1 Exercise 1: SVM

In the general case of solving a linear SVM with slack variables without a regularizer, the objective function is:

$$\begin{aligned} \underset{\alpha}{\text{minimize}} \quad & -\sum_{i=1}^{n}\alpha_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y^{(i)}y^{(j)}(x^{(i)}\cdot x^{(j)}) && \text{(1a)}\\ \text{subject to} \quad & 0 \le \alpha_i \le C \quad \forall i\,, && \text{(1b)}\\ & \sum_{i=1}^{n}\alpha_i y^{(i)} = 0 && \text{(1c)} \end{aligned}$$

Using the Python CVXOPT package, the general form of the objective function is:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \frac{1}{2}x^T P x + q^T x && \text{(2a)}\\ \text{subject to} \quad & Gx \le h && \text{(2b)}\\ & Ax = b && \text{(2c)} \end{aligned}$$

The general form for converting our slack variable objective function in Equation 1 to the CVXOPT objective function in Equation 2 is described in Table 1.

CVXOPT	Conversion from Equation 1
x	If there are n points in the training set, an $n\times 1$ vector equal to the values of x in the training data
P	An $n \times n$ matrix which is the kernel matrix between all pairs of training data x weighted by the corresponding value of y from the training data
q	An $n \times 1$ vector of -1s
G	A $2n\times n$ matrix where the top $n\times n$ is the identity matrix and the bottom $n \times n$ is the negative identity matrix
h	A $2n \times 1$ vector with the top $n \times 1$ vector of Cs and the bottom $n \times 1$ vector of 0s
A	An $1 \times n$ vectors with elements $y^{(i)}$ from the training set for all values of i
b	An 1×1 vector of 0s

Table 1: Conversion rule for deriving CVXOPT constraints

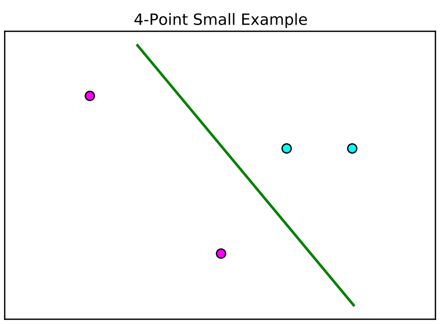
For the small example with $(1, 2), (2, 2)$ as positive examples and $(0, 0), (-2, 3)$ as negative examples, the constraints from CVXOPT as written above are written in Equation 3.

$$P = \begin{bmatrix} 5 & 6 & 0 & -4 \\ 6 & 8 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ -4 & -2 & 0 & 13 \end{bmatrix} \qquad q = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad h = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \end{bmatrix}$$

The decision boundary generated by the SVM code for the small example is shown in Figure 1a.



(a) Decision boundary for 4 points using CVXOPT and SVM with slack variables

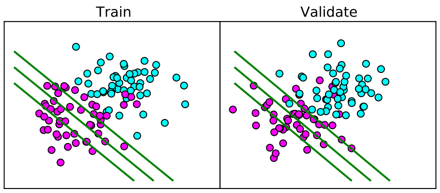
Dataset	Kernel	Training	Validation
smallOverlap	Linear	.24	.24
smallOverlap	Gaussian $\beta = 0.1$	.26	.25
smalloverlap	Gaussian $\beta = 1$	.26	.25
bigoverlap	linear	.305	.255
bigoverlap	Gaussian $\beta = 0.1$	.3	.255
bigoverlap	Gaussian $\beta = 1$	.3	.255
ls	linear	0	0.00375
ls	Gaussian $\beta = 0.1$	0.0625	0.0775
ls	Gaussian $\beta = 1$	0.0625	0.0775
nonsep2	linear	.485	.495
nonsep2	Gaussian $\beta = 0.1$	.48	.4975
nonsep2	Gaussian $\beta = 1$	.48	.4975
stdev1	Gaussian $\beta = 0.5$	0	.005
stdev1	Gaussian $\beta = 1$	0	.005

(b) Error rates for training and validation sets, $c = 1$, linear and Gaussian kernels

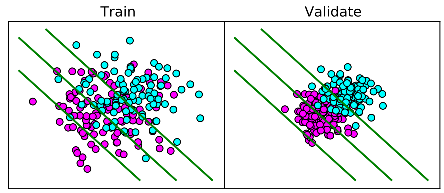
Setting $C = 1$, the error rates for the training and validation sets for different data sets and kernels is shown in Table 1b. If a data set did not come with a training / validation set pair, the dataset was randomly cut in half for each class to use as training and validation. In general, the more separable the data set is, the better the slack-variable SVM without a regularizer does. In the non-separable case, depending on the nature of the inseparability, the solution has a higher error rate.

Using a Gaussian kernel at different bandwidths does not change the error rates much unless the data is distributed with a Gaussian distribution, as shown in Table 1b.

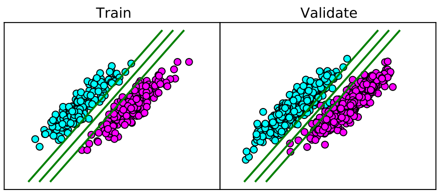
As c increases, the geometric margin decreases, as shown for various values of c, various kernels, and various datasets in figure 3a. this always happens as c increases, because this means there can be more slack in the final svm, which means the svm will be more tolerable to incorrect classifications for inseparable data. the number of support vectors first decreases, then increases as c increases, as shown in figure 3b. this means that the classifier is less overfit on the training data and will have smaller errors on the testing data. choosing c for maximizing the margin will yield a value of c equal to zero, which is the same as having a hard-margin svm that does not perform well on non-separable data. an alternate criteria for choosing c could be the minimum number of support vectors (since the number of support vectors eventually increases with a higher value of c as the classifier gets over-fit to the training data). Changing c has almost no effect on the training error unless the data is highly non-separable.



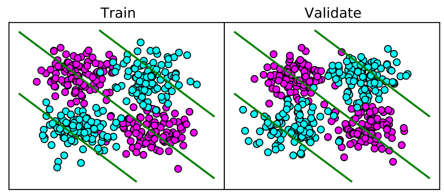
(a) smallOverlap



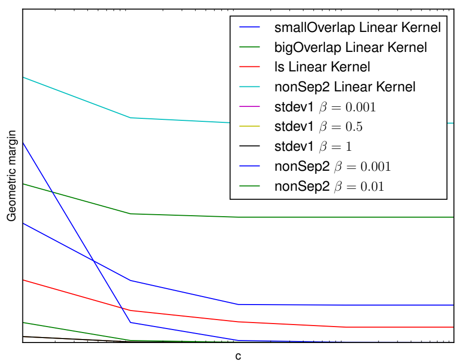
(b) bigOverlap



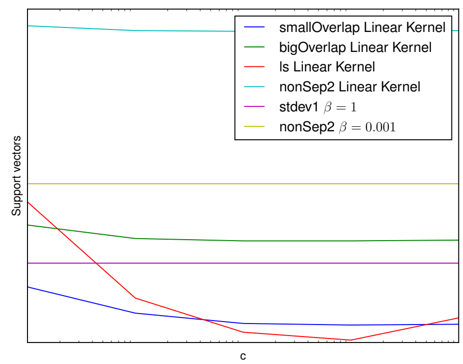
(c) ls



(d) nonSep2



(a) plot of the geometric margin as a function of c for various kernels and datasets



(b) plot of the number of support vectors as a function of c for various kernels and datasets

2 exercise 2: logistic regression

2.2: * do training set over some values of lambda * use that to pick lambda * test on 'validation' data and report an error value

2.3: will need to plot lambda versus sparsity

2.4: * do training set over some values of bandwidth with lambda = 0 * do validation set over some values of lambda * test on test and report results

2.5: * will need to think about this a little bit more.

3.1: * https://piazza.com/class/hzdfawvtilo7hf?cid=434 * l2 regularization, linear case * http://blog.datumbox.com/mac learning-tutorial-the-multinomial-logistic-regression-softmax-regression/

* maybe need to save coefficients or predictions or something?

3.1 - lr: * 2 features, 100 pts, 2 classes, l = 0.01 -¿ error = .485 * might need to randomly select subsets of data points

* bigoverlap - error is .26 on test, ,.305 on training with l = 0.1 * smalloverlap - error is .26 on test, .27 on training with l = 0.1

* tips for getting numerics to work better: * normalize data beforehand (then don't forget to re-normalize later) *

3.2 - multiclass svm: * used http://scikit-learn.org/stable/modules/generated/sklearn.svm.linearsvc.html * can do this so fast it doesn't even make sense to try to do this by myself. * tried an array of l values with l1 loss (multiclasssvm.py), best l with random partitioning of data into 3 sets. rigorous stopping criteria * hinge loss, l2 regularization: * validation error: .187 * test error: .261 * l: 0.01 * squared loss, l1 regularization: * validation error: .143 * test error: .143 * l: 7e-7