Lecture 03: Amplitude Modulation

Definition: A modulation process in which the amplitude of the carrier wave is varied in accordance with instantaneous value of the message signal is known as ***Amplitude Modulation***.

Time-domain Description:

Consider a sinusoidal carrier signal $c(t)$, defined as $c(t) = A_c\cos(\omega_c t + \varphi)$,

Where A_c is amplitude of unmodulated carrier,

ω_c is the carrier frequency and

φ is the phase of the carrier.

For our convenience let us consider $\varphi = 0$.

So, carrier signal $\mathbf{c(t) = A_c\cos(\omega_c t)}$.

Now the amplitude modulated wave $\mathbf{s(t) = A_c[1 + k_a m(t)]\cos(\omega_c t)}$.

Here, $m(t)$ = modulating message signal,

A_c = the amplitude of unmodulated carrier,

$A_c[1 + k_a m(t)]$ = the amplitude of modulated carrier and

$k_a = \frac{1}{A_C}$ the amplitude sensitivity in volt $^{-1}$.

This AM consists both modulated and unmodulated carrier signals.

Below figure shows a baseband signal $m(t)$, and the corresponding AM wave $s(t)$ for two values of k_a and a carrier amplitude Ac = 1 volt.

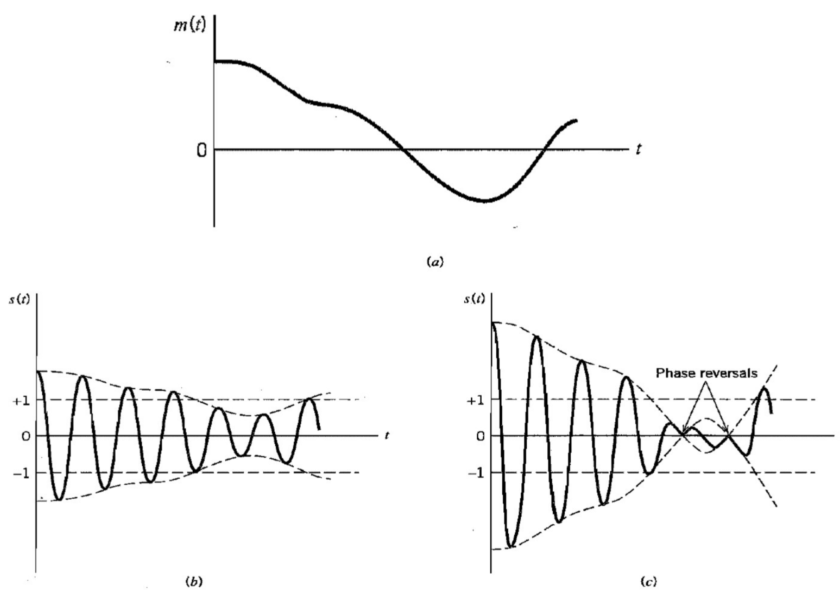


Fig. Illustrating amplitude modulation process: (a) Baseband signal $m(t)$, (b) AM wave for $|k_a m(t)| < 1$ for all t and (c) AM wave for $|k_a m(t)| > 1$ for some t.

We can observe that the envelope of $\mathrm{s(t)}$ has essentially the same shape as the baseband signal $\mathrm{m(t)}$ provided that two requirements are satisfied:

1. The amplitude of $\mathbf{k_a m(t)}$ is always less than unity, i.e. $|\mathbf{k_a m(t)}| < \mathbf{1}$ for all t
2. The carrier frequency f_c is much greater than the highest frequency component W of the message signal $\mathrm{m(t)}$, i.e. f_c >>W where W is the highest frequency in $\mathrm{m(t)}$.

Frequency-domain Description:

The amplitude modulated wave $s(t) = \mathrm{A_C}[1 + k_a\, \mathrm{m(t)}]\cos(\omega_C t)$

$$= \mathrm{A_C}\cos(\omega_C t) + \mathrm{A_C} k_a \mathrm{m(t)}]\cos(\omega_C t)$$

By applying F.T to this $\mathrm{s(t)}$, we get

$$\mathrm{S(f)} = \frac{\mathrm{A_C}}{2}\,[\delta(\mathrm{f - f_C}) + \delta(\mathrm{f + f_C})] + \frac{\mathrm{A_C}}{2}\, k_a[\mathrm{M(f - f_C) + M(f + f_C)}]$$

If $\mathrm{m(t)}$ is band-limited to the frequency range, $-\mathrm{W} \le \mathrm{f} \le \mathrm{W}$, from S(f) expression the spectrum is as follows:

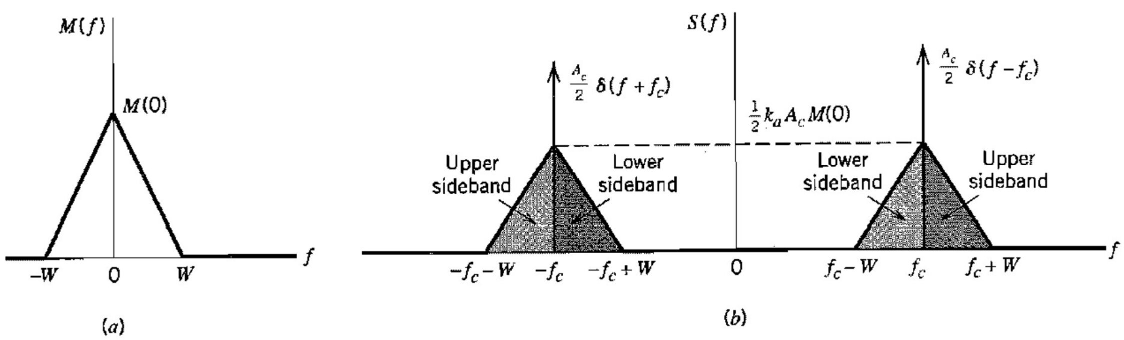


Fig. (a) Spectrum of baseband signal $\mathbf{m(t)}$ (b) Spectrum of AM wave

For positive frequencies, the portion of the spectrum of an AM wave lying above the carrier frequency f_C is referred to as ***the upper sideband***, whereas the symmetric portion below f_C is referred to as ***the lower sideband***. For negative frequencies, the upper sideband is represented by the portion of the spectrum below $-f_C$ and the lower sideband by the portion above $-f_C$.

So in AM spectrum there are two USB and two LSB bands: one USB is from f_C to $f_C + W$ another USB is from $-(\mathrm{f_C + W})$ to $-f_C$ and one LSB is from $\mathrm{f_C - W}$ to $\mathrm{f_C}$ another LSB is from $-\mathrm{f_C}$ to $-(\mathrm{f_C - W})$.

Transmission Bandwidth B_T : For positive frequencies, the highest frequency component of the AM wave equals $\mathrm{f_C + W}$, and the lowest frequency component equals $\mathrm{f_C - W}$. The difference between these two frequencies defines the ***transmission bandwidth*** B_T for an AM wave, which is exactly twice the message bandwidth W, i.e., $\mathbf{B_T=2W}$

Single tone modulation:

For a single tone modulating signal there is only one frequency component, so let us consider

$$\mathbf{m(t) = A_m \cos(2\pi f_m\, t) \rightarrow (1)}$$

Where, A_m − maximum amplitude of the modulating signal and

f_m − frequency of modulating signal

Consider a sinusoidal carrier signal $c(t)$, defined as

$$\mathbf{c(t) = A_C \cos(2\pi f_c\, t) \rightarrow (2)}$$

Where, A_C − maximum amplitude of the carrier signal and

f_c − frequency of carrier signal

The standard equation for AM wave is given by

$$\mathbf{s(t) = A_C[1 + k_a m(t)] \cos(2\pi f_c\, t) \rightarrow (3)}$$

Where, k_a is a constant called the amplitude sensitivity of the modulator generally expressed in $volt^{-1}$.

Substituting Equation (1) in Equation (3). We get

$s(t) = A_C[1 + k_a A_m \cos(2\pi f_m\, t)] \cos(2\pi f_c\, t)$

$\Longrightarrow s(t) = A_C[1 + \mu \cos(2\pi f_m\, t)] \cos(2\pi f_c\, t)$

Where $\mathbf{\mu = k_a m(t) =}$ **modulation index or modulation factor**

$$\mathbf{\Longrightarrow s(t) = A_C \cos(2\pi f_c\, t) + \mu A_C \cos(2\pi f_m\, t) \cos(2\pi f_c\, t) \rightarrow (4)}$$

Equation (4) can be further expanded, by means of trigonometric relation

$$\boxed{2\cos A \,.\cos B = \cos(A + B) + \cos(A - B)}$$

$$\mathbf{\Longrightarrow s(t) = A_C \cos(2\pi f_c\, t) + \frac{\mu A_C}{2} \cos(2\pi(f_c + f_m)\, t) + \frac{\mu A_C}{2} \cos(2\pi(f_c - f_m)\, t) \rightarrow (5)}$$

Equation (5) is the amplitude modulated signal, consist of three frequency components:

1. The first term is the carrier itself. It has a frequency f_c and amplitude A_C.
2. The second component is $\frac{\mu A_C}{2} \cos(2\pi(f_c + f_m)\, t)$. It has frequency $(f_c + f_m)$, called Upper Sideband f_{USB} and having amplitude $\frac{\mu A_C}{2}$.
3. The third component is $\frac{\mu A_C}{2} \cos(2\pi(f_c - f_m)\, t)$. It has frequency $(f_c - f_m)$, called Lower Sideband f_{LSB} and having amplitude $\frac{\mu A_C}{2}$.

Frequency domain description:

$$\mathbf{s(t) = A_C \cos(2\pi f_c\, t) + \frac{\mu A_C}{2} \cos(2\pi(f_c + f_m)\, t) + \frac{\mu A_C}{2} \cos(2\pi(f_c - f_m)\, t)}$$

Taking Fourier Transform on both sides, we get

$$\mathbf{S(f)} = \frac{\mathbf{A_C}}{\mathbf{2}}\,[\boldsymbol{\delta}(\mathbf{f}-\mathbf{f_c})+\boldsymbol{\delta}(\mathbf{f}+\mathbf{f_c})] + \frac{\boldsymbol{\mu}\mathbf{A_C}}{\mathbf{4}}\,[\boldsymbol{\delta}(\mathbf{f}-(\mathbf{f_c}+\mathbf{f_m}))+\boldsymbol{\delta}(\mathbf{f}+(\mathbf{f_c}+\mathbf{f_m}))]$$
$$+\frac{\boldsymbol{\mu}\mathbf{A_C}}{\mathbf{4}}\,[\boldsymbol{\delta}(\mathbf{f}-(\mathbf{f_c}-\mathbf{f_m}))+\boldsymbol{\delta}(\mathbf{f}+(\mathbf{f_c}-\mathbf{f_m}))] \rightarrow \mathbf{(6)}$$

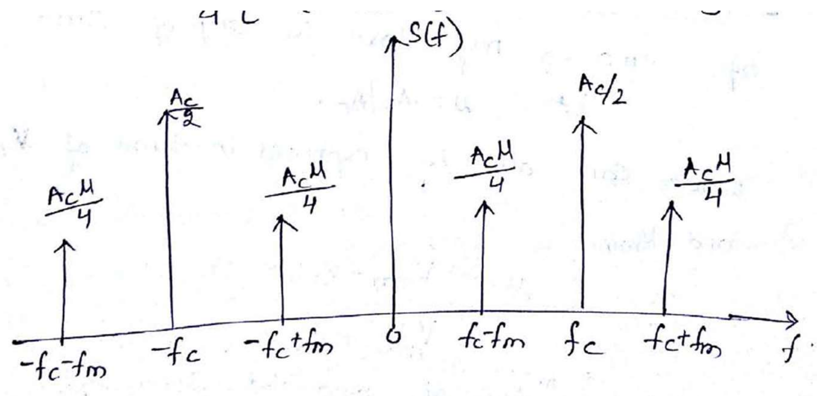


Fig. Single-tone AM wave spectrum

The frequency spectrum of the single-tone AM wave has two sidebands on either side of $\pm f_c$.

The highest frequency component of the AM wave= $f_{USB} = f_c + f_m$

The lowest frequency component of the AM wave = $f_{LSB} = f_c - f_m$

$\therefore$Transmission bandwidth $B_T = f_{USB} - f_{LSB} = 2f_m$

i.e., transmission bandwidth of AM wave is twice the message bandwidth.

<u>Modulation Index (μ or m):</u>

The product of the amplitude sensitivity of the modulator and maximum amplitude of the modulating signal is known as ***modulation index or modulation factor or modulation coefficient or depth of modulation***. Or

The ratio of maximum amplitude of the modulating signal to the amplitude of the carrier signal is known as ***modulation index or modulation factor or modulation coefficient or depth of modulation***.

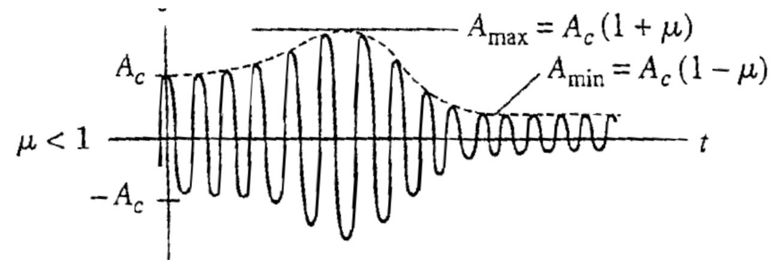
It is denoted by either μ or m.

$$\therefore \boldsymbol{\mu} = \mathbf{k_a m(t)} \text{ or } \boldsymbol{\mu} = \frac{\mathbf{A_m}}{\mathbf{A_c}}$$

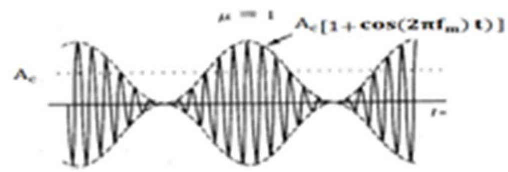
It is mostly expressed in percentage and %**modulation** $= \frac{\mathbf{A_m}}{\mathbf{A_c}}\ \mathbf{X\ 100\%}$

Based on the value of μ, there are three degrees of modulation:

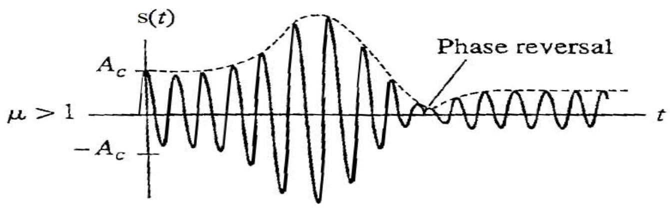
1. Under modulation (μ<1 or $A_m < A_c$)



2. Critical modulation (μ=1 or $A_m = A_c$)



3. Over modulation (μ>1 or $A_m > A_c$).



If $A_m > A_c$ that is in over modulation condition modulating signal cannot be recovered exactly because of occurrence of the envelope distortion in the system.

∴ For proper amplitude modulation and its recovery A_m must be less than A_c .

Modulation index calculation using AM wave:

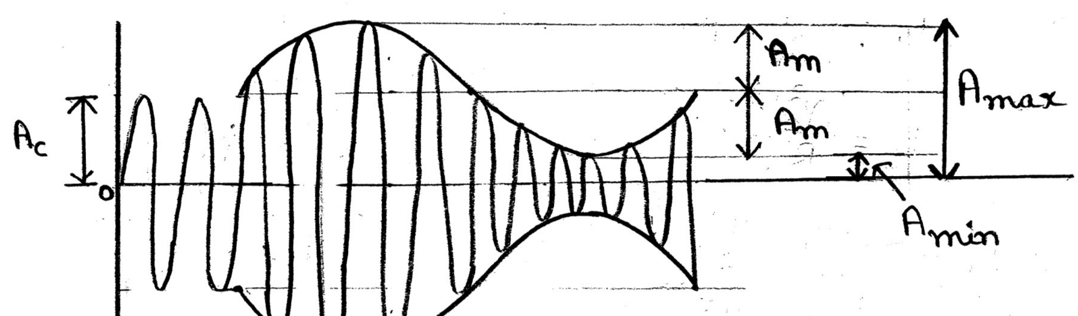


Fig : Amplitude modulated Wave

To find the expression for modulation index from AM wave it is necessary to express A_m&A_c in terms of A_{max}& A_{min}.

From the graph, $A_{max} - A_{min} = 2\,A_m \Rightarrow A_m = \frac{A_{max} - A_{min}}{2} \longrightarrow (1)$

And $A_c = A_m + A_{min} \Rightarrow A_c = \frac{A_{max} - A_{min}}{2} + A_{min}$

$$\Rightarrow 2A_c = A_{max} - A_{min} + 2A_{min} = A_{max} + A_{min}$$

$$\Rightarrow A_c = \frac{A_{max} + A_{min}}{2} \longrightarrow (2)$$

By the definition, $\mu = \frac{A_m}{A_c} \rightarrow (3)$

Substituting equation (1) and (2) in equation (3), we get

$$\boxed{\therefore \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

NOTE: If the value of μ is specified then $\mathbf{A_{max} = A_c(1+\mu)}$ and $\mathbf{A_{min} = A_c(1-\mu)}$

<u>Power relations in AM waves:</u>

The standard single-tone AM wave equation is

$$\mathbf{s(t) = A_C \cos(2\pi f_c\, t) + \frac{\mu A_C}{2}\cos(2\pi(f_c + f_m)\, t) + \frac{\mu A_C}{2}\cos(2\pi(f_c - f_m)\, t)} \ .$$

It has three components: unmodulated carrier, upper sideband and lower sideband components.

∴ The total power of AM wave is the sum of carrier power and the power of two sidebands, i.e. $P_t = P_c + P_{USB} + P_{LSB}$.

The average carrier power is $P_c = \frac{V_{rms}^2}{R}$ or $I_{rms}^2 . R = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R}$ or $\left(\frac{I_m}{\sqrt{2}}\right)^2 . R$

$$\Rightarrow P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

And the average sideband power

$$P_{USB} = P_{LSB} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}$$

∴ The average total power $P_t = P_c + P_{USB} + P_{LSB}$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{4R}$$

$$= \frac{A_c^2}{2R}(1 + \frac{\mu^2}{2})$$

$$\boxed{\therefore P_t = P_c(1 + \frac{\mu^2}{2})}$$

The maximum power will result if $\mu = 1$ and that maximum power is

$(P_t)_{max} = P_c\left(1 + \frac{1}{2}\right) = \frac{3}{2}P_c$

$\therefore (P_t)_{max} = 1.5P_c$

NOTE: $P_c = \frac{1}{1.5}(P_t)_{max} = 0.6667(P_t)_{max}$ i.e. in AM, 66.67% of the total power is used by the carrier and remaining 33.33% is used by the two sidebands.

Transmission efficiency (η): Transmission efficiency is defined as the ratio of the sideband power to the total transmitted power of the modulated wave.

$$\therefore \eta = \frac{P_{SB}}{P_t}$$

$$= \frac{P_{USB} + P_{LSB}}{P_t}$$

$$= \frac{P_c\left(\frac{\mu^2}{4} + \frac{\mu^2}{4}\right)}{P_c\left(1 + \frac{\mu^2}{2}\right)}$$

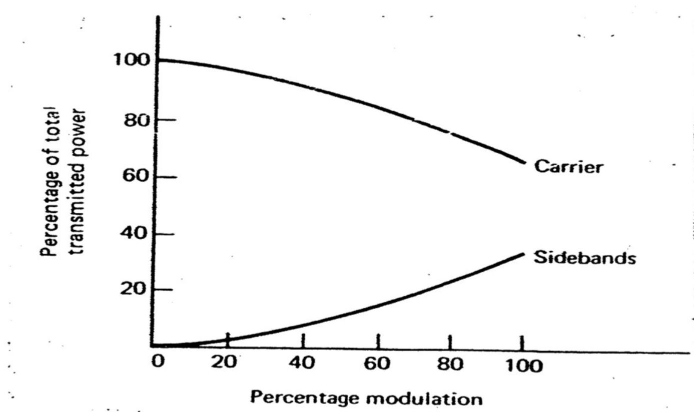
$$= \frac{\frac{P_c.\mu^2}{2}}{\frac{P_c.(2 + \mu^2)}{2}}$$

$$\therefore \text{Transmission efficiency } '\eta' = \frac{\mu^2}{2 + \mu^2}$$

For 100% modulation i.e.μ=1, ⇒$\eta = \frac{1}{3}$.

Therefore sidebands power is only 1/3rd of the total transmitted power.

For various percentage modulations the carrier and sideband powers associated are shown in fig. below



Current relations in AM waves:

$$\because \text{The average power} = I^2R$$

The total transmitted power$P_t = I_t^2.R$ & carrier $P_c = I_c^2.R$, where

I_t = Total transmitted current and I_c = carrier signal current.

As we know $P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$

$\Rightarrow I_t^2 . R = I_c^2 . R\left(1 + \frac{\mu^2}{2}\right)$

$\Rightarrow I_t^2 = I_c^2\left(1 + \frac{\mu^2}{2}\right)$

$$\therefore I_t = I_c\sqrt{(1 + \frac{\mu^2}{2})}$$

NOTE: Total transmitted voltage $V_t = V_c\sqrt{(1 + \frac{\mu^2}{2})}$