Optimizing welfare on linked supply chains through taxes.

Cleaning variables

```
ClearAll[qs, qb ]
ClearAll[ a, b, c, d, e, f, g, α]
ClearAll[i, t]
ClearAll[bs, bb, cs, cb, dbe, dbb, πs, πb]
ClearAll[UB, UE]
ClearAll[opti, optt]
ClearAll[BrOpti, EUOptt]
```

Defining market curves

Basic Mkt functions

```
bs = a * qs - b * qs^2
```

$a\,qs - b\,qs^2$

```
bb = c * qb - d * qb^2
```

$c\,qb - d\,qb^2$

```
cs = e * qs^2
```

$e\,qs^2$

```
cb = D[cs, {qs}] * qb
```

$2\,e\,qb\,qs$

```
dbe = f * qb^2
```

$f\,qb^2$

```
dbb = g * qb^2
```

$g\,qb^2$

```
πs = (D[bs, {qs}] - t) * qs - cs
```

$-e\,qs^2 + qs\,(a - 2\,b\,qs - t)$

```
πb = (D[bb, {qb}] - i) * qb - cb
```

$qb\,(c - i - 2\,d\,qb) - 2\,e\,qb\,qs$

Country Functions

Brazil

```
UB = πs + πb + bb + α * i * qb - dbb
```

$c\,qb - d\,qb^2 - g\,qb^2 + qb\,(c - i - 2\,d\,qb) - 2\,e\,qb\,qs - e\,qs^2 + qs\,(a - 2\,b\,qs - t) + i\,qb\,\alpha$

European Union

```
UE = bs - D[bs, {qs}] * qs - dbe + qs * t
```

$-f\,qb^2 + a\,qs - b\,qs^2 - qs\,(a - 2\,b\,qs) + qs\,t$

Solving reaction functions for companies

```
rb = Solve[(D[bb, {qb}] - i) == D[cb, {qb}], {qb}]
```

$$\left\{\left\{qb \to \frac{c - i - 2\,e\,qs}{2\,d}\right\}\right\}$$

```
qb = rb[[1]][[1]][[2]]
```

$$\frac{c - i - 2\,e\,qs}{2\,d}$$

```
rs = Solve[(D[bs, {qs}] - t) == D[cs, {qs}], {qs}]
```

$$\left\{\left\{qs \to \frac{a - t}{2\,(b + e)}\right\}\right\}$$

```
qs = rs[[1]][[1]][[2]]
```

$$\frac{a - t}{2\,(b + e)}$$

Finding i and t

```
opti = ∂_i UB == 0
```

$$-\frac{c}{2\,d} + \frac{c - i - \frac{e\,(a-t)}{b+e}}{2\,d} + \frac{g\left(c - i - \frac{e\,(a-t)}{b+e}\right)}{2\,d^2} - \frac{i\,\alpha}{2\,d} + \frac{\left(c - i - \frac{e\,(a-t)}{b+e}\right)\alpha}{2\,d} == 0$$

```
rUB = Solve[opti, {i}][[1]][[1]][[2]]
```

$$(-a\,d\,e + b\,c\,g - a\,e\,g + c\,e\,g + d\,e\,t + e\,g\,t + b\,c\,d\,\alpha - a\,d\,e\,\alpha + c\,d\,e\,\alpha + d\,e\,t\,\alpha)\,/\,((b + e)\,(d + g + 2\,d\,\alpha))$$

Simplify[rUB]

$(b\,c\,(g+d\,\alpha)+c\,e\,(g+d\,\alpha)-a\,e\,(d+g+d\,\alpha)+e\,t\,(d+g+d\,\alpha))\,/\,((b+e)\,(d+g+2\,d\,\alpha))$

Simplify[D[rUB, t]]

$$\frac{e\,(d+g+d\,\alpha)}{(b+e)\,(d+g+2\,d\,\alpha)}$$

Simplify[D[rUB, α]]

$(d\,(b\,c\,(d-g)+e\,(c\,(d-g)+a\,(d+g)-(d+g)\,t)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)$

∂_α((b c (g + d α) + c e (g + d α) − a e (d + g + d α) + e t (d + g + d α)) / ((b + e) (d + g + 2 d α)))

$$\frac{b\,c\,d-a\,d\,e+c\,d\,e+d\,e\,t}{(b+e)\,(d+g+2\,d\,\alpha)}-$$
$$(2\,d\,(b\,c\,(g+d\,\alpha)+c\,e\,(g+d\,\alpha)-a\,e\,(d+g+d\,\alpha)+e\,t\,(d+g+d\,\alpha)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)$$

$\text{Simplify}\left[\frac{b\,c\,d-a\,d\,e+c\,d\,e+d\,e\,t}{(b+e)\,(d+g+2\,d\,\alpha)}-(2\,d\,(b\,c\,(g+d\,\alpha)+c\,e\,(g+d\,\alpha)-a\,e\,(d+g+d\,\alpha)+e\,t\,(d+g+d\,\alpha)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)\right]$

$(d\,(b\,c\,(d-g)+e\,(c\,(d-g)+a\,(d+g)-(d+g)\,t)))\Big/\left((b+e)\,(d+g+2\,d\,\alpha)^2\right)$

optt = ∂_tUE == 0

$$-\frac{a}{2\,(b+e)}+\frac{a-\frac{b\,(a-t)}{b+e}}{2\,(b+e)}-\frac{e\,f\left(c-i-\frac{e\,(a-t)}{b+e}\right)}{2\,d^2\,(b+e)}+\frac{a-t}{2\,(b+e)}-\frac{t}{2\,(b+e)}==0$$

rUE = Solve[optt , t][[1]][[1]][[2]]

$$\left(\frac{a\,b}{2\,(b+e)^2}-\frac{a}{2\,(b+e)}-\frac{a\,e^2\,f}{2\,d^2\,(b+e)^2}+\frac{c\,e\,f}{2\,d^2\,(b+e)}-\frac{e\,f\,i}{2\,d^2\,(b+e)}\right)\Big/\left(\frac{b}{2\,(b+e)^2}-\frac{1}{b+e}-\frac{e^2\,f}{2\,d^2\,(b+e)^2}\right)$$

Simplify[rUE]

$$\frac{e\left(a\left(d^2+e\,f\right)-(b+e)\,f\,(c-i)\right)}{b\,d^2+e\left(2\,d^2+e\,f\right)}$$

D[rUE, i]

$$-\left((e\,f)\Big/\left(2\,d^2\,(b+e)\left(\frac{b}{2\,(b+e)^2}-\frac{1}{b+e}-\frac{e^2\,f}{2\,d^2\,(b+e)^2}\right)\right)\right)$$

```
α = 1; a = 3.5; b = 0.1; c = 3; d = 0.6; e = 0.1;
f = 2.65;  g = 1.14; (*g= 1.14, f = 2.65;*)
sol = Solve[rUB - i == 0 && rUE - t == 0, {i, t}]
```

{{i → 0.663487, t → 0.705687}}

Results

Benchmark

```
UE /. {i → 0, t → 0}
```

4.78082

```
UB /. {i → 0, t → 0}
```

8.89323

```
dbb /. {i → 0, t → 0}
```

1.23698

```
dbe /. {i → 0, t → 0}
```

2.87543

```
totalwelfare = UE + UB /. {i → 0, t → 0}
```

13.674

```
totaldamage = dbb + dbe /. {i → 0, t → 0}
```

4.11241

```
qb /. {i → 0, t → 0}
```

1.04167

```
qs /. {i → 0, t → 0}
```

8.75

Optimal Tariffs

```
Simplify[UE /. sol]
```

{8.18605}

```
Simplify[UB /. sol]
```

{6.68166}

```
Simplify[dbb /. sol]
```

{0.698558}

```
dbe /. sol
```

{1.62384}

```
Simplify[totalwelfare = UE + UB /. sol]
```

{14.8677}

```
totaldamage = dbb + dbe /. sol
```

{2.3224}

```
Simplify[qb /. sol]
{0.782796}
```

```
qs /. sol
{6.98578}
```

Results if Brazil sets the tariff first t(i*)

```
BrOpti = rUB /. t → 0
0.382653
```

```
EUOptt = rUE /. i → BrOpti
0.595023
```

```
UE /. {i → BrOpti, t → EUOptt}
7.09856
```

```
UB /. {i → BrOpti, t → EUOptt}
6.91832
```

```
dbb /. {i → BrOpti, t → EUOptt}
1.07421
```

```
dbe /. {i → BrOpti, t → EUOptt}
2.49706
```

```
totalwelfare = UE + UB /. {i → BrOpti, t → EUOptt}
14.0169
```

```
totaldamage = dbb + dbe /. {i → BrOpti, t → EUOptt}
3.57127
```

```
qb /. {i → BrOpti, t → EUOptt}
0.970715
```

```
qs /. {i → BrOpti, t → EUOptt}
7.26244
```

Results if EU sets the tariff first i(t*)

```
NeEUOptt = rUE /. i → 0
0.444238
```

```
NeBrOpti = rUB /. t → EUOptt
0.619448
```

```
UE /. {i → NeBrOpti, t → NeEUOptt}
```

7.89179

```
UB /. {i → NeBrOpti, t → NeEUOptt}
```

7.52937

```
dbb /. {i → BrOpti, t → EUOptt}
```

1.07421

```
dbe /. {i → NeBrOpti, t → NeEUOptt}
```

1.33797

```
totalwelfare = UE + UB /. {i → NeBrOpti, t → NeEUOptt}
```

15.4212

```
totaldamage = dbb + dbe /. {i → NeBrOpti, t → NeEUOptt}
```

1.91355

```
qb /. {i → NeBrOpti, t → NeEUOptt}
```

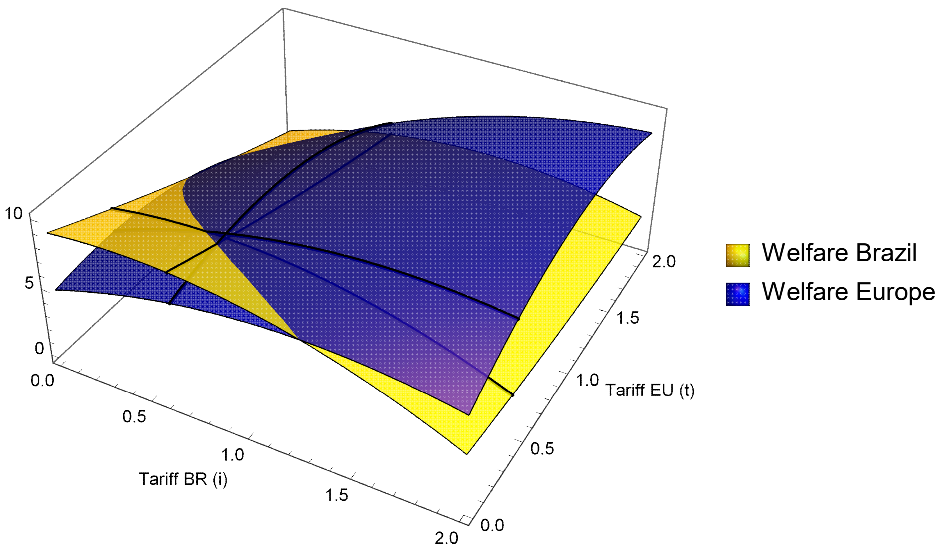
0.710559

```
qs /. {i → NeBrOpti, t → NeEUOptt}
```

7.63941

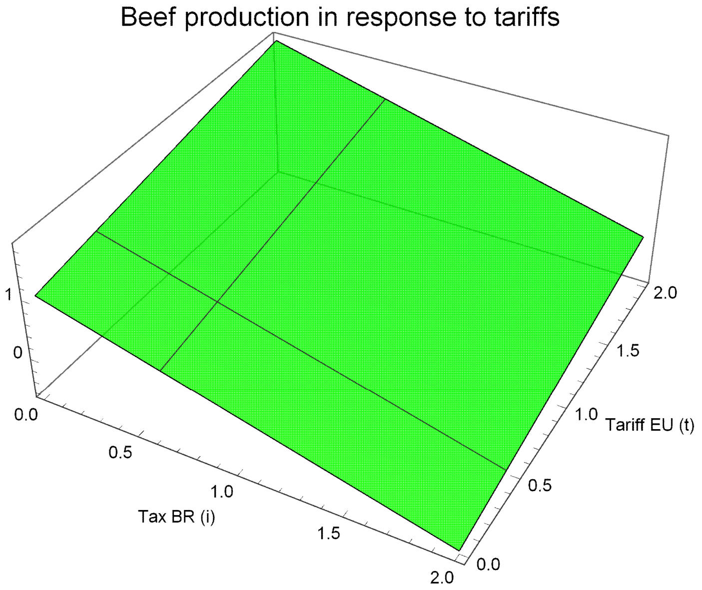
Graphing the welfare to find the best parameters.

```
welfarePlot =
 Plot3D[{UB, UE}, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tariff BR (i)", "Tariff EU (t)"},
  Mesh → {{NeBrOpti}, {NeEUOptt}}, MeshStyle → {Thick, Thick},
  PlotTheme → "FullAxes", PlotLegends → {"Welfare Brazil", "Welfare Europe"},
  PlotStyle → {Directive[Opacity[0.8], Yellow, Specularity[White, 50]],
    Directive[Opacity[0.8], Blue, Specularity[White, 50]]}]
```

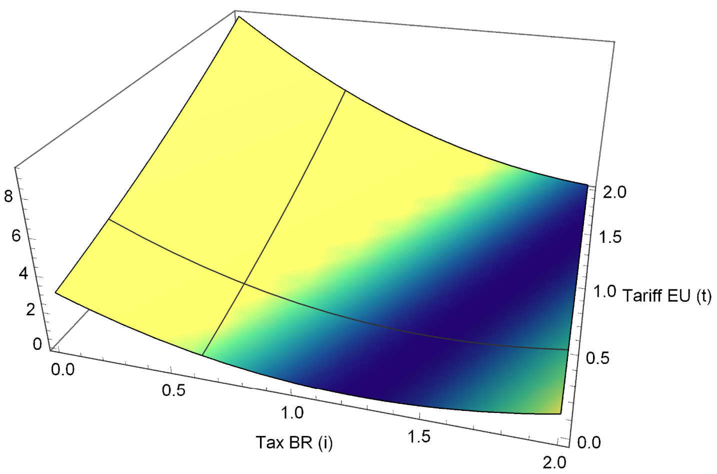


```
Plot3D[qs, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tax BR (i)", "Tariff EU (t)"},
    Mesh → {{sol[NeBrOpti}, {NeEUOptt}}, PlotTheme → "FullAxes",
 PlotStyle → Directive[Opacity[0.8], Green, Specularity[White, 50]],
 PlotLabel → Style["Soy production in response to tariffs", FontSize → 14]]
```

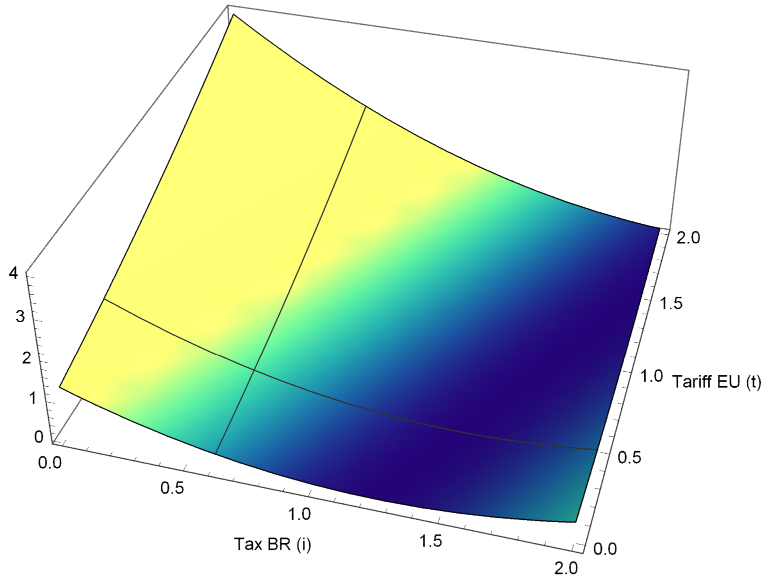
```
Plot3D[qb, {i, 0, 2}, {t, 0, 2}, AxesLabel → {"Tax BR (i)", "Tariff EU (t)"},
 Mesh → {{NeBrOpti}, {NeEUOptt}}, PlotTheme → "FullAxes",
 PlotStyle → Directive[Opacity[0.8], Green, Specularity[White, 50]],
 PlotLabel → Style["Beef production in response to tariffs", FontSize → 14]]
```




```
Plot3D[dbe, {i, 0, 2}, {t, 0, 2},
 AxesLabel → {"Tax BR (i)", "Tariff EU (t)"}, Mesh → {{NeBrOpti}, {NeEUOptt}},
 PlotTheme → "FullAxes", ColorFunction → "BlueGreenYellow",
 ColorFunctionScaling → False, ViewPoint → {1.3, -2.4, 2.}]
```



```
Plot3D[dbb, {i, 0, 2}, {t, 0, 2},
 AxesLabel → {"Tax BR (i)", "Tariff EU (t)"}, Mesh → {{NeBrOpti}, {NeEUOptt}},
 PlotTheme → "FullAxes", ColorFunction → "BlueGreenYellow",
 ColorFunctionScaling → False, ViewPoint → {1.3, -2.4, 2.}]
```



Proposition 1: increasing t increases qb

```
ClearAll[a, b, c, d, e, f, g, α]
```

```
qb
```

$$\frac{c - i - \frac{e (a-t)}{b+e}}{2 d}$$

```
D[qb, {t}]
```

$$\frac{e}{2 d (b + e)}$$

Graphical representations of basic curves

```
ClearAll[qs, qb]
```

```
α = 1; a = 3.5; b = 0.1; c = 3; d = 0.6; e = 0.1; f = 2.65; g = 1.14;
```

```
Derbs = D[bs, {qs}]
```

3.5 - 0.2 qs

```
Dercs = D[cs, {qs}]
```

0.2 qs

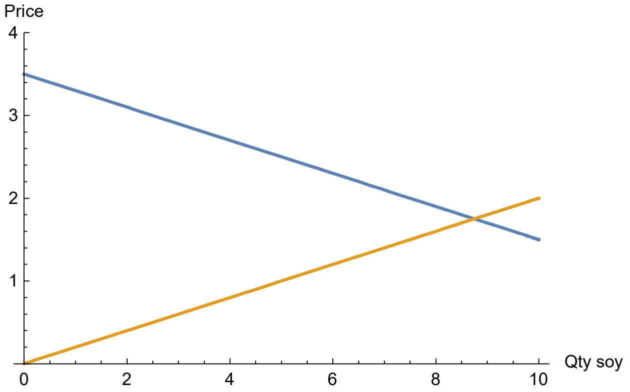
```
Derbb = D[bb, {qb}]
```

3 - 1.2 qb

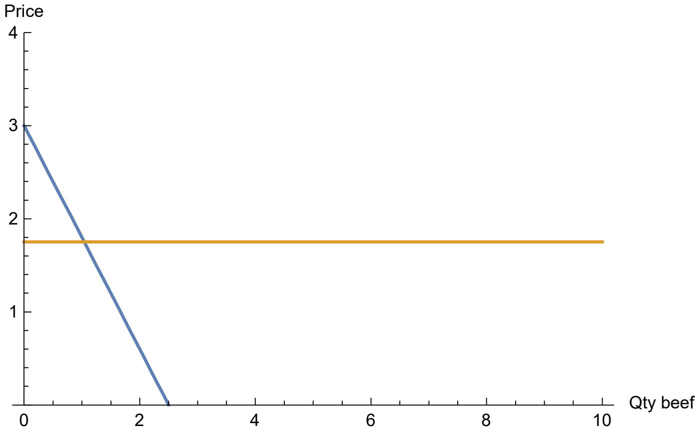
```
Dercb = D[cb, {qb}]
```

0.2 qs

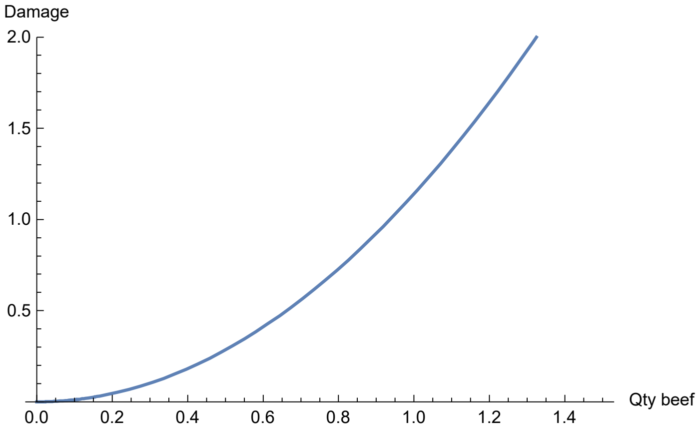
```
Plot[{Derbs, Dercs}, {qs, 0, 10}, AxesLabel → {"Qty soy", "Price"}, PlotRange → {0, 4}]
```



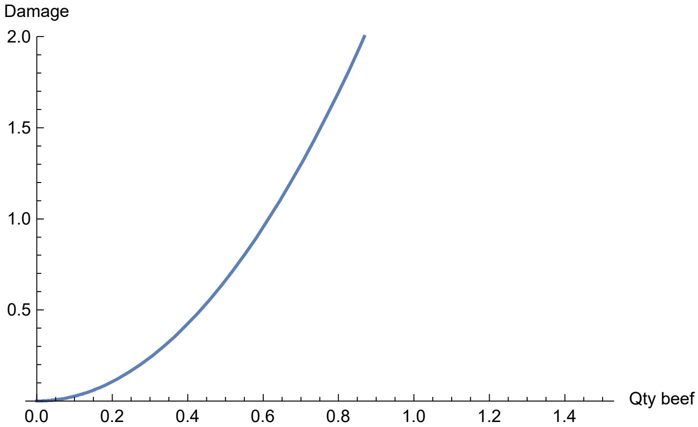

```
Plot[{Derbb, 0.2 * 8.75}, {qb, 0, 10},
 AxesLabel → {"Qty beef", "Price"}, PlotRange → {0, 4}]
```



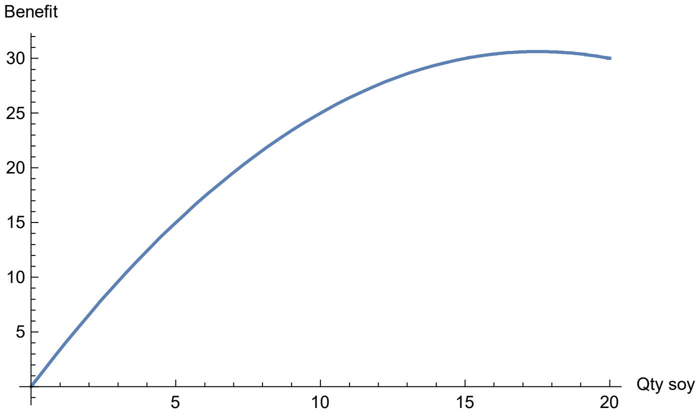

```
Plot[dbb, {qb, 0, 1.5}, AxesLabel → {"Qty beef", "Damage"}, PlotRange → {0, 2}]
```



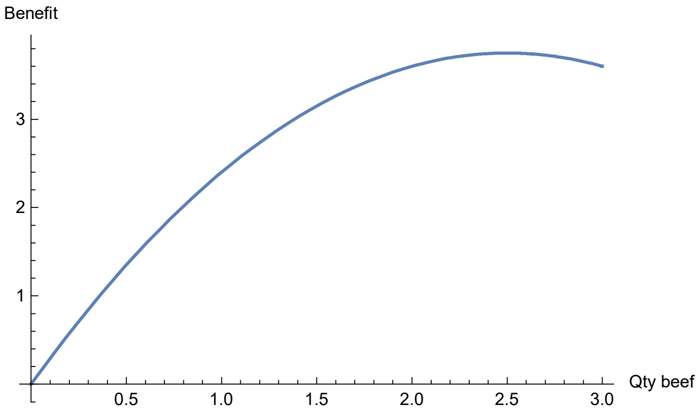
```
Plot[dbe, {qb, 0, 1.5}, AxesLabel → {"Qty beef", "Damage"}, PlotRange → {0, 2}]
```



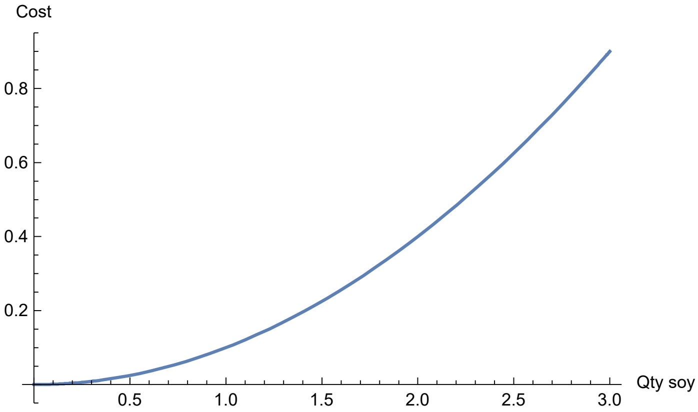

```
Plot[bs, {qs, 0, 20}, AxesLabel → {"Qty soy", "Benefit"}]
```




```
Plot[bb, {qb, 0, 3}, AxesLabel → {"Qty beef", "Benefit"}]
```



```
Plot[cs, {qs, 0, 3}, AxesLabel → {"Qty soy", "Cost"}]
```




```
Plot[cb /. qs → 8.75, {qb, 0, 3}, AxesLabel → {"Qty beef", "Cost"}]
```