(A)

~~EJEMPLO 1~~

~~El sistema~~ $\cancel{\begin{cases} 3x_1 + 2x_2 = 5 \\ 2x_1 + 4x_2 = 6 \end{cases}}$ ~~es equivalente a:~~

$$\cancel{\begin{cases} 2x_1 + 4x_2 = 6 \\ 3x_1 + 2x_2 = 5 \end{cases}}$$

$\downarrow$

~~Intercambio~~

EJEMPLOS CAP. 3

EJEMPLO 1

El sistema $\begin{cases} x_1 - x_2 + x_3 = -4 \\ 5x_1 - 4x_2 + 3x_3 = -12 \\ 2x_1 + x_2 + x_3 = 11 \end{cases}$ es equivalente a:

- Intercambio: $\begin{cases} 2x_1 + x_2 + x_3 = 11 \\ 5x_1 - 4x_2 + 3x_3 = -12 \\ x_1 - x_2 + x_3 = -4 \end{cases}$

- Escalado ~~E3 ↔ 3 × E3~~ $E_1 = 3 \times E_1$ $\begin{cases} 3x_1 - 3x_2 + 3x_3 = -12 \\ 5x_1 - 4x_2 + 3x_3 = -12 \\ 2x_1 + x_2 + x_3 = 11 \end{cases}$

- Sustitución $E_3 = E_2 - 3E_1$ $\begin{cases} x_1 - x_2 + x_3 = -4 \\ 5x_1 - 4x_2 + 3x_3 = -12 \\ 2x_1 - x_2 + 0x_3 = 0 \end{cases}$

EJEMPLO 2:

$$\begin{cases} 3x = 6 \\ 2y = 10 \\ 5z = 2 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & 2 & 0 & 10 \\ 0 & 0 & 5 & 2 \end{array}\right] \Rightarrow \begin{cases} x = 2 \\ y = 5 \\ z = 0.4 \end{cases}$$

EJEMPLO 3

$$\begin{cases} \text{i)} & 4x_1 - x_2 + 2x_3 + 3x_4 = 20 \\ \text{ii)} & -2x_2 + 7x_3 - 4x_4 = -7 \\ \text{iii)} & 6x_3 + 5x_4 = 4 \\ \text{iv)} & 3x_4 = 6 \end{cases} \qquad \left[\begin{array}{cccc|c} 4 & -1 & 2 & 3 & 20 \\ 0 & -2 & 7 & -4 & -7 \\ 0 & 0 & 6 & 5 & 4 \\ 0 & 0 & 0 & 3 & 6 \end{array}\right]$$

i) $x_4 = 2$ ii) $6x_3 + 10 = 4$ iii) $-2x_2 - 7 - 8 = -7$ iv) $4x_1 + 4 - 2 + 6 = 20$

$x_3 = -1$ $x_2 = -4$ $x_1 = 3$ ✓

Siguiendo la fórmula:

$\boxed{n=4}$

$$x_4 = \frac{6}{3} = 2 \qquad x_2 = \frac{-7 - \overbrace{(7.(-1) - 4.2)}^{-15}}{-2} = -4$$

$$x_3 = \frac{4 - (5.2)}{6} = -1 \qquad x_1 = \frac{20 - [-1.(-4) + 2.(-1) + 3.2]}{4} = 3$$

$\boxed{\text{Obs}}$

$$\sum_{j=k+1}^{n} a_{kj} x_j = A[k, k+1:n] * x[k+1:n]$$

En general

$$U = [u_1, \ldots, u_n] \quad V = \begin{bmatrix} v_1 \\ \vdots \\ \vdots \\ v_n \end{bmatrix} \qquad U * V = \sum_{j=1}^{n} u_j v_j .$$

EJEMPLO 4

$$\begin{cases} x + 2y - z - 3t = -8 \\ 2x + 2z - t = 13 \\ -x + y + z - t = 8 \\ 3x + 3y - z + 2t = -1 \end{cases}$$

Sistema equiv. triangular:

$$\begin{cases} x + 2y - z + 3t = -8 \\ -4y + 4z - 7t = 29 \\ -12z + 13t = -87 \\ -136t = 408 \end{cases}$$

$$\Rightarrow \boxed{x = 1 \quad y = 2 \quad z = 4 \quad t = -3}$$

◎ Primero pondría a ellos que lo resuelvan, seguramente se plantea algo así: (multipl. cruzada y resta)

x	y	z	t	b_i	
1	2	-1	3	-8	1º PASO
2	0	2	-1	13	
-1	1	1	-1	8	
3	3	-1	2	-1	
	-4	4	-7	29	2º PASO
	3	0	2	0	
	-3	2	-7	23	
		-12	13	-87	3º PASO
		4	7	-5	
			-136	408	

◎ Luego hacer ver que son multiplicaciones cruzadas y estos son la sistematización de hacer:

$\boxed{\text{1º PASO}}$

$$\begin{cases} \text{Fila } 2 = \text{Fila } 2 - 2\ \text{Fila } 1 \\ \text{Fila } 3 = \text{Fila } 3 - (-1)\ \text{Fila } 1 \\ \text{Fila } 4 = \text{Fila } 4 - 3\ \text{Fila } 1 \end{cases}$$

"multiplicadores"

$$\left.\begin{aligned} m_{21} &= 2 = 2/1 \\ m_{31} &= -1 = -1/1 \\ m_{41} &= 3 = 3/1 \end{aligned}\right\} m_{r1} = \frac{a_{r1}}{a_{11}}$$

fila a cambiar — fila pivote

$\boxed{\text{2º PASO}}$

$$\begin{cases} \text{Fila } 3 = \text{Fila } 3 - (-3/4)\ \text{Fila } 2 \\ \text{Fila } 4 = \text{Fila } 4 - 3/4\ \text{Fila } 2 \end{cases}$$

multiplicadores

$$\left.\begin{aligned} m_{32} &= -3/4 \\ m_{42} &= 3/4 \end{aligned}\right\} m_{r2} = \frac{a_{r2}}{a_{22}}$$

$\boxed{3^{o}\ \text{PASO}}$ Fila 4 = Fila 4 − $\underbrace{(-4/12)}$ Fila 3

multiplicador $m_{43} = \dfrac{a_{43}}{a_{33}} = -\dfrac{4}{12}$

EJEMPLO 5

$$\begin{cases} x - 2y + z = -4 \\ -2x + 4y - 3z = 3 \\ x - 3y - 4z = -1 \end{cases}$$

x	y	z	ti
-1	-2	1	-4
-2	4	-3	3
1	-3	-4	-1
	0	-1	-5
	-1	-5	3
	-1	-5	3
	0	-1	-5
		1	5

$$\begin{cases} x - 2y + z = -4 \\ -y - 5z = 3 \\ z = 5 \end{cases} \Downarrow \begin{cases} x = -65 \\ y = -28 \\ z = 5 \end{cases}$$

EJEMPLO 6

$$\begin{cases} 1{,}133\,x + 5{,}281\,y = 6{,}414 \\ 24{,}14\,x - 1{,}210\,y = 22{,}93 \end{cases} \qquad \left[\begin{array}{cc|c} 1{,}133 & 5{,}281 & 6{,}414 \\ 24{,}14 & -1{,}210 & 22{,}93 \end{array}\right]$$

multiplicador: $m_{21} = \dfrac{24{,}14}{1{,}133} = 21{,}31$

$\boxed{\text{Fila 2} = \text{Fila 2} - 21{,}31\ \text{Fila 1}}$

$24{,}14 - 21{,}31 \times 1{,}133$	$-1{,}210 - 21{,}31 \times 5{,}281$	$22{,}93 - 21{,}31 \times 6{,}414$
$24{,}14 - 24{,}14$	$-1{,}210 - 112{,}5$	$22{,}93 - 136{,}7$
$0{,}0000$	$-113{,}7$	$-113{,}8$

$$\begin{cases} 1{,}133\,x + 5{,}281\,y = 6{,}414 \\ \cancel{0{,}0000\,x}\ -113{,}7\,y = -113{,}8 \end{cases} \Rightarrow \begin{cases} y = \dfrac{-113{,}8}{-113{,}7} = 1{,}001 \\ x = \dfrac{6{,}414 - 5{,}281 \times 1{,}001}{1{,}133} = \dfrac{6{,}414 - 5{,}286}{1{,}133} = \dfrac{1{,}128}{1{,}133} = 0{,}9956 \end{cases}$$

observar magnitud

pequeño pivote ⇒ gran multiplicador

Intercambiando el orden

$$\begin{cases} 24{,}14\,x - 1{,}210\,y = 22{,}93 \\ 1{,}133\,x + 5{,}281\,y = 6{,}414 \end{cases}$$

multiplicador $m_{21} = 1{,}133/24{,}14 = 0{,}04693$ $\quad \boxed{F_2 = F_2 - 0{,}04693\,F_1}$

$1{,}133 - 0{,}04693 \times 24{,}14$	$5{,}281 - 0{,}04693 \times 1{,}210$	$6{,}414 - 0{,}04693 \times 22{,}93$
$1{,}133 - 1{,}133$	$5{,}281 - 0{,}05679$	$6{,}414 - 1{,}076$
$0{,}0000$	$5{,}338$	$5{,}338$

$$\begin{cases} 24{,}14\,x - 1{,}210\,y = 22{,}93 \\ 5{,}338\,y = 5{,}338 \end{cases} \Rightarrow \quad y = 5{,}338 / 5{,}338 = 1{,}000$$

$$x = \frac{22{,}93 + 1{,}210 \times 1{,}000}{24{,}14} = \frac{22{,}93 + 1{,}210}{24{,}14} = \frac{24{,}14}{24{,}14} = 1{,}000$$

D

observar magnitud.

pivote grande → multiplicador chico.

→ menos error.

EJEMPLO 7

Pivoteo parcial

$$\begin{cases} x_1 - x_2 + x_3 = -4 \\ 5x_1 - 4x_2 + 3x_3 = -12 \\ 2x_1 + x_2 + x_3 = 11 \end{cases}$$

x_1	x_2	x_3	t_i
1	-1	1	-4
5	-4	3	-12
2	1	1	11
5	-4	3	-12
1	-1	1	-4
2	1	1	11
	-1	2	-8
	13	-1	79
	13	-1	79
	-1	2	-8
		25	-25

Cambio xq hay otro con mayor valor absoluto.

Cambio xq hay otro con > valor abs.

⇐

$$\begin{cases} x_1 - x_2 + x_3 = -4 \\ 13x_2 - x_3 = 79 \\ 25\,x_3 = -25 \end{cases}$$

$$\begin{cases} x_3 = -1 \\ x_2 = 78/13 = 6 \\ x_1 = \underline{-4 + 1 + 6} = 3 \end{cases}$$

EJEMPLO 8

Sobre el ejemplo anterior:

				max abs A	$\lvert a_{i1}/\max\rvert$
1	-1	1	-4	1	1
5	-4	3	-12	5	1.25
2	1	1	11	2	2
2	1	1	11		
5	-4	3	-12		
1	-1	1	-4		
	-13	1	-79	13	1
	-3	1	-19	3	1
		-10	10		

$x_3 = -1$

$-13x_2 - 1 = -79 \Rightarrow x_2 = 6$

$x_1 = 3$

EJEMPLO 4 cont

- En el ejemplo 4 llegamos a

$$\begin{array}{c} \overbrace{\qquad\qquad\qquad}^{A} \quad \overbrace{\quad}^{b} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & +3 & -8 \\ 0 & -4 & 4 & -7 & 29 \\ 0 & 0 & -12 & 13 & -87 \\ 0 & 0 & 0 & -136 & 408 \end{array}\right]$$

- Debemos continuar hasta que A se transforme en diagonal:

Fila 2 = Fila 2 / (-4)
F3 = F3 / (-12)
F4 = F4 / (-136)

$$\begin{array}{ccccc} x & y & z & t & \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & -8 \\ 0 & 1 & -1 & 7/4 & -29/4 \\ 0 & 0 & 1 & -13/12 & 87/12 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

F1 = F1 + (-3) F4
F2 = F2 - 7/4 F4
F3 = F3 + 13/12 F4

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & -2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

F1 = F1 + F3
F2 = F2 + F3

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

F1 = F1 - 2F2

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \Rightarrow \begin{cases} x = 1 \\ y = 2 \\ z = 4 \\ t = -3 \end{cases}$$

$-8 - 3(-8)$

$-\frac{29}{4} + \frac{7}{4}(-3) = \frac{25}{2}$

$-\frac{29}{4} - \frac{7}{4}(-3) =$

$-8 + 3(-3) \quad -2 - \frac{7}{4}($

$-27,5 - 2 + 2,5 = -32,5$

$\frac{87}{12} + \frac{13}{12}(-3)$

$3 - \frac{3}{2}3 = 3 - \frac{9}{2} = \frac{6-9}{2} = -\frac{3}{2}$

$-4 - \frac{3}{2} = \frac{-8-3}{2} = -\frac{11}{2}$

EJEMPLO 8 → cuentas complicadas, usar el sgte.

$$\begin{cases} 2x + 3y + z = 1 \\ 3x - 2y - 4z = -3 \\ 5x - y - z = 4 \end{cases}$$

$$\left[\begin{array}{ccc|c|ccc} 2 & 3 & 1 & 1 & 1 & 0 & 0 \\ 3 & -2 & -4 & -3 & 0 & 1 & 0 \\ 5 & -1 & -1 & 4 & 0 & 0 & 1 \end{array}\right]$$

F1 = F1/2
F2 = F2 - 3/2 F1
F3 = F3 - 5/2 F1

$$\left[\begin{array}{ccc|c|ccc} 1 & 3/2 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -13/2 & -11/2 & -9/2 & -3/2 & 1 & 0 \\ 0 & -17/2 & -7/2 & 3/2 & -5/2 & 0 & 1 \end{array}\right]$$

$F_1 = F_1 + \frac{3}{2}\frac{2}{13} F_2$
$F_2 = F_2 / (-13/2)$
$F_3 = F_3 - \frac{17}{2}\frac{2}{13} F_2$

$\frac{1}{2} + \frac{3}{13}\frac{11}{2} = \frac{13-33}{26} = \frac{-20}{26} = -\frac{10}{13}$

$\frac{1}{2} - \frac{3}{13}\frac{9}{2} = \frac{13+27}{26} = \frac{-14}{26} = \frac{-7}{13}$

$\frac{1}{2} - \frac{3}{13} \quad -\frac{7}{2} + \frac{17}{13}\frac{11}{2}$

$$\left[\begin{array}{ccc|c|ccc} 1 & 0 & -10/13 & -7/13 & 2/13 & \frac{3}{13} & 0 \\ 0 & 1 & 11/13 & 9/13 & 3/13 & -2/13 & 0 \\ 0 & 0 & 48/13 & 96/13 & -7/13 & -17/13 & 1 \end{array}\right]$$

$F_1 = F_1 + \frac{10}{13}\frac{13}{48} F_3$
$F_2 = F_2 - \frac{11}{13}\frac{13}{48} F_3$
$F_3 = F_3 / (48/13)$

$$\rightarrow \left[\begin{array}{ccc|c|ccc} 1 & 0 & 0 & 1 & 1/24 & -1/24 & 5/24 \\ 0 & 1 & 0 & -1 & 17/48 & 7/48 & 11/48 \\ 0 & 0 & 1 & 2 & -7/48 & -17/48 & \frac{13}{48} \end{array}\right]$$

$$\underbrace{\qquad\qquad}_{A^{-1}}$$

$\frac{221}{624} \quad -\frac{7}{13}\frac{5}{24}\frac{96}{13} \quad \frac{3}{13}\frac{11}{48} \quad \frac{96}{13}\frac{13}{48} \quad \frac{7}{13}\frac{13}{48} \quad \frac{17}{48}$

$\boxed{\text{EXEMPLO 9}}$ F

$$\begin{cases} x_1 - x_2 + x_3 = -4 \\ 5x_1 - 4x_2 + 3x_3 = -12 \\ 2x_1 + x_2 + x_3 = 11 \end{cases}$$

$$\left(\det(A) = 5 \neq 0 \Rightarrow A \text{ invertible} \quad A = \begin{pmatrix} 1 & -1 & 1 \\ 5 & -4 & 3 \\ 2 & 1 & 1 \end{pmatrix} \right)$$

(no importa) (sí, pero bueno).

$$\left(\begin{array}{ccc|c|ccc} 1 & -1 & 1 & -4 & 1 & 0 & 0 \\ 5 & -4 & 3 & -12 & 0 & 1 & 0 \\ 2 & 1 & 1 & 11 & 0 & 0 & 1 \end{array}\right) \rightarrow \begin{cases} F_1 = F_1 \\ F_2 = F_2 - 5F_1 \\ F_3 = F_3 - 2F_1 \end{cases} \rightarrow$$

$$\left(\begin{array}{ccc|c|ccc} 1 & -1 & 1 & -4 & 1 & 0 & 0 \\ 0 & 1 & -2 & 8 & -5 & 1 & 0 \\ 0 & 3 & -1 & 19 & -2 & 0 & 1 \end{array}\right) \rightarrow \begin{cases} F_2 = F_2 \\ F_1 = F_1 + F_2 \\ F_3 = F_3 - 3F_2 \end{cases} \rightarrow \left(\begin{array}{ccc|c|ccc} 1 & 0 & -1 & 4 & -4 & 1 & 0 \\ 0 & 1 & -2 & 8 & -5 & 1 & 0 \\ 0 & 0 & 5 & 19 & 13 & -3 & 1 \end{array}\right)$$

NO HACERLO AHORA

no hacerlo

$$\rightarrow \begin{cases} F_3 = F_3/5 \\ F_1 = F_1 + 1/5\, F_3 \\ F_2 = F_2 + 2/5\, F_3 \end{cases} \rightarrow \left(\begin{array}{ccc|c|ccc} 1 & 0 & 0 & 3 & 7/5 & 2/5 & 1/5 \\ 0 & 1 & 0 & 6 & 1/5 & -1/5 & 2/5 \\ 0 & 0 & 1 & -1 & 13/5 & -3/2 & 1/5 \end{array}\right) \rightarrow \begin{cases} x_1 = 3 \\ x_2 = 6 \\ x_3 = -1 \end{cases}$$

A^{-1} no hacerlo.

EJEMPLO JACOBI

$$\begin{cases} x_{k+1} = \dfrac{7 + y_k - z_k}{4} \\ y_{k+1} = \dfrac{21 + 4x_k + z_k}{8} \\ z_{k+1} = \dfrac{15 + 2x_k - y_k}{5} \end{cases}$$

$x_0 = (1,2,2)$

k	x_k	y_k	z_k
0	1	2	2
1	3,5000	3,3750	3,0000
2	1,8438	3,8750	3,0250
⋮			
10	2,0000	4,0000	3,0000

$$D^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & -1/8 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$$

Matricialmente:

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ -21 \\ 15 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad R = \begin{bmatrix} 0 & -1 & 1 \\ 4 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$X_1 = D^{-1}(b - RX_0) = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & -1/8 & 0 \\ 0 & 0 & 1/5 \end{bmatrix} \left(\begin{bmatrix} 7 \\ -21 \\ 15 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & -1 & 1 \\ 4 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}_{\begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}} \right) = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & -1/8 & 0 \\ 0 & 0 & 1/5 \end{bmatrix} \begin{bmatrix} 7 \\ -27 \\ 15 \end{bmatrix} = \begin{bmatrix} 1,7500 \\ 3,3750 \\ 3,0000 \end{bmatrix}$$

$$X_2 = D^{-1}(b - RX_1) = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & -1/8 & 0 \\ 0 & 0 & 1/5 \end{bmatrix} \left(\begin{bmatrix} 7 \\ -21 \\ 15 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & -1 & 1 \\ 4 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1,7500 \\ 3,3750 \\ 3,0000 \end{bmatrix}}_{\begin{bmatrix} -0,375 \\ 10 \\ -0,125 \end{bmatrix}} \right) = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & -1/8 & 0 \\ 0 & 0 & 1/5 \end{bmatrix} \begin{bmatrix} 7,375 \\ -31 \\ 15,125 \end{bmatrix} = \begin{bmatrix} 1,8438 \\ 3,8750 \\ 3,0250 \end{bmatrix}$$

Teorema:

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{bmatrix}$$

$|4| > |-1| + |1| = 2$ ✓

$|-8| > |4| + |1| = 5$ ✓

$|5| > |-2| + |1| = 3$ ✓

El proceso converge.

Intercambiando 1° y 3°:

$$A = \begin{bmatrix} -2 & 1 & 5 \\ 4 & -8 & 1 \\ 4 & -1 & 1 \end{bmatrix}$$

$|-2| < |1| + |5| = 6$ ✗

$|-8| > |4| + |1| = 5$ ✓

$|1| < |4| + |-1| = 5$ ✗

No se puede asegurar que ~~No converge~~ converja.

El sistema era:

$$\begin{cases} 4x - y + z = 7 \\ 4x - 8y + z = -21 \\ -2x + y + 5z = 15. \end{cases}$$

EXEMPLO GAUSS-SEIDEL

$$\begin{cases} x_{k+1} = \dfrac{7 + y_k - z_k}{4} \\[2ex] y_{k+1} = \dfrac{21 + 4x_{k+1} + z_k}{8} \\[2ex] z_{k+1} = \dfrac{15 + 2x_{k+1} - y_{k+1}}{5} \end{cases}$$

$\underline{x_0} = (1,2,2)$

k	x_k	y_k	z_k
0	1	2	2
1	1,7500	3,7500	2,9500
2	1,9500	3,9688	2,9862
⋮	⋮	⋮	⋮
6	2,0000	4,0000	3,0000

Matricialmente

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ -21 \\ 15 \end{bmatrix} \quad L = \begin{bmatrix} 4 & 0 & 0 \\ 4 & -8 & 0 \\ -2 & 1 & 5 \end{bmatrix} \quad U = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad L^{-1} = \begin{bmatrix} 0{,}25 & 0 & 0 \\ 0{,}125 & -0{,}125 & 0 \\ 0{,}075 & 0{,}025 & 0{,}2 \end{bmatrix}$$

$$\underline{x_1} = L^{-1}(b - Ux_0) = L^{-1}\left(\begin{bmatrix} 7 \\ -21 \\ 15 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}_{\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}} \right) = \begin{bmatrix} 0.25 & 0 & 0 \\ 0{,}125 & -0{,}125 & 0 \\ 0{,}075 & 0{,}025 & 0{,}2 \end{bmatrix} \begin{bmatrix} 7 \\ -23 \\ 15 \end{bmatrix} = \begin{bmatrix} 1{,}75 \\ 3{,}75 \\ 2{,}95 \end{bmatrix}$$

$$x_2 = L^{-1}(b - Ux_1) = L^{-1}\left(\begin{bmatrix} 7 \\ -21 \\ 15 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1{,}75 \\ 3{,}75 \\ 2{,}95 \end{bmatrix}}_{\begin{bmatrix} -0{,}8 \\ 2{,}95 \\ 0 \end{bmatrix}} \right) = \begin{bmatrix} 0{,}25 & 0 & 0 \\ 0{,}125 & -0{,}125 & 0 \\ 0{,}075 & 0{,}025 & 0{,}2 \end{bmatrix} \begin{bmatrix} 7{,}8 \\ -23{,}95 \\ 15 \end{bmatrix} = \begin{bmatrix} 1{,}95 \\ 3{,}9688 \\ 2{,}9863 \end{bmatrix}$$

✓