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The Bayesian viewpoint

An introduction

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Deductive reasoning

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- Generally credited to Aristotle's *Organon*.

Deductive reasoning

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- Generally credited to Aristotle's *Organon*.
- If A is true, then B is true.

Deductive reasoning

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- Generally credited to Aristotle's *Organon*.
- If A is true, then B is true.
- Its inverse: If B is false, then A is false.

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Deductive reasoning

- Generally credited to Aristotle's *Organon*.
- If A is true, then B is true.
- Its inverse: If B is false, then A is false.
- Do we always have the right kind of information to allow this kind of reasoning?

Plausible reasoning

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- Not quite. Sometimes, we need weaker syllogisms.

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Plausible reasoning

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Plausible reasoning

- Not quite. Sometimes, we need weaker syllogisms.
- If A is true, then B is true.
- What if we only know that B is true?

Plausible reasoning

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- Not quite. Sometimes, we need weaker syllogisms.

- If A is true, then B is true.
- What if we only know that B is true?

- We would like to say: Then, A becomes more plausible.

Plausible reasoning

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- Not quite. Sometimes, we need weaker syllogisms.

- If A is true, then B is true.
- What if we only know that B is true?

- We would like to say: Then, A becomes more plausible.

- Similar reasoning: If A is false, then B becomes less plausible.

Modes of plausible reasoning

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- Reasoning from consequence.

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Modes of plausible reasoning

- Reasoning from consequence.
- Reasoning from randomness.

Modes of plausible reasoning

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- Reasoning from consequence.
- Reasoning from randomness.
- Reasoning from analogy.

Modes of plausible reasoning

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- Reasoning from consequence.
- Reasoning from randomness.
- Reasoning from analogy.

- In the calculus of plausibility, our **prior** assessments are all important!

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Prior, likelihood & posterior

- Observed data D.

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Prior, likelihood & posterior

- Observed data D.
- Want to know something about a variable θ.

Prior, likelihood & posterior

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- Observed data D.
- Want to know something about a variable θ.

- Our interest is then in the quantity:

$$p(\theta|D) = \frac{p(D|\theta)\, p(\theta)}{p(D)} = \frac{p(D|\theta)\, p(\theta)}{\int_{\theta} p(D|\theta)\, p(\theta)}$$

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Prior, likelihood & posterior

- Observed data D.
- Want to know something about a variable θ.

- Our interest is then in the quantity:

$$p(\theta|D) = \frac{p(D|\theta)\,p(\theta)}{p(D)} = \frac{p(D|\theta)\,p(\theta)}{\int_\theta p(D|\theta)\,p(\theta)}$$

- $p(\theta|D)$ is the *posterior* distribution of the variable θ in light of the observed data, $p(\theta)$ is the *prior*, and $p(D|\theta)$ is the *generative model/likelihood* of the dataset.

Exact binomial probability inference

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- Outcome of a single flip given by a function of parameter θ:

$$p(\gamma|\theta) = \theta^{\gamma}\,(1-\theta)^{(1-\gamma)}$$

Exact binomial probability inference

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- Outcome of a single flip given by a function of parameter θ:

$$p(\gamma|\theta) = \theta^{\gamma} \, (1-\theta)^{(1-\gamma)}$$

- So, if you have z heads out of N flips:

$$p(\{\gamma_i\}|\theta) = \theta^{z} \, (1-\theta)^{(N-z)}$$

where z is $\sum_i \gamma_i$

Specifying the prior

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- Beta distribution:

$$
\begin{aligned}
p(\theta|a,b) &= \mathsf{beta}(\theta|a,b) \\
&= \theta^{(a-1)}(1-\theta)^{(b-1)}/B(a,b)
\end{aligned}
$$

Specifying the prior

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- Beta distribution:

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\begin{aligned}
p(\theta|a,b) &= \text{beta}(\theta|a,b) \\
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\end{aligned}
$$

- In terms of the mode ω and concentration κ,

$$a = \omega(\kappa - 2) + 1 \text{ and } b = (1-\omega)(\kappa - 2) + 1$$

where $\kappa > 2$

The posterior

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- Posterior is also a beta:

$$p(\theta|z,N) = \frac{p(z,N|\theta)\,p(\theta)}{p(z,N)}$$

$$= \theta^{z}\,(1-\theta)^{N-z}\;\frac{\theta^{(a-1)}(1-\theta)^{(b-1)}}{B(a,b)}\Big/ p(z,N)$$

$$= \text{beta}(\theta|z+a, N-z+b)$$

The posterior

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- Posterior is also a beta:

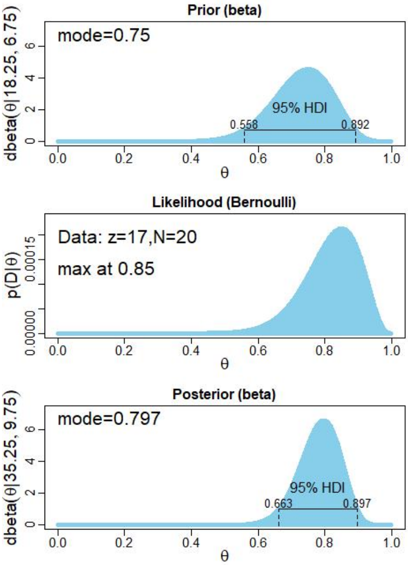
$$p(\theta|z,N) = \frac{p(z,N|\theta)\,p(\theta)}{p(z,N)}$$

$$= \theta^z\,(1-\theta)^{N-z}\ \frac{\theta^{(a-1)}(1-\theta)^{(b-1)}}{B(a,b)}\Big/ p(z,N)$$

$$= \mathsf{beta}(\theta|z+a, N-z+b)$$

- The posterior is a compromise of prior and likelihood.

Example 1



Example1.jpg

Example 2

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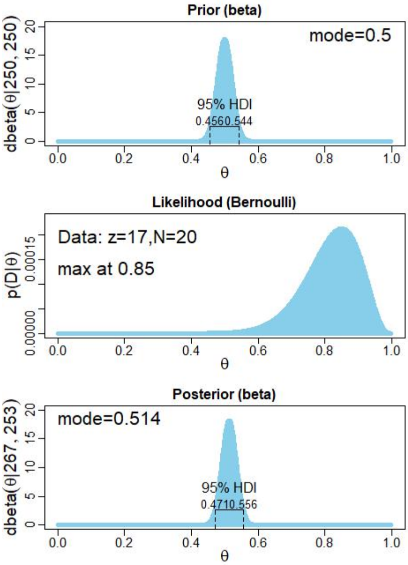
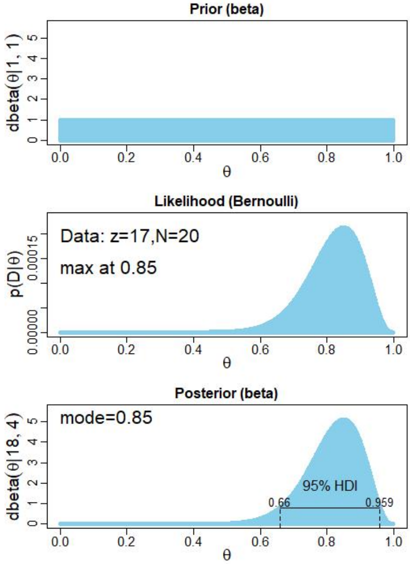


Example2.jpg

Example 3



Example3.jpg

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What should we watch out for?

- What are your priorities?

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What should we watch out for?

- What are your priorities?
- Subjective vs. objective priors.

What should we watch out for?

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- What are your priorities?
- Subjective vs. objective priors.
- Are analytical solutions always viable?

What should we watch out for?

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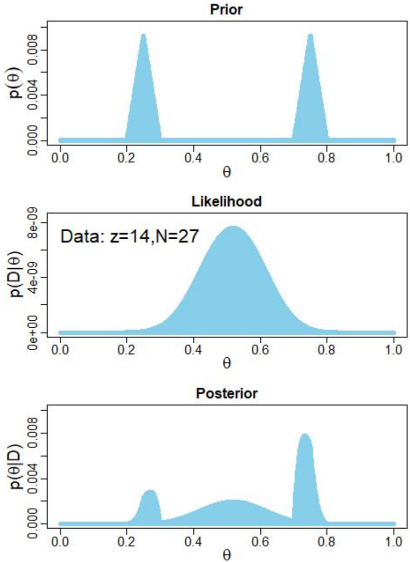
- What are your priorities?
- Subjective vs. objective priors.
- Are analytical solutions always viable?
- (Nope!) MCMC methods to the rescue.

Non-beta prior



Conjugate prior distributions

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- Let $f_\mu(x) = e^{n[\alpha\,\bar{x} - \psi(\alpha)]} f_0(x)$, and

Conjugate prior distributions

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- Let $f_\mu(x) = e^{n[\alpha\,\bar{x} - \psi(\alpha)]} f_0(x)$, and
- $g_{n_0,x_0}(\mu) = c\, e^{n_0[\alpha\, x_0 - \psi(\alpha)]} \Big/ V(\mu)$.

Conjugate prior distributions

- Let $f_\mu(x) = e^{n[\alpha\,\bar{x} - \psi(\alpha)]} f_0(x)$, and
- $g_{n_0,x_0}(\mu) = c\, e^{n_0[\alpha\, x_0 - \psi(\alpha)]} \Big/ V(\mu)$.
- Then,

$$g(\mu|x) = g_{n_+,\bar{x}_+}(\mu), \text{ where}$$

$$n_+ = n_0 + n \text{ and } \bar{x}_+ = \frac{n_0}{n_+}\, x_0 + \frac{n}{n_+}\, \bar{x}$$

Robbins' Formula

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- Consider following claims data for a European automobile insurance company circa 1950s:

Claims	0	1	2	3	4	5	6	7
Counts y_x	7840	1317	239	42	14	4	4	1

Robbins' Formula

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- Consider following claims data for a European automobile insurance company circa 1950s:

Claims	0	1	2	3	4	5	6	7
Counts y_x	7840	1317	239	42	14	4	4	1

- **Key idea** Large data sets of parallel situations carry within them their own Bayesian information.

Last but not least

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- Bayesian Learning (BIC, etc.)

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Last but not least

- Bayesian Learning (BIC, etc.)
- Frequentist vs. Bayesian comparisons