

Optimizing welfare on linked supply chains through tariffs.

Defining market curves

```
ClearAll[qs, qb ]
ClearAll[ a, b, c, d, e, f, g, α]
ClearAll[i, t]
ClearAll[bs, bb, cs, cb, dbe, dbb, πs, πb]
ClearAll[UB, UE]
ClearAll[opti, optt]
ClearAll[BrOpti, EUOptt]
```

Basic Mkt functions

```
bs = a * qs - b * qs^2
a qs - b qs^2

bb = c * qb - d * qb^2
c qb - d qb^2

cs = e * qs^2
e qs^2

cb = D[cs, {qs}] * qb
2 e qb qs

dbe = f * qb^2
f qb^2

dbb = g * qb^2
g qb^2

πs = (D[bs, {qs}] - t) * qs - cs
-e qs^2 + qs (a - 2 b qs - t)

πb = (D[bb, {qb}] - i) * qb - cb
qb (c - i - 2 d qb) - 2 e qb qs
```

Country Functions

Brazil

$$UB = \pi s + \pi b + bb + \alpha * i * qb - dbb$$

$$c qb - d qb^2 - g qb^2 + qb (c - i - 2 d qb) - 2 e qb qs - e qs^2 + qs (a - 2 b qs - t) + i qb \alpha$$

Eropean Union

$$UE = bs - D[bs, \{qs\}] * qs - dbe + qs * t$$

$$- f qb^2 + a qs - b qs^2 - qs (a - 2 b qs) + qs t$$

Solving reaction functions for companies

$$rb = \text{Solve}[(D[bb, \{qb\}] - i) == D[cb, \{qb\}], \{qb\}]$$

$$\left\{ \left\{ qb \rightarrow \frac{c - i - 2 e qs}{2 d} \right\} \right\}$$

$$qb = rb[[1]][[1]][[2]]$$

$$\frac{c - i - 2 e qs}{2 d}$$

$$rs = \text{Solve}[(D[bs, \{qs\}] - t) == D[cs, \{qs\}], \{qs\}]$$

$$\left\{ \left\{ qs \rightarrow \frac{a - t}{2 (b + e)} \right\} \right\}$$

$$qs = rs[[1]][[1]][[2]]$$

$$\frac{a - t}{2 (b + e)}$$

Finding i and t

$$\text{opti} = \partial_i UB == 0$$

$$-\frac{c}{2 d} + \frac{c - i - \frac{e (a - t)}{b + e}}{2 d} + \frac{g \left(c - i - \frac{e (a - t)}{b + e} \right)}{2 d^2} - \frac{i \alpha}{2 d} + \frac{\left(c - i - \frac{e (a - t)}{b + e} \right) \alpha}{2 d} == 0$$

$$rUB = \text{Solve}[\text{opti}, \{i\}][[1]][[1]][[2]]$$

$$\frac{(-a d e + b c g - a e g + c e g + d e t + e g t + b c d \alpha - a d e \alpha + c d e \alpha + d e t \alpha)}{(b + e) (d + g + 2 d \alpha)}$$

$$\text{Simplify}[rUB]$$

$$(b c (g + d \alpha) + c e (g + d \alpha) - a e (d + g + d \alpha) + e t (d + g + d \alpha)) / ((b + e) (d + g + 2 d \alpha))$$

Simplify[D[rUB, t]]

$$\frac{e (d + g + d \alpha)}{(b + e) (d + g + 2 d \alpha)}$$

Simplify[D[rUB, α]]

$$(d (b c (d - g) + e (c (d - g) + a (d + g) - (d + g) t))) / ((b + e) (d + g + 2 d \alpha)^2)$$

$$\partial_{\alpha} ((b c (g + d \alpha) + c e (g + d \alpha) - a e (d + g + d \alpha) + e t (d + g + d \alpha)) / ((b + e) (d + g + 2 d \alpha)))$$

$$\frac{b c d - a d e + c d e + d e t}{(b + e) (d + g + 2 d \alpha)} -$$

$$(2 d (b c (g + d \alpha) + c e (g + d \alpha) - a e (d + g + d \alpha) + e t (d + g + d \alpha))) / ((b + e) (d + g + 2 d \alpha)^2)$$

$$\text{Simplify}\left[\frac{b c d - a d e + c d e + d e t}{(b + e) (d + g + 2 d \alpha)} -$$

$$(2 d (b c (g + d \alpha) + c e (g + d \alpha) - a e (d + g + d \alpha) + e t (d + g + d \alpha))) / ((b + e) (d + g + 2 d \alpha)^2)\right]$$

$$(d (b c (d - g) + e (c (d - g) + a (d + g) - (d + g) t))) / ((b + e) (d + g + 2 d \alpha)^2)$$

$$\text{optt} = \partial_t \text{UE} == 0$$

$$-\frac{a}{2 (b + e)} + \frac{a - \frac{b (a - t)}{b + e}}{2 (b + e)} - \frac{e f (c - i - \frac{e (a - t)}{b + e})}{2 d^2 (b + e)} + \frac{a - t}{2 (b + e)} - \frac{t}{2 (b + e)} == 0$$

$$\text{rUE} = \text{Solve}[\text{optt}, t][[1]][[1]][[2]]$$

$$\left(\frac{a b}{2 (b + e)^2} - \frac{a}{2 (b + e)} - \frac{a e^2 f}{2 d^2 (b + e)^2} + \frac{c e f}{2 d^2 (b + e)} - \frac{e f i}{2 d^2 (b + e)} \right) /$$

$$\left(\frac{b}{2 (b + e)^2} - \frac{1}{b + e} - \frac{e^2 f}{2 d^2 (b + e)^2} \right)$$

Simplify[rUE]

$$\frac{e (a (d^2 + e f) - (b + e) f (c - i))}{b d^2 + e (2 d^2 + e f)}$$

D[rUE, i]

$$-\left((e f) / \left(2 d^2 (b + e) \left(\frac{b}{2 (b + e)^2} - \frac{1}{b + e} - \frac{e^2 f}{2 d^2 (b + e)^2} \right) \right) \right)$$

$$\alpha = 1; a = 3.5; b = 0.1; c = 3; d = 0.6; e = 0.1;$$

$$f = 2.65; g = 1.14; (*g = 1.14, f = 2.65;*)$$

$$\text{sol} = \text{Solve}[\text{rUB} - i == 0 \ \&\& \ \text{rUE} - t == 0, \{i, t\}]$$

$$\{\{i \rightarrow 0.663487, t \rightarrow 0.705687\}\}$$

Results

Benchmark

UE /. {i → 0, t → 0}

4.78082

UB /. {i → 0, t → 0}

8.89323

dbb /. {i → 0, t → 0}

1.23698

dbe /. {i → 0, t → 0}

2.87543

totalwelfare = UE + UB /. {i → 0, t → 0}

13.674

totaldamage = dbb + dbe /. {i → 0, t → 0}

4.11241

qb /. {i → 0, t → 0}

1.04167

qs /. {i → 0, t → 0}

8.75

Optimal Tariffs

Simplify[UE /. sol]

{8.18605}

Simplify[UB /. sol]

{6.68166}

Simplify[dbb /. sol]

{0.698558}

dbe /. sol

{1.62384}

Simplify[totalwelfare = UE + UB /. sol]

{14.8677}

totaldamage = dbb + dbe /. sol

{2.3224}

Simplify[qb /. sol]

{0.782796}

qs /. sol

{6.98578}

Results if Brazil sets the tariff first $t(i^*)$

BrOpti = rUB /. t → 0

0.382653

EUOptt = rUE /. i → BrOpti

0.595023

UE /. {i → BrOpti, t → EUOptt}

7.09856

UB /. {i → BrOpti, t → EUOptt}

6.91832

dbb /. {i → BrOpti, t → EUOptt}

1.07421

dbe /. {i → BrOpti, t → EUOptt}

2.49706

totalwelfare = UE + UB /. {i → BrOpti, t → EUOptt}

14.0169

totaldamage = dbb + dbe /. {i → BrOpti, t → EUOptt}

3.57127

qb /. {i → BrOpti, t → EUOptt}

0.970715

qs /. {i → BrOpti, t → EUOptt}

7.26244

Results if EU sets the tariff first $i(t^*)$

NeEUOptt = rUE /. i → 0

0.444238

NeBrOpti = rUB /. t → EUOptt

0.619448

```
UE /. {i → NeBrOpti, t → NeEUOptt}
```

```
7.89179
```

```
UB /. {i → NeBrOpti, t → NeEUOptt}
```

```
7.52937
```

```
dbb /. {i → BrOpti, t → EUOptt}
```

```
1.07421
```

```
dbe /. {i → NeBrOpti, t → NeEUOptt}
```

```
1.33797
```

```
totalwelfare = UE + UB /. {i → NeBrOpti, t → NeEUOptt}
```

```
15.4212
```

```
totaldamage = dbb + dbe /. {i → NeBrOpti, t → NeEUOptt}
```

```
1.91355
```

```
qb /. {i → NeBrOpti, t → NeEUOptt}
```

```
0.710559
```

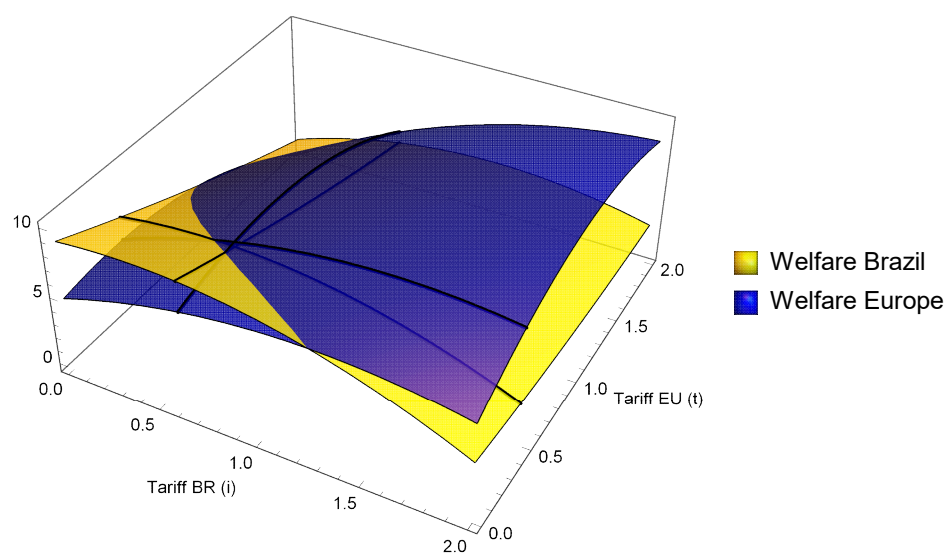
```
qs /. {i → NeBrOpti, t → NeEUOptt}
```

```
7.63941
```

Graphing the welfare to find the best parameters.

```
welfarePlot =
```

```
Plot3D[{UB, UE}, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tariff BR (i)", "Tariff EU (t)"},  
Mesh → {{NeBrOpti}, {NeEUOptt}}, MeshStyle → {Thick, Thick},  
PlotTheme → "FullAxes", PlotLegends → {"Welfare Brazil", "Welfare Europe"},  
PlotStyle → {Directive[Opacity[0.8], Yellow, Specularity[White, 50]],  
Directive[Opacity[0.8], Blue, Specularity[White, 50]]}]
```

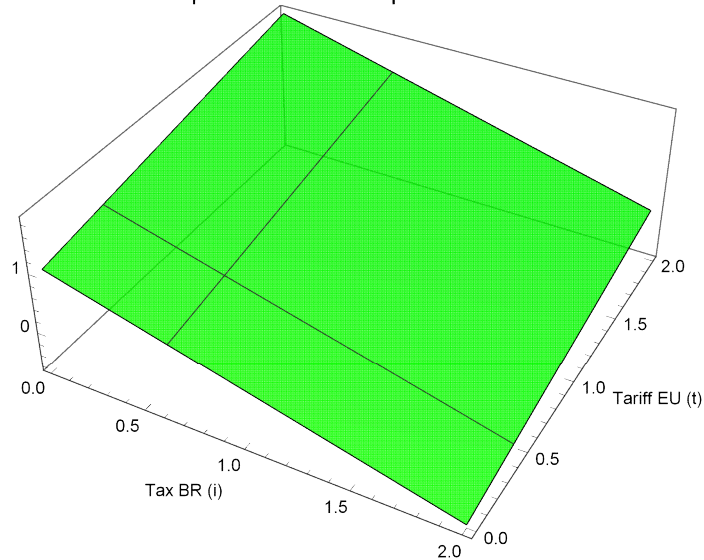


```
Plot3D[qs, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tax BR (i)", "Tariff EU (t)"},
  Mesh → {{sol[NeBrOpti]}, {NeEUOptt}}, PlotTheme → "FullAxes",
  PlotStyle → Directive[Opacity[0.8], Green, Specularity[White, 50]],
  PlotLabel → Style["Soy production in response to tariffs", FontSize → 14]]
```

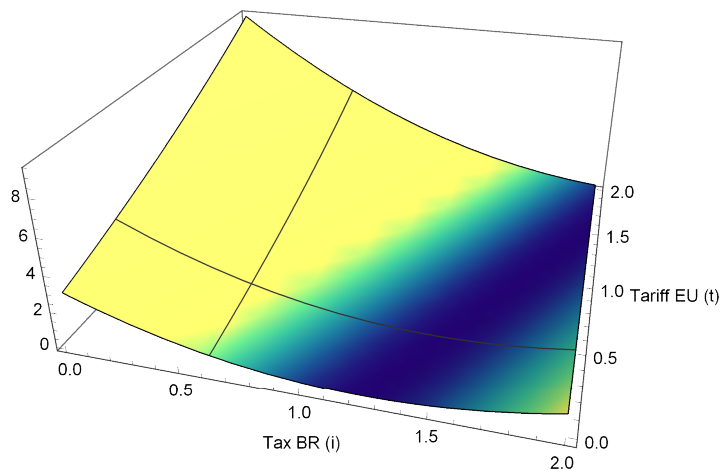


```
Plot3D[qb, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tax BR (i)", "Tariff EU (t)"},
  Mesh → {{NeBrOpti}, {NeEUOptt}}, PlotTheme → "FullAxes",
  PlotStyle → Directive[Opacity[0.8], Green, Specularity[White, 50]],
  PlotLabel → Style["Beef production in response to tariffs", FontSize → 14]]
```

Beef production in response to tariffs



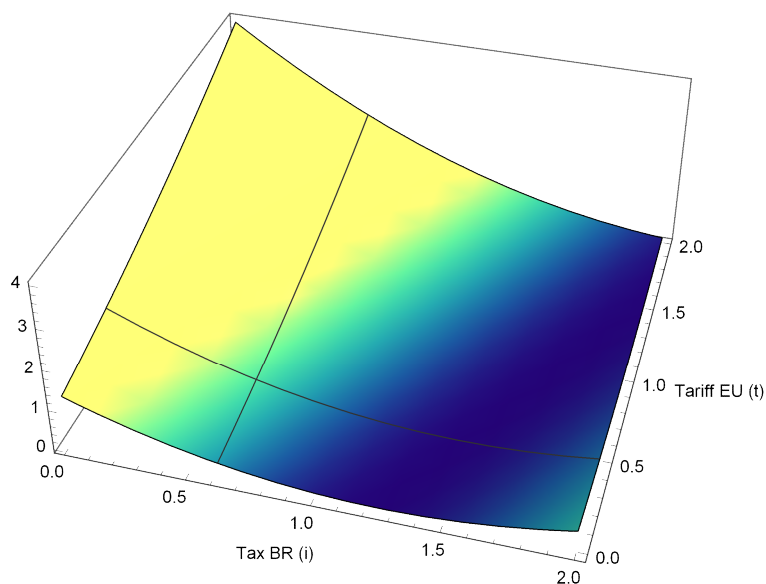
```
Plot3D[dbbe, {i, 0, 2}, {t, 0, 2},
  AxesLabel → {"Tax BR (i)", "Tariff EU (t)"}, Mesh → {{NeBrOpti}, {NeEUOptt}},
  PlotTheme → "FullAxes", ColorFunction → "BlueGreenYellow",
  ColorFunctionScaling → False, ViewPoint → {1.3, -2.4, 2.}]
```



```

Plot3D[dbb, {i, 0, 2}, {t, 0, 2},
  AxesLabel → {"Tax BR (i)", "Tariff EU (t)"}, Mesh → {{NeBrOpti}, {NeEUOptt}},
  PlotTheme → "FullAxes", ColorFunction → "BlueGreenYellow",
  ColorFunctionScaling → False, ViewPoint → {1.3, -2.4, 2.}]

```



Proposition 1: increasing t increases qb

```
ClearAll[ a, b, c, d, e, f, g, α]
```

qb

$$\frac{c - i - \frac{e(a-t)}{b+e}}{2d}$$

D[qb, {t}]

$$\frac{e}{2d(b+e)}$$

Graphical representations of basic curves

```
ClearAll[qs, qb]
```

```
α = 1; a = 3.5; b = 0.1; c = 3; d = 0.6; e = 0.1; f = 2.65; g = 1.14;
```

```
Derbs = D[bs, {qs}]
```

$$3.5 - 0.2 \text{ qs}$$

```
Derbs = D[cs, {qs}]
```

$$0.2 \text{ qs}$$

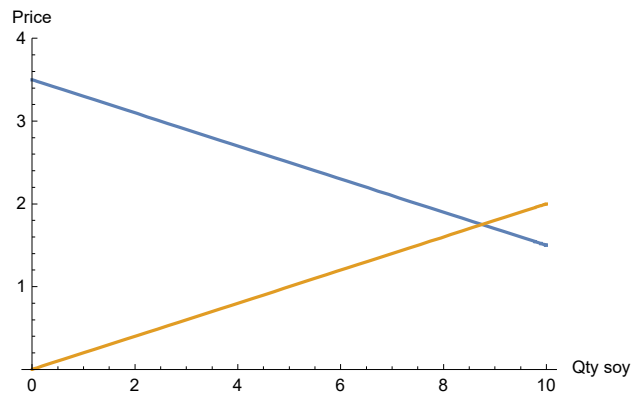
Derbb = D[bb, {qb}]

$3 - 1.2 \text{ qb}$

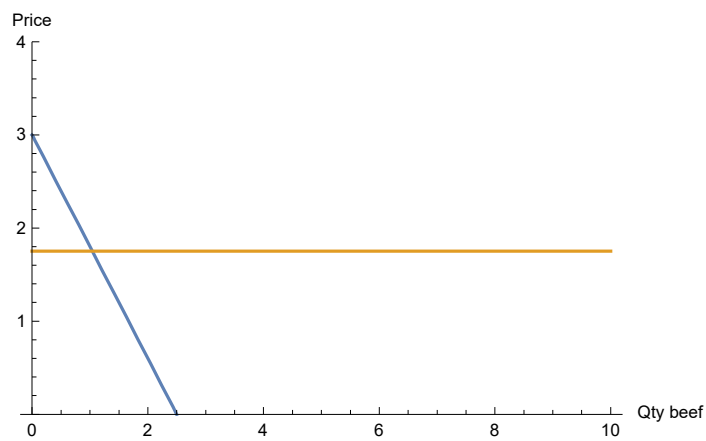
Dercb = D[cb, {qb}]

0.2 qs

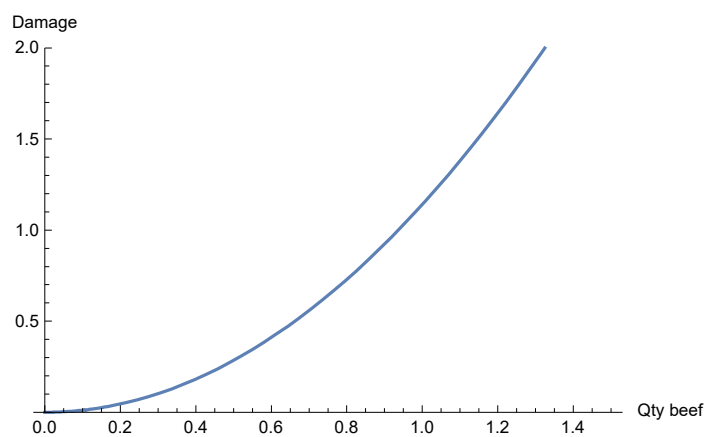
Plot[{Derbs, Dercs}, {qs, 0, 10}, AxesLabel → {"Qty soy", "Price"}, PlotRange → {0, 4}]



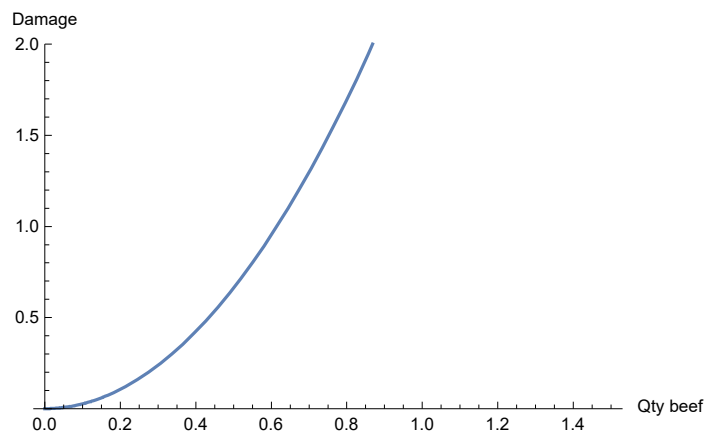
**Plot[{Derbb, 0.2 * 8.75}, {qb, 0, 10},
AxesLabel → {"Qty beef", "Price"}, PlotRange → {0, 4}]**



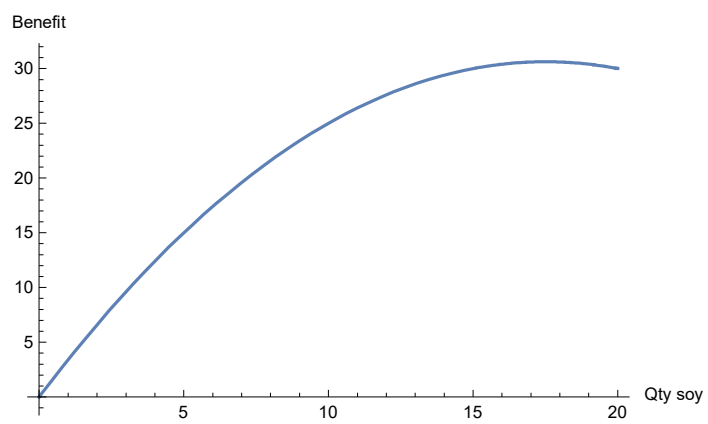
Plot[dbb, {qb, 0, 1.5}, AxesLabel → {"Qty beef", "Damage"}, PlotRange → {0, 2}]



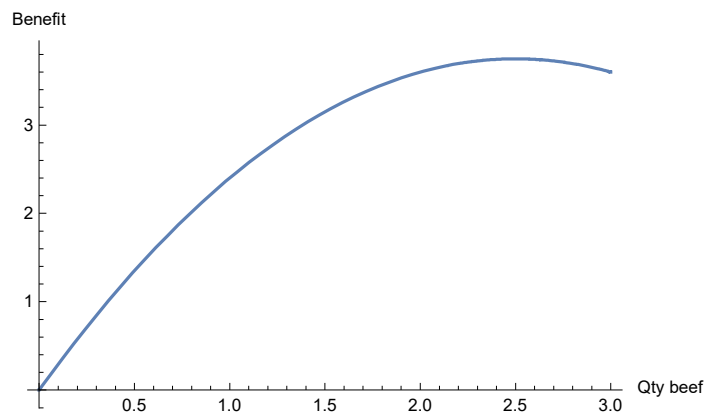
```
Plot[dbe, {qb, 0, 1.5}, AxesLabel → {"Qty beef", "Damage"}, PlotRange → {0, 2}]
```



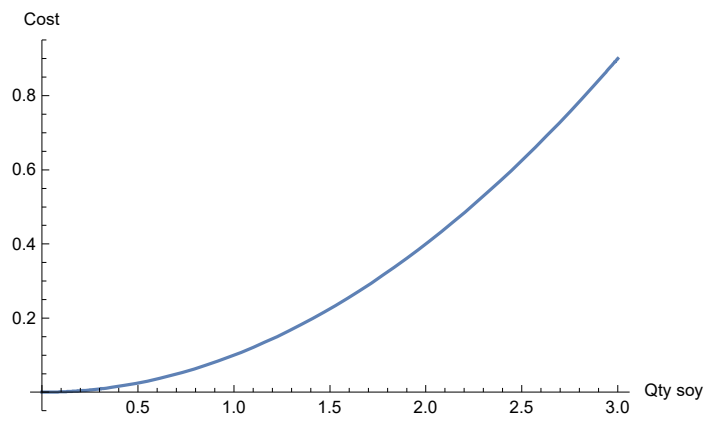
```
Plot[bs, {qs, 0, 20}, AxesLabel → {"Qty soy", "Benefit"}]
```



```
Plot[bb, {qb, 0, 3}, AxesLabel → {"Qty beef", "Benefit"}]
```



```
Plot[cs, {qs, 0, 3}, AxesLabel → {"Qty soy", "Cost"}]
```



```
Plot[c b /. qs → 8.75, {qb, 0, 3}, AxesLabel → {"Qty beef", "Cost"}]
```

