# A Note on Derivations: Outage Analysis in Directional NTN Links With UAV Platform Instabilities

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This supplementary note discusses the detailed mathematical derivations of outage analysis in directional NTN links with UAV platform instabilities. The rest of this note is organized as follows. Section I discusses the mathematical formulation of the ULA transmit antenna gain. Sections II discusses the outage probability derivation in UAV-to-gNB P2P directional links.

## I. Modeling ULA Transmit Antenna Gain

In this section, we provide a detailed derivation of the ULA transmit antenna gain (Eq. (7) of the main manuscript). We consider a UAV equipped with a ULA antenna of order $N_T \times 1$ for transmit beamforming. All the antenna elements are identical and equally spaced with inter-element spacing of $d$ (usually half wavelength $\frac{\lambda}{2}$). For far-field observations of a ULA, the phase difference between signals received from two adjacent antenna elements is approximately $d\cos\phi_{Tx}$ in the direction $\phi_{Tx}$, as shown in the Fig. 1. This direction-dependent phase difference across the signals received from each antenna element influences the radiation pattern of the ULA towards the receiver, leading to either constructive or destructive interference. Considering the signal received by the first antenna element ($r_1$) as the reference, the phase difference ($\psi_i,\ i \in [1, N_T]$) of the signal $r_i$ received from antenna element $i$ can be expressed using Eq. (1) given below.

$$\psi_i = \frac{2\pi}{\lambda} \times (i-1)d\cos\phi_{Tx} = k(i-1)d\cos\phi_{Tx}, \quad (1)$$

where $k = \frac{2\pi}{\lambda}$ represents the wave number. Using Eq. (1), phase differences of the signals received from each antenna element with respect to $r_1$ in the far-field can be expressed as an array manifold vector. Whereas the complex weight vector added to steer the beam in the desired direction is termed the steering vector. The array manifold vector ($AMV_{Tx}(\phi_{Tx})$) in the direction $\phi_{Tx}$ is expressed below in Eq. (2), whereas the steering vector ($SV_{Tx}(\phi_{St,Tx})$) to steer the beam in $\phi_{St,Tx}$ direction is expressed in Eq. (3).

$$\begin{aligned} AMV_{Tx}(\phi_{Tx}) &= \begin{bmatrix} \mathrm{e}^{\psi_1} & \mathrm{e}^{\psi_2} & \cdots & \mathrm{e}^{\psi_{N_T}} \end{bmatrix} \\ &= \begin{bmatrix} 1 & \mathrm{e}^{jkd\cos\phi_{Tx}} & \cdots & \mathrm{e}^{j(N_T-1)kd\cos\phi_{Tx}} \end{bmatrix}, \end{aligned} \quad (2)$$

$$\begin{aligned} &SV_{Tx}(\phi_{St,Tx}) = \\ &\quad \frac{1}{\sqrt{N_T}} \begin{bmatrix} 1 & \mathrm{e}^{-jkd\cos(\phi_{St,Tx})} & \dots & \mathrm{e}^{-j(N_T-1)kd\cos(\phi_{St,Tx})} \end{bmatrix}^T. \end{aligned} \quad (3)$$

Using Eqs. (2) and (3), the transmitter antenna gain can now be calculated as shown in Eq. (4) below.

$$\begin{aligned} &G_{Tx}(\phi_{Tx}, \phi_{St,Tx}) \\ &\quad = |AMV_{Tx}(\phi_{Tx}) \times SV_{Tx}(\phi_{St,Tx})|^2 \\ &\quad = \left| \frac{1}{\sqrt{N_T}} \sum_{n=0}^{N_T-1} \mathrm{e}^{jnkd(\cos(\phi_{Tx}) - cos(\phi_{St,Tx}))} \right|^2; \\ &\qquad\qquad \text{where } \phi_{St,Tx} = \phi_{M,Tx} + \phi_{Tx}, \end{aligned} \quad (4)$$

where $\phi_{M,Tx}$ is the misalignment introduced due to platform instabilities. Considering the receiver is positioned towards the broadside of the ULA mounted on the UAV ($\phi_{Tx} = 90°$, for ease of analysis and without loss of generality) and beam steered towards the receiver, the transmit antenna gain in terms of $\phi_{M,Tx}$ can be expressed as shown in Eq. (4) below.

$$\begin{aligned} G_{Tx}(\phi_{M,Tx}) &= G_{Tx}(\phi_{Tx} = 90°, \phi_{St,Tx} = 90° + \phi_{M,Tx}) \\ &= \left| \frac{1}{\sqrt{N_T}} \sum_{n=0}^{N_T-1} \mathrm{e}^{jnkd(\cos(90°) - \cos(90° + \phi_{M,Tx}))} \right|^2 \\ &= \left| \frac{1}{\sqrt{N_T}} \sum_{n=0}^{N_T-1} \mathrm{e}^{-jnkd\cos(90° + \phi_{M,Tx})} \right|^2. \end{aligned} \quad (5)$$

Taking $d = \lambda/2$ and considering the typical range of $\phi_{M,Tx}$, Eq. (5) can be further simplified using geometric progression as shown in Eq. (6) below.

$$\begin{aligned} G_{Tx}(\phi_{M,Tx}) &= \left| \frac{1}{\sqrt{N_T}} \sum_{n=0}^{N_T-1} \mathrm{e}^{-jn\pi\cos(90° + \phi_{M,Tx})} \right|^2 \\ &= \left| \frac{1}{\sqrt{N_T}} \sum_{n=0}^{N_T-1} \mathrm{e}^{jn\pi\sin(\phi_{M,Tx})} \right|^2 \\ &\approx \left| \frac{1}{\sqrt{N_T}} \sum_{n=0}^{N_T-1} \mathrm{e}^{jn\pi\phi_{M,Tx}} \right|^2 \\ &\approx \left| \frac{1}{\sqrt{N_T}} \frac{(1 - e^{jN_T\pi\phi_{M,Tx}})}{(1 - e^{j\pi\phi_{M,Tx}})} \right|^2. \end{aligned} \quad (6)$$

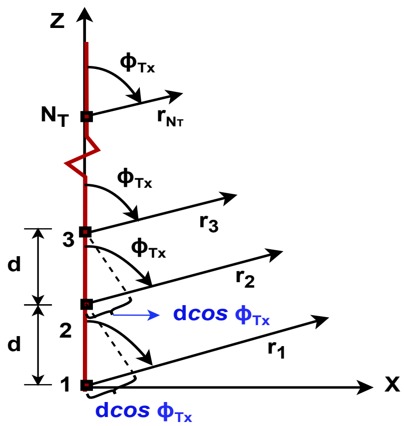


Fig. 1: ULA antenna geometry considered for modeling.

Eq. (6) can be further solved as given below.

$$
\begin{aligned}
&G_{Tx}(\phi_{M,Tx}) \\
&= \left[\frac{1}{\sqrt{N_T}}\sqrt{\frac{(1-cos(N_T\pi\phi_{M,Tx}))^2+sin^2(N_T\pi\phi_{M,Tx})}{(1-cos(\pi\phi_{M,Tx}))^2+sin^2(\pi\phi_{M,Tx})}}\right]^2 \\
&= \left[\frac{1}{\sqrt{N_T}}\sqrt{\frac{(1-\cos(N_T\pi\phi_{M,Tx}))}{(1-\cos(\pi\phi_{M,Tx}))}}\right]^2 \\
&= \left[\frac{1}{\sqrt{N_T}}\frac{\sin(\frac{N_T\pi\phi_{M,Tx}}{2})}{\sin(\frac{\pi\phi_{M,Tx}}{2})}\right]^2 . \qquad (7)
\end{aligned}
$$

By adjusting the coefficients of the numerator and denominator, Eq. (7) can be rewritten in terms of the $sinc(.)$ function, as shown in Eq. (8) below.

$$
\begin{aligned}
&G_{Tx}(\phi_{Tx},\phi_{St,Tx}) \\
&= \left[\frac{1}{\sqrt{N_T}} \times \frac{\frac{\frac{N_T\pi\phi_{M,Tx}}{2}\times\sin(\frac{N_T\pi\phi_{M,Tx}}{2})}{\frac{N_T\pi\phi_{M,Tx}}{2}}}{\frac{\frac{\pi\phi_{M,Tx}}{2}\times\sin(\frac{\pi\phi_{M,Tx}}{2})}{\frac{\pi\phi_{M,Tx}}{2}}}\right]^2 \\
&= \left[\sqrt{N_T}\times\frac{\mathrm{sinc}(\frac{N_T\pi\phi_{M,Tx}}{2})}{\mathrm{sinc}(\frac{\pi\phi_{M,Tx}}{2})}\right]^2 \\
&= N_T \times \frac{\mathrm{sinc}^2(\frac{N_T\pi\phi_{M,Tx}}{2})}{\mathrm{sinc}^2(\frac{\pi\phi_{M,Tx}}{2})}. \qquad (8)
\end{aligned}
$$

## II. Scenario 1 (S1): Analytical Formulation of Outage Probability in a UAV-to-gNB P2P Directional Link

In this section, the detailed mathematical formulation of the outage probability in a UAV-to-gNB P2P directional link is discussed.

### A. Case 1 (S1-C1) - $\phi_{M,Tx} = 0°$

The outage probability in this case is given by Eq. (9) below (Eq. (10) in main manuscript).

$$
\begin{aligned}
P_{\mathrm{o}} &= \mathbb{P}\left[SNR_I(h) < SNR_{Th}\right] \\
&= \mathbb{P}\left[|h|^2 N_T SNR_B < SNR_{Th}\right] \\
&= \mathbb{P}\left[|h|^2 < \frac{SNR_{Th}}{N_T \times SNR_B}\right]. \qquad (9)
\end{aligned}
$$

As $h$ follows the Rician fading, the probability density function ($f_{|h|^2}(z)$) of channel fading power ($|h|^2$) is given below in Eq. (10).

$$
f_{|h|^2}(z) = \frac{(K+1)}{\Omega}e^{\left(-K-\frac{z(K+1)}{\Omega}\right)}I_0\left\{2\sqrt{\frac{K(K+1)z}{\Omega}}\right\}, \qquad (10)
$$

where $K$ and $\Omega$ represent relative strength of the line of sight (LoS) component with respect to the non-line of sight (NLoS) and average power, respectively. Considering $\Omega = 1$ and using Eq. (10), Eq. (9) can be solved as shown in Eq. (11) below.

$$
\begin{aligned}
P_{\mathrm{o}} &= \int_0^{\frac{SNR_{Th}}{N_T\times SNR_B}} f_{|h|^2}(z)\mathrm{d}z \\
&= \int_0^{\frac{SNR_{Th}}{N_T\times SNR_B}} (K+1)e^{-K-z(K+1)}I_0\left\{\sqrt{4K(K+1)z}\right\}\mathrm{d}z. \qquad (11)
\end{aligned}
$$

In above Eq. (11), we consider $z = \frac{x^2}{2(K+1)}$, therefore $\mathrm{d}z = \frac{x\ \mathrm{d}x}{(K+1)}$. Substituting the variable change and adjusting the upper and lower limit of integration accordingly, Eq. (11) can be solved as shown in Eq. (13) below.

$$
\begin{aligned}
P_o &= \int_0^{\sqrt{\frac{2(K+1)SNR_{Th}}{N_T\times SNR_B}}} (K+1)e^{-K-\frac{x^2}{2(K+1)}}(K+1) \\
&\qquad \times I_0\left\{\sqrt{4K(K+1)\tfrac{x^2}{2(K+1)}}\right\}\frac{x\ \mathrm{d}x}{(K+1)} \\
&= \int_0^{\sqrt{\frac{2(K+1)SNR_{Th}}{N_T\times SNR_B}}} x\ e^{-\frac{2K+x^2}{2}} I_0\left\{\sqrt{2K}\ x\right\}\mathrm{d}x. \qquad (12)
\end{aligned}
$$

Eq. (12) can be further simplified to Eq. (13) shown below using first order Marcum-Q function described in Eq. (14).

$$
\begin{aligned}
P_o &= 1 - \int_{\sqrt{\frac{2(K+1)SNR_{Th}}{N_T\times SNR_B}}}^{0} x\ e^{-\frac{2K+x^2}{2}} I_0\left\{\sqrt{2K}\ x\right\}\mathrm{d}x \\
&= 1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(K+1)SNR_{Th}}{N_T\times SNR_B}}\right) \qquad (13)
\end{aligned}
$$

$$
Q_1(a,b) = \int_b^{\infty} xe^{-\frac{x^2+a^2}{2}}I_0(ax)\,dx, \qquad (14)
$$

### *B. Case 2 (S1-C2) - $\phi_{M,Tx} \neq 0°$, with $\phi_{M,Tx}$ varying at a rate faster than $h$ and $T_{Sym}$*

The outage probability in this case is given by Eq. (15) below (Eq. (15) in main manuscript).

$$\begin{aligned} P_{\mathrm{o}} &= \mathbb{P}\left[SNR_I(h) < SNR_{Th}\right] \\ &= \mathbb{P}\left[|h|^2 E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})]SNR_B < SNR_{Th}\right] \\ &= \mathbb{P}\left[|h|^2 < \frac{SNR_{Th}}{E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})]SNR_B}\right], \end{aligned} \tag{15}$$

The above Eq. (15) can be further simplified and expressed in terms of first order Marcum-Q function as shown in Eq. (16) below.

$$\begin{aligned} P_o &= \mathbb{P}\left[|h|^2 < \frac{SNR_{Th}}{E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})]SNR_B}\right] \\ &= 1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(K+1)SNR_{Th}}{SNR_B \times E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})]}}\right). \end{aligned} \tag{16}$$

To further solve $E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})]$, we consider the probability density function of $\phi_{M,Tx}$ given below in Eq. (17) (Eq. (12) of the main manuscript).

$$f_{\phi_{M,Tx}}(\theta_{M,Tx}) = \frac{\sqrt{\frac{2}{\pi\sigma_{Tx}^2}} e^{\frac{-\theta_{M,Tx}^2}{2\sigma_{Tx}^2}}}{erf(\frac{\theta_{U,Tx}}{\sqrt{2}\sigma_{Tx}}) - erf(\frac{\theta_{L,Tx}}{\sqrt{2}\sigma_{Tx}})}; \quad \theta_{M,Tx} \in [\theta_{L,Tx}, \theta_{U,Tx}], \tag{17}$$

where $\theta_{L,Tx}$, $\theta_{U,Tx}$, and $\sigma_{Tx}$ represent lower limit, upper limit, and standard deviation of $\phi_{M,Tx}$, respectively. Also, $erf(.)$ represents the error function, and is expressed using Eq. (18) below.

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\ dt. \tag{18}$$

Hence, using Eq. (17), $E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})]$ can be solved as shown in Eq. (19) below.

$$\begin{aligned} &E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})] \\ &\approx \int_{\theta_{L,Tx}}^{\theta_{U,Tx}} \tilde{G}_{Tx}(\theta_{M,Tx}) f_{\phi_{M,Tx}}(\theta_{M,Tx}) \mathrm{d}\theta_{M,Tx}. \end{aligned} \tag{19}$$

In Eq. (19), $\tilde{G}_{Tx}(\phi_{M,Tx})$ is given by Eq. (20) below.

$$\tilde{G}_{Tx}(\phi_{M,Tx}) = N_T e^{\frac{-\phi_{M,Tx}^2}{2a_t^2}}, \tag{20}$$

where, $a_t = \frac{\phi_{M,Tx}^{3dB}}{\sqrt{2\log(2)}}$ is a constant and $\phi_{M,Tx}^{3dB}$ is the misalignment at which the gain drops to half of the maximum achievable (can be referred to as 3 $dB$ gain point, *i.e.* $G_{Tx}(\phi_{M,Tx} = \phi_{M,Tx}^{3dB}) = \frac{N_T}{2}$). Using Eq. (20), Eq. (19) can be further solved as shown below.

$$\begin{aligned} &E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})] \\ &\approx \int_{\theta_{L,Tx}}^{\theta_{U,Tx}} N_T e^{\frac{-\theta_{M,Tx}^2}{2a_t^2}} \frac{\sqrt{\frac{2}{\pi\sigma_{Tx}^2}} e^{\frac{-\theta_{M,Tx}^2}{2\sigma_{Tx}^2}}}{erf(\frac{\theta_{U,Tx}}{\sqrt{2}\sigma_{Tx}}) - erf(\frac{\theta_{L,Tx}}{\sqrt{2}\sigma_{Tx}})} \mathrm{d}\theta_{M,Tx} \\ &\approx \frac{N_T \sqrt{\frac{2}{\pi\sigma_{Tx}^2}} \times}{erf(\frac{\theta_{U,Tx}}{\sqrt{2}\sigma_{Tx}}) - erf(\frac{\theta_{L,Tx}}{\sqrt{2}\sigma_{Tx}})} \\ &\qquad \times \int_{\theta_{L,Tx}}^{\theta_{U,Tx}} e^{-\theta_{M,Tx}^2(\frac{1}{2a_t^2} + \frac{1}{2\sigma_{Tx}^2})} \mathrm{d}\theta_{M,Tx}. \end{aligned} \tag{21}$$

Considering $B_{Tx} = \frac{\sqrt{\frac{2}{\pi\sigma_{Tx}^2}}}{erf(\frac{\theta_{U,Tx}}{\sqrt{2}\sigma_{Tx}}) - erf(\frac{\theta_{L,Tx}}{\sqrt{2}\sigma_{Tx}})}$ and $C_{2,Tx} = \frac{1}{2a_t^2} + \frac{1}{2\sigma_{Tx}^2}$, Eq. (21) can be expressed as shown in Eq. (22) below

$$\begin{aligned} &E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})] \\ &\qquad \approx B_{Tx} N_T \times \int_{\theta_{L,Tx}}^{\theta_{U,Tx}} e^{-C_{2,Tx}\theta_{M,Tx}^2} \mathrm{d}\theta_{M,Tx}. \end{aligned} \tag{22}$$

In Eq. (22), to solve the integration, we assume $p = \sqrt{C_{2,Tx}}\theta_{M,Tx}$, therefore $\mathrm{d}\theta_{M,Tx} = \frac{\mathrm{d}p}{\sqrt{C_{2,Tx}}}$. Substituting the variable change along with the lower and upper limit of the integration accordingly, Eq. (22) can be expressed as below.

$$E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})] \approx B_{Tx} N_T \int_{\sqrt{C_{2,Tx}}\theta_{L,Tx}}^{\sqrt{C_{2,Tx}}\theta_{U,Tx}} e^{-p^2} \frac{\mathrm{d}p}{\sqrt{C_{2,Tx}}}. \tag{23}$$

Integration in Eq. (23) can be separated into two parts as shown below in Eq. (24).

$$\begin{aligned} &E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})] \approx \frac{B_{Tx} N_T}{\sqrt{C_{2,Tx}}} \\ &\times \left(\int_0^{\sqrt{C_{2,Tx}}\theta_{U,Tx}} e^{-p^2} \mathrm{d}p - \int_0^{\sqrt{C_{2,Tx}}\theta_{L,Tx}} e^{-p^2} \mathrm{d}p\right). \end{aligned} \tag{24}$$

Multiplying with $\sqrt{\frac{4}{\pi}}$ in both numerator and denominator, Eq. (24) can be further expressed in terms of the error function as shown in below Eq. (25).

$$\begin{aligned} &E_{\phi_{M,Tx}}[G_{Tx}(\phi_{M,Tx})] \\ &\approx \frac{B_{Tx} N_T (erf(\theta_{U,Tx}\sqrt{C_{2,Tx}}) - erf(\theta_{L,Tx}\sqrt{C_{2,Tx}}))}{\sqrt{\frac{4C_{2,Tx}}{\pi}}}. \end{aligned} \tag{25}$$

Using Eqs. (25) and (16), the closed-form expression for outage probability can be derived as shown in Eq. (26).

$$P_o = 1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(K+1)\sqrt{\frac{4C_{2,Tx}}{\pi}}SNR_{Th}}{SNR_B \times B_{Tx}N_T(erf(\theta_{U,Tx}\sqrt{C_{2,Tx}}) - erf(\theta_{L,Tx}\sqrt{C_{2,Tx}}))}}\right). \tag{26}$$

### C. *Case 3 (S1-C3) - $\phi_{M,Tx} \neq 0°$, with $\phi_{M,Tx}$ varying at a rate similar to or slower than $h$ and $T_{Sym}$*

The outage probability in this case can be expressed as shown in below Eq. (27) (Eq. (21) of the main manuscript).

$$P_o = A \times E_{\phi_{M,Tx}}\left[\int_0^{\eta_T^{-1}(\phi_{M,Tx})} e^{-z(K+1)} I_0\left\{\sqrt{4K(K+1)z}\right\}\mathrm{d}z\right], \tag{27}$$

where $A = (K+1)e^{-K}$ is a constant used for ease of representation. In Eq. (27), the modified Bessel function of the first kind with zero order ($I_o(.)$) can be expressed using the power-series expansion shown in Eq. (28) below.

$$\begin{aligned} I_0(m) &= \sum_{l=0}^{\infty}\left(\frac{1}{l!}\right)^2 \times \left(\frac{m}{2}\right)^{2l} \\ I_0\left(\sqrt{4K(K+1)z}\right) &= \sum_{l=0}^{\infty}\left(\frac{1}{l!}\right)^2\left(\frac{4K(K+1)z}{4}\right)^l. \end{aligned} \tag{28}$$

Substituting Eq. (28) in Eq. (27), the outage probability can now be further expressed as below in Eq. (29).

$$\begin{aligned} P_\mathrm{o} &= A \times E_{\phi_{M,Tx}}\Bigg[\int_0^{\eta_T^{-1}(\phi_{M,Tx})} e^{-z(K+1)} \\ &\qquad \times \sum_{l=0}^{\infty}\left(\frac{1}{l!}\right)^2\left(\frac{4K(K+1)z}{4}\right)^l \mathrm{d}z\Bigg] \\ &= A \times E_{\phi_{M,Tx}}\Bigg[\sum_{l=0}^{\infty}\left(\frac{1}{l!}\right)^2 (K(K+1))^l \\ &\qquad \times \int_0^{\eta_T^{-1}(\phi_{M,Tx})} e^{-z(K+1)} z^l \mathrm{d}z\Bigg]. \end{aligned} \tag{29}$$

Here, we consider $p = z(K+1)$, therefore $\mathrm{d}z = \dfrac{\mathrm{d}p}{(K+1)}$. Substituting the variable change and adjusting the upper and lower limits, the integration in Eq. (29) can be further simplified as shown in Eq. (30) below.

$$\begin{aligned} &\int_0^{\eta_T^{-1}(\phi_{M,Tx})} e^{-z(K+1)} z^l \mathrm{d}z \\ &= \int_0^{(K+1)\eta_T^{-1}(\phi_{M,Tx})} e^{-p}\left(\frac{p}{K+1}\right)^l \frac{\mathrm{d}p}{(K+1)} \\ &= (K+1)^{-(l+1)}\int_0^{(K+1)\eta_T^{-1}(\phi_{M,Tx})} e^{-p}p^l \mathrm{d}p. \end{aligned} \tag{30}$$

Eq. (30) can then be expressed in terms of lower incomplete gamma function ($\gamma(.,.)$) as shown below in Eq. (31).

$$\begin{aligned} &\int_0^{\eta_T^{-1}(\phi_{M,Tx})} e^{-z(K+1)} z^l \mathrm{d}z \\ &= (K+1)^{-(l+1)}\int_0^{(K+1)\eta_T^{-1}(\phi_{M,Tx})} e^{-p}p^{(l+1-1)}\mathrm{d}p \\ &= (K+1)^{-(l+1)} \times \gamma\big(l+1, \eta_T^{-1}(\phi_{M,Tx})(K+1)\big), \end{aligned} \tag{31}$$

where $\gamma(.,.)$ is given by Eq. (32).

$$\gamma(a,b) = \int_0^b t^{a-1}e^{-t}\ dt. \tag{32}$$

Using Eq. (31), Eq. (29) can be expressed as shown in Eq. (33) below.

$$\begin{aligned} P_o = A \times E_{\phi_{M,Tx}}\Bigg[&\sum_{l=0}^{\infty}\left(\frac{1}{l!}\right)^2 (K+1)^{-(l+1)} \\ &\times \gamma\big(l+1, \eta_T^{-1}(\phi_{M,Tx})(K+1)\big)\Bigg]. \end{aligned} \tag{33}$$

For a large fade margin when $SNR_{Th} \ll SNR_B$ or $\eta_T^{-1} \ll 1$, higher-order summation terms with respect to $l$ in Eq. (33) can be neglected. Hence, considering only the summation term $l = 0$, the Eq. (33) can be further reduced to Eq. (34) shown below.

$$P_\mathrm{o} = E_{\phi_{M,Tx}}[A(K+1)^{-1}\gamma(1, \eta_T^{-1}(\phi_{M,Tx})(K+1))]. \tag{34}$$

In Eq. (34), $\gamma(1, \eta_T^{-1}(\phi_{M,Tx})(K+1))$ can be further simplified using the following identity in Eq. (35) (where $b = \eta_T^{-1}(\phi_{M,Tx})(K+1)$).

$$\gamma(1,b) = \int_0^b e^{-t}\ dt = (1 - e^{-b}). \tag{35}$$

For smaller value of $b$, the exponential term can also be approximated as shown below in Eq. (36).

$$e^{-b} \approx 1 - b. \tag{36}$$

Hence, using Eqs. (35) and (36) for smaller $m$, Eq. (34) can be further simplified as shown in Eq. (37) (this approximation is valid in this case as we consider a high $SNR_B$ regime).

$$\begin{aligned} P_\mathrm{o} &= E_{\phi_{M,Tx}}[A(K+1)^{-1}(1 - e^{-\eta_T^{-1}(\phi_{M,Tx})(K+1)})] \\ &= E_{\phi_{M,Tx}}[A(K+1)^{-1}(1 - (1 - \eta_T^{-1}(\phi_{M,Tx})(K+1)))] \\ &= E_{\phi_{M,Tx}}[A\eta_T^{-1}(\phi_{M,Tx})] \\ &= E_{\phi_{M,Tx}}\left[(K+1)e^{-K}\frac{SNR_{Th}}{SNR_B \times G_{Tx}(\phi_{M,Tx})}\right] \\ &= \frac{(K+1)e^{-K}SNR_{Th}}{SNR_B}E_{\phi_{M,Tx}}\left[\frac{1}{G_{Tx}(\phi_{M,Tx})}\right]. \end{aligned} \tag{37}$$

Using Eq. (20), $E_{\phi_{M,Tx}}\left[\frac{1}{G_{Tx}(\phi_{M,Tx})}\right]$ in Eq. (37) can be derived as shown below in Eq. (38). Here, we consider $E_{\phi_{M,Tx}}\left[\frac{1}{G_{Tx}(\phi_{M,Tx})}\right] \approx E_{\phi_{M,Tx}}\left[\frac{1}{\tilde{G}_{Tx}(\phi_{M,Tx})}\right]$, as $G_{Tx}(\phi_{M,Tx})$ closely follows $\tilde{G}_{Tx}(\phi_{M,Tx})$.

$$
\begin{aligned}
&E_{\phi_{M,Tx}}\left[\frac{1}{G_{Tx}(\phi_{M,Tx})}\right] \\
&\approx \int_{\theta_{L,Tx}}^{\theta_{U,Tx}} \frac{1}{\tilde{G}_{Tx}(\theta_{M,Tx})} f_{\phi_{M,Tx}}(\theta_{M,Tx}) \mathrm{d}\theta_{M,Tx} \\
&\approx \int_{\theta_{L,Tx}}^{\theta_{U,Tx}} \frac{1}{N_T} e^{\frac{\theta_{M,Tx}^2}{2a_t^2}} \frac{\sqrt{\frac{2}{\pi\sigma_{Tx}^2}} e^{\frac{-\theta_{M,Tx}^2}{2\sigma_{Tx}^2}}}{erf(\frac{\theta_{U,Tx}}{\sqrt{2}\sigma_{Tx}}) - erf(\frac{\theta_{L,Tx}}{\sqrt{2}\sigma_{Tx}})} \mathrm{d}\theta_{M,Tx} \\
&\approx \frac{\sqrt{\frac{2}{\pi\sigma_{Tx}^2}}}{erf(\frac{\theta_{U,Tx}}{\sqrt{2}\sigma_{Tx}}) - erf(\frac{\theta_{L,Tx}}{\sqrt{2}\sigma_{Tx}})} \times \frac{1}{N_T} \\
&\qquad \times \int_{\theta_{L,Tx}}^{\theta_{U,Tx}} e^{-\theta_{M,Tx}^2\left(\frac{1}{2\sigma_{Tx}^2} - \frac{1}{2a_t^2}\right)} \mathrm{d}\theta_{M,Tx}. \quad (38)
\end{aligned}
$$

Considering $B_{Tx} = \frac{\sqrt{\frac{2}{\pi\sigma_{Tx}^2}}}{erf(\frac{\theta_{U,Tx}}{\sqrt{2}\sigma_{Tx}}) - erf(\frac{\theta_{L,Tx}}{\sqrt{2}\sigma_{Tx}})}$ and $C_{2,Tx} = \frac{1}{2a_t^2} + \frac{1}{2\sigma_{Tx}^2}$ and $C_{3,Tx} = (\frac{1}{2\sigma_{Tx}^2} - \frac{1}{2a_t^2})$ constants used for ease of representation, the integration term in Eq. (38) can be further simplified as shown in Eq. (39) below.

$$
\begin{aligned}
E_{\phi_{M,Tx}}\left[\frac{1}{G_{Tx}(\phi_{M,Tx})}\right] &= \frac{B_{Tx}}{N_T}\sqrt{\frac{\pi}{4C_{3,Tx}}} \\
&\times \left(erf(\theta_{U,Tx}\sqrt{C_{3,Tx}}) - erf(\theta_{L,Tx}\sqrt{C_{3,Tx}})\right), \quad (39)
\end{aligned}
$$

Finally, substituting Eq. (39) in Eq. (37), the closed-form outage probability can now be expressed as shown in Eq. (40) below.

$$
\begin{aligned}
P_{\mathrm{o}} &= \frac{(K+1)e^{-K} SNR_{Th}}{SNR_B} \times \\
&\frac{B_{Tx}\left(erf(\theta_{U,Tx}\sqrt{C_{3,Tx}}) - erf(\theta_{L,Tx}\sqrt{C_{3,Tx}})\right)}{N_T\sqrt{\frac{4C_{3,Tx}}{\pi}}} \quad (40)
\end{aligned}
$$