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Development of New Wall Friction Factor and Interfacial Friction Factor Correlations for Gas-Liquid Stratified Flow in Wells and Pipelines

Liang-Biao Ouyang, SPE, and Khalid Aziz, SPE, Stanford University

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Abstract

Stratified flow is one of the most basic flow pattern in the analysis of gas-liquid two-phase flow in pipes. In the present paper, different interfacial friction factor correlations were used to predict the liquid holdup and pressure gradients. The comparison of predictions with experimental observations shows that most existing correlations for the interfacial friction factor can lead to large deviations from measurements and that the standard method underestimates the liquid phase wall friction factors. New correlations for both the liquid phase wall friction factor and the interfacial friction factor were developed based on large amounts of available experimental data. The resulting correlations were used to predict liquid holdup and pressure gradient for different experiments. Considerable improvement in predictions was observed.

Introduction

Gas-liquid flow in pipes is of practical importance in petroleum, chemical, nuclear and geothermal industries, and has been the subject of intensive research for dozens of years. Theoretical analyses and experiments have been conducted to predict key factors, such as pressure drop, liquid holdup, gas-liquid interfacial area, heat and mass transfer and temperature distribution.

In general, gas-liquid two-phase flow is much more complicated than single phase flow, due primarily to the existence of a phase interface and the spatial distribution of

the phases. Some consequences of the complex nature of gas-liquid pipe flow are:

- Pressure drop in transmission lines increases significantly even with the presence of very small quantities of condensate. Significant energy loss occurs as a result of acceleration and transportation of the liquid which wets the pipe wall and creates waves at the gas-liquid interface. The liquid film reduces the area of the pipe and increases the effective roughness.
- Whereas in hilly terrain the pressure loss because of gravity head on the uphill side of a pipe would be recovered on the downhill side for single phase flow, this is often not the case for gas-liquid two-phase flow.
- In hilly terrains and in submerged river crossings, energy is lost in lifting the liquid over natural obstacles. If the pressure is insufficient to overcome the obstacle, there may be flooding of the pipe, thus further reducing the area accessible for gas flow and therefore creating an even larger pressure drop.
- In single phase flow, pressure drop decreases as pipe diameter increases, but this is not necessarily so in gas-liquid flow. Under certain conditions, pressure drop exhibits a minimum with pipe diameter¹.

Due to the complex nature of gas-liquid two-phase flow, the problem was first approached through empirical or engineering methods. This approach is based on empirical or semi-empirical relations and has dominated practical design procedures. It leads to correlations that should be extrapolated with caution outside the range of parameters investigated in the experimental work. Most of the available correlations deal with air-water or steam-water systems, their reliability for other systems of industrial importance is questionable.

Recently the trend has shifted towards the modeling approach, or mechanistic models. This approach is based on the fundamentals of multiphase flow and fluid dynamics. The fundamental postulate in this kind of approach is the existence of flow patterns or flow regimes and it has greatly contributed to the basic understanding of the nature of gas-liquid two-phase flow. The resulting models can be utilized for a wider range of flow conditions than is possible with

empirical correlations. But, so far the modeling approach has not been widely accepted in design practice.

When gas and liquid flow simultaneously in a pipe, both phases can be distributed in various flow patterns or regimes. It is recognized that there are many different flow regimes existing in gas-liquid two-phase flow in pipes. Among them are four basic flow regimes: *stratified flow*, *annular flow*, *intermittent flow* and *bubble flow*. The total number of flow regimes can reach up to sixteen if transition regimes are also counted².

Interfacial friction shear is an intrinsic characteristic of gas-liquid two-phase flow and it has a profound influence on the properties and the nature of the flow processes. For example, interfacial shear in gas-liquid flow has a significant effect on mass transfer rates. It can also play an important role in two-phase heat transfer problems, especially in thin liquid films that exist in condensers or reboilers. The surface roughness increases dramatically due to the presence of interfacial shear, affecting the pressure drop and adding additional length scales to an already complex design problem. The relationship for the interfacial friction shear is an indispensable condition to complete any mechanistic models for gas-liquid two-phase flow in pipes. Notwithstanding many different correlations reported in the literature for calculating the interfacial friction shear, this remains largely an unresolved area. Most, if not all, available correlations are deduced from specific range of experimental conditions, such as gas Reynolds number, liquid Reynolds number, pipe diameter, liquid holdup, inclination angle and flow patterns. Such correlations may only apply to a particular range of flow conditions. Significant errors may result when these correlations are used under flow conditions that are different from those used to develop them. Furthermore, existing mechanistic models typically exhibit discontinuities in pressure gradient and liquid holdup predictions across flow pattern boundaries. These discontinuities may cause serious problems during simulation of production facilities. One of the main reasons for the discontinuities is the inclusion of inappropriate interfacial friction shear correlation.

Direct and accurate measurements of wall friction and interfacial friction shears are the first choice for the evaluation of different wall friction factor equations and interfacial friction factor correlations. Unfortunately, few experiments belonging to this category have been reported in the literature. However, both the wall friction and interfacial friction shears may be deduced indirectly from the following measurements:

- gas velocity profile measurements;
- turbulent kinetic energy profiles;
- the momentum balance using the wall-to-gas shear stress, liquid level, and pressure drop measurements;
- the extrapolation of the shear stress profiles at the gas-liquid interface;
- the Reynolds shear stress measurements; and

- the flow momentum balance using wall shear stresses and void fraction measurements³.

Among these measurements, liquid holdup and pressure gradient are the two most commonly-measured parameters in experimental investigations. They will be applied in the present study to examine an existing stratified flow model and determine both the wall friction and interfacial friction shears, and thus evaluate different interfacial friction factor correlations.

Stratified Flow Model

Stratified flow in pipes refers to the flow pattern where the gas phase flows at the top of the pipe, while the liquid phase flows at the bottom (Fig. 1). It is divided further into stratified smooth flow and stratified wavy flow, depending on the shape of the interface.

If we assume:

- steady, one-dimensional, developed two-phase flow;
- no mass transfer between gas and liquid phases;
- negligible acceleration effects;
- constant physical properties;
- no interfacial level gradient (ILG);
- same pressure gradient in both phases; and
- isothermal flow;

then the momentum balance equations for stratified flow in pipes can be derived for both gas and liquid parts^{4,5}

$$-A_L \left(\frac{dp}{dL} \right) - \tau_{wL} S_L + \tau_i S_i - \rho_L A_L \frac{g}{g_c} \sin \theta = 0 \quad (1)$$

$$-A_G \left(\frac{dp}{dL} \right) - \tau_{wG} S_G - \tau_i S_i - \rho_G A_G \frac{g}{g_c} \sin \theta = 0 \quad (2)$$

where the friction shears are expressed in the form of the Fanning friction factors

$$\tau_{wL} = \frac{f_{wL} \rho_L V_L^2}{2 g_c} \quad (3a)$$

$$\tau_{wG} = \frac{f_{wG} \rho_G V_G^2}{2 g_c} \quad (3b)$$

$$\tau_i = \frac{f_i \rho_G (V_G - V_L) |V_G - V_L|}{2 g_c} \quad (3c)$$

In order to complete the stratified flow model, empirical correlations are required to determine the wall and the interfacial friction shears. The friction factors for the gas and liquid phases are usually evaluated from a single phase equation, such as the Colebrook-White, or the Blasius equation, based on the Reynolds number defined as a function of local velocities and hydraulic diameters (this method is termed as the standard method in this paper). The interfacial friction factor is obtained from empirical correlations. While there are relatively few methods to calculate wall friction factors, dozens of correlations have been proposed for the determination of interfacial friction factor. These correlations can be classified into four basic types⁶:

- Type I: $f_i = \text{const}$ (4a)

- Type 2: $\frac{f_i}{f_{wG}} = \text{const} \dots\dots\dots(4b)$

- Type 3: $\frac{f_i}{f_{wG}} = F_n(\text{flow parameters}) \dots\dots\dots(4c)$

- Type 4: $f_i = F_n(\text{flow parameters}) \dots\dots\dots(4d)$

As shown in Fig. 2, predictions vary significantly among different correlations⁶.

Consistency Check and Model Evaluation

As aforementioned, the main purpose of this study is to evaluate the existing interfacial friction factor correlations based on experimental observations. In order to obtain reliable results, it is very important to make sure that the experimental data are consistent and the stratified flow model used is reasonable. Due to the presence of measurement difficulties, it is not so easy to obtain experimental data which are completely consistent. Hence we have looked into the degree of consistency in experimental data. Tables 1a and 1b compare results of the interfacial friction shears obtained from the two momentum balance equations (Eqs. 1 & 2) for all available data sets. The average absolute error (AAE) is used in assessing the order of magnitude of the difference, the average absolute relative error (AARE) is taken as the accuracy criterion which best expresses the average difference, while the standard deviation (SDE and SRDE) measures the scatter of results around the average value.

Table 1a and 1b show that for most data sets there exist significant differences between the two interfacial friction shears calculated from gas and liquid momentum balance equations. The high errors appearing in the consistency check for both stratified smooth and stratified wavy flows suggest the need for further investigation. Two possibilities exist which may cause high error in the consistency check. One is measurement errors in the experimental data, and the other is the possible inappropriateness of the stratified flow model itself.

To check the influence of measurement error on the consistency study, errors are introduced artificially in the measured liquid holdup data and the measured pressure gradient data of sets SU-101 and SU-205 for stratified smooth flow and of sets SU-96 and SU-200 for stratified wavy flow in the Stanford Multiphase Flow Database (SMFD). Six types of artificial errors are used, i.e., $\pm 10\%$ changes in measured liquid holdup data, $\pm 5\%$ and ± 0.0001 psi/ft changes in measured pressure gradient data. It was found that AAEs, AAREs and SDEs do not change significantly for all cases of error and for all data sets considered.

Therefore, it seems that the influence of measurement errors in experimental data cannot be the main reason for the inconsistency observed. In other words, the model used for the consistency check, though the most widely-used model for describing gas-liquid flow in pipes, may be inappropriate.

Recall the stratified flow model discussed above. The only parts in this model which may cause problems in the

consistency check are the determination of the wall friction shears which are obtained through the calculation of the wall friction factors. The Colebrook-White equation or the Blasius-type equations are usually applied to calculate wall friction factors for both gas and liquid phases in turbulent flow. As we know, the Colebrook-White equation and the Blasius-type equation were proposed to calculate the friction factor in single-phase flow and whether they can be used for two-phase or three-phase flow is questionable. Several researchers, such as Andreussi & Persen⁷, Andritsos & Hanratty⁸, and Kowalski^{3,9} argued that the determination of the wall friction shear for gas phase is more reliable than the wall friction shear for the liquid phase. Direct measurements of the wall friction shears verified this observation^{9,10,11}. So, on the assumption that most of the experimental data available are consistent, the liquid phase wall friction factor can be calculated from the pressure equation⁶, such that the predicted liquid holdup and the predicted pressure gradients match the experimental results. The wall friction factor for the liquid phase obtained in this way is termed experimental f_{wL} .

Fig. 3 compares the experimental wall friction factor with the predicted value determined from the standard method. It can be easily seen that the standard method underestimates the wall friction factor for liquid phase for both stratified smooth and stratified wavy flows. This is expected considering the fact that the Colebrook-White equation and the Blasius equation (or other equivalent forms) were proposed for single phase flow in pipes. A different mechanism dominates the fluid flow characteristics for multiphase flow in pipes. Hence use of the standard method to describe the wall friction is inappropriate and leads to errors. Govier and his co-workers^{10,11} determined the liquid phase wall friction factor from the measured pressure drop and liquid holdup and found that, depending upon the relative volume of gas and liquid present, the liquid phase wall friction factor was 2.0 to 2,000 times larger than the value obtained from the single phase relationship. Unfortunately, enough attention has not been paid to their observations.

Comparison with Experimental Data

In this study, 18 different interfacial friction factor correlations, including all the widely-used correlations reported in the literature, have been applied in the stratified flow model to predict the liquid holdup and the pressure gradient. The predictions have been done separately for stratified smooth flow and stratified wavy flow data.

For stratified smooth flow, Table 2a shows that the modified Andreussi & Persen⁷ correlation gives the best prediction for liquid holdup, since the AARE (57.22%) is lower than those from other correlations and the AAE (0.065) only second to the lowest value (0.0632 for the Cheremisinoff & Davis correlation¹²). The Andritsos & Hanratty⁸ and Hanratty & Andritsos¹³ correlations rank as second and third best in the list. These three correlations are also the best ones

for the prediction of pressure gradient. Their AAEs are 0.0038, 0.0035 and 0.0036 psi/ft respectively, which are much smaller than the corresponding results from other correlations. Unfortunately, even for these three correlations, the deviations of predicted liquid holdup and the pressure gradient from experiments are still quite large. The primary reason for this is that the predicted interfacial friction factors are different from those required for the correct prediction of experimental data. Fig. 4a displays an example of the deviations from experiments for the Andritsos & Hanratty⁸ correlation. It is instructive to note that the $f_i = f_{wG}$ correlation, even though it is commonly used by researchers for stratified smooth flow, does not predict pressure gradients and liquid holdups that are sufficiently close to the measured values.

For stratified wavy flow, most existing correlations overestimate both the liquid holdup and the pressure gradient (Table 2b). Among the correlations considered, the modified Andreussi & Persen⁷ correlation² predicts the closest liquid holdup to the measurements (AAE equals to 0.0607). The Andritsos & Hanratty⁸ correlation as well as the Cheremisinoff & Davis¹² correlation also give good predictions of liquid holdup. But, for pressure gradients, the modified Andreussi & Persen⁷ correlation does not predict good results, whereas the Andritsos & Hanratty⁸, the Hanratty & Andritsos¹³ correlations provide best predictions. The Baker et al.¹⁴ correlation yields fairly good results. Comparisons between predicted interfacial friction factors and the corresponding values obtained from experimental parameters for stratified wavy flow are shown in Fig. 4b.

Hence, it can be concluded that none of the existing interfacial friction factor correlations can predict accurate results for liquid holdup and pressure gradient.

Development of New Correlations

Considerations for the Selection of Variables and Functional Forms of New Correlations. The choice of variables and the functional form of new correlations are worth a special mention. For gas-liquid simultaneous flow in pipes, the variables governing the flow characteristics include pipe diameter, pipe roughness, pipe inclination angle, densities, viscosities and superficial velocities of both phases, surface tension, gravity acceleration, and wall contact angle. Since problem must be reduced to manageable proportions whenever we construct models, only a limited number of independent or predictor variables can be included in a regression model for the situation of interest. A central problem therefore is that of choosing, for our new correlation, a set of variables which is 'good' in some sense for the purpose of the analysis. A major consideration in making this

choice is the extent to which a chosen variable contributes to reducing the remaining variation in response (i.e., the liquid phase wall friction factor and the interfacial friction factor for our case) after allowance is made for the contributions of other variables that have tentatively been included in the regression model. Other considerations include the importance of the variable's role in the process under analysis, and the degree to which observations on the variable can be obtained more accurately, or quickly, or economically than on competing variables¹⁵.

Note that mechanisms involved in any flow situation are governed by dimensionless groups, the number of which equals to three less than the number of independent quantities required to fully describe the system and the boundary conditions. We only consider the case where the flow mechanism is independent of the way the gas and liquid are supplied at the entrance or flow out at the exit, i.e., we assume that the test section is not too close to either the inlet or the outlet. Under these conditions, the number of independent variables, as mentioned in the above paragraph, reaches up to twelve. That means we can construct nine independent dimensionless groups. Ros¹⁶ chose the diameter number, the relative pipe roughness, the pipe inclination angle, the gas-liquid density ratio, the liquid viscosity number, the gas viscosity number, the liquid velocity number, the in situ gas-liquid velocity ratio (i.e., gas-liquid volume ratio) as well as the wall contact angle as the nine dimensionless groups. This set is only one among many possible choices. Other dimensionless groups may also be selected provided all the dimensionless groups entered are independent.

The selection of the functional form of the regression correlation is related to the choice of the independent variables or dimensionless groups. Sometimes, relevant theory may indicate the appropriate functional form. However, the functional form of the regression correlations for liquid phase wall friction factor and interfacial friction factor is not known in advance and must be decided upon once the data have been collected and analyzed. The functional form is determined by means of analyzing existing correlations and considering the coefficient of multiple determination and the error mean square (*MSE*) for the trial correlation.

It should be noted that a large coefficient of multiple determination does not necessarily imply that the fitted model is correct. For instance, only small number of observations may have been taken, or the data points may be related to only a few of the independent variables. Despite the fact that a high value of the coefficient of multiple determination is obtained, the fitted correlation may not be useful because most predictions would require extrapolations outside the region of experiment data. Furthermore, even though the coefficient of multiple determination is large, the error mean square (*MSE*) may still be too large for inferences to be useful. On the other hand, adding more independent variables to the regression correlation can only increase the coefficient

² Strictly speaking, this correlation has been revised for our comparison, since the original correlation was proposed for horizontal pipe flow and has some problems which restrict its applying to vertical or inclined pipe flow. Modifications have been made to make this correlation applicable for all pipe inclinations²².

of multiple determination and never reduce it, it is thus suggested that the adjusted coefficient of multiple determination, denoted by R_a^2 be used. The adjusted coefficient of multiple determination may actually become smaller when another independent variable is introduced into the regression correlation, because the decrease in error sum of squares or residual sum of squares (SSE) may be more than offset by the loss of a degree of freedom¹⁵.

Liquid Phase Wall Friction Factor Correlation. From the experimental data available from the Stanford Multiphase Flow Database (SMFD), the experimental wall friction factor data for liquid phase have been computed and applied to develop a new correlation by means of regression. The regression was primarily done based on the dimensionless groups which affect gas-liquid two-phase flow in pipes. Two regression schemes were used for our investigation, one is the pseudo-linear regression (nonlinear regression which can be transformed into a linear one, it is also called general multiple linear regression), the other is the Polytope method¹⁷.

Different forms of correlations which combine different dimensionless variables were tested to get the best fit of experimental liquid phase wall friction factor. Regression leads to the following new correlation

$$f_{wL} = 1.6291 R_{vL}^{-0.5161} R_v^{0.0926} \dots \dots \dots (5)$$

where R_v is the gas-liquid volume ratio

$$R_v = \frac{\text{Volumetric Flow Rate of Gas Phase}}{\text{Volumetric Flow Rate of Liquid Phase}} = \frac{V_{sG}}{V_{sL}} \dots \dots \dots (6)$$

Fig. 5 shows the comparison between the experimental and predicted liquid phase wall friction factor by the new correlation (Eq. 5). The figure indicates that the new correlation gives much closer results for the liquid phase wall friction factor to the experimental values than the standard method (Fig. 3), but it still predicts unsatisfactory friction factors for some of the data points. Fortunately, the number of data points falling into this category is small considering the fact that about 800 data points are shown in Fig. 5. It should be noted that there exist same problems with the new liquid phase wall friction factor correlation as is the case for any regression model, in that they are dependent upon the data available. When more data over a wider range of fluid properties, pipe sizes and inclination angles are considered, the results may change. It is, however, expected that the change will not be too large because the experimental data included in our investigation come from many different sources and the number of data points is quite large. Moreover, these data have rather different gas and liquid properties, flow parameters and pipe geometries.

Interfacial Friction Factor Correlation. As shown by Fig. 4, the predicted interfacial friction factors are substantially different from experimental values even for the best available correlations, such as the modified Andreussi & Persen⁷, and the Andritsos & Hanratty⁸ relations. Furthermore,

discontinuities in liquid holdup and pressure gradient often occur at the transition of different flow patterns when available interfacial friction factor correlations are applied in mechanistic models¹⁸. Such discontinuities can cause problems for simulation of production facilities. Hence, it is necessary to develop new correlations for interfacial friction factor which can hopefully predict satisfactory results for liquid holdup and pressure gradient, and provide smooth flow pattern transitions.

The same regression approach as used for the liquid phase wall friction factor case leads to the following new correlation for the interfacial friction factor

$$f_i = 10^{-8.0942 + 4.2893 E_L \sin \theta} \frac{f_{wL}^{0.8732} N_{vL}^{0.3072} N_D^{1.0365}}{N_G^{1.9140} H^{0.9783}} \dots \dots \dots (7)$$

where E_L is the liquid holdup, θ the pipe inclination angle, f_{wL} the wall friction factor for liquid phase. Other dimensionless groups used are:

$$\text{Liquid velocity number } N_{vL} = V_{sL} \sqrt{\rho_L / g \sigma} \dots \dots \dots (8a)$$

$$\text{Gas viscosity number } N_G = \mu_G \sqrt{g / \rho_L \sigma^3} \dots \dots \dots (8b)$$

$$\text{Pipe diameter number } N_D = D \sqrt{\rho_L g / \sigma} \dots \dots \dots (8c)$$

$$\text{Holdup ratio } H = V_G / V_L = \frac{C_G}{C_L} \frac{E_L}{1 - E_L} \dots \dots \dots (8d)$$

Fig. 6 compares the predicted values of interfacial friction factor from the new correlation with experimental data. Satisfactory agreement is observed.

Test of New Correlations

No matter how strong the statistical relations, no cause-and-effect pattern is necessarily implied by a regression model. Even if the new correlations are developed on the basis of some underlying mechanisms, the new correlations still need to be examined with experimental data.

Calculations indicate that the new correlations can predict more satisfactory liquid holdup and pressure gradient for the experimental data used for developing the correlations than other correlations. Fig. 7 show the predicted liquid holdup and pressure gradient for stratified flow against experimental data. Notwithstanding the fact that large deviations from experimental values still exist for some data points, the number of data points falling into this category is small compared to the amount of data considered. But we should note that for some special data sets, predictions by the new correlations are not so satisfactory as expected.

Application of New Correlations to Predict the Liquid Holdup of Minami & Brill's Experiment. A good test of the new correlations is to apply them to predict either liquid holdup or pressure gradient and compare these predictions with reliable measurements not used in developing the correlations. Minami & Brill¹⁹ experiment, in which only the liquid holdup data were measured, is selected for the test. This experiment consists of two types of fluid combinations. One is an air-water system where 54 measurements are

provided and the other is an air-kerosene system where 57 measurements are reported.

Fig. 8 shows the comparison between measured and predicted liquid holdup by the new correlations for the liquid phase wall friction factor and for the interfacial friction factor. It is found that the new correlations provide satisfactory predictions of liquid holdup for the air-kerosene case but slightly underpredict the liquid holdup for the air-water case.

Use of New Correlations in a Mechanistic Model. As mentioned above, the interfacial friction factor correlations may introduce discontinuities around the flow pattern transition boundaries. We have applied the new correlations for both the liquid phase wall friction factor and the interfacial friction factor to the Stanford Mechanistic Model¹⁸ to predict flow pattern map, pressure gradient and liquid holdup. It was found that the combination of new liquid phase wall friction factor correlation and the Baker et al.¹⁴ interfacial friction factor correlation predicts smoother transitions of the liquid holdup and the pressure gradient between stratified flow and its adjacent flow patterns than other correlations. Unfortunately, no significant improvement was observed in the reduction of the discontinuities of pressure gradient and liquid holdup during the transition of flow patterns if both new correlations are applied. This is probably because the number and the range of the data points used to get the best fit by regression are not sufficiently large and wide to account for all possible ranges of flow parameters, including pipe diameter, pipe inclination angle, pipe roughness, viscosities and velocities of both phases, fluid types, surface tension, and so on. Also, it may be that the mechanistic model needs further refinement.

Conclusions and Recommendations

Different interfacial friction factor correlations have been used to predict liquid holdup and pressure gradient and compare with experimental observations. The comparison results show that: (a) most existing correlations can lead to large deviations from measurements; (b) among available correlations, the modified Andreussi & Persen⁷ correlation, the Andritsos & Hanratty⁸ correlation, as well as the Hanratty & Andritsos¹³ correlation, are the best choices to determine the interfacial friction factor for stratified flow.

Consistency checks of experimental data indicate that the widely-used stratified flow model is inconsistent with experimental observations. It must be noted that this conclusion is independent of the interfacial friction factor correlation. On the basis of reported experiments with direct measurements of wall friction shear stress, we conclude that the liquid phase wall friction shear calculation should be reconsidered. The standard method (i.e., the single phase method) is found to underestimate the liquid phase wall friction factor.

New correlations for both the interfacial friction factor and the liquid phase wall friction factor have been developed

based on experimental observations. Satisfactory coincidence is observed when new correlations are applied to predict the liquid holdup for the Minami & Brill¹⁹ experiments.

Additional work is recommended in areas related to: (a) interfacial friction factor correlation; (b) definition of interfacial friction shear and determination of interface velocity; (c) interfacial level gradient; (d) small liquid holdup; (e) gas-liquid flow with heat transfer; (f) mass transfer between gas and liquid phases; (g) unsteady flow; and (h) two-dimensional flow or three-dimensional flow.

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Nomenclature

A_G	cross-sectional area available to the gas phase
A_L	cross-sectional area available to the liquid phase
C_G	volume fraction of the gas phase at the inlet
C_L	volume fraction of the liquid phase at the inlet
D	pipe diameter
E_L	liquid holdup
f_i	interfacial friction factor
f_{nG}	Fanning friction factor for the gas phase
f_{nL}	Fanning friction factor for the liquid phase
g	acceleration due to gravity
g_c	dimensional conversion constant
H	holdup ratio
h_L	liquid height
N_D	pipe diameter number
N_{VL}	liquid velocity number
N_G	gas viscosity number
p	pressure
R_a	adjusted coefficient of multiple determination
R_{eL}	liquid phase Reynolds number
S_G	wall perimeter wetted by the gas phase
S_i	interfacial perimeter
S_L	wall perimeter wetted by the liquid phase
V_G	average velocity of the gas phase
V_i	interface velocity
V_L	average velocity of the liquid phase
V_{sG}	superficial velocity of the gas phase
V_{sL}	superficial velocity of the liquid phase
ϵ	absolute pipe roughness
θ	inclination angle of the pipe

ρ_G	gas density
ρ_L	liquid density
τ_i	interfacial friction shear
τ_{wG}	wall friction shear for the gas phase
τ_{wL}	wall friction shear for the liquid phase

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Table 1a Consistency Check of the Available Experimental Data (Stratified Smooth Flow)

Data Set	Number of Data	AE (psf)	AAE (psf)	RMS (psf)	SDE (psf)	ARE (%)	AARE (%)	RMSR (%)	SRDE (%)
SU-24	9	-0.0918	0.0918	0.0998	0.0416	-102.98	102.98	105.77	25.63
SU-25	11	-0.0366	0.0366	0.0476	0.0321	-112.66	112.66	117.58	35.31
SU-26	3	-0.0462	0.0462	0.0489	0.0196	-94.92	94.92	98.68	33.03
SU-28	1	-0.1323	0.1323	0.1323	—	-104.83	104.83	104.83	—
SU-29	1	-0.1349	0.1349	0.1349	—	-109.58	109.58	109.58	—
SU-96	29	-0.0124	0.0124	0.0192	0.0149	-136.73	136.73	139.83	29.77
SU-101	45	0.0005	0.0006	0.0009	0.0008	45.14	64.32	77.49	63.70
SU-109	2	0.0326	0.0875	0.0933	0.1237	-16.35	82.15	82.78	114.76
SU-110	9	-0.0476	0.0476	0.0495	0.0145	-102.40	102.40	106.06	29.28
SU-184	7	0.0378	0.0389	0.0458	0.0278	78.26	84.36	89.57	47.04
SU-188	8	0.6729	0.6729	1.0883	0.9144	88.15	88.15	103.15	57.26
SU-189	14	2.6010	2.6010	2.7356	0.8798	87.62	87.62	87.91	7.46
SU-190	9	0.2638	0.2638	0.3896	0.3041	105.09	105.09	105.57	10.64
SU-191	11	0.3461	0.4096	0.6016	0.5161	58.99	111.23	115.13	103.69
SU-192	9	0.8979	0.9322	1.2987	0.9952	83.93	120.94	126.73	100.71
SU-193	2	9.6615	9.6615	10.0383	3.8531	162.01	162.01	165.16	45.35
SU-194	19	4.5296	4.5296	5.0903	2.3862	113.49	113.49	114.74	17.34
SU-195	1	8.2426	8.2426	8.2426	—	118.93	118.93	118.93	—
SU-196	15	4.9025	4.9087	6.2529	4.0175	92.79	106.04	107.27	55.71
SU-197	11	-0.1188	0.4619	0.5799	0.5953	-42.30	103.06	104.50	100.22
SU-198	13	5.3311	5.3311	5.5865	1.7379	100.32	100.32	100.41	4.37
SU-199	74	-0.0340	0.0346	0.0588	0.0484	-64.66	88.95	93.49	67.99
SU-200	23	-0.0729	0.0729	0.1503	0.1344	-53.25	53.25	56.09	18.02
SU-201	5	-0.0179	0.0192	0.0260	0.0211	-13.08	14.69	17.49	12.98
SU-202	1	0.0118	0.0118	0.0118	—	5.48	5.48	5.48	—
SU-203	1	0.1031	0.1031	0.1031	—	48.43	48.43	48.43	—
SU-205	10	0.1098	0.1819	0.4635	0.4747	-113.28	153.89	164.24	125.36
SU-206	37	5.7107	5.8654	13.3760	12.2625	-56.99	119.67	122.52	109.95
SU-207	41	25.7271	25.8997	40.8723	32.1540	2.56	111.26	114.50	115.90
SU-208	38	9.1726	9.6558	29.7337	28.6632	-74.85	105.07	107.48	78.17

Table 1b Consistency Check of the Available Experimental Data (Stratified Wavy Flow)

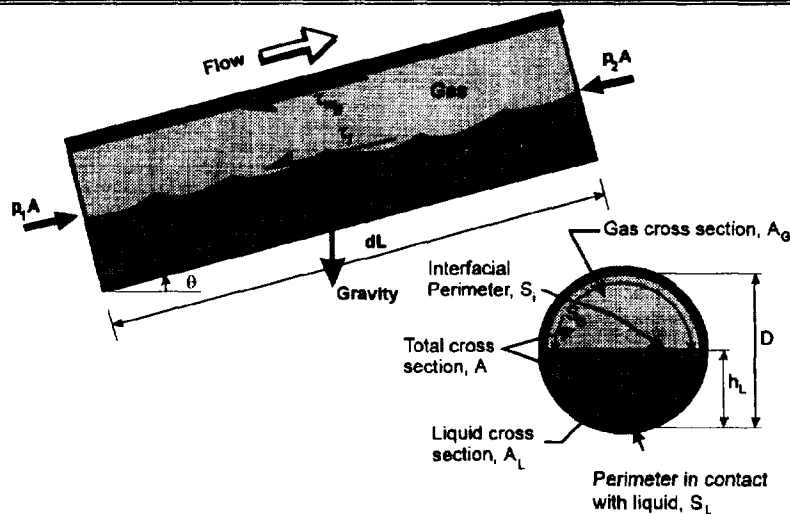
Data Set	Number of Data	AE (psf)	AAE (psf)	RMS (psf)	SDE (psf)	ARE (%)	AARE (%)	RMSR (%)	SRDE (%)
SU-21	14	0.0018	0.2473	0.3581	0.3716	-17.46	52.01	55.54	54.71
SU-23	37	-0.0524	0.4103	0.5624	0.5677	-39.64	70.94	80.15	70.63
SU-24	61	-0.1312	0.1352	0.3286	0.3038	-63.10	65.79	76.54	43.69
SU-25	33	2.8126	3.0949	11.8744	11.7154	-47.09	64.68	69.85	52.39
SU-26	38	-0.2457	0.2484	0.4213	0.3469	-65.89	66.24	77.08	40.54
SU-28	4	-0.2313	0.2313	0.2723	0.1659	-71.49	71.49	85.66	54.49
SU-29	4	-0.2422	0.2422	0.2959	0.1963	-72.03	72.03	84.61	51.26
SU-53	5	-0.0478	0.0478	0.0519	0.0227	-47.72	47.72	48.31	8.46
SU-54	3	0.0059	0.0300	0.0346	0.0417	-8.17	67.19	68.87	83.76
SU-96	10	-0.0404	0.0404	0.0536	0.0371	-80.85	80.85	81.37	9.65
SU-109	2	0.7755	0.7755	1.0817	1.0665	75.28	75.28	92.88	76.94
SU-110	1	1.5209	1.5209	1.5209	—	141.57	141.57	141.57	—
SU-199	27	-0.2208	0.2213	0.3273	0.2463	-47.34	60.15	62.80	42.04
SU-200	29	-0.8053	0.8075	0.8877	0.3801	-66.41	66.66	68.79	18.23
SU-201	17	-0.6811	0.6831	0.9375	0.6641	-44.61	45.32	51.62	26.76
SU-202	11	-0.2962	0.2962	0.4000	0.2819	-29.44	29.44	32.58	14.63
SU-203	10	-0.3529	0.4078	0.7501	0.6978	-13.80	26.73	30.50	28.68
SU-204	50	-0.7828	0.8047	1.0664	0.7316	-33.81	36.30	40.47	22.48
SU-205	52	-0.1414	0.2079	0.3687	0.3439	-62.42	81.07	89.89	65.32
SU-206	39	1.4571	1.8597	5.3518	5.2170	-7.04	94.02	101.75	102.84
SU-207	18	8.8404	9.4962	17.4175	15.4424	3.8847	115.46	121.96	125.44
SU-208	5	26.1374	26.6737	41.7535	36.4039	-13.52	125.10	125.77	139.80
SU-209	63	50.9893	52.5793	251.825	248.590	-48.85	128.40	131.52	123.10
SU-213	16	-0.0524	0.0523	0.0586	0.0270	-139.17	139.17	142.87	33.35
SU-215	25	1.7081	1.9508	5.5383	5.3769	-66.78	125.95	129.54	113.28

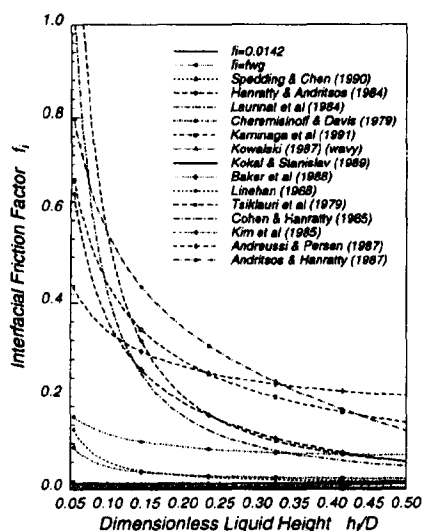
Table 2a Comparison between the Measured and Predicted Liquid Holdup and Pressure Gradient (Stratified Smooth Flow)

Correlation	Liquid Holdup				Pressure Gradient			
	ARE (%)	AARE (%)	AE	AAE	ARE (%)	AARE (%)	AE	AAE
$f_i = 0.0142$	145.47	157.79	0.0718	0.0893	-10.94	167.01	0.0042	0.0052
Andritsos & Hanratty (1987) ⁸	58.28	74.11	0.0583	0.0759	-17.79	139.09	0.0027	0.0036
Andreussi & Persen (1987) ⁷	39.54	57.22	0.0436	0.0650	-42.71	186.77	0.0023	0.0038
Baker et al. (1988) ($V_i = V_G - V_L$) ¹⁴	166.53	177.88	0.0711	0.0890	-12.47	171.00	0.0044	0.0055
Baker et al. (1988) ($V_i = V_L$) ¹⁴	155.22	166.57	0.0694	0.0873	-15.92	162.52	0.0039	0.0051
Cheremisinoff & Davis (1979) ¹²	96.70	116.01	0.0403	0.0632	299.59	753.12	0.0057	0.0073
Cohen & Hanratty (1965) ²⁰	101.33	120.10	0.0467	0.0689	268.29	688.78	0.0058	0.0072
$f_i = f_{wG}$	235.43	246.62	0.0879	0.1051	-3.61	182.93	0.0047	0.0059
Hanratty & Andritsos (1984) ¹¹	49.42	67.42	0.0560	0.0743	-29.45	137.29	0.0026	0.0035
Kowalski (1984) ⁹	334.31	345.48	0.1119	0.1290	-3.56	187.68	0.0052	0.0064
Kowalski (1987) ¹	379.64	390.66	0.1142	0.1309	38.91	245.56	0.0055	0.0067
Kaminaga et al. (1991) ²¹	173.77	186.09	0.0888	0.1059	-10.16	172.85	0.0043	0.0054
Kim et al. (1985) ²²	99.36	113.51	0.0554	0.0750	12.15	200.54	0.0040	0.0049
Kokal & Stanislav (1989) ²	318.87	330.09	0.0939	0.1115	1.99	192.76	0.0051	0.0063
Laurinat et al. (1984) ²³	204.84	216.36	0.0968	0.1138	10.54	200.85	0.0052	0.0063
Linehan (1968) ²⁴	141.78	154.21	0.0669	0.0844	29.37	232.05	0.0045	0.0054
Spedding & Hand (1990) ²⁵	281.18	292.36	0.0907	0.1079	3.90	195.07	0.0049	0.0061
Tsiklauri et al. (1979) ²⁶	101.07	121.83	0.0453	0.0690	334.84	819.61	0.0062	0.0079

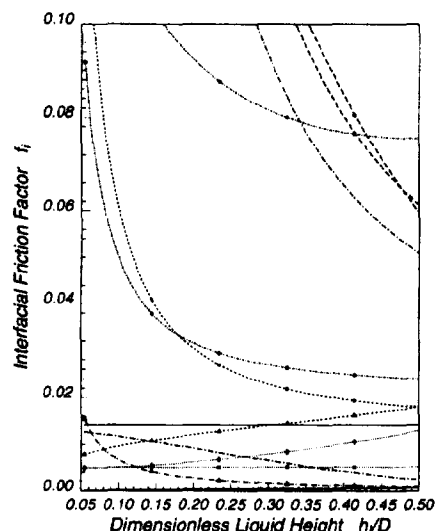
Table 2b Comparison between the Measured and Predicted Liquid Holdup and Pressure Gradient (Stratified Wavy Flow)

Correlation	Liquid Holdup				Pressure Gradient			
	ARE (%)	AARE (%)	AE	AAE	ARE (%)	AARE (%)	AE	AAE
$f_i = 0.0142$	248.66	254.06	0.1788	0.1897	82.17	188.42	0.0305	0.0507
Andritsos & Hanratty (1987) ⁸	76.98	94.50	0.0786	0.1010	97.29	203.49	0.0191	0.0286
Andreussi & Persen (1987) ⁷	15.32	60.31	0.0075	0.0607	240.96	405.78	0.0716	0.0770
Baker et al. (1988) ($V_i = V_G - V_L$) ¹⁴	286.12	291.28	0.1942	0.2050	88.39	197.12	0.0349	0.0558
Baker et al. (1988) ($V_i = V_L$) ¹⁴	242.93	248.10	0.1732	0.1840	73.79	181.88	0.0273	0.0484
Cheremisinoff & Davis (1979) ¹²	31.10	95.72	0.0263	0.1056	601.87	1098.12	0.0673	0.0723
Cohen & Hanratty (1965) ²⁰	38.66	101.58	0.0353	0.1117	557.27	1021.80	0.0667	0.0718
$f_i = f_{wG}$	318.24	322.85	0.2226	0.2315	84.52	198.23	0.0361	0.0581
Hanratty & Andritsos (1984) ¹¹	136.81	146.26	0.1198	0.1338	89.79	188.30	0.0240	0.0382
Kowalski (1984) ⁹	322.74	327.21	0.2331	0.2415	90.91	205.04	0.0381	0.0601
Kowalski (1987) ¹	279.20	284.12	0.2161	0.2250	137.57	271.43	0.0389	0.0578
Kaminaga et al. (1991) ²¹	233.93	238.85	0.1828	0.1918	89.12	197.80	0.0298	0.0492
Kim et al. (1985) ²²	152.74	163.59	0.1202	0.1382	168.77	306.88	0.0294	0.0425
Kokal & Stanislav (1989) ²	353.85	358.31	0.2441	0.2525	88.52	204.57	0.0393	0.0617
Laurinat et al. (1984) ²³	330.04	334.73	0.2358	0.2444	109.46	225.61	0.0427	0.0637
Linehan (1968) ²⁴	154.73	170.23	0.1230	0.1467	207.51	369.43	0.0331	0.0436
Spedding & Hand (1990) ²⁵	254.39	259.44	0.1843	0.1957	87.57	205.03	0.0289	0.0495
Tsiklauri et al. (1979) ²⁶	28.81	97.88	0.0258	0.1090	624.45	1155.10	0.0759	0.0810

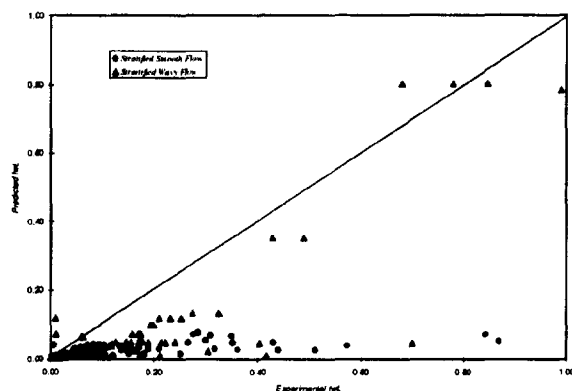
**Figure 1** Schematic of Stratified Flow in Pipes



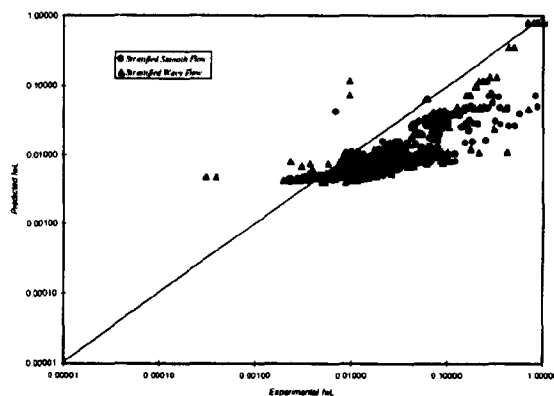
(a) Large Interfacial Friction Factor Range



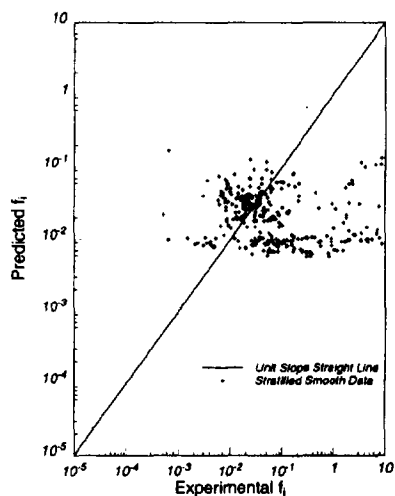
(b) Small Interfacial Friction Factor Range

Figure 2 Interfacial Friction Factor Comparison Based on Different Correlations

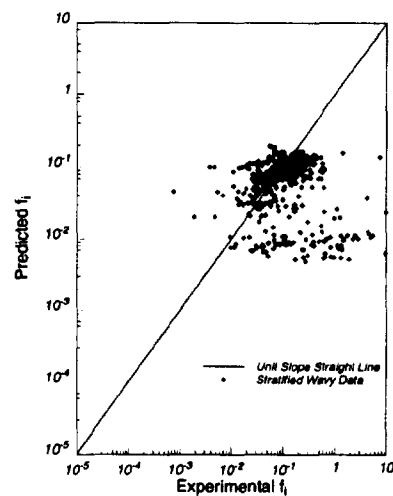
(a) Normal Scale



(b) Logarithmic Scale

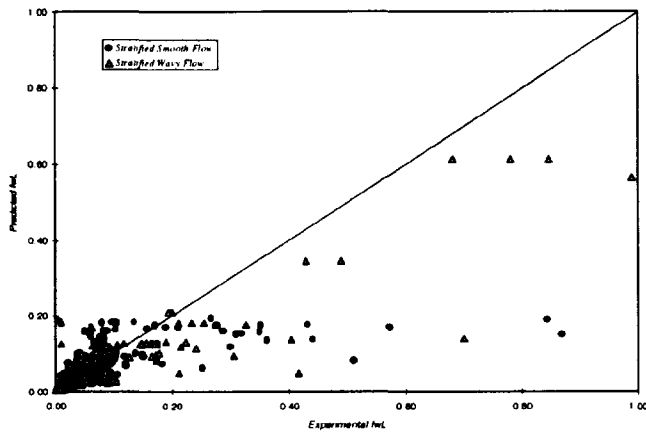
Figure 3 Comparison between the Experimental Liquid Phase Wall Friction Factor and the Predicted Value from Standard Method

(a) Stratified Smooth Flow

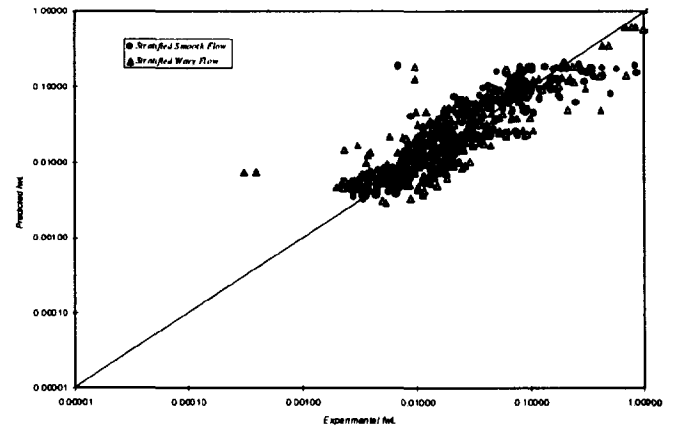


(b) Stratified Wavy Flow

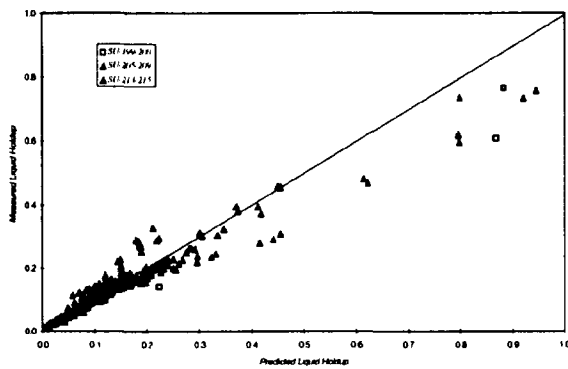
Figure 4 Comparison between Predictions and Experiments of the Interfacial Friction Factors for Andritsos & Hanratty (1987) Correlation³



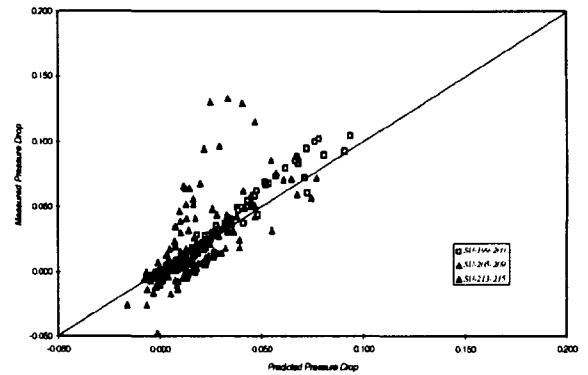
(a) Normal Scale



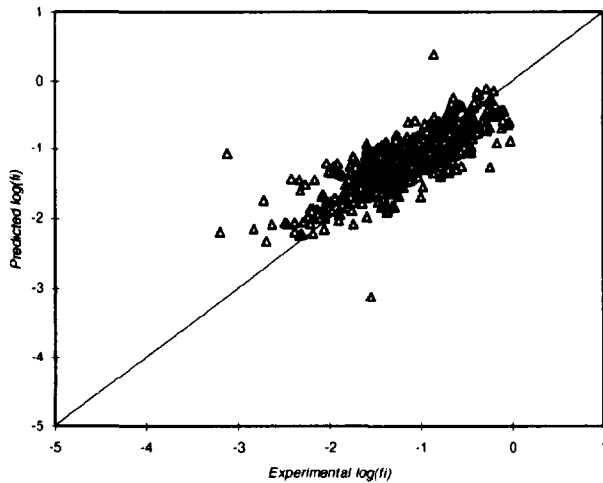
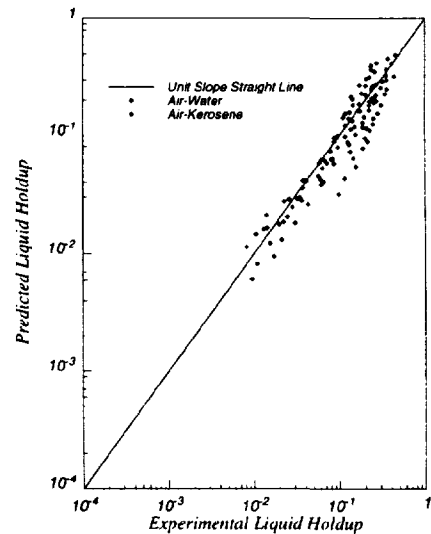
(b) Logarithmic Scale

Figure 5 Comparison between the Experimental Liquid Phase Wall Friction Factor with the Predicted Value from New Correlation

(a) Liquid Holdup



(b) Pressure Drop

Figure 7 Liquid Holdup and Pressure Drop Predicted from New Correlations**Figure 6.** Comparison between the Experimental Interfacial Friction Factors and the Predictions by the New Correlation**Figure 8** Comparison between Measured and Predicted Liquid Holdup by New Correlations