Optimizing welfare and external damage on linked supply chains: An hypothetical example for the beef-soybeans supply chain in international trade.

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1 Introduction

Tariffs are the preferred tool for countries to regulate markets, both in local or international trade arenas. The effects of tariffs are felt on prices and quantities of goods traded and, ultimately, on the welfare of the parties involved. Therefore, the enforcement of tariffs plays a role in welfare maximization strategies.

To study this role, we developed a hypothetical scenario where Brazil and the European Union (EU) trade soybeans. In this example, Brazil is the only exporter of soybeans to the EU, and the EU is the only buyer from Brazil. Therefore, establishing a bilateral relationship.

Additionally, Brazil's economy also depends on the production of beef meat that is exclusively consumed domestically and take soybeans as an input. Furthermore, This production is responsible for environmental externalities that, ultimately, hurts both countries welfares.

As Brazil and the EU interact only through the soybeans trade, the EU is developing an import tariff on soybeans aiming to maximize its welfare. Additionally, Brazil also uses tariffs to optimize its welfare and counterbalance the effects of the environmental damages caused by the beef industry.

The objective of this paper is to examine how by optimizing their welfares with tariffs, Brazil and the European Union will impact the production and trade volumes, global welfare and the external damages caused by meat production.

2 The model

In the next section, we present the functions that describe the environment where the two players of the game are inserted.

2.1 Market assumptions

2.1.1 Soy Market

$$B_s(q_s) = a \cdot q_s - b \cdot q_s^2 \tag{1}$$

$$C_s(q_s) = e \cdot qs^2 \tag{2}$$

Where $B_s(q_s)$ represents the utility function of soy consumers and is a concave quadratic function, $C_s(q_s)$ represents the convex cost function of the soy industry, q_s is the quantity of soy produced/consumed, while a, b and e are constants defining the shape of the curves.

2.1.2 Beef Market

$$B_b(q_b) = c \cdot q_b - d \cdot q_b^2 \tag{3}$$

$$C_b(q_b, q_s) = C_s'(q_s) \cdot q_b \tag{4}$$

Where $B_b(q_b)$ is the concave utility function for beef consumers, $C_b(q_b, q_s)$ the linear cost function of the beef industry, q_b is the quantity of beef produced/consumed, and c and d are constants defining the shape of the functions. Notice here that the cost function $C_b(q_b, q_s)$ is the link between the production of soybeans and meat. As can be seen in the formula 4 the costs of beef production depend exclusively on the price of soybeans $C_s'(q_s)$ and the quantity produced q_b.

2.2 Externalities assumptions

$$D_b E(q_b) = f \cdot q_b^2 \tag{5}$$

$$D_b B(q_b) = g \cdot q_b^2 \tag{6}$$

Where $D_b E(q_b)$ represents the external damage caused by the beef industry to the EU and $D_b B(q_b)$ represents the estimated damage caused by the beef industry to Brazilian welfare.

2.3 Coefficients assumptions

For the remainder of the article, we assume that the coefficients a, b, c, d, e, f and g, on the above equations, $\in \mathbb{R} > 0$ in order to maintain the shapes of the curve economically meaningful.

2.4 Profits and Welfare functions

This next section describes the welfare functions (payoff functions) that are driving the decisions of the industries and the regions interacting in this model.

Profit functions for the soy and beef industries:

$$\pi_s(q_s, t) = (B_s'(q_s) - t) \cdot q_s - C_s(q_s) \tag{7}$$

$$\pi_b(q_b, q_s) = (B_b'(q_b) - i) \cdot q_b - C_b(q_b, q_s) \tag{8}$$

Expression 7 represents the profits of the soy industry and is translated as the quantity of soybeans traded (q_s) multiplied by the equilibrium price of soybeans ($B_s'(q_s)$, marginal benefit at q_s) adjusted by the specific import tariff (t), minus the production costs ($C_s(q_s)$). Equation 8 expresses the profits of the beef industry and is structurally similar to equation 7, being the difference the internal tariff imposed by Brazilian Government (i).

Welfare functions for the EU (U_E) and Brazil (U_B):

$$U_E(q_b, q_s, t) = B_s + q_s \cdot t - B'_s \cdot q_s - D_b E \tag{9}$$

$$U_B(q_b, q_s, i) = \pi_s + \pi_b + B_b + \alpha \cdot i \cdot q_b - D_b B \tag{10}$$

The welfare of the EU is composed of the benefit from consuming soybeans (B_s) plus the tariff revenues ($q_s \cdot t$) minus the importing costs ($B'_s \cdot q_s$) and the external damage of beef production in Brazil ($D_b E$).

Brazil's government welfare is composed of the profits of both industries (π_s, π_b) plus the benefit from consuming beef (B_b) and the tax revenue ($i \cdot q_b$) adjusted by α, a coefficient that measure the importance of the tariff revenue for the Brazilian Government.

3 Game steps

The game has three steps, first, the European Union and Brazil will decide the order in which they will apply the tariffs. Then they will calculate the value of the tariffs to be implemented. Lastly, each industry will define simultaneously how much to produce.

Step 1 - Players sequence: Brazil and EU decide if they will apply the tariffs simultaneously or in sequential order and who is applying the tariffs first.

Step 2 - Tariffs definition: European Union and Brazil decide the optimum tariff value each country will apply.

Step 3 - Market equilibrium: Soybeans industry and beef industry decide how much to produce in response to the tariffs imposed.

4 Finding players strategies

To find the players strategies we will solve the model by backward induction, as to say, first we reason about the later step of the game and only then, about the earlier step.

4.1 Market equilibrium - Step 3

As it is well known from the *microeconomic theory*, in a perfect market equilibrium, production will reach the point where the marginal benefit of consumers (B') equals the marginal cost of producers (C'). Based on this assumption, we can calculate the equilibrium quantities for q_s and q_b by setting the marginal benefit functions equal to the marginal cost functions for each industry.

4.1.1 Reaction function of the soy industry

$$(B'_s(q_s) - t) = C'_s(q_s) \tag{11}$$

By solving equation 11 we get the quantity of soy produced as a function of tariff t. In our example, this is equivalent to:

$$q_s(t) = \frac{a - t}{2(b + e)} \tag{12}$$

In equation 12, $q_s(t)$ stands for the reaction of the soy industry to t. In other words, the amount of soy produced depends on the value of the tariff imposed by the EU.

4.1.2 Reaction function of the beef industry

$$B'_b(q_b) = C'_b(q_b, q_s) \tag{13}$$

By solving equation 13 we get the reaction function of the beef industry q_b as a function of q_s and i.

$$q_b(q_s, i) = \frac{c - 2 \cdot e \cdot q_s - i}{2d} \tag{14}$$

As we already know the value of q_s we can substitute it in expression 14 and get q_b as a function of t and i.

$$q_b(t, i) = \frac{-\frac{e(a-t)}{b+e} + c - i}{2d} \tag{15}$$

Proposition 1 - The European tariff on soybeans t increase the production of beef in Brazil, and the Brazilian tariff i decreases the production of beef, as can be seen on the proofs below:

$$\frac{\partial q_b}{\partial t} = \frac{e}{2d(b+e)} > 0 \tag{16}$$

$$\frac{\partial q_b}{\partial i} = \frac{-1}{2d} < 0 \tag{17}$$

4.2 Brazil and European Union decide the optimum tariffs - Step 2

4.2.1 Optimal tariff calculation by the European Union

For simplicity, we decided to evaluate only the adoption of Specific Tariffs for both markets. The optimal tariff that maximizes EU's welfare can be calculated by maximizing U_E as shown below:

$$\max_{\{t\}} U_E(i, t) = t^* \tag{18}$$

$$\max_{\{t\}} [-f \cdot q_b^2 + a \cdot q_s - b \cdot q_s^2 - q_s(a - 2b \cdot q_s) + q_s \cdot t] = t^* \tag{19}$$

$$t(i^*) = \frac{e\left(a\left(d^2 + e \cdot f\right) - f(b+e)(c-i)\right)}{b \cdot d^2 + e\left(2d^2 + e \cdot f\right)} \tag{20}$$

4.2.2 Optimal tariff calculation by Brazil

Considering the tariff imposed by the EU on soybeans, Brazil can expect a reduction on its internal price, which will allow the production of beef to increase, as shown in *Proposition 1.* Therefore, by setting the tariff i Brazil attempts to balance the benefits and the damages caused by the increased production of beef.

$$\max_{\{i\}} U_B(i,t) = i^* \tag{21}$$

$$\begin{aligned} \max_{\{t\}} [c \cdot q_b - d \cdot q_b^2 - g \cdot q_b^2 + q_b(c - i - 2d \cdot q_b) \\ - 2e \cdot q_b \cdot q_s - e \cdot q_s^2 + q_s(a - 2bqs - t) + iq_b \cdot \alpha] = i^* \end{aligned} \tag{22}$$

$$\begin{aligned} i(t^*) = & \frac{-a \cdot e(\alpha \cdot d + d + g) + b \cdot c(\alpha \cdot d + g)}{(b+e)(2\alpha \cdot d + d + g)} \\ & + \frac{c \cdot e(\alpha \cdot d + g) + e \cdot t(\alpha \cdot d + d + g)}{(b+e)(2\alpha \cdot d + d + g)} \end{aligned} \tag{23}$$

4.3 Players decide the order in which they will add the tariffs - Step 1

As the definition of the optimal tariff t by the EU depends on the definition of the optimal tariff i by Brazil (equations 20 and 23), the order in which those tariffs are defined is important for the end result. Therefore, the last stage of the game was designed to evaluate the effects of the order in which the tariffs are applied in the final results.

Suppose the players have the option to choose the timing of their tariff implementation. The two options they have, could be called "wait" and "move". If both players decide to wait, no tariff is implemented. However, if both decide to move at the same time, the tariff will be simultaneously defined. If one decides to move and the other decides to wait, then we have a sequential game where the mover sets its optimal tariff first.

A payoff table was designed to find the equilibrium of this final step and is presented in the Results section.

5 Results

As we are not able to explore the results purely on analytical terms, to gain understanding of possible outcomes of the game, we empirically selected a set of coefficient values that approximate conventional microeconomic curves. The set of values selected can be found in Table 1.

Table 1: Selected coefficient values

a = 3.5	b = 0.1	c = 3
d = 0.6	e = 0.1	f = 2.65
g = 1.14		

5.1 Basic curves

With the coefficients selected in Table 1, the basic functions presented in section 2 produce the curves illustratively shown in Figure 1. Also, we can plot the marketing equilibrium for both markets before the implementation of tariffs as in Figure 2.

5.2 Nash Equilibrium

To find the final Nash Equilibrium (NE) in this game we developed 4 different scenarios. For all scenarios, we evaluated the welfare, the external damages and the quantity of soybeans and beef meat produced. However, to determine the NE, only the welfare of the countries were taken into account as they represent the net result of all the other indicators.

- For the first scenario we considered that both countries decided to wait and no tariffs were implemented. This scenario was identified as $i, t = 0$.
- The second scenario, considers that the EU decided to move first and set an import tariff on soybeans. Brazil reacts and sets the optimal beef tariff in response. That scenario is identified as $i(t^*)$.
- The third scenario considered that Brazil moves first and set its beef tariff, then EU reacted by setting the optimal import tariff on soybeans. That scenario is identified as $t(i^*)$.
- The forth scenario simulates both countries moving at the same time and setting the optimal tariff simultaneously. This scenario is identified as i^*, t^*.

In addition to these 4 scenarios, we also tested the model for two values of α (0 and 1). In that way, we could assess the importance of the tariff revenue in Brazil's decisions. Next, we present the summary table with the resulting values of i and t for each scenario and value of α.

Table 2: Values of i and t in the 4 different scenarios

	$\alpha = 1$		$\alpha = 0$	
	i	t	i	t
Both countries wait, $i, t = 0$	0	0	0	0
EU moves first, $i(t^*)$	0.559442	0.444238	0.437636	0.444238
Brazil moves first, $t(i^*)$	0.382653	0.595023	0.215517	0.529163
Both players move, i^*, t^*	0.663487	0.705687	0.545019	0.659004

After finding the tariffs in each scenario, we can calculate the resulting welfare and create the payoff tables for players strategies as shown in Tables 3 and 4.

Table 3: Step 1 - Nash Equilibrium results for $\alpha = 1$

		European Union			
		Wait $t()$		Move $t*$	
Brazil	Wait $i()$	(b) 8.89323	(e) 4.78082	(b) 7.53672	(e) 7.69685
	Move i^*	(b) 6.91832	(e) 7.09856	(b) 6.68166	(e) 8.18605

Where (b) represents the welfare of Brazil and (e) the welfare of Europe.

Table 4: Step 1 - Nash Equilibrium results for $\alpha = 0$

		European Union			
		Wait $t()$		Move $t*$	
Brazil	Wait $i()$	(b) 8.89323	(e) 4.78082	(b) 7.12915	(e) 7.26038
	Move i^*	(b) 6.72496	(e) 7.34072	(b) 6.33764	(e) 7.75572

Where (b) represents the welfare of Brazil and (e) the welfare of Europe.

The Nash Equilibrium is found when Europe moves first and Brazil optimizes its welfare next regardless of α. NE = $i(t^*)$

The results of welfare, external damages and quantities produced are presented in Tables 5 and 6, below:

Table 5: Results, $\alpha = 1$

		$i, t = 0$	$i(t^*)$	$t(i^*)$	i^*, t^*
Welfare	Brazil	8.89323	7.53672	6.91832	6.68166
	E. Union	4.78082	7.69685	7.09856	8.18605
	Total	13.6740	15.2336	14.0169	14.8677
External damages	Brazil	0.65944	0.59556	1.07421	0.69855
	E. Union	1.53291	1.38442	2.49706	1.62384
	Total	2.19236	1.97999	3.57127	2.32240
Production	Soybeans	8.75000	7.63941	7.26244	6.98578
	Beef meat	1.04160	0.76056	0.97071	0.78279

Table 6: Results, $\alpha = 0$

		$i, t = 0$	$i(t^*)$	$t(i^*)$	i^*, t^*
Welfare	Brazil	8.89323	7.12915	6.72469	6.33764
	E. Union	4.78082	7.26038	6.34072	7.75572
	Total	13.6740	14.3895	13.0654	14.0934
External damages	Brazil	1.23698	0.84720	1.33599	0.84720
	E. Union	2.87543	1.96938	3.10559	1.96938
	Total	4.11241	2.81659	4.44158	2.81659
Production	Soybeans	8.75000	7.63940	7.42700	7.10240
	Beef meat	1.04160	0.82668	1.08250	0.86206

6 Discussion

6.1 Welfare analysis

The payoff tables show that both countries have dominant strategies. Brazil will always wait for the decision of the EU to determine the optimum tariff on beef production, and the EU will always choose to move and implement the tariff regardless of Brazil's decision.

In this specific setting, the implementation of tariffs reduced Brazilian welfare and increases the welfare of the EU. However, NE will allow for the best solution in terms of total welfare (15.2336). Also, it is possible to observe that the equilibrium is kept the same regardless of the value of α. When comparing tables 3 and 4 we observe that, although the absolute welfare values changed, the NE remained the same.

Figure 3 shows a graphical representation of the two welfare functions and allow for intuitive visualization of the Nash Equilibrium solution.

6.2 External damages analysis

Similar to what happened with the welfare, the NE provided the lowest total external damages results. And that can be explained by the fact that the combined tariffs resulting from scenario $i(t^*)$ cause the greatest suppression in beef production. Showing that, as long as EU takes the initiative, the minimum damage will be achieved.

Additionally, we can observe that when Brazil regards the tariff income ($\alpha = 1$), the production of beef is more suppressed in both scenarios $i, t = 0$ and $i^*(t^*)$, as can be observed by comparing the tables 6 and 5.

6.3 Production and trade volumes

As expected after the implementation of tariffs, the volume traded internationally is reduced. Also, we can observe in equation 12 that tariff t and quantity of soybeans produced qs are inversely proportional. Therefore, considering that the production of soybeans is exclusively traded with the EU, a reduction in soybeans production means a reduction in the volumes traded.

Regarding the production of beef meet, according to *Proposition 1*, we could expect an increase in q_b proportional to t and a decrease inversely proportional to an increase in i. However, we can observe that in the NE, the effect of i overpowers the effect of t and the amount of beef meat produced is also reduced when compared with the scenario with no tariffs.

7 Conclusions

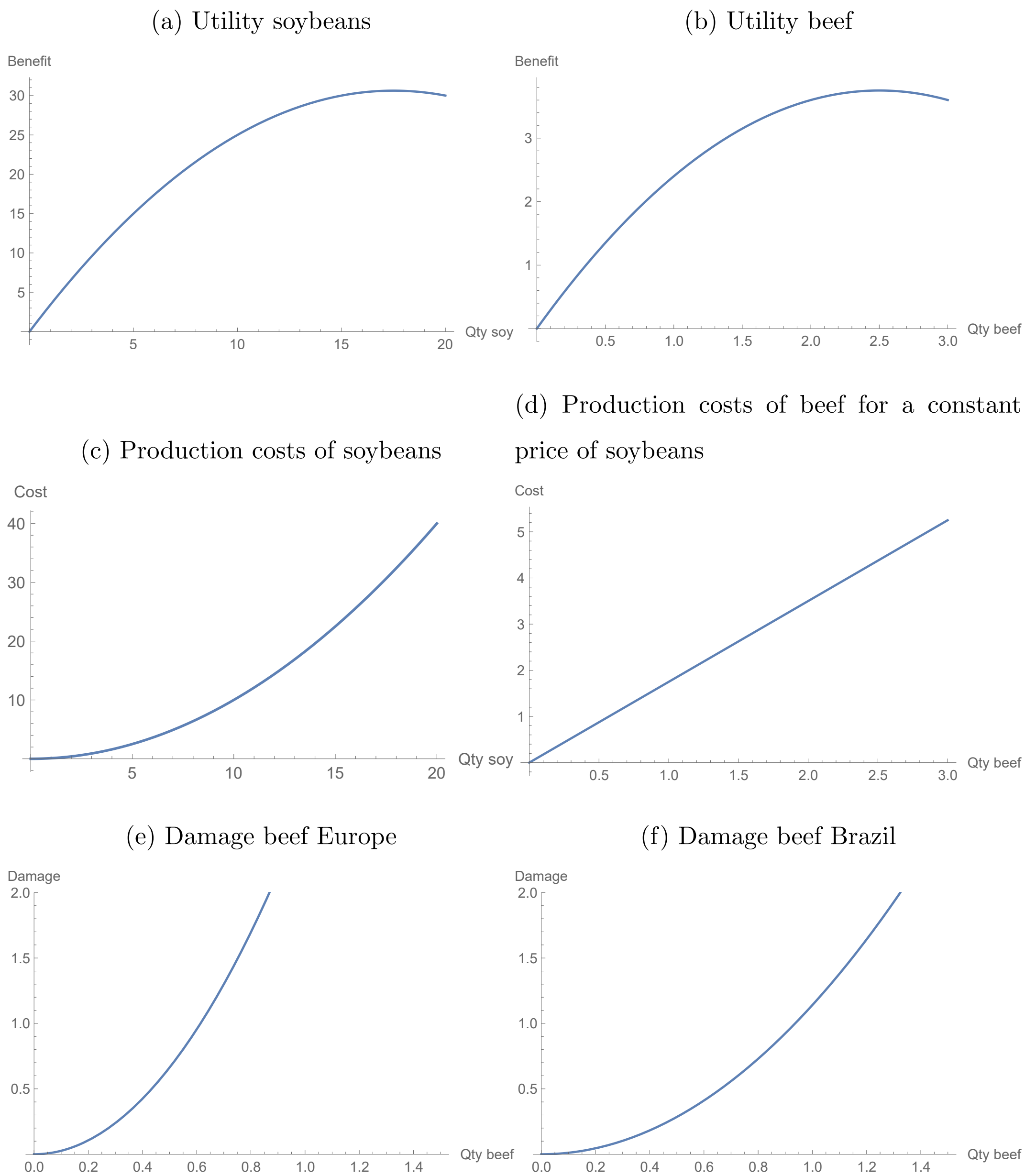
The implementation of tariffs in this setting reduced the total amount of commodities produced leading to less environmental damages and less trade. However, there was an increase in total welfare resulting from the increase in the EU's welfare that was proportionally bigger than the reduction in the welfare of Brazil.

This simple model limited the evaluation to only two trading regions and one traded commodity. Therefore, it is difficult to come to general conclusions. In that sense, an interesting extension would allow for other countries to participate in the international trade and, improve the input factor relationship description between the commodities and the environmental damages.

A Appendixes

A.1 Figures

Figure 1: Basic functions



(a) Utility soybeans

(b) Utility beef

(c) Production costs of soybeans

(d) Production costs of beef for a constant price of soybeans

(e) Damage beef Europe

(f) Damage beef Brazil

Figure 2: Market equilibrium for $i, t = 0$

(a) Market soybeans (b) Market beef (constant price of soybeans)

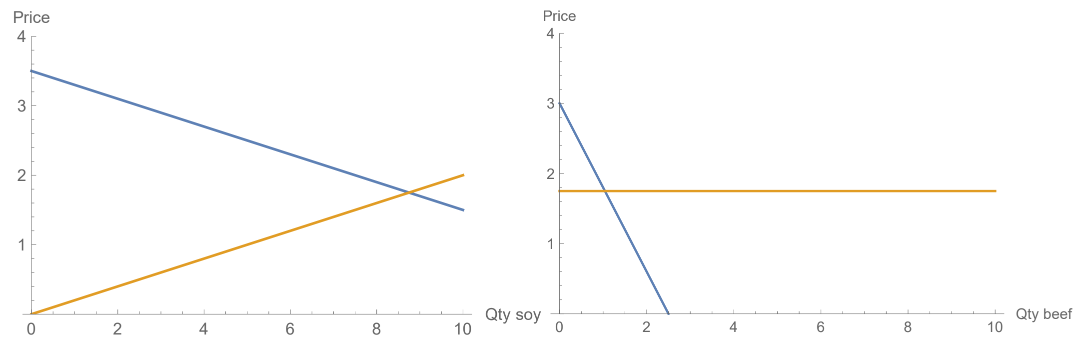
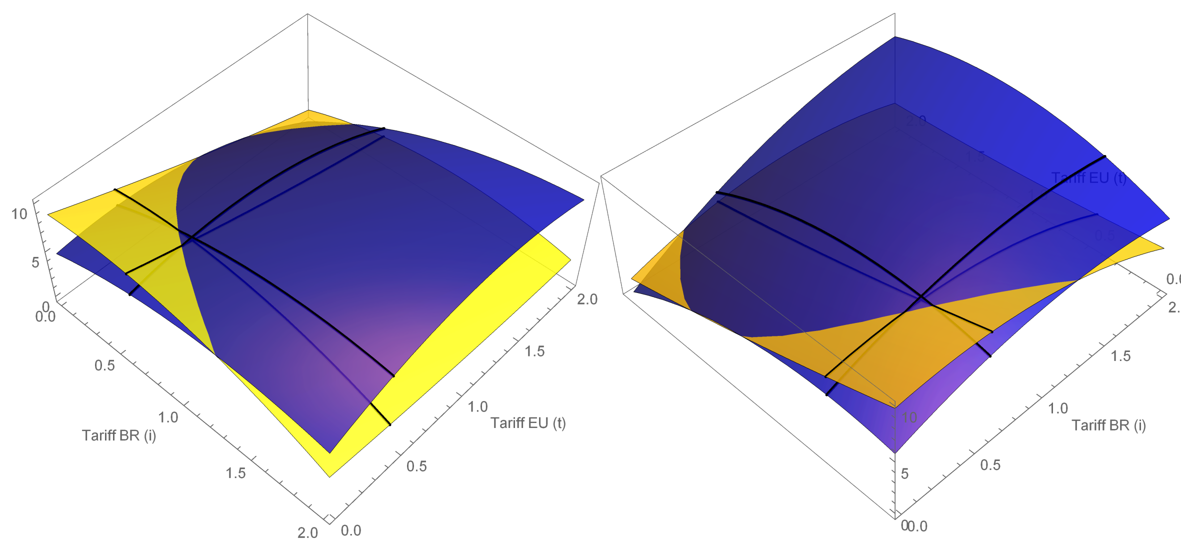


Figure 3: Welfare planes

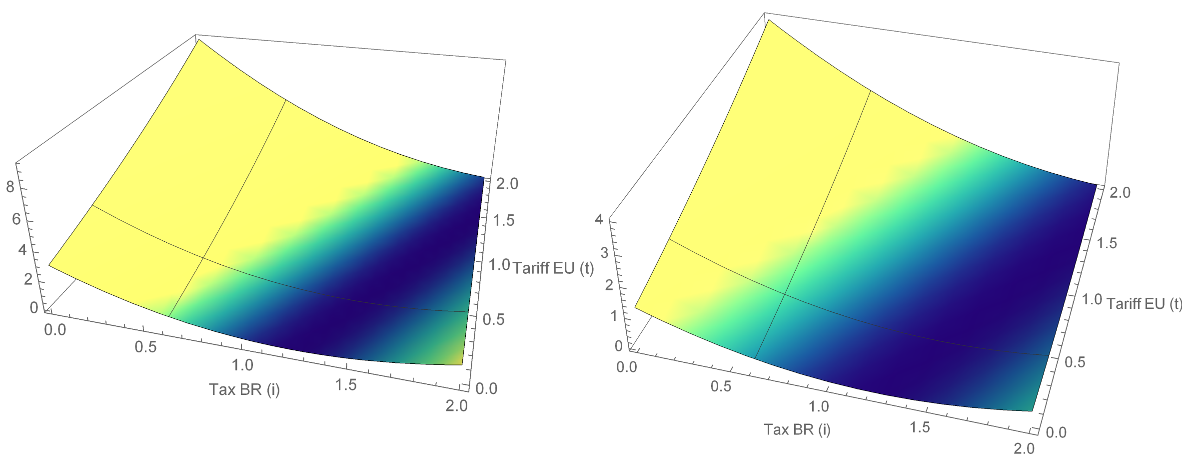
(a) View a (b) View b



The planes represent the welfare changes of Brazil (yellow) and the EU (blue) in response to the changes in the tariffs i and t. The lines across the planes represent the values of t and i in the Nash Equilibrium for $\alpha = 1$.

Figure 4: External damage planes

(a) Damages in EU (b) Damages in Brazil



The plans represent the external damage varying with both i and t. The lines across the planes represent the values of t and i in the Nash Equilibrium for $\alpha = 1$.