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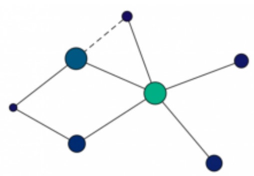
Project Overview

This is an open science project that explores a computational model of international relations, called **quantitative realism**. The international system is represented as an abstract power structure, like this:



Larger nodes are more powerful, solid lines represent cooperation, and dashed lines represent conflict.

The major questions are:

1. How do power structures evolve in time?
2. Does the abstract model say anything meaningful about actual historical power struggles?
3. What is the nature of the current world power structure?

The project uses the Wolfram Language and C++ to investigate these questions. The code lives in this Github repository (https://github.com/mpoulshock/QuantitativeRealism/).

Content Summary

The material in this wiki is organized more or less like a long scientific article. It can be read in a linear way, like a book, using the "Next" links at the bottom of each page.

A detailed table of contents can be found here.

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Abstract

Political realism aims to describe the interaction of agents involved in struggles for political power. This project attempts to formulate realism in terms of quantitative postulates that depict political power as a fluid-like substance flowing through a network of agents. Agent preferences within this framework resemble those of nation states competing for power. By combining these preferences with game-playing techniques such as Monte Carlo tree search, we can simulate balance of power dynamics in the international system.

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Preface

What happens when a big country and a small country interact with each other? If they were at war, how would that reduce their levels of capital? And if they were to cooperate, how much would each grow? If their relationship was friendly, or positive, how would they enlarge? If the relationship was negative, at what rates would they shrink?

And if you added other countries to the situation, how would that change things? With a system composed of many states, a country would have to take into consideration its relationships not just with its allies and enemies, but with their allies and enemies as well. It would also have to figure out how to allocate its power among the various states in the system. For instance, it might want to expend a bit of its positive energy interacting with one country, but focus most of its destructive power on fighting with a rival. States would have to figure out what to do at each moment, but they'd also have to anticipate what the other states were going to do, in a kind of never-ending, multiplayer chess game. And after each round of moves, the power levels of the countries would change. Some would come to dominate; others might be destroyed.

The conceptual model just sketched has only one variable, something vaguely corresponding to the notion of capital or political power. Is there a way to formulate it mathematically, to be able to say the degree to which power levels change as a result of the agents' interactions? And is there some optimal way that the agents should behave in order to survive?

This simple model provides a template applicable to a wide range of situations in the social world, not just among countries but also to political networks more generally. In any venue where there are agents struggling for power, this conceptualization seems to capture something essential about the situation.

The purpose of this wiki is to provide **a technical introduction to a computational model of power** and to explore how it may be applicable to problems of world order: that is, to the understanding of empires, international relations, and historical change. This is an ongoing science project. It is presented in its current state in the spirit of discovery, acknowledging that it is incomplete and in some respects likely incorrect. Consider this your invitation to participate.

The world is rife with injustice, real and perceived, and power is at the root of it. There is much to be angry about, and political anger is sometimes a constructive force. But it can also be beneficial to temporarily set aside both the moral repulsion and undeniable attraction that we feel towards power, and instead to try to gather our observations into a logical framework — abstract, austere, universal — that has no larger political purpose other than as an attempt at scientific understanding.

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A Few Quotes

A few quotes to set the mood...

> [I]f it is possible to attain an internal knowledge of history...we shall be able to see in all historic phenomena the expression of a moulding force behind the play of circumstance.
> — Christopher Dawson, *The Dynamics of World History*

> As subjects become more understood, they become more mathematical.
> — John Ball, *The Oxford Guide to Mathematics*

> The concepts and conclusions of arithmetic, which generalize an enormous amount of experience, reflect in abstract form those relationships in the actual world that are met with constantly and everywhere. It is possible to count the objects in a room, the stars, people, atoms, and so forth. Arithmetic considers certain of their general properties, in abstraction from everything particular and concrete, and it is precisely because it considers only these general properties that its conclusions are applicable to so many cases. The possibility of wide application is guaranteed by the very abstractness of arithmetic, although it is important here that this abstraction is not an empty one but is derived from long practical experience. The same is true for all mathematics, and for any abstract concept or theory...At the same time every abstract concept...is limited in its significance as a result of its very abstractness.
> — Aleksandrov, Kolmogorov, and Lavrent'ev, *Mathematics: Its Content, Methods, and Meaning, Vol. 1*

> The object of power is power.
> — George Orwell, *1984*

But also bearing in mind that...

> Many a man has cherished for years as his hobby some vague shadow of an idea, too meaningless to be positively false.
> — Charles Sanders Peirce, *Chance, Love, and Logic: Philosophical Essays*

Power and World Order

When we look back over the history of humanity, regardless of the time period, place, or culture that we examine, we find people engaged in power struggles. We can see these struggles in the rising and falling of civilizations and empires. We see them in acts of war, genocide, and revolution, and in the games nations play in the international arena. We see these struggles as politicians attempt to gain control of the levers of government, as citizens strive to win rights and benefits, as ideological movements coalesce and dissipate, and as interest groups try to reshape society in their own image. We see power struggles among corporate titans, within civic institutions, and between street gangs. We experience them firsthand in our workplaces, in our neighborhoods, in the organizations where we volunteer, and sometimes at the hands of our friends and family. At all of these levels, across all of history, the existence of power struggles is a constant.

Contents

Power and History

Power provides a unifying interpretive framework for understanding the primary forces that drive historical processes forward. When we reflect upon history, we tend to focus on the operation of power and about things that were shaped by power. Our curiosity about the past gravitates naturally towards leaders, presidents, empires, dominant civilizations, and the conflicts and wars that have had a formative effect on the events that followed. We want to know what powerful people did, how they became powerful, how they were vanquished, and what happened when they clashed with others in power. We are intrigued by individuals and institutions which have agglomerated authority or capital, and which influenced the events of their day. We are entranced by conflict, and the bigger and more destructive it is, the more it commands our attention.

In contrast, we are less interested in the powerless, such as ordinary ancient Sumerians, nameless toilers who built the Great Wall, or basket weavers of the pre-Columbian Amazon. This is not to say that no one cares about them (scholars certainly do), or that their lives had any less worth than our own. It is instead to say that they are not the focal points of our collective awareness of the past. To the extent that we consider them at all, it's more about them as a class, or about what their lives reveal about the power structures that controlled and dominated them. Even social history, a discipline which was invented to contextualize the experiences of ordinary people and demonstrate their historical agency, frames such groups in the context of the struggle for power.

The struggle for power is a driver of events, unlike other determinants of history to which we are primarily reactive or which we control only obliquely. Humankind has little command over many of the phenomena that affect the course of events. Our fate is fairly sensitive to natural climatic changes, ice ages, earthquakes, tsunamis, volcanoes, drought, severe storms, floods, and asteroid impacts. Events like these have shaped the rise and fall of civilizations since time immemorial. We can be more or less prepared for these events, marshalling our surpluses with foresight, or not, but we are otherwise at their mercy. Similarly, we can prepare for epidemics and plagues, but they are basically calamities that fall upon us and which we do our best to survive. Sometimes they manage to destabilize societies, but they are not the primary plot line of the human story, thankfully.

Technological innovation and advances in knowledge are other factors that can result in enormous rearrangements of social and political power, and history at its most dramatic scale is the story of humankind's capacity to create its own change. But even if such advances could magically be halted, the struggle for power would go on, as it did in the places and centuries where few innovations were introduced. Conversely, it is extremely difficult to imagine the opposite situation: a world in which technology continued to advance but humans' struggle for power ceased.

Common terms from 50 random history articles in Wikipedia

Likewise, it cannot be denied that personality plays a role in history. One can see many branch points where events almost certainly would have been different if not for the influence of particular individuals. But were we to make personality the locus of our understanding of historical processes, we would be lost in a forest of detail without much substance to unite the mass of historical facts in front of us. Because power is a general phenomenon based upon human agency, it has explanatory potential that these other phenomena do not, or do to a much lesser degree.

Moreover, power tends to underlie other sociological forces that propel history, such as religion and ideology. Ideological and religious movements can be interpreted, fairly plausibly, as contenders for power and attempts to reshape the topology of power in their respective societies. For political ideologies, this is almost self-evident. Communism, socialism, conservatism, liberalism, anarchism, nationalism, and racism, for instance, set forth explicit visions of how power should be allocated and used. Ideologies advocate for power to be channeled towards certain individuals, groups, and institutions, and away from others. Their objectives are to alter existing power structures, reshaping them towards their respective ideals, or conversely to defend those structures against attack. And it's not only in the abstract that political ideologies are fundamentally about power. They invariably entail networks of individuals and organizations mobilizing to shift, constrain, or preserve social and political power, typically in an assertive fashion. Ideologies are integral to the course of history and our understanding of it, and this is largely because of their effects on the operation of power in society.

That religion is deeply involved in the power dynamics of human societies is a slightly more difficult premise to accept. It's not always easy to see religion objectively due to its emotional and nostalgic hold over us. Obviously, religion fulfills many people's needs for existential meaning, moral guidance, and a sense of community. So it's not purely about power. Yet its political dimension cannot be denied. In the earliest civilizations, religious and political authority were closely intertwined, often as a singular form of order and control. Religious and political structures have coevolved for thousands of years and even today they remain fairly entangled. Religions have historically provided political and military leaders with the justifications for violence that they often require, and they have benefitted from the resulting victories by having had more opportunities to proliferate. Religious orders are concentrations of capital in their own right, some large, like the Catholic Church, and many small, and these loci tend to compel loyalty, deference, and obedience. Religious organizations also tend to encourage adherents to advance their agendas in the political arena. And they typically engage in communal self-help to provide social assistance and cooperation among their members, a low-level form of mutual support. Much more could be said on this topic, but the point is that religious behavior often has a political dimension resembling tribalism that results in self-interested changes to the way that power is distributed. To the extent that religion has relevance as a causative force in history, it is largely along this dimension.

The Ubiquity of Power

Power operates at many scales in the social world. It is coveted and contested by civilizations, nations, peoples, organizations, families, and individuals. It's difficult to think of social contexts in which power has no relevance at all, and in those it's often just that the machinations of power are drowned out by other competing psychological and sociological phenomena. Power is therefore a useful common thread for grasping the logic of human events.

Because it is relevant at so many levels of analysis, the desire for it is a plausible motivation of many historical actors. If individuals and organizations of various kinds all tend to have a lust for control, dominance, and influence, we can gain insight into historical processes at various scales of social organization by understanding the logic of power struggles. Put differently, one way to start understanding a particular historical situation is to look at the power relations among the various agents involved and ask what each is trying to achieve. More likely than not, they are motivated by the pursuit of power in some form or another. Obviously, not every individual or institution is power hungry. However, those who are tend to be much more likely to influence events, and thus more relevant to the overall story.

Power is also closely related to the notion of scarcity and how we respond to it. Presumably, in a world of infinite supply, where everyone's desires could always be fulfilled, there would be no need for conflict. That is not the world we inhabit. Our capacity for desire is fathomless, and its objects therefore necessarily scarce. When things are scarce, people fight over them. Sometimes the market resolves problems of scarcity through the operation of supply and demand, allocating scarce items to those with the greatest capacity to pay for them. But often people resort to using whatever power is at their disposal to compel others to relinquish their claims over contested resources. In extreme cases, they use physical violence. More frequently, they find ways of imposing some kind of negative consequences upon their competitors. Typically, contestants over scarce resources form alliances in order to use their collective power to win control over the resource, meaning that scarcity also helps forge cooperation.

It is not the case that all human interaction is about power, or that if you understood power you'd understand everything about society and history. The social world is obviously extremely complex and numerous forces exert determinative effects upon it. However, the struggle for power is central to a coherent interpretation of events.

It's not always easy for us to see this. Certain things conspire against that perception. Usually the history that we're interested in is what's relevant to the present, and specifically to our present. We're vested in the interpretation and meaning of that history, because it helps define us and legitimize our place in the world. If those historical processes were fundamentally about power, it might suggest that any privileges we derived from them were unfairly obtained. So it's hard to get a sense of moral proportion about any part of the past from which we see ourselves emerging, as doing so has the potential to undermine our political identities.

In contrast, when looking at the remote past, remote in the sense of time, place, or culture, we're more likely to view events as the power struggles that they actually were. Ancient kingdoms fought with each other not over any great principles, certainly none relevant to us, but in order to conquer or resist being conquered. We can see these processes fairly objectively. The meanings of these distant events are open to debate and they have multiple plausible interpretations. Nonetheless, our ability to identify raw power and its pursuit in these remote contexts is not compromised by what the interpretation of these events implies about us now.

The same challenge applies to appreciating the role of power in the present. If power is one of the central concepts for understanding history, then it must play the same role in illuminating the present, since the present is feverishly generating tomorrow's history, being propelled by the same impetus. When we benefit from a power structure, we're disinclined to view our privileges as deriving from it (although those who are less privileged may have fewer illusions about this). The present is difficult to see clearly because we consume media diets of spin, deception, and ideology, and are taught fictionalized narratives of history from an early age. Moreover, the significance of events that occur in our lifetimes is not always fully clear, as it can take generations for the historical record to be sufficiently developed, for long-held secrets to be unearthed, and for causal patterns to be identified and given their appropriate weight. So while we may be able to recognize the past as primarily a struggle for power, we're likely to mistakenly believe that the present is primarily a struggle for principle.

The Dynamic Nature of Power

So what is power, this curious, ubiquitous thing that everyone keeps fighting over? Where does it come from? How do you get it? Why is it so ephemeral?

The existence of power has been recognized since time immemorial, but its nature remains a mystery. The concept is intuitively known to all of us, perceptible even by young children, a kind of animal instinct. It has been pondered for centuries, generating a literature so vast that it's tiresome merely to list all of the academic disciplines that have grappled with the subject: political science, international relations, sociology, psychology, geography, economics, law, military science, postcolonial studies, and philosophy. Yet despite all of this research, there are still gaps in our understanding of how power operates. Most social scientists define power as the ability to influence or control someone, or to compel probable obedience, or something along those lines. But how is that ability acquired? How is it lost? Why do some people seem to have so much of it, and others virtually none? And why are some power structures and organizational forms so stable and resistant to disturbance, while others are rickety and weak?

What we lack is an understanding of the dynamic nature of power. Power is both a stock, a quantity that can be possessed, and a flow, a quantity in motion. It spends much of its time on the move, circulating around human networks like a fluid. Though its course may shift rapidly, often quite surprisingly, it doesn't come out of nowhere. It is a substance created, nurtured, transferred, and destroyed by people. We can use it to make others more powerful or, conversely, to diminish their power, and in choosing how to deploy it, we alter the course of its movement. Power shifts as a consequence of the way that agents interact. And if power moves around, then there must be patterns that describe it. Its flow can't be entirely random. Too much is at stake. Power is simply too valuable for its conduct to be haphazard. There must be some explanation for its behavior, some logic to the way it's transmitted through social networks.

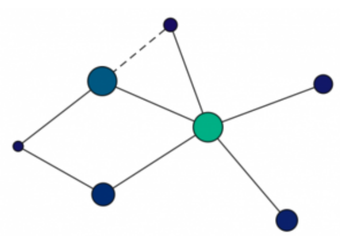
Power struggles at every level of the social world are generated by a single underlying process, regardless of whether the scale is that of world history or the office water cooler, and regardless of whether the struggle occurs over the course of centuries or during an elementary school recess. If we conceive of power at the proper level of abstraction, as a substance that oozes through human networks, we can gain insight into a wide range of social and historical phenomena, because it turns out that this diversity of situations is actually governed by the same core principles.

We know that power is desired, that people want to attract and possess it. They tend to want as much of it as they can reasonably get, and they'd prefer that competitors have as little of it as possible. To accumulate it, they rearrange and restructure their relationships: forming alliances, defending themselves, and coordinating aggression. Ultimately, people tend to use what power they have at their disposal to accumulate more of it. This is the essence of a power struggle: the use of power to obtain even more power, in competition with others who are trying to do exactly the same thing. Power is both the means and the object.

The Abstraction of Quantitative Realism

Turning rough ideas like these into clearly defined axioms, we can devise a model describing the dynamic nature of power. We call this model quantitative realism, because it's an outgrowth of political realism, discussed later on. It attempts to capture quantitatively what political realism describes qualitatively.

Quantitative realism seeks to answer the following question: If we had a set of agents and knew how powerful each one was, as well as their alliance structure, such as in the figure below, how would that network tend to evolve in time? Who would grow stronger, and who would weaken? From that minimal amount of data — the relative strengths of the agents and their network topology — can we predict how a particular power struggle is likely to unfold?



A random abstract **power structure**. Solid lines represent cooperation and dashed lines indicate conflict. The larger the node, the more powerful that agent is.

Quantitative realism is abstract, a deliberate over-simplification. It leaves out numerous features of the real world that determine the course of human events. It says nothing about institutions, technology, culture, ideology, religion, the environment, climate, race, justice, economics, personality, and disease. It ignores these and other factors, focusing its attention exclusively on the behavior of a single variable: power.

Though it's counterintuitive, often the simpler a model is, the more useful it becomes. Simple models winnow away things that are irrelevant or that have trivial effects. They help feeble human minds comprehend complex phenomena. And they yield approximations that make computation practicable. In contrast, complex models are generally too complex to be useful. Even if a complex model were able to make sufficiently accurate predictions about a phenomenon, it would not provide much insight into why it worked, as one would have to disentangle the many variables generating the behavior in order to understand what was going on. This minimalistic approach is referred to as parsimony, the belief that a scientific theory should be based on as few concepts and assumptions as possible.

The abstract nature of a simple model is what helps makes it applicable to many situations. Because quantitative realism is independent of institutions, technology, and the like, it allows us to say something about the ways of power regardless of whether we're talking about ancient tribal violence, city council politics, or the Israeli-Palestinian conflict. The abstraction lets us tease out pure patterns, and the fact that these patterns may not exist perfectly in the real world is about as unimportant as the fact that perfect spheres don't exist in the real world. Despite their nonexistence, mathematical spheres are useful idealizations of real world shapes, and we can use their properties to infer information that we wouldn't otherwise know, such as the approximate surface area of the Earth.

The abstraction of quantitative realism can help us unearth patterns in human conflict. If the pattern is hard to recognize, it is perhaps because we're so deeply embedded in it that it's hidden in plain sight. Power struggles are a ubiquitous feature of the social world and, regardless of who's engaging in them or the means by which they're carried out, they all share similar dynamic tendencies. By studying these fluid dynamics as abstractions, we can come to understand how political power moves, how that movement forges networks, and how those networks shape world order.

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About This Project

This wiki and the accompanying code repository investigate a theory of power structures: how they work, what logic underlies them, and what relevance they have to world order and historical change. Here we provide some general project information.

Contents

The Basics

Code

The primary programming language for this project is the **Wolfram Language** (formerly *Mathematica*). Some functions will be replicated in **C++** for performance reasons and because it has a larger user base.

The Wolfram Language code snippets shown in this wiki strive for clarity, attempting to make the main ideas simple and readily understood. These snippets will execute, but they're not particularly fast and they don't handle edge cases. The code in the repository (https://github.com/mpoulshock/QuantitativeRealism/tree/main/Code/) contains the actual implementation, which is more robust and contains a variety of other functions not described here.

Documentation

The documentation in this wiki is organized like a (very long) scientific article, with a focus on methods, results and discussion, and conclusions. It can be read linearly using the "Next" hyperlinks at the bottom of each page.

Copyright and Licensing

The source code is subject to the MIT open source license. The content of this wiki is currently the copyright of Michael Poulshock, until an appropriate "copyleft" regime has been researched and selected.

Institutional Affiliations

TBD

Principal Investigator

The principal investigator is Michael Poulshock (https://www.linkedin.com/in/mpoulshock/). I'm an adjunct professor at Drexel Law School, but otherwise not a professional academic. This is a side project.

Q&A

Why are you doing this as a wiki?

That is, why not follow a traditional academic publishing route? The wiki format provides a way to organize a large body of thoughts on the subject of quantitative realism. It also is a way of facilitating contributions from a wider community of interested people. Some of the investigations discussed in this wiki will likely give rise to traditional academic articles.

How far along is this project?

At this point, the theory of quantitative realism has a pretty solid core. The thrust now is to test it against empirical data and better understand its abstract dynamics. See this list of major loose ends to get a more detailed sense of what remains to be done.

What's with the logo?

It's the first one that came up in an online logo generator. It doesn't have any more intrinsic meaning than that. Someday, we'll get a new one.

Research Questions

We can pose a variety of research questions related to quantitative realism. They seem to fall into three categories:

1. Implications of the abstract theory in and of itself
2. Verification of the theory in light of historical events
3. Analysis and interpretation of contemporary power structures

Subsequent wiki sections provide additional context on the questions listed below.

Abstract Implications

Questions about the abstract theory as an abstraction include:

1. What patterns emerge when computational agents, animated by minimax game tree search, interact?
2. In what ways do these patterns correspond with theories of political realism and balance of power?
3. What general conclusions can be drawn about equilibrium and disequilibrium?
4. What are the effects of move order?
5. What are the effects of changing parameters?
6. What are the effects of using continuous vs. discrete tactics?
7. Is there a relationship between quantitative realism and the four classic strategic maxims: *the enemy of my enemy is my friend*, *the enemy of my friend is my enemy*, *the friend of my friend is my friend*, and *the friend of my enemy is my enemy*?
8. What happens when agent moves are determined by a machine learning algorithm trained via self-play?
9. What are the effects of extending the model to include things like physical distance, competition for scarce resources, and institutional constraints?
10. Is there a relationship between quantitative realism and gravity models of trade (https://en.wikipedia.org/wiki/Gravity_model_of_trade/)?
11. Is there a relationship between quantitative realism and Bruce Bueno de Mesquita's political theories (https://en.wikipedia.org/wiki/The_Logic_of_Political_Survival/)?
12. Can quantitative realism be made into an abstract board or computer game? Would it be any fun?

Historical Interpretation and Model Verification

Questions related to the application of the theory to historical events include:

1. What empirical definitions of national power align most closely with quantitative realism?
2. What does the model have to say about various historical situations? For example:
    1. The Greco-Persian Wars
    2. The Peloponnesian War
    3. The rise and fall of the Roman Empire
    4. The rise and fall of various Chinese empires
    5. Patterns in European imperialism
    6. The collapse of the Aztec and Maya empires
    7. World War I and II
3. Does historical evidence support the law of motion?
4. What parameter combinations best reflect various historical episodes?
5. What is the empirical relationship between system polarity and stability?
6. What general techniques and best practices can be found for modeling historical situations?
7. Does quantitative realism apply to massively multiplayer online games like EVE Online (https://www.eveonline.com/)?
8. Does quantitative realism reflect anything meaningful about power struggles among non-human primates?

Analysis of Contemporary Power Structures

Questions pertaining to current-day power structures include:

1. What is the current world power structure?
    1. Who are the major actors in it?
    2. What are their relative levels of power?
    3. What are their relationships?
2. What data is available to determine this structure and update it in real time?
    1. Trade and economic data
    2. Military information
    3. Diplomatic data
    4. News event coding and social media analysis
    5. Cyber event monitoring
    6. Aggregation of expert opinion, prediction markets, etc.
3. What are the risks and opportunities in the world power structure? Which relationships present the greatest threat to peace?
4. What are the implications of quantitative realism for international order and global governance?

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Conceptual Foundations

Quantitative realism rests upon a small set of definitions and axioms from which all of its consequences derive. The goal is to describe a wide variety of phenomena in a logical way, by making as few assumptions as possible and by being explicit about those assumptions. This axiomatic structure provides a way for all of the mathematical bits and pieces to fit together, ultimately offering an account of how power structures change over time.

The following sections describe the objects that are the subject of our investigation and the axioms that govern how those objects change:

Definitions

There are five fundamental definitions that serve as the foundation of quantitative realism.

Contents

Agent

An **agent** is an individual actor or a collection of individuals presumed to be acting in concert, in other words, a political unit. An agent can represent any number of real world political entities that are able to amass and utilize power. It could be a country, an institution, a group of people, or an individual person. It could even be a planet, should intergalactic warfare become a concern. China, the Persian Empire, the Egyptian Pharaoh, the British Labour party, a high school principal, a oil company lobbyist, a terrorist cell, and Singapore could all be agents in a power structure. That the definition of agent can be used at so many levels of social magnification goes a long way towards explaining the wide applicability of quantitative realism. Because this wiki focuses on world order, an agent in the models here is likely to be an empire, a nation, or some other large-scale political entity. Sometimes we use the term **player** as a synonym for agent, particularly when referring to the model as if it were a formal game.

We can expect that the more an agent reflects an aggregation of smaller entities, the more likely it will be to follow the assumptions soon to be delineated below. Conversely, the smaller the agent - at an extreme, a single individual - the more likely it is that other variables external to the model will intervene, causing the agent's behavior to deviate from the simplistic assumptions below. One could analogize this to statistical mechanics, which describes the behavior of gases in the aggregate, ignoring the contribution of individual molecules. Such physical laws are assumed to hold in the limit, meaning that they are more accurate when there are more individual particles in the system being analyzed. We take a similar approach here: the larger the agent, the more likely it is to be described by quantitative realism, such that we would expect the theory to reasonably model empires and countries, but perform worse when describing the behavior of individual people.

We sometimes use the term **focal agent** to refer to a particular agent who is the focus of our attention in a given situation.

Power

Power is a quantity reflecting an agent's ability to affect the power of other agents. This is a deliberately recursive definition that strikes at the heart of what is obvious about power: those who have it can render others powerless, or conversely, empower them. Power is essentially the capacity to administer rewards and punishments, carrots and sticks. It could be loosely thought of as well-being or capital. It's a broad, deliberately abstract definition, not meant to correspond at all times to a single real-world quantity. In some contexts, it may not necessarily be measurable at all, such as where its means of use are unquantifiable or where there are many of them. Whatever specific form it may take, power must allow for a fungibility between wealth and violence.

One might object to this definition as being circular or reductive, or even meaningless. Before leaping to such a conclusion, it's important to understand the definition in the context of the wider theory. On its own, this definition doesn't give much indication of how power works. Without saying how it works, we can't say what it is. The definition only makes sense as part of the larger axiomatic structure. This is typical of mathematical objects: their meaning is often determined by virtue of their relationships to other objects.

Another objection is that this definition of power is not in line with how power is conceptualized in the social sciences. While the concept is indeed slippery, there is a fairly strong consensus in political science, international relations, sociology, anthropology, and psychology that power essentially has to do with the ability of one agent to compel another to do something it would not otherwise do [CITE]. Glossing over a host of nuances and semantic squabbles, there is not necessarily a conflict between this traditional definition and the one proposed here. The definition here is simply more abstract, encompassing both the means of compulsion, be it carrots or sticks, and the act being compelled. This more abstract definition will allow us to better understand power relations within a network of agents.

Action

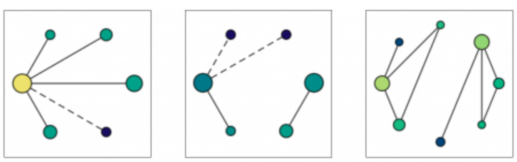
An **action** is an allocation or transfer of power from one agent to another. Actions are what cause other agents' power levels to change. They are essentially the relationships that agents form with each other, and they can be either friendly or antagonistic, to varying degrees.

Tactic

One agent's action toward the other agents in the system is called a **tactic**. A tactic is essentially an agent's foreign policy, expressed numerically. A tactic is a list of numbers indicating the agent's actions with respect to the other agents.

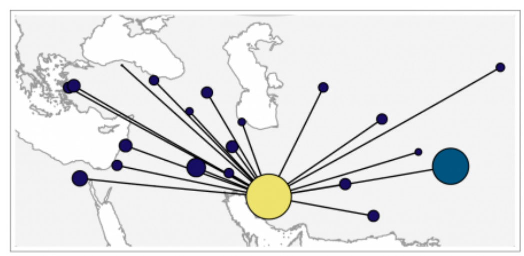
Power structure

The final definition is that of a **power structure**. This is our primary object of concern. In everyday life, this term refers to a system of relationships in which power and authority are distributed among people or organizations. We use it in exactly the same way, but as an abstraction that can be applied to a variety of real world situations.



Three random power structures

Power structures are readily comprehended visually, as graphs. A power structure is composed of agents and their interrelationships. Each agent is represented by one of the nodes in the graph. The larger the node, the more powerful the agent is; conversely, smaller nodes have less power. For this reason, we use the term **size** synonymously with power. Friendly relationships are indicated by solid lines connecting two agents. Hostile relationships are denoted by dashed lines. When two agents have the same attitude toward each other, the relationship is displayed as an undirected edge (one without arrows). Asymmetrical relationships are displayed as directed edges. Vertex placement is not based on spatial or geographical distances, unless so indicated. Node colors represent how happy each agent is with its place in the network.



A model of the Persian Empire c. 500 BC, based on the tribute paid by each satrapy. The data comes from Herodotus 3.89-3.95 (https://github.com/mpoulshock/QuantitativeRealism/wiki/References#h), but we rely on the converted amounts listed in Wikipedia (https://en.wikipedia.org/wiki/Districts_of_the_Achaemenid_Empire/). This is possibly the first geopolitical power structure for which we have quantitative data.

Power structures pull together the first four definitions: agents, power, actions, and tactics. Power structures are composed of agents, each of whom has some level of power, who take actions toward other agents by allocating their power in a tactic. If someone were to look at a power structure diagram like the one above, without knowing any other historical or political details about a situation, they would have a decent first approximation of the power struggle at hand.

A power structure exists at a point in time. What we want to know, and what the axioms describe, is how a given power structure will evolve as time unfolds.

Axioms

Power structures are not static in time. There are two processes at work that make them evolve. First, agent relationships cause power to be transferred, and these transfers make the agents stronger or weaker. Second, agents continually alter their relationships to respond to the state of the power structure, in a ceaseless quest to improve their position within it. Since a power structure is a graph, the first process relates to how vertices change in size, and the second describes how edges change.

These two processes boil down to seven core principles that describe how power structures change over time. Six of these principles are fundamental assumptions, or *axioms*. One of the principles is a heuristic made for convenience. Each axiom is associated with a model parameter that specifies the degree to which the axiom has its intended effect. The parameters allow the axioms' qualitative statements to be quantified.

Contents

Principles of Vertex Change

The first three axioms describe how the vertices of a power structure change in size as a result of agent interactions.

Axiom 1: Effect of constructive action

Constructive action is an expenditure of power that increases the power of another agent by more than the amount expended. Constructive action among states might entail trading goods or services, forming a military alliance, supporting an ally with troops and weapons, paying tribute, providing disaster relief, engaging in cultural exchange, or giving other forms of assistance.

As a result of a constructive action, the acting agent loses power and the receiving agent gains power. For instance, in a transfer of one unit of constructive power, the acting agent will lose one unit and the receiving agent will gain more than one unit. When agents cooperate by reciprocating constructive exchanges, they make each other more powerful, with each becoming better off than they were before the exchange. When one agent allocates a positive amount of power to another, the amount received by the other agent is increased by a factor of β, where $\beta > 1$. The parameter β, called the constructive multiplier, reflects the idea that, regardless of what has been given, it has more value to the recipient than it did to the giver. β can be thought of as an imaginary interest rate.

The effect of this axiom is that when two agents both cooperate by acting constructively towards each other, they both increase slightly in strength as a result of the cooperation. Consider a sale of oil between two countries. We can assume that to the purchaser, the oil was more valuable than the money it paid, otherwise it wouldn't have purchased it. Likewise, to the seller, the money must have been more valuable than the oil, or else it would not have put the oil on the market. The transaction served to increase the well-being of both parties. In the case of economic transactions, this increase is a manifestation of the division of labor and the benefits of exchange, and similar justifications can be put forward for other types of transactions. Power is not zero sum: agents can work together to create more of it.

Constructive power is expressed as a positive number.

Axiom 2: Effect of destructive action

Destructive action is an expenditure of power that decreases the power of another agent, with more impact than constructive action. Destructive action entails the use of violence or the imposition of unwanted consequences. It includes actions like military assault, siege, bombardment, killing, destruction of property and infrastructure, and terrorism. Both the acting agent and the receiving agent lose power as a result of destructive action. Since it is easier to harm another agent than to help them, and easier to destroy value than it is to create it, destructive action has a greater impact than constructive action. For example, it costs much less to raze a city than it does to build (or rebuild) it, and it's much easier to harm someone than to help them to the same degree.

When a unit of destructive power is transferred, the acting agent loses one unit and the receiving agent loses more than they would have gained had the transfer been constructive. Like constructive action, destructive action is subject to a multiplier, μ, the destructive multiplier, such that $\mu > \beta$. That is, when an agent uses power negatively, for every unit of power it expends, it causes a reduction in the recipient's power by μ. Destructive power is expressed as a negative number.

Axiom 2 is essentially a statement about entropy and the ease with which states of disorder can be reached, as compared to the amount of effort required to create or impose order.

Axiom 3: Effect of unused power

Power that is not used constructively or destructively, but that is instead retained by an agent as a stock, either depreciates or has a lower growth rate than power used constructively. Without this axiom, the incentive for agents to cooperate with each other would be undermined. The amount of growth/depreciation is governed by the parameter λ, such that $\lambda < \beta$.

Principles of Edge Change

The remaining principles govern how agent relationships in a power structure change over time.

Axiom 4: Pursuit of absolute and relative power

Agents prefer other agents to be relatively weak. They are preoccupied with how much power other agents have, wanting to be relatively powerful while keeping others relatively powerless. Given the choice, agents would prefer that their competition be weak and divided, rather than strong and united. For example, an agent would generally prefer five competitors with one unit of power each, to one competitor with five units of power. At the same time, agents want to become more powerful in absolute terms, meaning that they want the amount of power they have to increase.

Later we define a utility function that quantifies these preferences, taking into account the absolute and relative sizes of the agents. Within this function, the parameter α controls the importance of absolute versus relative power.

Axiom 5: Social inertia

It cannot be the case that agents are free to choose whatever tactics they like. In the real world, the past, or more specifically the present, binds their options. For example, a country could not one day suddenly alter its entire foreign policy. Processes like trade agreements, peace talks, mergers and acquisitions, and divorces all take time, because social relationships have a kind of stickiness that resists rapid change. This phenomenon is called social inertia (https://en.wikipedia.org/wiki/Social_inertia/), and it makes it less likely that agents will be able to effect dramatic tactical changes. The parameter ρ expresses the degree of social inertia present in a power structure.

Axiom 6: Ongoing interaction

Agents engage in ongoing interaction. They do not interact merely one time, nor do they necessarily know how long the interaction will continue. However, they expect to continue interacting with each other in the future. There is no terminal state or end game; agents just continue the struggle for power forever. How far ahead agents look in time is influenced by the parameter δ, or the discount rate of intertemporal utility, a concept which is explained later.

Heuristic 1: Reciprocity

Though agents can behave asymmetrically towards each other, with one engaging in a constructive action and the other engaging in a destructive one, such asymmetrical actions are likely to be short-lived. A constructive action will likely only be continued if it is reciprocated, and an agent who is attacked will tend to attack back (Axelrod 1984 (https://github.com/mpoulshock/QuantitativeRealism/wiki/References#a)). So relationships are presumed to be reciprocal. This means that agent actions towards each other typically have the same polarity, even if each agent does not necessarily allocate the same amount of power. To resolve cases in which agents cannot agree on a relationship, this axiom assumes that one agent can start a conflict unilaterally, whereas cooperation requires mutual consent.

There is no parameter associated with the reciprocity axiom. This may suggest that this axiom is not actually fundamental, and that it can instead be derived from the other axioms.

Recap

To recap, the principles and their associated parameters are:

- **A1.** Effect of constructive action ($\beta > 1$)
- **A2.** Effect of destructive action ($\mu > \beta$)
- **A3.** Dissipation of unused power ($\lambda < \beta$)
- **A4.** Pursuit of absolute and relative power ($2 \leq \alpha \leq 3$)
- **A5.** Social inertia ($0 < \sigma < 1$)
- **A6.** Ongoing interaction ($0 < \delta < 1$)
- **H1.** Reciprocity

The Axioms and Political Realism

Though on the surface it might appear that these starting assumptions are a radical departure from realism as traditionally conceived, they can be interpreted as a generalization of traditional realism. The traditional assumptions that states have military capacity and interact under anarchy are, here, abstracted into the simple idea that agents can use their power to render each other powerless, and possibly even nonexistent. And in both paradigms, agents interact repeatedly and are assumed to take self-help action to promote their own survival in light of the current distribution of power. What this new formulation adds are the following: First, the conception of power is more general. It's not solely about destructive capacity and it's not zero sum: power can be created as a result of cooperation. Second, agents make tactical decisions based upon their interrelationships and not solely upon the distribution of power, as in traditional realism. Finally, agents' use of power has quantitative effects on other agents' power levels, causing the distribution of power to change in potentially measurable ways. Overall, this generalization allows quantifiable consequences to follow directly from the starting assumptions.

Data Structures

Here we show how quantitative realism can be represented formally, using mathematical notation and the Wolfram Language.

Contents

Power Structure

A power structure is an object composed of a size vector **s** and a tactic matrix **T**. In the Wolfram Language, a power structure is represented using an Association object of the form:

```
<|"s"→ s, "T"→ T|>
```

The function `PowerStructure[s,T]` instantiates a power structure object that takes the form of the expression above.

The variable *n* represents the number of agents in the power structure. It is often convenient to refer to agents by an index number *i* that indicates the position of that agent's data in each data structure. (In the Wolfram Language, list indexes start at 1, not 0.)

Size Vector

The variable *s*, for size, is a nonnegative number representing the amount of power an agent has. The larger the number, the greater the agent's power. It is always the case that $s \geq 0$. Zero power means that the agent has no influence and is effectively dead. The vector **s** lists the sizes of all agents in a power structure. For example, in a power structure with three agents, s = {.8, 1, .5} means that agent #3 has a size of 0.5.

By convention, size vectors are normalized at the beginning of a simulation such that the largest agent has a size of 1, using:

```
NormalizeS[s_] := s/Max[s]
```

Normalization ensures that size vectors with the same proportions but at different scales give rise to the same behavior in the course of a simulation. This is necessary due to Axiom 4, which recognizes that agents want their power to increase in an absolute sense. For example, two power structures with the size vectors {.8, 1, .5} and {1.6, 2, 1}, *ceteris parabus*, will not necessarily evolve the same way.

A random size vector can be generated using the function:

```
RandomS[n_] := NormalizeS[RandomReal[{0, 1}, n]]
```

Random power structures are useful points of departure for exploring the consequences of quantitative realism.

Tactic Matrix

A tactic matrix **T** is composed of *n* tactic vectors. A tactic vector τ is expressed as a list of real numbers, indicating how an agent's power is to be allocated to the other agents and to itself. The element τ_j represents the amount of power being allocated from the agent whose tactic vector it is to agent *j*. The slot in the tactic vector at the agent's own index number indicates the amount of power that the agent is self-allocating, in other words, keeping to itself and not transferring to other agents. Constructive action in a tactic vector is represented by a positive number; destructive action by a negative number.

Conventions

There are three conventions related to tactic vectors.

Convention 1

Every element of a tactic vector must be between -1 and 1 and an agent cannot use more (or less) power than it has. Therefore, the sum of the absolute values of the numbers in the vector must be 1. Expressing this convention mathematically, we get:

$$\sum_{j=1}^{n} |\tau_j| = 1, \qquad \tau_j \in [-1, 1]$$

In other words, a tactic vector shows how an agent's power is to be distributed, by percentage. For example, if agent #2's tactic vector is {.1, .8, .04, .06}, it is allocating 4% of its power to agent #3 and 80% to itself. We can test a tactic vector for compliance with these constraints using:

```
LegalTacticQ[tactic_] := Total[Abs[tactic]] == 1
```

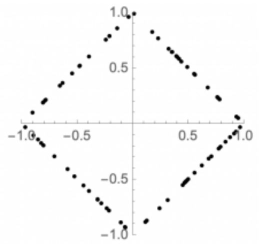
A tactic vector can be legalized by normalizing it with:

```
NormalizeTactic[tactic_] := tactic / Total[Abs[tactic]]
```

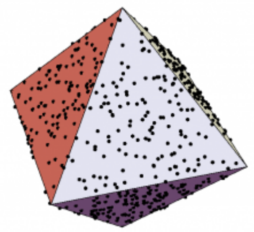
We can use this normalization function to generate random tactic vectors. A simple approach is to pick a random point on a sphere of dimension (https://en.wikipedia.org/wiki/N-sphere) n-1 and then normalize it:

```
RandomTactic[n_] := NormalizeTactic[RandomPoint[Sphere[Table[0, n]]]]
```

Normalization essentially adjusts the length of the vector such that it falls on the surface of a cross-polytope of dimension n. We can see this by generating random tactics and plotting them. For example, randomly generated n=2 tactics form the outline of a two dimensional cross-polytope, or square:



If the tactics generated above were for agent #1, the x-axis would represent the agent's self-allocation and the y-axis would represent the amount of power allocated to the other agent. For n=3, legal tactics fall upon the surface of an octahedron:



Convention 2

There is a minimum percentage of power that each agent must allocate to itself. This convention helps simulations maintain a higher level of fidelity to the real world, by limiting the amount of power that the agents can expend in any given time step. Without such a limit, they could conceivably expend all of their power at one time, and the result would be erratic simulations that bore little resemblance to any realistic situation. We can correct for this using an additional parameter ρ, which establishes a minimum self-allocation percentage. For example, when ρ=0.9, agent #2's tactic vector might be {0, 0.9, 0.02, -0.05, 0.03}. The self-allocation percentage controls the tempo of a simulation: the more power that agents allocate to themselves, the less they allocate to others, and therefore the less change occurs from one time step to the next. Experience has shown that suitable values for ρ are typically in the range ρ ≥ 0.9.

Convention 3

Agents are assumed not to engage in self-harm and therefore their self-allocation is never negative. This assumption obviates the need to search for self-destructive tactics that an agent will likely never adopt.

Random Generation

It is often useful to randomly generate tactic vectors and matrices, both to initialize simulations and to explore an agent's possible moves.

Random Tactic Vectors

We can generate random tactic vectors that fulfill all of the above conventions using:

```
RandomTactic[n_, i_, ϱ_] := With[{a=RandomReal[{ϱ, 1}]}, Insert[(1-a) RandomTactic[n-1], a, i]]
```

This function uses our definition of `RandomTactic` above to build a tactic vector for a given agent *i*. For instance, to generate a random four-agent vector for agent #2, one would evaluate `RandomTactic[4, 2, .9]` to get, for example {-0.0098, 0.9158, 0.0135, -0.0607}.

Note that ρ does not set a fixed amount, but is instead a floor below which the self-allocation percentage cannot fall.

Random Tactic Matrices

The tactic vectors of the agents are aggregated together into a tactic matrix in which each tactic vector is a column vector. The tactic matrix encodes all information about how power is allocated within the power structure. We generate random tactic matrices using the function:

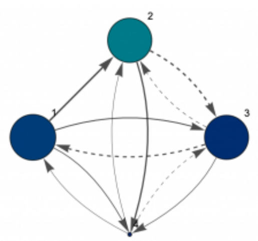
```
RandomT[n_] := Transpose[Table[RandomTactic[n, i], {i, n}]]
```

at which point we are now able to generate random power structures such as:

$$\left\langle \left| s \to \begin{pmatrix} 1. \\ 0.939 \\ 0.93 \\ 0.007 \end{pmatrix}, T \to \begin{pmatrix} 0.927 & 0. & -0.029 & 0.028 \\ 0.034 & 0.935 & -0.013 & 0.019 \\ 0.022 & -0.033 & 0.943 & -0.03 \\ 0.017 & 0.032 & 0.015 & 0.923 \end{pmatrix} \right| \right\rangle$$

The main diagonal of the tactic matrix indicates the self-allocation percentages of the agents.

Even though the data structures and algorithms in the model are fundamentally numeric, it's easier to comprehend a power structure by looking at a picture of it. Here's a depiction of the power structure above:



Even though this is easier to understand than a raw numeric data structure, the diagram is still fairly busy, showing every transaction among the agents. It also violates Heuristic 1's assumption that all relationships are reciprocal in polarity.

Ternary Tactics

For these and other reasons, it is easier to initialize and interpret simulations using ternary matrices, which use 1 to represent constructive power, -1 for destructive power, and 0 for null relationships.

Conversion

Tactic vectors and matrices composed of only the elements {-1, 0, 1} can then be converted to legal (continuous) tactic objects, with power being allocated equally among the downstream agents. A ternary tactic matrix can be legalized using the function:

```
NormalizeTernaryTactic[tactic_, i_] := ReplacePart[NormalizeTactic[ReplacePart[tactic, i → 0]] * (1-ϱ), i → ϱ]
```

This function can be applied to all columns in a tactic matrix to convert it from a ternary to continuous matrix. By convention, an agent's self-allocation in ternary is always 0.

Conversely, a continuous tactic matrix can be converted to ternary using:

```
ToTernary[T_] := ReplacePart[Sign[T], {i_, i_} → 0]
```

Using this function, the tactic matrix in the power structure above becomes:

$$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Symmetrization

Notice that the matrix above is not symmetrical about the main diagonal, as is also evident from its visualization. We can symmetrize a matrix based on the concept in Heuristic 1 that one party in a relationship can singlehandedly start a conflict, whereas it requires two agents to cooperate:

```
SymmetrizeTernaryT[T_] := MapThread[Min, {T, Transpose[T]}, 2]
```

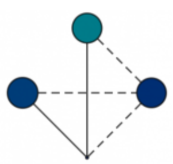
This function symmetrizes the tactics used by each pair of agents by taking the lower of the two tactical values. For example, using ternary, if one agent is giving constructive power (+1) and the other agent is neutral (0), the symmetrized relationship becomes neutral (0). If one agent is attacking (-1) and the other is neutral, the symmetrized relationship becomes a mutual conflict (-1). Symmetrizing the matrix above would yield:

$$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ -1 & -1 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

The power structure above, converted to ternary and symmetrized, is now:

$$\langle \vert s \to \begin{pmatrix} 1. \\ 0.939 \\ 0.93 \\ 0.007 \end{pmatrix}, T \to \begin{pmatrix} 0.9 & 0. & -0.033 & 0.033 \\ 0. & 0.9 & -0.033 & 0.033 \\ -0.05 & -0.05 & 0.9 & -0.033 \\ 0.05 & 0.05 & -0.033 & 0.9 \end{pmatrix} \vert \rangle$$

and it looks like:



This interpretation tells a simpler story: an alliance of three agents attacking a fourth.

Ternary vectors are preferable because they're easier to set up and interpret, and because they dramatically narrow the state space to be explored. (There are an infinite number of continuous tactics that can be considered, even within the limitations of social inertia.) However, ternary tactics fail to capture situations where an agent has some relationships that are more intense than others, because an agent's outgoing power is divided equally among its recipients. This limitation glosses over some behavioral nuances and has to be borne in mind when evaluating the apparent preferences of agents in a given simulation.

"Gravitized" Tactics

As noted above, when ternary tactics are normalized with `NormalizeTernaryTactic`, an agent's outgoing power is allocated equally among its downstream recipients. An alternative approach is to make the outgoing allocations proportional to the sizes of the recipients, thereby emulating the relationship structure of a gravity model of trade (https://en.wikipedia.org/wiki/Gravity_model_of_trade/). A ternary tactic can be "gravitized" with the function:

```
GravitizeTernaryT[T_, s_] := NormalizeTernaryT[s*T]
```

where `NormalizeTernaryT` is just `NormalizeTernaryTactic` mapped over each column in the tactic matrix.

A gravitized ternary tactic would look something like this, where the amount of power transferred is proportional to the thickness of each arrow:

An alternative way to gravitize tactics is to have each relationship represent a fixed amount of potential power that is proportional to the other agents' sizes. The tactic vector is then normalized by allocating all unused power back to each agent:

```
GravitizeTernaryTactic[tactic_, s_, i_] := InsertSelfAllocation[tactic*s*(1 - ϱ)/Total[s], i]
```

With this approach, agents would have an incentive to diversify their relationships: the more cooperative relationships, the faster they would grow. There would still be jealousy when a third party gets empowered, as well as a disincentive to two-front wars, which would rapidly wear down an agent.

In summary, there are some intricacies in how tactic vectors and matrices are defined, manipulated, and permuted. These finer points generally follow from the requirements imposed by the axioms and by our need to create realistic simulations that are reasonable to initialize and interpret.

Time

We model time *t* discretely. The model proceeds in time steps and the other parameters together control how much activity can occur at each step. For example, we could configure the parameters such that a lot of power is transferred at each time step, or an incremental amount. This might correspond to the system changing every decade or every year, respectively. Such decisions are not merely about the "speed" at which events in the simulations unfold. They may yield qualitatively different results. Ultimately, parameters must be chosen so as to give rise to intuitive behavior that corresponds with the real world phenomena being modeled.

It is possible to devise a continuous-time model of quantitative realism, replacing the difference equations below with differential equations. However, for computational purposes, discrete time models are more manageable.

Information

It is assumed that agents have complete information, meaning that they know the entire power structure. This is obviously a significant simplification, one that is almost never true in the real world, in which the management of information is the primary art of statecraft and politics. There, agents may not know the entire power structure with certainty. Their knowledge is approximate, gleaned from signals, and often subject to misinformation, deception, and cognitive errors. Nonetheless, in the international context, it is generally reasonable to assume that actors for the most part operate with a shared understanding of the salient power relationships.

Deviations from perfect information can be explored by assuming that each agent has their own model of the power structure, and that they make decisions based upon that subjective representation.

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Parameters

Summary of Parameters

The parameters used in the model are aggregated in the chart below:

Symbol	Parameter	Range	Axiom
β	Constructive multiplier	β > 1	1
μ	Destructive multiplier	μ > β	2
λ	Decay multiplier	λ < β	3
α	Utility exponent	2 ≤ α ≤ 3	4
σ	Coefficient of social inertia	0 < ρ < 1	5
δ	Discount rate	0 < δ < 1	6
ρ	Self-allocation percentage	0 < ρ ≤ 1	-

Our simulations commonly use parameters in the neighborhood of:

```
β=2; μ=3; λ=0.99; α=2.25; δ=0.9; ϱ=0.9;
```

These values were not chosen for any special reason other than that they tend to give rise to reasonably intuitive behavior.

A Theoretical Approach to μ

While we later explore the economics of destruction using real world quantities, here we consider a purely theoretical approach to defining μ. Suppose you send one warrior to attack an opposing group of warriors, and that there is a 50% chance that your warrior will kill the first opponent she encounters before moving on to fight the next one. If she doesn't kill the opponent, then she will be killed. On average, how many opponents can we expect our warrior to kill? The answer is given by the infinite series:

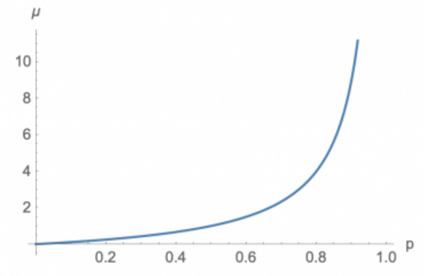
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n=1.$$

In other words, under these conditions, the expected destruction from using 1 unit of power (the warrior) is equal to 1 (warrior). In quantitative realism, this is equivalent to saying that μ = 1.

What happens when the warrior's likelihood of success is something other than 50%? Let that likelihood be p. The expected destruction is then:

$$\mu=\sum_{n=1}^{\infty}p^n.$$

Intuitively, μ is higher when it is more likely that a unit of power will destroy a unit of the opponent's power. Plotting μ as a function of p, we get:



In international relations theory, the **offense-defense balance** (see Glaser 1998 (https://github.com/mpoulshock/QuantitativeRealism/wiki/References#g)) reflects whether offense or defense has the advantage in a given situation — for example, whether it's easier to conquer territory or to defend it. When offense has the relative advantage, there is less of a deterrent to aggression, and vice versa. Glaser defines the offense-defense balance (ODB) as the ratio of the cost of attacking forces required to prevail to the cost of the defender's forces. This ratio may be different for each dyad of states, and it varies by direction (i.e. which state is attacking versus defending). Based on Glaser's definition:

$$ODB = \frac{cost_{attack}}{cost_{defense}} = \frac{1}{\mu}$$

For example, if it costs state A \$10 to prevail against state B, which spends \$100 on defense, $ODB_{AB} = 0.1$ and $\mu_{AB} = 10$.

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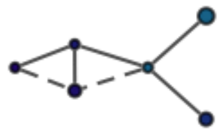
This page was last edited on 20 December 2020, at 10:30.

Power in Motion

The axioms of quantitative realism are based on the notion that power is a fluid-like substance that flows through a network of agents. Agents can use their power to make others more powerful or, conversely, to diminish their power. In choosing how to deploy their power, agents alter the course of its movement. Consequently, power is both a stock (a quantity possessed by an agent) and a flow (a quantity in motion between agents).



Power structures are not static but evolve in time due to two interacting processes. First, as depicted above, the relationships among the agents cause power to flow through the network, affecting agent strength levels: some agents get stronger, and others get weaker. Second, agents alter their foreign policies in a ceaseless quest to improve their position within the power structure. Agent sizes change as a result of their relationships, and in response the agents readjust those relationships, in an unfolding strategic dance ad infinitum. The first process entails a change to the vertices of a power structure; the second to its edges. The interplay of these two processes is what drives the model and brings power structures to life in ways that resemble, in abstract form, the kinds of power struggles that we see in the real world.

In this section, we examine the first process: how agents grow stronger or weaker as a function of the relationships they have with other agents. We call this the **law of motion**. After describing this movement of power and illustrating it visually, we consider refinements that take into account physical distances among the agents and their possession of scarce resources. Later, we explore the second process: the incentives that shape how agents alter their relationships with each other — in other words, how they determine their foreign policies.

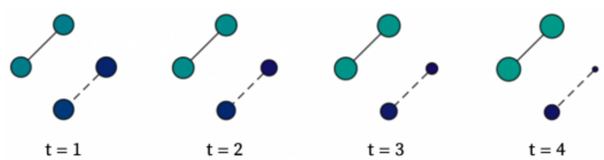
Subtopics:

The Law of Motion

Contents

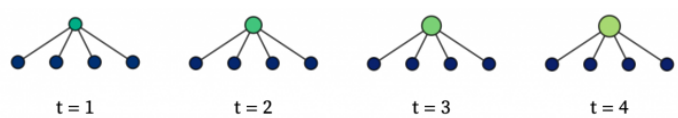
Description

The relationships in a power structure determine how the sizes of the agents change over time. Constructive relationships cause the agents to grow; destructive ones cause them to shrink. As power flows around the network, some agents rise in strength and other agents weaken. This process is depicted visually below, acting over four time steps:

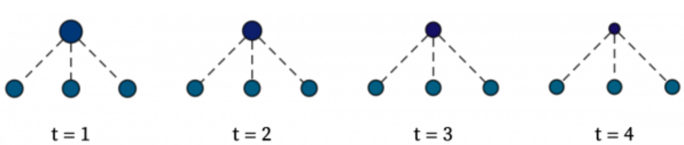


In the figure above, two of the agents are cooperating, and getting stronger over time as a result. The other two agents are attacking each other, and getting weaker as a result of their conflict.

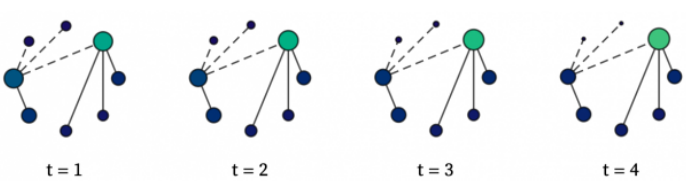
This process is called the **law of motion**, and it describes what happens when power flows through the network. The law of motion tells us how the power of agents evolves in time as a result of transactions among the agents. Its significance is that it allows us to calculate and visualize the systemic effects of individual agents' tactics, playing them out over time. It captures something intuitive and important about power. For instance, below a hegemonic agent at the hub of a network gets stronger over time:



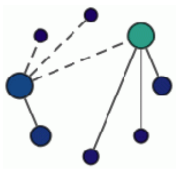
And here's how the sizes of the agents would change during a rebellion:



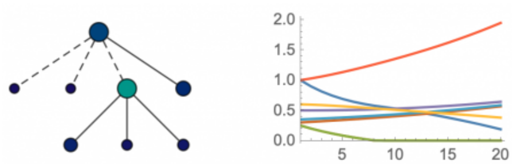
Scenarios often involve combinations of positive and negative relationships that are intertwined with each other, such as this one:



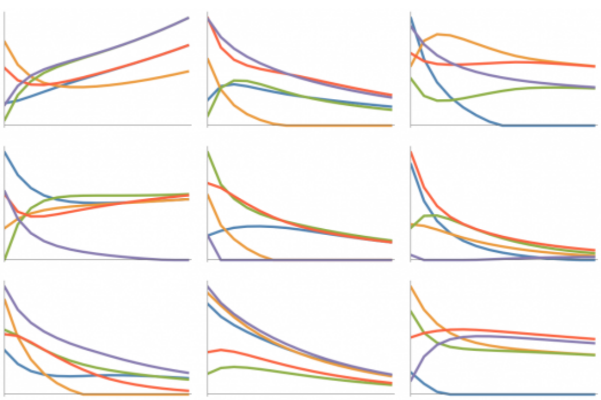
The law of motion operates upon the entire network of a power structure. Its effects on any given agent are not based solely upon the agents with whom it is immediately interacting. Agents in other parts of the network can also affect an agent's fate. Power sloshes around like a fluid, enlarging some agents who become beneficiaries to their downstream clients, and weakening other agents due to conflicts occurring in remote corners of the graph. This fluid-like nature of the law of motion may be more evident when we look at an animation:



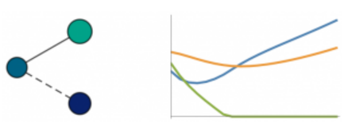
Everyone's destinies are interconnected, and while it's not always obvious from looking at a power structure how any given agent will fare, the process is deterministic and predictable. Consider the power structure below left:



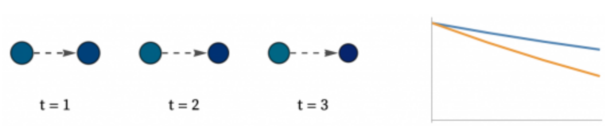
The plot on the right shows how the sizes of the agents in this power structure change over 20 time steps, according to the law of motion. Looking at similar size plots from randomly generated, five-agent power structures, we observe that power can move around a network quite dynamically:



Where there are reversals of fortune — agents growing and then shrinking, or vice versa — it is due to the fact that agents are affected by indirect connections with other agents. Agents several degrees of separation away can affect an agent's growth rate, such as when one's ally is attacked by a third party:



Another phenomenon to note, one that is a consequence of Axiom 2, is that even if an agent were to launch a unilateral attack, it would still lose power:



The attack itself expends power, so the attacking agent shrinks, albeit at a slower rate than the agent being attacked.

Derivation

The mathematical formula for the law of motion follows directly, and trivially, from the first three axioms. Axiom 1 says that a constructive action is a transfer of power from one agent that increases the power of another agent. If agent #1 engages in a constructive action, of magnitude x, with agent #2, then agent #1's power decreases by x and agent #2's power increases by at least x. The resulting changes to the agents' sizes can be represented as:

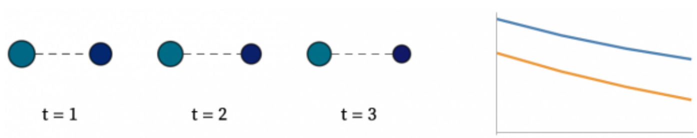
$$\Delta s_1 = -x$$
$$\Delta s_2 = \beta x$$

where the parameter β quantifies the magnitude of agent #2's increase.

Axiom 2 says that a destructive action is a transfer of power from one agent that decreases the power of another agent. So if agent #1 acts destructively towards agent #2 with a magnitude of x, then agent #1's power decreases by x and agent #2's power decreases by at least x. These changes in size can be expressed as:

$$\Delta s_1 = -x$$
$$\Delta s_2 = -\mu x$$

where the parameter μ quantifies the magnitude of agent #2's decrease. The law of motion for destructive action is similar to Lanchester's laws (https://en.wikipedia.org/wiki/Lanchester%27s_laws) of the changing strength of opposing military forces in combat. This theory, expressed as a set of differential equations, describes the rate at which the strength of each party diminishes as the conflict continues. An example of the analogous process in our model is:



Axiom 3 says that agents tend to dissipate power. Any power that they don't use will decrease, in each time step, at a rate defined by the parameter λ, where λ < β.

The variable x used above in the formalization of represents the amount of power expended during an action. It is the percentage of power devoted to the action times the agent's size:

$$x = \mathbf{T}_{ij} m \mathbf{s}_j$$

where destructive actions are represented by negative values in the tactic matrix **T**, and m is either β, μ, or λ, as appropriate.

Formalization

The equation for the law of motion uses matrix multiplication to update the sizes of all agents at once:

$$\mathbf{s}_{t+1} = \text{Ramp}((\mathbf{T}_t \circ \mathbf{M}_t).\,\mathbf{s}_t)$$

The ^{o} operator is the Hadamard product (element-wise matrix multiplication) and **M** is a multiplier matrix defined as:

$$(m_{ij}) = \begin{cases} \lambda & i = j \\ \beta & \tau_{ij} \geq 0 \\ \mu & \tau_{ij} < 0 \end{cases}$$

For example, here's a ternary tactic matrix and its associated multiplier matrix when β=2, μ=3, and λ=0.99:

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \qquad M = \begin{pmatrix} 0.99 & 2. & 2. & 2. & 2. \\ 2. & 0.99 & 3. & 2. & 2. \\ 2. & 3. & 0.99 & 2. & 2. \\ 2. & 2. & 2. & 0.99 & 2. \\ 2. & 2. & 2. & 2. & 0.99 \end{pmatrix}$$

When an agent dies, its power cannot be less than 0. The existence of negative sizes, besides being meaningless, would cause subsequent update operations to behave counterintuitively. The Ramp function sets any negative sizes that result from the law of motion to 0.

One way to implement the law of motion in code is:

```
UpdateS[s_, T_] := Ramp[(T*M[T]).s]
```

The multiplier matrix is determined by:

```
M[T_] := Table[Which[i==j, λ, T[[i, j]] >= 0, β, True, μ], {i, Length[T]}, {j, Length[T]}]
```

Alternative Formulation

We could also express the law of motion by decomposing **T** into positive, negative, and self-allocating parts:

$$\mathbf{T} = \mathbf{T}^{+} + \mathbf{T}^{-} + \mathbf{T}^{\mathbf{0}}.$$

The law of motion can then be expressed as:

$$\mathbf{s}_{t+1} = \mathrm{Ramp}((\beta \circ \mathbf{T}^{+}{}_{t} + \mu \circ \mathbf{T}^{-}{}_{t} + \lambda \circ \mathbf{T}^{\mathbf{0}}{}_{t}).\, \mathbf{s}_{t})$$

where β, μ, and λ are each $n \times n$ matrices in which each relationship (and direction) can have a unique value. This version of the law of motion is useful when modeling real world situations in which a relationship is both positive and negative at the same time — for example, if two countries are at war but nonetheless continue trading with each other.

Simulation

The law of motion can be simulated over a number of time steps by applying it iteratively to an initial power structure:

```
Simulate[s_, T_, t_] := Nest[UpdateS[#, T] &, s, t]
```

The function above returns a size vector representing the strengths of the agents at time t.

Note that the implementation on this page has been simplified for explanatory purposes. A more performant version, capable of handing edge cases, is available in the code repository (https://github.com/mpoulshock/QuantitativeRealism).

Physical Distance

In the basic model, agents don’t exist at specific locations in space and their behavior doesn’t take into account how near or far they are from each other. The model can be extended using a decay function for power transfers, such that transfers over long distances are attenuated.

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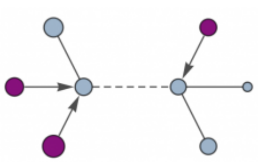
Competition for Scarce Resources

Contents

Description

In quantitative realism, the power struggles in the model are over nothing more than power itself. In the real world, however, power struggles are often waged over the control of scarce resources. A resource could be many things, such as land, oil, minerals, food, or water. Here we show how the basic model can be extended to include resources that can be disputed and possessed.

One way to add resources to a power structure is to treat them as nodes that emit a certain amount of power to whichever agent exclusively possesses them.



In the power structure above, agents are shown in gray and resources in purple. At each time step, the resource nodes give power to the agents who possess them. The directed edges show the flow of "income" from these resources, and the downstream agents will tend to grow at a faster rate than others. This model extends the abstraction of power to encompass the benefits given off by resources: a resource is simply a source of power. In the real world, these sources might be of varying types; but in the model, they are abstracted into a single fluid-like substance.

With resource nodes introduced into a power structure, how is it decided which agent will get access to each resource? There are a variety of rules we could come up with to govern this competitive process. A simple one is: An agent is awarded with possession of a resource if the agent **currently possesses the resource and continues to claim it, or is the sole claimant of the resource**. This rule gives a right of continued possession to whichever agent already has a resource, and any resource claimed by only one agent remains in the possession of that agent.

The rule implies that when two agents both want to control a resource, two scenarios are possible: First, if one of them already possesses it, the other one has to create incentives for the owner to give up possession. Second, if neither of them own it, they have to fight it out until one of them gives up their claim. Again, other rules are possible, but this one is minimalistic in terms of both the assumptions it makes and the data structures it requires.

Formal Representation

Data Structures

To implement the rule above, we need to add three data objects to a power structure: a resource vector, a possession matrix, and a contention matrix.

A **resource vector r** indicates the amount of power given off by each resource to the possessor at each time step. For example, a vector of four resources might be: r = {0.029, 0.15, 0.03, 0.041}. Given this resource vector, whichever agent possesses resource #1 will get 0.029 units of power at each time step.

A **possession matrix P** keeps track of which agents possess which resources. Each row in the possession matrix corresponds to an agent, indicating with a 1 or 0 which resources that agent controls. (Note that this is a different convention from that used in tactic matrices, in which each *column* is an agent's tactic vector.) For example, in the possession matrix shown below, agent #1 possesses resources 1 and 4 (highlighted in orange):

$$P = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

A **contention matrix C** tracks which agents have a claim over which resources. Like the possession matrix, each row represents an agent, and each column represents a resource. A 1 indicates that a particular agent claims that resource; a 0 indicates no claim. In the example below, agents #2 and #3 both have claims over resource 2 (highlighted in orange):

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

The resource vector, possession matrix, and contention matrix can then be appended to a power structure object. For instance, here's a power structure with three agents and two resources:

$$\left\langle \left| s \to \begin{pmatrix} 0.8 \\ 1. \\ 0.7 \end{pmatrix}, T \to \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, r \to \begin{pmatrix} 0.03 \\ 0.11 \end{pmatrix}, P \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, C \to \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \right| \right\rangle$$

Law of Motion

Given this extended power structure, when the law of motion is applied, two things need to happen. First, the possession matrix needs to be updated by applying the possession rule above. Second, the law of motion needs to take into account the power emitted by each of the resources. We'll illustrate each of these in turn.

Updating the Possession Matrix

To update the possession matrix, we need to know who currently possesses what, and who is staking a claim on what. The following function applies the possession rule described above:

```
UpdateP[P_, C_] := MapThread[Max, {P*C, MapTransposed[If[#.# > 1, 0 #, #] &, C]}, 2]
```

Though it's a bit dense, this code awards each resource to whichever agent is the sole claimant, or to whichever one possesses it and continues to claim it. Applying the `UpdateP` function to the sample possession and contention matrices above returns:

$$P = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \quad \text{UpdateP}[P,C] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Resource 1 (highlighted in orange) is both possessed and claimed by agent #1, so agent #1 gets to keep it. Resource 4 (highlighted in green) is claimed by agent #2 and possessed by agent #1, who doesn't want to claim it; so it is awarded to agent #2.

Updating the Size Vector

Now we can consider the second update process: updating the size vector in light of both the basic law of motion and the additive power given off by the resources:

```
UpdateS[s_, T_, r_, P_, C_] := UpdateS[s, T] + UpdateP[P, C].r
```

This function overloads `UpdateS` to account for resource possession.

Everything just mentioned is an extension of the law of motion. Along with it, at every time step, agents still need to craft their foreign policies, which would now include a claim vector indicating which resources they want to make claims upon.

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Foreign Policy Formation

The law of motion captures our intuition about how the consequences of conflict and cooperation unfold in the real world. It formalizes something about how we naturally think about power struggles, even if we've never visualized it in that particular way before. The law of motion alone, however, is limited in what it can tell us about a system of agents, because it is based on the artificial assumption that none of the agents ever changes their relationships. In the real world, agents constantly alter their foreign policies in order to improve their position within a power structure. It is to this process — foreign policy formation — that we now turn.

This section covers:

There are other ways of reasoning about foreign policy formation that we have not (yet adequately) explored, such as machine learning, Axelrod-style computational tournaments, and mathematical game theory.

Our implementation assumes that when deciding what move to make, agents consider only the current power structure. One might generalize this and instead allow agents to consider not just the present power structure but its full history as well.

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Quantitative Realism as a Game

The law of motion is deterministic; it doesn't depend on human decision making or agency. Its consequences follow from the network topology of agents interacting with each other. When two countries engage in protracted conflict with each other, they tend to reduce each other's strength. That is simply a fact about the world that the law of motion describes.

In contrast, the formation of a foreign policy entails volition, calculation, and judgment. It also has a moral aspect because it involves the infliction of negative consequences upon others. So what does it mean to propound a theory about foreign policy formation? Is it a statement about what real world actors *actually* do (or did, or will do), or is it a statement about what they *should* do? In other words, is it descriptive or prescriptive?

The answer is: it's neither. It's not descriptive because we do not expect whatever algorithm we come up with to represent how countries actually behave. Life is too complex for that. On the other hand, it would be a mistake to treat the theory as prescriptive because doing so would elevate its dark assumptions about human preferences into moral assertions, turning the theory into a self-fulfilling prophecy.

If our theory of foreign policy formation is neither descriptive nor prescriptive, then what is it?

Gamelike Nature

The most straightforward approach is to think of quantitative realism as a game, like chess. Chess establishes rules, such as about what pieces are used, how they are allowed to move, the consequences of moves, and how the game ends. But the decision about what moves to make is up to each player, and over the centuries elaborate theories have been developed based on the exploration of various lines of moves and countermoves. This theory of chess does not describe what moves players do (or will) make, and it does not tell players what moves they should make. It is neither descriptive nor prescriptive. It instead tries to establish general principles based upon an exploration of the space of all possible moves, seeking to understand the consequences of player decisions in light of the opposing player's interests and scope of action.

The "rules of the game" in quantitative realism are: (1) the law of motion, (2) social inertia, and (3) the assumption of ongoing interaction. To figure out strategies of gameplay, we lean on the assumption of how power is pursued and the tendency for reciprocal relationships to form. We then want to understand the space of moves and countermoves to see if we can perceive general patterns within the game. We are looking for a function that takes a power structure as input and returns a tactic vector for a given agent, indicating their preferred move. We'll call this function `UpdateTactic[s, T, i]`, where **s** and **T** make up the power structure, and *i* is the index of the focal agent. This function is what animates agent behavior.

We assume for the sake of simplicity that when determining what to do, agents look only at the current power structure. Because they are embedded in a dynamic process that unfolds over time, we could just as well generalize move search to allow agents to consider not just the present power structure but its history as well. In that case, agents could bear in mind their particular histories with other agents, the 'personalities' of the other agents in terms of how they tend to respond to situations, and the pattern of interaction among third parties with each other (see Axelrod 1984). This would make for rich and nuanced reasoning about what foreign policies to adopt, and it is obviously how real world agents go about their calculations. A full understanding of quantitative realism will require exploring this idea and taking into account deep interaction history in order to determine player action in the present.

Game Tree Complexity

Even with this simplification, there is an intimidating level of complexity afoot. When agents strategize about what to do next, they have to consider the potential moves of their competitors, anticipate what they might do, and in turn think about what those agents think they might do. As in chess, this implies a chain of reasoning composed of many if *I do that, she'll do that* links, in which numerous game states have to be evaluated mentally before anyone makes their next move. This reasoning process produces a game tree, which starts from an origin state — the current power structure — and looks at the agents' possible moves and countermoves. The resulting tree grows at an exponential rate. If an agent fails to carry out this analysis, or if they do so with insufficient rigor, they will be at a disadvantage compared with competitors who plan more thoroughly and logically.

Unlike chess, which has just two players, a power structure can have any number of agents, and the number of game states to be examined increases at a truly astonishing rate. In chess, each player has around 30 possible moves in a typical a middlegame position. Looking four moves ahead in chess, there are about a million board positions to contend with. In contrast, in a power structure, if agents had 30 possible moves at their disposal, and they wanted to look just three moves ahead against a dozen competitors, they would have to consider over 10^{3245} individual states. This game tree complexity is beyond astronomical — literally, as there are only 10^{80} atoms in the known universe.

With continuous tactics it's even worse, because agents in the model would have more than 30 moves available to them. They would have an infinite number. Ternary tactics therefore impose useful constraints. By limiting agents to altering a single ternary relationship at each move, which is often sufficient to generate an illuminating simulation, the game tree branching factor is not more than 2^{n-1}. That still can grow quite rapidly, especially as the number of agents increases, but it is on par with the computational complexity of other well-known games, especially when some sort of game tree pruning is used.

Unlike chess, in quantitative realism there's no notion of when the game might end, because there's no terminal condition that defines when someone has won a power struggle. Winning just means surviving to play the next round; there is no last round. Agents just continue to interact indefinitely in an unrelenting competition for survival. This indefinite interaction eliminates the possibility of working backwards from some specific goal. For example, in chess, a player can reason backwards from a checkmate, trying to figure out what combination of moves will entrap her opponent. That strategy is not available here.

And yet, despite the apparent impossibility, humans seem to solve this problem every day, in numerous social contexts, without melting their CPUs. Presumably we have developed, by virtue of biological evolution, an instinct for how to navigate power struggles through the intelligent use of constructive and destructive action. It stands to reason that there must be a way to approximate this natural sense computationally.

Despite these and other difficulties that we'll address as we proceed, the approach we will take is conceptually similar to that of a chess engine. Chess engines are based on a board evaluation function, which allows game states to be compared and ranked, and a search algorithm, which plays out hypothetical games to find the most desirable path forward for a given player.

In the sections that follow, we develop both of these components, building upon the axioms to see how simulated "games" can help us make generalizations about the trajectories of power struggles.

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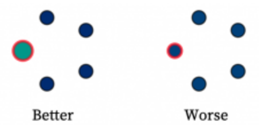
Utility Function

A utility or objective function defines a value that a particular model seeks to optimize. It is a way to quantify an individual's preferences over some domain of choices. Here we define a utility function for agents based on their presumed desire for both absolute and relative power.

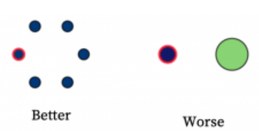
Motivation

Per Axiom 4, agents generally want to increase their power in an absolute sense. They want their well-being, capital, and capacity to affect events to rise whenever possible. For example, given the two power structures below, the focal agent (highlighted in red) would prefer the structure on the left, in which it is stronger:



Similarly, agents do not merely want to amass as much power as possible. They also are preoccupied with how much power everyone else has and they generally prefer other agents to be relatively weak. For instance:

Along the same lines, agents want their competitors to be weak and divided, rather than strong and united. For example, an agent would generally prefer to have five competitors with one unit of power each instead of one competitor with five units of power:



In short, part of an agent's fantasy is to be very large, with tiny competitors, or to have 100% of the market share of power:

But again, agents don't want to dominate so much that they can't grow in an absolute sense. These two flavors of greed — absolute and relative — are sometimes in tension, such as when one agent can grow in absolute terms only by allowing another agent to grow even more, or when one agent can dominate another in a relative sense only by suffering a reduction in absolute power.

Derivation and Definition

We can encode these conflicting objectives into the utility function **u**, which is derived as follows. We approximate the combination of absolute and relative greed with:

$$\mathrm{u}_i \approx \frac{\mathrm{s}_i^2}{\sum_{j=1}^{n} \mathrm{s}_j^2} \sqrt{\mathrm{s}_i}$$

where u_i is the utility to the i th agent. The first component on the right side of this equation reflects dominance: it is the ratio of an agent's size squared to the total of all the agents' sizes squared. This component embodies the idea that the smaller and more divided one's competition, the better off one is. The second component on the right side of the equation provides an incentive for absolute growth. The square root of size is used in order to reflect the diminishing marginal utility of acquiring power, which would mean that one additional unit of power is more valuable to a small agent than to a large one. This is a common facet of economic utility functions.

The equation above is equivalent to:

$$u_i \approx \frac{s_i^{2.5}}{\sum_{j=1}^{n} s_j^2}.$$

But we don't want to assume that 2.5 is the correct exponent. To give ourselves more flexibility, we replace it with the exponent α and posit that $2 \leq \alpha \leq 3$. The utility function can then be expressed as:

$$u_i = \frac{s_i^{\alpha}}{\sum_{j=1}^{n} s_j^2}.$$

As α decreases, relative power is incentivized and agents become more apt to use violence to cut other agents down to size, so they can hoard market share. As α increases, they're more prone to pursue absolute growth, which requires mutual constructive action.

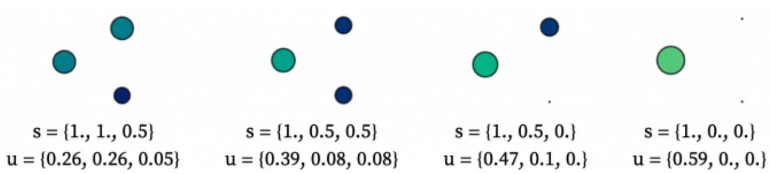
Implemented in code, the utility function takes the size vector as an input and returns a vector representing the utility of all of the agents:

```
Utility[s_] := s^α/Total[s^2]
```

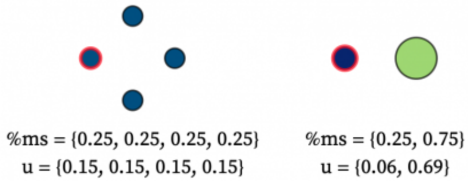
In this function, α can be a vector in which each agent has its own preference for absolute versus relative power. Allowing α to vary for each agent might better reflect the heterogeneity of the international system, in which some agents are said to have "revisionist" intentions (i.e. they are eager to seek power for themselves).

Behavior

To get a feel for how the utility function appraises different situations, here are the size and utility vectors of a few small power structures, where α=2.25:



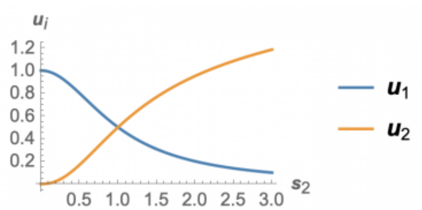
It's worth noting that this is not how utility is typically defined by international relations theorists. For one thing, power is not always measured numerically, partly due to longstanding problems in defining exactly what it is, and partly because IR has historically been a qualitative discipline. But even when power has been quantitatively assessed, other sets of assumptions have been applied. For example, Mearsheimer (2014) asserts that states in the international system seek to maximize their "market share" or percentage of total power, or `s/Total[s]`. While apparently reasonable on the surface, such a metric would be unable to distinguish between the following two scenarios, in which the focal agent has the same market share:



Clearly, however, the focal agent would prefer to be in the scenario on the left, as the utility function recognizes. Even though there has been much ink spilled in the debate over whether nation states are motivated to seek absolute versus relative gains (see Powell 1991), we are not aware of a mathematical expression having been devised to mediate between those two preferences.

This utility function not only puts a definite value on those preferences, but also provides a way to identify equivalent outcomes. For example, consider the size vectors s_A = {1.1, 1.1} and s_B = {1, 0.976}, where α=2.25. Agent #1 has the same utility in both of these situations. It can grow by 10% along with agent #2, or reduce #2's size by around 2.4%, and be equally happy either way. Because destruction is easier to accomplish than cooperation — due to Axiom 2 and because it can be done unilaterally — agents often turn to it as a way to satisfy their utility.

The utility function has a few other desirable properties worth mentioning. First, agents that are the same size have the same payoff, dead agents have a payoff of zero, and the largest agents will have the largest payoff. Adding agents with a size of zero to the population doesn't affect the payoffs to the existing agents. Further, when there are two agents whose sizes differ by a constant amount, their payoffs will tend to converge as their sizes increase by the same amount. For example, the payoffs to agents with **s** = {100, 101} will be closer together than those with **s** = {1, 2}. Finally, the utility function is smooth and well-behaved, except when all agents have a size of zero, in which case there's no one left to care.



Utility of two agents when $\mathbf{s}_1$=1 and $\mathbf{s}_2$ varies

Probably there are other utility functions that could be devised with properties similar to this one. For example, why not multiply the factors for absolute and relative power, as opposed to adding them linearly or combining them with weighted exponents, as in a Cobb-Douglas production function (https://en.wikipedia.org/wiki/Cobb%E2%80%93Douglas_production_function)?

However, this function seems fairly simple and elegant enough, and is basically fit for purpose. It is conceptually similar to the Herfindahl-Hirschman Index (HHI) (https://en.wikipedia.org/wiki/Herfindahl%E2%80%93Hirschman_Index), a standard measure of competition within a given market which can be computed from a size vector using `Total[(s/Total[s])^2]`.

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PrinceRank

PrinceRank is a metric that quantifies an agent's level of satisfaction within a given power structure. It is named after *The Prince*, Machiavelli's handbook for aspiring power-mongers. This page discusses the rationale for and definition of PrinceRank. A subsequent page applies PrinceRank to a variety of power structures.

Contents

Motivation

The utility function considers only the sizes of the agents in a power structure. It is clearly onto something important and interesting in the way it enables the comparison of different power structures. However, it would be very useful to have a metric that also took into consideration the agents' interrelationships. That way, we would be able to see how the alliance structure of the entire network affects each agents' preferences. For example, is an agent in a beneficial position within the network? Or is its security being compromised, with one of its allies being attacked by another agent? We have no way to know these things from the utility function alone.

To illustrate, the four power structures below all have the same size vector, $\mathbf{s}$ = {1, 1, 1, 1, 1}, and therefore the utility function gives the agents the same utility rating in each case. Yet the agents in these structures are in very different predicaments. But he utility function is simply blind to these distinctions.

What we need is a metric that quantifies an individual agent's preference for a particular power structure, that takes into account the entire structure: both the size vector and the tactic matrix. We can achieve this objective by combining the utility function with the law of motion.

Definition

We define an agent's PrinceRank $\mathbf{p}_i$ as follows: For a given power structure, run the law of motion, find the agents' utility at each time step, and then add up the total utility over time, weighting future time steps less heavily than those closer to the present. Expressed mathematically:

$$\mathbf{p}_i = (1-\delta)\sum_{t=0}^{\infty}\delta^t \mathbf{u}_i(t).$$

This "intertemporal utility" represents an agent's naive appraisal of utility as it accrues over time, naive in the sense that the agent assumes implausibly that all of the agents, including itself, will hold their tactics constant over those future time steps. The equation is a common method of discounting utility, used in the economics literature in the context of infinitely repeated games (see Ratliff 1996).

PrinceRank produces a complete ordering of all possible power structures, reflecting the preferences of a given agent. Equivalently, it also provides a preferential ordering of all agents within a single power structure, making it possible to know how happy each agent is with its position in the structure, relative to the others.

To make it easy to see the agents' PrinceRank in a power structure diagram, we color the agents' vertices along a blue-green-yellow spectrum. Yellow agents have the highest PrinceRank, while blue agents have the lowest. Green agents are somewhere in the middle.

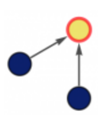
PrinceRank can be implemented as:

```
PrinceRank[s_, T_] := (1-δ) Sum[δ^t * Utility[Simulate[s, T, t+1]], {t, 0, Infinity}]
```

which has the same structure as the equation above. Note that this implementation is rather inefficient due to the repeated invocation of `Simulate`. Also, for practical purposes, `Infinity` in the above function can be replaced with `Floor[Log[δ,x]]`, which gives the number of time steps required to approximate PrinceRank by ignoring negligible future states, with the degree of negligibility determined by x (0.1 should suffice).

Use of Symmetrization

The usefulness of PrinceRank is not immediately obvious if one is focused on asymmetric tactics like this one:



In the diagram above, two agents are allocating constructive power at a focal agent who is not reciprocating. While the focal agent is clearly in a strong position, that good fortune is unlikely to persist because the other agents are not going to keep giving away their power without getting something in return. This may at first seem like a flaw in PrinceRank: it should not rate this focal agent's position so highly, because the assumption that the others will not change their tactics is tenuous (Heuristic 1).

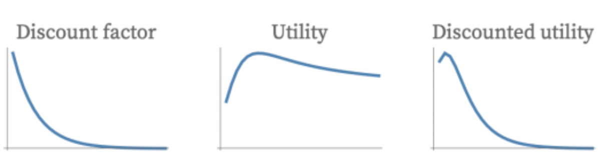
For this reason, it is preferable to correct such asymmetries before applying PrinceRank, using the `SymmetrizeTernaryT` function. Symmetrizing the power structure above and then computing PrinceRank, one now gets:

which is a much sounder appraisal of the situation at hand.

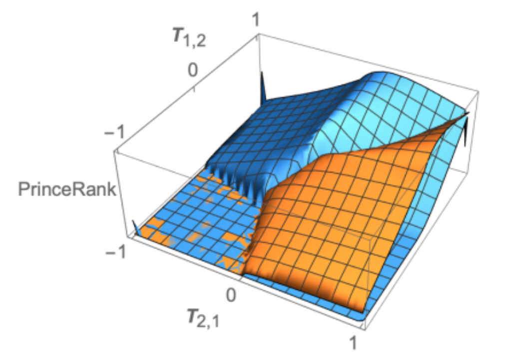
Properties

Here we mention a few (mathy) properties of PrinceRank. A more intuitive discussion of PrinceRank's behavior is on the next page.

First, consider the three plots below. On the left is the discounting function, which weighs near-term values more heavily than values farther in the future. In the middle is the utility of an agent in a random simulation of the law of motion. And on the right is discounted utility, which is what you get when you multiply the utility curve by the discounting function. The area under the discounted utility curve, multiplied by 1-δ, is the agent's PrinceRank.



Next, consider the simplest case of two equally sized agents. In the plot below, the PrinceRank payoffs for each agent are shown as a function of the power that each agent allocates to the other. Agent #1's tactic is $\mathbf{T}_{2,1}$ and agent #2's is $\mathbf{T}_{1,2}$. The blue surface represents agent #1's PrinceRank and the orange surface #2's PrinceRank.



PrinceRank as a function of **T** where **s**={1,1}

Note that the plot above assumes that there's no preset self-allocation percentage: an agent can allocate 100% of their power.

The next section uses visualizations to illustrate PrinceRank in a less technical manner.

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Implications of PrinceRank

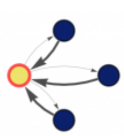
In this section, we explore the implications of the PrinceRank metric. Most of the examples use ternary tactics, although PrinceRank also works on continuous and asymmetric power structures.

Contents

Structural Ideals

An Agent's Fantasy

What would an agent's ideal world would be like according to PrinceRank? That is, if an agent could structure the relations of a system of equally-sized agents however it wished, what would that power structure be? When we sample continuous tactic matrices and select the one that gives the focal agent the highest PrinceRank, we get something like this:



The focal agent's ideal arrangement is for every agent in the system to be giving it constructive power, and for it to be reciprocating to some lesser degree. This pattern is reminiscent of imperial systems and their tributary states, and of hierarchical relationships generally. Historically, weak nations often paid tribute to a strong one, which in turn protected the weak from attack. This type of power relationship has been known to exist since the dawn of recorded history and even though it doesn't exist in the same formal sense in modern international relations, it is still the case that less powerful political entities often provide a disproportionate volume of benefits to more powerful ones, who seek to prevent the disruption of that flow of benefits. These tributary links form the backbone of hierarchical power structures. If this kind of hub-and-spoke structure above is every agent's fantasy, we would expect that when they have the ability to shape networks, they will generally try to impose this topology upon it.

What is considered optimal in an agent's ideal power structure depends upon the specific parameter values used, particularly δ and α. When an agent is shortsighted (low δ) or not interested in absolute growth (low α), then its ideal structure is just to receive constructive power from every other agent and not reciprocate anything in return. In contrast, forward-thinking agents reciprocate in order to create a positive feedback loop of mutual growth, albeit one based upon unequal exchange that allows them to grow at a faster rate than their minions.

Pairwise Preferences

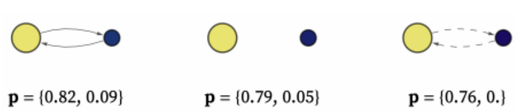
Preferences in Dyads

When there are only two actors in a power structure, what relationships do they prefer as their relative sizes vary? When the two agents are approximately equal in strength and ternary tactics are used, PrinceRank shows that they have an incentive to cooperate with each other. As indicated by the three possible relationships below — positive, neutral, and negative — both agents' PrinceRanks are maximized when they cooperate:

On the other hand, when there is a slight size difference between the two agents, as shown below, the stronger one has an incentive to attack as a way to keep its rival at bay, dominating it while it still can. Here one cannot help but be reminded of Machiavelli's repeated injunctions that power politics requires the destruction of rivals before they present a danger.

When there is a significant size difference between the two entities and the weaker one is no longer considered a threat, conflict is not worth the expense to the stronger agent, and neutrality appears to be the preferred option:

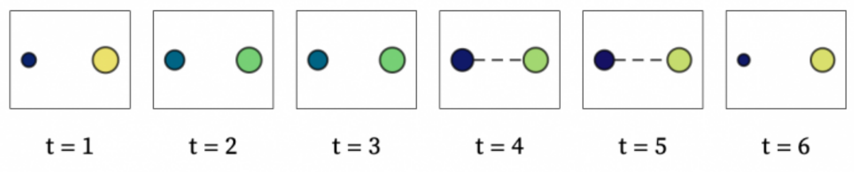
This last example gives the impression that a large agent and a much smaller one will not engage with each other, a conclusion that would appear to be in tension with the incentives noted above regarding hierarchy formation. The reason for the apparent inconsistency is that the scenario above uses ternary tactics, which are not sensitive enough to contemplate small allocations of power when there are only two agents. With more agents, the large agent can subdivide its outgoing allocation among numerous recipients. With only two agents, however, the large one's allocation would increase the size of the smaller one disproportionately. In a more nuanced, non-ternary set up, the stronger party could allocate a smaller amount of power, in which case a cooperative protectorate-tributary relationship would then be in both agents' interests:



As shown above, when continuous tactics are used, exchange is the preferred option, not neutrality as was the case with ternary tactics. So the incentive structure of PrinceRank does support hierarchy formation, even if it's not always apparent when ternary tactics are used.

Alignment with Realist Theory

We can crudely summarize the bilateral preferences just described as: *generally engage in positive exchange, but cut down potential rivals*. They allow quantitative realism to model the phenomenon what some international relations scholars call the *Thucydides Trap* (Allison 2017). This is the observation that a rising power presents a threat to an established power, which feels pressure to attack, as was the case at the outset of the Peloponnesian War. As famously noted by Thucydides, who chronicled and interpreted that war, "What made war inevitable was the growth of Athenian power and the fear which this caused in Sparta." We can portray a Thucydides Trap as a sequence like this:

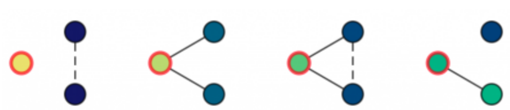


As the rising power (left) starts to grow (t=1,2,3), the established power's PrinceRank turns green, reflecting its mounting displeasure. At some point (t=4), the rising power is considered a genuine threat, and the established power attacks it while it can still prevail. Both parties are reduced by the conflict (t=5), until the established power regains a comfortable size difference, at which point it ceases the attack. A tributary relationship may very well form in the aftermath of this conflict (not shown). The Peloponnesian War was obviously much more complex than this, involving numerous parties, many years of combat, and a variety of plot twists that made the result far from inevitable. Nonetheless, in the end, the upstart Athenian Empire was destroyed.

These bilateral incentives also align with certain interpretations of political realism, such as that of Henry Kissinger. In *Diplomacy*, Kissinger asserts that two types of international systems are possible: a universal empire with a single dominant state, and a balance of power system in which states form coalitions to counterbalance against potentially dominant or aggressive states. PrinceRank reveals incentives for both types of systems. First, it shows that states with large power differences have incentives to form tributary relations and, by extension, imperial structures. Second, it posits that relatively equal entities have disincentives to fight with each other and that those with slight size differences have incentives to fight. Each of these entities can be a coalition of allied states, meaning that balanced coalitions have disincentives to fight — the essential claim of balance of power theory. So quantitative realism is not only in accord with Kissinger-style realism, but provides a theoretical foundation for it.

Triadic Structures

How would an agent order three-agent, or triadic, power structures according to PrinceRank? Let's consider all triadic, ternary structures in which all agents have the same size. There are 18 such scenarios that are unique from a focal agent's perspective. When we sort these by PrinceRank, there is a fair amount of variation in the ordering based on whatever parameters are chosen. However, the most preferred and least preferred structures tend to remain in the same general place in the list, regardless of the parameter choices. Here are four of the most preferred structures:

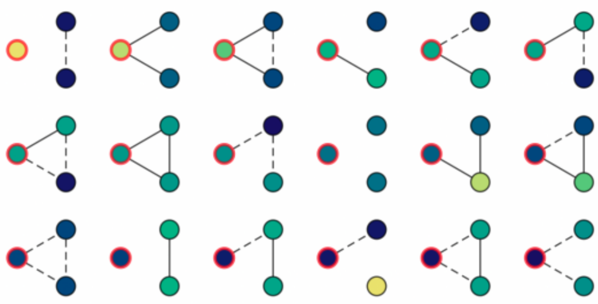


The first scenario is the *schadenfreude* structure, in which the focal agent takes numerical pleasure in the suffering that the other agents are inflicting upon each other. Their conflict is causing the focal agent's PrinceRank to increase. Some realists call this pattern *bloodletting*, which is when a state happily watches two rivals weaken each other. The second structure we've seen before: the hub-and-spoke pattern is every agent's fantasy. The third scenario is a composite: the focal agent has allies who are helping it grow, but the allies are weakening each other. And in the fourth, the focal agent has an ally, resulting in growth.

Now let's look at the triadic structures that tend to be among the least preferred, using the same parameters:

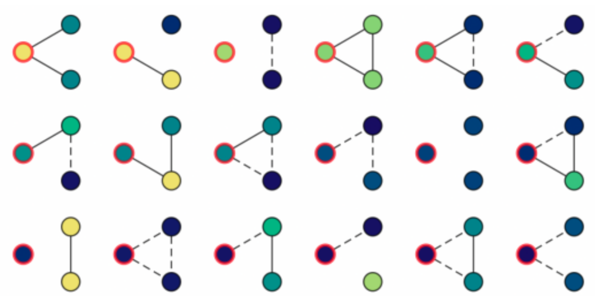
What these power structures have in common is that the focal agent is being attacked, sometimes by two agents and sometimes when the other agents have formed an alliance. These examples align fairly well with intuition. In the most preferred scenarios, the focal agent is not being attacked, there are no rivals in a position to surpass it, and it is poised to grow in absolute terms. In the least preferred structures, the focal agent is the object of violence, including by hostile alliances.

Here are all 18 triadic, ternary structures (with symmetries eliminated), ordered by the focal agent's PrinceRank, along with the parameters used:



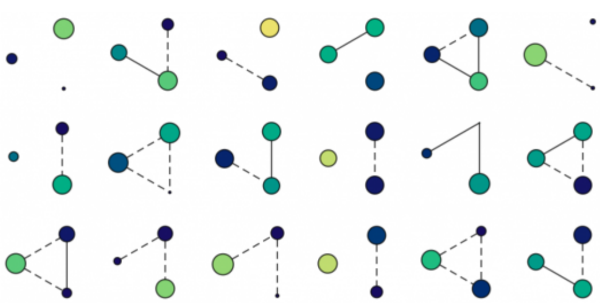
β=2., μ=3., λ=1., α=2.25, ρ=0.9, δ=0.9

The ordering is most sensitive to changes in α and δ. For example, if we raise those two parameters, we see the focal agent's preferences shift slightly in favor of structures that will give it real growth:



β=2., μ=3., λ=1., α=2.75, ρ=0.9, δ=0.95

The agents in the power structures above all have the same size. Here's what PrinceRank has to say about triadic, ternary structures with random relationships and agent sizes:



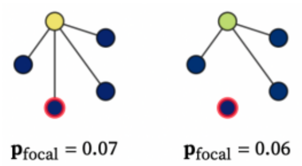
Hierarchy

PrinceRank reflects agent preferences for being at the epicenter of hierarchical power structures. This section examines the basic incentives that lead to the establishment, maintenance, and overthrow of hierarchies.

Establishing Hierarchy

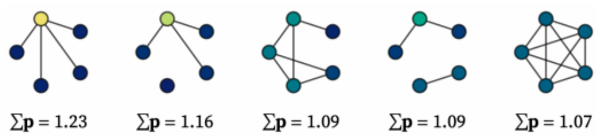
As is evident from an agent's ideal power structure and from the triadic preferences shown above, agents have strong affinities for hierarchy — specifically, hierarchies in which they are the hub. Why do hegemon-tributary relations form? When one agent is significantly more powerful than another and the flow of mutual power balanced such that the stronger agent gets an even or disproportionate benefit out of the relationship, the stronger agent need not be threatened by the growth of the weaker one. From the perspective of the weaker agent, it's better to pay tribute than to be disconnected, particularly when resistance to the hegemon is futile. Additionally, the weaker agent can still grow in an absolute sense, even under conditions of unequal exchange. Under such circumstances, a cooperative hegemon-tributary relationship is preferable for both parties.

For example, the power structures below show two possible outcomes for the focal agent. It can join the hierarchy as a tributary state (left) or it can remain unaffiliated (right):



It is obviously in the hegemon's interest for the focal agent to join. It is in the focal agent's interest as well, as indicated by the slight increase in its PrinceRank. This is typical of the mutual incentives that lead to the creation of hierarchies.

Notably, *a hierarchy maximizes the total PrinceRank in a system*. Consider the following power structures, which all have the same size vector:



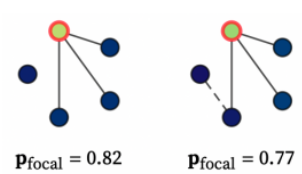
The hierarchy (far left) has the highest total PrinceRank out of the examples given. Presumably, this is a universal rule, due to the fact that hub agents are rewarded with extreme levels of PrinceRank. If PrinceRank is equated with well-being, then a hierarchical structure is the topology that maximizes the population's well-being — setting aside the fact that that well-being is not distributed equitably. This structural factor tends to push systems towards hierarchies.

Maintaining Hierarchy

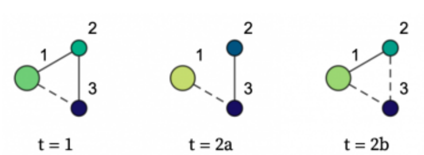
Since at least Machiavelli, one strand of realist thought has concerned itself with the maintenance of hierarchy. The idea is that a hegemonic agent must be vigilant about maintaining its position against rivals and must discipline coalition members in order to maintain its supremacy. PrinceRank provides a glimpse into what motivates this.

Mutual Defense

First, PrinceRank indicates that hub agents typically don't like when their tributary states are attacked by third parties. As shown below, the hegemon's PrinceRank decreases in the scenario on the right, in which one of its sources of power is being diminished.



PrinceRank also reflects incentives for tributary states to join in the hegemon's campaigns against third parties. For example, in the sequence below, at t=1, agents #1 and #2 are allies but they have inconsistent policies towards agent #3. At the next time step, agent #2 is faced with a choice: continue its alliance with #3 and lose support from #1 (t=2a), or harmonize its foreign policy with agent #1 (t=2b). PrinceRank shows that agent #1's threat of playing time branch 2a has the potential to coerce agent #2 into time branch 2b.

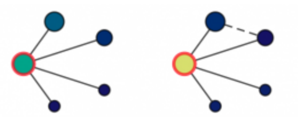


These preferences can create a kind of mutual defense arrangement in which a strong state defends a weak one to protect its supply of power, and the weak acts as a mercenary to the strong — a point we develop in more detail later on.

Infighting and Collusion

Given a hub-and-spoke structure dominated by a powerful agent, two general challenges present themselves to the leader: coalition members fighting with each other and coalition members colluding with each other.

Infighting is a problem because it drains the system of constructive power that would otherwise enrich the hegemon, and collusion is a problem because it threatens the hegemon with the rise of an alternate power center. PrinceRank shows that such behavior is generally displeasing to hegemonic agents. The hegemon facing infighting or collusion will have to apply carrots and sticks to correct these situations, such as by temporarily withholding support from the offending parties or by inflicting punishment. On the other hand, a hegemon may not mind infighting if it serves to weaken potential rivals, as shown here:

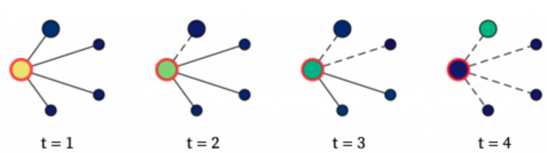


The focal agent's PrinceRank is higher in the structure on the right, where the infighting is occurring.

Rebellion

The desire to rebel against a power structure, and the converse desire of powerful agents to resist disturbance, forms a theme that permeates all of history. The representation of those desires in the context of quantitative realism is straightforward, and PrinceRank reflects the kinds of incentives that one would naturally expect to see.

Consider the sequence below. The canonical starting point is a hub dominated by a powerful agent (t=1). Rebellions are only successful when a critical mass of agents act in concert to overthrow the powerful, and the subsequent time steps show how the preferences of the various parties change as more agents participate in the revolt. The PrinceRank of the focal (hub) agent plummets as more agents join the fray.



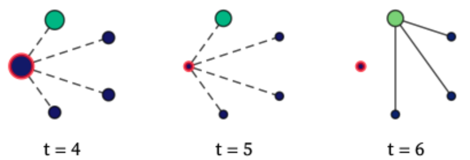
PrinceRank also reflects the preference of agents to free ride on another's aggression. For example, the PrinceRank of those who don't rebel at t=3 is higher than it would be if no one had rebelled at all (t=1), though this is not distinguishable in the diagrams above. The non-rebelling agents reap the benefits of the fight without incurring any of its costs. Moreover, the PrinceRank of those who do rebel is generally lower than it would have been had they not rebelled at all, reflecting the reality that there's a cost to rebelling. Part of the calculus of revolution is for agents to determine whether those short-term costs are worth taking in light of the probability of overall success.

The PrinceRank of a rebel goes up when more agents rebel along with it, because the hegemon then has to fight on multiple fronts and weakens more quickly. In the example above, the largest rebel benefits the most when everyone rebels. Not only is its PrinceRank higher than that of all the other agents (above, at t=4), including the soon-to-be former hegemon, it is also higher than it was in the original, hierarchical power structure. This "beta" agent is the only one who would ultimately prefer a full rebellion.

PrinceRank shows that rebels sometimes have an incentive to cooperate with each other. For example, even though one can't discern it in the image below, the smaller agents have incentives to support the beta agent as a counterbalance to the hegemon:

Notice also how the focal agent is unhappy about this support, for two reasons: it bolsters a rival and it diverts power that would otherwise be allocated to the hegemon. These arrangements are likely to be short-lived if the hegemon has the ability to quell the uprising and reimpose order, and it seems natural that a threatened hegemon would want to fight back. However, the hegemon could also increase the amount power allocated to the disgruntled agents, essentially buying their loyalty.

The final stage of a successful rebellion or revolution is when a new political order is established. Suppose we play out the initial scenario, with all of the agents rebelling simultaneously (t=4,5). After the hegemon is defeated (t=5), the victors form a new hierarchy (t=6). In this particular example, all of the rebels end up with a higher PrinceRank than they did when the revolution began, despite the fact they they have all been diminished in absolute size. This is because they have eliminated agent the original hegemon, a giant that was dominating them.

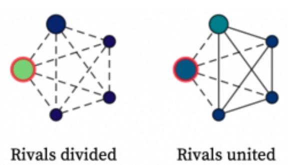


PrinceRank can account for the basic incentive patterns of revolution: agents willing to absorb the short-term pain of fighting in order to achieve the long-term benefits of a newly balanced order, and established powers struggling to maintain the status quo.

Divide and Rule

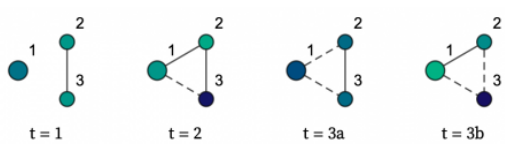
Divide and rule, also known as divide and conquer, is a classic strategy of political control. It entails breaking up larger concentrations of power by sparking rivalries and preventing smaller agents from linking up. In quantitative realism, an agent can't interfere directly with the relationship of two other parties, and it can't create dissension by spreading lies, as there are simply no mechanisms in the model to accomplish that. However, agents can nonetheless choose tactics that use rewards and punishments to break up rival coalitions.

What kind of opposing coalition would an agent would prefer to deal with? As the figure below illustrates, agents would generally prefer a group of rivals to be divided by infighting rather than unified against it, all else being equal:



Assuming that an agent finds itself in a situation like the one on the left, how might it attempt to turn it into one more like that on the right? One possibility is to introduce discord into the coalition by attacking some members and co-opting others. This strategy can imbalance relationships, triggering waves of strife and punishment within the coalition, because the co-opted agents have an incentive to turn on each other.

Let's examine the simplest possible case of divide and rule. Suppose that agents #2 and #3 have an alliance, and that agent #1 is stronger and excluded from it (t=1). Agent #1 commences divide and rule operations by co-opting agent #2 and attacking #3 (t=2). Agent #2 then has to choose between two outcomes: remain with #3 and be attacked by #2 (t=3a), or betray #2 and align with #1 (t=3b). As agent #2's PrinceRank indicates, the latter option is preferred, enabling #1 to accomplish its objective of fracturing its opposition.



In the example shown, the agent carrying out divide and rule (#1) used a combination of positive and negative tactics. However, divide and rule can also be more subtle, entailing any kind of differential treatment among similarly situated agents, as well as flip-flopping between temporary favorites. Agents can also use divide and rule to exploit existing divisions within an opposing coalition, causing some members to get sucked into the fray and diverting destructive power away from the instigating agent.

Even though this glosses over some of the finer points — for example, the sizes of the agents here do not change at the various time steps — it should be evident that PrinceRank helps illuminate the essential logic underlying divide and rule: both the preference structure that motivates it and the mechanism of its attainment.

Competing Alliances

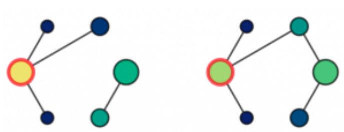
Most of the examples above involved hierarchical power structures, even if some of them were undergoing processes of degeneracy. But multiple hierarchies often coexist together in a single system, forming alliances that face off in potential competition. This section explores PrinceRank in that context.

General

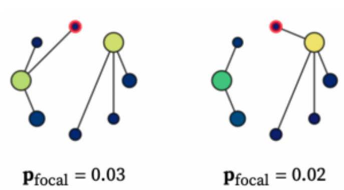
Each of the three power structures below depicts multiple alliances in what are arguably bipolar worlds. PrinceRank nevertheless manages to reasonably reflect each agent's level of satisfaction within these structures.

Exclusivity of Tribute

The first observation is that hegemonic agents do not like when their tributary states have positive relationships with other powerful agents. They prefer that tribute be given exclusively to them. For example, in the diagram below, the focal agent's PrinceRank declines when one of its tributaries also pays allegiance to a rival power.



Given this hegemonic preference for exclusivity of tribute, weaker members of a multipolar system are often faced with a choice of which major power to align with. In the scenarios below, the focal agent can choose between joining either of two alliances:



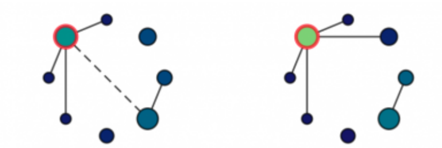
The variations in PrinceRank show that each hegemon would prefer that the focal agent join its gang. For them, every alternate alignment is a double loss: less power for themselves, and more power for a competitor. In the example given, it would be better for the focal agent to align with the alliance on the left. However, the choice might be different if the major power on the right were significantly stronger.

Balancing, Bandwagoning, and Buck-Passing

The international relations literature often refers to idiomatic behavior such as balancing, bandwagoning, and buck-passing. *Balancing* is a broad term that seems to refer to any conduct that attempts to reestablish a balance of power. It could involve shifting alliances to oppose an aggressor state, direct confrontation with an aggressor, internal strengthening, or some sort of signaling. The term *bandwagoning* is used to describe situations where a weaker state joins with an aggressor, rather than helping to balance against it. And *buck-passing* is when a state gets another state to do the balancing, either by providing it with support or, more commonly, by doing nothing, i.e. "passing the buck."

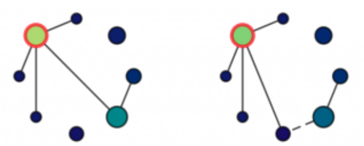
Balancing, bandwagoning, and buck-passing can for the most part be represented in the context of abstract power structures. Consider a scenario in which there is one hierarchy, led by a focal agent, and a separate, rival structure:

The question is what options the focal agent has for containing its large rival in the southeast. Balancing here can be construed in two different ways: as the initiation of aggression to cut down a rival or as the formation of an alliance that counterbalances the rival by building strength:



Balancing (aggression) Balancing (alliance)

Bandwagoning and buck-passing are depicted below:



Bandwagoning Buck–passing

Even without a full game tree to help us reason through this situation, PrinceRank exposes the preferences of the relevant parties to help us understand what outcomes are possible and mutually desirable.

Latent Tension

Some power structures have intrinsic stress due to the incentives of certain agents to initiate conflict. We can explore this latent tension by looking at power structures in which there is no active conflict, but where PrinceRank suggests that a party has an incentive to start one. Consider the following power structure, in which the great powers that have a slight size difference:

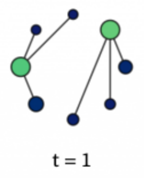
t = 1

At the next time step, the great powers can form one of the three possible ternary relationships:

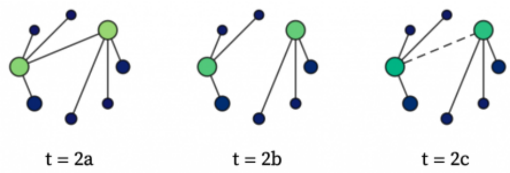
t = 2a t = 2b t = 2c

PrinceRank suggests that the focal agent can improve its position at t=2 by instigating a fight with the other great power, which is slightly weaker. It doesn't matter that the other agent's PrinceRank is lowest in this scenario, because of Heuristic 1: if one agent wants to fight, a fight will occur. Accordingly, the power structure at t=1 has a latent tendency for great power conflict — an instability inherent within it that is not immediately obvious on the surface.

Compare the situation above with a variation in which the two most powerful agents are roughly equal in size:



Here the incentives of the great powers are different, and PrinceRank indicates that it is in their mutual interest to cooperate. Accordingly, there is less latent potential for great power conflict.

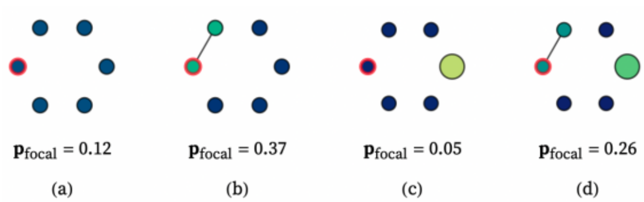


The same latent tensions can exist between any two agents in a system, and not just between great powers. It doesn't mean that two agents will necessarily fight merely because some structural tension is present. Agents have to consider all of their other possible relationships and any counter-reactions to their actions. So the discussion above is not meant to suggest that conflict is inevitable under any given set of circumstances.

Power structures with latent tension illustrate the fear and insecurity that agents experience in an anarchic setting, and PrinceRank highlights the fact that structures that appear to be in equilibrium may actually be fraught with destructive potential.

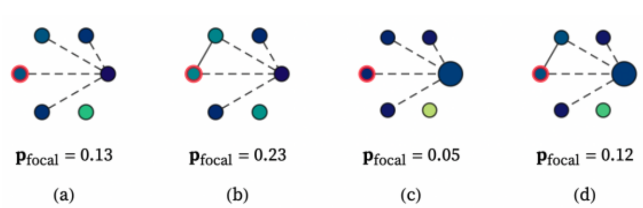
Unity under Threat

Another theme in power politics is that agents tend to unify in the face of a common threat. To see how PrinceRank accounts for this incentive, consider the four power structures below:



In diagrams (a) and (b), in which all agents are the same size, when two agents unify it increases their PrinceRank by a factor of 2.6 (=.37/.12). In contrast, in diagrams (c) and (d), in which there is a more powerful agent in the system, cooperating increases PrinceRank by a factor of 4.3 (=.26/.05). The mere presence of a dominant agent intensifies the incentives of smaller agents to unite with each other. This sort of four-way comparison is necessary because agents generally like cooperation and prefer to be the hub of the network, and so would be inclined to cooperate regardless. However, the presence of a more powerful agent has the causal effect of amplifying that desire.

The same incentive exists when the powerful agent is not merely present but also behaving aggressively towards weaker ones:



Here the focal agent's PrinceRank increases by a factor of 1.8 (=.23/.13) when transitioning from (a) to (b), and a factor of 2.3 (=.12/.05) when going from (c) to (d). Note that this assumes that the power that the smaller agents allocate to cooperation does not detract from the power needed to defend themselves against their attacker.

Other Notable Characteristics

PrinceRank has a few other characteristics worth mentioning.

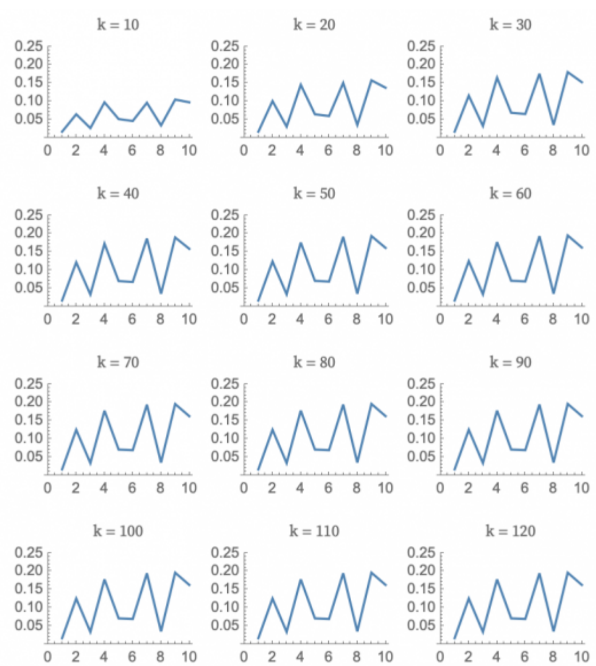
Network Centrality Measure

PrinceRank is a kind of network centrality measure, vaguely similar to degree, Katz, Bonacich, and PageRank centrality (Valente 2008). These other measures generally move in the same direction when the graph links are positive. When there are negative edges in the graph, PrinceRank goes its own way, emphasizing different aspects and always keeping the result nonnegative. PrinceRank also takes into account information about graph vertices (the size vector), which these other measures do not. These differences are probably not so significant that one of these other measures couldn't be used as a coarse position evaluation function. However, PrinceRank derives from an axiomatic foundation that has a meaning consistent with the metric's purpose and results, and a conceptual coherence about what power is and how it behaves.

Computational Shortcut

PrinceRank can be computationally intensive to calculate, especially when δ requires it to simulate a significant number of time steps. For instance, when δ=0.9, PrinceRank has to compute 29 steps, each of which applies the law of motion and the utility function. Raising δ to 0.95 requires 59 time steps. While a single PrinceRank calculation can be done in a thousandth of a second, game trees grow exponentially and require PrinceRank to be run at each leaf state, so we want to be frugal.

To make PrinceRank more efficient, we can approximate it by limiting the number of steps run to a preset parameter *k*. This way, even if δ=0.99 (299 steps), we need only compute the first, say, *k*=25 steps of it. Even though the result for an individual agent may be different when we truncate the simulation, experiments suggest that the agents' approximate PrinceRanks relative to each other will be similar enough to be workable:



Each of the plots above shows the approximated PrinceRank as a function of the agent index, i.e. their slot in the PrinceRank vector. Each graph is the "signature" of that vector. As suggested by these plots, the signature has basically the same contours starting around *k*=20, and anything beyond that is simply wasted computation. By cutting off the long tail of discount function, we are dropping only trivial values.

A caveat is that *k* should be at least as large as the graph radius of the power structure being analyzed. Because power flows through the graph one step at a time, an agent will not feel the effects of an upstream agent that is farther away than the number of time steps elapsed. So if *k* is too small, PrinceRank won't reflect all of the interlinkages among the agents.

Handling an Edge Case in the Utility Function

Another notable aspect of PrinceRank is that it helps correct a deficiency in the utility function. An agent can get a high utility score by being large and alone. But due to the law of motion, an agent would not want to be the only one in existence, because then it would stagnate or shrink (Axiom 3), depending on how λ was set:

PrinceRank recognizes that a lone agent is destined to decay, because it looks ahead whereas the utility function looks only at the present state.

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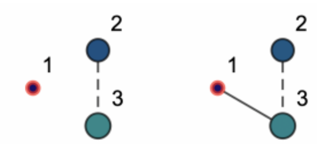
Legal Moves

This section introduces the concept of a **legal move**. Legal moves should be distinguished from legal tactics: a "move" is a change in an agent's tactics, whereas a tactic vector is legal if it meets certain conventions.

The Concept of Legality

When a power structure has symmetric, ternary tactics and one player makes a move, they also alter an opposing player's relationship to them, due to Heuristic 1. The problem is that if the focal agent is allowed to make whatever move she wants, she could force the other player into a situation that is against their interests. For example, in the following sequence, the focal agent initiates cooperation with a larger competitor whose PrinceRank drops as a result.



t=1, $\mathbf{p}_3$= 0.3   t=2, $\mathbf{p}_3$= 0.28

To avoid this sort of situation, we introduce the concept of a legal move. A move is only considered legal if, *where a tactic requires increased positivity, it also increases the PrinceRank of the other agent*. In other words, if cooperation increases, it has to be in the other player's interest. On the other hand, a player can start a conflict unilaterally. This rule enforces consistency with Heuristic 1.

One (inefficient) way to implement legality checking is with the following function, which takes two power structures and an agent index as input, and returns a Boolean indicating whether the proposed move from `ps1` to `ps2` is legal.

```
LegalMoveQ[ps1_, ps2_, i_] := Block[{j},
  For[j = 1, j <= Length[ps1["s"]], j++,
    If[i != j && T2[[i, j]] > T1[[i, j]] && pr2[[i]] <= PrinceRank[ps1][[i]],
      Return[False]]
    ];
    Return[True];
  ]
```

The need for move legality is due to the use of symmetric tactics, which are used due to the challenge of modeling reciprocity in a game (see Axelrod 2006). This challenge arises in the context of minimax game tree search because of the backward induction problem: players have an incentive to defect in the last round, making it difficult for cooperation to be sustained. It would be interesting to explore how reciprocity might emerge in a quantitative realist model, perhaps using machine learning, instead of assuming it.

What percentage of moves are legal?

TODO

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Move Selection

In foreign policy formation, an agent decides what move to make in a given power structure. **Move selection** in quantitative realism uses a game engine that is similar to the way computers play chess (https://www.chessprogramming.org/Main_Page).

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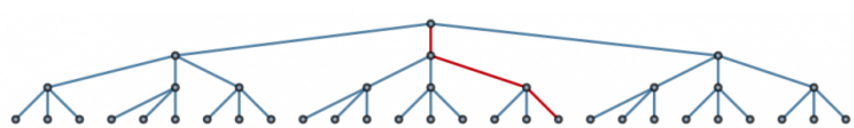
Minimax Game Tree Search

In combinatorial game theory (https://en.wikipedia.org/wiki/Combinatorial_game_theory), the minimax algorithm (https://en.wikipedia.org/wiki/Minimax#Example_2) provides a way to search a game tree in order to find the best move for a given player. From a given game state, the algorithm explores various lines of play, evaluates the terminal states from the perspective of each player, and then reasons backwards about which moves are best for each player who took a turn along each line. The minimax algorithm thus looks for the sequence of moves through the game tree that represents the players' best responses to each other's moves. The resulting sequence is called a principal variation (https://www.chessprogramming.org/Principal_Variation) and it can be visualized as the red path in the imaginary game tree below. The first step in the principal variation is the moving player's best move.



Because minimax game tree search is a well-known algorithm (see Russell and Norvig, ch. 6, which includes an example for multiplayer games), we do not explain it here but merely highlight the features that are specific to quantitative realism.

Evaluation Function

Minimax game tree search requires an evaluation function that assigns a value to each player for any given game state, or position. In chess, for instance, the evaluation function looks at the pieces each player has and their positioning on the board, and reduces that information to a number between -1 and 1. In quantitative realism, PrinceRank is the logical choice for an evaluation function, because it intuitively reflects agent preferences in various power structures and provides a complete ordering of all possible power structures from the perspective of a given agent. The utility function could also conceivably serve as this evaluation function; however, it does not consider agent relationships and experiments suggest that it does not lead to hierarchy formation as reliably as PrinceRank does.

Move Order

Minimax search presumes that players move sequentially, rather than simultaneously. Simultaneous play would be a more natural choice for quantitative realism, because everyone would move, the law of motion would update agent sizes, and then everyone would move again. In contrast, sequential play introduces a fiction into the already abstract model, as we must somehow decide in what order the players are to move. Should they go "around the table," or completely at random, or with each round randomized? Should a player be able to move twice in a row? Should everyone get an equal number of turns, or should more powerful agents move more frequently? And should the law of motion update after each individual turn, or after each round?

These questions are significant because the answers can drastically change the behavior of the agents in the model. We have not yet resolved these questions, however, our current approach to simulating move sequences is as follows: Every player has an equal opportunity to move, but is chosen randomly. Within the game tree search of a particular move, a player's probability of moving is proportional to their PrinceRank, causing them to pay extra attention to the more important players. This topic is discussed in more detail here.

Tree Traversal

Pruning

Tree traversal is based upon depth first search, using a single processor. The quality of the search for a principal variation depends upon the ability to prune away unpromising branches of the game tree. Game engines for two-person games, like chess, use alpha-beta pruning (https://en.wikipedia.org/wiki/Alpha%E2%80%93beta_pruning) to reduce the size of the tree to be searched. However, alpha-beta pruning is not available for games involving three or more players. As a substitute, we use PrinceRank-based pruning: a node and its children are pruned from the game tree if the value of the node plus some delta value Δ is less than the best known minimax value for the applicable branch. The assumption is that PrinceRank is a reasonable proxy for the minimax value, and if that value is too low, it is unlikely that that node will be a best move. The value Δ is set as a global parameter. Leaf nodes are not pruned. More work is needed to refine this pruning algorithm, and a major goal is to rewrite the game tree search in C++ for maximum performance.

Breadth

Because PrinceRank-based pruning is used, a maximum breadth can be set for each game tree node. Candidate moves are tested in order of their PrinceRank for the moving player, and a limit on the number of moves to be tested can be imposed.

Depth

The depth of the tree search is limited by the amount of time one is willing to wait for a result. Iterative deepening (https://en.wikipedia.org/wiki/Iterative_deepening_depth-first_search#Example_2) is not used because it degrades performance, presumably because the current implementation does not cache the game tree, which has to be re-traversed with each iteration.

Randomness and Indeterminacy

Because minimax search is based on random move orders, it is the primary source of randomness in the model. Randomization is also used within the minimax algorithm to break ties when two moves are of equal value. However, this turns out to rarely occur unless more than one agent dies, and tie-breaking situations can usually be avoided by making sure that no two agents have the same exact size when initiating a simulation.

Social Inertia

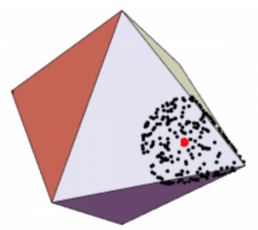
Social inertia limits the tactical choices available to an agent, forcing them to choose tactic vectors that are in some way *near* the agent's current tactic. How we implement social inertia depends upon whether continuous or ternary tactics are used.

Continuous Tactics

A natural way to define *tactical distance* for continuous tactics is to use the Euclidean distance between the two tactic vectors, either `EuclideanDistance[a,b]` or `Norm[a-b]`. The parameter σ sets a maximum acceptable distance and we can then generate random tactics within that distance from some starting tactic:

```
RandomTacticNear[tactic_, i_, σ_] := NormalizeTactic[MapAt[Abs, RandomPoint[Sphere[tactic, σ]], i]]
```

The function above works by: (1) starting from a point represented by the original tactic, (2) picking a random point on a sphere of radius σ, centered at the original point, (3) ensuring that the self-allocation is nonnegative, and (4) normalizing the tactic. To give an example where n=4, this provides a way to find random tactics (black) near a given starting tactic (red):



The implementation above is admittedly quick-and-dirty. For one thing, it does not enforce the self-allocation percentage ρ, so any nonconforming tactic vectors will have to be discarded. For another, as illustrated in the image above, the random points are not distributed evenly across the surface of the cross-polytope, but instead tend to cluster at the perimeter of the circle defined by σ. Finally, it is possible that when we make the self-allocation positive, the resulting tactic has a radius greater than σ. At any rate, more complex versions of this function could be created that address these deficiencies.

Ternary Tactics

To find ternary tactics that are somehow near a given vector, a natural way to think about distance is based on the number of elements that are altered. This can be computed as the Hamming distance, for example `HammingDistance[{-1, 0, 0, 1, 1}, {0, 0, -1, 0, 1}]` is 3. Given a ternary tactic vector and an agent index *i*, the following function returns all ternary tactic vectors within a given Hamming distance *dx*:

```
TernaryTacticsNear[tactic_, i_, dx_] :=
   DeleteDuplicates[Flatten[Map[Tuples[ReplacePart[Partition[tactic, 1],
      Map[# → {-1, 0, 1} &, #]]] &, Subsets[Delete[Range[Length[tactic]
   ], i], {dx}]], 1]]
```

For example, `TernaryTacticsNear[{0, 1, -1}, 1, 1]` will return {{0, -1, -1}, {0, 0, -1}, {0, 1, -1}, {0, 1, 0}, {0, 1, 1}}. We use *dx* as opposed to σ, even though they both express notions of social inertia, so as not to confuse the two approaches.

Because of the symmetry imposed by Heuristic 1, we generally want to find nearby *symmetric* tactic matrices, which are generated by:

```
SymmetricTsNear[T_, i_, dx_] := Map[ReplacePart[ReplaceColumn[T, i, #], i → #] &, TernaryTacticsNear[T[[ All, i]], i, dx]]
```

For instance, the following symmetric tactic matrices are within a Hamming distance of 1, for agent #1, starting from the **0** matrix.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.9 & -0.1 & 0. \\ -0.1 & 0.9 & 0. \\ 0. & 0. & 1. \end{pmatrix} \begin{pmatrix} 0.9 & 0.1 & 0. \\ 0.1 & 0.9 & 0. \\ 0. & 0. & 1. \end{pmatrix} \begin{pmatrix} 0.9 & 0. & -0.1 \\ 0. & 1. & 0. \\ -0.1 & 0. & 0.9 \end{pmatrix} \begin{pmatrix} 0.9 & 0. & 0.1 \\ 0. & 1. & 0. \\ 0.1 & 0. & 0.9 \end{pmatrix}$$

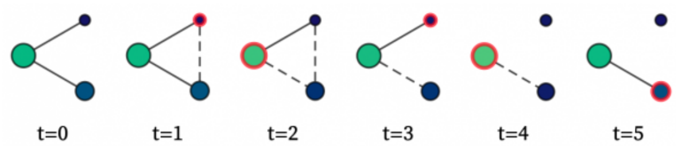
Other notions of distance are possible with ternary tactics. For example, we could start with all possible ternary tactics and then filter out ones that are beyond a Euclidean distance defined by σ. Or we could forbid changing a -1 to a 1, or vice versa, without first making the tactic a 0.

Move Sequences and Distributions

Two tools at our disposal for understanding the dynamics inherent in a given power structure are **move sequences** and **move distributions**.

Move Sequences

Move sequences show agents moving in turn. In these sequences, such as the one below, the moving or "focal" player is highlighted in red:

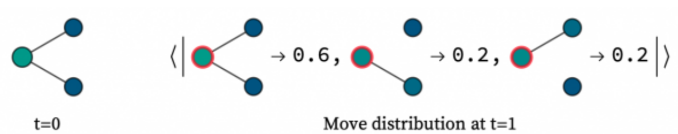


Sometimes move sequences are created by hand, such as to show the progression of an actual historical event. But they can also be generated by simulations in which each move is made based on minimax game tree search. As discussed elsewhere, the behavior that emerges from these simulations is very sensitive to the order in which the players move. Unless otherwise indicated, we use random move orders in which the players have an equal probability of moving. However, when generating the game tree to determine each individual move, a player's probability of moving is proportional to their PrinceRank. In other words, every player has equal opportunities to move, but when deciding what their move should be, they pay extra attention to the more important players.

In move sequences, the sizes of the agents are typically normalized such that the largest agent always has a size of 1.

Move Distributions

A second way to explore the model is to look at the best moves of a given player, in a given scenario, over a variety of random move orders. These "move distributions" are presented as follows:



The initial state is shown on the left, at t=0. On the right are the likely moves of the focal player, with their associated probabilities based on the move orders that were randomly sampled. Move distributions estimate the strength of a player's move options.

The Effect of Institutions

Institutions can be thought of as constraining power relations. We can interpret them as power structures in which certain relationships are not permitted to change, while others can. The simplest way to represent an institution in the context of quantitative realism is therefore as a set of constraints on a tactic matrix. For example, an institution might cause a particular element of a tactic vector to be frozen, or it might require it to remain within certain bounds. Agents would be free to alter other, unconstrained tactics. Simulations could then be run based on those assumptions.

When analyzing international relations, one is often interested in military alliances and defense agreements, and specifically whether one country will defend another under certain conditions. These coordinated foreign policies can be represented using tactic matrices that assume coordinated action.

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Systemic Metrics

PrinceRank is a measure of a particular agent's satisfaction within a given power structure. There are also metrics that are not specific to individual agents but that characterize power structures *as a whole*.

Contents

Polarity

In the academic literature on international relations, the **polarity** of the international system is often an attribute of concern. This metric is typically determined by counting the number of great powers which dominate a system. For instance, a system dominated by one large state is a *unipolar* system; a system with two relatively strong powers is *bipolar*; and a system with more than two is considered *multipolar*.

Polarity is usually not defined in a rigorous way. It typically entails an informal counting of the great powers in the system at any given time, based on states' "market share" of total power — or equivalently, `MarketShare[s_] := s/Total[s]` when size vectors are used. It may very well be that this procedure is sufficient and that no additional sophistication is needed; in fact, there are rarely debates about what the polarity is at any given point in time. On the other hand, if quantification is possible, we should like to know how.

Singer Concentration

To this end, a definition of power concentration was proposed by Singer (1972). Here Singer concentration is expressed using normalized size vectors:

```
SingerConcentration[s_] := Sqrt[(Total[MarketShare[s]^2] - 1/Length[s])/(1 - 1/Length[s])]
```

In other words:

$$\text{SingerConcentration} = \sqrt{\frac{\sum \frac{s^2}{(\sum s)^2} - \frac{1}{n}}{1 - \frac{1}{n}}}$$

This formula returns a number between 0 (least concentrated) and 1 (most concentrated). Unipolar systems are thought to have values from 0.4-0.5, and bipolar and multipolar systems have values from 0.2-0.4. Though conceptually coherent, this formula is never used in the literature.

Herfindahl-Hirschman Index

Another missed opportunity is that international relations theorists have not repurposed the Herfindahl-Hirschman Index (HHI) of market concentration as a way to quantify polarity. The formula for HHI, `Total[MarketShare[s]^2]`, shares the same beating heart as Singer's formula, but it is simpler and well-understood from its use in the antitrust context. Though international relations scholars could have easily adapted the HHI for this purpose, no such reference in the literature has yet been found.

However, we can do better than HHI. HHI always returns a value between 0 and 1, and it is interpreted to have three ranges of market concentration, essentially low, medium, and high. But with a slight hack, and by applying it to international power as opposed to market dominance, we can make it approximate the number of relatively large agents in the system. We do this by dividing each agent's market share *ms* of power by the market share of the largest agent, before squaring:

$$\text{ModifiedHHI} = \sum_{i=1}^{n} \frac{ms_i^2}{\text{Max}(ms)^2} = \sum_{i=1}^{n} s_i^2$$

or in code:

```
ModifiedHHI[s_] := Total[s^2]
```

In other words, if we square a normalized size vector and then total it, the result is a summary of the number of large agents in a given power structure. The table below shows the modified HHI for a few simple size vectors. As indicated, this is a simple way to get an objective count of the number of great powers.

s	ModifiedHHI
{1}	1
{1, 1}	2
{1, 0.2}	1.04
{1, 1, 1}	3
{1, 0.5, 0.5}	1.5
{1, 0.1, 0.1}	1.02
{1, 0.75, 0.2}	1.6
{1, 1, 0.05, 0.05}	2

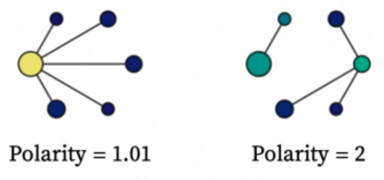
Polarity

But we can do better still. The problem with Singer's formula, the HHI, and the modified HHI is not so much that they aren't used, but that they only take into account the sizes of the relevant entities and not the relationships among them. The figure below illustrates why this is an issue. In the two power structures shown, the agent sizes are exactly the same. However, it is clear to the eye that the system on the left represents a unipolar reality while the one on the right represents a bipolar system.

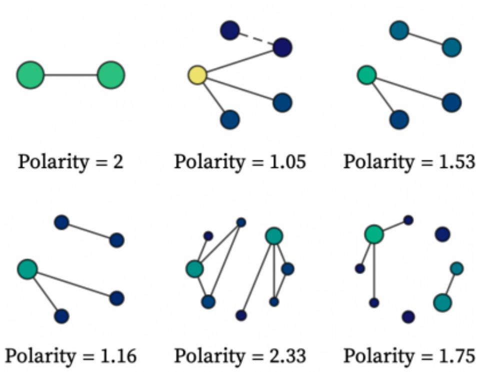
We can solve for this by building upon PrinceRank to create a measure of polarity that takes into account the entire power structure — both the sizes and the agents' relationships:

```
Polarity[s_, T_] := Total[NormalizeList[PrinceRank[s, T]]^2]
```

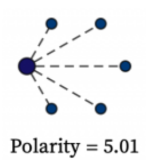
This function yields a fairly intuitive, albeit non-integer, value of how many poles or hubs there are in the network. Applying this definition to the two examples above, we get:



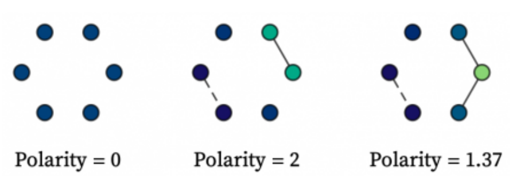
This new polarity function provides a more holistic classification of power structures. Of course, the specific results are dependent upon six parameters: β, μ, λ, ρ, δ, and α. Let's look at some examples using the parameters β=2, μ=3, λ=1, ρ=0.9, δ=0.9, and α=2.25:



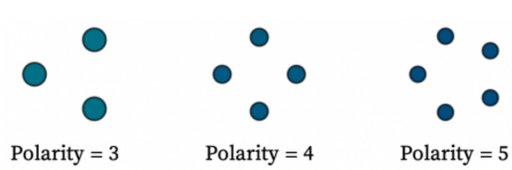
Roughly speaking, polarity is the number of agents that are tied for first in PrinceRank. In a way, it is more a statement about the potential future of the system rather than its immediate present, because PrinceRank looks ahead by simulating the flow of power through the network. So `Polarity` recognizes that a state conventionally believed to have unipolar dominance can have that superiority eroded due to the realignment of the smaller agents in the system. To illustrate, in the example of rebellion below, the polarity value anticipates a future composed of five equally dominating agents:



Because the polarity function is forward-looking, the relationships among the agents play a large role in how it characterizes them, as shown in these other examples:



One final feature to note about the polarity function is the correct way that it handles equal-sized agents with no interrelationships:



Stability

An important feature we would like to know about the international system is how stable it is at any given time. *Stability* can mean one of two things: the system's propensity to resist change of any kind or, perhaps more importantly, its tendency towards violence and disorder. International relations scholars often take the polarity of the system as an independent variable, and the question is how stable the system is, given its polarity. This is a pivotal issue for the field, one of whose primary objectives is to be able to predict how stable the international system is under various configurations. There are a variety of opinions as to whether unipolar, bipolar, or multipolar systems are the most stable, and indeed whether there is any correlation at all. Here we attempt to establish quantitative measures of stability.

Pariwise Stability

A power structure is considered pairwise stable if, at the next time step, none of the agents have an incentive to change their relationships with any of the other agents, based on PrinceRank alone. Pairwise stability and stress only work with discrete (ternary) tactics. All of the other functions presented above work with either continuous or discrete ones.

Global Stability

TODO

Destructive Potential

TODO

Stress

A power structure's stress refers to its inherent tendencies towards conflict versus cooperation. It can be approximated by considering all Nash equilibria power structures at the next time step, and then computing the average change in power for all agents in those next states.

```
Stress[s_, T_] := Mean[Map[(Total[UpdateS[#]]-Total[s]) / Total[s] &, NashEquilibria[s,T]]]
```

A negative result indicates destructive tendencies; a positive value suggests that there are opportunities for growth latent in the power structure. If there are no Nash equilibria, stress equals 0 by convention. Nash equilibria are based on the following additional assumptions: (1) only ternary tactic vectors are considered; (2) agents cannot alter more than one relationship per move (a kind of social inertia); (3) pairwise relationships are defined by the minimum value of the two agents' individual tactics; and (4) agents seek to maximize PrinceRank.

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Abstract Results

How does the quantitative realist model behave in the abstract, separate from any connection to real world phenomena? That is, when we run simulations under various conditions, what kinds of things tend to happen?

The results of investigations into these questions are organized as follows:

- Types of Moves
- Game Dynamics
- Stability and Polarity
- Effects of the Parameters
- Four Maxims of Strategy

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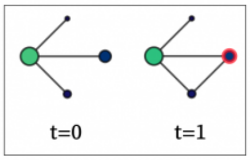
Types of Moves

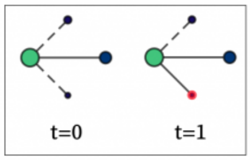
Moves can be interpreted as falling into different categories, depending on the context in which they occur. These categories are not mutually exclusive and any given move can have more than one of the interpretations below.

Constructive Moves

Constructive moves can serve two purposes: to **centralize** the focal agent's position at the hub of the network, or to **appease** another agent that is attacking it.



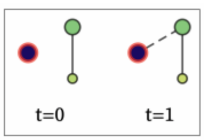
Centralizing



Appeasing

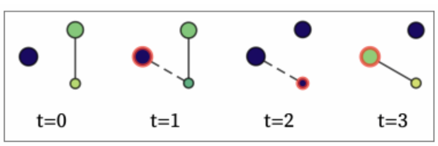
Destructive Moves

Destructive moves fall into three general categories. The first, and most straightforward, is when the primary purpose is to diminish or **reduce** the power of a competitor.



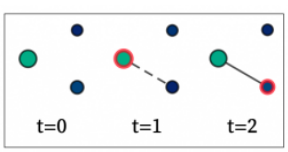
Reducing

The second is a **punishing** move used to get another player to alter its relationship with a third party. For example, in the sequence below, the large agent gives the small one a hard incentive to switch allegiance.



Punishing

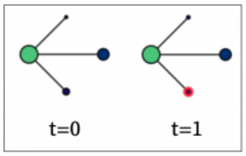
The third is a **softening** move designed to elicit appeasement.



Softening

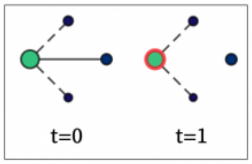
Other

Other common types of moves include **status quo** moves, in which the focal player decides that no change is needed to the power structure.



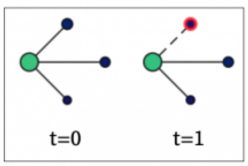
Status quo

Players also sometimes change a relationship to neutral in order to **redirect** their power into other relationships.



Redirecting

And sometimes they simply make a mistake or **blunder**, such as by attacking a stronger player (perhaps due to a random minimax game tree suggesting that other players might join in a rebellion).



Blunder

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Game Dynamics

What happens in a given simulation depends upon the initial power structure, the parameters chosen, and the random move order generated. Consequently, a wide variety of behavior can emerge. Despite this variation, general principles can be distilled that informally characterize the dynamic tendencies that arise from the axioms of quantitative realism. The following dynamics are generally observed.

Contents

Agents attack near rivals but otherwise cooperate, even if on unequal terms.

TODO

Agents prefer to form hierarchies in which they are the hub.

TODO

In a unipolar system, the hegemon pursues hierarchy while other agents try to unify as a counterbalance.

In a unipolar system, the essential struggle is between the large agent, which wants to form a hierarchy, and the smaller agents, which want to unify with each other as a counterbalance...

In bipolar systems, the great powers compete to control tributaries.

TODO

Power structures are in near-perpetual disequilibrium.

TODO

Four Maxims of Strategy

There are four ancient maxims of political behavior, first suggested by the *Arthashastra*, a 4th century BC Sanskrit treatise on statecraft. These maxims are:

> *The enemy of my enemy is my friend.*
> *The enemy of my friend is my enemy.*
> *The friend of my enemy is my enemy.*
> *The friend of my friend is my friend.*

Though far from iron laws of politics, these principles are generally accepted and common sensical. They also form the basis of a well-known sociological theory called *structural balance theory*. The questions we want to answer are:

1. Does quantitative realism provides a justification for these maxims? If so, that would mean that quantitative realism ultimately underpins structural balance theory.
2. If quantitative realism is not the underlying rationale for the maxims, why not? And is it at least consistent with them?

We first explain structural balance and its relationship to the maxims, and then investigate these questions.

Contents

Structural Balance Theory

Structural balance theory looks at agent triads in which all of the relationships are symmetric (and not neutral), and it makes assertions about which triads are balanced and which are not.

[image]

Structural balance theory has a variety of implications for larger social networks, predicting phenomena that have been observed among humans and animal communities. Here we explain how SBT is underpinned by certain strategic principles, which are in turn underpinned by quantitative realism.

These maxims explain why two of the triads above are considered balanced, and the other two unbalanced. In the balanced triads, the maxims are satisfied for every agent, and none of the maxims are violated. Conversely, in the unbalanced triads, none of the maxims are satisfied for any of the agents.

Quantitative Realism

Interpretation of the Maxims

Depicted as power structures from the perspective of the focal agent, the four maxims are:

[image]

Basis of the Maxims

Does quantitative realism provide the foundation for the four maxims?

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Historical Modeling

In this section, we apply quantitative realism to actual social processes and historical events and assess its usefulness. First we explore general empirical aspects of the theory:

- General Modeling Considerations
- Estimating National Power
- The Cost of Destruction
- Evidence of the Law of Motion

Then we apply the theory to specific events:

- Ancient China
- The Greco-Persian Wars
- The Roman Empire
- Fall of the Aztec Empire
- European Imperialism
- World War I
- World War II
- The Cold War

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General Modeling Considerations

Time and Data

Is it easier to model recent historical episodes, or ancient ones? For historical events that are far in the past, simpler technologies reduce the complexity of the situation. Moreover, we have have less of a vested interest in the interpretation of these events, making the essential power struggles more obvious. On the other hand, the farther back one goes in time, the less quantitative data is available to populate our power structure models, leaving us with a lot of guesswork as to what actually happened.

Subjectivity and Granularity

Historical situations can be characterized in various ways, making it debatable how particular individuals and coalitions should be grouped into abstract agents. This raises the question of the level of granularity at which the model should be applied. For example, in the Persian War (c. 500 BC), one might simplify the situation to be a case of two agents: Persia and Greece. Alternately, one might decompose Persia into its provinces, some of which rebelled during the conflict, and model the Greeks as being composed of a Spartan coalition and a Athenian one. One could go further and model each Greek city state as an agent. With more detail, one gets more nuanced and interesting political behavior, but one also begins to run into computational limits due to the exponential growth of the game tree.

Since models simplify reality, there are numerous ways to represent a real world situation within the model. A real world power structure, such as the United Nations, is composed of many interacting states and agencies. Representing each of these in an abstract power structure would be overly complex and burdensome. Decisions have to be made about which entities should be included and excluded from the model, and about how to group those entities into agglomerations. It is rarely clear cut how to do this, and the results of these choices may cause the outputs of the model to vary widely. This difficulty seems inherent to the endeavor, and can be managed only by being explicit about one's choices and assumptions.

Modeling Treaties

Treaties can relate to a wide variety of subjects, but here we will concern ourselves with those related to defense and trade. Defense-related treaties can often be represented as constraints on possible tactic matrices, as discussed in the context of institutions.

- Mutual defense pacts
- Non-aggression pacts
- Other defense cooperation agreements
- Trade agreements

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Estimating National Power

Over the last seventy years, there have been numerous efforts to quantify national power. Here we propose a definition that derives from the ideas of quantitative realism.

Contents

Definition

In quantitative realism, power is the ability to affect the power of another state either constructively or destructively. When applied to international relations, how does this correspond to real world quantities?

Constraints

There are several constraints this correspondence should satisfy:

1. The measured quantities should allow for *absolute increases* in power, since the model allows for absolute growth. This breaks with many definitions of national power, which are based on states' shares of the world total of some quantity, such as GDP.
2. Quantities that are combined into a composite metric should have *compatible units of measure*. The common method of using shares of world total sidesteps this problem, but due to #1, that is not available to us. We will use U.S. dollars baselined to some relevant year.
3. There should be a *coherence* between the quantities used for power in motion and power at rest. Power at rest is what is possessed by agents at a point in time; power in motion constitutes transactions of trade, exchange, and violence that occur over some interval of time. The various quantities used for these activities should be commensurable. Ideally, they would be based on some sort of accounting identity (https://en.wikipedia.org/wiki/Accounting_identity) that is true by definition.
4. Quantitative realism assumes an immediate *fungibility between constructive and destructive power*, in that each agent has a single bucket of power that can be used for either purpose. In the real world, this is obviously not the case. A state with immense wealth but no military would have a limited ability to apply destructive power. So when estimating expenditures of positive and negative power in the context of international relations, we should take into account each state's capacity for these two activities. The two primary dimensions of interest are overall economic strength and military capacity. While there are numerous indicia of positive and negative relations among states, we assume that destructive power is based primarily upon military capacity, and constructive power is based primarily upon trading capacity.

Definition

One formulation which meets the above criteria is to define national power in terms of **national wealth**. National wealth is a stock quantity, estimated at a point in time, and it has a standardized definition in the United Nations System of National Accounts, section 13.4 (2008) (https://unstats.un.org/unsd/nationalaccount/docs/SNA2008.pdf#page=315). By explicitly separating out the military and non-military components, we could define power at rest as:

$$s = m + c$$

where m is the value of a state's military assets and c is the value of its non-military (civilian) assets. The amount m represents the limit of a state's destructive capacity.

Destructive action could then be measured as state A's **military expenditure** on a particular conflict with state B over some time interval. In other words, state A expends some amount of its resources on violence towards state B, and state B's wealth is reduced accordingly. This expenditure, when multiplied by the destructive multiplier μ, would be equal to state B's reduction in wealth over that same time period, *ceteris parabus*. This reduction would have to be apportioned over state B's military and non-military assets, the simplest approach being to reduce each by the same factor. If we allow the parameter μ to vary by situation, which is reasonable due to technological and other differences, we will have established an accounting identity per constraint #3 above.

Constructive action could be measured in terms of the **trade volume** between states A and B over some time interval. Since for our purposes there is no qualitative difference between imports and exports, the average of the two should be used. One-way transfers of **foreign assistance** could also be included. When state A transfers this outgoing amount, it reduces A's national wealth accordingly, and increases the national wealth of state B by that amount times the constructive multiplier β. State B's military and non-military assets would increase by the same factor. This multiplier effect is arguably not instantaneous; state B's economy may need time to productively metabolize the new assets, thereby increasing its wealth, but this effect is presumed to occur within the relevant time interval. The parameter β can float to satisfy constraint #3.

What happens when a bilateral relationship entails simultaneous trade and conflict, the alternative formulation of the law of motion is apt. That way, when two nations engage in trade and conflict at the same time, the positive and negative effects partially cancel each other out.

One consequence of using wealth, military expenditure, and trade is that it obviates the need to arbitrarily set the self-allocation percentage ρ. Instead, the percentage of power allocated by a state will be

```
(total dyadic military expenditures + total trade volume) / national wealth.
```

The conception above could be extended to non-state actors: power at rest would correspond to wealth, destructive action would correspond to the amount of assets expended on violence, and constructive action would correspond to commercial activity.

Example

In 2019 the national wealth of the U.S. was approximately $92T (World Inequality Database (https://wid.world/data/)), military assets were worth around $2.9T (Dept. of Defense Financial Report 2019, p. 34 (https://comptroller.defense.gov/Portals/45/Documents/afr/fy2019/DoD_FY19_Agency_Financial_Report.pdf)), and the average of imports plus exports was $2.8T (Census Dept. (https://www.census.gov/foreign-trade/Press-Release/current_press_release/exh1.pdf)). If all military assets had been directed towards a single conflict, it would represent an expenditure of 3% of national power. Since not all assets can be deployed toward a single enemy, this figure is an upper ceiling, with less than 1% of national power being a more likely magnitude for a dyadic conflict. In this example, trade flows constitute a constructive transfer of 3% of national power.

Availability of Data

Given the above, the data we need (by year) is:

- **National wealth**, preferably separated into military and non-military components
- **Military expenditure** per dyadic conflict
- Dyadic **trade volume** and **foreign assistance**

Unfortunately, the availability of this data is rather limited at present. Currently known sources include:

Variable	Source	Data set	Years	Units
National wealth	World Inequality Lab	World Inequality Dataset (https://wid.world/data/)	Varies by country	2019 USD
Military expenditure				
Trade volume	CEPII	TRADEHIST trade history database (http://www.cepii.fr/CEPII/en/bdd_modele/presentation.asp?id=32)	1827-1970	Current GBP
Trade volume	Correlates of War	Trade data set v4.0 (https://correlatesofwar.org/data-sets/bilateral-trade)	1870-2014	Current USD (millions)
Foreign assistance	David Roodman	Net Aid Transfers data set (https://davidroodman.com/data/)	1960-2016	Current USD (millions)

If the variables listed above are indeed the real world manifestations of power in international relations, then it will be challenging to demonstrate the soundness of that interpretation until the historical time series for that data is more fully developed. In the meantime, our models will have to rely upon proxy data and reasonable methods of estimation, seeking approximations rather than exact quantities. The approach in each case will depend upon what data is available for the time period at issue: for example, modeling the Peloponnesian War will require different sources and techniques than, say, World War I.

Methods of Approximation

How can we make back-of-the-envelope approximations of national wealth, military expenditure, bilateral trade, and foreign assistance, using general data suggestive of national power?

Estimating national wealth from GDP

Since GDP data is plentiful for the postwar era, we can estimate national wealth from it using world average wealth-income ratios from Piketty (2014) and Bauluz (2019), Fig. 3. For example, to estimate the national wealth of Argentina in 1970, we would multiply its GDP of 31.6B (1970 USD) by 3.5, the average capital-income ratio at the time, to get 110.6B (1970 USD). This is obviously a very rough approximation, almost certainly subject to future revision, but it should enable us to tinker with various historical scenarios.

Other

- Baselining military expenditure as percentage of national wealth
- Baselining trade volume and foreign assistance as percentage of national wealth

The Cost of Destruction

In this section, we ask: How much does destruction cost? That is, if you had \$100 of capital or power, what is the value of the capital you could destroy with it? Various ways of answering this question offer baseline values of the destructive multiplier μ.

TODO

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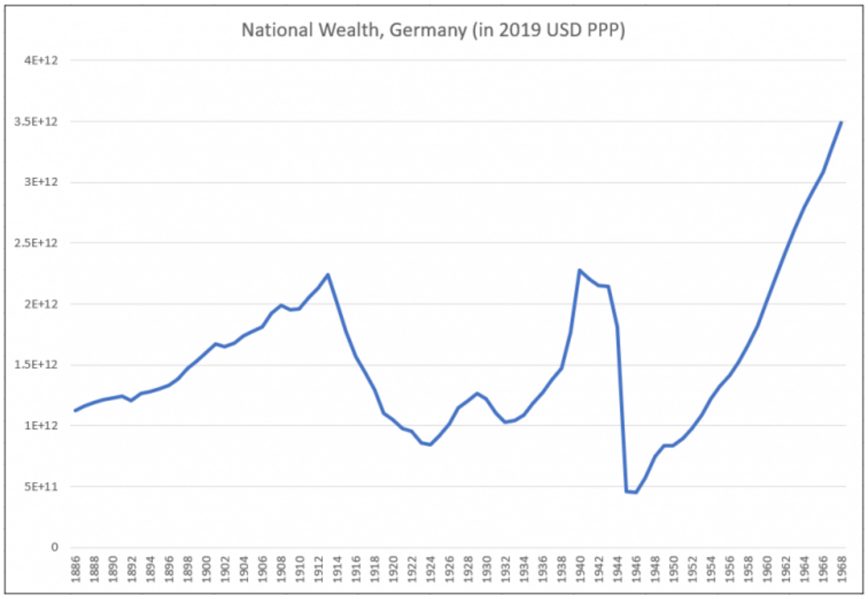
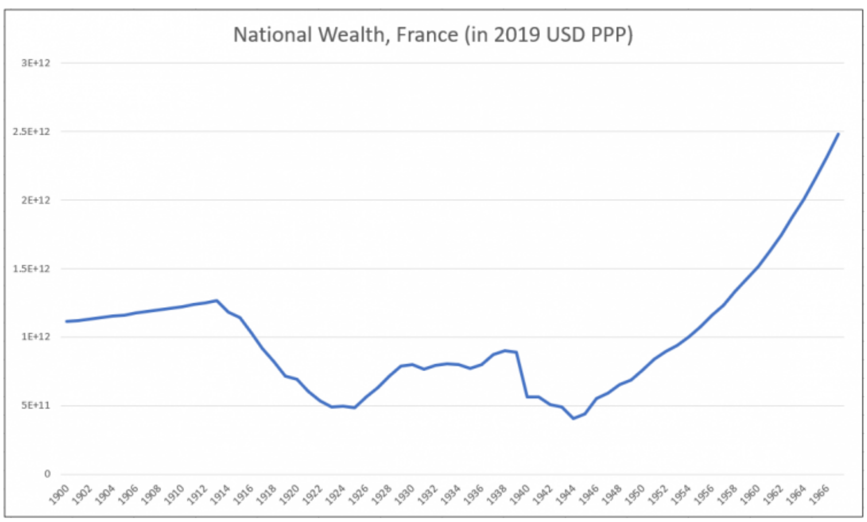
Evidence of the Law of Motion

Changes in National Wealth Due to War

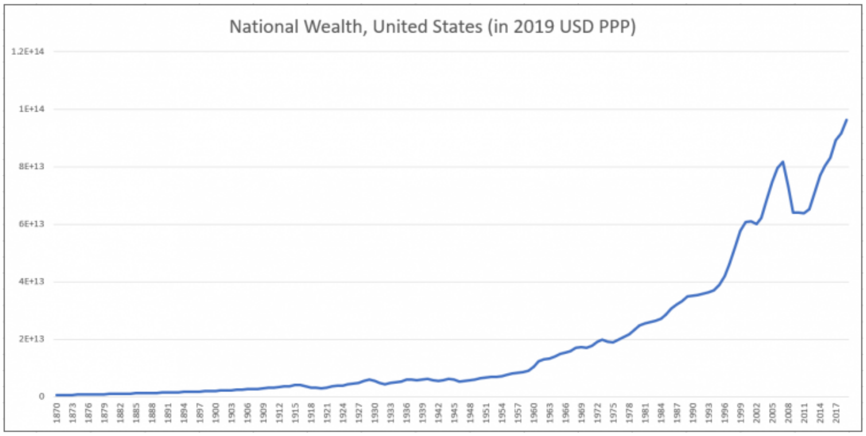
Some initial evidence for the law of motion can be found in historical time series of national wealth, in which major destructive events tend to leave their mark. Consider the following chart of German national wealth from 1886 to 1968. It features two major dips coinciding with each of the World Wars, 1914-1918 and 1939-1945, when German strength was severely weakened due to the combat. Germany's wealth plummeted rapidly around 1943-1944, after the Soviet Union turned the tide at Stalingrad and the other Allies opened up a second front in the west.



A similar patterns holds for France, which also suffered devastation during those wars:



But obviously not every decrease in national wealth can be attributed to war. For example, in the chart below of U.S. national wealth from 1870 to 2017, the major dip occurring from 2008 to 2013 was due to financial collapse rather than international conflict. There are dips coinciding with the World Wars, but these are minor, reflecting the fact that the war was for the most part not fought on U.S. soil.



Though more impressionistic than dispositive, these time series suggest the almost tautological observation that the application of destructive power tends to decrease the recipient's power, as measured by national wealth.

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World War II

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Interpretation

World War II was a global conflagration involving dozens of countries, hundreds of battles, 100 million military personnel, and at least 70 million fatalities, an astonishing 50 million of whom were civilians. Its causes are still debated and the forces that propelled it are varied and complex. It was one of the defining events of the 20th century, affecting virtually everyone alive at that time and leaving an aftermath of international relationships many of which persist to this day. Any model that we create to interpret this eruption of human history will necessarily be a drastic oversimplification.

We make our model as simple as possible, including only five entities — the U.S., the British Empire, the Soviet Union, Germany, and Japan — and trying to interpret their interactions in light of quantitative realism. While obviously many other countries were involved, these five were the among most powerful and exercised the greatest agency in determining the course of events. We assume that relationships are always reciprocal, leaving only 10 to keep track of. We omit the geographic realities and the various theaters of operations, focusing instead on the belligerents' relative power levels, alliance structure, and periods of hostility.

To estimate the strength of the parties, we use the Composite Index of National Capability (CINC) as calculated by the Correlates of War Project. To estimate destructive power, we use data from the Correlates of War Project's database of Militarized Interstate Disputes. These disputes are coded via the *hihosta* variable using the numbers 1-5, with 5 representing war and 4 representing a variety of aggressive and destructive actions (MID 5.0 Codebook (https://correlatesofwar.org/data-sets/MIDs)). We use values of 4 and 5, as explained further below, but not 1-3. This data set indicates the start and end date of the various hostility levels.

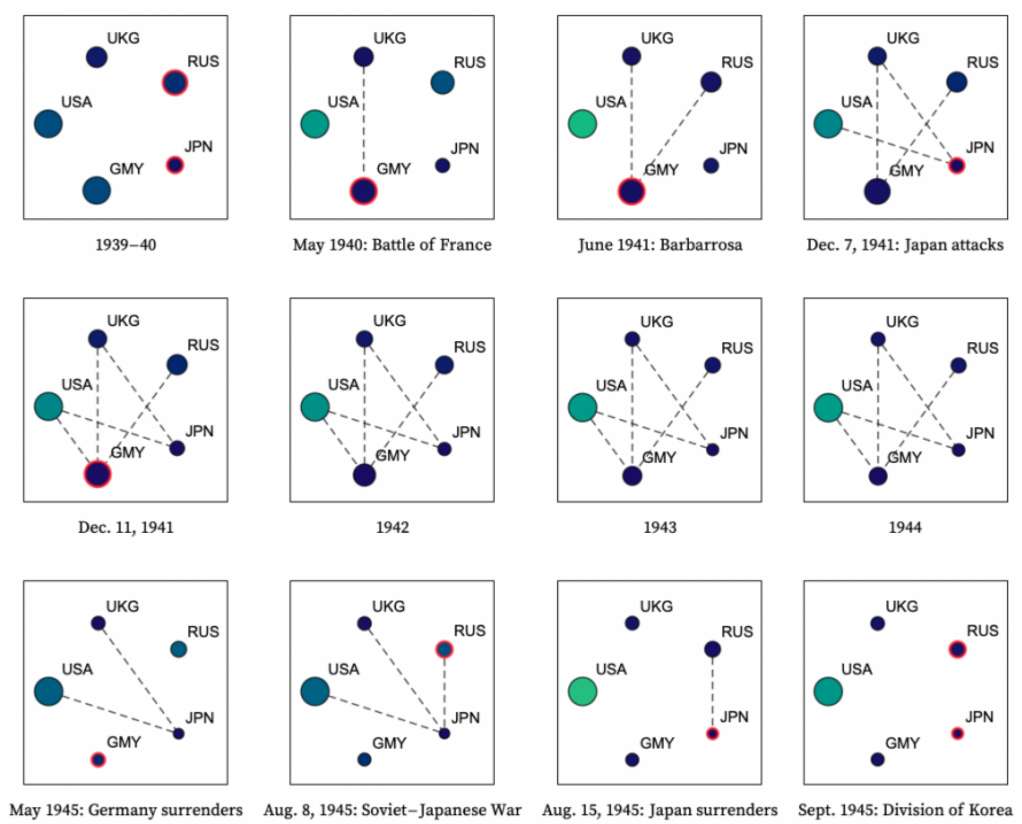
- Data for cooperation
- Parameters used

Sequence of Events

Setting aside for the moment the effect of cooperation and considering only destructive transfers of power, the following timeline summarizes the key events of the war:

Date	Caption	Relationships changed	Event
5/10/1940	May 1940	{GMY,UKG} → -1	Beginning of the Battle of France (https://en.wikipedia.org/wiki/British_Expeditionary_Force_(World_War_II))
6/22/1941	June 1941	{GMY,RUS} → -1	Germany attacks Russia (https://en.wikipedia.org/wiki/World_War_II#Axis_attack_on_the_Soviet_Union_(1941))
12/7/1941	Dec. 7, 1941	{JPN,USA} → -1, {JPN,UKG} → -1	Japan attacks UK and US (https://en.wikipedia.org/wiki/World_War_II#War_breaks_out_in_the_Pacific_(1941))
12/11/1941	Dec. 11, 1941	{GMY,USA} → -1	Germany declares war on US (https://en.wikipedia.org/wiki/World_War_II#War_breaks_out_in_the_Pacific_(1941))
5/7/1945	May 1945	{GMY,UKG} → 0, {GMY,RUS} → 0, {GMY,USA} → 0	Germany surrenders (https://en.wikipedia.org/wiki/World_War_II#Axis_collapse,_Allied_victory_(1944%E2%80%9345))
8/8/1945	Aug. 8, 1945	{RUS,JPN} → -1	Soviet-Japanese War (https://en.wikipedia.org/wiki/Soviet%E2%80%93Japanese_War)
8/15/1945	Aug. 15, 1945	{JPN,USA} → 0, {JPN,UKG} → 0	Japan surrenders (https://en.wikipedia.org/wiki/Surrender_of_Japan)
9/15/1945	Sept. 1945	{RUS,JPN} → 0	Division of Korea (https://en.wikipedia.org/wiki/Division_of_Korea#World_War_II)

We can visualize this as a move sequence, with the countries placed in a circular arrangement rather than geographically:



1939–40 | May 1940: Battle of France | June 1941: Barbarrosa | Dec. 7, 1941: Japan attacks

Dec. 11, 1941 | 1942 | 1943 | 1944

May 1945: Germany surrenders | Aug. 8, 1945: Soviet–Japanese War | Aug. 15, 1945: Japan surrenders | Sept. 1945: Division of Korea

The Law of Motion

Key Decision Points

Appeasement vs Resistance of German Aggression

German Prioritization of UK vs Soviets

- Germany underestimated Russian military and economic strength, for example, expecting 200 Russian divisions and then encountering over 360 (Shirer pp. 1021-22).

Soviet Alliance with Germany vs the U.S. and UK

In August 1939, The Soviet Union signed a nonaggression pact with Germany. As Shirer 1959 notes (p. 656-7):

> [T]here might not have been any war at all if Hitler had known he must take on Russia as well as Poland, England and France. Even the politically timid Germany generals, if one can judge from their later testimony at Nuremberg, might have put their foot down against embarking on a war against such a formidable coalition.

We have omitted Poland and France from our calculations, but let's consider what would have happened if the Soviets had instead made a pact with the British to counter German aggression together:

[image]

Timing of Japanese Provocation of U.S. and UK

- Hitler underestimated the strength of the U.S. and overestimated that of Japan (Shirer p. 1069).

U.S. Prioritization of Japan vs Germany

The World Power Structure

Data and Methodology

TODO

Trends 1980-2020

TODO

Tensions in the System

TODO

<< Prev | Next >>

Tentative Conclusions

Recap

TODO

The Meaning of this Work

TODO

Realism and Idealism

TODO

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References

A

- Ackerman, Peter and Jack DuVall. *A Force More Powerful: A Century of Nonviolent Conflict*. Palgrave Macmillan, 2001.
- Allison, Graham. *Destined for War: Can America and China Escape Thucydides's Trap?* Houghton Mifflin Harcourt, 2017.
- Almog, Assaf, et al. *Enhanced Gravity Model of Trade: Reconciling Macroeconomic and Network Models* (https://www.frontiersin.org/article/10.3389/fphy.2019.00055). Frontiers in Physics. Vol. 7, 2019.
- Anderton, Charles H. and John R. Carter. *Principles of Conflict Economics: A Primer for Social Scientists*. Cambridge University Press, 2009.
- Aristotle. *The Politics*. Penguin Classics, 1981.
- Art, Robert J. and Robert L. Jervis. *International Politics: Anarchy, Force, Political Economy, and Decision Making*. Little, Brown, 1985.
- Asimov, Isaac. *Foundation*. Bantam Spectra Books, 1991.
- Asimov, Isaac. *Prelude to Foundation*. Spectra, 2012.
- Axelrod, Robert. *The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration*. Princeton University Press, 1997.
- Axelrod, Robert. *A Model of the Emergence of New Political Actors* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/Political%20Science%20and%20Sociology/Simulation/Building%20New%20Political%20Actors%20-%20Axelrod.pdf), in Nigel Gilbert and Rosaria Conte (eds.), *Artificial Societies: the Computer Simulation of Social Life*. London: University College Press, 1995.
- Axelrod, Robert and D. Scott Bennett. *A Landscape Theory of Aggregation* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Quantitative%20IR/Axelrod%20-%20A%20Landscape%20Theory%20of%20Aggregation.pdf). British Journal of Political Science, Vol. 23, No. 2, pp. 211-233, 1993.
- Axelrod, Robert. *The Evolution of Cooperation*. Basic Books, 1984.

B

- Barbieri, Katherine, Omar M. G. Keshk, and Brian Pollins. *TRADING DATA: Evaluating our Assumptions and Coding Rules*. Conflict Management and Peace Science. 26(5): 471-491, 2009. Version 4.0.
- Barbieri, Katherine and Jack S. Levy. *Does War Impede Trade? A Response to Anderton and Carter* (https://fas-polisci.rutgers.edu/levy/articles/2001%20Does%20War%20Impede%20Trade.pdf). Journal of Peace Research, Vol. 38, No. 5, 2001.
- Barbieri, Katherine and Jack S. Levy. *Sleeping with the Enemy: The Impact of War on Trade* (https://www.semanticscholar.org/paper/Sleeping-with-the-Enemy%3A-The-Impact-of-War-on-Trade-Barbieri-Levy/8b2ad4013878dbe9a953f417ebcf1acac05a6283?p2df). Journal of Peace Research (1999).
- Barnett, Michael and Raymond Duvall. *Power in International Politics* (https://www.jstor.org/stable/3877878?seq=1). International Organization, Vol. 59, No. 1 (Winter 2005), pp. 39-75.
- Bates, Robert. *Prosperity and Violence: The Political Economy of Development*. W. W. Norton & Company, 2009.
- Battacharya, K. et al. *The International Trade Network: Weighted Network Analysis and Modelling* (https://arxiv.org/abs/0707.4343). J. Stat. Mech. (2008).
- Batty, Michael. *The New Science of Cities*. MIT Press, 2013.

- Bauluz, Luis. *Revised National Income and Wealth Series: Australia, Canada, France, Germany, Italy, Japan, UK, and USA* (https://wid.world/document/revised-national-income-wealth-australia-canada/). WID.world Working Paper No. 2017/23 (2019).
- Berle, Adolf A. *Power*. Harcourt, 1969.
- Berlin, Isaiah. *Concepts and Categories*. Princeton University Press, 1999.
- Black, Edwin. *IBM and the Holocaust: The Strategic Alliance Between Nazi Germany and America's Most Powerful Corporation*. Dialog Press, 2012.
- Bonacich, Phillip. *Introduction to Mathematical Sociology*. Princeton University Press, 2012.
- Bonacich, Phillip. *Power and Centrality: A Family of Measures* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/Network%20Theory%20and%20the%20Economics%20of%20Networks/Bonacich%20-%20Power%20and%20Centrality.pdf). American Journal of Sociology, Vol. 92, No. 5 (Mar., 1987), pp. 1170-1182.
- Boo, Katherine. *Behind the Beautiful Forevers: Life, Death, and Hope in a Mumbai Undercity*. Random House, 2014.
- Boot, Max. *The Savage Wars of Peace: Small Wars and the Rise of American Power*. Basic Books, 2014.
- Bramoullé, Yann, Andrea Galeotti, and Brian Rogers. *The Oxford Handbook of the Economics of Networks*. Oxford University Press, 2016.
- Braudel, Fernand. *The Mediterranean and the Mediterranean World in the Age of Philip II*. University of California Press, 1996.
- Braudel, Fernand. *On History*. University of Chicago Press, 1982.
- Brown, Vincent. *Tacky's Revolt: The Story of an Atlantic Slave War*. Belknap Press, 2020.
- Bueno de Mesquita, Bruce, and David Lalman. *War and Reason: Domestic and International Imperatives*. Yale University Press, 1994.
- Bueno de Mesquita, Bruce, et al. *The Logic of Political Survival*. MIT Press, 2004.
- Bueno de Mesquita, Bruce and Alastair Smith. *The Dictator's Handbook: Why Bad Behavior is Almost Always Good Politics*. PublicAffairs, 2012.

C

- Caro, Robert A. *The Years of Lyndon Johnson: Master of the Senate*. Vintage, 2003.
- Caro, Robert A. *The Power Broker: Robert Moses and the Fall of New York*. Vintage, 1975.
- Carr, E.H. *The Twenty Years' Crisis: 1919-1939: An Introduction to the Study of International Relations*. Harper Perennial, 1964.
- Case, James. *Competition: The Birth of a New Science*. Hill and Wang, 2008.
- Césaire, Aimé. *Discourse on Colonialism*. Monthly Review Press, 2001.
- Cederman, Lars-Erik. *Emergent Actors in World Politics: How States & Nations Develop & Dissolve*. Princeton University Press, 1997.
- Chagnon, Napoleon A. *Noble Savages: My Life Among Two Dangerous Tribes -- the Yanomamo and the Anthropologists*. Simon & Schuster, 2014.
- Chang, Chin-Lung. *A Measure of National Power* (https://www.analytickecentrum.cz/upload/soubor/original/measure-power.pdf).
- Chomsky, Noam. *Manufacturing Consent: The Political Economy of the Mass Media*. Pantheon, 2002.
- Chomsky, Noam. *Understanding Power: The Indispensable Chomsky*. The New Press, 2002.
- Churchman, David. *Why We Fight: The Origins, Nature, and Management of Human Conflict*. UPA, 2013.

- Cioffi-Revilla, Claudio. *The Long-Range Analysis of War*. Journal of Interdisciplinary History, Vol. 21, No. 4 (Spring 1991), pp. 603-629.
- Clark, Christopher. *The Sleepwalkers: How Europe Went to War in 1914*. Harper Perennial, 2014.
- Clausewitz, Carl von. *On War*. Penguin Classics, 1968.
- Cline, Eric H. *1177 BC: The Year Civilization Collapsed*. Princeton University Press, 2015.
- Coll, Steve. *Directorate S: The CIA and America's Secret Wars in Afghanistan and Pakistan*. Penguin Books, 2019.
- Coquide, Celestin, et al. *Crisis Contagion in the World Trade Network* (https://arxiv.org/abs/2002.07100). Appl Netw Sci 5, 67 (2020).
- Coulomb, Fanny. *Economic Theories of Peace and War*. Routledge, 2004.

D

- Daggett, Stephen. *Costs of Major U.S. Wars* (https://fas.org/sgp/crs/natsec/RS22926.pdf), Congressional Research Service 7-5700, 2010.
- Dale, Stephen Frederic. *The Orange Trees of Marrakesh: Ibn Khaldun and the Science of Man*. Harvard University Press, 2015.
- Dalrymple, William. *The Anarchy: The Relentless Rise of the East India Company*. Bloomsbury Publishing, 2019.
- Davies, Norman. *Europe: A History*. Harper Perennial, 1998.
- De Benedictis, Luca, and Lucia Tajoli. *The World Trade Network*. The World Economy, 2011, vol. 34, issue 8, 1417-1454.
- de Jouvenel, Bertrand. *On Power: The Natural History of Its Growth*. Liberty Fund, Inc., 1993.
- de Waal, Frans. *Chimpanzee Politics: Power and Sex among Apes*. Johns Hopkins University Press, 2007.
- Debeck, Charles. *The Correlates of Cyber Warfare: A database for the modern era* (https://lib.dr.iastate.edu/etd/12062). Graduate Theses and Dissertations, 12062, 2011.
- Demange, Gabrielle and Myrna Wooders. *Group Formation in Economics: Networks, Clubs, and Coalitions*. Cambridge University Press, 2012.
- Deutsch, Karl W. *The Analysis of International Relations*. Prentice-Hall, 1968.
- Diamond, Jared. *Collapse: How Societies Choose to Fail or Succeed*. Penguin Books, 2011.
- Diamond, Jared. *Guns, Germs, and Steel: The Fates of Human Societies*. W. W. Norton & Company, 2017.
- DiAngelo, Robin. *White Fragility: Why It's So Hard for White People to Talk about Racism*. Beacon Press, 2018.
- Dickson, Keith. *World War II for Dummies*. For Dummies, 2020.
- Domhoff, William G. *The Corporate Rich and the Power Elite in the Twentieth Century*. Routledge, 2017.
- Domhoff, William G. *Studying the Power Elite: Fifty Years of Who Rules America?*. Routledge, 2019.
- Douglass, Frederick. *My Bondage and My Freedom*. Penguin Classics, 2003.
- Doyle, Michael W. *Empires*. Cornell University Press, 1986.

E

- Easley, David and Jon Kleinberg. *Networks, Crowds, and Markets: Reasoning about a Highly Connected World*. Cambridge University Press, 2010.
- East, W. Gordon. *The Geography Behind History*. W. W. Norton & Company, 1999.
- Elkins, Caroline. *Imperial Reckoning: The Untold Story of Britain's Gulag in Kenya*. Holt Paperbacks, 2005.

- Ellsberg, Daniel. *The Doomsday Machine: Confessions of a Nuclear War Planner*. Bloomsbury Publishing, 2018.
- Erickson, Paul. *The World the Game Theorists Made*. University of Chicago Press, 2015.
- Ermann, Leonardo and Dima L. Shepelyansky. *Google Matrix of the World Trade Network* (https://arxiv.org/abs/1103.5027). Acta Physica Polonica A, vol. 120 (6A), A-158 (2011).
- Evans, Richard J. *In Defense of History*. W. W. Norton & Company, 2000.

F

- Fagiolo, G., Reyes, J. & Schiavo, S. *The Evolution of the World Trade Web: a Weighted-Network Analysis* (https://doi.org/10.1007/s00191-009-0160-x). J Evol Econ 20, 479–514 (2010).
- Fanon, Frantz. *Wretched of the Earth*. Grove Press, 2005.
- Ferguson, Niall. *The Square and the Tower: Networks, Hierarchies, and the Struggle for Global Power*. Pengin Books, 2019.
- Ferguson, Yale H. and Richard W. Mansbach. *The Elusive Quest Continues: Theory and Global Politics*. Prentice Hall, 2002.
- Flint, Colin and Peter Taylor. *Political Geography: World-Economy, Nation-State and Locality*. Prentice Hall, 1999.
- Fogel, David. *Blondie24: Playing at the Edge of AI*. Morgan Kaufmann, 2001.
- Fossey, Dian. *Gorillas in the Mist*. Mariner Books, 2000.
- Foucault, Michel. *Power/Knowledge: Selected Interviews & Other Writings, 1972-1977*. Vintage Books, 1980.
- Freeden, Michael. *Ideology: A Very Short Introduction*. Oxford University Press, 2003.
- Friedkin, Noah E. *Theoretical Foundations for Centrality Measures* (http://core.ecu.edu/biol/luczkovichj/biocomplexity/Friedkin.pdf). AJS Volume 96, No. 6 (May 1991): 1478-1504.
- Frijters, Paul and Gigi Foster. *An Economic Theory of Greed, Love, Groups, and Networks*. Cambridge University Press, 2013.
- Fukuyama, Francis. *Political Order and Political Decay: From the Industrial Revolution to the Globalization of Democracy*. Farrar, Straus and Giroux, 2015.
- Fukuyama, Francis. *The Origins of Political Order*. Farrar, Straus and Giroux, 2012.
- Fukuyama, Francis. *The End of History and the Last Man*. Free Press, 2006.

G

- Galbraith, John Kenneth. *The Anatomy of Power*. Houghton Mifflin Harcourt, 1983.
- Galbusera, Luca and Georgios Giannopoulos. *On input-output economic models in disaster impact assessment* (https://www.sciencedirect.com/science/article/pii/S2212420918305521). International Journal of Disaster Risk Reduction. Volume 30, Part B, September 2018, Pages 186-198.
- Garcia-Algarra, Javier, et al. *A Stochastic Generative Model of the World Trade Network* (https://www.nature.com/articles/s41598-019-54979-1). Scientific Reports volume 9, Article number: 18539 (2019).
- Garlaschelli, D. et al. *Interplay between Topology and Dynamics in the World Trade Web* (https://arxiv.org/abs/physics/0701030). Eur. Phys. J. B 57,159-164 (2007).
- Garlaschelli D. and M.I. Loffredo. *Structure and Evolution of the World Trade Network* (https://arxiv.org/abs/physics/0502066). Physica A 355/1, 138-144 (2005).
- Gewen, Barry. *The Inevitability of Tragedy: Henry Kissinger and His World*. W. W. Norton & Company, 2020.
- Gibbon, Edward. *The Decline and Fall of the Roman Empire*. Modern Library, 2003.

- Gibler, Douglas M. *International military alliances, 1648-2008*. CQ Press, 2009. Version 4.1.
- Gilpin, Robert. *War and Change in World Politics*. Cambridge University Press, 1983.
- Gilpin, Robert. *Global Political Economy: Understanding the International Economic Order*. Princeton University Press, 2001.
- Gintis, Herbert. *The Bounds of Reason: Game Theory and the Unification of the Behavioral Sciences*. Princeton University Press, 2014.
- Glaser, Charles. *Rational Theory of International Politics: The Logic of Competition and Cooperation*. Princeton University Press, 2010.
- Glaser, Charles and Chaim Kaufmann. *What is the offense-defense balance and can we measure it?* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Quantitative%20IR/Glaser%20-%20Offense-Defense%20Balance.pdf) International Security, Spring 1998, Vol. 22, No. 4.
- Glick, Reuven and Alan M. Taylor. *Collateral Damage: Trade Disruption and the Economic Impact of War* (https://www.frbsf.org/economic-research/files/wp05-11bk.pdf). Federal Reserve Bank of San Francisco, Working Paper 2005-11, 2005.
- Goldsmith, Raymond W. *The National Balance Sheet of the United States of America* (http://piketty.pse.ens.fr/files/Goldsmith1955.pdf). Income and Wealth, Series IV (1955).
- Gordon, Deborah. *Ants at Work: How an Insect Society is Organized*. Free Press, 2011.
- Graeber, David. *Debt: The First 5,000 Years*. Melville House, 2012.
- Green, Anna and Kathleen Troup. *The Houses of History: A Critical Reader in 20th Century History and Theory*. New York University Press, 1999.
- Groen, Andrew. *Empires of EVE: A History of the Great Wars of EVE Online*.

H

- Harper, Kyle. *The Fate of Rome: Climate, Disease, and the End of an Empire*. Princeton University Press, 2019.
- Hastings, Max. *Winston's War: Churchill 1940-1945*. Vintage, 2010.
- Hedges, Chris. *War is a Force that Gives Us Meaning*. PublicAffairs, 2014.
- Hegel, Georg Wilhelm Friedrich. *Introduction to the Philosophy of History*. Hackett Publishing Co., 1988.
- Herodotus and Robert Strassler. *The Landmark Herodotus: The Histories*. Anchor Books, 2009.
- Hibbing, John R., Kevin B. Smith, and John R. Alford. *Predisposed: Liberals, Conservatives, and the Biology of Political Differences*. Routledge, 2013.
- Hirschman, Albert O. *National Power and the Structure of Foreign Trade*. University of California Press, 1981.
- Hobbes, Thomas. *Leviathan*. Penguin Classics, 1982.
- Hokkanen, George, et al. *Structure and Properties of Trade Networks* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/World%20Trade%20Modeling/Hokkanen%20-%20Structure%20and%20Properties%20of%20Trade%20Networks.pdf). 2012.
- Homer-Dixon, Thomas F. *Environment, Scarcity, and Violence*. Princeton University Press, 2001.
- Hughes-Warrington, Marnie. *Fifty Key Thinkers on History*. Routledge, 2014.
- Huremovic, Kenan. *Rent Seeking and Power Hierarchies: A Noncooperative Model of Network Formation with Antagonistic Links* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/Network%20Theory%20and%20the%20Economics%20of%20Networks/Huremovic%20-%20Rent%20Seeking%20and%20Power%20Hierarchies.pdf). FEEM Working Paper No. 45.2014, 2014.

I

- Isaacson, Walter. *Kissinger: A Biography*. Simon & Schuster, 2005.
- Ibn Khaldun. *The Muqaddimah: An Introduction to History*. Princeton University Press, 2015.
- Immerwahr, Daniel. *How to Hide an Empire: A History of the Greater United States*. Farrar, Straus and Giroux, 2019.

J

- Jackson, Matthew O. *Social and Economic Networks*. Princeton University Press, 2010.
- Jang, Yeongkyun and Jae-Suk Yang. *Network Analysis of the US-China Hegemonic Transition* (https://ssrn.com/abstract=3535056) (2020).
- Jennings, Francis. *The Ambiguous Iroquois Empire*. Norton, 2013.
- Jervis, Robert. *Hans Morgenthau, Realism, and the Scientific Study of International Politics*. Social Research, Vol. 61, No. 4, 1994.
- Josephus. *The Jewish Wars*. Penguin Classics, 1984.

K

- Kagan, Donald. *The Fall of the Athenian Empire*. Cornell University Press, 1991.
- Kagan, Donald. *The Peloponnesian War*. Penguin Books, 2004.
- Kalyvas, Stathis N. *The Logic of Violence in Civil War*. Cambridge University Press, 2006.
- Kaplan, Morton. *System and Process in International Politics*. Wiley, 1957.
- Kaur, Mankirat. *Analyzing Negative Ties in Social Networks: A Survey* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/Network%20Theory%20and%20the%20Economics%20of%20Networks/Kaur%20-%20Analyzing%20Negative%20Ties%20in%20Social%20Networks.pdf). Egyptian Informatics Journal, Volume 17, Issue 1, March 2016, Pages 21-43.
- Keay, John. *China: A History*. Basic Books, 2011.
- Keltner, Dacher. *The Power Paradox: How We Gain and Lose Influence*. Penguin Books, 2017.
- Keohane, Robert. *Neorealism and Its Critics*. Columbia University Press, 1986.
- Keohane, Robert. *After Hegemony: Cooperation and Discord in the World Political Economy*. Princeton University Press, 2005.
- Kershaw, Ian. *Hitler: A Biography*. W. W. Norton & Company, 2010.
- Khanna, Parag. *Connectography: Mapping the Future of Global Civilization*. Random House, 2016.
- Kissinger, Henry. *Diplomacy*. Simon & Schuster, 1995.
- Kissinger, Henry. *On China*. Penguin Books, 2012.
- Kissinger, Henry. *World Order*. Penguin Books, 2015.
- Klare, Michael. *Resource Wars: The New Landscape of Global Conflict*. Holt Paperbacks, 2002.
- Kinne, Brandon J. *The Defense Cooperation Dataset (DCAD)*. The Journal of Conflict Resolution (forthcoming).
- Krakauer, David C., John Gaddis, and Kenneth Pomeranz. *History, Big History, and Metahistory*. CreateSpace Independent Publishing Platform, 2017.
- Krapivsky, P.L. and S. Redner. *Wealth Distributions in Asset Exchange Models* (https://arxiv.org/abs/1006.4595). 2010.
- Krasner, Stephen D. *Sovereignty: Organized Hypocrisy*. Princeton University Press, 1999.

L

- Laine, James W. *Meta-Religion: Religion and Power in World History*. University of California Press, 2015.
- Lee, Alexander. *Machiavelli: His Life and Times*. Picador, 2020.
- Lepenies, Philipp. *The Power of a Single Number: A Political History of GDP*. Columbia University Press, 2016,
- Liska, George. *The Ways of Power: Pattern and Meaning in World Politics*. Blackwell Publishers, 1990.
- Luo, Shaowen and Tsang, Kwok Ping. *China and World Output Impact of the Hubei Lockdown During the Coronavirus Outbreak* (https://ssrn.com/abstract=3543760) (April 24, 2020). Forthcoming, Contemporary Economic Policy.
- Luce, R. Duncan and Howard Raiffa. *Games and Decisions: Introduction and Critical Survey*. Dover Publications, 1989.
- Luttwak, Edward M. *The Grand Strategy of the Byzantine Empire*. Belknap Press, 2011.

M

- Machiavelli, Niccolò and Peter Constantine. *The Essential Writings of Machiavelli*. Modern Library, 2007.
- Madison, James. *The Federalist Papers*. Signet, 2003.
- Maestripieri, Dario. *Macachiavellian Intelligence: How Rhesus Macaques and Humans Have Conquered the World*. University of Chicago Press, 2007.
- MacQuarrie, Kim. *The Last Days of the Incas*. Simon & Schuster, 2008.
- Malcolm, Noel. *Kosovo: A Short History*. Harper Perennial, 1999.
- Malthus, Thomas. *An Essay on the Principle of Population and Other Writings*. Penguin Classics, 2015.
- Mandela, Nelson. *Long Walk to Freedom*. Back Bay Books, 1995.
- Mann, Charles. *1491: New Revelations about the Americas before Columbus*. Vintage Books, 2006.
- Mann, Charles. *1493: Uncovering the New World Columbus Created*. Vintage Books, 2012.
- Maoz, *Network Power* (https://www.researchgate.net/profile/Zeev_Maoz/publication/228906529_Network_Power/links/004635307670c93660000000.pdf) (2011).
- Maoz, *Network Centrality and International Conflict* (https://www.researchgate.net/profile/Ranan_Kuperman/publication/228562817_Network_Centrality_and_International_Conflict_1816-2001_Does_it_Pay_to_Be_Important/links/0c96052139b08bc2cd000000.pdf) (2004).
- Mayer, Jane. *Dark Money: The Hidden History of the Billionaires Behind the Rise of the Radical Right*. Anchor, 2017.
- Martin, Philippe, Thierry Mayer, and Mathias Thoenig. _Make Trade Not War_. Review of Economic Studies 75, 2008, pp. 865-900.
- Marshall, Peter. *Demanding the Impossible: A History of Anarchism*. PM Press, 2010.
- McCain, Roger. *Game Theory: A Nontechnical Introduction to the Analysis of Strategy*. World Scientific Publishing, 2014.
- McNeill, William H. *Plagues and Peoples*. Anchor, 1976.
- McNeill, William H. *Arnold J. Toynbee: A Life*. Oxford University Press, 1990.
- Mearsheimer, John. *The Tragedy of Great Power Politics*. W. W. Norton & Company, 2014.
- Mearsheimer, John. *The Great Delusion: Liberal Dreams and International Reality*. Yale University Press, 2018.
- Memmi, Albert. *The Colonizer and the Colonized*. Beacon Press, 1991.

- Minnis, Jack. *The Care and Feeding of Power Structures* (https://www.crmvet.org/docs/65_minnis_power-r.pdf). New England Free Press, 1965.
- Morgenthau, Hans. *Politics Among Nations: The Struggle for Power and Peace*. Knopf, 1954.
- Morris, Benny. *Righteous Victims: A History of the Zionist-Arab Conflict, 1881-2001*. Vintage, 2001.
- Morris, Ian. *Why the West Rules - For Now: The Patterns of History and What They Reveal about the Future*. Picador, 2011.
- Mosca, Gaetano. *What is Mafia?* M&J Publishing House, 2019.

N

- Naim, Moises. *The End of Power: From Boardrooms to Battlefields and Churches to States, Why Being in Charge Isn't What It Used to Be.* Basic Books, 2014.

O

- Obama, Barack. *A Promised Land*. Crown Publishing Group, 2020.

P

- Paine, Lincoln. *The Sea and Civilization: A Maritime History of the World*. Vintage, 2015.
- Piketty, Thomas. *Capital in the Twenty-First Century*. Belknap Press, 2014.
- Piketty, Thomas and Gabriel Zucman. *Capital is Back: Wealth-Income Ratios in Rich Countries 1700-2010* (http://piketty.pse.ens.fr/files/PikettyZucman2013WP.pdf) (2013).
- Pilisuk, Marc and Jennifer Achord Rountree. *The Hidden Structure of Violence: Who Benefits from Global Violence and War*. Monthly Review Press, 2015.
- Pinker, Steven. *The Better Angels of Our Nature: Why Violence Has Declined*. Penguin Books, 2012.
- Polybius. *The Rise of the Roman Empire*. Penguin Classics, 1980.
- Popper, Karl. *The Poverty of Historicism*. Routledge, 2002.
- Poulshock, Michael. *An Abstract Model of Historical Processes* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Quantitative%20IR/Poulshock%20-%20An%20Abstract%20Model%20of%20Historical%20Processes.pdf). Cliodynamics: The Journal of Quantitative History and Cultural Evolution, issue 8(1), 2017.
- Poundstone, William. *The Prisoner's Dilemma: John von Neumann, Game Theory, and the Puzzle of the Bomb*. Anchor, 1993.
- Powell, Robert. *Absolute and Relative Gains in International Relations Theory* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Quantitative%20IR/Powell%20-%20Absolute%20and%20Relative%20Gains%20in%20IR.pdf). The American Political Science Review, Vol. 85, No. 4, 1991, pp. 1303-1320.

R

- Rapoport, Anatol. *N-Person Game Theory: Concepts and Applications*. Dover Publications, 2013.
- Rashevsky, Nicholas. *Looking at History Through Mathematics*. MIT Press, 1968.
- Ratliff, Jim. *Infinitely Repeated Games with Discounting* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/Network%20Theory%20and%20the%20Economics%20of%20Networks/Ratliff%20-%20Infinitely%20Repeated%20Games.pdf). 1996.
- Reus-Smit, Christian, and Duncan Snidal. *The Oxford Handbook of International Relations*. Oxford University Press, 2010.

- Reynolds, Julia. *Blood in the Fields: Ten Years Inside California's Nuestra Familia Gang*. Chicago Review Press, 2017.
- Rich, Nathaniel. *Losing Earth: A Recent History*. MCD, 2019.
- Roberts, J. M. and Odd Arne Westad. *The Penguin History of the World*. Penguin Books, 2014.
- Rodrik, Dani. *Economics Rules: The Rights and Wrongs of the Dismal Science*. W. W. Norton & Company, 2016.
- Roehner, Bertrand M. and Tony Syme. *Pattern and Repertoire in History*. Harvard University Press, 2002.
- Russell, Bertrand. *Power: A New Social Analysis*. Routledge, 2004.
- Russell, Stuart and Peter Norvig. *Artificial Intelligence: A Modern Approach*. Pearson, 2009.
- Ryan, Alan. *On Politics: A History of Political Thought (Vols. 1-2)*. Liveright Publishing Corp., 2012.

S

- Samuelson, Paul. *Foundations of Economic Analysis*. Harvard University Press, 1983.
- Sanger, David. *The Perfect Weapon: War, Sabotage, and Fear in the Cyber Age*. Crown, 2018.
- Sarkees, Meredith Reid and Frank Wayman. *Resort to War: 1816 - 2007*. Washington DC: CQ Press, 2010. Version 4.0.
- Scheidel, Walter. *The Shape of the Roman World*. Journal of Roman Archaeology 27, 2014.
- Schelling, Thomas. *Micromotives and Macrobehavior*. W. W. Norton & Company, 2006.
- Schlefer, Jonathan. *The Assumptions Economists Make*. Belknap Press, 2012.
- Schroeder, Paul. *Historical Reality vs. Neo-Realist Theory*. International Security, Vol. 19, No. 1, 1994, pp. 108-148.
- Shirer, William. *The Rise and Fall of the Third Reich: A History of Nazi Germany*. Pan Books Ltd., 1959.
- Singer, J. David, Stuart Bremer, and John Stuckey. *Capability Distribution, Uncertainty, and Major Power War, 1820-1965*. in Bruce Russett (ed), _Peace, War, and Numbers_, Beverly Hills: Sage, 1972, pp. 19-48. Version 5.0.
- Scott, John. *Power*. Polity, 2002.
- Sheehan, Michael. *Balance of Power: History & Theory*. Routledge, 1995.
- Smil, Vaclav. *Energy and Civilization: A History*. MIT Press, 2018.
- Smith, Jason M., et al. *Power in Politically Charged Networks* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/Network%20Theory%20and%20the%20Economics%20of%20Networks/Smith%20-%20Power%20in%20Politically%20Charged%20Networks.pdf).
- Smith, Michael Joseph. *Realist Thought from Weber to Kissinger*. Louisiana State University Press, 1990.
- Spanier, John W. and Robert L. Wendzel. *Games Nations Play*. Congressional Quarterly, 1996.
- Squazzoni, Flaminio. *Agent-Based Computational Sociology*. Wiley, 2012.
- Stiglitz, Joseph and Linda Bilmes. *The Three Trillion Dollar War: The True Cost of the Iraq Conflict*. Norton, 2008.

T

- Tai, Allen H., et al. *On Computing Prestige in a Network with Negative Relations*. International Journal of Applied Mathematical Sciences, ISSN 0973-0176 Vol.2 No.1 (2005), pp. 97-104.
- Tang, Jiliang. *A Survey of Signed Network Mining in Social Media* (https://arxiv.org/abs/1511.07569). 2016.
- Tellis, Ashley J., et al. *Measuring National Power in the Postindustrial Age* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Measuring%20Power/RAND%20-%20Measuring%20National%20Power%20in%20the%20Post-Industrial%20Age%20-%20Ch.%203.pdf). RAND Corporation, 2000.

- Tellis, Ashley J. [*Reconstructing Political Realism: The Long March to Scientific Theory* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Realism%20and%20the%20Balance%20of%20Power/Tellis%20-%20Reconstructing%20Political%20Realism.pdf)]. Security Studies, Vol. 5, No. 2, 1995, pp. 3-94.
- Thompson, Nato. *Culture as Weapon: The Art of Influence in Everyday Life*. Melville House, 2017.
- Thucydides. *History of the Peloponnesian War*. Penguin Classics, 1954.
- Tolstoy, Leo. *War and Peace*. Vintage, 2008.
- Toynbee, Arnold. *A Study of History: Abridgement of Volumes I-X*. Oxford University Press, 1987.
- Traag V.A., Nesterov Y.E., Van Dooren P. (2010) [*Exponential Ranking: Taking into Account Negative Links* (https://doi.org/10.1007/978-3-642-16567-2_14)]. In: Bolc L., Makowski M., Wierzbicki A. (eds) Social Informatics. SocInfo 2010. Lecture Notes in Computer Science, vol 6430. Springer, Berlin, Heidelberg.
- Treverton, Gregory F. and Seth G. Jones. [*Measuring National Power* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Measuring%20Power/RAND%20-%20Measuring%20National%20Power.pdf)]. RAND National Security Research Division, 2005.
- Trouillot, Michel-Rolp. *Silencing the Past: Power and the Production of History*. Beacon Press, 2015.
- Turchin, Peter, et al. [*Seshat: The Global History Databank* (https://doi.org/10.21237/C7clio6127917)]. Cliodynamics 6(1): 77-107, 2015.
- Turchin, Peter. *War and Peace and War: The Rise and Fall of Empires*. Plume, 2007.

U

- United Nations, Department of Economic and Social Affairs, Statistics Division. [*Handbook of Input-Output Table Compilation and Analysis* (https://unstats.un.org/unsd/publication/SeriesF/SeriesF_74E.pdf)]. ST/ESA/STAT/SER.F/94, 1999.
- Urbina, Ian. *The Outlaw Ocean: Journeys Across the Last Untamed Frontier*. Vintage, 2020.

V

- Vahabi, Mehrdad. *The Political Economy of Destructive Power*. Edward Elgar Pub, 2004.
- Valente, Thomas W. [*How Correlated Are Network Centrality Measures?* (https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2875682/)]. Connect (Tor). 2008 Jan 1; 28(1): 16–26.
- Vasquez, John A. and Colin Elman. *Realism and the Balancing of Power: A New Debate*. Pearson, 2002.
- Venkatesh, Sudhir. *Gang Leader for a Day: A Rogue Sociologist Takes to the Streets*. Penguin Books, 2008.
- Victor, Jennifer Nicholl, et al. *The Oxford Handbook of Political Networks*. Oxford University Press, 2017.

W

- Wachsmann, Nikolaus. *KL: A History of the Nazi Concentration Camps*. Farrar, Straus and Giroux, 2016.
- Wallerstein, Immanuel. *World-Systems Analysis: An Introduction*. Duke University Press, 2008.
- Walther, [*Wars Without Beginning or End: Violent Political Organizations and Irregular Warfare in the Sahel-Sahara* (https://arxiv.org/ftp/arxiv/papers/1606/1606.02705.pdf)] (2015).
- Waltz, Kenneth. *Theory of International Politics*. McGraw-Hill, 1979.
- Ward, Colin. *Anarchism: A Very Short Introduction*. Oxford University Press, 2004.
- Wärneryd, Karl (ed.). *The Economics of Conflict: Theory and Empirical Evidence*. MIT Press, 2014.
- Weatherford, Jack. *Genghis Khan and the Making of the Modern World*. Broadway Books, 2005.

- West, Geoffrey. *Scale: The Universal Laws of Life, Growth, and Death in Organisms, Cities, and Companies*. Penguin Books, 2018.
- Wilson, Edward O. *On Human Nature*. Harvard University Press, 2004.
- Wilson, Edward O. *Sociobiology: The New Synthesis*. Belknap Press, 2000.
- Wohlforth, William C. (et al). *Testing Balance-of-Power Theory in World History* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/International%20Relations/Realism%20and%20the%20Balance%20of%20Power/Wohlforth%20-%20Testing%20Balance%20of%20Power%20Theory.pdf). European Journal of International Relations, Vol. 13(2), 2007, pp. 155-185.
- Woodruff, William. *The Struggle for World Power, 1500-1980*. Palgrave Macmillan, 1981.
- Wright, Lawrence. *The Looming Tower: Al-Qaeda and the Road to 9/11*. Vintage, 2007.
- Wright, Robert. *Nonzero: The Logic of Human Destiny*. Vintage, 2001.

Y

- Yergin, Daniel. *The Prize: The Epic Quest for Oil, Money & Power*. Free Press, 2008.

Z

- Zakaria, Fareed. *The Post-American World: Release 2.0*. W. W. Norton & Company, 2012.
- Zagorin, Perez. *Thucydides: An Introduction for the Common Reader*. Princeton University Press, 2008.
- Zeev Maoz, Paul L. Johnson, Jasper Kaplan, Fiona Ogunkoya, and Aaron Shreve. *The Dyadic Militarized Interstate Disputes (MIDs) Dataset: Logic, Characteristics, and Comparisons to Alternative Datasets*. Journal of Conflict Resolution, 2018, Version 4.3.
- Zestos, George K. *Trade and GDP Growth: Causal Relations in the United States and Canada* (https://www.researchgate.net/publication/46568591_Trade_and_GDP_Growth_Causal_Relations_in_the_United_States_and_Canada). Southern Economic Journal 68(4):859-874, 2002.
- Zinn, Howard. *A People's History of the United States*. HarperCollins, 1995.

Data Sources

This page lists data sources for the quantitative investigation of power structures.

Contents

World Power Structure

The following data sets relate to international relations, conflict, and trade:

National Power

- Correlates of War Project, National Material Capabilities (https://correlatesofwar.org) (v5.0) (population, number of military personnel, military expenditure, and Composite Index of National Capability (CINC)).
- Credit Suisse, Global Wealth Report (https://www.credit-suisse.com/about-us/en/reports-research/global-wealth-report.html) (total household wealth in USD by country, 2010-2020).
- International Monetary Fund, World Economic Outlook Database (GDP PPP from 1980).
- Maddison Project Database (https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2018) (2018) (historical per capita GDP).
- Penn World Table (https://www.rug.nl/ggdc/productivity/pwt/) (v9.1) (historical GDP).
- Seshat Project, Seshat Global History Databank (http://seshatdatabank.info) (data on historical regions and civilizations).
- World Bank. The Changing Wealth of Nations (https://datacatalog.worldbank.org/dataset/wealth-accounting) (1995, 2000, 2005, 2010, 2014) (national household wealth in 2014 USD millions).
- World Bank, GDP in current USD (https://data.worldbank.org/indicator/NY.GDP.MKTP.PP.KD) (1960-2018).
- World Bank, Historical GDP Purchasing Power Parity (https://data.worldbank.org/indicator/NY.GDP.MKTP.PP.KD) (1990-2019; current USD).
- World Bank, Military expenditure as percentage of GDP (https://data.worldbank.org/indicator/MS.MIL.XPND.GD.ZS) (1960-2019).

International Conflict

- Correlates of War Project, COW War Data, 1816-2007 (https://correlatesofwar.org) (v4.0).
- Correlates of War Project, Militarized Interstate Disputes (https://correlatesofwar.org) (v4.3) (1816-2010).
- Digital Attack Map (https://www.digitalattackmap.com/#anim=1&color=0&country=ALL&list=0&time=18285&view=map) (real time analytics of cyber conflict).
- Integrated Crisis Early Warning System (ICEWS) Dataverse (https://dataverse.harvard.edu/dataverse/icews) (event coding of international political news).
- PRIO, International conflict data sets (https://www.prio.org/Data/).
- SIPRI, International conflict data sets (https://www.sipri.org/databases), including military expenditures (1949-2018; USD millions).
- UCDP, International conflict data sets (https://ucdp.uu.se/downloads/index.html), including external support (i.e. proxy wars) (1975-2009).
- Wikipedia, List of proxy wars (https://en.wikipedia.org/wiki/List_of_proxy_wars) (1839-present).

- Wikipedia, List of wars by death toll (https://en.wikipedia.org/wiki/List_of_wars_by_death_toll).

International Trade, Assistance, and Cooperation

- CEPII, Bilateral Trade History Dataset (http://www.cepii.fr/CEPII/en/bdd_modele/presentation.asp?id=32) (1827-2014, also contains GDP in current GBP and population).
- Correlates of War Project, Defense Cooperation Agreement Dataset (https://correlatesofwar.org) (v1) (1980-2010).
- Correlates of War Project, Interstate Alliances (https://correlatesofwar.org/data-sets/formal-alliances) (v4.1) (1816-2012).
- Correlates of War Project, Trade (https://correlatesofwar.org) (v4.0) (1870-2014; USD millions).
- The RICardo Project, Trade between Nations (http://ricardo.medialab.sciences-po.fr/#!/) (1800-1938).
- Roodman, David. Net Aid Transfers data set (https://davidroodman.com/data/) (1960–2016).
- UCDP, Dyadic Dataset version 19.1 (https://www.ucdp.uu.se/downloads/) (1946-2018).
- UNESCAP, Gravity Model of Trade Data (https://trade.unescap.org/analytics/).
- William & Mary, AidData (https://www.aiddata.org/datasets) (data on international aid transfers).

Other

- Dotlan, EVE maps (https://evemaps.dotlan.net/) (real-time alliance and conflict information for the EVE Online computer game).

Prior Work

Quantitative realism touches a variety of academic disciplines. This section describes some of the prior work related to it, organized as follows:

- Grand Theories of History and Historical Change
- International Relations Theory
- Theories of Power
- Economics and Network Science
- Political Philosophy and the Philosophy of History

For prior work specifically about quantitative realism, see:

- Poulshock, Michael. *The Foundations of Political Realism* (https://arxiv.org/abs/1910.04785) (2019).
- Poulshock, Michael. *An Abstract Model of Historical Processes* (https://escholarship.org/uc/item/9sv1d0h8). Cliodynamics: The Journal of Quantitative History and Cultural Evolution, 8(1) (2017).

A collection of relevant academic articles is hosted at this sister site (https://github.com/mpoulshock/QuantitativeRealismResearch).

Grand Theories of History and Historical Change

- Chinese Historiography
- Ibn Khaldun
- Toynbee
- Asimov

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International Relations Theory

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Political Realism

Precepts

Historical Development

Claims

Measuring Power

Since World War II, extensive efforts have been made to devise a quantitative measurement of national power (see Tellis 2000). These metrics typically use or combine statistics related to the economic, military, demographic, and geographic characteristics of a country into a single number to facilitate inter-country comparisons. There are many formulas that have been developed, taking different factors into consideration and often based upon data that is readily available. Despite this diversity, most definitions produce similar rank-order results (Tellis, ch. 3, citing Merritt and Zinnes).

One well known example is Singer's (1972) Composite Indicator of National Capability, or CINC:

```
CINC = (ME + MP + E + TP + UP + ISP) / 6
```

where `ME` is military expenditure, `MP` is military personnel, `E` is energy consumption, `TP` is total population, `UP` is urban population, and `ISP` is iron and steel production. All values are the individual state's percentages of total world capacity. CINC enables comparisons across a broad time span, and the Correlates of War Project (https://correlatesofwar.org/) has calculated it for every country from 1870 to 2012. On the other hand, the inclusion of iron and steel production seems arbitrary in light of subsequent technological developments, and CINC may not be the appropriate metric for analyzing contemporary situations.

Also noteworthy, due to who created it, the CIA's Strategic Assessments Group uses GDP, population, defense spending, and technological innovation to determine a nation's share of global power (Treverton 2005).

These traditional approaches do not seem to derive from any overarching theory of power or the dynamics of how power balances change over time.

Liberalism, etc.

Network Theory in International Relations

- Jang (2020) applies network centrality measures to trade data to evaluate centrality of the U.S. and China in various sectors.
- Maoz (2011) combines network centrality with voting theory (n-person cooperative games) to develop a metric and apply it to international conflict and voting.
- Maoz (2004) seeks to fuse realism and liberalism around the concept of status inconsistency.
- Walther (2016) applies network theory to a violent conflict.

Theories of War

World Systems Theory

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Theories of Power

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Themes

The literature on political power can be organized around the following themes:

1. **Conception** - How has “power” been defined throughout history?
2. **Maintenance** - How is power to be attained and used?
3. **Justification and critique** - What’s the history of moral critiques of power, and prescriptions for its redistribution?

Major thinkers on each of these themes are listed in the sections below.

Definitions of Power

- Robert Dahl
- Friedkin, Noah E. *A Formal Theory of Social Power* (http://www.communicationcache.com/uploads/1/0/8/8/10887248/a_formal_theory_of_social_power.pdf). Journal of Mathematical Sociology, 1986, Vol. 12(2) pp. 103-126. Friedkin assumes that power is based on influence and he analyzes small power structures and their effects on generating agreement. His power structures do not have any negative edges.
- Barnett (2005) challenges the traditional definition of power as A causing B to do something B wouldn’t otherwise do, arguing that “power is the production, in and through social relations, of effects that shape the capacities of actors to determine their circumstances and fate."

The Acquisition and Maintenance of Power

- Machiavelli
- Caro

Critiques of Power

- Max Weber
- Bertrand Russel
- Orwell
- C. Wright Mills, The Power Elite
- Power Structure Analysis
- Postmodernism (Foucault)

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Economics and Network Science

Contents

Network Centrality Measures

There are a variety of network centrality measures that are vaguely similar to PrinceRank, including PageRank. Of particular interest are measures that address signed (aka ternary) graphs, or those with negative links. Some representative lines of inquiry include:

- Bonacich (1987) defines a function c(α, β) which provides a generalized measure of network centrality. While not directly applicable to quantitative realism, this is a seminal article in eigenvector centrality, which is similar to PrinceRank. Bonacich centrality seems to work with negative edges, although the meaning is not entirely clear.
- Friedkin (1991) argues that it is preferable for centrality measures to be theoretically grounded.
- Kaur (2016) provides a very good summary of centrality measures in signed networks.
- Smith, et al, in *Power in Politically Charged Networks* (https://github.com/mpoulshock/QuantitativeRealismResearch/blob/main/Network%20Theory%20and%20the%20Economics%20of%20Networks/Smith%20-%20Power%20in%20Politically%20Charged%20Networks.pdf), define the Political Independence Index, which quantifies political power as a function of network structure. The PII is based on the idea of adding up the centrality of positive links to each node and then subtracting the centrality of the negative links. The motivation here is very similar to that of quantitative realism.
- Tai (2005) uses a dummy node to represent aggregate effects of negative relations.
- Tang (2016) discusses the handling of negative links by subtracting the PageRank of the negative links from that of the positive links, effectively breaking the network in two.
- Traag (2010) describes "exponential ranking," which accounts for negative links by analogy to trust and reputation metrics.
- Valente (2008) observes that despite the variety of network centrality measures, they are strongly correlated, especially in non-directed graphs.

Models of International Trade

The international trade network provides clues to the nature of the world power structure. The literature on this subject seems to be clustered around four general types of models:

- Gravity models and the study of trade frictions
- Application of statistical network analysis to trade networks
- Network wealth/asset exchange and input-output models, which have a similar flow concept to quantitative realism
- Regression (correlation) models between GDP and trade

Gravity Models

- Almog (2019) tries to improve upon gravity models using a probability distribution related to bilateral trade volume.

- Garlaschelli (2007) infers a fitness function between two nodes indicating the likelihood of connection and finds that this fitness function is related to GDP.

Statistical Properties of the International Trade Network (ITN)

- Algarra (2019) uses a stochastic network formation model to approximate empirical trade network data.
- Benedictis (2009) summarizes history of network analysis for international trade, which dates back to the 1940s, and then analyzes the ITN over time using common network statistics.
- Ermann (2011) applies PageRank-CheiRank to the ITN, sorting countries into three tiers.
- Fagiolo (2009) applies statistical network theory to trade networks.
- Hokkanen (2012) applies common graph metrics to the ITN and finds correlations between some of these metrics and GDP.

Asset Exchange and Input-Output Models

In an asset exchange model, agents trade resources stochastically to see how they become distributed. This exchange concept is similar to the way the law of motion handles constructive transfers. Economic input-output models, particularly those based upon a single commodity, also bear a resemblance to the flow of power in the law of motion. Of note are:

- Coquide (2020) models a contagion doomsday scenario using PageRank-CheiRank, considering both countries and products.
- Galbusera (2018), describing ways that I-O models can be used “in reverse” to quantify the impact of some disaster on production.
- Krapivsky (2010), while not specifically about the international trade network, tries to account for the wealth distribution that results from random exchanges among agents. Unlike quantitative realism, this model "neglects the possibility of wealth growth...by both agents benefiting in exchanges."
- Luo and Tsang (2020), which uses input-output analysis to determine labor supply shock to China's GDP from Covid-19 and then calculates the effects on other countries' GDPs.
- United Nations (1999), providing a primer on the mathematics of I-O models.

Relationship Between Trade and GDP

Does international trade tend to cause a state's GDP to rise? In one sense, the question is a tautology, because the expenditure method (https://en.wikipedia.org/wiki/Gross_domestic_product#Expenditure_approach) of calculating GDP takes into account net exports (exports minus imports), so by definition trade has a causal effect on GDP. That technicality aside, does trade expand a nation's economy?

- Bhattacharya (2009) examines distribution of bilateral trade flows and finds some positive relationship between trade volume and GDP exists
- Garlaschelli (2007) discusses effects of GDP on the trade network, and vice versa. For the latter, hypothesizes a stochastic network wealth exchange model as the cause of GDP distribution.
- Zestos (2002) applies time series causality tests between trade and growth, for the U.S. and Canada. Does not look at network effects.

The Relationship between Trade and War

One of the assumptions of liberalism is that the benefits of trade creates a disincentive for conflict. Is this sound?

- Barbieri and Levy (1999) and (2001) look at case studies and conclude that “there is no consistent, systematic, and substantial reduction in trade between belligerents during wartime, and trade between adversaries appears to recover quickly after the termination of war.”
- On the other hand, Glick (2005) concludes: “We search for and find a very strong impact of war on trade volumes. Moreover this effect has two important aspects: first, it is persistent, meaning that even after conflicts end, trade does not resume its pre-war level for many years, exacerbating the total costs; second, the effect has a multilateral

dimension and, unlike direct costs, which largely effect only the belligerents, trade destruction affects neutral parties as well, generating a negative externality.”

Philosophy

Numerous ideas in political philosophy and the philosophy of history touch quantitative realism. Here are some of the major ones.

1. Aristotle
2. Hobbes's *Leviathan*
3. Hegel
4. Kant
5. Theory of Government
    1. Magna Carta
    2. Tripartite governmental powers
6. Marx
7. Anarchism

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Essays, etc.

This section contains some short essays and other thoughts on the subject of power. These are evolving ideas that may not stand the test of time, but they represent a cross-section into my thinking on this fascinating topic.

A Board Game Version of Quantitative Realism

One could take the general ideas underlying quantitative realism and turn them into a simple, multi-player board game that can be played with friends and family. We call this game *Clausewitz*, after the Prussian military theorist (https://en.wikipedia.org/wiki/Carl_von_Clausewitz) known for describing war as "the continuation of politics by other means."

Rules

1. **Goal:** *Clausewitz* is an accumulation game for three or more people in which players collect stones and eliminate other players' stones. The object of the game is to have more stones than all of the other players combined.
2. **Pieces:** There are two dishes, one with white stones (worth one unit each) and one with black stones (worth two units each).
3. **Commencement:** Each player starts with five white stones and one black stone from the dishes.
4. **Moves:** Players move one at a time, starting with the youngest player and proceeding clockwise. On each turn, a player can do one of the following:
    1. Give a white stone to another player, along with an additional white stone taken from the dish.
    2. Give a black stone to another player. The other player must then put that stone, plus three units' worth of her own stones, back into the dishes. The other player chooses which of her stones to forfeit. If she only has black stones, she must forfeit two of them.
    3. Trade in two white stones for a black stone, or one black stone for two white stones.
    4. Pass.
5. **Termination:** Players are not eliminated from the game when they run out of stones. The game ends when one player has more white stones than all of the other players combined. Draws are possible.
6. **Knowledge:** Players must keep their stones visible at all times, but they can communicate with each other privately via their mobile devices.

Commentary

In *Clausewitz*, players amass stones and try to destroy other players' collections of stones. Piles of stones are analogous to national power or wealth, and players can build up these collections by engaging in mutual exchanges of white stones with other players. They can also attack other players by spending their black stones, which reduces the material capacity of the other players. These basic mechanics of setting power in motion and consequently affecting the power levels of other agents is arguably the essence of international relations.

Unlike the formal representation of quantitative realism, in which relationships persist over time, in *Clausewitz* player interaction is based on a series of individual transactions. Nonetheless, players still have to make choices about reciprocity, cooperation, and revenge, forming and betraying alliances as the game unfolds.

Players must also maintain an optimal balance between white and black stones, in other words, between their capacity to increase their stock via mutual exchange and their ability to impose destruction. These choices are loosely analogous to balancing economic growth and military expenditure.

Quantitative realism assumes that states want to maximize their absolute and relative levels of power, but it doesn't articulate a terminal game state per se. Board games need some way of ending, and *Clausewitz* ends when one player totally dominates all of the others.

Because players can establish private channels of communication and engage in side deals limited only by their imaginations, it is possible that the most compelling aspects of the game take place "off board" in the diplomatic structures that players create as a result of these interactions.

Notable Books

Here are some books that either directly address the nature of power or otherwise illuminate its workings, in no particular order.

***The Power Broker*, Robert Caro.** Caro's epic biography of the urban planner Robert Moses, an intricate case study in the nature of power.

***The Years of Lyndon Johnson: Master of the Senate*, Robert Caro.** My favorite book of Caro's, this traces LBJ's transformation of the Senate from a dysfunctional institution to one that he effectively controlled (and which was still dysfunctional).

***The Rise and Fall of the Third Reich*, William Shirer.** A classic and gripping account of the rise of Hitler and the Nazis.

***How to Hide an Empire: A History of the Greater United States*, Daniel Immerwahr.** In what should be required reading for every American, Immerwahr unearths the history of the U.S. overseas empire and shows how white supremacy was one of the driving forces behind it.

***A Penguin History of the World*, J.M. Roberts and Odd Arne Westad.** My all-time favorite book, this blew my mind when I read it in my 20s and I've reread it four or five times since then. Though slightly Eurocentric, it eloquently takes you from the dawn of humanity to the present, exhibiting great erudition and depth about the historical experience of our species.

***My Bondage and My Freedom*, Frederick Douglass.** A brilliant and beautiful autobiography of Douglass's life as a slave and his ultimate escape. Should also be required reading for every American.

***The Evolution of Cooperation*, Robert Axelrod.** Axelrod's famous exploration of the Iterated Prisoner's Dilemma, describing the results of a computer tournament he organized in search of the optimal strategy.

***The Perfect Weapon: War, Sabotage, and Fear in the Cyber Age*, David Sanger.** This will keep you up at night.

***Gang Leader for a Day: A Rogue Sociologist Takes to the Streets*, Sudhir Venkatesh.** An academic rides shotgun with a local gang for a few years and chronicles its power struggles. You come away understanding that gang violence is not random and irrational, but subject to an underlying logic.

***Genghis Khan and the Making of the Modern World*, Jack Weatherford.** If anyone knew how to wield power, it was Genghis Khan. Weatherford's portrait is a bit hagiographic and overly forgiving of Khan's gratuitous violence, but it is nevertheless an extremely illuminating look at the life of someone who had an incredible impact on history.

***Behind the Beautiful Forevers*, Katherine Boo.** Boo spends a few years covering life in the Annawadi slum in Mumbai, revealing how humans treat each other under anarchical conditions.

***Imperial Reckoning: The Untold Story of Britain's Gulag in Kenya*, Caroline Elkins.** Elkins conducted numerous interviews and scoured British archives to paint a portrait of the true nature of British imperialism in Kenya.

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Major Loose Ends

Here are the major topics that still need to be sorted out, more or less in priority order:

- Finish section on μ and the offense-defense balance
- Methods of estimating and data for national power
- Modeling WWII (data and decision points)
- Empirical evidence for the law of motion, including the cost of destruction
- Game tree pruning
- Experiments with gravitized tactics
- Understanding game dynamics
- Digging into the four maxims of strategy
- Data of the world power structure
- Testing polarity on real world data
- Fleshing out various definitions of stability
- Understanding the relationship between stability and polarity, both theoretically and empirically
- Simultaneous moves?
- Percentage of moves that are legal
- Code that implements institutional constraints
- Distance attenuation function
- Modeling other historical episodes
- Summarizing prior work

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