Network And Circuits Theory Second Assignment 2019-2020

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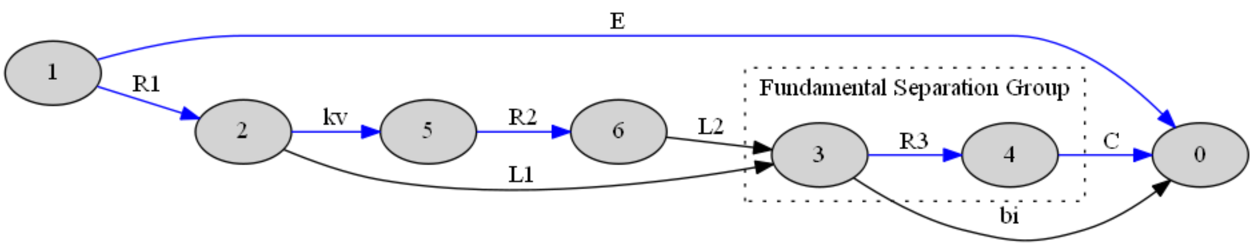
1 Exercise One



taking into account that

$$i = i_{L2} \tag{1}$$

$$v = E_s - (I_{L1} + I_{L2})R1 \tag{2}$$

From the conservation of current on the fundamental separation group, and from Kirchoffs voltage law across the two fundamental loops we have

$$I_{L1} + I_{L2} = C\dot{V_C} + bI_{L2} \tag{3}$$

$$(E_s - (I_{L1} + I_{L2})R1)(k-1) + I_{L2}(R3 + R2) + L2\dot{I}_{L2} + V_C = 0 \tag{4}$$

$$(E_s - (I_{L1} + I_{L2})R1) + I_{L1}R3 + L1\dot{I}_{L1} + V_C = 0 \tag{5}$$

Organizing matrix format we get

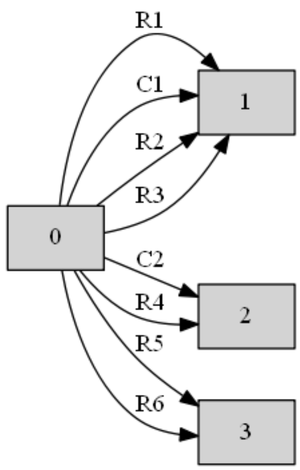
$$\dot{x}(t) = \overset{A}{\begin{pmatrix} 0 & \frac{1}{C} & \frac{1-b}{C} \\ -\frac{1}{L2} & \frac{R1-R3}{L1} & \frac{R1}{L1} \\ -\frac{1}{L2} & \frac{(k-1)R1}{L2} & \frac{(k-1)R1-R2-R3}{L2} \end{pmatrix}} \overset{x(t)}{\begin{pmatrix} V_C(t) \\ I_{L1}(t) \\ I_{L2}(t) \end{pmatrix}} + \overset{B}{\begin{pmatrix} 0 \\ \frac{-1}{L1} \\ -\frac{(k-1)}{L2} \end{pmatrix}} \overset{u(t)}{(E_s(t))}$$

and

$$y(t) = \overset{C}{\begin{pmatrix} 0 & 0 & b \\ 0 & -R1 & -R1 \end{pmatrix}} \overset{x(t)}{\begin{pmatrix} V_C(t) \\ I_{L1}(t) \\ I_{L2}(t) \end{pmatrix}} + \overset{D}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} E$$

2 Exercise Two

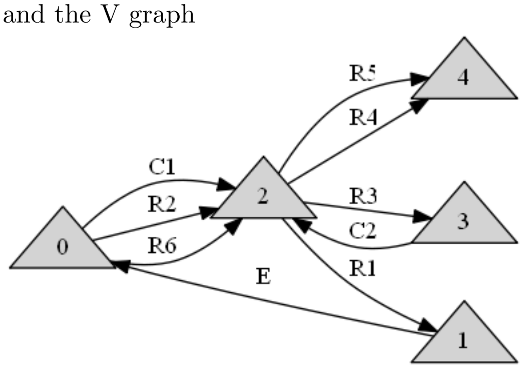
The I graph



Where

$$\begin{aligned} \boxed{0} &= 0,1,3,5 \\ \boxed{1} &= 2 \\ \boxed{2} &= 4 \\ \boxed{3} &= 6 \end{aligned} \tag{6}$$

and the V graph



Where the notes correspond to

$$
\begin{aligned}
\triangle_0 &= 0 \\
\triangle_1 &= 1 \\
\triangle_2 &= 2, 4, 6 \\
\triangle_3 &= 3 \\
\triangle_4 &= 5
\end{aligned}
\tag{7}
$$

with the output being

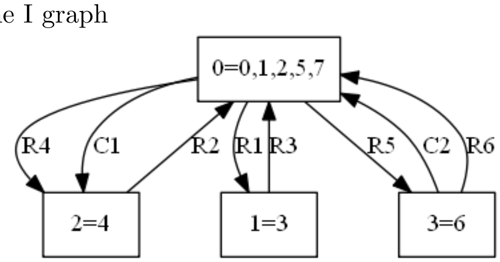
$$V_{out} = E_2 \tag{8}$$

and we arrive at the equations

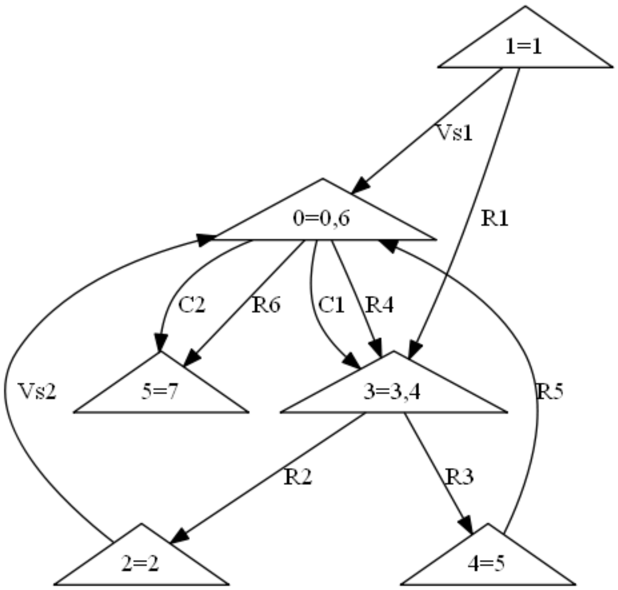
$$
\begin{array}{c}
\\ \boxed{1} \\ \boxed{2} \\ \boxed{3} \\ E
\end{array}
\begin{pmatrix}
\triangle_1 & \triangle_2 & \triangle_3 & \triangle_4 \\
R1 + sL1 + R2 & R1 + sL1 & 0 & -sL1 \\
R1 + sL1 - r1 & r2 + R1 + sL1 - r1 + R3 + R4 & -R3 & -sL1 + r1 - R3 \\
r1 & -R3 + r1 & R3 + sL2 & R3 - r1 \\
1 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix} E1 \\ E2 \\ E3 \\ E4 \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ 0 \\ E \end{pmatrix}
$$

3 Exercise Three

The I graph



and the V graph



Where the notes correspond to
with the output being

$$y = v_{out} = E_5 \tag{9}$$

and we arrive at the equations

$$\begin{array}{c c}
 & \begin{array}{ccccc} \triangle 1 & \triangle 2 & \triangle 3 & \triangle 4 & \triangle 5 \end{array} \\
\begin{array}{c} \boxed{1} \\ \boxed{2} \\ \boxed{3} \\ Vs1 \\ Vs2 \end{array} &
\begin{pmatrix}
-\frac{1}{R1} & 0 & \frac{1}{R1} + \frac{1}{R3} & -\frac{1}{R3} & 0 \\
0 & -\frac{1}{R2} & \frac{1}{R4} + sC1 + \frac{1}{R2} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{R5} & -sC2 - \frac{1}{R6} \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{pmatrix}
\end{array}
\begin{pmatrix} E1 \\ E2 \\ E3 \\ E4 \\ E5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ Vs1 \\ Vs2 \end{pmatrix}$$

As state variables we consider the voltage of the capacitors

$$x = \begin{pmatrix} v_{C1}(t) \\ v_{C2}(t) \end{pmatrix} = \begin{pmatrix} E_3 \\ -E_5 \end{pmatrix}$$

from those equations it must follow

$$sC_1 E_3 = -(\frac{1}{R4} + \frac{1}{R2})E3 + \frac{V_{s2}}{R2} \tag{10}$$

and

$$sC2E5 = -\frac{1}{R6}E5 + \frac{R3}{R5}(-\frac{V_{s1}}{R1} + (\frac{1}{R3} + \frac{1}{R1})E3) \tag{11}$$

So going back to the time domain we obtain

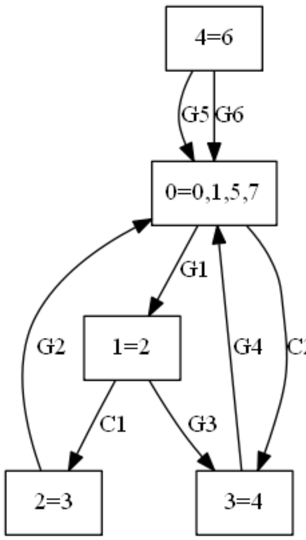
$$\dot{x}(t) = \overset{A}{\begin{pmatrix} -(\frac{1}{R4C1} + \frac{1}{R2C1}) & 0 \\ \frac{R3}{R5C2}(\frac{1}{R3} + \frac{1}{R1}) & -\frac{1}{R6C2} \end{pmatrix}} \overset{x(t)}{\begin{pmatrix} V_{C1}(t) \\ V_{C2}(t) \end{pmatrix}} + \overset{B}{\begin{pmatrix} 0 & \frac{1}{C1R2} \\ \frac{R3}{R5R1C2} & 0 \end{pmatrix}} \overset{u(t)}{\begin{pmatrix} V_{s1}(t) \\ V_{s2}(t) \end{pmatrix}}$$

and for the output it must hold

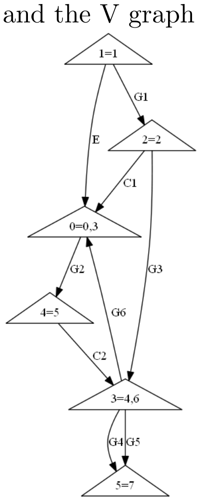
$$y(t) = \overset{C}{\begin{pmatrix} 0 & 1 \end{pmatrix}} \overset{x(t)}{\begin{pmatrix} V_{C1}(t) \\ V_{C2}(t) \end{pmatrix}} + \overset{D}{\begin{pmatrix} 0 & 0 \end{pmatrix}} \overset{u(t)}{\begin{pmatrix} V_{s1}(t) \\ V_{s2}(t) \end{pmatrix}}$$

4 Exercise Four

The I graph



and the V graph



Where the notes correspond to
with the output being

$$V_{out} = E_2 \tag{12}$$

and we arrive at the equations

$$
\begin{array}{cc}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} \boxed{1} \\ \boxed{2} \\ \boxed{3} \\ \boxed{4} \\ E \end{array} &
\begin{pmatrix}
-G1 & G1 + sC1 + G3 & -G3 & 0 & 0 \\
0 & -sC1 & 0 & -G2 & 0 \\
0 & -G3 & G3 + sC2 + G4 & -sC2 & -G4 \\
0 & 0 & G5 + G6 & 0 & -G5 \\
1 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
\begin{pmatrix} E1 \\ E2 \\ E3 \\ E4 \\ E5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ E \end{pmatrix}
$$