

# TikZ tensor network diagrams

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Finite MPS:

$$|\Psi\rangle = \begin{array}{c} A_1 \quad A_2 \quad A_3 \quad \dots \quad A_{N-1} \quad A_N \\ \circ - \circ - \circ - \dots - \circ - \circ \\ | \quad | \quad | \quad \quad \quad | \quad | \end{array} . \quad (1)$$

Gauge transform:

$$\begin{array}{c} A_i \\ \circ - \end{array} \rightarrow \begin{array}{c} G_{i-1}^{-1} \quad A_i \quad G_i \\ \circ - \circ - \circ \\ | \quad | \quad | \end{array} . \quad (2)$$

Left-orthogonal form:

$$\begin{array}{c} A_i \\ \circ - \end{array} \rightarrow \begin{array}{c} L_i \\ \triangleleft - \end{array} , \quad \left[ \begin{array}{c} L_i \\ \triangleleft \\ \triangleleft \\ L_i \end{array} \right] = \left[ \right] . \quad (3)$$

Right-orthogonal form:

$$\begin{array}{c} A_i \\ \circ - \end{array} \rightarrow \begin{array}{c} R_i \\ - \triangleright \end{array} , \quad \left[ \begin{array}{c} R_i \\ \triangleright \\ \triangleright \\ R_i \end{array} \right] = \left[ \right] . \quad (4)$$

SVD:

$$\begin{array}{c} A_i \\ \circ - \end{array} = \begin{array}{c} U \\ \triangleleft - \end{array} \begin{array}{c} D \\ \diamond - \end{array} \begin{array}{c} V^\dagger \\ \circ - \end{array} . \quad (5)$$

Mixed-canonical form:

$$|\Psi\rangle = \begin{array}{c} L_1 \quad L_2 \quad \dots \quad L_{i-1} \quad C_i \quad R_{i+1} \quad \dots \quad R_{N-1} \quad R_N \\ \triangleleft - \triangleleft - \dots - \triangleleft - \circ - \triangleleft - \dots - \triangleleft - \triangleleft \\ | \quad | \quad \quad \quad | \quad | \quad \quad \quad | \quad | \end{array} . \quad (6)$$

$$|\Psi\rangle = \begin{array}{c} L_1 \quad L_2 \quad \dots \quad L_{i-1} \quad L_i \quad D_i \quad R_{i+1} \quad \dots \quad R_{N-1} \quad R_N \\ \triangleleft - \triangleleft - \dots - \triangleleft - \triangleleft - \circ - \triangleleft - \dots - \triangleleft - \triangleleft \\ | \quad | \quad \quad \quad | \quad | \quad \quad \quad | \quad | \end{array} . \quad (7)$$

Unitary gauge transformation:

$$\begin{array}{c} L_i \\ \triangleleft - \end{array} \rightarrow \begin{array}{c} U_{i-1}^\dagger \\ \circ - \end{array} \begin{array}{c} L_i \\ \triangleleft - \end{array} \begin{array}{c} U_i \\ \circ - \end{array} . \quad (8)$$

Expectation value:

$$\langle \Psi | O_i | \Psi \rangle = \begin{array}{c} \text{\scriptsize $A_{i-1}$} \quad \text{\scriptsize $A_i$} \quad \text{\scriptsize $A_{i+1}$} \\ \begin{array}{ccccccc} \circ & \cdots & \circ & \circ & \circ & \cdots & \circ \\ | & & | & \square & | & & | \\ \circ & \cdots & \circ & \circ & \circ & \cdots & \circ \end{array} \end{array} \quad (9)$$

$$\langle \Psi | O_i | \Psi \rangle = \text{Diagram with } L_{i-1}, C_i, R_{i+1}, O \text{ and } C_i \text{ blocks} = \text{Diagram with } C_i \text{ block} \quad (10)$$

Multi-site expectation value:

$$\langle \Psi | O | \Psi \rangle = \left[ \begin{array}{c} L_{i-1} \quad C_i \quad R_{i+1} \\ \text{Diagram of a quantum circuit with two horizontal lines, a central box, and control lines} \\ \end{array} \right]. \quad (11)$$

MPO:

$$\text{[Diagram: A large rectangle with 6 vertical lines (3 on each side)]} = \text{[Diagram: Three squares connected in series, each with 2 vertical lines]} . \quad (12)$$

MPO expectation value:

$$\langle \Psi | H | \Psi \rangle = \begin{array}{c} \text{Diagram 1: A sequence of gates } L_{i-1}, C_i, R_{i+1} \text{ on three lines, with ancilla lines and measurement symbols.} \end{array} = E_{i-1} \begin{array}{c} \text{Diagram 2: A gate } C_i \text{ on three lines, with ancilla lines.} \end{array} F_{i+1} = E_i \begin{array}{c} \text{Diagram 3: A gate } D_i \text{ on three lines, with ancilla lines.} \end{array} F_{i+1} . \quad (13)$$

Environment tensors:

$$E_1 \begin{array}{|c|} \hline \text{ } \\ \hline \end{array} \equiv \begin{array}{c} L_1 \\ \text{ } \\ \text{ } \\ \text{ } \end{array}, \quad E_i \begin{array}{|c|} \hline \text{ } \\ \hline \end{array} \equiv E_{i-1} \begin{array}{|c|} \hline \text{ } \\ \hline \end{array} \begin{array}{c} L_i \\ \text{ } \\ \text{ } \\ \text{ } \end{array}. \quad (14)$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \left[ \text{---} \right] F_N \equiv \begin{array}{c} R_N \\ \text{---} \end{array} \left[ \text{---} \right] \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} , \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \left[ \text{---} \right] F_i \equiv \begin{array}{c} R_i \\ \text{---} \end{array} \left[ \text{---} \right] \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \left[ \text{---} \right] F_{i+1} . \quad (15)$$

iMPS:

$$|\Psi\rangle = \dots - \text{---} \bigcirc - \text{---} \bigcirc - \text{---} \bigcirc - \text{---} \bigcirc - \text{---} \bigcirc - \text{---} \dots, \quad (16)$$

Transfer matrix:

$$T = \begin{array}{c} \text{---} \bigcirc \text{---} \\ | \\ \text{---} \bigcirc \text{---} \end{array} . \quad (17)$$

MPS norm:

$$\langle \Psi | \Psi \rangle = \begin{array}{ccccccccc} \dots & \text{---} & \bigcirc & \text{---} & \bigcirc & \text{---} & \bigcirc & \text{---} & \bigcirc & \text{---} & \dots \\ & & | & & | & & | & & | & & \\ \dots & \text{---} & \bigcirc & \text{---} & \bigcirc & \text{---} & \bigcirc & \text{---} & \bigcirc & \text{---} & \dots \end{array} . \quad (18)$$

Left-orthogonal form:

$$\begin{array}{c} \text{Diagram 1} \end{array} = \left[ \begin{array}{c} \text{Diagram 2} \end{array} \right], \quad \begin{array}{c} \text{Diagram 3} \end{array} \rho_L = \begin{array}{c} \text{Diagram 4} \end{array} \rho_L. \quad (19)$$

Right-orthogonal form:

$$\begin{array}{c} \text{Diagram 1} \end{array} = \left[ \begin{array}{c} \text{Diagram 2} \end{array} \right], \quad \rho_R \begin{array}{c} \text{Diagram 3} \end{array} = \rho_R \begin{array}{c} \text{Diagram 4} \end{array}. \quad (20)$$

Mixed-canonical form:

$$|\Psi\rangle = \dots \text{Diagram 1} \text{Diagram 2} \text{Diagram 3} \text{Diagram 4} \text{Diagram 5} \dots \quad (21)$$

$$= \dots \text{Diagram 1} \text{Diagram 2} \text{Diagram 3} \text{Diagram 4} \text{Diagram 5} \dots. \quad (22)$$

iMPS expectation value:

$$\langle \Psi | O_i | \Psi \rangle = \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \end{array} \rho_L. \quad (23)$$

$$\langle \Psi | O_i | \Psi \rangle = \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \end{array}. \quad (24)$$

Environment tensor recursion relation:

$$E(n+1) \left[ \begin{array}{c} \text{Diagram 1} \end{array} \right] \alpha = E(n) \left[ \begin{array}{c} \text{Diagram 2} \end{array} \right] \alpha + \sum_{\beta < \alpha} E(n) \left[ \begin{array}{c} \text{Diagram 3} \end{array} \right] \alpha. \quad (25)$$