Drift-Flux Parameters for Three-Phase Steady-State Flow in Wellbores

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Summary

Drift-flux models represent multiphase flow in wellbores or pipes in terms of a number of empirically determined parameters. Because of the lack of data for two- and three-phase flow in large-diameter inclined pipes, existing parameters are commonly based on small-diameter pipe experiments, which can lead to significant errors when the models are applied to wellbore flows. In this work, we use recent large-diameter experimental data for the determination of drift-flux parameters for oil/water/gas flow. The parameters are computed through application of an optimization procedure. It is shown that in-situ gas volume fraction in three-phase systems can be estimated using a two-phase flow model by viewing the system as an effective gas/liquid system, with oil and water constituting the "liquid" phase. This approach is, however, generally inaccurate for the determination of oil and water holdups, in which case the effect of gas must be taken into account. Specifically, for pipe inclinations away from horizontal, even small amounts of gas can act to eliminate the slip between oil and water. As the pipe deviation approaches horizontal, however, oil/water slip persists, even in the presence of gas. We develop and apply a unified two- and three-phase flow model to capture this gas effect. The new model is shown to provide much more accurate predictions for oil and water holdups in three-phase systems than were achievable with previous models.

Introduction

Drift-flux modeling techniques are commonly used to represent two- and three-phase flow in pipes and wellbores. These models are well-suited for use in reservoir simulators because they are relatively simple, continuous, and differentiable.[1,2] Drift-flux models require a number of empirical parameters. Most of the parameters used in current simulators were determined from experiments in small-diameter (5 cm or less) pipes and may therefore not be appropriate for large-diameter wellbores.[3,4]

In recent work,[5] we described a new research program, which includes experimental and modeling components, aimed at the determination of drift-flux parameters for large-diameter deviated wells. The experimental work entailed water/gas, oil/water, and oil/water/gas flows in a 15-cm-diameter, 11-m-long pipe at eight deviations ranging from vertical to 2° downward. Unique steady-state holdup data were measured using several different experimental techniques.[6] Our previous work provided optimized drift-flux parameters for two-phase water/gas and oil/water flows. Here, we extend the analysis to three-phase flows.

Even though the simultaneous flow of oil, water, and gas is very common in wellbores and pipelines, systems of this type are not fully understood. Most of the studies to date have focused on horizontal or near-horizontal flows.[7–12] Açikgöz *et al.*[7] classified the observed 10 flow patterns in a horizontal 1.9-cm pipe into two categories: oil-based and water-based flows, depending on which phase is dominant in the liquid. No three-layer stratified flows were observed in their experiments. Following the work of Açikgöz *et al.*, Lahey *et al.*[8] used the same experimental facility to collect three-phase holdup data. These data were then used to determine the drift-flux parameters C_0 and V_d for each of the 10 flow patterns. They found that the values of C_0 and V_d could be significantly different from one flow pattern to another and, as expected, the drift velocities were quite small compared with those in vertical flows. It is, however, clear that general simulation models are still lacking for three-phase stratified flows.[13] The state of three-phase flow modeling is even less developed for deviated pipes and wells.

Because comprehensive three-phase flow models are lacking, one treatment for three-phase flow is to combine oil and water into a single "liquid" phase and to then model the system as a two-phase liquid/gas flow. In this treatment, the slip between oil and water is ignored, and a homogeneous mixture is assumed for the liquid phase. Some studies indicate that this simple treatment can lead to significant errors in phase holdup predictions,[10,14] while other observations suggest that this approach is valid.[15,16] In this work, we will use our experimental data and model to clearly quantify the range of validity of this approach.

An alternate two-stage technique was proposed to model three-phase flow in wellbores.[1,2] This approach uses two-phase liquid/gas and oil/water flow models. In the first stage, oil/water/gas flow is treated as a liquid/gas flow with flow-weighted average properties for the liquid phase. The liquid/gas drift-flux model is applied to determine the gas volume fraction and liquid holdup. In the second stage, the oil/water drift-flux model is applied to compute the oil and water holdups within the liquid phase. This idealized approach ignores the effect of the third phase on the two-phase flow models. Nevertheless, it does produce the expected qualitative behavior in some cases. For example, it enables a stagnant three-phase mixture to separate into gas, oil, and water zones through countercurrent flow.

In this work, we first evaluate the use of our optimized two-phase drift-flux parameters for the modeling of the 15-cm-diameter oil/water/gas volume fraction data collected by Oddie *et al.*[6] We show that the two-phase water/gas drift-flux parameters can be used directly to provide gas volume fraction in three-phase systems. However, direct application of the two-phase oil/water parameters leads to considerable error in the predicted oil and water holdups. This error is shown to be caused by the effect of even small amounts of gas on the slip between the oil and water phases. We demonstrate that, for gas-volume fractions greater than a certain critical value, the slip between the water and oil phases vanishes (except at inclinations very near horizontal, in which the gas is separated from the oil/water mixture). We determine this critical gas volume fraction as a function of inclination angle from the experimental measurements and introduce an additional parameter into the drift-flux model to capture this effect. The resulting three-phase drift-flux model provides predictions in close agreement with the experimental data over the entire range of inclinations and additionally reduces to our previous two-phase model if one of the phases is not present.

Although the model developed in this work is specific to the system studied, we expect it to be applicable for other (similar) three-phase systems. The parameter values may require some retuning, however, particularly if the fluid properties differ substantially from those considered here. It would of course be of interest to test the model with other data sets. However, because we do not have access to any other three-phase experimental data for large-

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This paper (SPE 89836) was first presented at the 2004 SPE Annual Technical Conference and Exhibition, Houston, 26–29 September, and revised for publication. Original manuscript received for review 17 February 2005. Revised manuscript received 9 March 2005. Paper peer approved 20 March 2005.

diameter inclined pipes, we are unable to perform such comparisons at this time.

The outline of this paper is as follows. We first briefly describe the experimental setup and then review the drift-flux formulation. We describe our optimization procedure for the determination of drift-flux parameters from the data and briefly discuss our earlier results for two-phase flow systems. We then establish drift-flux parameters for three-phase flow over a wide range of flow rates and pipe inclinations. Conclusions and recommendations for simplified modeling approaches are also provided.

Experimental Data

A detailed description of the experimental study was previously presented by Oddie *et al.*[6] A sampling of the two-phase data for water/gas and oil/water systems was shown in our previous two-phase modeling work.[5] Here we briefly explain the experimental setup and present representative three-phase data, which will be used for the three-phase drift-flux parameter determination.

Experimental Setup. The experiments were carried out for two-phase water/gas and oil/water systems and for three-phase oil/water/gas flows in a 15-cm-diameter, 11-m-long plexiglass pipe at deviations of 0, 5, 45, 70, 80, 88, 90, and 92° from upward vertical. The fluids used were kerosene (with a viscosity of 1.5 cp and a density of 810 kg/m^3 at 18°C), tapwater, and nitrogen. Steady-state holdup data were measured using three different experimental techniques (electrical probes, nuclear densitometer, and shut-in). We use the shut-in holdup data to represent the steady-state holdup because it is a direct measurement with very small experimental error (less than 1%).

Because the test section includes the inlet, some portion of the flow is not fully developed, and this would be expected to affect the shut-in holdup measurements. As discussed in detail in our earlier papers,[5,6] although the flow is not fully developed everywhere in the test section, there is nonetheless a high degree of consistency between the three different holdup measurements at deviations from 0 to 88°. This indicates that the impact of flow development on the holdup results is small. See Refs. 5 and 6 for a quantitative assessment of this issue.

Three-Phase Steady-State Holdup Data. In our determination of three-phase model parameters, we include data only for the 6 deviations from 0 to 88°. This is consistent with our treatment of the two-phase systems and is done because the experimental holdup data for 90 and 92° display relatively large errors because of the end effect.[6] Also, we include data only for α_g>0.06 (where α_g is the in-situ gas fraction), because $\alpha_g \leq 0.06$ could not be measured.[6]

Fig. 1 shows a sample of the three-phase experimental data. Here we combine water and oil into a single liquid phase and plot the liquid holdup (α_l) vs. input liquid volume fraction (c_l) to show the slip between liquid and gas. The slip between oil and water will be considered in detail below. For an oil/water/gas system, $c_l=(Q_w+Q_o)/(Q_w+Q_o+Q_g)$, where Q represents volumetric flow rate. The error bars in the figures are ±5%, as determined from repeated experiments; this is taken to be the overall experimental error value. The diagonal line in Fig. 1 is the no-slip line, which represents a homogeneous liquid/gas flow (i.e., $\alpha_l=c_l$). The vertical distance from any point to the line gives the slip between the two phases. The data are for vertical and 45° deviation for different gas flow rates.

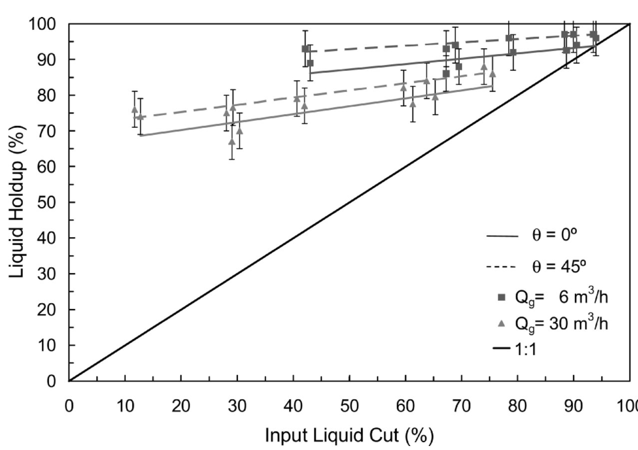

Fig. 1—Holdup for oil/water/gas system for θ=0° and θ=45°.

It is apparent from the figure that the slip between liquid and gas is quite significant. For example, for an input liquid cut of about 10%, the in-situ liquid percentage can be as high as 70%. The effect of deviation, gas flow rate, and c_l are clearly displayed in the figure. As the pipe is deviated from vertical to 45°, the slip increases. With increasing gas flow rate, the slip decreases for a constant input fraction. The slip also decreases as c_l increases. Comparing the data shown here with data at 70° deviation,[6] we see that there is a similar level of slip at both 45 and 70° inclinations. Trends similar to those shown in Fig. 1 were also observed in the water/gas data,[5,6] which suggests that the slip between liquid and gas in three-phase flow may be similar to that in water/gas flow.

Two-Phase Model and Parameter Determination

We now review the drift-flux models and the original parameters that were used for two-phase flows.[2] These models form the basis for the three-phase flow description. For full details of the two-phase model and parameters, see Shi *et al.*[5]

Liquid/Gas Model and Parameters. The drift-flux model for two-phase gas-liquid flow is given by[17]

$$V_g = C_0 V_m + V_d, \qquad (1)$$

where V_g is the average in-situ gas velocity, C_0 is the profile parameter, V_m is the mixture velocity, and V_d is the drift velocity. The mixture velocity is the sum of the gas and liquid superficial velocities (superficial velocity is given by the volumetric flow rate divided by the pipe cross-sectional area),

$$V_m = V_{sg} + V_{sl} = \alpha_g V_g + (1-\alpha_g)V_l. \qquad (2)$$

In our model,[1,2,5] the profile parameter C_0 involves three parameters: A, B, and F_v. Specifically,

$$C_0 = \frac{A}{1+(A-1)\gamma^2}, \qquad (3)$$

where γ is given by

$$\gamma = \frac{\beta - B}{1-B} \text{ subject to the limits } 0 \leq \gamma \leq 1 \qquad (4)$$

and β includes both α_g and V_m effects:

$$\beta = \max\left(\alpha_g, F_v \frac{\alpha_g |V_m|}{V_{sgf}}\right) \qquad (5)$$

Here, V_{sgf} is the "flooding" value of the gas superficial velocity (i.e., the value of V_{sg} at which a liquid film is prevented from falling back against the gas flow).

As shown in **Fig. 2,** the parameter A defines the maximum value for C_0. A value of $A=1.2$ is obtained from small-diameter data in the bubble and slug flow regimes in vertical pipes.[2] The parameter B defines the value of α_g at which C_0 starts to decrease; the original value used was $B=0.3$[2]. F_v represents the sensitivity to the velocity; this parameter was originally set to 1.0[2].

The derivation of the correlation for the gas-liquid drift velocity V_d was described in detail in our previous paper.[5] V_d is given by

$$V_d = \frac{(1-\alpha_g C_0)C_0 K(\alpha_g) V_c}{\alpha_g C_0 \sqrt{\frac{\rho_g}{\rho_l}} + 1 - \alpha_g C_0}, \qquad (6)$$

where $K(\alpha_g)$ is given by $K(\alpha_g)=1.53/C_0$ when $\alpha_g \leq a_1$, and $K(\alpha_g)=K_u(\hat{D})$ when $\alpha_g \geq a_2$.

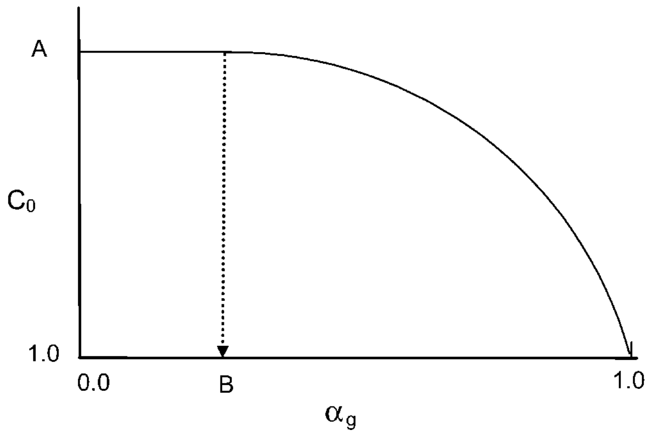

Fig. 2—Profile parameter for water/gas system.

$K_u(\hat{D})$ is the critical Kutateladze number, which is a function of the dimensionless pipe diameter $\hat{D}$. The dependency on $\hat{D}$ is given in Shi *et al.*[5] A linear interpolation is performed between a_1 and a_2 to connect the two curves representing bubble rise and liquid flooding. The original values of a_1 and a_2 are 0.2 and 0.4, respectively. V_c is the characteristic bubble rise velocity determined by Harmathy.[18] He observed that the rise velocity of a bubble through a stagnant liquid was 1.53 V_c, with V_c given by

$$V_c = \left(\frac{\sigma_{gl} g (\rho_l - \rho_g)}{\rho_l^2} \right)^{1/4}, \qquad (7)$$

where σ_{gl} is the gas/liquid interfacial tension, g is the acceleration due to gravity, and ρ_l and ρ_g are the liquid and gas densities.

Oil/Water Model and Parameters. The drift-flux model for oil/water flow has the same form as for liquid/gas flow:

$$V_o = C_0' V_l + V_d'. \qquad (8)$$

The original value for C_0' is 1.2 as suggested by Hasan and Kabir,[19] and when oil becomes the continuous phase ($\alpha_o > 0.7$), it reduces to 1.0. In our model, to avoid discontinuities, C_0' is a continuous function of the oil volume fraction:

$$C_0' = A' \text{ when } \alpha_o \leq B_1'$$
$$C_o' = 1.0 \text{ when } \alpha_o \geq B_2' \qquad (9)$$
$$C_0' = A' - (A' - 1) \left(\frac{\alpha_o - B_1'}{B_2' - B_1'} \right) \text{ when } B_1' < \alpha_o < B_2'.$$

The original values for A', B_1', and B_2' are 1.2, 0.4, and 0.7, respectively.[2]

The drift velocity correlation used in our model is also from Hasan and Kabir[19]:

$$V_d' \; 1.53 \; V_c' \, (1 - \alpha_o)^{n'}, \qquad (10)$$

where V_c' is given by

$$V_c' = \left(\frac{\sigma_{ow} g \, (\rho_w - \rho_o)}{\rho_w^2} \right)^{1/4}. \qquad (11)$$

The original value of n' is 2, as suggested by Hasan and Kabir.[19] For α_o>0.7, they recommend a no-slip model, though we apply Eq. 10 over the full range of α_o in order to maintain continuity.

Inclined Flows. The previously described correlations for the profile parameter and drift velocity are for vertical liquid/gas and oil/water flows. For inclined wells and pipes, we modify V_d as follows:

$$V_{d\theta} = V_{d0} m(\theta), \qquad (12)$$

where θ is the angle of deviation from the vertical. $V_{d\theta}$ represents the drift velocity at a deviation θ, and V_{d0} is the drift velocity defined above for vertical flow. The same procedure is applied for oil/water flow. We use the oil/water flow correlation of Hasan and Kabir[19] as the original deviation effect for both water/gas and oil/water systems:

$$m(\theta) = (\cos\theta)^{0.5} \, (1 + \sin\theta)^2. \qquad (13)$$

Parameter Determination. We describe the parameter determination for the two-phase case here (see Ref. 5 for full details) because these results form the basis for the three-phase model. The parameters to be optimized for two-phase flow were selected according to their sensitivity in the mathematical model and the physics they represent.

To apply our optimization method, in which data at all deviations are considered together, we first assigned a functional form to $m(\theta)$ (for water/gas systems) and to $m'(\theta)$ (for oil/water systems). We used the following functional form for water/gas flow[5]:

$$m(\theta) = m_0 (\cos\theta)^{n1} \, (1 + \sin\theta)^{n2}, \qquad (14)$$

where m_0, the multiplier for vertical flow, and n_1 and n_2 are adjustable parameters. The parameters (included in the vector of unknowns, X_p) to be optimized for liquid/gas systems were then as follows: $X_p = [A \; B \; a_1 \; a_2 \; m_0 \; n_1 \; n_2]^T$.

For oil/water flow, we fitted $m'(\theta)$ using the following expression[5]:

$$m'(\theta) = n_1' \cos\theta + n_2' \sin 2\theta + n_3' \sin 3\theta, \qquad (15)$$

where n_1', n_2', and n_3' are adjustable parameters. The X_P' vector of unknowns for this case contained seven parameters: $X_P' = [A' \; B_1' \; B_2' \; n' \; n_1' \; n_2' \; n_3']^T$.

The profile parameter is defined by A and B for liquid/gas flow and by A', B_1', and B_2' for oil/water flow. In the liquid/gas drift velocity correlation, a_1 and a_2 define the transitions between the different flow regimes. The parameter n' in Eq. 10 directly determines V_d' and is also influenced by the size of oil droplets[20] and thus the viscosity of the liquid mixture.[21,22]

We determined the optimized parameters by minimizing the difference between the experimental volume fraction data and the model predictions. For water/gas flows, the objective function is

$$E_{\alpha_g} = \sum_{i=1}^{N} w_i |\alpha_{gi} - \alpha_{gi}^*|^2, \qquad (16)$$

where N is the total number of experimental data points, w_i is the weight assigned to each point, and α_{gi} represent the experimental gas volume fraction and α_{gi}^* the calculated volume fraction. The weights allow us to properly account for repeated experiments (i.e., w_i = 0.5 for each of two repeated tests). In Eq. 16, α_{gi}^* is calculated using the drift-flux model as:

$$\alpha_{gi}^* = \frac{V_{sg_i}}{C_o(\alpha_{gi}^*, X_p) V_{m_i} + V_d(\alpha_{gi}^*, X_p)}. \qquad (17)$$

Because α_{gi}^* appears implicitly in Eq. 17, we apply a successive substitution technique.

The minimization of E_{α_g} in Eq. 16 represents a nonlinear least-square problem. We used the trust region reflective Newton algorithm provided in the MATLAB[23] optimization toolbox for this minimization. This algorithm allows bounds to be set for the parameters in X_p. We specified these bounds based on physical limitations.

In applying the optimization procedure, we used multiple starting points (on the order of several thousand different initial guesses for the parameters) to ensure a more complete coverage of the parameter space. The optimization results presented in Ref. 5 (and summarized below) represent the best parameter values (in terms of minimizing the objective function) achieved over all of the optimization runs. Corresponding to the least-square minimization in Eq. 16, we quantify the error as the root mean square average of the absolute error at each point.

The optimum parameter values for the water/gas system were found to be[5]

$A = 1.0$, $a_1 = 0.06$, $a_2 = 0.21$,

and $m(\theta) = 1.85(\cos\theta)^{0.21}\,(1+\sin\theta)^{0.95}$. . . . . (18)

Note that when $A = 1.0$, B is not needed in the model because $C_0 = 1.0$ over the entire range. The high degree of accuracy of the model with these parameters is evident from the comparison between experimental and predicted α_g, shown in Fig. 13 in Ref. 5. We also investigated the sensitivity of the overall fit to the value of A by specifying $A = 1.2$ and $B = 0.60$. A close fit of the data was achieved in this case, though the error was slightly higher than in the case where $A = 1.0$ (see Ref. 5 for details).

The results for the oil/water system are[5]

$A = 1.0$, $n' = 1.0$,

and $m'(\theta) = 1.07\cos\theta + 3.23\sin 2\theta - 2.32\sin 3\theta$. . . . . (19)

Note that this formula is valid up to 88°, and it does not go to zero as θ approaches 90° as was the case with Eq. 13. Here $C_0' = 1.0$, which is consistent with the results of Hill.[24] See Fig. 16 in Ref. 5 for an illustration of the accuracy of the model using these parameters. In contrast to $m(\theta)$ for water/gas flow, where the maximum value is at 50°, here the maximum in $m'(\theta)$ is at 70°. This suggests that different mechanisms determine drift velocities in oil/water systems from those in water/gas systems.

Three-Phase Parameter Determination

To model three-phase flow, we first apply a two-stage approach based purely on the two-phase flow models. As discussed in the Introduction, we treat the system first as a gas/liquid flow to determine α_g and then model the liquid as an oil/water system to determine α_o and α_w. We use the optimized parameters presented previously for these calculations.

This relatively simple approach is well suited for situations in which oil, water, and gas flow as three distinct layers (i.e., for stratified flow). It is also capable of modeling the separation of a three-phase mixture into pure phases of gas, oil, and water through transient countercurrent flow. However, it is known that in many steady-state flows,[6] the effect of the third phase cannot be ignored. We now assess the accuracy of this treatment for our three-phase flow data.

Application of Water/Gas Results to Three-Phase Flow. We now consider the determination of α_g in three-phase systems. For this calculation, we view the system as a gas/liquid flow and apply our optimized two-phase water/gas data. In these optimizations, we use the experimental values of α_{ol} and α_{wl}, where α_{ol} is the in-situ volume fraction of oil in the liquid phase [i.e., $\alpha_{ol} = \alpha_o/(\alpha_o + \alpha_w)$ and similarly for α_{wl}($\alpha_{wl} = 1 - \alpha_{ol}$)]. These values are used to compute average liquid phase properties. Following the three-phase optimization results, we will apply the overall procedure using only the phase flow rates (Q_w,Q_o,Q_g).

The results for α_g for inclinations over the range $0 \leq \theta \leq 88°$, using the original parameters,[2] are shown in **Fig. 3.** The average absolute error value is shown in this and subsequent figures. In addition, ±10% and ±20% relative error (dashed) lines are shown in the figures. Note that we only include data for $\alpha_g > 0.06$. We see a clear tendency toward overprediction over the entire range of experimental data ($\alpha_g < 0.35$). Most of the points are outside the 20% relative error line.

We next apply our optimized gas/water parameters ($A = 1.0$, $a_1 = 0.06$, $a_2 = 0.21$, Eq. 18) to compute α_g for three-phase flow. The results, shown in **Fig. 4,** are quite reasonable, with the average error reduced to 0.0361 (from 0.0822 using the original parameters). Several points, however, continue to fall outside the ±20% range.

We obtained similar results for α_g, but with slightly higher error (0.0385), using the two-phase model with $A = 1.2$, $B = 0.6$, $a_1 = 0.06$, $a_2 = 0.12$, $m(\theta) = 1.27(\cos\theta)^{0.24}\,(1+\sin\theta)^{1.08}$.

Similar levels of error were achieved with other sets of parameters (i.e., with $A = 1.1$). This indicates that our measurements of α_g can be matched to a reasonable degree of accuracy both with and without profile slip.

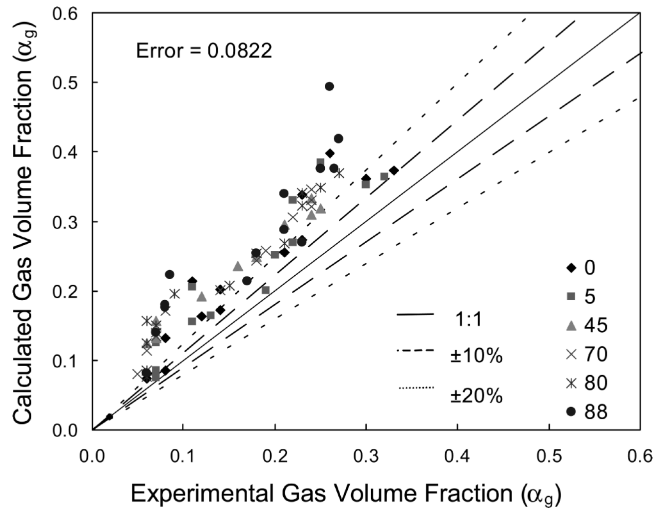

Fig. 3—Predicted gas volume fraction using original parameters[2] (A=1.2, B=0.30, a_1=0.20, a_2=0.40, Eq. 13).

From Fig. 4, we conclude that the optimized water/gas flow parameters can be applied to estimate α_g in three-phase flow. The prediction from a direct optimization of the experimental α_g for the three-phase system would provide improved results, but the results from this simplified procedure are satisfactory. An advantage of this approach is that the three-phase flow model reduces to the two-phase model when one of the liquid phases disappears.

Application of Oil/Water Results to Three-Phase Flow. We now use the two-stage procedure to determine the oil and water fractions in the combined "liquid" phase. Here we use $A' = 1.0$, $n' = 1.0$, and Eq. 19 to estimate the oil volume fraction in the liquid phase (α_{ol}).

The predictions for α_{ol} using the original parameters[2] are shown in **Fig. 5.** Significant scatter is evident (error of 0.149), with some points overpredicted and others underpredicted, for all six deviations. Applying the optimized oil/water parameters to obtain α_{ol} for three-phase flow gives the results shown in **Fig. 6.** The error here is still very large (0.141), and most of the points are underpredicted. From these results, we conclude that the optimized two-phase oil/water parameters are not suitable for the prediction of α_{ol}.

The underprediction evident in Fig. 6 indicates that V_d' for three-phase flow is much lower than that for two-phase oil/water

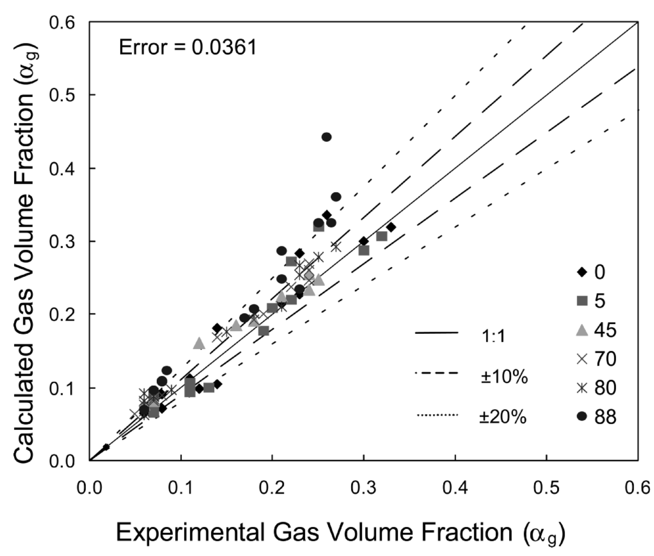

Fig. 4—Predicted gas volume fraction using two-phase optimized parameters (A=1.0, a_1=0.06, a_2=0.21, Eq. 18).

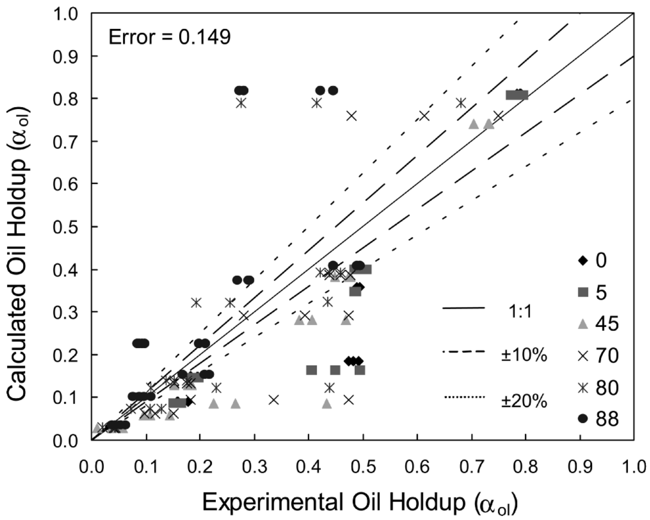

Fig. 5—Predicted oil holdup in liquid using original parameters[2] (*A*′=1.2, *B*$_1$′=0.40, *B*$_2$′=0.70, *n*′=2.0, Eq. 13).

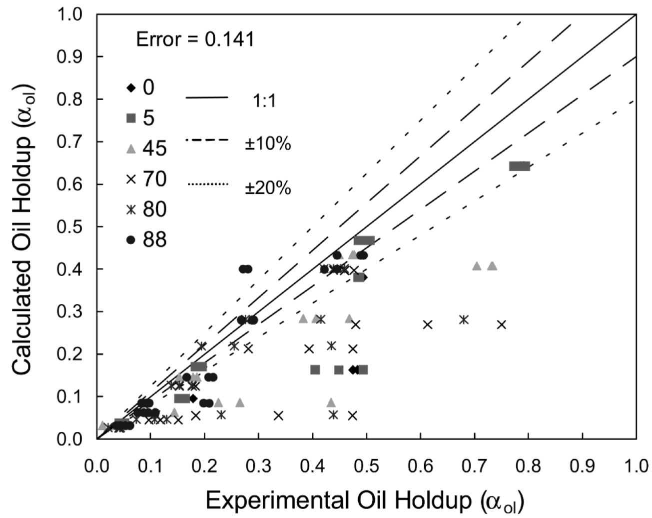

Fig. 6—Predicted oil holdup in liquid using two-phase optimized parameters (*A*′=1.0, *n*′=1.0, Eq. 19).

flow, except when the pipe is near horizontal (88°). It is worth reiterating that only V_d' affects these results, because there is no profile slip between oil and water ($C_0'=1.0$). The results suggest that additional effects arise in three-phase flows that must be specifically modeled.

The substantial reduction in slip between oil and water in three-phase flow is caused by the effect of the gas. To illustrate the effect of gas on oil/water slip in three-phase flows, we now compare the no-slip α_{ol} [i.e., input oil volume fraction in the liquid phase, c_{ol}, given by $Q_o/(Q_o+Q_w)$] with the experimentally determined α_{ol} at different deviations. **Fig. 7** shows these results for a vertical pipe. Here, the experimental α_{ol} are in close agreement with the input volume fraction c_{ol}, indicating that there is essentially no slip between oil and water in these experiments. Thus, we can treat this particular three-phase flow system as a two-phase liquid/gas flow with the oil in situ fraction given by

$$\alpha_o = (1-\alpha_g)\frac{Q_o}{Q_o+Q_w}.$$

This lack of slip between oil and water in three-phase flow is not, however, universal. To demonstrate this, we consider three-phase flow in a nearly horizontal pipe ($\theta=80°$). In the results for this case, shown in **Fig. 8,** we see significant slip between oil and water for most of the points, indicating that there is much less gas effect than was evident in Fig. 7. It is also interesting to see that the points align in five horizontal groups. Each group corresponds to a set of experimental tests with particular oil and water flow rates (i.e., the same input fractions). As gas flow rate increases, the experimental α_{ol} approach c_{ol}. This indicates that gas disrupts the slip between oil and water when there is sufficient gas in the system.

It is apparent from Figs. 7 and 8 that, in some three-phase systems, there is no slip between oil and water, though in other cases there is slip between oil and water. The effect of gas on oil/water flow is complicated, though it largely depends on the pipe deviation. We will now extend our drift-flux model to capture this important effect.

Oil/Water Model in Three-Phase Flow. Our goal here is to develop a unified model to predict oil and water holdup in three-phase flow systems. By unified model, we mean one that reduces to the two-phase oil/water model when there is no gas present in the system.

In order to develop this unified model, we first directly determine optimized parameters for oil and water within three-phase flow. In this optimization, we fix A' and n' to their optimum values for two-phase oil/water flow and optimize only the V_d' multiplier $m'(\theta)$. The optimization procedure is the same as that described above for the two-phase case. The resulting three-phase $m'(\theta)$ is shown in **Fig. 9** (square points). These results deviate considerably from the two-phase $m'(\theta)$ curve (Eq. 19), shown in the figure as the solid curve. It is apparent that $m'(\theta)$ for three-phase flow is

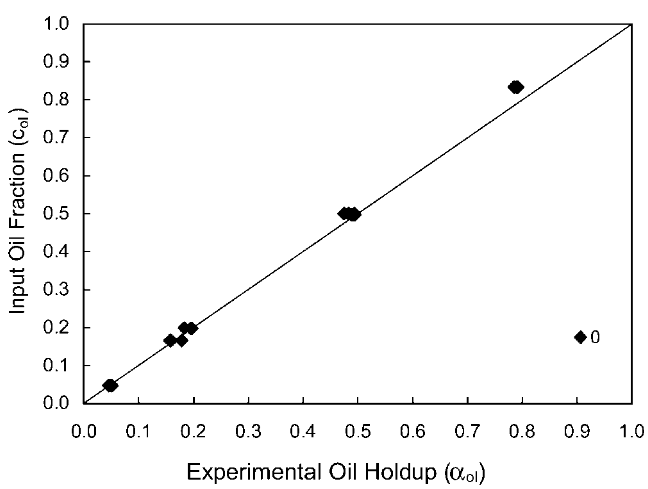

Fig. 7—Oil holdup in liquid for oil/water/gas system for θ=0°.

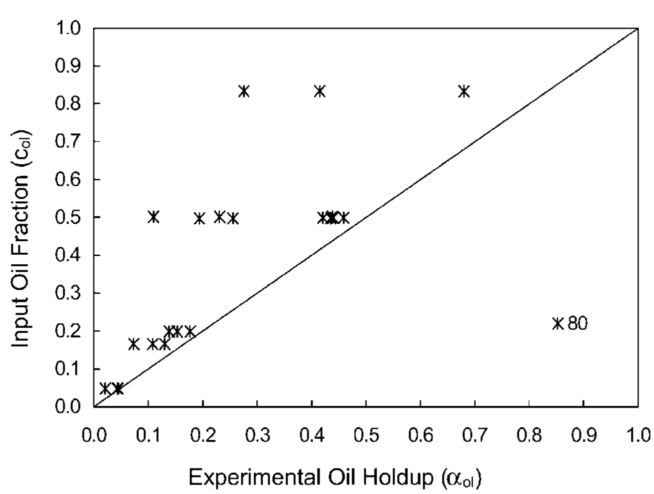

Fig. 8—Oil holdup in liquid for oil/water/gas system for θ=80° (5≤Q_g≤50 m^3/h).

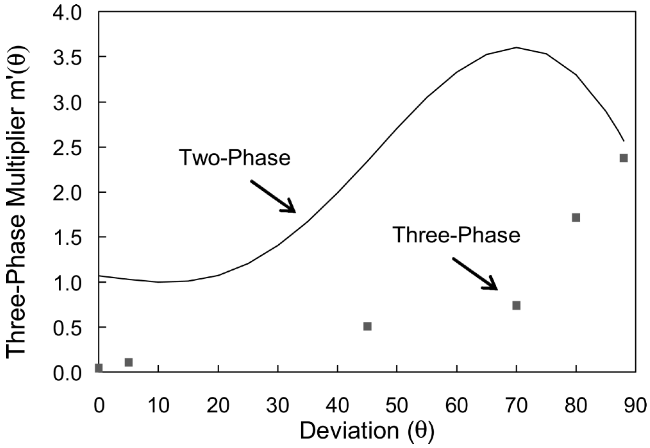

Fig. 9—Deviation effect for oil/water in three-phase system.

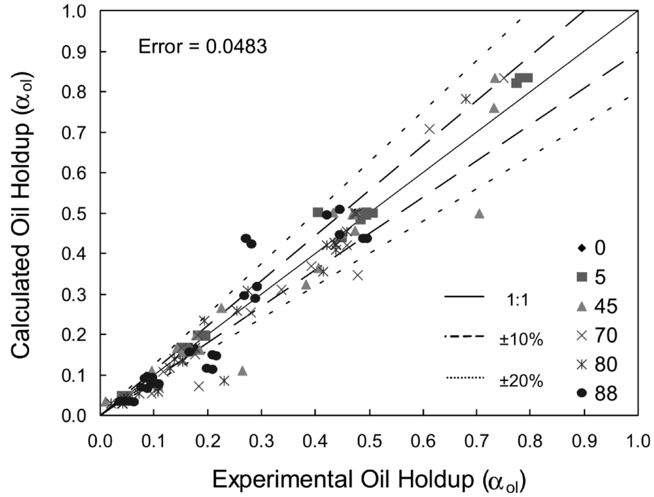

Fig. 10—Predicted oil holdup in liquid using three-phase model.

close to zero for vertical flow and is very small for $\theta = 5°$, indicating that there is no slip between oil and water at these two deviations. We observe that $m'(\theta)$ then increases with θ, with a sharper increase for $\theta > 70°$.

In the experiments, we observed that, for deviations of $0° \leq \theta \leq 70°$, gas is clearly entrained into the liquid. Because of this agitation of the liquid by the gas, oil and water are mixed to form a dispersion/emulsion.[6] As the pipe deviation approaches horizontal, by contrast, we observed that there was much less gas entrained in the liquid. In fact, most of the flow patterns observed for deviations in the range of $80° \leq \theta < 90°$ were elongated bubble flows with gas segregated from the liquid.[6] Therefore, the relatively high $m'(\theta)$ values for deviations near horizontal are caused by gas segregation.

Based on the results in Fig. 9, we propose the following unified model:

$$m^{ow}_{3\text{-phase}} = m^{ow}_{2\text{-phase}} m_g, \qquad (20)$$

where m_g captures the gas effect. Because the two-phase function $m^{ow}_{2\text{-phase}}$ depends only on θ, m_g must depend on both θ and α_g to provide the required functionality in $m^{ow}_{3\text{-phase}}$. We prescribe the following simple functional form for m_g:

$$m_g(\theta,\alpha_g) = 1 - \frac{\alpha_g}{a_3(\theta)} \quad \text{when} \quad \alpha_g < a_3$$

$$= 0 \quad \text{when} \quad \alpha_g \geq a_3 \qquad (21)$$

Note that we have introduced a new parameter, $a_3(\theta)$. When the gas volume fraction decreases to zero, the model reverts smoothly to the two-phase oil/water model. However, when the gas volume fraction exceeds a_3, the oil/water slip vanishes. The model behaves linearly between these two limits. We note that a_3 should technically also be a function of velocity, as the results in Fig. 8 suggest. However, we do not have sufficient data at high α_g to model this effect. Were we to include the velocity effect, a_3 would be smaller for near-horizontal flows when gas is entrained in liquid. This would, however, introduce only a small correction, because V'_d is less important at higher flow velocities.

The equation for V'_d for oil and water in a three-phase system is now given by

$$V'_d = 1.53 V'_c (1 - \alpha_{ol}) m^{ow}_{2\text{-phase}}(\theta) m_g(\theta,\alpha_g). \qquad (22)$$

This equation reduces to the result for two-phase oil/water flow when $\alpha_g \to 0$, indicating that the two- and three-phase flow models are unified.

Fig. 10 shows the results for α_{ol} obtained using this new model. The error here is about one-third of that obtained with the original parameters or with the optimized two-phase parameters. **Fig. 11** shows the gas effect in terms of the variation of a_3 with deviation. We see that, from vertical up to 70° deviation, only a small amount of gas ($\alpha_g \approx 0.1$) is required to eliminate the slip between oil and water. But for near-horizontal flow, gas has a much smaller effect on the oil/water slip. As discussed above, this results from the gas being segregated from the liquid at near-horizontal deviations in our experiments. The points in Fig. 11 can be approximated by the following equation (this is the curve shown in the figure):

$$\alpha_3(\theta) = 0.017\exp(\theta^{3.28}), \qquad (23)$$

where θ is in radians. Note that this equation is valid from vertical up to 88° deviation, where $a_3(\theta) \leq 1$.

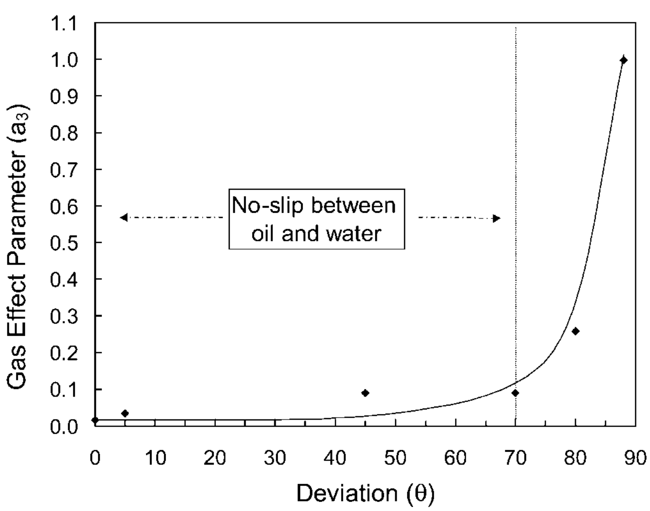

Fig. 11—Gas effect on oil/water in three-phase system.

Holdup Prediction From Volumetric Flow Rates. In the determination of α_g in the optimized model developed previously, we require knowledge of α_{ol} and α_{wl} (in order to compute average liquid-phase properties). However, the determination of α_{ol} and α_{wl} requires us to know α_g (to compute the effect of gas on oil/water slip). Thus, given only the phase flow rates (Q_g, Q_o, Q_w), and not α_{ol} and α_{wl}, the calculation of holdups must be accomplished iteratively. Our procedure is as follows: We first compute α_g using liquid phase properties approximated under the assumption of no slip between the oil and water phases [i.e., $\alpha_{ol} = c_{ol} = Q_o/(Q_o+Q_w)$]. Then, using this α_g, we compute α_{ol} and α_{wl} using our new three-phase model. We then recompute α_g using the updated values for α_{ol} and α_{wl}. This procedure is continued until the holdups no longer change. In cases with no slip between oil and water, only one iteration is required; in cases with oil/water slip, two or three iterations are typically required.

The accuracy of the predicted holdups is fairly close to that achieved in the optimized results shown in Figs. 4 and 10. Specifically, the average error in α_g is 0.0379 (compared to 0.0361 in Fig. 4), and the average error in α_{ol} is 0.0643 (compared to 0.0483 in Fig. 10). The error in α_{ol} increases somewhat because of the overprediction of α_g for near-horizontal flows (evident in Fig. 4). This in turn causes some of the oil/water flows to be treated with no slip when there is in fact some slip between the oil and water. The accuracy of the predicted α_{ol} could be improved if we enhanced the model for α_g directly for three-phase systems. This does not appear to be necessary, however, because the level of accuracy of the predicted holdups is quite acceptable.

Conclusions and Recommendations

In this work, a set of recently collected 15-cm-diameter, three-phase flow data were used to determine drift-flux modeling parameters. Data at six inclinations from vertical to 2° above horizontal were used for the parameter determination. The following conclusions can be drawn from this study:

1. Model parameters are best determined using a unified approach that includes the full set of data (for all inclinations) in a single optimization. Optimized parameters computed in this way were presented for water/gas and oil/water systems.
2. The optimized parameters from two-phase water/gas systems can be applied to estimate in-situ gas volume fraction in three-phase flow.
3. A small amount of gas ($\alpha_g \approx 0.1$) in three-phase flow acts to eliminate the slip between oil and water for deviations up to 70° from vertical.
4. For near-horizontal flows, gas ($\alpha_g \leq 0.35$) affects the oil/water slip much less because the gas is separated from the oil/water mixture.
5. A new gas effect parameter (a_3) was introduced to capture the reduction of oil/water slip as a function of the in-situ gas volume fraction and the deviation. This enabled the development of a unified model for both two-phase and three-phase flows. The results using this model were significantly more accurate than those from the original model.

This work suggests that the proper treatment of three-phase flow systems depends largely on the pipe inclination and gas volume fraction. For conventional (vertical or near-vertical) wells, with gas volume fraction greater than 0.1, three-phase flow can be modeled as a two-phase liquid/gas flow with a homogeneous liquid phase in which there is no slip between the oil and water. For horizontal or near-horizontal wells, by contrast, the slip between oil and water can be significant. Our three-phase model can be applied in these cases to calculate the oil and water holdups.

Nomenclature

a_1 = drift velocity ramping parameter
a_2 = drift velocity ramping parameter
a_3 = gas effect parameter
A = profile parameter term, value in bubble/slug regimes for liquid/gas flows
A' = profile parameter term for oil/water flows
B = profile parameter term, gas volume fraction at which C_0 begins to reduce
B_1 = profile parameter term, oil volume fraction at which C_0' begins to reduce
B_2 = profile parameter term, oil volume fraction at which C_0' falls to 1.0
c = input volume fraction
C_o = profile parameter
$\hat{D}$ = pipe internal diameter
F_v = velocity sensitivity parameter for liquid/gas flows
g = gravitational acceleration
K_u = Kutateladze number
L = test section length
m = drift velocity multiplier for water/gas flows
m' = drift velocity multiplier for oil/water flows
m_0 = drift velocity multiplier for vertical water/gas flows
n_1 = deviation effect parameter for water/gas flows
n_2 = deviation effect parameter for water/gas flows
n' = drift velocity exponent for oil/water flows
n_1' = deviation effect parameter for oil/water flows
n_2' = deviation effect parameter for oil/water flows
n_3' = deviation effect parameter for oil/water flows
N = number of experimental points
Q = volumetric flow rate
V = velocity
V_c = characteristic velocity
V_d = gas/liquid drift velocity
V_d' = oil/water drift velocity
V_m = mixture velocity
V_s = superficial velocity
X_P = vector of parameters optimized
α = in-situ fraction or holdup
β = profile parameter reduction term
γ = profile parameter reduction term
θ = deviation from vertical
ρ = density
σ = interfacial tension/surface tension

Subscripts

g = gas
l = liquid
m = mixture
o = oil
ol = oil in liquid phase
w = water
wl = water in liquid phase

Acknowledgments

The support from Schlumberger and the other industrial affiliates of the Stanford Project on the Productivity and Injectivity of Advanced Wells (SUPRI-HW) is greatly appreciated.

References

1. Holmes, J.A., Barkve, T., and Lund, O.: "Application of a Multisegment Well Model To Simulate Flow in Advanced Wells," paper SPE 50646 presented at the 1998 SPE European Petroleum Conference, The Hague, 20–22 October.
2. Schlumberger, *ECLIPSE Technical Description Manual* (2001).
3. Jepson, W.P. and Taylor, R.E.: "Slug Flow and Its Transitions in Large-Diameter Horizontal Pipes," *Int. J. Multiphase Flow* (1993) **19,** 411.
4. Abduvayt, P., Manabe, R., and Arihara, N.: "Effects of Pressure and Pipe Diameter on Gas-Liquid Two-Phase Flow Behavior in Pipelines," paper SPE 84229 presented at the 2003 SPE Annual Technical Conference and Exhibition, Denver, 5–8 October.
5. Shi, H., Holmes, J.A., Durlofsky, L.J., Aziz, K., Diaz, L.R., Alkaya, B., and Oddie, G.: "Drift-Flux Modeling of Two-Phase Flow in Wellbores," *SPEJ* (March 2005) 24.
6. Oddie, G., Shi, H., Durlofsky, L.J., Aziz, K., Pfeffer, B., and Holmes, J.A.: "Experimental Study of Two and Three Phase Flows in Large Diameter Inclined Pipes," *Int. J. Multiphase Flow* (2003) **29,** 527.
7. Açikgöz, M., Franca, L., and Lahey, R.T.: "An Experimental Study of Three-Phase Flow Regimes," *Int. J. Multiphase Flow* (1992) **18,** 327.
8. Lahey, R.T., Açikgöz, M., and Franca, F.: "Global Volumetric Phase Fractions in Horizontal Three-Phase Flow," *AIChE J.* (1992) **38,** 1049.
9. Lee, A.H.: "A Study of Flow Regime Transitions for Oil/water/gas Mixture in Large Diameter Horizontal Pipelines," MS thesis, Ohio U., Athens, Ohio (1993).
10. Taitel, Y., Barnea, D., and Brill, J.P.: "Stratified Three Phase Flow in Pipes," *Int. J. Multiphase Flow* (1995) **21,** 53.

11. Wilkens, R.J.: "Prediction of the Flow Regime Transitions in High Pressure, Large Diameter Inclined Multiphase Pipelines," PhD dissertation, Ohio U., Athens, Ohio (1997).
12. Wu, H., Zhou, F., and Wu, Y.: "Intelligent Identification System of Flow Regime of Oil-Gas-Water Multiphase Flow," *Int. J. Multiphase Flow* (2001) **27,** 459.
13. Donnelly, G.F., Spedding, P.L., and McBride, W.J.: "Prediction of Two- and Three-Phase Flow in the Horizontal Configuration," paper OTC 7743 presented at the 1995 Offshore Technical Conference, Houston, 1–4 May.
14. Fairhurst, C.P. and Barrett, N.: "Oil/Water/Gas Transport in Undulating Pipelines—Field Observations, Experimental Data, and Hydraulic Model Comparisons," paper SPE 38811 presented at the 1997 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 5–8 October.
15. Ozon, P.M., Ferschneider, G., and Chwetzoff, A.: "A New Multiphase Flow Model Predicts Pressure and Temperature Profiles in Wells," paper SPE 16535 presented at the 1987 SPE Offshore Europe Conference, Aberdeen, 8–11 September.
16. Danielson, T.J., Brown, L.D., and Bansal, K.M.: "Flow Management: Steady-State and Transient Multiphase Pipeline Simulation," paper OTC 11965 presented at the 2000 Offshore Technology Conference, Houston, 1–4 May.
17. Zuber, N. and Findlay, J.A.: "Average Volumetric Concentration in Two-Phase Flow Systems," *J. Heat Transfer Trans. ASME* (1965) **87,** 453.
18. Harmathy, T.Z.: "Velocity of Large Drops and Bubbles in Media of Restricted Extent," *AIChE J.* (1960) **6,** 281.
19. Hasan, A.R. and Kabir, C.S.: "A Simplified Model for Oil/Water Flow in Vertical and Deviated Wellbores," *SPEPF* (February 1999) 56.
20. Nicolas, Y. and Witterholt, E.J.: "Measurements of Multiphase Fluid Flow," paper SPE 4023 presented at the 1972 SPE Annual Meeting of AIME, San Antonio, Texas, 8–11 October.
21. Pal, R.: "Effect of Droplet Size on the Rheology of Emulsions," *AIChE J.* (1996) **42,** 3181.
22. Shi, H.: "A Study of Oil/water Flows in Large Diameter Horizontal Pipelines," PhD dissertation, Ohio U., Athens, Ohio (2001).
23. The MathWorks Inc., *MATLAB* (2002).
24. Hill, A.D.: "A Comparison of Oil/Water Slip Velocity Models Used for Production Log Interpretation," *J. Pet. Sci. & Eng.* (1992) **8,** 181.

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