

Definition of variables

Thrust T : The thrust axis \vec{n}_T maximises the following quantity:

$$T = \max_{\vec{n}_T} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|} \right) ,$$

where the sum extends over all particles in the event.

Thrust Major T_{Ma} : The thrust major vector, \vec{n}_{Ma} , is defined in the same way as the thrust vector, but with the additional condition that \vec{n}_{Ma} must lie in the plane perpendicular to \vec{n}_T :

$$T_{\text{Ma}} = \max_{\vec{n}_{\text{Ma}} \perp \vec{n}_T} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}_{\text{Ma}}|}{\sum_i |\vec{p}_i|} \right) .$$

Thrust Minor T_{Mi} : The minor axis is perpendicular to both the thrust axis and the major axis $\vec{n}_{\text{Mi}} = \vec{n}_T \times \vec{n}_{\text{Ma}}$. The value of thrust minor is given by:

$$T_{\text{Mi}} = \frac{\sum_i |\vec{p}_i \cdot \vec{n}_{\text{Mi}}|}{\sum_i |\vec{p}_i|} .$$

Oblateness O : The oblateness is defined as the difference between thrust major and thrust minor:

$$O = T_{\text{Ma}} - T_{\text{Mi}} .$$

Sphericity S : The sphericity is calculated from the ordered eigenvalues of the quadratic momentum tensor:

$$\begin{aligned} M^{\alpha\beta} &= \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i |\vec{p}_i|^2} , \quad \alpha, \beta = 1, 2, 3 ; \\ \lambda_1 &\geq \lambda_2 \geq \lambda_3 , \quad \lambda_1 + \lambda_2 + \lambda_3 = 1 ; \\ S &= \frac{3}{2}(\lambda_2 + \lambda_3) . \end{aligned}$$

The sphericity axis \vec{n}_S is defined along the direction of the eigenvector associated to λ_1 .

Aplanarity A : The aplanarity is calculated from the third eigenvalues of the quadratic momentum tensor

$$A = \frac{3}{2}\lambda_3 .$$

Planarity P : The planarity is a linear combination of sphericity and aplanarity

$$P = \frac{2}{3}(S - 2A) .$$

Heavy Jet Mass M_H : A plane through the origin and perpendicular to \vec{n}_T divides the event into two hemispheres, H_1 and H_2 , from which one obtains the corresponding normalised hemisphere invariant masses:

$$M_i = \frac{1}{E_{vis}^2} \left(\sum_{k \in H_i} p_k \right)^2, \quad i = 1, 2, \quad ,$$

where E_{vis} is the total visible energy in the event. The larger of the two hemisphere masses is called the heavy jet mass :

$$M_H = \max(M_1, M_2) \quad .$$

Light Jet Mass M_L : Similarly, the light jet mass is the smaller of the two normalised hemisphere masses:

$$M_L = \min(M_1, M_2) \quad .$$

Jet Mass Difference M_D : The difference between M_H and M_L is called the jet mass difference:

$$M_D = M_H - M_L \quad .$$

Single Inclusive Jet Mass M_S : Each event has two entries in the single inclusive jet mass distribution: The heavy jet mass M_H and the light jet mass M_L . The integral of the distribution is normalised to the 'hemisphere multiplicity', i.e. 2.

Wide Jet Broadening B_W : A measure of the broadening of particles in transverse momentum with respect to the thrust axis can be calculated for each hemisphere H_i using the relation:

$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}, \quad i = 1, 2, \quad ,$$

where j runs over all of the particles in the event. The wide jet broadening is the larger of the two hemisphere broadenings:

$$B_W = \max(B_1, B_2) \quad .$$

Narrow Jet Broadening B_N : The narrow jet broadening is the smaller of the two hemisphere broadenings:

$$B_N = \min(B_1, B_2) \quad .$$

Total Jet Broadening B_T : The total jet broadening is the sum of the wide jet broadening and the narrow jet broadening:

$$B_T = B_W + B_N \quad .$$

Single Inclusive Jet Broadening B_S : Similar to the single inclusive jet mass, each event has two entries in the single inclusive jet broadening distribution: the wide jet broadening B_W and the narrow jet broadening B_N .

C-parameter C : The C-parameter is derived from the eigenvalues of the linearised momentum tensor $\Theta^{\alpha\beta}$:

$$\Theta^{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i |\vec{p}_i|} , \quad \alpha, \beta = 1, 2, 3 .$$

The three eigenvalues λ_j of this tensor define C with:

$$C = 3 \cdot (\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1) .$$

D-parameter D : The D-parameter is calculated with the eigenvalues of the linearised momentum tensor:

$$D = 27 \lambda_1 \lambda_2 \lambda_3 .$$

Jet Rates R_n : Jet rates are defined by means of a clustering algorithm. For each pair of particles i and j in an event one computes:

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{E_{vis}^2} \quad (\text{Durham algorithm}) .$$

The pair of particles with the smallest value of y_{ij} is replaced by a pseudo-particle (cluster). The four-momentum of the cluster is taken to be the sum of the four momenta of particles i and j , $p^\mu = p_i^\mu + p_j^\mu$ ('E' recombination scheme). The clustering procedure is repeated until all y_{ij} values exceed a given threshold y_{cut} . The number of clusters remaining at this point is defined to be the number of jets. For each value of y_{cut} the number of n-jet events is normalised to the total number of events to give the jet rates, i.e. $\sum_{n=1}^{\infty} R_n(y_{cut}) = 1$.

Jet resolution parameter y_n : The jet resolution parameters y_n are defined as the particular values of y_{cut} at which an event switches from a n-1-jet configuration to a n-jet configuration. The same clustering algorithm as for jet rates is applied.

Energy-Energy-Correlation EEC : Two-particle correlation defined in terms of the angle χ_{ij} between two particles, weighted with their scaled energy:

$$EEC(\chi) = \frac{1}{N} \frac{1}{\Delta\chi} \sum_N \sum_{i,j} \frac{E_i E_j}{E_{vis}^2} \int_{\chi - \frac{\Delta\chi}{2}}^{\chi + \frac{\Delta\chi}{2}} \delta(\chi' - \chi_{ij}) d\chi' ,$$

where N is the number of events and $\Delta\chi$ is the angular bin width. The range of χ is taken from 0° to 180° .

Energy-Energy-Correlation Asymmetry AEC : The asymmetry of the energy-energy correlation is defined as:

$$AEC(\chi) = EEC(180^\circ - \chi) - EEC(\chi) .$$

Charged Particle Momentum ξ : The inclusive charged particle momentum distribution is presented in terms of the logarithmic variable ξ :

$$\xi = -\log(x_p) , \quad x_p = \frac{p}{p_{beam}} .$$

The energy evolution of the peak position ξ^* of the ξ distribution is predicted in modified leading-log approximation, the integral of the ξ distribution is equal to the mean charged particle multiplicity $\langle N_{ch} \rangle$.

Further charged particle inclusives: rapidity $y = \frac{1}{2} \ln(E + p_{\parallel}) / \ln(E - p_{\parallel})$ with p_{\parallel} measured with respect to the thrust axis and the projections *in* resp. *out* the plane defined by thrust and major axes, p_{\perp}^{in} resp. p_{\perp}^{out} .
New measurements are presented at LEP II, LEP I measurements have been published in the meaga-paper.

Pre-clustered near-to-planar event shapes: On request of the Milano group, Thrust Minor and D-parameter are measured for 3 different $y_{\text{cut}} = 0.01, 0.005, 0.0025$.