Set theory - Famous Infinite Sets

$\mathbb{N} = \{1, 2, 3, \ldots\}$ - roster format

(can't compare the sizes of infinite sets)

$\mathbb{N}$ - symbol for all natural numbers (gothic N)

$\mathbb{W} = \{0, 1, 2, 3, \ldots\}$ -> infinite and countable

$\mathbb{W} = \{k-1 \mid k \in \mathbb{N}\}$

$\mathbb{Z}$ - set of all integers

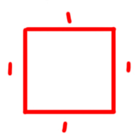
$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$= \{\pm(k-1) \mid k \in \mathbb{N}\}$

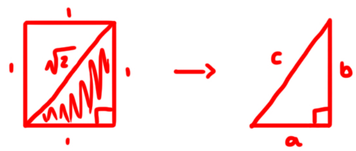
$\mathbb{Q}$ - set of all rational numbers

$\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\}$ (, means "and")

Proving there can be irrational numbers



-> unit squared (1^2)



$c^2 = a^2 + b^2$
$c^2 = 1^2 + 1^2$
$c^2 = 1 + 1$
$c^2 = 2$
$c = \sqrt{2}$

Assume to the contrary that $\sqrt{2} \in \mathbb{Q}$ (is rational)

$\sqrt{2} = \frac{a}{b}$ $a, b \in \mathbb{Z}$ $b \neq 0$

WLOG (without loss of generality)
assume $\frac{a}{b}$ is simplified

$\sqrt{2} = \frac{a}{b} \rightarrow 2 = \left(\frac{a}{b}\right)^2 \rightarrow 2 = \frac{a^2}{b^2}$

$\rightarrow 2b^2 = \underbrace{a^2}_{\text{even}}$ $\rightarrow$ a is even

$(2 \cdot x)^2 = 4 \cdot x^2$

$a^2 \rightarrow$ even
$a \rightarrow$ even
$b^2 \rightarrow$ even
$b \rightarrow$ even

$2b^2 = a^2$

$b^2 = \frac{a^2}{2}$

b^2 is even

$\sqrt{2} = \frac{a}{b}$ a fully simplified fraction
(how can they be even?)

$\sqrt{2} \in \mathbb{Q}$ is a false statement

$\sqrt{2} \in \mathbb{Q}$

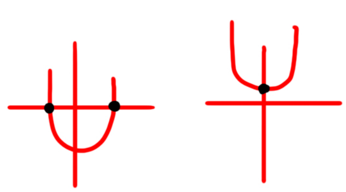
Irrationals are the compliment $\mathbb{Q}$

$\mathbb{Q}^c$

$\mathbb{Q} \cup \mathbb{Q}^c = \mathbb{R}$ reals

$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$\mathbb{Q}^c \subset \mathbb{R}$



$D = \mathbb{R}$

$x^2 + 1 = 0$ $x \in \varnothing$

$D = \mathbb{C}$ - complex numbers

$x \in \{i, -i\}$

$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$