus. We won't see limits every day, but they play some role in every topic that we'll cover in this class, so we want to understand the concept.

Example: Consider the following graph of the function $f(x) = \dfrac{x^2-4}{x-2}$.

Note that $x = 2$ is not in the domain of f!

$$f(1.9) = \frac{1.9^2-4}{1.9-2} = \frac{3.61-4}{1.9-2} = \frac{-0.39}{-0.1} = 3.9$$



if an x makes only denominator 0, it is x-asymptote, only numberator is y-asymptote, both is a hole asymptote

x	$f(x) = \dfrac{x^2-4}{x-2}$
1.9	
1.99	
1.999	

Should be getting closer and closer to the same number which in this case should be getting closer to 4. Should never equal 4 in this case.

x	$f(x) = \dfrac{x^2-4}{x-2}$
2.1	
2.01	
2.001	

Question: What seems to be true? If we *had to* give a value to $f(2)$, what *should* it be? What makes the most sense?

$f(2) = 4$

In the previous example, we would say

$$\lim_{x \to 2} f(x) = 4$$

Definition of Limit:
We write

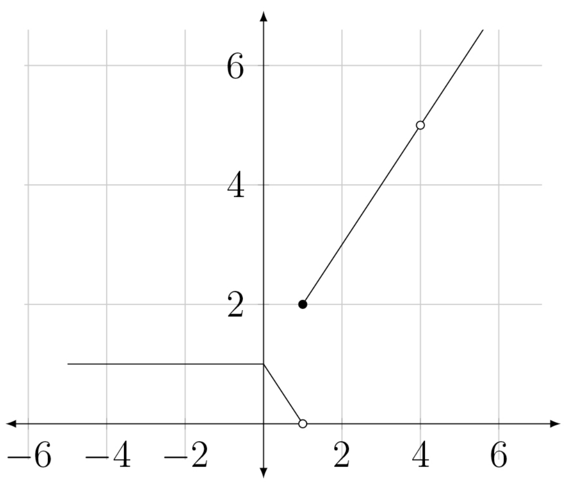
$$\lim_{x \to c} f(x) = L$$

or

$$f(x) \to L \text{ as } x \to c$$

if the functional value $f(x)$ is close to the *single* real number L whenever x is close, but not equal, to c (on either side of c).

Subtle but important point: The function doesn't have to be defined at $x = c$ but it does have to be defined on both sides of $x = c$. Said another way: $x = c$ doesn't have to be in the domain but all values on both sides of $x = c$ do need to be in the domain.



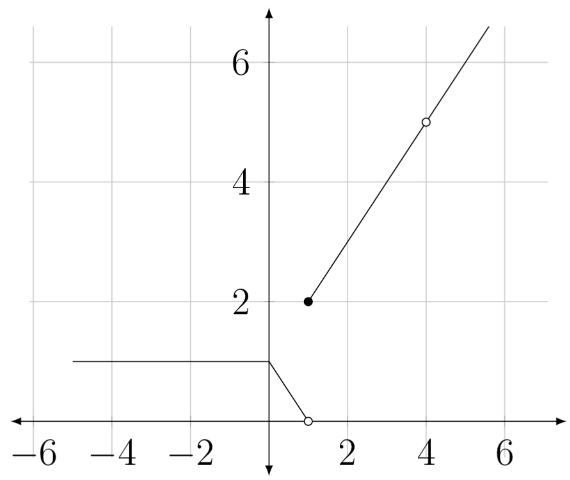
Examples: Evaluate (if possible) the limits.

$\lim_{x \to -2} f(x)$ = 0

$\lim_{x \to 0} f(x)$

$\lim_{x \to 4} f(x)$

What can we say about this one?



Does $\lim_{x \to 1} f(x)$ exist?

Definition of One-sided Limits:
We write

$$\lim_{x \to c^-} f(x) = K$$

and call K the **left-hand limit** or **limit from the left** if $f(x)$ is close to K whenever x is close, but *to the left* of, c on the real number line.
We write

$$\lim_{x \to c^+} f(x) = L$$

and call L the **right-hand limit** or **limit from the right** if $f(x)$ is close to L whenever x is close, but *to the right* of, c on the real number line.

Example In the example above

$\lim_{x \to 1^-} f(x) = \ ?$

$\lim_{x \to 1^+} f(x) = \ ?$

$$\lim_{x \to c} f(x) = L \text{ if and only if } \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

Properties of limits

Let f and g be two functions, and assume that

$$\lim_{x\to c} f(x) = L \text{ and } \lim_{x\to c} g(x) = M$$

where L and M are both real numbers.

1. $\lim_{x\to c} k = k$ for any constant k
2. $\lim_{x\to c} x = c$
3. $\lim_{x\to c}[f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = L + M$
4. $\lim_{x\to c}[f(x) - g(x)] = \lim_{x\to c} f(x) - \lim_{x\to c} g(x) = L - M$
5. $\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x) = kL$
6. $\lim_{x\to c}[f(x) \cdot g(x)] = [\lim_{x\to c} f(x)][\lim_{x\to c} g(x)] = LM$
7. $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \dfrac{L}{M}$ if $M \neq 0$
8. $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)} = \sqrt[n]{L}$, where $L > 0$ when n is even

Note: these same rules apply for one-sided limits too.

Example: Find

$$\lim_{x\to 3}(x^2 - 5x)$$

Example: Find
$$\lim_{x \to -1} (3x + 5)$$

Example: Find
$$\lim_{x \to 3} (x^2 + 3x + 5)$$

Example: Find
$$\lim_{x \to 2} \frac{2x - 1}{x^3 + 4}$$

Theorem 3:

1. $\lim_{x \to c} f(x) = f(c)$ for any polynomial function f
2. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$ for any rational function $\frac{f}{g}$ with $g(c) \neq 0$.

Example: Find

$$\lim_{x \to 3} \sqrt{x+1}$$

Example: Find

$$\lim_{x \to 3} \frac{1}{\sqrt{x+1}}$$

Indeterminate form

Example: Find

$$\lim_{x\to 1} \frac{x^2-1}{x-1}$$

If $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)}$$

is said to be **indeterminate**.
If $\lim_{x\to c} f(x) \neq 0$ and $\lim_{x\to c} g(x) = 0$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)}$$

does not exist.

Evaluate the following limits

Example: $\lim_{x\to 0}(3x^4 - 2x^3 + 7)$

Example: $\lim_{x\to 1}(2x - 5\sqrt{x+3})$

Example: $\lim_{x\to 1}\dfrac{x-1}{x^2+1}$

Example: $\lim_{x\to 1}\dfrac{x-1}{x^2-1}$

Example: $\lim_{x\to 1}\dfrac{x+1}{x^2-1}$