An Abstract Model of Historical Processes

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August 1, 2016

Abstract

A game theoretic model is presented which attempts to simulate, at a very abstract level, power struggles in the social world. In the model, agents can benefit or harm each other, to varying degrees and with differing levels of influence. The agents play a dynamic, noncooperative, perfect information game where the goal is to maximize payoffs based on positional utility and intertemporal preference, while being constrained by social inertia. Agents use the power they have in order to get more of it, both in an absolute and relative sense. More research is needed to assess the model's empirical validity.

Keywords: Theoretical history; political sociology; agent-based simulation; game theory; balance of power theory; conflict theory; and interdependence

1 Introduction and Motivation

When we look back over the history of humanity, regardless of the time period, place, or culture that we examine, we find agents engaged in power struggles. These agents might be states jockeying for primacy in the international arena. They might be groups or factions grappling for control of some market or political apparatus. They might be individuals competing for supremacy within some institution or spatial region. At all of these various levels, across all of history, the existence of power struggles like these has been invariant, presumably a function of the fact that throughout all of human history, *there have been humans present.*

What do we mean by *power struggles*? Simply put, they are conflicts that arise due to the tendency of agents to use whatever power they have to get *more* of it. In other words,

*Thanks to Pavel Grinfeld, Roger McCain, and Constantinos Rokas for their counsel and suggestions. All mistakes are my own.

they are struggles over power itself. When struggling for power, agents must decide who to cooperate with, and whom to harm, allocating whatever power is at their disposal and taking into consideration the anticipated actions and reactions of their competitors. They tend to seek a combination of both absolute and relative power. The political theorist Hans Morgenthau (1954) had this sort of process in mind when he described power, in the international context, as "that untamed and barbaric force which finds its laws in nothing but its own strength and its sole justification in its own aggrandizement."

This paper presents a toy model of power struggles. It's a toy model because it ignores many of the things that affect real world conflict dynamics, like culture, technology, the environment, ideology, geography, migration, institutions, disease, and resource scarcity. Instead, the model focuses on the fluid dynamics of a single thing: power. It seeks to answer the question: Given an initial configuration of relationships among agents, each of whom has a starting amount of power, how will the system tend to evolve in time? The model illustrates how the struggle for power in and of itself might give rise to conflict and alliance dynamics that resemble those that we see in the real world. Future work will refine the model and attempt to apply it to historical situations in order to gauge its empirical utility; at the moment, it is merely hypothetical.

2 The Model

The model is presented here in mathematical form. A source code implementation in the *Mathematica* programming language is available at `www.github.com/mpoulshock/AMHP`.

2.1 Basic Mechanism

We begin by describing the model's main data structures and variables. We use a network, or graph, structure to model agents and their interrelationships. Each node, or agent, has a characteristic called *power*, which represents its ability to influence events. Power, also referred to as *size*, is represented in this model as a number s, where it is always the case that

$$s \geq 0 \tag{1}$$

The larger the number, the greater the agent's power. Zero power means that the agent has no influence, even over itself, and is effectively dead.

An agent can direct its power positively or negatively towards other agents. Using it positively entails giving it to another agent. However, when one agent gives a positive amount of power to another, the amount received by the other agent is increased by a factor of β, where

$$\beta > 1 \tag{2}$$

This *benevolence multiplier* corresponds to the notion that, regardless of what has been given, it has more value to the recipient than it did to the giver. For example, if S sells B a pound of apples for \$1, we can assume that to B the apples were more valuable than \$1, and that to S the \$1 was more valuable than apples. The increase factor can be thought of as possibly corresponding in the real world to the benefits of exchange and the division of labor. The benevolence multiplier allows agents in the model to achieve mutual benefit (growth in power) by cooperating with each other.

The transfer of negative power is similarly subject to a *malevolence multiplier*, μ, where

$$\mu > \beta \tag{3}$$

That is, when an agent uses power negatively, for every unit of power it expends, it causes a reduction in the recipient's power by μ. This reflects the idea that it is easier to destroy value than to create it. Exactly how much easier is left undefined, as one of the model's parameters. To the extent that this model has real world correspondence, it may be that an empirical value for μ bears some relationship to the concept of entropy, since destruction is ultimately about increasing disorder.

An agent can never use more power than it has. That is, the sum of the absolute value of an agent's outgoing allocations to other agents must equal the agent's total power. A *tactic vector* τ represents an agent's allocation of its power, where for each element

$$\tau_j \in [-1, 1] \tag{4}$$

and

$$\sum_{j=1}^{n} |\tau_j| = 1 \tag{5}$$

where n is the number of agents. The tactic vector expresses an agent's relationships as positive and negative percentages whose absolute values must sum to one. It can be thought of as a kind of a foreign policy. An example of a legal tactic vector in a three player game is

$$(0.7, -0.1, 0.2)$$

All of the agents' tactic vectors together form a *tactic matrix*, $\mathbf{T}$. Here's an example:

$$\begin{pmatrix} 0.7 & 0.1 & 0 \\ -0.1 & 0.8 & 0 \\ 0.2 & 0.1 & 1 \end{pmatrix}$$

Each column in this matrix represents a tactic vector and satisfies equation (5). The matrix diagonal represents the amount of power that the agents are allocating to themselves. Any power that an agent allocates to itself is not subject to any multiplier.

The agents' sizes are stored in a *size vector*, $\mathbf{s}$, for example

$$(1.1, 4.3, 2.7)$$

The tactic matrix and the size vector together comprise the *game state*. The game state can be visualized as a graph in which the node sizes show the power of the agents and the directed edges show the agents' tactical allocations. Figure 1 shows an example using the size vector and tactic matrix above. Green represents positive (benevolent) transfers and red negative (malevolent) ones.

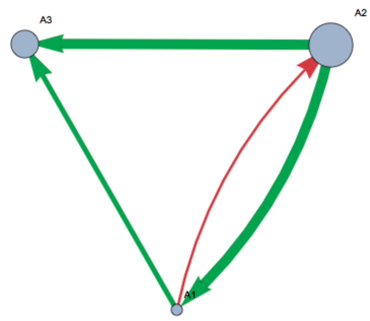

Figure 1: Game state graph

The state of the model is updated in time steps, with all players moving simultaneously. At each step, power is transferred among the agents according to their tactic vectors and as it flows around the network, the power of each agent increases or decreases. An agent's updated power is the sum of the power coming to it from other agents, subject to the appropriate multiplier, plus the power that it kept to itself. We can represent this updating procedure mathematically as

$$\mathbf{s}(t) = (\mathbf{T} \circ \mathbf{M}) \cdot \mathbf{s}(t-1) \tag{6}$$

where $\circ$ represents the Hadamard product (element-wise matrix multiplication), $\cdot$ is the dot product, and $\mathbf{M}$ is a matrix representing the benevolence and malevolence multipliers defined as

$$(m_{ij}) = \begin{cases} 1 & i = j \\ \beta & \tau_{ij} \geq 0 \\ \mu & \tau_{ij} < 0 \end{cases} \tag{7}$$

It's important to add that in equation (6), for every element of $\mathbf{s}$, $s_i \geq 0$, due to the constraint imposed by equation (1). If, as a result of updating, an agent's size becomes less than or equal to zero, they are considered dead and assigned a size of 0. In *Mathematica*, this can be implemented as

```
s[t_] := Map[Max[#,0]&, (T*M).s[t-1]]
```

See the source code for details.

Using the example data above, and assuming that $\beta = 1.2$ and $\mu = 3$, the new size vector produced by equation (6) would be

$$\left(\begin{pmatrix} 0.7 & 0.1 & 0 \\ -0.1 & 0.8 & 0 \\ 0.2 & 0.1 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 1.2 & 1.2 \\ 3 & 1 & 1.2 \\ 1.2 & 1.2 & 1 \end{pmatrix}\right) \cdot \begin{pmatrix} 1.1 \\ 4.3 \\ 2.7 \end{pmatrix} = \begin{pmatrix} 1.29 \\ 3.11 \\ 3.48 \end{pmatrix}$$

The formalism of this model provides a flexible way of expressing a variety of interrelationships. Actions can be constructive or destructive, and in varying degrees. Relationships need not be symmetrical, as they frequently are not in real life. And some agents have more influence than others. A variety of situations can be modeled this way.

2.2 The Game

Because the possible actions of each agent, in choosing a tactic, depend upon the possible actions of other agents, it is appropriate to treat this as a problem in game theory. The game is a noncooperative, dynamic game in which each player can change its tactics at each time step, and all players have complete information. Again, all players move simultaneously.

An agent's *strategy* is a sequence of tactics. Each agent chooses a number of random strategies and then we play out all combinations of all agents' strategies. We are effectively randomly sampling the game tree using Monte Carlo tree search. Random sampling is necessary because the game has a *continuous action space*: tactic vectors are continuous rather than discrete, so at each time step, each agent has an infinity of moves to choose from.

How do do agents prevail in this game? How do they decide what moves are preferable? Their decisions are based upon three considerations: positional utility, social inertia, and intertemporal payoffs. These ideas are explained in the sections that follow, after which we describe how these concepts combine to generate a simulation.

2.3 Positional Utility

At each time step, each agent adjusts its tactic vector. These choices can lead to a wide variety of game states, and each agent needs a way to evaluate whether a given game state is better or worse than any other state. Agents do not merely seek to maximize their own power. They instead maximize a utility function approximated by

$$u_i \approx \frac{s_i^{2.5}}{\sum_{j=1}^{n} s_j^2}$$

where u_i is the utility to the ith agent.

Basically, two things matter to agents: they want to be powerful in an absolute sense, and they want to dominate other agents by being powerful in a relative sense. These two desires are in tension: to dominate, they may have to shrink by causing others to shrink even more; to grow in size, they may have to cease dominating. The utility function encodes these conflicting objectives. This is easier to see when the right side of the utility function is split into two components:

$$u_i \approx \frac{s_i^2}{\sum_{j=1}^{n} s_j^2} \sqrt{s_i}$$

The first component on the right side of this equation reflects dominance: specifically, the ratio of an agent's *market share* (their proportion of all power in the system) squared to the total of each agent's market share squared. In economic terms, this expresses each agent's contribution to the Herfindahl-Hirschman Index, which is a measure of competition within a given market. This component embodies the idea that the smaller and more divided one's competition, the better off one is. (By way of comparison, John Mearsheimer (2001) asserts that states in the international system seek to maximize their *market share* of power.)

The second component of the equation above provides an incentive for absolute growth. The square root of size is taken in order to reflect the diminishing marginal utility of acquiring power, since one additional unit of power is more valuable to a small agent than to a large one. (This is a common facet of economic utility functions.) This could be generalized to a cube or nth root, such that

$$u_i = \frac{s_i^\alpha}{\sum_{j=1}^{n} s_j^2} \tag{8}$$

where the utility exponent α is in the range

$$\alpha \in [2, 3] \tag{9}$$

As α decreases, relative power is incentivized and agents become more apt to use violence to cut other agents down to size, so they can hoard market share. As α increases, they're more prone to pursue absolute growth, which requires cooperation (mutual benevolence). We could allow α to vary for each individual, which might better reflect the heterogeneity of the social world. However, for the sake of simplicity, we'll assume that this parameter is the same for all agents.

This utility function has a few other desirable properties worth mentioning. First, agents that are the same size have the same payoff, dead agents have a payoff of zero, and the largest agents will have the largest payoff. Adding agents with a size of zero to the population doesn't affect the payoffs to the existing agents. Further, when there are two agents whose sizes differ by a constant amount, their payoffs will tend to converge as their sizes increase by the same amount (e.g. the payoffs to agents with $\mathbf{s} = (100, 101)$ will be closer together than those with $\mathbf{s} = (1, 2)$). Finally, the utility function is smooth and well-behaved, except when all agents have a size of 0, in which case there's no longer a game.

2.4 Social Inertia

Recall that agents must decide, at each time step, how to allocate their power among the other agents. It cannot be the case that they are free to choose *whatever* allocations they like. The past, or more specifically the present, binds their options. For example, a country could not one day suddenly alter its entire foreign policy. Processes like trade agreements, peace talks, mergers and acquisitions, and divorces all take time, because social relationships have a kind of stickiness that resists rapid change. This phenomenon is called *social inertia* (Wikipedia 2016).

Social inertia makes it less likely that agents will be able to effect dramatic tactical changes. Faraway tactics are unlikely to be reached; ones more similar to the current tactic are more plausible. An agent's expected payoff p from a position, then, is the position's utility to the agent times the probability of getting to the current tactic matrix $\mathbf{T}_1$ from the previous one $\mathbf{T}_0$:

$$p_i = u_i f(d(\mathbf{T}_0, \mathbf{T}_1), \sigma) \tag{10}$$

Here d is the distance between the two tactic matrices, defined as

$$d(\mathbf{A}, \mathbf{B}) = \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} (a_{ij} - b_{ij})^2} \tag{11}$$

and f is the probability density function of the normal distribution

$$f(x, \sigma) = \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \tag{12}$$

In other words, the expected payoff is the utility times the height of the bell curve as a function of the distance between the two tactic matrices. The variable σ is used as a model parameter that controls the intensity of social inertia (when σ is close to 0, inertia is high; as σ increases, inertia decreases).

2.5 Intertemporal Payoffs

The agents assume that the game is played indefinitely into the future, which opens up the possibility of reciprocity. We define the *intertemporal payoff* P of a strategy as the total payoffs that an agent will receive over future time steps, subject to discounting such that payoffs in the near future are weighted more heavily than those in the distant future:

$$P_i = (1 - \delta) \sum_{t=1}^{\infty} \delta^t p_i(t) \tag{13}$$

where δ is the *discount factor* and

$$\delta \in (0, 1) \tag{14}$$

Ratliff (1996) explains how equations like (13) work and how to interpret δ. We can summarize by saying that when δ increases, future payoffs are given more weight and players can be thought of as having more patience for a longer-term view, enabling greater cooperation. Conversely, when δ decreases, players are more short-sighted and eager for immediate rewards. The term $(1 - \delta)$ normalizes the result so it can be compared with the payoffs from individual time steps.

Having calculated the intertemporal payoff for a given sequence of moves, we then compare it to the equilibrium solution of the starting game state, or the *stage Nash equilibrium*. The stage equilibrium is a state at the next time step in which no player could get a higher payoff by altering its tactic vector. For every game state, there may be one or more stage Nash equilibria at the next time step, or there may be none at all.

Once we know the stage Nash equilibria of the starting game state, we can determine the minimum payoff that each player is guaranteed to receive. As mentioned, we then look for game paths (strategy combinations) in which every player's intertemporal payoff is greater than their minimum payoff. Per the Folk Theorem, all such outcomes are Nash equilibria.

2.6 Probabilistic Outcomes

There are many intertemporal equilibria (theoretically an infinity of them, although Monte Carlo search generates a finite number). However, we need not feel overwhelmed. We can make statements about the likelihood of various outcomes being reached, again using the concept of social inertia. The greater the tactical variation in a particular game path, the less likely we can presume the outcome to be. We'll define the likelihood l of a game path as

$$l = f\left((1 - \delta) \sum_{t=1}^{\infty} \delta^t d(\mathbf{T}_{t-1}, \mathbf{T}_t), \sigma\right) \tag{15}$$

where d is defined in (11) and f is the probability function (12), with σ as a parameter that can be used to calibrate probabilities with empirical data. Discounting is used so that irrelevant future game states are ignored.

We now have a model that is nondeterministic. We can look at the space of possible outcomes and associate them with their likelihood of occurring.

2.7 Simulations

The mathematical machinery above can project what agents are likely to do at the next time step. However, it does not provide the necessary infrastructure for playing an initial situation forward in time indefinitely. Ideally, we want to be able to configure a starting scenario, press *play*, and then watch events in this universe unfold. We'll generate these simulations as follows.

Given an initial game state, we'll carry out the procedure described in sections 2.1-2.6 above, which will result in a list of game paths, ordered by likelihood. We'll take the most likely of those paths and then take the first game state on that path (the one that comes after the initial game state). This game state will be our first *frame*. We will then take that frame as our new initial game state and repeat the process, obtaining a second frame. Continuing this frame-by-frame generation, we'll create a *reel* that we can play like a film.

With more computing power at our disposal, we need not restrict ourselves to choosing the most likely game path when we identify each frame. We could do another level of Monte Carlo simulation and explore many such paths, ultimately developing numerous reels with associated probability profiles. This would give us a broad sense of the possible futures of a starting game state, along with their corresponding likelihoods.

3 Discussion and Future Work

We have defined a game theoretic simulation model in which agents capable of inflicting harm on others seek to maximize their own power. The model is represented more or less by the numbered equations above. To loosely summarize its basic assumptions:

1. Agents have an attribute called power that they can expend to benefit or harm other agents. Benefiting another agent has a greater impact than keeping power to oneself. Harming another agent has a greater impact than giving them a benefit.

2. Agents seek power in absolute and relative terms, according to a utility function that incentivizes both.

3. Agents are constrained by social inertia, which makes drastic changes in their relationships less likely and which reduces expected positional utility.

4. Agents play a dynamic, iterative, non-cooperative game in which they must allocate their power so as to maximize payoffs not just at the next time step, but taking into account future payoffs as well.

The question remains: Does the real world work this way? Certainly, the basic assumptions of this model seem to align with intuitive notions of power and its pursuit. Historical agents are continually rearranging their relationships in order to prosper, dominate, and avoid being dominated, and these rearrangements simultaneously cause changes to the wider network and are reactions to it. The universality of power struggles at many levels of the social world suggests that they may be generated by a single underlying process. So it doesn't seem insane to wonder whether the model has any empirical merit. How would we test it?

One would expect that agents that are aggregates of many individuals - for example, nations - would be more likely to conform to the idealization of the model. The same holds true of power struggles that unfold over longer time scales. We also have to be wary about applying the model to historical eras in whose interpretation we have a vested interest. The farther we look from our own historical myths and ideologies, the more objectively we can appreciate the role that raw power plays in history.

With those considerations in mind, what needs to be done is to initialize simulations with historical scenarios and see whether they resemble reality. This would require tinkering with the model parameters (α, β, μ, δ, and σ) to see what ranges lead to the most plausible behavior. We should not expect perfect fidelity, as the model's simplistic assumptions - perfect information, perfect rationality, perfect computational ability, and perfectly homogeneous agents - obviously do not hold in the real world. Three aspects of this effort merit comment.

First, quite a lot of computation is required as the number of agents increases. The number of game paths equals the number of strategies per player to the power of n, and that's just for a single frame of the simulation. For nontrivial historical situations, this is more than can be carried out on a lone machine.

Second, one needs to simultaneously explore how *power* in the model is reflected in measurable real world quantities. The definition has to permit a fungibility between benevolence and malevolence: capital needs to be convertible into violence and vice versa. In the international context, many measures of national power are available (Höhn 2011).

Finally, we might consider extending the model to include other variables, like resource scarcity, the location and movement of agents in space, the role of institutions in constraining and channeling power, and information asymmetries, the last of these being particularly interesting.

4 Data structures and variables

For convenience:

Symbol	Meaning
n	Number of agents
$\mathbf{s}$	Size (power) vector
$\tau, \mathbf{T}$	Tactic vector, matrix
$\mathbf{M}$	Multiplier matrix
t	Time step
β, μ	Benevolence, malevolence multipliers
α	Utility function exponent
σ	Coefficient of social inertia
δ	Discount factor

Figure 2: The model's data structures and variables. The last four rows are parameters that can be tuned when searching for correspondence with real world behavior.

5 References

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