Ultra light dark matter ringing earth normal modes

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1 Ultra Light Dark Matter

2 Earth Normal Modes

Following the derivation by Aki and Richards [1], to derive the displacement due to a generic force we start by analyzing the equation of motion of the αth particle in a discrete collection of the point particles:

$$m_a\ddot{\vec{u}}_\alpha + \gamma\dot{\vec{u}}_\alpha + \sum_\beta^N c_{\alpha\beta}\vec{u}_\beta = \vec{f}_\alpha$$

where $c_{\alpha\beta}$ is the spring constant between the αth and βth particle. Now decomposing into normal modes and taking the Fourier transform:

$$-m_a\omega^2\sum_i u_{\alpha,i}\hat{n}_{\alpha,i} + i\omega\sum_i \gamma_i u_{\alpha,i}\hat{n}_{\alpha,i} + \sum_\beta^N c_{\alpha\beta}\sum_i u_{\beta,i}\hat{n}_{\beta,i} = \vec{f}_\alpha$$

where $u_{\beta,i}$ and $\hat{n}_{\beta,i}$ are the amplitude in the ith mode and it's respective normal mode vector. Using the orthogonality and normalization given my Aki and Richards:

$$\vec{u}_\alpha(\vec{r},\omega) = \sum_i \frac{\sum_\beta \hat{n}(\vec{r}_\beta)^*_{\beta,i}\cdot\vec{f}(\vec{r}_\beta,\omega)_\beta}{\omega^2 + i\frac{\omega\omega_i}{Q} + \omega_i^2}\ \hat{n}(\vec{r})_{\alpha,i}$$

Generalizing to continuum:

$$\vec{u}(\vec{r},\omega) = \sum_i \frac{\int \hat{n}(\vec{r}\,')^*_i\cdot\vec{f}(\vec{r}\,',\omega)dV'}{\omega^2 + i\frac{\omega\omega_i}{Q} + \omega_i^2}\ \hat{n}(\vec{r})_i$$

3 References

[1] Quantitative Seismology 2nd Edition, Keiiti Aki and Paul G. Richards

[2] Constraining the gravitational wave energy density of the Universe using Earth's ring, Michael Coughlin and Jan Harms, Phys. Rev. D 90, 042005 (2014)

[3] Sound of Dark Matter: Searching for Light Scalars with Resonant-Mass Detectors, Asimina Arvanitaki, Savas Dimopoulos, and Ken Van Tilburg, Phys. Rev. Lett. 116, 031102 (2016)

[4] Weiss, R., and B. Block (1965), A gravimeter to monitor the 0 S 0 dilational mode of the Earth, J. Geophys. Res., 70(22), 5615–5627, doi:10.1029/JZ070i022p05615.