Notes on Dark Matter and Data Analysis

Mateus P. Otto

Abstract

These are my personal notes on dark matter (DM) physics and data analysis developed to substantiate my undergraduate monograph on the *Fermi GeV excess*. Particularly, I will be using these to write my research assignments and to perform my own incursions on the above mentioned topics.

1 Dark Matter profiles

- NFW [NFW95]: based on N-body simulations.

$$\rho_{\rm NFW}(r) = \rho_s \frac{r_s}{r}\left(1+\frac{r}{r_s}\right)^{-2} \tag{1}$$

- Einasto [TWPS09]: based on N-body simulations.

$$\rho_{\rm Ein}(r) = \rho_s \exp\left\{-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]\right\} \tag{2}$$

- Isothermal [BBS91]: reproduces the naive r^{-2} dependence of the DM density.

$$\rho_{\rm Iso}(r) = \frac{\rho_s}{1+(r/r_s)^2} \tag{3}$$

- Burkert [SB00]: based on the observation of galactic RCs.

$$\rho_{\rm Bur}(r) = \frac{\rho_s}{(1+r/r_s)(1+(r/r_s)^2)} \tag{4}$$

- Moore [DMS04]: fit on N-body simulations of a NFW-like density profile.

$$\rho_{\rm Moo}(r) = \rho_s \left(\frac{r_s}{r}\right)^{1.16}\left(1+\frac{r}{r_s}\right)^{-1.84} \tag{5}$$

These profiles are depicted in Figure 1 for the SS, where I also included the angular dependence of the density, following [CCH+10].

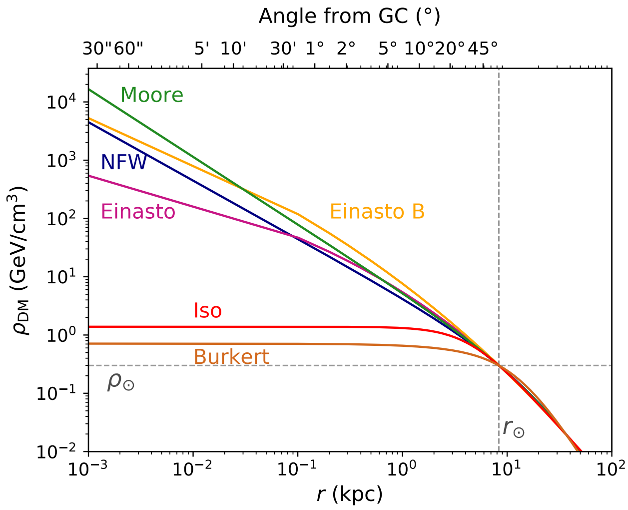

Figure 1: DM profiles and the predicted DM densities ($\rho_{\rm DM}$) for the Milky Way.

1.1 Discussion and Questions

- Spherical symmetry

  All these profiles contain only radial dependence, *i.e.* they are assumed to have spherical symmetry. [JS02] discusses, using simulated halos, that the profile is better described by triaxial ellipsoids. [LMJ09] uses the tidal stream from SagDEG disruption to determine the shape of the MW halo.

- Galactic Center

  Do these profiles provide a good description of the GC dark matter distribution? [CCH+10] discusses this under his Figure 1. Do simulations include baryonic feedback? Does feedback change the profile far from the GC?

- Substructure

  Cirelli [CCH+10] mentions that N-body simulations show that a hierarchy of DM halos is formed during galactic evolution, creating so-called *subhalos* or *DM substructure*. Is there a place in the MW where the DM density is greater than in the GC? Potential places can be explored with gravitational lensing? See [BMSH+19].

2 Dark Matter J-factors

Dark matter J-factors describe the astrophysical component of the observed differential photon flux, in the sense that it integrates the particle physics processes occurring along the line of sight (los) of the observer.

For annihilating DM, they are defined as:

$$J_\mathrm{a} = \int_\mathrm{los} \frac{ds}{r_\odot} \left(\frac{\rho(r(s,\theta))}{\rho_\odot} \right)^2 \tag{6}$$

Whilst for decaying DM:

$$J_\mathrm{d} = \int_\mathrm{los} \frac{ds}{r_\odot} \left(\frac{\rho(r(s,\theta))}{\rho_\odot} \right) \tag{7}$$

We can also define the effect of the detecting instrument aperture $\Delta\Omega$ on the total differential flux by introducing the mean J-factor:

$$\bar{J} = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} J(\theta) d\Omega \tag{8}$$

The geometry and the parameters involved in these integrals are represented in Figure 2, where I also included the galactic polar coordinates (ℓ, b), that are useful in the calculation of $\bar{J}$, as defined on (8).

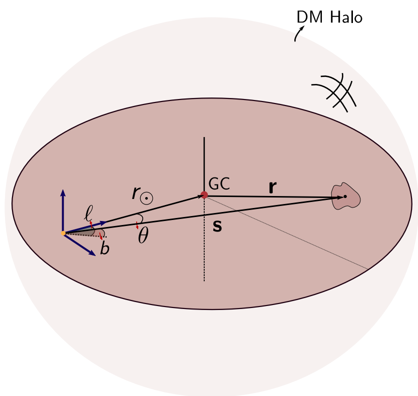

Figure 2: Schematic of the Milky Way disk and its DM halo.

In Figures 3 and 4, the calculated J-factors for annihilating and decaying DM are depicted assuming the common density profiles for DM in the MW.

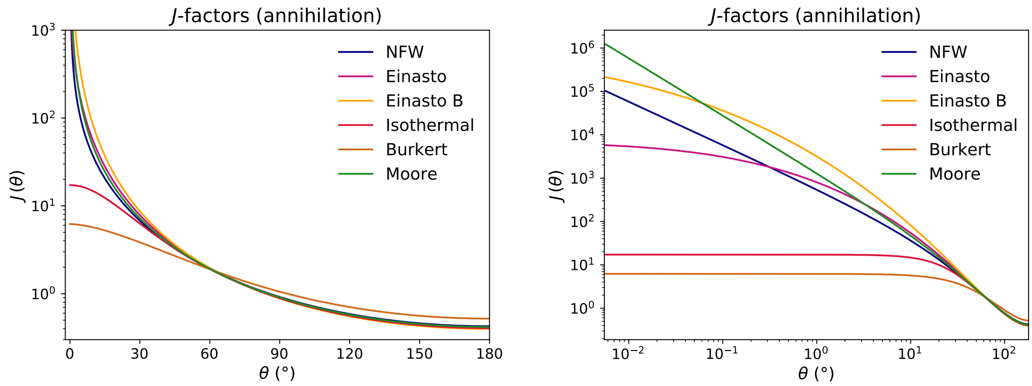

Figure 3: Annihilating DM J-factors for the common dark matter density profiles.

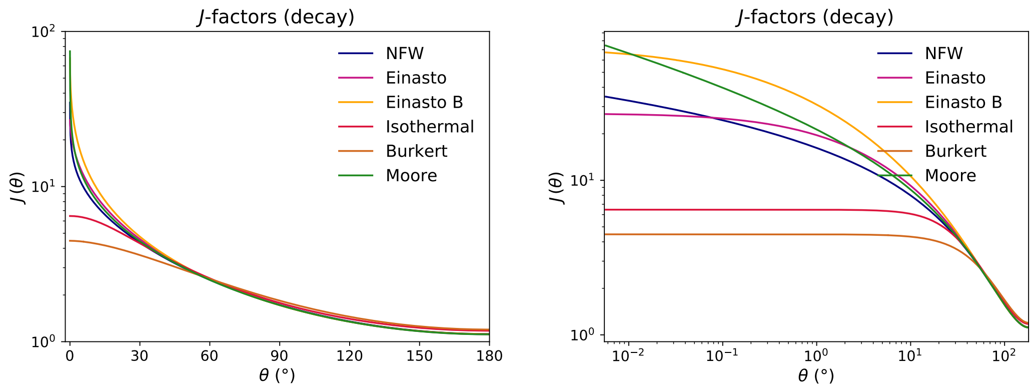

Figure 4: Decaying DM J-factors for the common dark matter density profiles.

2.1 Maps of J-factors

Our objective is to make two-dimensional maps of J-factors centered on the GC. Computationally, we adopt the following procedure:

1. Create a grid of galactic longitudes and latitudes (ℓ, b).
2. Convert the grid to the (s, θ) coordinate system.
3. Integrate, for each grid position, along the observer l.o.s.

Note that using the coordinates depicted in Figure 2, we can write:

$$\cos(\theta) = \frac{s_x}{s} = \frac{s\cos(b)\cos(\ell)}{s} = \cos(b)\cos(\ell)$$

So that the conversion between (ℓ, b) to the (s, θ) system is straightforward. For the calculations, I defined 3 regions:

Name	b	ℓ
GC1	$< 0.1\ ^\circ$	$< 0.1\ ^\circ$
GC2	$< 0.14\ ^\circ$	$< 0.14\ ^\circ$
GC3	$< 1\ ^\circ$	$< 1\ ^\circ$

Table 1: Nomenclature and definition of J-factors maps around GC.

The following Figures 5 and 6 depict the resulting maps for the region GC1 with NFW and Einasto profiles.

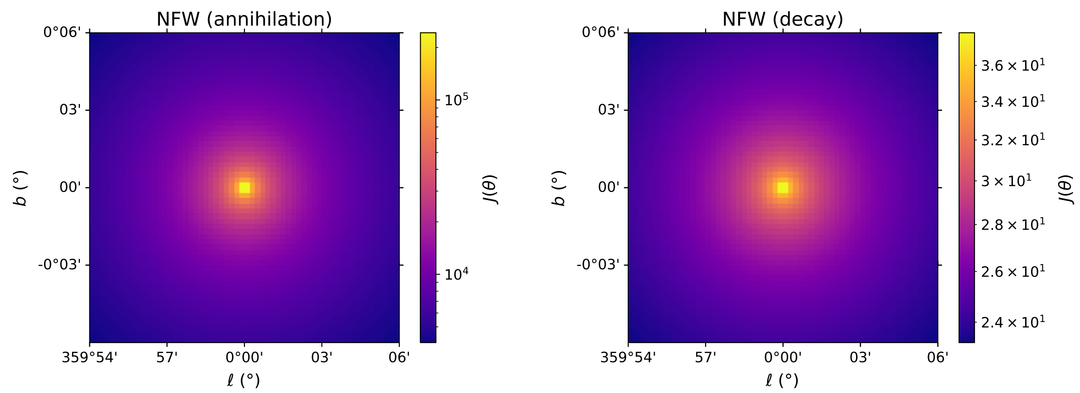

Figure 5: Map of J-factors for the NFW profile.

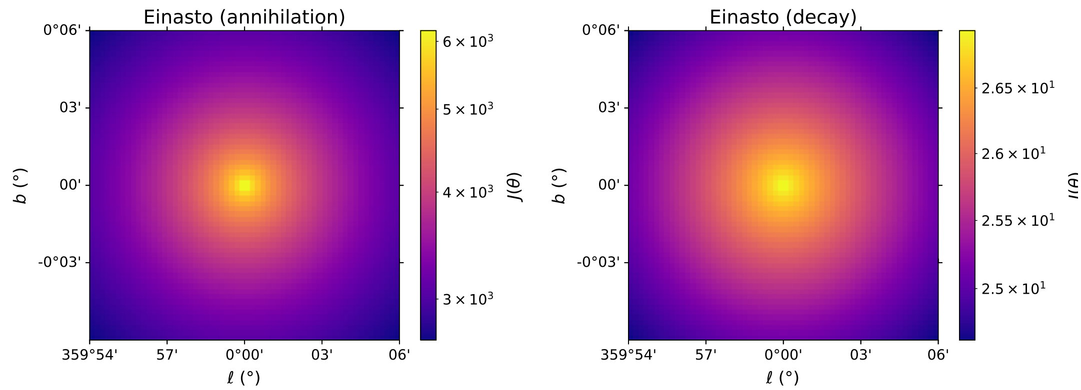

Figure 6: Map of J-factors for the Einasto profile.

3

A Notation

B Spherical Astronomy

Based on [RC82], see also [BGPW60].

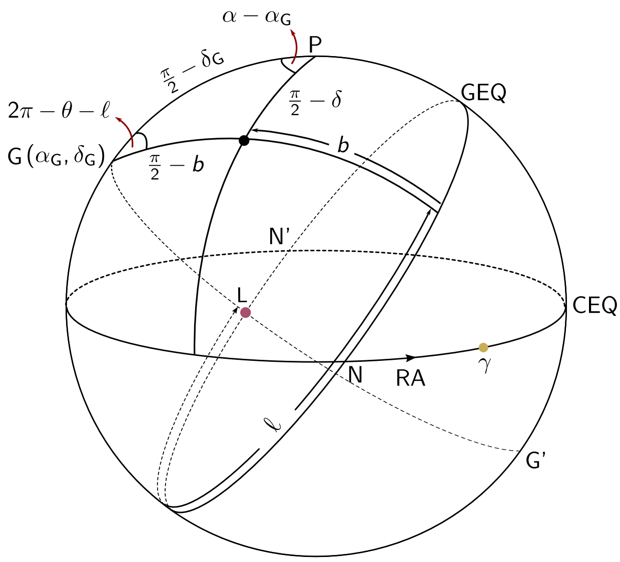

Figure 7: Celestial sphere geometry for the relation between the equatorial and galactic coordinate systems.

References

[BBS91] K. G. Begeman, A. H. Broeils, and R. H. Sanders, *Extended rotation curves of spiral galaxies: dark haloes and modified dynamics*, Monthly Notices of the Royal Astronomical Society **249** (1991), no. 3, 523–537.

[BGPW60] A. Blaauw, C. S. Gum, J. L. Pawsey, and G. Westerhout, *The new I. A. U. system of galactic coordinates (1958 revision)*, **121** (1960), 123.

[BMSH+19] Johann Brehmer, Siddharth Mishra-Sharma, Joeri Hermans, Gilles Louppe, and Kyle Cranmer, *Mining for dark matter substructure: Inferring subhalo population properties from strong lenses with machine learning*, 2019.

[CCH+10] Marco Cirelli, Gennaro Corcella, Andi Hektor, Gert Hütsi, Mario Kadastik, Paolo Panci, Martti Raidal, Filippo Sala, and Alessandro Strumia, *PPPC 4 DM ID: A poor particle physicist cookbook for dark matter indirect detection*, 2010.

[DMS04] Juerg Diemand, Ben Moore, and Joachim Stadel, *Convergence and scatter of cluster density profiles*, 2004.

[JS02] Y. P. Jing and Yasushi Suto, *Triaxial modeling of halo density profiles with high-resolution n-body simulations*, 2002.

[LMJ09] David R. Law, Steven R. Majewski, and Kathryn V. Johnston, *Evidence for a triaxial milky way dark matter halo from the sagittarius stellar tidal stream*, 2009.

[NFW95] Julio F. Navarro, Carlos S. Frenk, and Simon D. M. White, *The structure of cold dark matter halos*, 1995.

[RC82] A.E. Roy and D. Clarke, *Astronomy: Principles and Practice*, A. Hilger, 1982.

[SB00] Paolo Salucci and Andreas Burkert, *Dark matter scaling relations*, 2000.

[TWPS09] Patricia B. Tissera, Simon D. M. White, Susana Pedrosa, and Cecilia Scannapieco, *Dark matter response to galaxy formation*, 2009.

Glossary

DM	Dark Matter
MW	Milky Way
NFW	Navarro-Frenk-White Density Profile
RC	Rotation Curves
SS	Solar System