Optimizing welfare on linked supply chains through taxes: An hypothetical example for the beef-soybeans supply chains in international trade.

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February 23, 2019

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1 Introduction

Taxes are involuntary fees levied on individuals or corporations and enforced by the government in order to finance its activities. Tariffs are a special type of taxes imposed by one country on the goods and services imported from another country. Tariffs are a common tool for countries to regulate markets in international trade arenas. The effects of tariffs are felt on prices and quantities of goods traded and, ultimately, on the welfare of the parties involved. Therefore, the enforcement of tariffs plays a role in welfare maximization strategies (Henderson, 2008). For readability, in this paper taxes and tariffs will be referred to only as taxes.

To study this role, an hypothetical scenario where Brazil and the European Union (EU) trade soybeans is developed. In this example, Brazil is the only exporter of soybeans to the EU, and the EU is the only buyer from Brazil in a bilateral relationship.

Additionally, Brazil's economy also depends on the production of beef meat that is exclusively consumed domestically and take soybeans as an input. Furthermore, This production is responsible for environmental externalities that, ultimately, hurts both countries welfares.

As Brazil and the EU interact only through the soybeans trade, the EU is developing an import tax on soybeans aiming to maximize its welfare. Additionally, Brazil can also use taxes to optimize its welfare and counterbalance the effects of the environmental damages caused by the beef industry.

The objective of this paper is to examine how by optimizing their welfares with taxes, Brazil and the European Union will impact the production and trade volumes, global welfare and the external damages caused by meat production.

2 The model

The next section presents the functions that describe the environment where the players of the game interact.

2.1 Market assumptions

A market is any place where two or more parties can meet to engage in an economic transaction. This transaction presupposes the presence of at least one seller and one buyer of goods, services, information, currency, or any combination of these to complete an exchange of property rights between the parties. In a market in equilibrium, the price of this transaction is defined by equalizing the marginal benefit of consumers with the marginal cost of production by suppliers. Therefore, to study the trade impacts of taxes, is necessary to know how consumers and producers behave in respect to the benefits and costs perceived.

2.1.1 Soybeans Market

In describing the soybeans market, functions 1 and 2 represent the benefit of soybeans consumption (B_s) and the costs for soybeans production (C_s).

$$B_s(q_s) = a \cdot q_s - b \cdot q_s^2 \tag{1}$$

$$C_s(q_s) = e \cdot qs^2 \tag{2}$$

Where $B_s(q_s)$ is the utility function of soybeans consumers, a concave quadratic function, and $C_s(q_s)$ is a convex cost function of the soybeans industry. These two functions depend on the quantity of soybeans produced/consumed q_s, while a, b and e are constants defining the shape of the curves.

2.1.2 Beef Market

Similarly with the soybeans market, the beef market is also defined in terms of the benefit of beef consumption (B_b), and the costs of beef production (C_b) as shown by equations 3 and 4.

$$B_b(q_b) = c \cdot q_b - d \cdot q_b^2 \tag{3}$$

$$C_b(q_b, q_s) = C_s'(q_s) \cdot q_b \tag{4}$$

Where $B_b(q_b)$ is the concave utility function for beef consumers and $C_b(q_b, q_s)$ is the linear cost function of the beef industry, that depends on the quantity of beef produced/consumed (q_b) and the quantity of soybeans produced (q_s). The constants c and d define the shape of the functions. In this scenario, the cost function $C_b(q_b, q_s)$ is the link between the production of soybeans and beef. As shown in formula 4, the costs of beef production depend exclusively on the price of soybeans $C_s'(q_s)$ and the quantity produced q_b.

2.2 Externalities assumptions

An externality is an economic term referring to a cost or benefit incurred or received by a third party that has no control over the creation of that cost or benefit.

An externality can be both positive or negative and the costs and benefits can be both private or social.

In this example, the externalities of beef production in Brazil that causes negative effects globally through emissions of greenhouse gases (GHG) and environmental damages are going to be explored. These negative effects are translated to costs to society both by direct damages on health due to pollution or by indirect climate change and environment-related welfare losses.

2.2.1 Beef externalities

In describing beef externalities, functions 5 and 6 represent the damage cost of beef production for the EU (D_bE) and the damage cost of beef production in Brazil (D_bB).

$$D_bE(q_b) = f \cdot q_b^2 \tag{5}$$

$$D_bB(q_b) = g \cdot q_b^2 \tag{6}$$

In the proposed scenario, $D_bE(q_b)$ and $D_bB(q_b)$ are convex functions correlating the estimated welfare losses to the quantity of beef produced. Therefore, f and g are coefficients describing the sensitivity of each region to beef production.

2.3 Coefficients assumption

For the remainder of the article, it is assumed that the coefficients a, b, c, d, e, f and $g \in \mathbb{R} > 0$ in order to maintain the shapes of the curve economically meaningful.

2.4 Profits and Welfare functions

Bellow are described the welfare functions (payoff functions) driving the decisions of the industries and the regions interacting in this model. These welfare functions are inspired by the work of Harstad (2012).

Profit functions for the soy and beef industries:

$$\pi_s(q_s, t) = (B'_s(q_s) - t) \cdot q_s - C_s(q_s) \tag{7}$$

$$\pi_b(q_b, q_s, i) = (B'_b(q_b) - i) \cdot q_b - C_b(q_b, q_s) \tag{8}$$

Expression 7 represents the profits of the soy industry and is translated as the quantity of soybeans traded (q_s) multiplied by the equilibrium price of soybeans ($B'_s(q_s)$, marginal benefit at q_s) adjusted by the specific import tax (t), minus the production costs ($C_s(q_s)$). Equation 8 expresses the profits of the beef industry and is structurally similar to equation 7, as to say: quantity (q_b) times adjusted price ($B'_b(q_b) - i$) minus production cost ($C_b(q_b, q_s)$). Being i the Brazilian tax on beef production.

Welfare functions for the EU (U_E) **and Brazil** (U_B)**:**

$$U_E(q_b, q_s, t) = B_s + q_s \cdot t - B'_s \cdot q_s - D_bE \tag{9}$$

$$U_B(q_b, q_s, i) = \pi_s + \pi_b + B_b + \alpha \cdot i \cdot q_b - D_bB \tag{10}$$

The welfare of the EU is composed of the benefit from consuming soybeans (B_s) plus the tax revenues ($q_s \cdot t$) minus the importing costs ($B'_s \cdot q_s$) and the external damage of beef production in Brazil (D_bE).

Brazil's government welfare is composed of the profits of both industries (π_s, π_b) plus the benefit from consuming beef (B_b) and the tax revenue $(i \cdot q_b)$ adjusted by α, a coefficient that measure the importance of the tax revenue for the Brazilian Government and is evaluated in the [0,1] interval.

3 Game steps

The game has three steps, first the European Union and Brazil will decide whether to apply the taxes or not and in which order (sequentially or simultaneously). Then the taxes values to be implemented are calculate. Lastly, each industry will define how much to produce in the market equilibrium.

Step 1 - Player's sequence: Brazil and EU decide if the implementation of the taxes are beneficial to their respective welfare. In a positive case, they will decide what is the best order in which to implement (sequentially or simultaneously), and who makes the first move.

Step 2 - Taxes definition: European Union and Brazil decide the optimum tax value that each country should apply.

Step 3 - Market equilibrium: The soybeans and beef industries decide how much to produce in response to the taxes imposed.

4 Finding players strategies

To find the players strategies the model is solved by backward induction, as to say, the evaluation begins at the last steps and follows a decreasing order until the first step.

4.1 Market equilibrium - Step 3

In a perfect market equilibrium, production will reach the point where the marginal benefit of consumers (B') equals the marginal cost of producers (C'). Based on this assumption, it possible to calculate the equilibrium quantities for q_s and q_b by setting the marginal benefit functions equal to the marginal cost functions for each industry.

4.1.1 Reaction function of the soy industry

$$(B_s'(q_s) - t) = C_s'(q_s) \tag{11}$$

By solving equation 11 we get the quantity of soy produced as a function of tax t. In this example, that is equivalent to:

$$q_s(t) = \frac{a-t}{2(b+e)}, \ for \ a \geqslant t \tag{12}$$

In equation 12, $q_s(t)$ stands for the reaction of the soy industry to t. In other words, the amount of soy produced depends on the value of the tax imposed by the EU.

4.1.2 Reaction function of the beef industry

$$B_b'(q_b) = C_b'(q_b, q_s) \tag{13}$$

By solving equation 13 we get the reaction function of the beef industry q_b as a function of q_s and i.

$$q_b(q_s, i) = \frac{c - 2 \cdot e \cdot q_s - i}{2d} \tag{14}$$

As the value of q_s is known, it can be substituted in expression 14 for the calculation of q_b as a function of t and i.

$$q_b(t, i) = \frac{-\frac{e(a-t)}{b+e} + c - i}{2d} \tag{15}$$

Proposition 1 - The European tax on soybeans t increases the production of beef in Brazil, and the Brazilian tax i decreases the production of beef as can be seen on the proofs below:

$$\frac{\partial q_b}{\partial t} = \frac{e}{2d(b+e)} > 0 \tag{16}$$

$$\frac{\partial q_b}{\partial i} = \frac{-1}{2d} < 0 \tag{17}$$

4.2 Brazil and the European Union decide the optimum taxes - Step 2

4.2.1 Optimal tax calculation for the European Union

For simplicity, only the adoption of specific taxes for both markets are evaluated. The optimal tax that maximizes EU's welfare can be calculated by maximizing U_E in respect to t as shown below:

$$\max_{\{t\}} U_E(i,t) = t^* \tag{18}$$

$$\max_{\{t\}} [-f \cdot q_b^2 + a \cdot q_s - b \cdot q_s^2 - q_s(a - 2b \cdot q_s) + q_s \cdot t] = t^* \tag{19}$$

$$t(i^*) = \frac{e\left(a\left(d^2 + e \cdot f\right) - f(b+e)(c-i)\right)}{b \cdot d^2 + e\left(2d^2 + e \cdot f\right)} \tag{20}$$

Proposition 2 - From equation 20 it is possible to draw that an increase in the beef tax i in Brazil will lead to an increase of tax t by the EU as demonstrated in equation 21.

$$\frac{\partial t}{\partial i} = \frac{e \cdot f(b+e)}{b \cdot d^2 + e\left(2d^2 + e \cdot f\right)} > 0 \tag{21}$$

This proposition can be intuitively explained by the fact that a reduction in the beef production caused by tax i will decrease the price of soybeans and increase the volume traded with Europe. This will open up the opportunity for increased import tax revenues.

4.2.2 Optimal tax calculation for Brazil

If the EU enforces an import tax on soybeans, Brazil can expect a reduction on its internal price, which will allow the production of beef to increase, as shown in *Proposition 1.* Therefore, by setting the tax i, Brazil can increase tax revenues and reduce the damages caused by a bigger beef production. To find the optimal tax value, U_B has to be maximized in respect to i as shown below:

$$\max_{\{i\}} U_B(i,t) = i^* \tag{22}$$

$$\begin{aligned} \max_{\{i\}} [c \cdot q_b - d \cdot q_b^2 - g \cdot q_b^2 + q_b(c - i - 2d \cdot q_b) \\ - 2e \cdot q_b \cdot q_s - e \cdot q_s^2 + q_s(a - 2bq_s - t) + iq_b \cdot \alpha] = i^* \end{aligned} \tag{23}$$

$$\begin{aligned} i(t^*) = \frac{-a \cdot e(\alpha \cdot d + d + g) + b \cdot c(\alpha \cdot d + g)}{(b+e)(2\alpha \cdot d + d + g)} \\ + \frac{c \cdot e(\alpha \cdot d + g) + e \cdot t(\alpha \cdot d + d + g)}{(b+e)(2\alpha \cdot d + d + g)} \end{aligned} \tag{24}$$

Proposition 3 - From equation 24 it is noticeable that an increase in tax t will lead to an increase in tax i as shown below.

$$\frac{\partial i}{\partial t} = \frac{e(\alpha d + d + g)}{(b+e)(2\alpha d + d + g)} > 0 \tag{25}$$

The same conclusion could be achieved intuitively by reasoning that an increase in tax t will reduce the internal price of soybeans in Brazil, leading to an increase in beef production and creating room for a stronger tax adjustment by the Brazilian government .

Proposition 4 - The relative importance of the tax revenue represented by α depends on the value of tax t. The bigger the tax t the smaller the effect of α in the Brazilian decision of tax i, $\forall$ $\frac{\partial i}{\partial \alpha} > 0$. Another possible observation is that as $\alpha \to \infty$, $\frac{\partial i}{\partial \alpha} \to 0$, therefore, Brazil will maximize its welfare when α equals 1. For $\alpha = [0,1]$.

$$\frac{\partial i}{\partial \alpha} = \frac{d(e(a(d+g)+c(d-g)-t(d+g))+b\cdot c(d-g))}{(b+e)(2\alpha\cdot d+d+g)^2} \tag{26}$$

4.3 Players decide the order in which they will add the taxes - Step 1

As the definition of the optimal tax t and tax i is interdependent (prepositions 2 and 3), the order in which they are defined is important for the end result. Therefore, the last stage of the game was designed to evaluate the effects tax implementation order.

Suppose the players have the option to choose the timing of their tax implementation. The two options they have could be called *wait* and *move*. If both players decide to *wait*, no tax is implemented. However, if both decide to *move* at the same time, the tax will be simultaneously defined. If one decides to *move* and the other decides to *wait*, a sequential game where the mover sets its optimal tax first will unfold.

A payoff table was designed to find the equilibrium of this final step and is presented in the Results section.

5 Results

As it is not possible to explore the results purely on analytical terms, to gain understanding of potential outcomes of the game, a set of coefficients were empirically selected to approximate conventional microeconomic curves. The values selected are found in Table 1.

Table 1: Selected coefficient values

a = 3.5	b = 0.1	c = 3
d = 0.6	e = 0.1	f = 2.65
g = 1.14		

5.1 Basic curves

With the coefficients selected in Table 1, the basic functions presented in section 2 produce the curves illustratively shown in **Figure 1**. The marketing equilibrium for both markets before the implementation of taxes are shown in **Figure 2**.

5.2 Nash equilibrium

To find the final Nash Equilibrium (NE) in this game 4 different scenarios were developed. For all scenarios, the welfare, the external damages and the quantity of soybeans and beef meat produced were evaluated. However, to determine the NE, only the welfare of the regions were taken into account as Brazil and EU are considered welfare maximizing states.

- The first scenario explores the results in case both countries decided to *wait*, i.e no tax were implemented. This scenario was identified as $i, t = 0$ and can be understood as a benchmark or static scenario.

- The second scenario, considers that the EU decided to *move* first and set an import tax on soybeans. Brazil reacts and sets the optimal beef tax in response. That scenario is identified as $i(t^*)$.

- The third scenario considered that Brazil *moves* first and set its beef tax, then EU reacted by setting the optimal import tax on soybeans. That scenario is identified as $t(i^*)$.

- The forth scenario simulates both countries moving at the same time and setting the optimal tax simultaneously. This scenario is identified as i^*, t^*.

In addition to these 4 scenarios, the model was tested for two values of α (0 and 1). In that way, the importance of the tax revenue in Brazil's decisions could be assessed. Below, a summary table with the resulting values of i and t for each scenario and value of α is presented.

Table 2: Values of i and t in the 4 different scenarios

	$\alpha = 1$		$\alpha = 0$	
	i	t	i	t
Both countries wait, $i, t = 0$	0	0	0	0
EU moves first, $i(t^*)$	0.559442	0.444238	0.437636	0.444238
Brazil moves first, $t(i^*)$	0.382653	0.595023	0.215517	0.529163
Both players move, i^*, t^*	0.663487	0.705687	0.545019	0.659004

As discussed in Proposition 4, the scenarios with $\alpha = 1$ have greater values of i compared to scenarios where $\alpha = 0$. In other words, for the same level of t Brazil maximize its welfare with a bigger tax i when it accounts for the tax revenues.

After finding the tax in each scenario, the resulting welfare could be calculated to create the payoff tables for players' strategies in step 1 as shown in Tables 3 and 4.

Table 3: Step 1 - Bimatrix game. Welfare results for $\alpha = 1$

		European Union			
		Wait $t()$		Move $t*$	
Brazil	Wait $i()$	(b) 8.89323	(e) 4.78082	(b) 7.53672	(e) 7.69685
	Move i^*	(b) 6.91832	(e) 7.09856	(b) 6.68166	(e) 8.18605

Where (b) represents the welfare of Brazil and (e) the welfare of Europe.

Table 4: Step 1 - Bimatrix game. Results for $\alpha = 0$

		European Union			
		Wait $t()$		Move $t*$	
Brazil	Wait $i()$	(b) 8.89323	(e) 4.78082	(b) 7.12915	(e) 7.26038
	Move i^*	(b) 6.72496	(e) 7.34072	(b) 6.33764	(e) 7.75572

Where (b) represents the welfare of Brazil and (e) the welfare of Europe.

The Nash Equilibrium is found in the second scenario when Europe moves first and Brazil optimizes its welfare next. The value of α does not change the equilibrium. NE $= i(t^*)$

The results of welfare, external damages and quantities produced are presented in Tables 5 and 6, below:

Table 5: Results, $\alpha = 1$

		$i, t = 0$	$i(t^*)$	$t(i^*)$	i^*, t^*
Welfare	Brazil	8.89323	7.53672	6.91832	6.68166
	E. Union	4.78082	7.69685	7.09856	8.18605
	Total	13.6740	15.2336	14.0169	14.8677
External damages	Brazil	1.23698	0.59556	1.07421	0.69855
	E. Union	2.87543	1.38442	2.49706	1.62384
	Total	4.11241	1.97999	3.57127	2.32240
Production	Soybeans	8.75000	7.63941	7.26244	6.98578
	Beef meat	1.04160	0.76056	0.97071	0.78279

Table 6: Results, $\alpha = 0$

		$i, t = 0$	$i(t^*)$	$t(i^*)$	i^*, t^*
Welfare	Brazil	8.89323	7.12915	6.72469	6.33764
	E. Union	4.78082	7.26038	6.34072	7.75572
	Total	13.6740	14.3895	13.0654	14.0934
External damages	Brazil	1.23698	0.84720	1.33599	0.84720
	E. Union	2.87543	1.96938	3.10559	1.96938
	Total	4.11241	2.81659	4.44158	2.81659
Production	Soybeans	8.75000	7.63940	7.42700	7.10240
	Beef meat	1.04160	0.82668	1.08250	0.86206

6 Discussion

6.1 Welfare analysis

The payoff tables show that both countries have dominant strategies. Brazil will always wait for the decision of the EU to determine the optimum tax on beef production, and the EU will always choose to move and implement the import tax regardless of Brazil's decision.

In this specific setting, the implementation of taxes reduced Brazilian welfare and increases the welfare of the EU. However, the NE will allow for the social optimum solution once it results in the maximum welfare achievable (15.2336 for $\alpha = 1$ and 14.3895 for $\alpha = 0$). Also, it is possible to observe that the NE choices are not altered by the value of α.

Figure 3 shows a graphical representation of the two welfare functions and allow for intuitive visualization of the Nash Equilibrium solution.

6.2 External damages analysis

Similar to what happened with the welfare, the NE provided the lowest total external damages results. That emerges as a result of the combined taxes from scenario $i(t^*)$ that causes the greatest suppression in beef production. Therefore, as long as EU takes the initiative, the minimum external costs will be achieved.

Additionally, when Brazil accounts for the tax income ($\alpha = 1$), the production of beef has a greater suppression in almost all scenarios ($i(t^*) = 0$, $t(i^*)$ and $i^*(t^*)$), in comparison with the $\alpha = 0$ scenario. This observation reflects the importance of accounting for the tax revenue in the total welfare for government decisions.

6.3 Production and trade volumes

As expected, after the implementation of the taxes the volume traded internationally is reduced. Also, tax t and the quantity of soybeans produced qs are inversely proportional as seen in equation 12. Therefore, considering that the production of soybeans is exclusively traded with the EU, a reduction in soybeans production means a reduction in the volumes traded.

Regarding the production of beef meet, according to *Proposition 1*, an increase in q_b proportional to t and a decrease proportional to i was expected. However, in the NE the effect of i overpowers that of t and the amount of beef meat produced is also reduced when compared with the no-taxes scenario.

6.4 External damages in linked supply chains

A possible generalization of this model would imply that in linked supply chain scenarios as the one described in this paper, a tax on non-polluting goods can lead to an increase in polluting activities due to the reduction of local prices of production factors. However, the external damages caused by such polluting activities can be counter-acted by the implementation of local taxes in the polluting region.

7 Conclusions

The implementation of taxes in this settings reduced the total amount of commodities produced leading to less environmental damages and less trade. However, there was an increase in total welfare resulting from the increase in the EU's welfare that outweighed the reduction in the welfare of Brazil.

This simple model limited the evaluation to only two trading regions and one traded commodity. Therefore, an interesting extension would allow for other countries and other commodities to participate in the international trade and improve the input factor relationship description between the commodities as well as their environmental damages.

References

D.R. Henderson. *The Concise Encyclopedia of Economics*. Liberty Fund, 2008. ISBN 9780865976665. URL `https://books.google.de/books?id=vq9aAAAAYAAJ`.

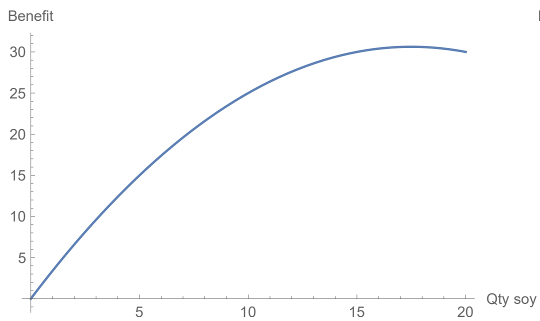
Bård Harstad. Buy coal! a case for supply-side environmental policy. *Journal of Political Economy*, 120(1):77–115, 2012.

A Appendixes

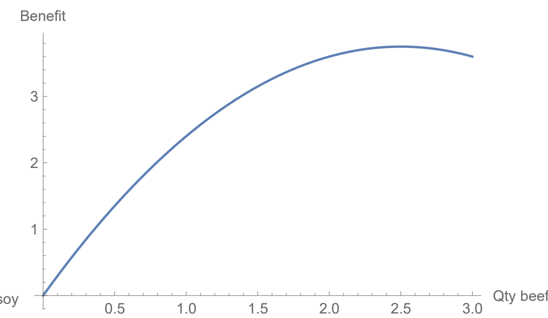
A.1 Figures

Figure 1: Basic functions

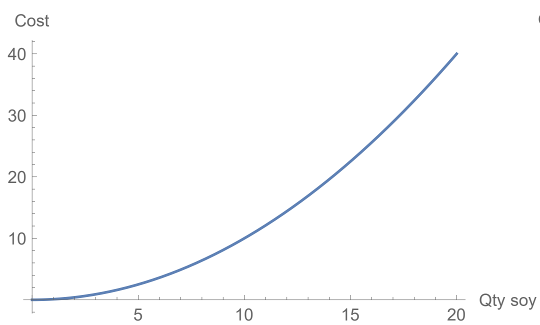
(a) Utility soybeans



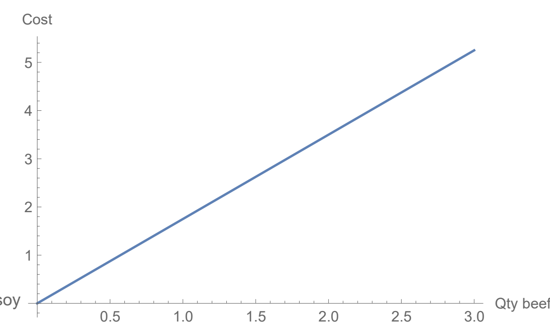
(b) Utility beef



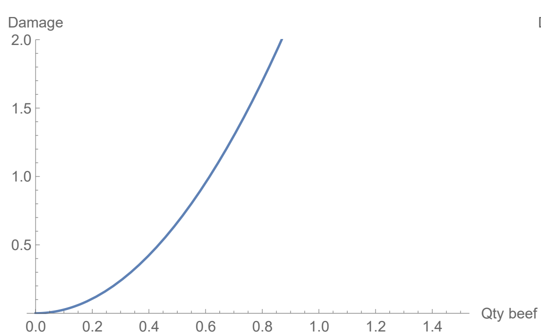
(c) Production costs of soybeans



(d) Production costs of beef for a constant price of soybeans



(e) Damage beef Europe



(f) Damage beef Brazil

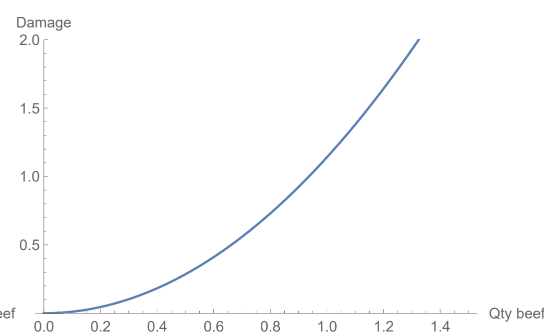


Figure 2: Market equilibrium for $i, t = 0$

(a) Market soybeans (b) Market beef (constant price of soybeans)

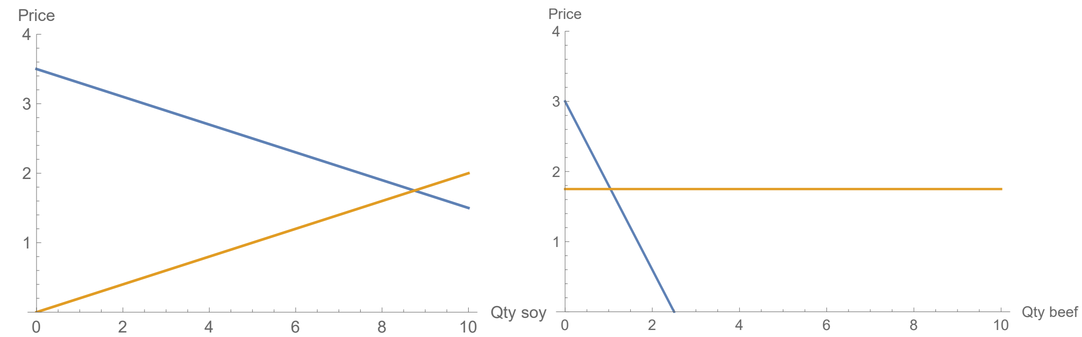
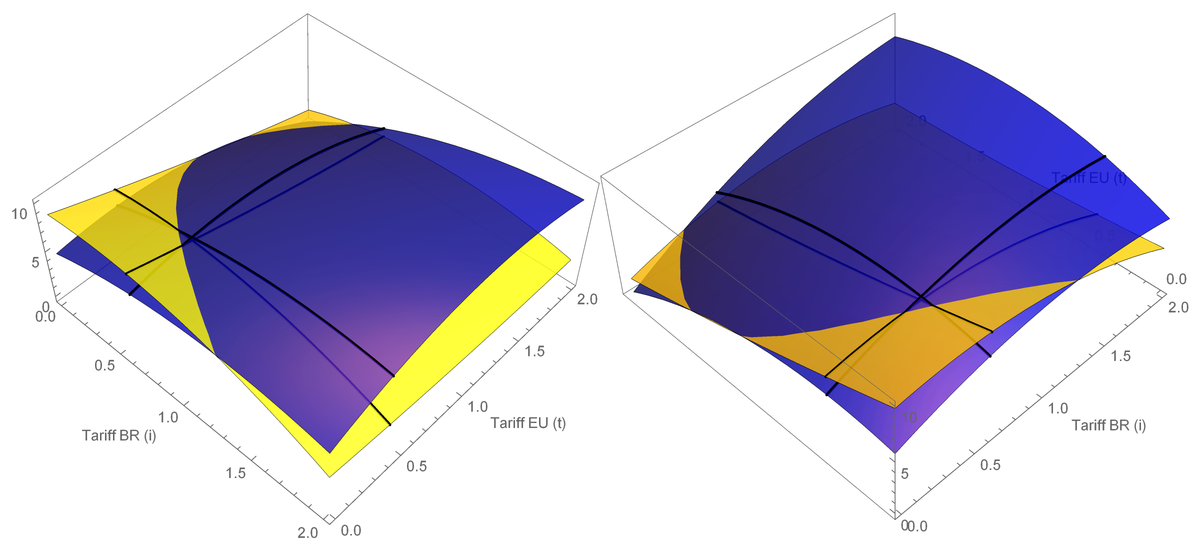


Figure 3: Welfare planes

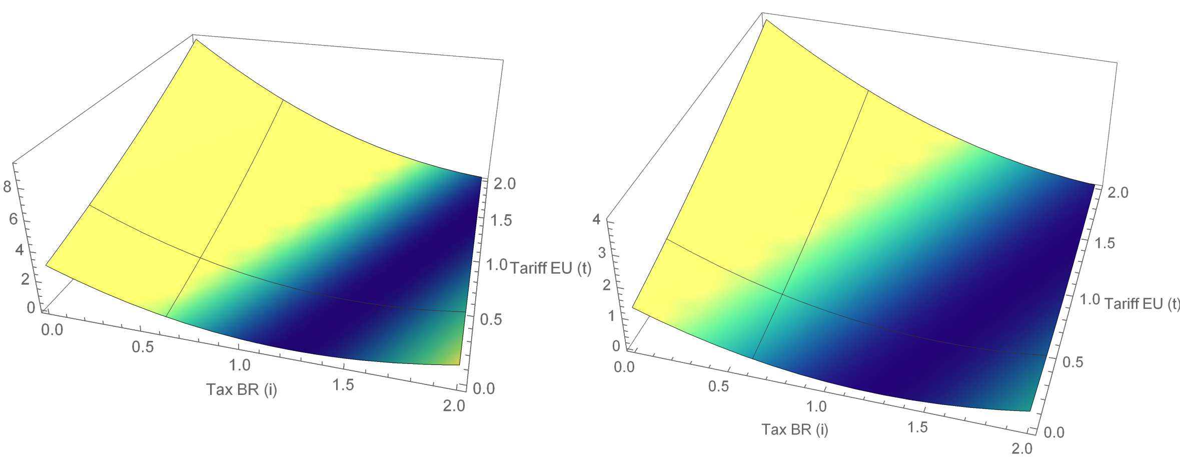
(a) View a (b) View b



The planes represent the welfare changes of Brazil (yellow) and the EU (blue) in response to the changes in the taxes i and t. The lines across the planes represent the values of t and i in the Nash Equilibrium for $\alpha = 1$.

Figure 4: External damage planes

(a) Damages in EU (b) Damages in Brazil



The plans represent the external damage varying with both i and t. The lines across the planes represent the values of t and i in the Nash Equilibrium for $\alpha = 1$.

A.2 Mathematica notebook

Optimizing welfare on linked supply chains through taxes.

Cleaning variables

```
ClearAll[qs, qb ]
ClearAll[ a, b, c, d, e, f, g, α]
ClearAll[i, t]
ClearAll[bs, bb, cs, cb, dbe, dbb, πs, πb]
ClearAll[UB, UE]
ClearAll[opti, optt]
ClearAll[BrOpti, EUOptt]
```

Defining market curves

Basic Mkt functions

```
bs = a * qs - b * qs^2
```

$a\,qs - b\,qs^2$

```
bb = c * qb - d * qb^2
```

$c\,qb - d\,qb^2$

```
cs = e * qs^2
```

$e\,qs^2$

```
cb = D[cs, {qs}] * qb
```

$2\,e\,qb\,qs$

```
dbe = f * qb^2
```

$f\,qb^2$

```
dbb = g * qb^2
```

$g\,qb^2$

```
πs = (D[bs, {qs}] - t) * qs - cs
```

$-e\,qs^2 + qs\,(a - 2\,b\,qs - t)$

```
πb = (D[bb, {qb}] - i) * qb - cb
```

$qb\ (c - i - 2\ d\ qb) - 2\ e\ qb\ qs$

Country Functions

Brazil

```
UB = πs + πb + bb + α * i * qb - dbb
```

$c\ qb - d\ qb^2 - g\ qb^2 + qb\ (c - i - 2\ d\ qb) - 2\ e\ qb\ qs - e\ qs^2 + qs\ (a - 2\ b\ qs - t) + i\ qb\ \alpha$

European Union

```
UE = bs - D[bs, {qs}] * qs - dbe + qs * t
```

$-f\ qb^2 + a\ qs - b\ qs^2 - qs\ (a - 2\ b\ qs) + qs\ t$

Solving reaction functions for companies

```
rb = Solve[(D[bb, {qb}] - i) == D[cb, {qb}], {qb}]
```

$$\left\{\left\{qb \to \frac{c - i - 2\ e\ qs}{2\ d}\right\}\right\}$$

```
qb = rb[[1]][[1]][[2]]
```

$$\frac{c - i - 2\ e\ qs}{2\ d}$$

```
rs = Solve[(D[bs, {qs}] - t) == D[cs, {qs}], {qs}]
```

$$\left\{\left\{qs \to \frac{a - t}{2\ (b + e)}\right\}\right\}$$

```
qs = rs[[1]][[1]][[2]]
```

$$\frac{a - t}{2\ (b + e)}$$

Finding i and t

```
opti = ∂_i UB == 0
```

$$-\frac{c}{2\ d} + \frac{c - i - \frac{e\ (a-t)}{b+e}}{2\ d} + \frac{g\ \left(c - i - \frac{e\ (a-t)}{b+e}\right)}{2\ d^2} - \frac{i\ \alpha}{2\ d} + \frac{\left(c - i - \frac{e\ (a-t)}{b+e}\right)\ \alpha}{2\ d} == 0$$

```
rUB = Solve[opti, {i}][[1]][[1]][[2]]
```

$(-a\ d\ e + b\ c\ g - a\ e\ g + c\ e\ g + d\ e\ t + e\ g\ t + b\ c\ d\ \alpha - a\ d\ e\ \alpha + c\ d\ e\ \alpha + d\ e\ t\ \alpha)\ /$
$((b + e)\ (d + g + 2\ d\ \alpha))$

```
Simplify[rUB]
```

$$(b\,c\,(g+d\,\alpha)+c\,e\,(g+d\,\alpha)-a\,e\,(d+g+d\,\alpha)+e\,t\,(d+g+d\,\alpha))\,/\,((b+e)\,(d+g+2\,d\,\alpha))$$

```
Simplify[D[rUB, t]]
```

$$\frac{e\,(d+g+d\,\alpha)}{(b+e)\,(d+g+2\,d\,\alpha)}$$

```
Simplify[D[rUB, α]]
```

$$(d\,(b\,c\,(d-g)+e\,(c\,(d-g)+a\,(d+g)-(d+g)\,t)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)$$

$$\partial_\alpha((b\,c\,(g+d\,\alpha)+c\,e\,(g+d\,\alpha)-a\,e\,(d+g+d\,\alpha)+e\,t\,(d+g+d\,\alpha))\,/\,((b+e)\,(d+g+2\,d\,\alpha)))$$

$$\frac{b\,c\,d-a\,d\,e+c\,d\,e+d\,e\,t}{(b+e)\,(d+g+2\,d\,\alpha)}-(2\,d\,(b\,c\,(g+d\,\alpha)+c\,e\,(g+d\,\alpha)-a\,e\,(d+g+d\,\alpha)+e\,t\,(d+g+d\,\alpha)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)$$

$$\text{Simplify}\Big[\frac{b\,c\,d-a\,d\,e+c\,d\,e+d\,e\,t}{(b+e)\,(d+g+2\,d\,\alpha)}-(2\,d\,(b\,c\,(g+d\,\alpha)+c\,e\,(g+d\,\alpha)-a\,e\,(d+g+d\,\alpha)+e\,t\,(d+g+d\,\alpha)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)\Big]$$

$$(d\,(b\,c\,(d-g)+e\,(c\,(d-g)+a\,(d+g)-(d+g)\,t)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)$$

$$\text{optt} = \partial_t \text{UE} == 0$$

$$-\frac{a}{2\,(b+e)}+\frac{a-\frac{b\,(a-t)}{b+e}}{2\,(b+e)}-\frac{e\,f\left(c-i-\frac{e\,(a-t)}{b+e}\right)}{2\,d^2\,(b+e)}+\frac{a-t}{2\,(b+e)}-\frac{t}{2\,(b+e)}==0$$

```
rUE = Solve[optt , t][[1]][[1]][[2]]
```

$$\left(\frac{a\,b}{2\,(b+e)^2}-\frac{a}{2\,(b+e)}-\frac{a\,e^2\,f}{2\,d^2\,(b+e)^2}+\frac{c\,e\,f}{2\,d^2\,(b+e)}-\frac{e\,f\,i}{2\,d^2\,(b+e)}\right)\Big/\left(\frac{b}{2\,(b+e)^2}-\frac{1}{b+e}-\frac{e^2\,f}{2\,d^2\,(b+e)^2}\right)$$

```
Simplify[rUE]
```

$$\frac{e\left(a\left(d^2+e\,f\right)-(b+e)\,f\,(c-i)\right)}{b\,d^2+e\left(2\,d^2+e\,f\right)}$$

```
D[rUE, i]
```

$$-\left((e\,f)\Big/\left(2\,d^2\,(b+e)\left(\frac{b}{2\,(b+e)^2}-\frac{1}{b+e}-\frac{e^2\,f}{2\,d^2\,(b+e)^2}\right)\right)\right)$$

```
α = 1; a = 3.5; b = 0.1; c = 3; d =0.6; e = 0.1;
f =2.65;  g = 1.14; (*g= 1.14, f = 2.65;*)
sol = Solve[rUB - i == 0 && rUE - t == 0, {i, t}]
```

{{i → 0.663487, t → 0.705687}}

Results

Benchmark

```
UE /. {i → 0, t → 0}
4.78082

UB /. {i → 0, t → 0}
8.89323

dbb /. {i → 0, t → 0}
1.23698

dbe /. {i → 0, t → 0}
2.87543

totalwelfare = UE + UB /. {i → 0, t → 0}
13.674

totaldamage = dbb + dbe /. {i → 0, t → 0}
4.11241

qb /. {i → 0, t → 0}
1.04167

qs /. {i → 0, t → 0}
8.75
```

Optimal Tariffs

```
Simplify[UE /. sol]
{8.18605}

Simplify[UB /. sol]
{6.68166}

Simplify[dbb /. sol]
{0.698558}

dbe /. sol
{1.62384}

Simplify[totalwelfare = UE + UB /. sol]
{14.8677}

totaldamage = dbb + dbe /. sol
{2.3224}
```

```
Simplify[qb /. sol]
```
{0.782796}

```
qs /. sol
```
{6.98578}

Results if Brazil sets the tariff first t(i*)

```
BrOpti = rUB /. t → 0
```
0.382653

```
EUOptt = rUE /. i → BrOpti
```
0.595023

```
UE /. {i → BrOpti, t → EUOptt}
```
7.09856

```
UB /. {i → BrOpti, t → EUOptt}
```
6.91832

```
dbb /. {i → BrOpti, t → EUOptt}
```
1.07421

```
dbe /. {i → BrOpti, t → EUOptt}
```
2.49706

```
totalwelfare = UE + UB /. {i → BrOpti, t → EUOptt}
```
14.0169

```
totaldamage = dbb + dbe /. {i → BrOpti, t → EUOptt}
```
3.57127

```
qb /. {i → BrOpti, t → EUOptt}
```
0.970715

```
qs /. {i → BrOpti, t → EUOptt}
```
7.26244

Results if EU sets the tariff first i(t*)

```
NeEUOptt = rUE /. i → 0
```
0.444238

```
NeBrOpti = rUB /. t → EUOptt
```
0.619448

```
UE /. {i → NeBrOpti, t → NeEUOptt}
```

7.89179

```
UB /. {i → NeBrOpti, t → NeEUOptt}
```

7.52937

```
dbb /. {i → BrOpti, t → EUOptt}
```

1.07421

```
dbe /. {i → NeBrOpti, t → NeEUOptt}
```

1.33797

```
totalwelfare = UE + UB /. {i → NeBrOpti, t → NeEUOptt}
```

15.4212

```
totaldamage = dbb + dbe /. {i → NeBrOpti, t → NeEUOptt}
```

1.91355

```
qb /. {i → NeBrOpti, t → NeEUOptt}
```

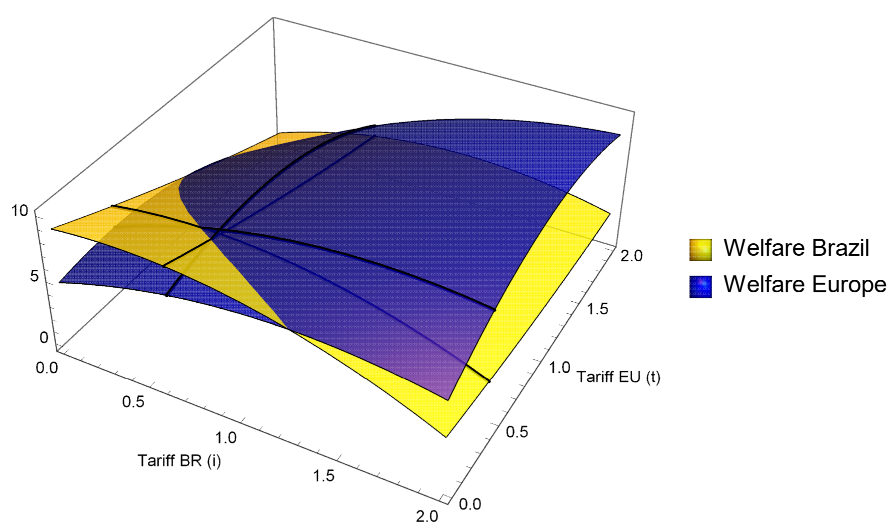
0.710559

```
qs /. {i → NeBrOpti, t → NeEUOptt}
```

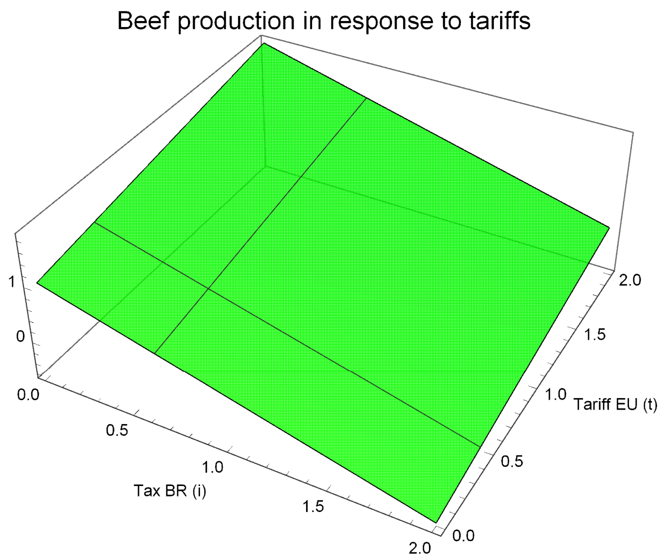
7.63941

Graphing the welfare to find the best parameters.

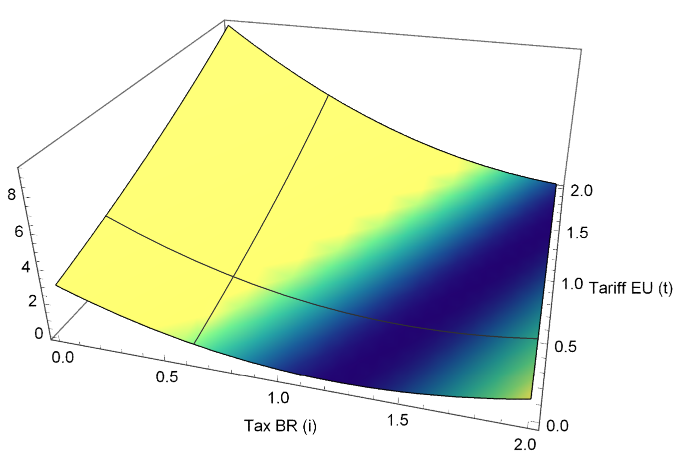
```
welfarePlot =
 Plot3D[{UB, UE}, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tariff BR (i)", "Tariff EU (t)"},
  Mesh → {{NeBrOpti}, {NeEUOptt}}, MeshStyle → {Thick, Thick},
  PlotTheme → "FullAxes", PlotLegends → {"Welfare Brazil", "Welfare Europe"},
  PlotStyle → {Directive[Opacity[0.8], Yellow, Specularity[White, 50]],
    Directive[Opacity[0.8], Blue, Specularity[White, 50]]}]
```



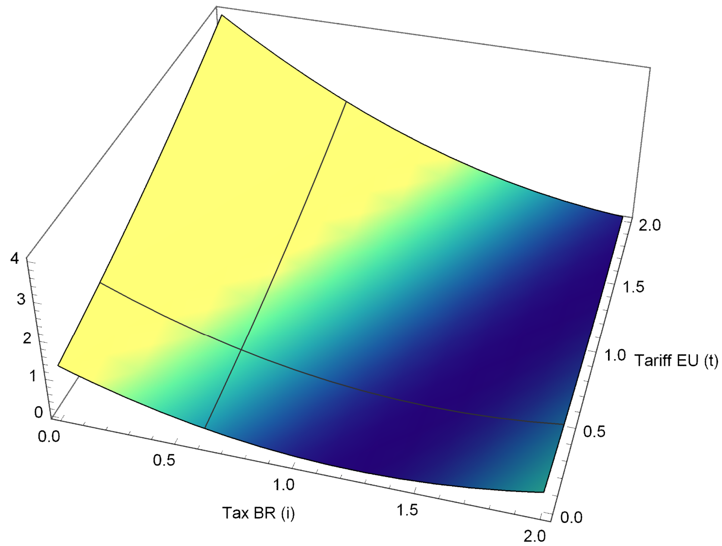
```
Plot3D[qs, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tax BR (i)", "Tariff EU (t)"},
    Mesh → {{sol[NeBrOpti}, {NeEUOptt}}, PlotTheme → "FullAxes",
 PlotStyle → Directive[Opacity[0.8], Green, Specularity[White, 50]],
 PlotLabel → Style["Soy production in response to tariffs", FontSize → 14]]

Plot3D[qb, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tax BR (i)", "Tariff EU (t)"},
 Mesh → {{NeBrOpti}, {NeEUOptt}}, PlotTheme → "FullAxes",
 PlotStyle → Directive[Opacity[0.8], Green, Specularity[White, 50]],
 PlotLabel → Style["Beef production in response to tariffs", FontSize → 14]]
```




```
Plot3D[dbe, {i, 0, 2}, {t, 0, 2},
 AxesLabel → {"Tax BR (i)", "Tariff EU (t)"}, Mesh → {{NeBrOpti}, {NeEUOptt}},
 PlotTheme → "FullAxes", ColorFunction → "BlueGreenYellow",
 ColorFunctionScaling → False, ViewPoint → {1.3, -2.4, 2.}]
```



```
Plot3D[dbb, {i, 0, 2}, {t, 0, 2},
 AxesLabel → {"Tax BR (i)", "Tariff EU (t)"}, Mesh → {{NeBrOpti}, {NeEUOptt}},
 PlotTheme → "FullAxes", ColorFunction → "BlueGreenYellow",
 ColorFunctionScaling → False, ViewPoint → {1.3, -2.4, 2.}]
```



Proposition 1: increasing t increases qb

```
ClearAll[ a, b, c, d, e, f, g, α]
```

```
qb
```

$$\frac{c - i - \frac{e\,(a-t)}{b+e}}{2\,d}$$

```
D[qb, {t}]
```

$$\frac{e}{2\,d\,(b+e)}$$

Graphical representations of basic curves

```
ClearAll[qs, qb ]
```

```
α = 1; a = 3.5; b = 0.1; c = 3; d = 0.6; e = 0.1; f = 2.65;  g = 1.14;
```

```
Derbs = D[bs, {qs}]
```

3.5 - 0.2 qs

```
Dercs = D[cs, {qs}]
```

0.2 qs

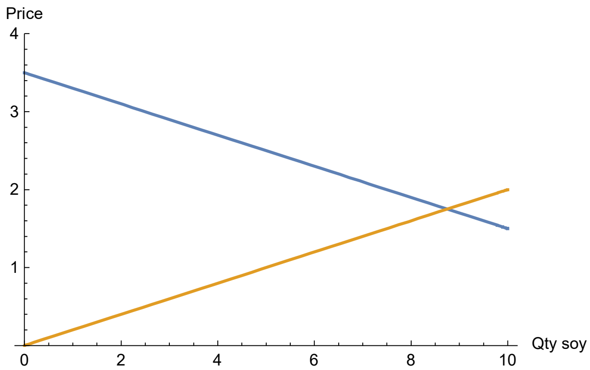
```
Derbb = D[bb, {qb}]
```

3 − 1.2 qb

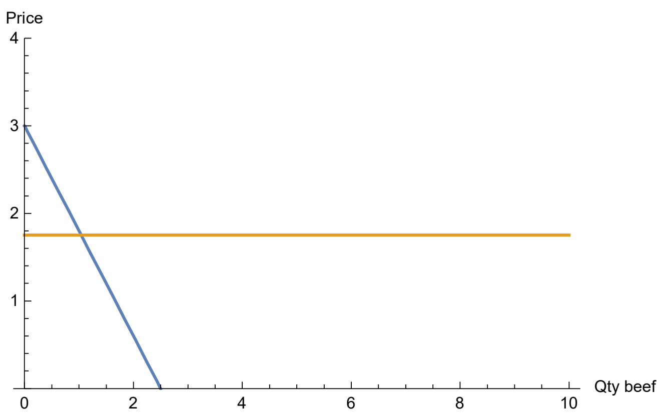
```
Dercb = D[cb, {qb}]
```

0.2 qs

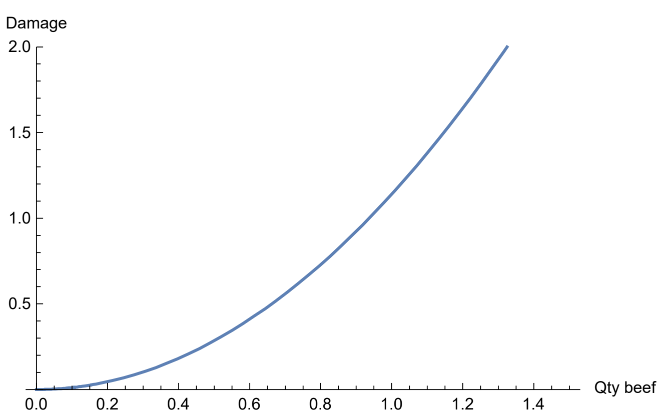
```
Plot[{Derbs, Dercs}, {qs, 0, 10}, AxesLabel → {"Qty soy", "Price"}, PlotRange → {0, 4}]
```



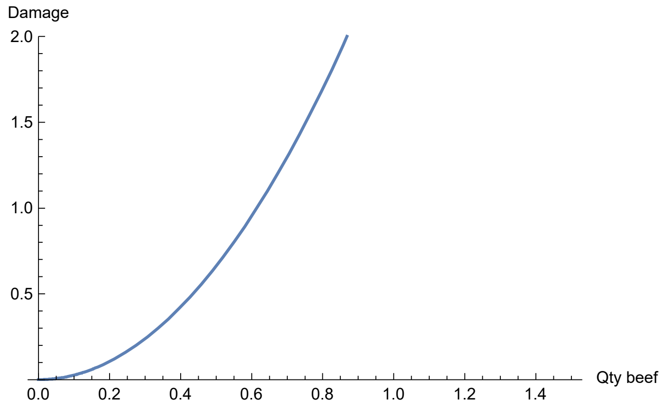

```
Plot[{Derbb, 0.2 * 8.75}, {qb, 0, 10},
 AxesLabel → {"Qty beef", "Price"}, PlotRange → {0, 4}]
```



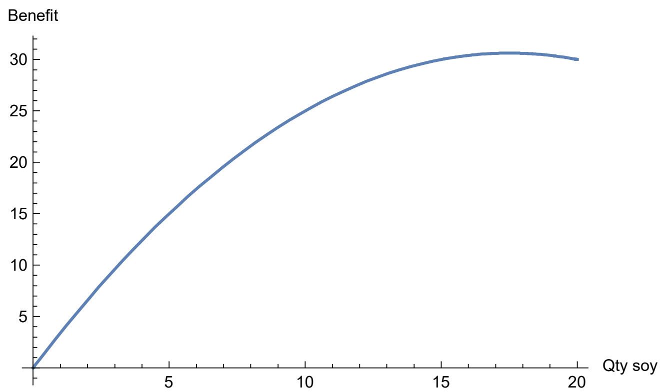

```
Plot[dbb, {qb, 0, 1.5}, AxesLabel → {"Qty beef", "Damage"}, PlotRange → {0, 2}]
```



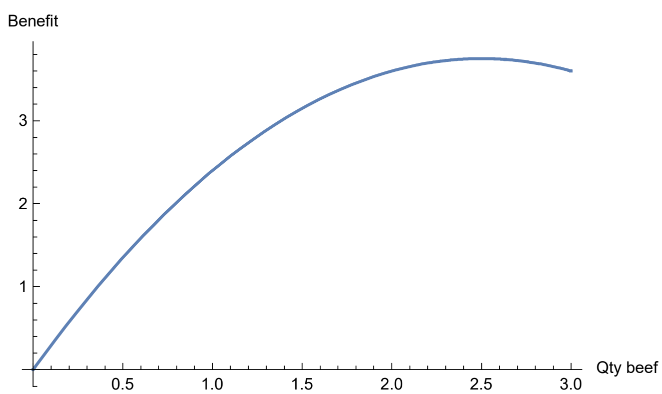
```
Plot[dbe, {qb, 0, 1.5}, AxesLabel → {"Qty beef", "Damage"}, PlotRange → {0, 2}]
```



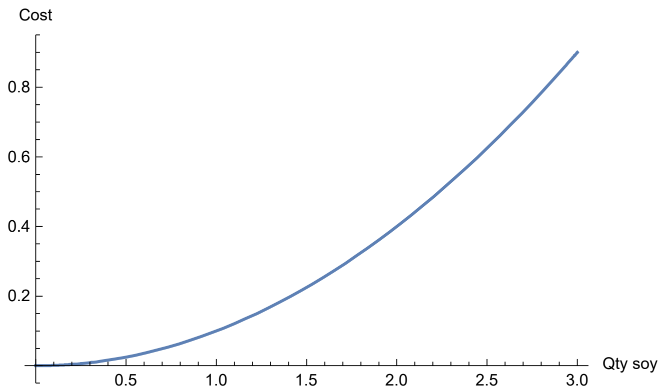

```
Plot[bs, {qs, 0, 20}, AxesLabel → {"Qty soy", "Benefit"}]
```




```
Plot[bb, {qb, 0, 3}, AxesLabel → {"Qty beef", "Benefit"}]
```



```
Plot[cs, {qs, 0, 3}, AxesLabel → {"Qty soy", "Cost"}]
```




```
Plot[cb /. qs → 8.75, {qb, 0, 3}, AxesLabel → {"Qty beef", "Cost"}]
```