

TikZ tensor network diagrams

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Finite MPS:

$$|\Psi\rangle = \begin{array}{c} A_1 \quad A_2 \quad A_3 \quad \cdots \quad A_{N-1} \quad A_N \\ \circ - \circ - \circ - \cdots - \circ - \circ \\ | \quad | \quad | \quad \quad \quad | \quad | \end{array} . \quad (1)$$

Gauge transform:

$$\begin{array}{c} A_i \\ \circ - \\ | \end{array} \rightarrow \begin{array}{c} \quad G_{i-1}^{-1} \quad A_i \quad G_i \\ \circ - \circ - \circ - \\ | \quad | \quad | \end{array} . \quad (2)$$

Left-orthogonal form:

$$\begin{array}{c} A_i \\ \circ - \\ | \end{array} \rightarrow \begin{array}{c} L_i \\ \square - \\ | \end{array}, \quad \left(\begin{array}{c} L_i \\ \square \\ | \\ \square \\ L_i \end{array} \right) = \left[\cdot \right] . \quad (3)$$

Right-orthogonal form:

$$\begin{array}{c} A_i \\ \circ - \\ | \end{array} \rightarrow \begin{array}{c} R_i \\ \square - \\ | \end{array}, \quad \left(\begin{array}{c} R_i \\ \square \\ | \\ \square \\ R_i \end{array} \right) = \left] \cdot \right] . \quad (4)$$

SVD:

$$\begin{array}{c} A_i \\ \circ - \\ | \end{array} = \begin{array}{c} U \\ \square - \\ | \end{array} \begin{array}{c} D \\ \diamond \\ | \end{array} \begin{array}{c} V^\dagger \\ \square - \\ | \end{array} . \quad (5)$$

Mixed-canonical form:

$$|\Psi\rangle = \begin{array}{c} L_1 \quad L_2 \quad \cdots \quad L_{i-1} \quad C_i \quad R_{i+1} \quad \cdots \quad R_{N-1} \quad R_N \\ \square - \square - \cdots - \square - \circ - \square - \cdots - \square - \square \\ | \quad | \quad \quad \quad | \quad | \quad | \quad \quad \quad | \quad | \end{array} . \quad (6)$$

$$|\Psi\rangle = \begin{array}{c} L_1 \quad L_2 \quad \cdots \quad L_{i-1} \quad L_i \quad D_i \quad R_{i+1} \quad \cdots \quad R_{N-1} \quad R_N \\ \square - \square - \cdots - \square - \square - \circ - \square - \cdots - \square - \square \\ | \quad | \quad \quad \quad | \quad | \quad | \quad \quad \quad | \quad | \end{array} . \quad (7)$$

Unitary gauge transformation:

$$\begin{array}{c} L_i \\ \square - \\ | \end{array} \rightarrow \begin{array}{c} U_{i-1}^\dagger \quad L_i \quad U_i \\ \circ - \square - \circ - \\ | \quad | \quad | \end{array} . \quad (8)$$

$$\langle \Psi | O_i | \Psi \rangle = \text{Diagram of a 2D tensor network with two rows of circles. The top row has nodes labeled A_{i-1}, A_i, and A_{i+1} above them. The bottom row has nodes labeled A_{i-1}, A_i, and A_{i+1} below them. A square node labeled O is connected to the A_i nodes in both rows. The diagram is part of equation (9).$$

$$\langle \Psi | O | \Psi \rangle = \text{Diagram with two rows of nodes and a central box. Top row: } L_{i-1} \text{ (square), } C_i \text{ (circle), } R_{i+1} \text{ (square). Bottom row: } L_{i-1} \text{ (square), } C_i \text{ (circle), } R_{i+1} \text{ (square). A central box connects the two rows. Vertical lines connect } L_{i-1} \text{ to } L_{i-1}, C_i \text{ to } C_i, \text{ and } R_{i+1} \text{ to } R_{i+1}. \text{ The diagram is enclosed in large parentheses.} \quad (11)$$

$$\text{Diagram 1} = \text{Diagram 2} \cdot \quad (12)$$
$$\langle \Psi | H | \Psi \rangle = \begin{array}{ccccccc} & & L_{i-1} & C_i & R_{i+1} & & \\ & & \text{---} & \text{---} & \text{---} & & \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ & & \text{---} & \text{---} & \text{---} & & \end{array} = E_{i-1} \begin{array}{c} C_i \\ \text{---} \\ \text{---} \\ \text{---} \end{array} F_{i+1} = E_i \begin{array}{c} D_i \\ \text{---} \\ \text{---} \\ \text{---} \end{array} F_{i+1} . \quad (13)$$
$$\begin{array}{ccc}
\begin{array}{c} L_1 \\ \text{AND} \\ \text{AND} \\ \text{AND} \end{array} & \begin{array}{c} E_1 \end{array} & \equiv \\
\begin{array}{c} L_i \\ \text{AND} \\ \text{AND} \\ \text{AND} \end{array} & \begin{array}{c} E_i \end{array} & \equiv \begin{array}{c} E_{i-1} \end{array}
\end{array} \quad (14)$$

$$\text{iMPS:} \quad |\Psi\rangle = \cdots \text{---} \bigcirc \text{---} \bigcirc \text{---} \bigcirc \text{---} \bigcirc \text{---} \bigcirc \text{---} \cdots \quad (16)$$

$$T = \begin{array}{c} \text{---} \bigcirc \text{---} \\ | \\ \text{---} \bigcirc \text{---} \end{array} . \quad (17)$$
$$\langle \Psi | \Psi \rangle = \begin{array}{c} \dots - \text{O} - \text{O} - \text{O} - \text{O} - \text{O} - \dots \\ | \quad | \quad | \quad | \quad | \\ \dots - \text{O} - \text{O} - \text{O} - \text{O} - \text{O} - \dots \end{array} . \quad (18)$$

Left-orthogonal form:

$$\begin{array}{c} \text{Diagram 1} \end{array} = \left[\begin{array}{c} \text{Diagram 2} \end{array} \right], \quad \begin{array}{c} \text{Diagram 3} \end{array} \rho_L = \left[\begin{array}{c} \text{Diagram 4} \end{array} \right] \rho_L. \quad (19)$$

Right-orthogonal form:

$$\begin{array}{c} \text{Diagram 1} \end{array} = \left[\begin{array}{c} \text{Diagram 2} \end{array} \right], \quad \rho_R \begin{array}{c} \text{Diagram 3} \end{array} = \rho_R \left[\begin{array}{c} \text{Diagram 4} \end{array} \right]. \quad (20)$$

Mixed-canonical form:

$$|\Psi\rangle = \dots \begin{array}{c} \text{Diagram 1} \end{array} \begin{array}{c} \text{Diagram 2} \end{array} \begin{array}{c} \text{Diagram 3} \end{array} \begin{array}{c} \text{Diagram 4} \end{array} \begin{array}{c} \text{Diagram 5} \end{array} \dots \quad (21)$$

$$= \dots \begin{array}{c} \text{Diagram 1} \end{array} \begin{array}{c} \text{Diagram 2} \end{array} \begin{array}{c} \text{Diagram 3} \end{array} \begin{array}{c} \text{Diagram 4} \end{array} \begin{array}{c} \text{Diagram 5} \end{array} \dots. \quad (22)$$

iMPS expectation value:

$$\langle \Psi | O_i | \Psi \rangle = \begin{array}{c} \dots \begin{array}{c} \text{Diagram 1} \end{array} \begin{array}{c} \text{Diagram 2} \end{array} \begin{array}{c} \text{Diagram 3} \end{array} \begin{array}{c} \text{Diagram 4} \end{array} \begin{array}{c} \text{Diagram 5} \end{array} \dots \\ \dots \begin{array}{c} \text{Diagram 1} \end{array} \begin{array}{c} \text{Diagram 2} \end{array} \begin{array}{c} \text{Diagram 3} \end{array} \begin{array}{c} \text{Diagram 4} \end{array} \begin{array}{c} \text{Diagram 5} \end{array} \dots \end{array} = \left[\begin{array}{c} \text{Diagram 6} \end{array} \right] \rho_L. \quad (23)$$

$$\langle \Psi | O_i | \Psi \rangle = \begin{array}{c} \dots \begin{array}{c} \text{Diagram 1} \end{array} \begin{array}{c} \text{Diagram 2} \end{array} \begin{array}{c} \text{Diagram 3} \end{array} \begin{array}{c} \text{Diagram 4} \end{array} \begin{array}{c} \text{Diagram 5} \end{array} \dots \\ \dots \begin{array}{c} \text{Diagram 1} \end{array} \begin{array}{c} \text{Diagram 2} \end{array} \begin{array}{c} \text{Diagram 3} \end{array} \begin{array}{c} \text{Diagram 4} \end{array} \begin{array}{c} \text{Diagram 5} \end{array} \dots \end{array} = \left[\begin{array}{c} \text{Diagram 6} \end{array} \right]. \quad (24)$$

Environment tensor recursion relation:

$$E(n+1) \left[\begin{array}{c} \text{Diagram 1} \end{array} \right] \alpha = E(n) \left[\begin{array}{c} \text{Diagram 2} \end{array} \right] \alpha + \sum_{\beta < \alpha} E(n) \left[\begin{array}{c} \text{Diagram 3} \end{array} \right] \alpha. \quad (25)$$