Lecture 04: Multi Tone Amplitude Modulation & Generation of AM

Multi-tone modulation:

The standard AM wave equation is given by,

$$s(t) = A_C[1 + k_a\, m(t)]\cos(2\pi f_C t) \longrightarrow (1)$$

The multi-tone modulating signal

$$m(t) = A_{m1}\cos(2\pi f_{m1}\, t) + A_{m2}\cos(2\pi f_{m2}\, t) + A_{m3}\cos(2\pi f_{m3}\, t) + A_{m4}\cos(2\pi f_{m4}\, t) + \cdots$$

For our simplicity let us consider $m(t) = A_{m1}\cos(2\pi f_{m1}\, t) + A_{m2}\cos(2\pi f_{m2}\, t) \longrightarrow (2)$ with only two harmonic components.

Now,

$$s(t) = A_C[1 + k_a(A_{m1}\cos(2\pi f_{m1}\, t) + A_{m2}\cos(2\pi f_{m2}\, t))]\cos(2\pi f_C t) \longrightarrow (3)$$

$$\Rightarrow s(t) = A_C[1 + \mu_1\cos(2\pi f_{m1}\, t) + \mu_2\cos(2\pi f_{m2}\, t))]\cos(2\pi f_C t)$$

Where, $\mu_1 = k_a . A_{m1}$ & $\mu_2 = k_a . A_{m2}$

$$\Rightarrow s(t) = A_C\cos(2\pi f_C t) + \mu_1 A_C\cos(2\pi f_{m1}\, t)\cos(2\pi f_C t) + \mu_2 A_C\cos(2\pi f_{m2}\, t))\cos(2\pi f_C t) \longrightarrow (4)$$

By means of trigonometric relation, it can be expanded further

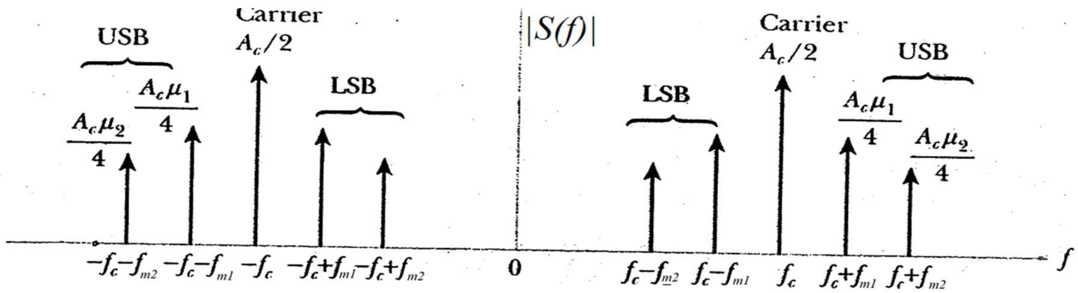
$$\boxed{\cos A . \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]}$$

$$\Longrightarrow s(t) = A_C\cos(2\pi f_c\, t) + \frac{\mu_1 A_C}{2}\cos(2\pi(f_c + f_{m1})\, t) + \frac{\mu_1 A_C}{2}\cos(2\pi(f_c - f_{m1})\, t) + \frac{\mu_2 A_C}{2}\cos(2\pi(f_c + f_{m2})\, t) + \frac{\mu_2 A_C}{2}\cos(2\pi(f_c - f_{m2})\, t) \longrightarrow (5)$$

By the Fourier transform we can get its frequency domain representation as

$$S(f) = \frac{A_C}{2}[\delta(f + f_c) + \delta(f - f_c)] + \frac{\mu_1 A_C}{4}[\delta(f + f_c + f_{m1}) + \delta(f - (f_c + f_{m1}))] + \frac{\mu_1 A_C}{4}[\delta(f + f_c - f_{m1}) + \delta(f - (f_c - f_{m1}))] + \frac{\mu_2 A_C}{4}[\delta(f + f_c + f_{m2}) + \delta(f - (f_c + f_{m2}))] + \frac{\mu_2 A_C}{4}[\delta(f + f_c - f_{m2}) + \delta(f - (f_c - f_{m2}))] \longrightarrow (6)$$

It has two Upper sidebands and two Lower sidebands in addition to the single-tone AM wave. The transmission bandwidth = $2f_m$, where f_m is the highest frequency component of modulating signal (in the spectrum shown above, $f_m = f_{m2}$).



Multi-tone AM wave spectrum

Total transmitted power $P_t = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$

$$\Rightarrow P_t = \frac{A_c^2}{2R} + \frac{\mu_1{}^2 A_c^2}{8R} + \frac{\mu_1{}^2 A_c^2}{8R} + \frac{\mu_2{}^2 A_c^2}{8R} + \frac{\mu_2{}^2 A_c^2}{8R} = \frac{A_c^2}{2R} + \frac{\mu_1{}^2 A_c^2}{4R} + \frac{\mu_2{}^2 A_c^2}{4R}$$

$$\Rightarrow P_t = \frac{A_c^2}{2R}[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}] = \frac{A_c^2}{2R}[1 + \frac{\mu_t^2}{2}]$$

Where, $\mu_t^2 = \mu_1^2 + \mu_2^2$ or $\mu_t = \sqrt{(\mu_1^2 + \mu_2^2)}$ = Effective modulation index

In general the effective modulation index is $\mu_t = \sqrt{(\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots + \mu_n^2\ )}$

∴ The total transmitted power of multi-tone AM wave is $P_t = P_c(1 + \frac{\mu_t^2}{2})$

Generation of AM waves:

The process of modulating the high-frequency carrier signal characteristic in accordance with the instantaneous value of modulating signal is known as modulation.

The circuit that generates the AM wave is known as amplitude modulator. There are two methods to generate AM wave:

1. Square-law modulator
2. Switching modulator

These two modulators use a nonlinear element like diode for their implementation. These two modulators are used for the low power modulation purpose.

1. Square-law modulator :

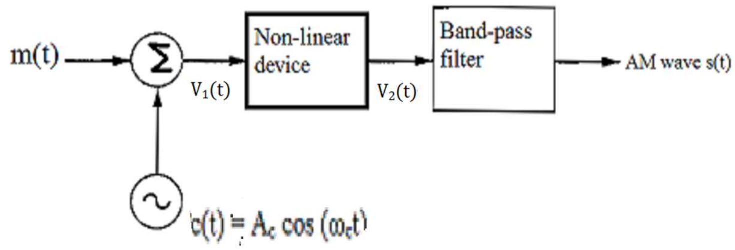
It consists the following three elements:

i) A means of summing the carrier and the modulating signal.

ii) A non-linear device.

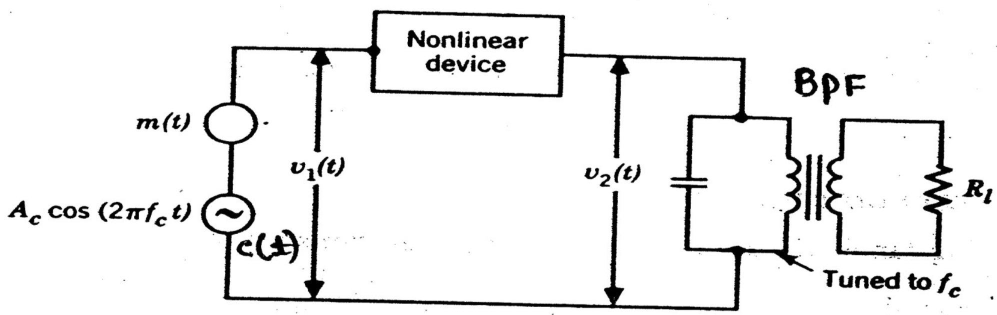
iii) A band-pass filter for extracting desired modulation products.

We assume that $|m(t) + A_c\cos(\omega_c t)| \leq 1$



A means of summing can be implemented by connecting modulating signal m(t) and the carrier signal c(t) in series with each other. The semiconductor diode or transistor can be used as a non-linear device and either single or double tuned circuit can be used as a band-pass filter.

The square-law modulator circuit is shown in fig. below:



The output of summer is $v_1(t) = m(t) + c(t) = m(t) + A_c\cos(2\pi f_c t) \rightarrow (1)$

The I/O relation for a non-linear device follows square law if the applied input is a low-level signal. Thus from the transfer characteristics of diode, its output by the square law is given by $v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \rightarrow (2)$, where a_1 &a_2 are the device constants.

Substituting equation (1) in equation (2), we get,

$$v_2(t) = a_1[m(t) + A_c\cos(2\pi f_c t)] + a_2[m(t) + A_c\cos(2\pi f_c t)]^2$$

$$\Rightarrow v_2(t) = a_1 m(t) + a_1 A_c\cos(2\pi f_c t) + a_2 m^2(t) + 2a_2 m(t) A_c\cos(2\pi f_c t) + a_2 {A_c}^2\cos^2(2\pi f_c t)$$

$$v_2(t) = a_1 m(t) + a_2 m^2(t) + [2a_2 m(t) + a_1]A_c\cos(2\pi f_c t) + a_2 {A_c}^2\frac{[1+\cos(4\pi f_c t)]}{2}$$

$$v_2(t) = [2a_2 m(t) + a_1]A_c\cos(2\pi f_c t) + a_1 m(t) + a_2 m^2(t) + \frac{a_2 {A_c}^2}{2} + \frac{a_2 {A_c}^2\cos(4\pi\ _c t)}{2} \rightarrow (3)$$

By passing this signal through the BPF which is tuned to the carrier frequency f_c, we can remove unwanted terms (DC, lower frequencies less than f_c and $2f_c$ terms) and the resulting output is

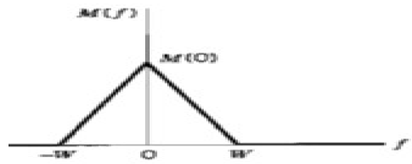
$$s(t) = [2a_2 m(t) + a_1]A_c \cos(2\pi f_c t)$$

$$\therefore s(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos(2\pi f_c t) \rightarrow (4)$$

This is the required AM wave with amplitude sensitivity $k_a = \frac{2a_2}{a_1}$.

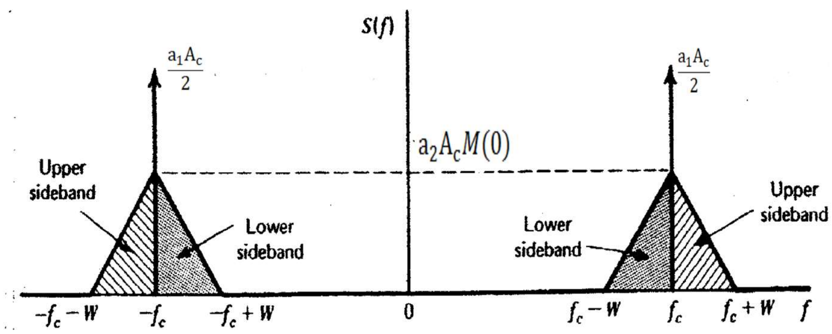
Thus the square-law modulator produces an AM wave.

Let us assume m(t) is band-limited to the interval $-W \le f \le W$, then its spectrum is as shown below:



From equation (4) by the Fourier transform,

$S(f) = \frac{a_1 A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + a_2 A_c[M(f - f_c) + M(f + f_c)]$. So the spectrum is as shown:



Note: To avoid undesired frequencies at the BPF output $f_c \gg 3W$

2. Switching Modulator:

Here we assume that

i) The forward resistance of diode is extremely small compared to R_L, and
ii) $|m(t)| \le 1$ and $A_c \gg 1$

Consider a semiconductor diode used as an ideal switch to which the carrier wave c(t) and the modulating signal m(t) are simultaneously applied as in fig.1 It is assumed that the carrier wave c(t) applied to the diode is large in amplitude.

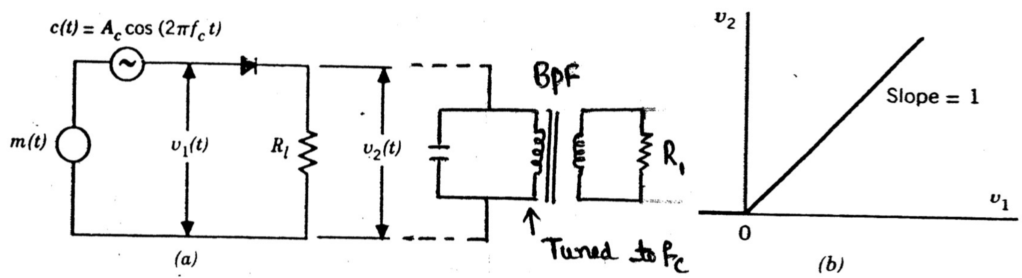


Fig.1.Switching modulator (a) Circuit diagram (b) Idealized input-output relation

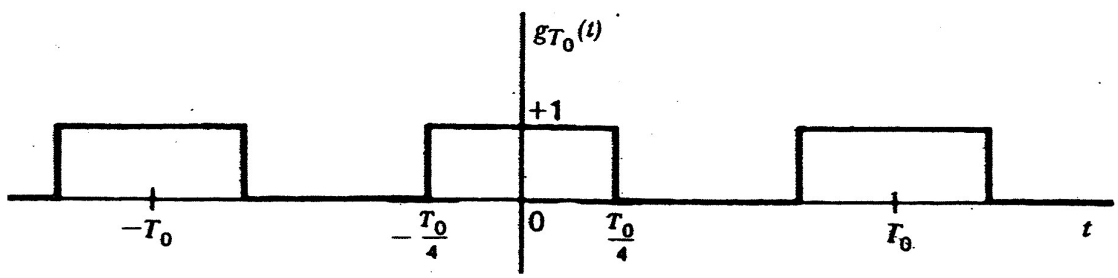


Fig.2. Periodic impulse train

The input applied at the diode is given by,

$v_1(t) = m(t) + c(t) = m(t) + A_c \cos(2\pi f_c t) \rightarrow (1)$, where $|\, m(t)| << A_c$.

The non-linear behaviour of the diode can be replaced by assuming the weak modulating signal compared with the carrier wave. Thus the output of the diode is approximately equivalent to the linear-time varying operation. So,

The output of the diode is $v_2(t) = \begin{cases} v_1(t), & \text{for } c(t) > 0 \\ 0, & \text{for } c(t) < 0 \end{cases}$

Thus the output of the diode varies between $v_1(t)$ and 0 at the rate of carrier frequency, $T_0 = \frac{1}{f_c}$

The turning ON and OFF the diode can be modelled as a rectangular train of pulses $g_p(t)$ with amplitude being unity and frequency f_c as in fig.2.

Mathematically the output of the diode can be written as $v_2(t) = v_1(t).\, g_p(t) \rightarrow (2)$

$\therefore v_2(t) = [m(t) + A_c \cos(2\pi f_c t)].\, g_p(t) \rightarrow (3)$

$g_p(t)$ can be represented using Fourier series as,

$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{(n-1)}}{2n-1} \cos(2\pi f_c (2n-1)t)$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(6\pi f_c t) + \frac{2}{5\pi}\cos(10\pi f_c t) - \frac{2}{7\pi}\cos(14\pi f_c t) + \cdots$$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \text{odd harmonic components} \rightarrow (4)$$

Substituting equation (4) in equation (3), we get,

$$v_2(t) = [m(t) + A_c\cos(2\pi f_c t)].[\tfrac{1}{2} + \tfrac{2}{\pi}\cos(2\pi f_c t) + \text{odd harmonic components}]$$

$$\Rightarrow v_2(t) = \left[\frac{m(t)}{2} + \frac{A_c}{2}\cos(2\pi f_c t)\right] + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{2A_c}{\pi}\cos^2(2\pi f_c t)$$

$$\Rightarrow v_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2A_c}{\pi}\frac{[1 + \cos(4\pi f_c t)]}{2}$$

$$\therefore v_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{A_c}{\pi} + \frac{A_c\cos(4\pi f_c t)}{\pi} \rightarrow (5)$$

By passing this through the band-pass filter having a centre frequency $'f_c'$ and bandwidth twice the message bandwidth i.e., $B_T = 2W$, we get the required AM wave.

$$\therefore \text{The output of the bandpass filter } s(t) = \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)$$

$$= \frac{A_c}{2}[1 + \frac{4}{\pi.A_c}m(t)]\cos(2\pi f_c t)$$

Where, $k_a = \frac{4}{\pi.A_c}$ = amplitude sensitivity.

∴ The obtained AM wave from the switching modulator is

$$\boxed{s(t) = \frac{A_c}{2}[1 + k_a m(t)]\cos(2\pi f_c t)\text{, with } k_a = \frac{4}{\pi.A_c}}$$