Three-Term Relation Neuro-Fuzzy Cognitive Maps

Michal Puheim[1], Ján Vaščák[2], Ladislav Madarász[3]
Department of Cybernetics and Artificial Intelligence
Technical University of Kosice
Letná 9, 042 00 Košice, Slovak Republic
[1]michal.puheim@tuke.sk, [2]jan.vascak@tuke.sk, [3]ladislav.madarasz@tuke.sk

***Abstract*—In this paper, we propose a novel approach to modeling using fuzzy cognitive maps, which we refer to as the Three-Term Relation Neuro-Fuzzy Cognitive Map or simply the TTR NFCM. The proposed method is mostly suited to model complex nonlinear technical systems with dynamic internal characteristics. With this method we aim to solve some of the most critical problems of the conventional fuzzy cognitive maps. We target two of these problems by hybridization with artificial neural networks. First of them is a linear nature of relations between the concepts. The second is a lack of mutual dependence between the relations connecting to the same concept. Finally, we tackle a problem of relation dynamics using an inspiration from the control engineering. While focusing on bringing these advanced additional methods to the design of cognitive maps, we also aim to keep the degree of dependency on expert knowledge on the same level as with the conventional fuzzy cognitive maps. We achieve this by utilizing the machine learning methods. However, since the proposed method is heavily dependent on automated data-driven learning, it is suitable mainly for systems which are well observable and can produce sufficient training datasets.**

Keywords—fuzzy cognitive map, artificial neural network, PID control, modelling, complex systems, concept relations.

I. INTRODUCTION

Cognitive Map (CM) [1] in general is an oriented graph, where nodes represent concepts (elements or attributes of the modeled system) and edges determine mutual relations between these concepts. Values of either concepts or edges are represented by three crisp values -1, 0 and 1. The advantage of the CM is clear and comprehensible knowledge representation, which can be easily visualized in graph form. The CM is a viable tool for modeling mostly because its design is very intuitive and simple.

However, usage of crisp values obstructs the capabilities of the CMs for modeling more complex relations between concepts. To tackle this problem, the Fuzzy Cognitive Maps (FCM) [2] were proposed by Kosko in 1986. The FCM is similar to the traditional CM since it is represented as an oriented graph (with possible feedback connections) emphasizing causal relations between concepts, which are either positive, negative or none [3][4]. In contrast to the traditional CM, the FMC also takes into account the degree of influence of the relation expressed by a respective real value in range [-1, 1].

Unsurprisingly, even the relations expressed by real numbers are not sufficient. The problem is, that most relations between system elements in the real world are highly nonlinear and dynamic [5]. These relations are often neither symmetric nor monotonic as it is the case of models implemented by basic FCMs. Furthermore, each individual pair of system elements may not necessarily have only one mutual relation. It is possible that there are several varying relations between elements [5]. Most of the real world systems have non-linear dynamics often higher than the second order. In contrast to this, the dynamics of the basic FCM are limited to the first order, where the current state of concepts within the FCM depends only upon their last previous state. The FCM therefore cannot handle randomness associated with the complex domains [5]. This may cause problems when the FCM is used to create models of complex systems.

On the contrary, the main advantage of the FMC is the simplicity of design and thus possibility to create quick model prototypes, even for systems which we have limited knowledge about. This way, it is possible even for non-experts to identify the crucial relations within the system internals. Even though the exact definition of these relations may not be initially known, the fact that the relation itself was discovered is enough to understand the basic principles of operation of the system. Sufficiently precise approximation of the relations can be found later employing techniques from the area of automated machine learning, hence exploiting the data gathered during the system operation.

In this paper, we propose adjustments to the basic structure of the FCM using ways and means of artificial intelligence and control theory, such as derivative and integral causal relations and hybridization of the FCM structure with use of artificial neural networks. With our approach we aim for an optimal combination of human knowledge and machine learning, which would keep the design process of the FMC as simple as it is, while improving its overall modeling capability.

II. FUZZY COGNITIVE MAPS

There are several formal definitions of the FCM, but the most common one is given by Chen [6], where the FCM (see Fig. 1) is defined as the quadruple:

$$\mathrm{FCM} = (C, W, \alpha, \beta), \tag{1}$$

where [7]:

- $C = \{C_1, C_2,..., C_n\}$ is a finite set of cognitive units (concepts),
- $W = \{w_{11}, w_{12},..., w_{nm}\}$ is a finite set of oriented connections between concepts,

978-1-4799-5338-7/14/$31.00 ©2014 IEEE

- $\alpha \rightarrow [-1, 1]$ is a membership function of a concept and its result is a grade of membership of real world values corresponding to this concept,
- $\beta \rightarrow [-1, 1]$ has the same meaning as α but for the edges.

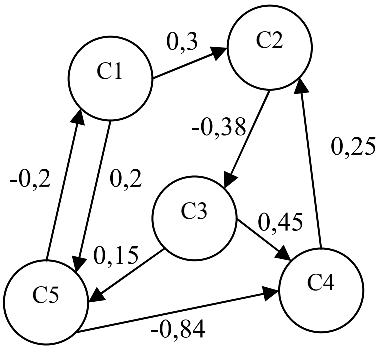

Fig. 1. The fuzzy cognitive map is a fuzzy oriented graph, which models the given system and its behavior using the concepts and its mutual relations.

A concept C_j (j = 1, 2,..., n) indicates a state, procedure, event or variable of the modeled system (in a symbolic, i.e. verbal form) and its activation value A_i is from a range [-1, 1] (or based on the implementation from a range [0, 1]). A directed connection between concepts C_i and C_j determines a causality of the influence between these concepts. It is represented by a weight w_{ij} which is determined by a fuzzy value from a range [-1, 1] [4].

A general rule to compute the value of the concept C_i in every simulation step is to calculate the influence of other concepts connected to the concept C_j. Therefore, if $A_j(t)$ is the value of the concept C_j in the time t, then the value $A_j(t+1)$ of the concept C_j in the time t+1 will be influenced by the values A_i(t) of the preceding concepts C_i in the time t as follows [4]:

$$A_j(t+1) = p\left(A_j(t) + \sum_{i=1; i \neq j}^{n} w_{ij} A_i(t) \right), \qquad (2)$$

where w_{ij} are the weights on the connections between concepts C_i and the concept C_j. A squashing (threshold) function $p(x)$ limits the resulting value x into a range [0, 1] (or [-1,1]) [4]. A behavior of the resulting FCM depends greatly upon the type of the used squashing function [3]. One of the most common squashing functions $p(x)$ is the logistic (sigmoid) function [3]:

$$p(x) = \frac{1}{1+e^{-mx}}, \qquad (3)$$

where the constant m determines the steepness of the sigmoid.

A. Advantages of FCM

According to Vliet et al. [8], main advantages of modeling approaches which utilize the fuzzy cognitive maps are:

- simplicity of map design and parameter setting,
- flexibility of representation (it is easy to add or remove new concepts),
- simplicity of usage, comprehensibility and transparency for non-technical experts,
- low computational complexity,
- ability to describe a variety of dynamic systems thanks to the feedback and recurrent structure of the map.

An advantage of FCMs over other modeling approaches such as the Bayesian Networks (BS) or the Petri Nets (PN) is, that it is easy to create FCM model even by a non-expert because of its comprehensibility and modularity. In case of the BS or the PN it is not evident for non-experts in the field how to construct a model of the system using these approaches [9][10]. It is also possible, that the single FCM is created by several experts using different approaches. This possibility is out of the question in case of the PN, because it is not well established how to combine different PNs which describe the same system [9][10].

B. Disadvantages of FCM

There are several limitations of conventional FCMs applied to the modeling of complex systems. These can be summarized as follows [5][11]:

- Relation weights are just linear (see Fig. 2).
- Models lack time delay in the interactions between nodes.
- The FCM cannot represent logical operators (AND, OR, NOT, and XOR) between ingoing nodes.
- It cannot model multi-meaning (grey) environments.
- Multi-state (quantum) of the concepts is not supported.
- No more than one relation between nodes is allowed.
- Many real world causal relations are neither symmetric nor monotonic as within the FCM model.
- The FCM dynamic is the first order, where the next state depends just on the previous one.

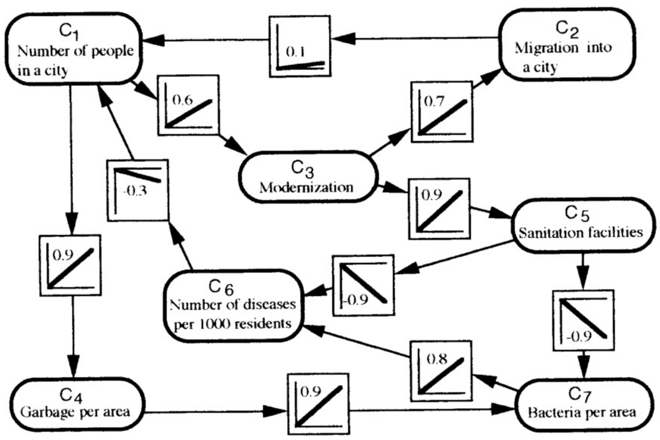

Fig. 2. A visualization of the concept relations within the FCM model of a city. It is clearly visible, that the relations can be represented only as linear functions [12].

III. ESTABLISHED FCM EXTENSIONS

If we take the shortcomings of the FCM from the previous section into account, it is clear that basic FCM are not sufficient to model complex systems. As a result, it is necessary to suggest some modifications to the basic structure of FMC. In order to overcome problems of the basic FMC, such as a linearity, first-order dynamics, a lack of time delay and an inability to represent logical operators, the following already established extensions can be taken into consideration (see Fig. 3) [12]:

- weights extended to nonlinear functions,
- conditional weights,
- time-delayed weights.

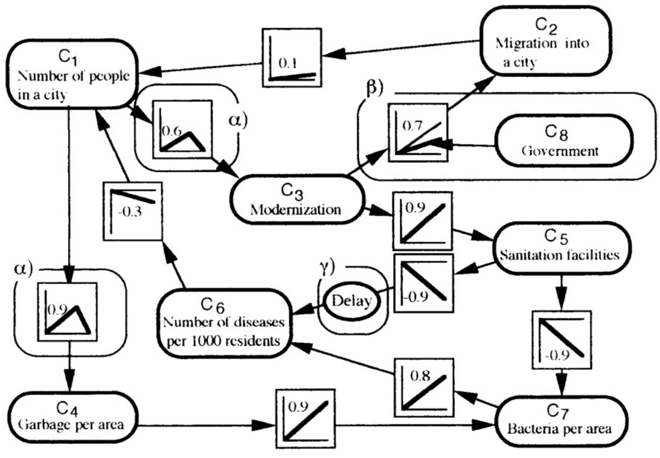

Fig. 3. An FCM extended by nonlinear (α), conditional (β) and time-delayed (γ) weights [12].

According to a study by Papageorgiou and Salmeron [5], several other extensions and methodologies emerged during the last decade, all of which improve the conventional FCM in various different ways. These include the Rule Based Fuzzy Cognitive Maps, the Fuzzy Grey Cognitive Maps, the Intuitionistic Fuzzy Cognitive Maps, the Dynamical Cognitive Networks, etc [5]. Although these methods solve most of the problems and disadvantages outlined in the previous section, they often complicate the process of the FCM design for experts because they increase the number of parameters which need to be adjusted. Furthermore, a mathematical background required to implement solutions using some of these methods is much more demanding, than a simple matrix algebra necessary to employ the conventional FCM.

IV. THREE-TERM RELATION NEURO-FUZZY CM

In this paper, we propose the *Three-Term Relation Neuro-Fuzzy Cognitive Map* (TTR NFCM) as a novel hybrid method designed particularly to model dynamic nonlinear complex systems. Nevertheless, it is applicable to modeling of various other systems as well. The proposed method enhances the conventional FCMs by adding two new features. The first one is the inclusion of a historical values or trends in the concept update formula using the *Three-Term Relations* (TTR), which are inspired by methods used within the control engineering, namely the PID controllers. As an additional alternative to the TTRs, *time-window* based modification is also proposed. This modification is inspired by similar methods used for prediction and basically changes the concept update formula by including concept activation values from the several previous states instead of only the last one. The second main feature is the replacement of simple linear weights between concepts by small feed-forward neural networks or multilayer perceptrons (MLPs), which modifies the FCM into the hybrid *Neuro-Fuzzy Cognitive Map* (NFCM).

Our proposal is motivated mainly by the *Extended Fuzzy Cognitive Maps* (E-FCMs – see Fig. 3) [12] by Hagiwara, but our approach is different and more general. The E-FCMs tackle the problems of the conventional FCMs by adding support for nonlinear relations, conditional relations and time delayed relations. However, the inclusion of these new relation types imposes additional requirements to knowledge of human experts who are tasked to design the E-FCMs. It can be difficult to determine the suitable type of the relation between the specific pair of concepts if the knowledge of the system is limited. In case of the time-delayed relations it is problematic to choose the correct value used for the delay. An application of conditional relations requires an exact knowledge of rules which determine the relations and also knowledge of the mutual dependence of the relations (the conditionals).

The NFCM solves the problems of nonlinear and conditional relations single-handedly by employing MLPs, which are well known to be able to approximate a wide variety of nonlinear functions, including the logical functions such as OR, AND, XOR, etc. The time-delay problems and lack of dynamics within conventional FCM relations is solved by employing TTR or time-window based approaches. Both components of the TTR NFCM can be used together or separately.

A. Nonlinear Relations Represented by MLPs

While being easy to understand, visualize and compute, the simple linear weights, which are used within the conventional FCMs, are naturally insufficient for modeling nonlinear relations within complex systems. One deficiency is the linear nature of the relation itself, but the other more important problem is the mutual independence of relations, where each relation acts as a linear function of one variable. The results of these functions are then combined using the concept update formula (2), which is basically the simple implementation of the OR rule. This is the only single rule allowed to determine the mutual dependence between relations in the conventional FCM. By replacing both the linear weights and the concept update process itself by the MLP, we can approximate all possible mutually interconnected relations which are influencing the single concept C_i, as a multivariable function, where input variables depend on the state of all concepts C_j preceding the concept C_i.

The applicability of the feed-forward MLP instead of conventional weights comes from the fact, that every single cognitive map, even one with several recursive connections, can be unwrapped into multiple simple feed-forward cognitive maps. These simple FCMs are mathematically equivalent to the former FCM and can be used to compute concept updates. The unwrapping process of the FCM is shown on the Fig. 4.

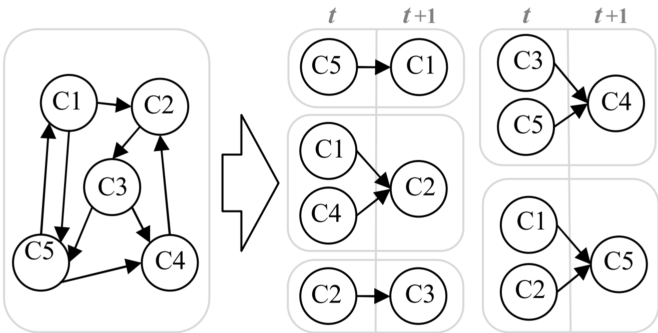

Fig. 4. An example of unwrapping of a simple FCM.

With an unwrapped FCM, the possibility for an application of MLPs is clearly evident (see Fig. 5). A rule of a thumb is to replace all relations which are preceding a single concept with an MLP. A number of input neurons in each MLP will be the same as a number of preceding concepts and there will be single output, as there is only one single concept which is being influenced. A topology of the deployed MLPs is to be considered, but generally even small and simple 2-hidden-layer topologies, with less than 5 neurons in each hidden layer, should perform better than the conventional linear FCM relations as their approximation capability is greater [13]. It is also possible to use MLPs with no hidden layer. This way the NFCM will operate equally to the conventional FCM.

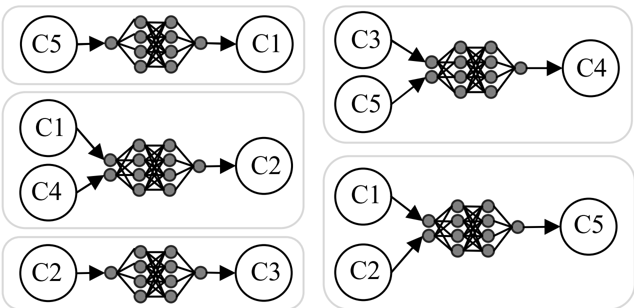

Fig. 5. A proposal of nonlinear relations between concepts using the MLP.

The replacement of the linear relations with the MLPs naturally leads to an increased computational complexity of the resulting FCM. This is obvious, as the increased approximation capability cannot be achieved without an increase in the model complexity [13]. The increase in the complexity is equal to the sum of the computational complexities of all used MLPs and grows exponentially with the size of the used MLPs. For small networks this is not the problem. Larger networks will slow computations down, so it is necessary to find the appropriate size/approximation ratio. However, the Fig. 5 clearly shows possible parallelization of computations, which may help to overcome the additional computational overhead caused by computationally demanding MLPs.

The general usage of the NFCM method for system modeling should be the following:

1. The expert designs the FCM by determining the key system variables and defining the corresponding concepts.
2. The expert determines which concepts should be connected (related). No further identification of the type of the relation is necessary as it will be a task for automated learning methods in the next stages.
3. The expert identifies domains of the system variables and declares membership functions for all the concepts. These functions will be used for the fuzzification (or the normalization) of the training data during the learning of the NFMC and for the defuzzification of the concept activations during the runtime.
4. The FCM is unwrapped into smaller feed-forward subsections, all of which are implemented as MLPs and trained using an arbitrary neural network supervised learning algorithm, using the data gathered during an operation of the system.

B. *Time Window Relations*

To tackle the problem of the time delay and the dynamics of the relations, we propose a time-window method (see Fig. 6), which is commonly used for prediction problems. Instead of a single relation between the two concepts, there are T relations, where T determines a size of the used time-window, i.e. the number of previous activation values of the preceding concepts which take part in the concept update. Each k-th activation value $A_j(t–k)$ of the concept C_i preceding the concept C_j has its own weight $w_{ij}(k)$ (or corresponding input neuron in case of using the NFCM). The concept update formula (2) is updated accordingly:

$$A_j(t+1) = p\left(\sum_{k=0}^{T}\sum_{i=1}^{n} w_{ij}(k)A_i(t-k)\right). \tag{4}$$

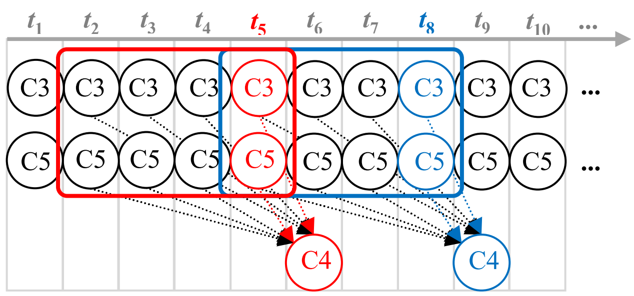

Fig. 6. A visualization of an expansion of the concept update function by inclusion of states of concepts from the past (time window).

C. Three-Term Relations

Another proposed solution to the problems of relation dynamics is inspired by methods from the control engineering, namely the PID controllers [14]. We gave the method a working title the *Three-Term Relations* (TTR), since it uses three main components to represent relations between concepts, similarly to the PID controller, which uses three components to modify input variables of the controlled system.

A *proportional-integral-derivative controller*, also known as a *PID controller* or a *three-term controller*, and its modifications are widely used in a feedback process control within countless industrial control systems [14]. The controller is used to keep a *selected system variable* within given limits using the difference between its *actual value* and its *desired value*. The controller attempts to minimize the difference (*error*) $e(t)$ between these values by changing the selected input process *variable*. The algorithm used to compute the input variable involves three separate components (see Fig. 7) [14]:

- *proportional component* (P), which depends merely on the present error $e(t)$,
- *integral component* (I), which accumulates all the error from the past,
- *derivative component* (D), which is based on the current rate of change of the error.

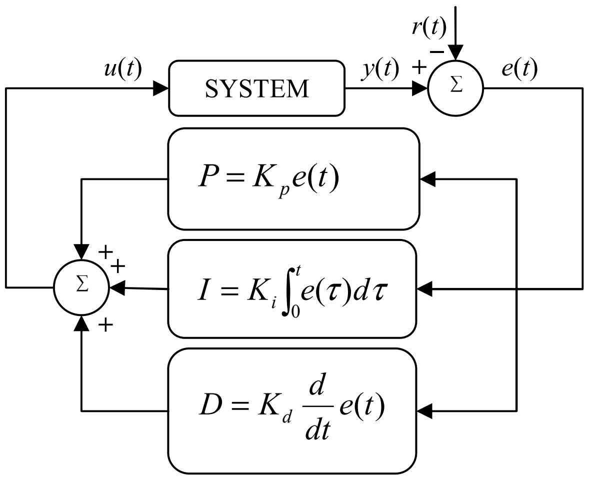

Fig. 7. A block diagram of a PID controller and a controlled system in a feedback loop: $y(t)$ is a value of the controlled system process variable; $r(t)$ is the value of the desired process variable; $e(t)$ is the difference (error) between $y(t)$ and $r(t)$; $u(t)$ is the value of the input process variable computed by the controller. The controller consists of three components – proportional (P), integral (I) and derivative (D).

An advantage of PID controllers is their applicability to a wide range of existing dynamic technical systems due to their ability to quickly react to sudden changes thanks to the derivative component and also to the undesirable long-term trends thanks to the integral component. This is also the main motivation to use a similar approach for FCM modeling. In addition, from the point of view of the control theory, every preceding concept can be seen as a simple controller of the following concept. Actually, a whole PID feedback loop (see Fig. 7) can be represented and also (with slight modifications) implemented as the simple FCM.

For the purpose of utilization within the FCM, we made slight changes to the three components. First of all, we removed the constant tunable parameters K_p, K_i and K_d from the component computation formulae (see Fig. 7) since they are already represented as weights within the FCM. Secondly, we transformed the formulae into a *numeric form*. Finally, we replaced the *integral term* with a *moving average term*, with a goal of diminishing the global stability issues. The reason for this slight modification is the fact, that the computations within FCM are supposed to produce values in range [0,1]. These limits cannot be met using the simple integral term, since it could diverge to positive or negative infinity very easily. Similarly, we also introduced a *threshold function p* to the *derivative term* to achieve the same limiting effect. The visualization of the single TTR along with the final component computation formulae is shown on Fig. 8.

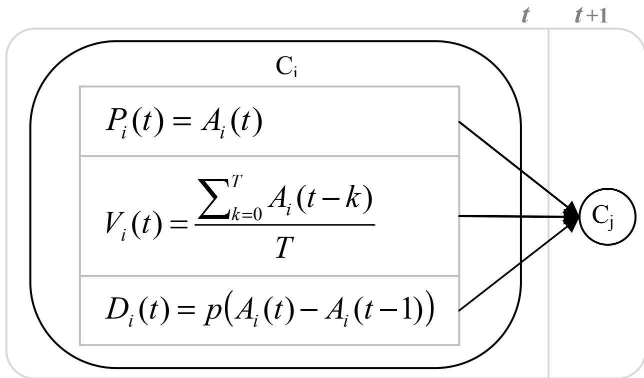

Fig. 8. An example of the three-term relation between concepts C_i and C_j with proportional component P_i, moving average component V_i and derivative component D_i. T is the size of the time window used to calculate the moving average.

With these modifications to the basic structure of the FMC, a computation of a concept updates consists of two stages. Firstly, the component activation values are computed using the formulae from Fig. 8. Secondly, the concept activation values are updated using the expansion of the concept update formula (2), which includes the outputs of all three components within the expression:

$$A_j(t+1) = p\left(\sum_{i=1}^{n} w_{ij}^P P_i(t) + w_{ij}^V V_i(t) + w_{ij}^D D_i(t)\right) \quad (5)$$

While utilizing the TTR, the concepts will *react to the last state* (or the last activation value of the concept) similarly to the conventional FCM, thanks to the *proportional relation component* P_i. The additional *moving average relation component* V_i enables the concepts to *react to the long term trends*. This can mitigate short-lived disturbances, noise, or implicit oscillations within the map. The concepts can also *react to the sudden changes* more quickly than within the conventional FCMs thanks to the *derivative relation component* D_i. Similarly to the time-window approach, there is only single tunable parameter T, which determines the number of previous states considered within the moving average V_i.

The TTR can be also viewed as a simplification of the time-window method. While it promises the same responsiveness to the dynamic behavior, it also reduces the number of required relations to only three per concept, which is often far lower than the number used for the time-window method.

V. Conclusion

In this paper, we proposed a new TTR NFCM methodology which can be used mainly (but not only) for modeling complex technical systems such as turbojet engines, which typically incorporate highly nonlinear behavior and dynamic characteristics of its internal components. Furthermore, we achieved the backward compatibility with the conventional FCMs. Our goal was to keep the human involvement in the TTR NFCM design as simple as with conventional FCM. This can be successfully achieved using this method, since the learning can be automated using data-driven techniques. However, the automated learning is very dependent on the quantity and quality of the training data. This is not a problem with systems which variables are simply observable. But for systems where it is unfeasible to obtain sufficiently large training datasets, it is advised to use more analytic-oriented FCM methods such as Fuzzy Grey Cognitive Maps [15], which are especially suited for discrete, incomplete and small datasets.

Acknowledgement

This paper is supported by VEGA, Grant Agency of Ministry of Education and Academy Science of Slovak Republic under Grant No. 1/0298/12 – “Digital control of complex systems with two degrees of freedom.” It is also supported by KEGA under Grant No. 018TUKE-4/2012 – “Progressive methods of education in the area of control and modeling of complex systems object oriented on aircraft turbo-compressor engines.”

References

[1] Axelrod, R. , “Structure of decision: The Cognitive Maps of political elites,”. Princeton, N.J.: Princeton University Press, 1976.

[2] Kosko, B., “Fuzzy Cognitive Maps,” International Journal on Man-Machine Studies, vol. 24, no. 1, pp 65-75, 1986.

[3] Papageorgiou, E.I., "Learning Algorithms for Fuzzy Cognitive Maps – A Review Study," IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews, vol.42, no.2, pp.150,163, March 2012. ISSN 1094-6977.

[4] Ketipi, M.K., Koulouriotis, D.E., Karakasis, E.G., Papakostas, G.A., Tourassis, V.D., "Nonlinear cause-effect relationships in Fuzzy Cognitive Maps," 2011 IEEE International Conference on Fuzzy Systems (FUZZ), pp.836,843, 27-30 June 2011. ISBN 978-1-4244-7315-1.

[5] Papageorgiou, E. I., Salmeron, J. L., "A Review of Fuzzy Cognitive Maps research during the last decade," IEEE Transactions on Fuzzy Systems, vol.21, no.1, pp.66,79, Feb. 2013. ISSN 1063-6706.

[6] Chen S. M., "Cognitive-Map-based Decision Analysis Based on NPN Logics", Fuzzy Sets and Systems, Elsevier, Vol. 71, No. 2, pp. 155-163, 1995

[7] Vaščák, J., Madarász L., "Adaptation of Fuzzy Cognitive Maps – a Comparison Study," Acta Polytechnica Hungarica, Vol. 7, No. 3, pp.109-122, 2010. ISSN 1785-8860.

[8] Vliet, M. van, Kok, K., Veldkamp, T., “Linking stakeholders and modellers in scenario studies: The use of Fuzzy Cognitive Maps as a communication and learning tool”, Futures, vol. 42, no. 1, pp. 1-14. 2010.

[9] León, M., Rodriguez, C., García, M.M., Bello, R., Vanhoof, K., “Fuzzy Cognitive Maps for Modeling Complex Systems”. Advances in Artificial Intelligence. 9th Mexican International Conference on Artificial Intelligence, MICAI 2010, Pachuca, Mexico, November 8-13, 2010, Proceedings, Part I, pp 166-174. Springer Berlin Heidelberg. ISBN 978-3-642-16760-7

[10] Li, X.: Dynamic Knowledge Inference and Learning under Adaptive Fuzzy Petri Net Framework. IEEE Transactions on Systems, Man, and Cybernetics – Part C: Applications and Reviews. vol.30, no.4, pp. 442-450, Nov 2000. ISSN 1094-6977

[11] Lazzerini, B., Mkrtchyan, L., "Risk Analysis Using Extended Fuzzy Cognitive Maps," 2010 International Conference on Intelligent Computing and Cognitive Informatics (ICICCI), pp.179,182, 22-23 June 2010. ISBN 978-1-4244-6640-5.

[12] Hagiwara, M., "Extended fuzzy cognitive maps," 1992 IEEE International Conference on Fuzzy Sysems, pp.795,801, 8-12 Mar 1992. ISBN 0-7803-0236-2.

[13] Puheim, M., Nyulászi, L., Madarász, L., Gašpar, V., "On Practical Constraints of Approximation Using Neural Networks on Current Digital Computers," IEEE 18th International Conference on Intelligent Engineering Systems : Proc., Tihany, Hungary, 3-5 July 2014, pp. 257-262, ISBN 978-1-4799-4616-7.

[14] Bennett, S., “A history of control engineering, 1930-1955,” IEE Control Engineering Series 47, 1993, ISBN 978-0-86341-299-8.

[15] Salmeron, J. L., “Modeling grey uncertainty with Fuzzy Grey Cognitive Maps”, Expert Systems with Applications, vol. 37, no. 12, pp. 7581–7588, 2010.