Unified Model for Gas-Liquid Pipe Flow via Slug Dynamics—Part 1: Model Development

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A unified hydrodynamic model is developed for predictions of flow pattern transitions, pressure gradient, liquid holdup and slug characteristics in gas-liquid pipe flow at all inclination angles from −90° to 90° from horizontal. The model is based on the dynamics of slug flow, which shares transition boundaries with all the other flow patterns. By use of the entire film zone as the control volume, the momentum exchange between the slug body and the film zone is introduced into the momentum equations for slug flow. The equations of slug flow are used not only to calculate the slug characteristics, but also to predict transitions from slug flow to other flow patterns. Significant effort has been made to eliminate discontinuities among the closure relationships through careful selection and generalization. The flow pattern classification is also simplified according to the hydrodynamic characteristics of two-phase flow. [DOI: 10.1115/1.1615246]

Introduction

During oil and gas production, fluids are transported upwards from vertical or deviated wells, through hilly-terrain pipelines to downstream processing facilities. Steam, water and gas injection are often used to boost production rate. Therefore, gas-liquid two-phase flows at all inclination angles from vertical downward to vertical upward are frequently encountered in the oil and gas industry. A unified model is required which can accurately predict two-phase flow behavior at all inclination angles.

Early predictive methods for gas-liquid two-phase flow were based on empirical correlations. Commonly used methods include the Dukler et al. [1] and Beggs and Brill [2] correlations for flow in pipelines, and the Hagedorn and Brown [3] and Duns and Ros [4] correlations for flow in wellbores. These approaches can predict pressure gradient (and sometimes liquid holdup) with a typical performance of ±30%. The generalization and accuracy improvement of these empirical correlations are limited due to simplifications of the complex physical mechanisms involved in two-phase flow.

Mechanistic models for gas-liquid pipe flows have been developed since the mid 1970's. The two-phase flow is formulated specifically for different flow patterns such as stratified, slug, annular and bubble flows in conjunction with a flow pattern prediction model. These models are expected to be more general and accurate than empirical correlations since they have incorporated the operating mechanisms and important parameters of two-phase flow. For flow pattern transitions, Barnea [5] proposed a unified model for the whole range of inclination angles. This model is a combination of different models developed for different flow conditions. Recently, several comprehensive mechanistic models were developed for prediction of the hydrodynamic behavior of two-phase flow, e.g., Xiao [6] for near-horizontal pipelines, Kaya [7] for vertical and deviated wells, and Gomez et al. [8] for inclination angles from horizontal to vertical upward.

There are still some areas not covered in previous mechanistic models, such as a hydrodynamic model for downward flow at large inclination angles. Also, improvements in mechanistic modeling are needed, including unification of different methods, correlations and models.

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A major problem exists due to the separation between flow pattern transition models and models for hydrodynamic behavior. In a comprehensive mechanistic model, the flow pattern is first predicted using simple correlations or models. Then, the multiphase flow behavior is calculated using flow pattern specific momentum equations and required closure relationships. Prediction of flow pattern transitions may not be accurate due to the significant simplifications made in modeling the complex transition mechanisms.

Basically, flow pattern transitions are hydrodynamic phenomena of gas-liquid two-phase pipe flow. They occur under certain flow conditions as the two-phase flow rates change, and the momentum exchanges between gas, liquid and their boundaries require a different phase distribution. Therefore, flow pattern transitions can (and should) be predicted by solving the momentum equations for gas-liquid two-phase flow.

Another difficulty is the unification of different models or closure relationships required for a unified model. These are the equations or correlations developed for different flow patterns, inclination angles, flow rates and/or other parameters. Discontinuities may exist when closure relationships meet at the boundaries of their application ranges. Therefore, careful selection and generalization must be considered for the models or closure relationships in unified modeling.

In previous studies, it appears that slug flow has not been adequately investigated, partially because of its complexity. Slug flow can be related to all other flow patterns. Every flow pattern is represented in slug flow, e.g., the film zone of slug flow resembles stratified or annular flow, and the slug body resembles bubbly or dispersed bubble flow. Also, slug flow is always found in the central part of a flow pattern map, surrounded by the other flow patterns. Therefore, significant progress can be made if the gas-liquid pipe flow is investigated based on slug dynamics as the starting point.

Zhang et al. [9] constructed the momentum and continuity equations for slug flow by considering the entire liquid film region as the control volume. As a result, the momentum exchange between the slug body and the film zone is introduced into the combined momentum equation. The existence condition for slug flow is embodied in these equations, and therefore the transition from slug flow to other flow patterns can be predicted. Based on this concept, the transition from slug to stratified flow was calculated and satisfactory results were obtained. In the present study, this scenario is extended for prediction of flow pattern transitions at different inclination angles.

Hydrodynamic Model

Equations for Slug Flow

Continuity Equations. As shown in Fig. 1, the entire liquid film zone (including liquid film and gas pocket) of a slug unit is used as the control volume. Continuity equations are derived relative to a coordinate system moving with the translational velocity, v_T. The input liquid mass flow rate at the left boundary and the output liquid mass flow rate at the right boundary of the film zone are

$$\rho_L H_{LF} A(v_F - v_T) + \rho_L H_{LC} A(v_C - v_T)$$

and

$$\rho_L H_{LS} A(v_S - v_T)$$

where H_{LF} and H_{LC} are the liquid holdups in the film and gas core, respectively. It is assumed that there is no slippage between the gas and liquid droplets in the gas core. v_S is the slug or mixture velocity $(v_{SL} + v_{SG})$.

For fully developed slug flow, the input mass flow rate should equal the output mass flow rate, and a continuity equation for the liquid phase in the film zone can be obtained as

$$H_{LS}(v_T - v_S) = H_{LF}(v_T - v_F) + H_{LC}(v_T - v_C) \tag{1}$$

For the gas phase, the input mass flow rate at the left boundary and the output mass flow rate at the right boundary of the film zone are

$$\rho_G(1 - H_{LF} - H_{LC}) A(v_C - v_T)$$

and

$$\rho_G(1 - H_{LS}) A(v_S - v_T)$$

Then, the continuity equation for the gas phase is

$$(1 - H_{LS})(v_T - v_S) = (1 - H_{LF} - H_{LC})(v_T - v_C) \tag{2}$$

The sum of Eqs. (1) and (2) gives

$$v_S = H_{LF} v_F + (1 - H_{LF}) v_C \tag{3}$$

Considering the passage of a slug unit at an observation point, the following relationships hold for the liquid and gas phases, respectively,

$$l_U v_{SL} = l_S H_{LS} v_S + l_F(H_{LF} v_F + H_{LC} v_C) \tag{4}$$

$$l_U v_{SG} = l_S(1 - H_{LS}) v_S + l_F(1 - H_{LF} - H_{LC}) v_C \tag{5}$$

The slug unit length given by

$$l_U = l_S + l_F \tag{6}$$

is not independent of the continuity equations, Eqs. (3, 4, 5). Therefore, only three of them can be used at the same time.

The liquid entrainment fraction in the gas core is defined as

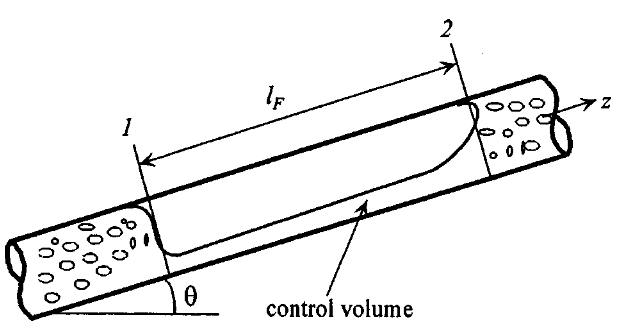

Fig. 1 Control Volume (entire film zone, l_F) Used in Modeling

$$F_E = \frac{H_{LC} v_C}{H_{LF} v_F + H_{LC} v_C} \tag{7}$$

Momentum Equations. Unlike the control volume for the continuity equations, the liquid film and the gas pocket in the film zone of a slug unit will be used as the control volumes separately to derive the momentum equations. For fully developed slug flow, the mass input and output rates at the left and right boundaries of the liquid film are both equal to

$$\rho_L H_{LF} A(v_F - v_T)$$

Therefore, in the conventional z direction shown in Fig. 1, the momentum exchange per unit time between the slug body and liquid film is

$$\rho_L H_{LF} A(v_T - v_F)(v_S - v_F)$$

In Zhang et al. [9], the forces acting on the left and right boundaries of the liquid film due to the change of static pressure were considered in the derivation of the momentum equations of slug flow. However, it was found that this term is negligible.

The frictional force acting on the film at the wall (in the opposite direction of z) is

$$-\tau_F S_F l_F$$

The frictional force acting on the film at the interface (in the same direction as z) is

$$\tau_I S_I l_F$$

The gravitational force is

$$-\rho_L H_{LF} A l_F g \sin\theta$$

All the above forces should be in balance for fully developed slug flow. Therefore, the momentum equation for the liquid film can be obtained,

$$\frac{(p_2 - p_1)}{l_F} = \frac{\rho_L(v_T - v_F)(v_S - v_F)}{l_F} + \frac{\tau_I S_I - \tau_F S_F}{H_{LF} A} - \rho_L g \sin\theta \tag{8}$$

Similarly, the momentum equation for the gas pocket can be written as

$$\frac{(p_2 - p_1)}{l_F} = \frac{\rho_C(v_T - v_C)(v_S - v_C)}{l_F} - \frac{\tau_I S_I + \tau_C S_C}{(1 - H_{LF}) A} - \rho_C g \sin\theta \tag{9}$$

The representative density of the gas core, ρ_C, is defined as

$$\rho_C = \frac{\rho_G(1 - H_{LF} - H_{LC}) + \rho_L H_{LC}}{1 - H_{LF}} \tag{10}$$

Equating the pressure drop in Eqs. (8) and (9) gives the combined momentum equation,

$$\begin{aligned} &\frac{\rho_L(v_T - v_F)(v_S - v_F) - \rho_C(v_T - v_C)(v_S - v_C)}{l_F} - \frac{\tau_F S_F}{H_{LF} A} \\ &+ \frac{\tau_C S_C}{(1 - H_{LF}) A} + \tau_I S_I\left(\frac{1}{H_{LF} A} + \frac{1}{(1 - H_{LF}) A}\right) \\ &- (\rho_L - \rho_C) g \sin\theta = 0 \end{aligned} \tag{11}$$

The first term on the *LHS* of Eq. (11) is due to the momentum exchange between the slug body and the film zone. The characteristics of the liquid film are governed by the above continuity and momentum equations.

Equations for Stratified and Annular Flows. The combined momentum equation for stratified flow is the same as the combined momentum equation for slug flow if the momentum exchange terms are removed from Eq. (11),

$$-\frac{\tau_F S_F}{H_{LF}A}+\frac{\tau_C S_C}{(1-H_{LF})A}+\tau_I S_I\left(\frac{1}{H_{LF}A}+\frac{1}{(1-H_{LF})A}\right)$$
$$-(\rho_L-\rho_C)g\sin\theta=0 \tag{12}$$

In annular flow, there is no contact between the gas core and the pipe wall. Eq. (12) then becomes

$$-\frac{\tau_F S_F}{H_{LF}A}+\tau_I S_I\left(\frac{1}{H_{LF}A}+\frac{1}{(1-H_{LF})A}\right)-(\rho_L-\rho_C)g\sin\theta=0 \tag{13}$$

Equation (12) or (13) can be solved along with Eqs. (4) and (5) for stratified and/or annular flows,

$$v_{SL}=H_{LF}v_F+H_{LC}v_C \tag{14}$$

$$v_{SG}=(1-H_{LF}-H_{LC})v_C \tag{15}$$

and Eq. (7).

Bubble Flow. Bubble flow includes dispersed bubble flow and bubbly flow. The liquid holdup and pressure gradient of dispersed bubble flow are calculated assuming that the gas and liquid phases are homogeneously mixed. For bubbly flow, the bubble rise velocity, v_o, relative to the liquid has to be considered. The bubble rise velocity is estimated by the Harmathy [10] correlation,

$$v_o=1.53\left(\frac{g(\rho_L-\rho_G)\sigma}{\rho_L^2}\right)^{1/4} \tag{16}$$

Discontinuities in the calculated liquid holdup and pressure gradient may exist at the boundary between bubbly flow and dispersed bubble flow due to the difference in the bubble rise velocity. A model still needs to be developed that ensures a continuous change of the bubble rise velocity from bubbly flow to dispersed bubble flow based on the turbulent intensity of the liquid phase.

Evaluation of Shear Stresses. The shear stresses in the combined momentum equations are evaluated as

$$\tau_F=f_F\frac{\rho_L v_F^2}{2} \tag{17}$$

$$\tau_C=f_C\frac{\rho_G v_C^2}{2} \tag{18}$$

$$\tau_I=f_I\frac{\rho_C(v_C-v_F)|v_C-v_F|}{2} \tag{19}$$

For a liquid film in laminar flow, the shear stress at the pipe wall can be related to the shear stress at the gas-liquid interface,

$$\tau_F=\frac{3\mu_L v_F}{h_F}-\frac{\tau_I}{2} \tag{20}$$

The average liquid film height h_F is defined as

$$h_F=\frac{2AH_{LF}}{S_F+S_I} \tag{21}$$

The friction factors, f_F and f_C, at the wall in contact with the liquid film or the gas pocket are estimated from

$$f=C\,\mathrm{Re}^{-n} \tag{22}$$

where $C=16$, $n=1$ for laminar flow if the Reynolds number is less than 2000, and $C=0.046$, $n=0.2$ for turbulent flow in smooth pipe if the Reynolds number is larger than 3000. For turbulent flow in a rough pipe, an alternative correlation for friction factor must be selected to take the wall roughness into account. In the Reynolds number range between 2000 and 3000, the friction factor is interpolated to prevent a discontinuity across the transition region.

The Reynolds numbers for the liquid film and the gas core are defined as

$$\mathrm{Re}_F=\frac{4A_F v_F \rho_L}{S_F\mu_L} \tag{23}$$

$$\mathrm{Re}_C=\frac{4A_C v_C \rho_G}{(S_C+S_I)\mu_G} \tag{24}$$

Here, A_F and A_C are the cross sectional areas occupied by the liquid film and the gas pocket, respectively,

$$A_F=H_{LF}A \tag{25}$$

$$A_C=(1-H_{LF})A \tag{26}$$

The estimation of the interfacial friction factor will be discussed in a following section.

Flow Pattern Transitions

Slug to Stratified and/or Annular Flow. When the transition from slug flow to stratified (or annular) flow occurs, the film length l_F becomes infinitely long. The momentum exchange term in the combined momentum equation, Eq. (11), becomes zero. Given the superficial gas velocity, v_{SG}, and making a guess for the superficial liquid velocity, v_{SL}, the liquid holdup of the film can be solved from Eqs. (1, 7, 14, 15),

$$H_{LF}=\frac{(H_{LS}(v_T-v_S)+v_{SL})(v_{SG}+v_{SL}F_E)-v_T v_{SL}F_E}{v_T v_{SG}} \tag{27}$$

Simultaneously, v_F, H_{LC} and v_C can be expressed as,

$$v_F=\frac{v_{SL}(1-F_E)}{H_{LF}} \tag{28}$$

$$H_{LC}=\frac{v_{SL}F_E(1-H_{LF})}{v_S-v_{SL}(1-F_E)} \tag{29}$$

$$v_C=\frac{v_S-v_{SL}(1-F_E)}{1-H_{LF}} \tag{30}$$

These parameters are needed to calculate some of the closure relationships before solving the combined momentum Eq. (11). A new value of v_F can be calculated from Eq. (11), and finally v_{SL} is obtained by

$$v_{SL}=\frac{v_F H_{LF}}{(1-F_E)} \tag{31}$$

Several iterations are required to obtain an accurate value of v_{SL}. The curve of v_{SL} versus v_{SG} in a flow pattern map defines the boundary between slug flow and stratified (or annular) flow.

Slug to Dispersed Bubble Flow. If a film length, l_F, as small as half of the pipe diameter is given, the transition from slug flow to dispersed bubble flow can be predicted using the combined momentum Eq. (11). However, in the present study, this transition boundary is predicted by use of a much simpler model, the unified mechanistic model developed by Zhang et al. [11].

The Zhang et al. [11] model is based on a hypothesis that dispersed bubble flow accommodates the maximum gas holdup at the transition boundary, which is the same as that in the slug body. The slug liquid holdup is predicted from the balance between the turbulent kinetic energy of the liquid phase and the surface free energy of dispersed gas bubbles. This model gives an accurate prediction for gas superficial velocities larger than 0.1 m/s. At lower gas velocities, the transition boundary can be predicted using the Barnea et al. [12] model, which is based on the Hinze [13] correlation at negligible gas void fraction.

Slug to Bubbly Flow. Bubbly flow occurs at low gas void fraction when the Taylor bubble velocity is greater than the rise velocity of small bubbles (Eq. 16) for near-vertical upward flow. The Taitel et al. [14] model will be used to predict this transition boundary. Thus, the transition occurs at a void fraction of 0.25.

Stratified to Annular Flow. The transition from stratified to annular flow cannot be defined at a distinct boundary. With an increase of gas velocity, the liquid film in stratified flow will spread to both sides of the pipe and the interface becomes concave. The wetted wall fraction becomes larger and larger, and finally the flow becomes annular. Therefore, in this study, the transition boundary will be estimated using the Grolman [15] correlation for wetted wall fraction, taking 0.9 as the transition criterion. This approach also assures a continuous transition between stratified and annular flows for the hydrodynamic calculations.

Closure Relationships

The performance of the unified model is dependent on both the basic equations and closure relationships. The closure relationships were selected from the open literature based on their validations and applications. A closure relationship is often developed for a certain range of flow conditions. It may give poor predictions outside the validated range, or a significant discontinuity may exist when it is changed at the boundary of its application range. A closure relationship can easily be replaced when a better one becomes available.

Interfacial Friction Factor. The interfacial friction factor correlations for stratified flow are very different from the correlations for annular flow, although the physical processes for both cases seem to be the same, namely gas flowing in contact with a liquid phase. At a certain superficial liquid velocity, the wetted wall fraction of stratified flow becomes larger with an increase in superficial gas velocity, and the flow will eventually become annular. A large discrepancy exists when the calculation of interfacial friction factor changes from using a stratified flow correlation to using an annular flow correlation. The criterion for this change is a controversial issue.

One solution for this problem is to find a way to make a continuous transition between the correlations for stratified and annular flows. A major difference between stratified and annular flows is the liquid film thickness. In stratified flow, the liquid film is often thick and the flow state is normally turbulent, whereas the film thickness in annular flow is normally very small and the flow state is mostly laminar. Therefore, it seems appropriate to use the critical Reynolds number of the liquid film as the criterion for whether the stratified or annular interfacial friction factor should be applied. This approach was also used for turbulent and laminar wall friction factors.

The following interfacial friction factors were selected from a number of correlations developed by different investigators based on their performances in comparisons with experimental results.

f_I for Stratified Flow. The Andritsos and Hanratty [16] correlation for interfacial friction factor,

$$f_I = f_C\left(1 + 15\left(\frac{\overline{h_F}}{d}\right)^{0.5}\left(\frac{v_{SG}}{v_{SG,t}} - 1\right)\right) \tag{32}$$

was developed for stratified flow assuming a flat interface. $\overline{h_F}$ is the film height and $v_{SG,t}$ is approximated as

$$v_{SG,t} = 5[\mathrm{m/s}]\left(\frac{\rho_{GO}}{\rho_G}\right)^{0.5} \tag{33}$$

where ρ_{GO} is the gas density at atmospheric pressure.

The actual interface can become concave at high gas flow rates or high inclination angles. In order to estimate the interfacial friction factor for a non-flat liquid film, Eq. (32) must be modified. One option is to convert $\overline{h_F}/d$ into holdup of the liquid film. The derivative of the cross sectional area of the liquid film with respect to the film height is

$$\frac{dA_F}{d\overline{h_F}} = \sqrt{1 - \left(1 - \frac{2\overline{h_F}}{d}\right)^2}\, d \tag{34}$$

for a flat film. Assuming the film height is typically $d/4$, corresponding to a uniformly distributed film thickness of about $0.05d$,

$$\left.\frac{dA_F}{d\overline{h_F}}\right|_{\overline{h_F}=d/4} = \sqrt{\frac{3}{4}}\, d \tag{35}$$

Then,

$$\frac{\overline{h_F}}{d} \approx 0.9 H_{LF} \tag{36}$$

Equation (32) becomes

$$f_I = f_C\left(1 + 14.3 H_{LF}^{0.5}\left(\frac{v_{SG}}{v_{SG,t}} - 1\right)\right) \tag{37}$$

f_I for Annular Flow. Asali [17] proposed a correlation for the interfacial friction factor in annular flow. The correlation was also tested by Willetts [18]. Ambrosini et al. [19] improved the Asali formulation as

$$f_I = f_G(1 + 13.8\,\mathrm{We}_G^{0.2}\,\mathrm{Re}_G^{-0.6}(h_F^+ - 200\sqrt{\rho_G/\rho_L})) \tag{38}$$

where the Weber number, We_G, and the Reynolds number, Re_G, are defined as

$$\mathrm{We}_G = \frac{\rho_G v_C^2 d}{\sigma} \tag{39}$$

and

$$\mathrm{Re}_G = \frac{\rho_G v_C d}{\mu_G} \tag{40}$$

h_F^+ is the dimensionless thickness of the liquid film,

$$h_F^+ = \frac{\rho_G h_F v_C^*}{\mu_G} \tag{41}$$

where

$$v_C^* = \sqrt{\tau_I/\rho_G} \tag{42}$$

f_G in Eq. (38) is the smooth pipe friction factor defined as

$$f_G = 0.046\,\mathrm{Re}_G^{-0.2} \tag{43}$$

Wetted Wall Fraction and Interfacial Perimeter. The liquid film will creep up the sides of the pipe at high gas flow rates or high inclination angles. This may significantly increase the interfacial and wetted wall perimeters. The wetted wall fraction can be predicted by use of the Grolman [15] correlation,

$$\Theta = \Theta_0\left(\frac{\sigma_{water}}{\sigma}\right)^{0.15} + \frac{\rho_G}{\rho_L - \rho_G}\frac{1}{\cos\theta}\left(\frac{\rho_L v_{SL}^2 d}{\sigma}\right)^{0.25}\left(\frac{v_{SG}^2}{(1 - H_{LF})^2 g d}\right)^{0.8} \tag{44}$$

where Θ_0 is the minimum wetted wall fraction corresponding to a flat interface.

Chen et al. [20] proposed a "double circle" model to estimate the perimeter of the concave interface. This is an implicit method and requires iteration to obtain an accurate solution. Therefore, this model will increase the computation time and may cause convergence problems. In this study, an explicit expression is proposed to estimate the interfacial perimeter,

$$S_I = \frac{S_F(A_{CD} - A_F) + S_{CD} A_F}{A_{CD}} \tag{45}$$

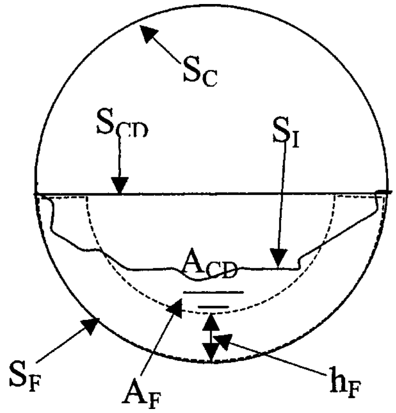

Fig. 2 Cross Section of a Stratified Flow with Curved Interface

where S_F is the wetted wall perimeter, S_{CD} is the chord length corresponding to the wetted wall fraction, and A_{CD} is the cross sectional area embraced by the wetted wall and its chord, as shown in Fig. 2,

$$S_F = \pi d \Theta \tag{46}$$

$$S_{CD} = d \sin(\pi\Theta) \tag{47}$$

$$A_{CD} = \frac{d^2}{4}\left(\pi\Theta - \frac{\sin(2\pi\Theta)}{2}\right) \tag{48}$$

Liquid Entrainment in Gas Core. An empirical correlation of liquid entrainment fraction was proposed by Oliemans et al. [21] using the AERE Harwell data bank (Whalley and Hewitt [22]),

$$\frac{F_E}{1-F_E} = 10^{\beta_0}\rho_L^{\beta_1}\rho_G^{\beta_2}\mu_L^{\beta_3}\mu_G^{\beta_4}\sigma^{\beta_5}d^{\beta_6}v_{SL}^{\beta_7}v_{SG}^{\beta_8}g^{\beta_9} \tag{49}$$

where the β parameters, as listed in Table 1, are regression coefficients with a constraint that they are chosen such that the *RHS* of Eq. (49) forms a dimensionless group. The above correlation can be reorganized as a function of six non-dimensional groups, since there are three dimensions involved,

$$\frac{F_E}{1-F_E} = 0.003\mathrm{We}_{SG}^{1.8}\mathrm{Fr}_{SG}^{-0.92}\,\mathrm{Re}_{SL}^{0.7}\,\mathrm{Re}_{SG}^{-1.24}\left(\frac{\rho_L}{\rho_G}\right)^{0.38}\left(\frac{\mu_L}{\mu_G}\right)^{0.97} \tag{50}$$

where

$$\mathrm{We}_{SG} = \frac{\rho_G v_{SG}^2 d}{\sigma} \tag{51}$$

Table 1 Parameters for Liquid Entrainment Fraction Correlation

Parameter	Physical Quantity	Parameter Estimate
β_0	Intercept	−2.52
β_1	ρ_L	1.08
β_2	ρ_G	0.18
β_3	μ_L	0.27
β_4	μ_G	0.28
β_5	σ	−1.80
β_6	d	1.72
β_7	v_{SL}	0.70
β_8	v_{SG}	1.44
β_9	g	0.46

$$\mathrm{Fr}_{SG} = \frac{v_{SG}}{\sqrt{gd}} \tag{52}$$

$$\mathrm{Re}_{SL} = \frac{\rho_L v_{SL} d}{\mu_L} \tag{53}$$

$$\mathrm{Re}_{SG} = \frac{\rho_G v_{SG} d}{\mu_G} \tag{54}$$

During the step by step reorganization, it was found that either β_3 should be changed to 0.26 or β_4 should be changed to 0.27 to satisfy the dimensionless requirement, although this makes no significant difference in predicting the liquid entrainment fraction. In Eq. (50), a value of 0.27 is used for β_4.

Slug Liquid Holdup. Recently, Zhang et al. [11] developed a unified mechanistic model for slug liquid holdup which can be used for the whole range of inclination angles. This model is based on a balance between the turbulent kinetic energy of the liquid phase and the surface free energy of dispersed gas bubbles in a slug body,

$$H_{LS} = \frac{1}{1 + \dfrac{T_{sm}}{3.16[(\rho_L - \rho_G)g\sigma]^{1/2}}} \tag{55}$$

where

$$T_{sm} = \frac{1}{C_e}\left(\frac{f_S}{2}\rho_S v_S^2 + \frac{d}{4}\,\frac{\rho_L H_{LF}(v_T - v_F)(v_S - v_F)}{l_S} + \frac{d}{4}\,\frac{\rho_C(1-H_{LF})(v_T - v_C)(v_S - v_C)}{l_S}\right) \tag{56}$$

and

$$C_e = \frac{2.5 - |\sin(\theta)|}{2} \tag{57}$$

The mixture density in the slug body is defined as

$$\rho_S = \rho_L H_{LS} + \rho_G(1 - H_{LS}) \tag{58}$$

and the friction factor, f_S, is calculated in the same way as for f_F with a slug Reynolds number given by

$$\mathrm{Re}_S = \frac{\rho_S v_S d}{\mu_L} \tag{59}$$

The prediction of slug liquid holdup is improved significantly by consideration of the momentum exchange between the liquid slug and the film zone in a slug unit. This model can also be used for prediction of the transition boundary between slug and dispersed bubble flows.

The experimental measurements of Roumazeilles [23] and Yang [24] show that the slug liquid holdup can become as low as about 0.24. Therefore, the slug liquid holdup is set at 0.24 if the model predicts a lower value.

Before solving the combined momentum equation, the slug liquid holdup must be estimated to calculate different closure relationships. This estimation can be made using the Gregory et al. [25] correlation,

$$H_{LS} = \frac{1}{1 + \left(\dfrac{v_S}{8.66}\right)^{1.39}} \tag{60}$$

where v_S is in m/s.

Translational Velocity and Slug Length. The translational velocity of the liquid slugs can be expressed as a function of mixture velocity, v_S, in the form proposed by Nicklin [26],

$$v_T = C_S v_S + v_D \tag{61}$$

where v_D is the drift velocity (or the velocity of the elongated bubble at the limit of $v_s \rightarrow 0$). The coefficient C_S is approximately the ratio of the maximum to the mean velocity of a fully developed velocity profile. Nicklin proposed a value of 1.2 for the parameter C_S for turbulent flow. For laminar flow, C_S is about 2 (Fabre [27]). For the drift velocity in horizontal and upward inclined pipe flow, Bendiksen [28] proposed the use of

$$v_D = 0.54\sqrt{gd}\cos\theta + 0.35\sqrt{gd}\sin\theta \tag{62}$$

Since there is no correlation available for the drift velocity in downward flow, the above correlation is also used for the translational velocity of downward slug flow in this study.

According to the analyses of Taitel et al. [14] and Barnea and Brauner [12], slug lengths of 32d and 16d are used for horizontal and vertical flows in this study. The estimation of slug length for horizontal flow is valid for relatively small pipe diameters. For inclined flow, it is proposed in this study that the slug length is estimated as

$$l_S = (32.0\cos^2\theta + 16.0\sin^2\theta)d \tag{63}$$

The closure relationship for slug length can be substituted with alternative models or correlations based on their validity.

Solution Procedure

Figure 3 shows an overall flow chart of the unified model for a pipe increment. Flow pattern is determined first, based on the input parameters. Then, the hydrodynamic behaviors of the multiphase flow are calculated using the corresponding momentum and continuity equations.

Figure 4 is a flow chart for stratified or annular flow calculations. Figure 5 is a flow chart for slug flow calculations. They are parts of the unified model (shown in Fig. 3). There is no flow chart provided for bubbly and dispersed bubble flows because of their simplicity.

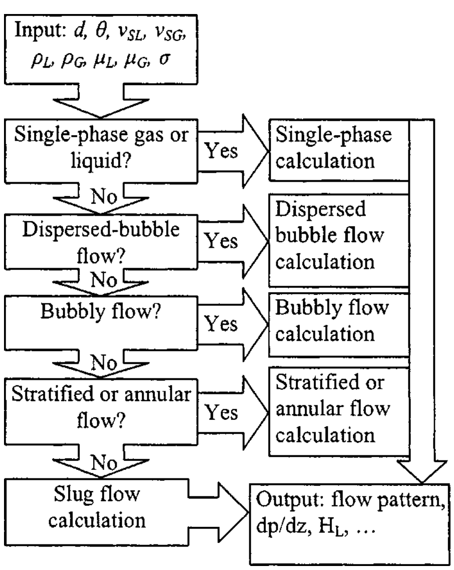

Fig. 3 Overall Flow Chart for the Unified Model

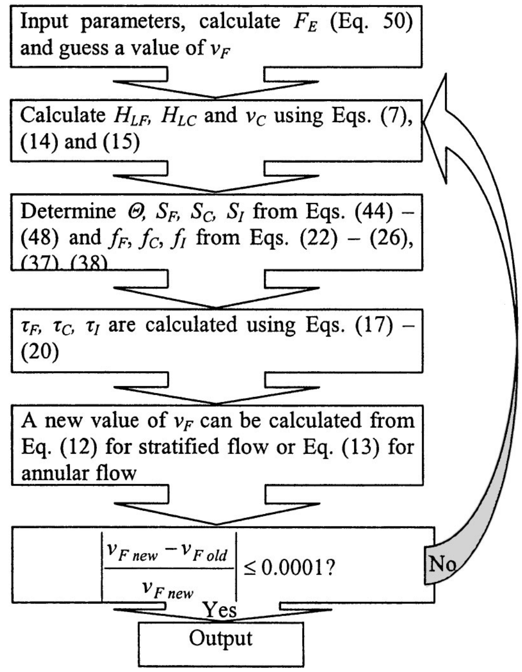

Fig. 4 Flow Chart for Stratified or Annular Flow Calculation

Concluding Remarks

A unified hydrodynamic model is developed for prediction of flow pattern transitions, liquid holdups, pressure gradient and slug characteristics in gas-liquid pipe flow at different inclination angles from −90 to 90 deg.

The flow pattern transition from slug flow to stratified and/or annular flow is predicted by solving the momentum equations for

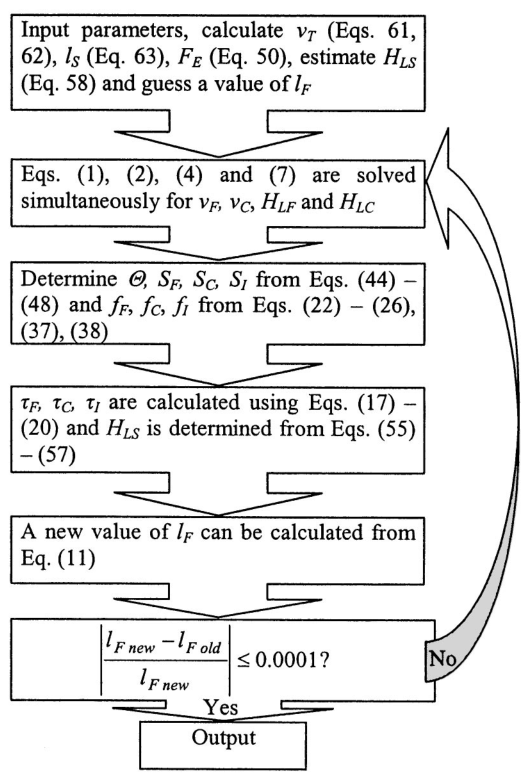

Fig. 5 Flow Chart for Slug Flow Calculation

slug flow rather than the traditional approach of using different mechanistic models and correlations. Therefore, every aspect of the two-phase flow is formulated under a single hydrodynamic model. Flow pattern transitions can also be used for validation of the hydrodynamic model along with other flow behaviors such as pressure gradient and liquid holdup.

All the flow patterns are grouped into three categories (bubble, intermittent, and stratified/annular) based on the hydrodynamic characteristics of each flow pattern. Due to this simplification, the flow pattern map is always composed of three areas under any set of flow conditions. Bubble flow includes bubbly and dispersed bubble flows. Intermittent flow includes elongated bubble, slug and churn flows. Stratified and annular flows are combined into one group, and the liquid film can be described by wetted wall fraction and liquid holdup.

The validation of the new unified model is carried out in [29].

Further improvements in the closure relationships are needed for the model to give better predictions of gas-liquid flow behaviors. Many closure relationships are correlated to the superficial gas and liquid velocities, and may be inconsistent with the hydrodynamic models. These closure relationships need to be reconstructed using the local flow parameters such as liquid and gas velocities, liquid holdups, etc.

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Nomenclature

A = cross-sectional area inside the pipe
C = coefficient
C_e = coefficient
d = pipe diameter
F_E = entrainment fraction of liquid in gas core
Fr = Froude number
f = friction factor
g = gravity acceleration
H_L = liquid holdup
h = height
$\bar{h}_F$ = liquid film height with flat interface
l = length
p = pressure
Re = Reynolds number
S = perimeter
v = velocity
v_o = gas bubble rise velocity relative to liquid
We = Weber number
z = axial length
β = exponents in Eq. (48)
Θ = wetted wall fraction
θ = inclination angle
μ = dynamic viscosity
ν = slug frequency
ρ = density
ρ_{GO} = gas density at atmospheric pressure
σ = surface tension
τ = shear stress

Subscripts

C = gas core
CD = chord
D = drift
exp = experimental
F = liquid film
G = gas
I = interface
L = liquid
pre = predicted
S = slug
SG = superficial gas
SL = superficial liquid
T = translational
t = transition
U = slug unit
W = wall

References

[1] Dukler, A. E., Wickes, M., and Cleveland, R. G., 1964, ‘‘Frictional Pressure Drop in Two-Phase Flow: B. An Approach Through Similarity Analysis,’’ AIChE J., **10**(1), pp. 44–51.
[2] Beggs, H. D., and Brill, J. P., 1973, ‘‘A Study of Two-Phase Flow in Inclined Pipes,’’ Trans. AIME, **255**, p. 607.
[3] Hagedorn, A. R., and Brown, K. E., 1965, ‘‘Experimental Study of Pressure Gradient Occurring During Continuous Two-Phase Flow in Small Diameter Vertical Conduits,’’ J. Pet. Technol., **17**, pp. 475–484.
[4] Duns, H., Jr., and Ros, N. C. J., 1963, ‘‘Vertical Flow of Gas and Liquid Mixtures in Wells,’’ Proceedings of the 6th World Petroleum Congress, p. 451.
[5] Barnea, D., 1987, ‘‘A Unified Model for Predicting Flow-Pattern Transitions for the Whole Range of Pipe Inclinations,’’ Int. J. Multiphase Flow, **13**, pp. 1–12.
[6] Xiao, J.-J., 1990, ‘‘A Comprehensive Mechanistic Model for Two-Phase Flow in Pipelines,’’ M.S. Thesis, U. of Tulsa, Tulsa, OK.
[7] Kaya, A. S., 1998, ‘‘Comprehensive Mechanistic Modeling of Two-Phase Flow in Deviated Wells,’’ M.S. Thesis, U. of Tulsa, Tulsa, OK.
[8] Gomez, L. E., Shoham, O., Schmidt, Z., and Chokshi, R. N., 1999, ‘‘A Unified Mechanistic Model for Steady-State Two-Phase Flow in Wells and Pipelines,’’ SPE J., **56520**, p. 307.
[9] Zhang, H.-Q., Jayawardena, S. S., Redus, C. L., and Brill, J. P., 2000, ‘‘Slug Dynamics in Gas-Liquid Pipe Flow,’’ ASME J. Energy Resour. Technol., **122**, pp. 14–21.
[10] Harmathy, T. Z., 1960, ‘‘Velocity of Large Drops and Bubbles in Media of Infinite or Restricted Extent,’’ AIChE J., **6**, p. 281.
[11] Zhang, H.-Q., Wang, Q., and Brill, J. P., 2003, ‘‘A Unified Mechanistic Model for Slug Liquid Holdup and Transition Between Slug and Dispersed Bubble Flows,’’ Int. J. Multiphase Flow, **29**, pp. 97–107.
[12] Barnea, D., and Brauner, N., 1985, ‘‘Hold-Up of the Liquid Slug in Two Phase Intermittent Flow,’’ Int. J. Multiphase Flow, **11**, pp. 43–49.
[13] Hinze, J. O., 1955, ‘‘Fundamentals of the Hydrodynamic Mechanism of Splitting in Dispersion Processes,’’ AIChE J., **1**, p. 289.
[14] Taitel, Y., Barnea, D., and Dukler, A. E., 1980, ‘‘Modeling Flow Pattern Transitions for Steady Upward Gas-Liquid Flow in Vertical Tubes,’’ AIChE J., **26**, pp. 345–354.
[15] Grolman, E., 1994, ‘‘Gas-Liquid Flow With Low Liquid Loading in Slightly Inclined Pipes,’’ Ph.D. Dissertation, U. of Amsterdam, The Netherlands, ISBN 90-9007470-8.
[16] Andritsos, N., and Hanratty, T. J., 1987, ‘‘Influence of Interfacial Waves in Stratified Gas-Liquid Flows,’’ AIChE J., **33**(3), pp. 444–454.
[17] Asali, J. C., 1984, ‘‘Entrainment in Vertical Gas-Liquid Annular Flow,’’ Ph.D. Dissertation, Dept of Chem Engng, U. of Illinois, Urbana.
[18] Willetts, I. P., 1987, ‘‘Non-Aqueous Annular Two-Phase Flow,’’ Ph.D. Dissertation, U. of Oxford, U.K.
[19] Ambrosini, W., Andreussi, P., and Azzopardi, B. J., 1991, ‘‘A Physically Based Correlation for Drop Size in Annular Flow,’’ Int. J. Multiphase Flow, **17**(4), pp. 497–507.
[20] Chen, X. T., Cai, X. D., and Brill, J. P., 1997, ‘‘Gas-Liquid Stratified-Wavy Flow in Horizontal Pipelines,’’ ASME J. Energy Resour. Technol., **119**, pp. 209–216.
[21] Oliemans, R. V., Pots, B. F. M., and Trompe, N., 1986, ‘‘Modeling of Annular Dispersed Two-Phase Flow in Vertical Pipes,’’ Int. J. Multiphase Flow, **12**(5), pp. 711–732.
[22] Whalley, P. B., and Hewitt, G. F., 1978, ‘‘The Correlation of Liquid Entrainment Fraction and Entrainment Rate in Annular Two-Phase Flow,’’ Report AERE-R 9187, UKAEA, Harwell, Oxon.
[23] Roumazeilles, P., 1994, ‘‘An Experimental Study of Downward Slug Flow in Inclined Pipes,’’ M.S. Thesis, U. of Tulsa, Tulsa, OK.
[24] Yang, J.-D., 1996, ‘‘A Study of Intermittent Flow in Downward Inclined Pipes,’’ Ph.D. Dissertation, U. of Tulsa, Tulsa, OK.
[25] Gregory, G. A., Nicholson, M. K., and Aziz, K., 1978, ‘‘Correlation of the Liquid Volume Fraction in the Slug for Horizontal Gas-Liquid Slug Flow,’’ Int. J. Multiphase Flow, **4**, pp. 33–39.
[26] Nicklin, D. J., 1962, ‘‘Two-Phase Bubble Flow,’’ Chem. Eng. Sci., **17**, pp. 693–702.
[27] Fabre, J., 1994, ‘‘Advancements in Two-Phase Slug Flow Modeling,’’ SPE 27961, U. of Tulsa Centennial Petroleum Engineering Symposium, August 29–31.
[28] Bendiksen, K. H., 1984, ‘‘An Experimental Investigation of The Motion of Long Bubbles in Inclined Tubes,’’ Int. J. Multiphase Flow, **10**, pp. 467–483.
[29] Zhang, H.-Q., Wang, Q., Sarica, C., and Brill, J. P., 2003, ‘‘Unified Model for Gas-Liquid Pipe Flow Via Slug Dynamics—Part 2: Model Validation,’’ Submitted to ASME J. Energy Resour. Technol.

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