

Lecture 03: Amplitude Modulation

Definition: A modulation process in which the amplitude of the carrier wave is varied in accordance with instantaneous value of the message signal is known as **Amplitude Modulation**.

Time-domain Description:

Consider a sinusoidal carrier signal $c(t)$, defined as $c(t) = A_c \cos(\omega_c t + \phi)$,

Where A_c is amplitude of unmodulated carrier,

ω_c is the carrier frequency and

ϕ is the phase of the carrier.

For our convenience let us consider $\phi = 0$.

So, carrier signal $c(t) = A_c \cos(\omega_c t)$.

Now the amplitude modulated wave $s(t) = A_c[1 + k_a m(t)] \cos(\omega_c t)$.

Here, $m(t)$ = modulating message signal,

A_c = the amplitude of unmodulated carrier,

$A_c[1 + k_a m(t)]$ = the amplitude of modulated carrier and

k_a = the amplitude sensitivity in volt^{-1} .

This AM consists both modulated and unmodulated carrier signals.

Below figure shows a baseband signal $m(t)$, and the corresponding AM wave $s(t)$ for two values of k_a and a carrier amplitude $A_c = 1$ volt.

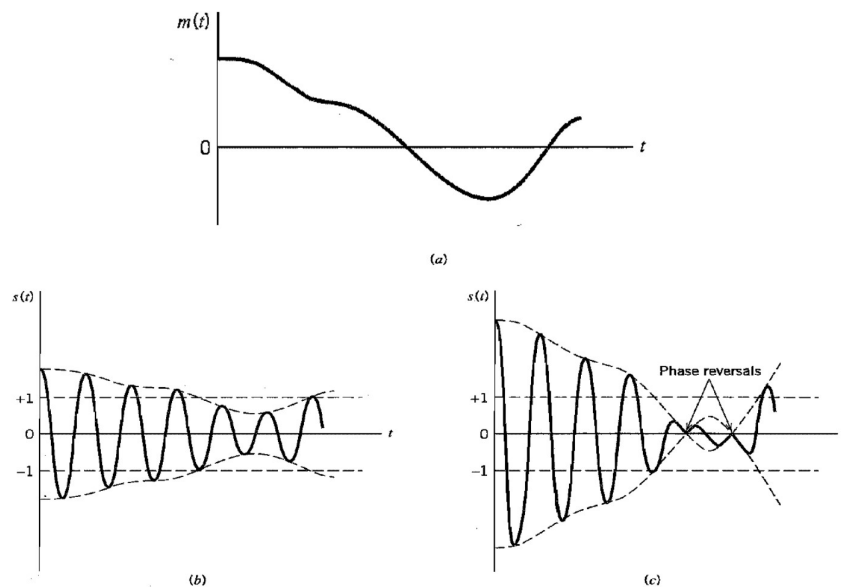


Fig. Illustrating amplitude modulation process: (a) Baseband signal $m(t)$, (b) AM wave

for $|k_a m(t)| < 1$ for all t and (c) AM wave for $|k_a m(t)| > 1$ for some t .

We can observe that the envelope of $s(t)$ has essentially the same shape as the baseband signal $m(t)$ provided that two requirements are satisfied:

1. The amplitude of $k_a m(t)$ is always less than unity, i.e. $|k_a m(t)| < 1$ for all t
2. The carrier frequency f_c is much greater than the highest frequency component W of the message signal $m(t)$, i.e. $f_c \gg W$ where W is the highest frequency in $m(t)$.

Frequency-domain Description:

The amplitude modulated wave $s(t) = A_c[1 + k_a m(t)]\cos(\omega_c t)$

$$= A_c \cos(\omega_c t) + A_c k_a m(t) \cos(\omega_c t)$$

By applying F.T to this $s(t)$, we get

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c}{2} k_a [M(f - f_c) + M(f + f_c)]$$

If $m(t)$ is band-limited to the frequency range, $-W \leq f \leq W$, from $S(f)$ expression the spectrum is as follows:

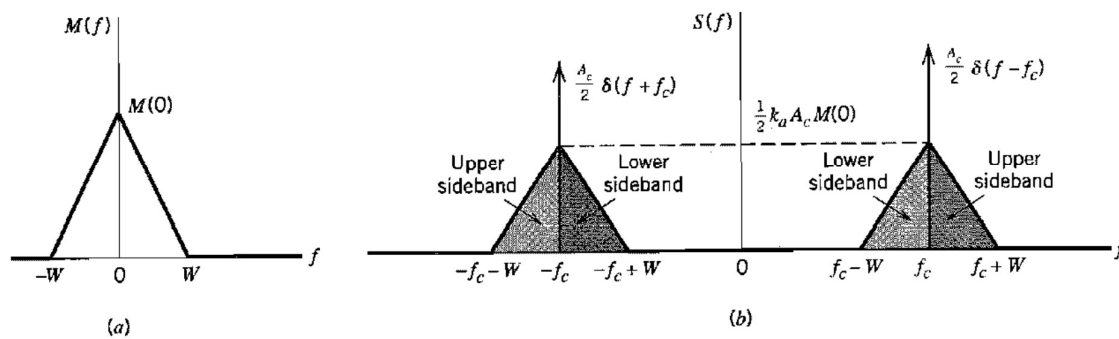


Fig. (a) Spectrum of baseband signal $m(t)$ (b) Spectrum of AM wave

For positive frequencies, the portion of the spectrum of an AM wave lying above the carrier frequency f_c is referred to as **the upper sideband**, whereas the symmetric portion below f_c is referred to as **the lower sideband**. For negative frequencies, the upper sideband is represented by the portion of the spectrum below $-f_c$ and the lower sideband by the portion above $-f_c$.

So in AM spectrum there are two USB and two LSB bands: one USB is from f_c to $f_c + W$ another USB is from $-(f_c + W)$ to $-f_c$ and one LSB is from $f_c - W$ to f_c another LSB is from $-f_c$ to $-(f_c - W)$.

Transmission Bandwidth B_T : For positive frequencies, the highest frequency component of the AM wave equals $f_c + W$, and the lowest frequency component equals $f_c - W$. The difference between these two frequencies defines the **transmission bandwidth** B_T for an AM wave, which is exactly twice the message bandwidth W , i.e., $B_T = 2W$

Single tone modulation:

For a single tone modulating signal there is only one frequency component, so let us consider

$$m(t) = A_m \cos(2\pi f_m t) \rightarrow (1)$$

Where, A_m – maximum amplitude of the modulating signal and

f_m – frequency of modulating signal

Consider a sinusoidal carrier signal $c(t)$, defined as

$$c(t) = A_c \cos(2\pi f_c t) \rightarrow (2)$$

Where, A_c – maximum amplitude of the carrier signal and

f_c – frequency of carrier signal

The standard equation for AM wave is given by

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t) \rightarrow (3)$$

Where, k_a is a constant called the amplitude sensitivity of the modulator generally expressed in volt^{-1} .

Substituting Equation (1) in Equation (3). We get

$$s(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

$$\Rightarrow s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

Where $\mu = k_a m(t) = \text{modulation index or modulation factor}$

$$\Rightarrow s(t) = A_c \cos(2\pi f_c t) + \mu A_c \cos(2\pi f_m t) \cos(2\pi f_c t) \rightarrow (4)$$

Equation (4) can be further expanded, by means of trigonometric relation

$$\boxed{2\cos A \cdot \cos B = \cos(A + B) + \cos(A - B)}$$

$$\Rightarrow s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos(2\pi(f_c + f_m) t) + \frac{\mu A_c}{2} \cos(2\pi(f_c - f_m) t) \rightarrow (5)$$

Equation (5) is the amplitude modulated signal, consist of three frequency components:

1. The first term is the carrier itself. It has a frequency f_c and amplitude A_c .
2. The second component is $\frac{\mu A_c}{2} \cos(2\pi(f_c + f_m) t)$. It has frequency $(f_c + f_m)$, called Upper Sideband f_{USB} and having amplitude $\frac{\mu A_c}{2}$.
3. The third component is $\frac{\mu A_c}{2} \cos(2\pi(f_c - f_m) t)$. It has frequency $(f_c - f_m)$, called Lower Sideband f_{LSB} and having amplitude $\frac{\mu A_c}{2}$.

Frequency domain description:

$$s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos(2\pi(f_c + f_m) t) + \frac{\mu A_c}{2} \cos(2\pi(f_c - f_m) t)$$

Taking Fourier Transform on both sides, we get

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{\mu A_c}{4} [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))] + \frac{\mu A_c}{4} [\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))] \rightarrow (6)$$

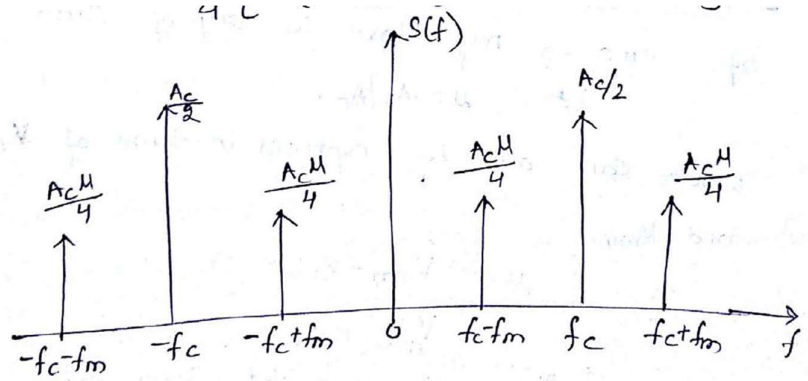


Fig. Single-tone AM wave spectrum

The frequency spectrum of the single-tone AM wave has two sidebands on either side of $\pm f_c$.

The highest frequency component of the AM wave = $f_{USB} = f_c + f_m$

The lowest frequency component of the AM wave = $f_{LSB} = f_c - f_m$

$$\therefore \text{Transmission bandwidth } B_T = f_{USB} - f_{LSB} = 2f_m$$

i.e., transmission bandwidth of AM wave is twice the message bandwidth.

Modulation Index (μ or m):

The product of the amplitude sensitivity of the modulator and maximum amplitude of the modulating signal is known as **modulation index or modulation factor or modulation coefficient or depth of modulation**. Or

The ratio of maximum amplitude of the modulating signal to the amplitude of the carrier signal is known as **modulation index or modulation factor or modulation coefficient or depth of modulation**.

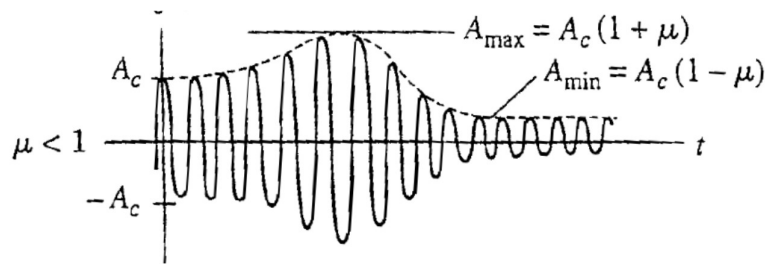
It is denoted by either μ or m .

$$\therefore \mu = k_a m(t) \text{ or } \mu = \frac{A_m}{A_c}$$

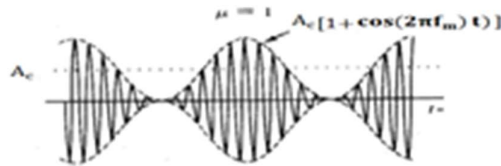
It is mostly expressed in percentage and **%modulation** = $\frac{A_m}{A_c} \times 100\%$

Based on the value of μ , there are three degrees of modulation:

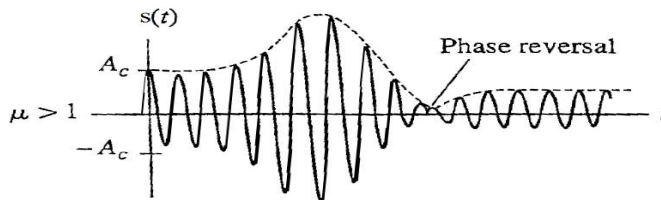
1. Under modulation ($\mu < 1$ or $A_m < A_c$)



2. Critical modulation ($\mu=1$ or $A_m = A_c$)



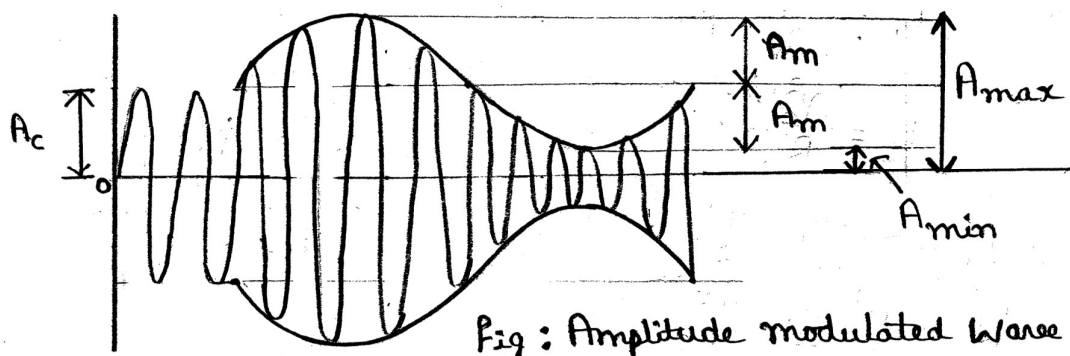
3. Over modulation ($\mu > 1$ or $A_m > A_c$).



If $A_m > A_c$ that is in over modulation condition modulating signal cannot be recovered exactly because of occurrence of the envelope distortion in the system.

\therefore For proper amplitude modulation and its recovery A_m must be less than A_c .

Modulation index calculation using AM wave:



To find the expression for modulation index from AM wave it is necessary to express A_m & A_c in terms of A_{max} & A_{min} .

$$\text{From the graph, } A_{max} - A_{min} = 2 A_m \Rightarrow A_m = \frac{A_{max} - A_{min}}{2} \rightarrow (1)$$

$$\begin{aligned}
 \text{And } A_c &= A_m + A_{min} \Rightarrow A_c = \frac{A_{max} - A_{min}}{2} + A_{min} \\
 &\Rightarrow 2A_c = A_{max} - A_{min} + 2A_{min} = A_{max} + A_{min} \\
 &\Rightarrow A_c = \frac{A_{max} + A_{min}}{2} \rightarrow (2)
 \end{aligned}$$

By the definition, $\mu = \frac{A_m}{A_c} \rightarrow (3)$

Substituting equation (1) and (2) in equation (3), we get

$$\therefore \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

NOTE: If the value of μ is specified then $A_{max} = A_c(1 + \mu)$ and $A_{min} = A_c(1 - \mu)$

Power relations in AM waves:

The standard single-tone AM wave equation is

$$s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos(2\pi(f_c + f_m) t) + \frac{\mu A_c}{2} \cos(2\pi(f_c - f_m) t) .$$

It has three components: unmodulated carrier, upper sideband and lower sideband components.

\therefore The total power of AM wave is the sum of carrier power and the power of two sidebands, i.e. $P_t = P_c + P_{USB} + P_{LSB}$.

$$\begin{aligned}
 \text{The average carrier power is } P_c &= \frac{V_{rms}^2}{R} \text{ or } I_{rms}^2 \cdot R = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R} \text{ or } \left(\frac{I_m}{\sqrt{2}}\right)^2 \cdot R \\
 &\Rightarrow P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}
 \end{aligned}$$

And the average sideband power

$$P_{USB} = P_{LSB} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}$$

\therefore The average total power $P_t = P_c + P_{USB} + P_{LSB}$

$$\begin{aligned}
 &= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R} \\
 &= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{4R} \\
 &= \frac{A_c^2}{2R} \left(1 + \frac{\mu^2}{2}\right)
 \end{aligned}$$

$$\therefore P_t = P_c \left(1 + \frac{\mu^2}{2}\right)$$

The maximum power will result if $\mu = 1$ and that maximum power is

$$(P_t)_{\max} = P_c \left(1 + \frac{1}{2}\right) = \frac{3}{2} P_c$$

$$\therefore (P_t)_{\max} = 1.5 P_c$$

NOTE: $P_c = \frac{1}{1.5} (P_t)_{\max} = 0.6667 (P_t)_{\max}$ i.e. in AM, 66.67% of the total power is used by the carrier and remaining 33.33% is used by the two sidebands.

Transmission efficiency (η): Transmission efficiency is defined as the ratio of the sideband power to the total transmitted power of the modulated wave.

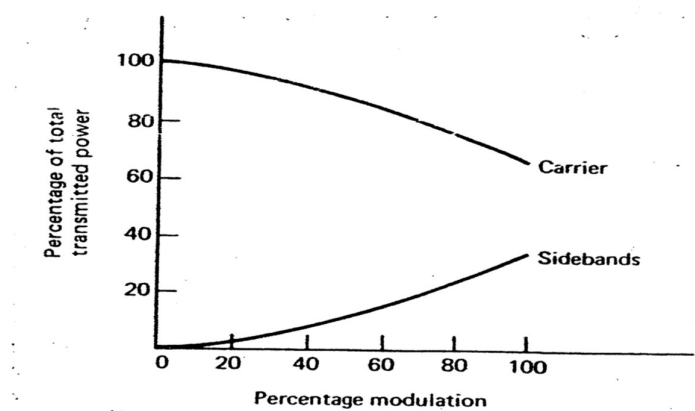
$$\begin{aligned} \therefore \eta &= \frac{P_{SB}}{P_t} \\ &= \frac{P_{USB} + P_{LSB}}{P_t} \\ &= \frac{P_c \left(\frac{\mu^2}{4} + \frac{\mu^2}{4} \right)}{P_c \left(1 + \frac{\mu^2}{2} \right)} \\ &= \frac{\frac{P_c \cdot \mu^2}{2}}{\frac{P_c \cdot (2 + \mu^2)}{2}} \end{aligned}$$

$$\therefore \text{Transmission efficiency } \eta' = \frac{\mu^2}{2 + \mu^2}$$

For 100% modulation i.e. $\mu=1$, $\Rightarrow \eta = \frac{1}{3}$.

Therefore sidebands power is only 1/3rd of the total transmitted power.

For various percentage modulations the carrier and sideband powers associated are shown in fig. below



Current relations in AM waves:

$$\therefore \text{The average power} = I^2 R$$

The total transmitted power $P_t = I_t^2 \cdot R$ & carrier $P_c = I_c^2 \cdot R$, where

I_t = Total transmitted current and I_c = carrier signal current.

As we know $P_t = P_c \left(1 + \frac{\mu^2}{2}\right)$

$$\Rightarrow I_t^2 \cdot R = I_c^2 \cdot R \left(1 + \frac{\mu^2}{2}\right)$$

$$\Rightarrow I_t^2 = I_c^2 \left(1 + \frac{\mu^2}{2}\right)$$

$$\therefore I_t = I_c \sqrt{\left(1 + \frac{\mu^2}{2}\right)}$$

NOTE: Total transmitted voltage $V_t = V_c \sqrt{\left(1 + \frac{\mu^2}{2}\right)}$