

Dynamic Treatment Regimes and Interference: Recent Developments in Estimation and Implementation

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Slide deck available at: mpwallace.github.io

Motivating example:

- Goal: Reduce cigarette dependence.
- Intervention: e-cigarette use.
- Challenge: personalized decision-making.



A personalized treatment rule example:

*"If patient is aged 35+, recommend e-cigarettes, otherwise
recommend alternative therapy."*

- Question: How do we choose the best decision rule?

Should our age cut-off be 25, 35, 45?

Some hypothetical data:

Participant	Age	e-cigarette use?	Dependence at 3 months
1	53	No	57
2	25	Yes	35
3	28	Yes	40
4	41	Yes	21
5	27	No	42
...



Goal: Identify treatment A that optimizes $E[Y|X, A]$

Identifying the best treatment regime

$$\underbrace{E[Y|X, A]}_{\text{Expected outcome (to be maximized)}} \quad A \in \{0, 1\}$$

- We might propose the following model

$$E[Y|X, A; \beta, \psi] = \beta_0 + \beta_1 X + A(\psi_0 + \psi_1 X)$$

“Recommend e-cigarettes ($A = 1$) if $\psi_0 + \psi_1 X > 0$ ”

- More generally:

$$\underbrace{E[Y|X, A; \beta, \psi]}_{\text{Expected outcome (to be maximized)}} = \underbrace{G(X; \beta)}_{\text{Treatment-free}} + \underbrace{\gamma(X, A; \psi)}_{\text{Blip}}$$

- Simplifies focus: choose A that maximizes $\gamma(X, A; \psi)$.

Identifying the best treatment regime

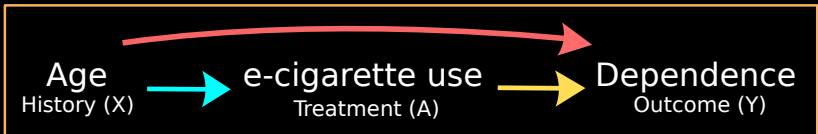
- Suppose the true outcome model is:

$$E[Y|X, A; \beta, \psi] = \beta_0 + \beta_1 X + \beta_2 X^2 + A(\psi_0 + \psi_1 X)$$

- But we propose:

$$E[Y|X, A; \beta, \psi] = \beta_0 + \beta_1 X + A(\psi_0 + \psi_1 X)$$

- Problem: A depends on $X \implies \psi_0, \psi_1$ mis-estimated.
- Solution: Account for this dependency.



$$E[Y|X, A; \beta, \psi] = G(X; \beta) + \gamma(X, A; \psi)$$

- Three models to specify:
 1. Treatment-free model: $G(X; \beta)$.
 2. Blip model: $\gamma(X, A; \psi)$.
 3. Treatment model: $P(A = 1|X; \alpha)$.
- Estimate ψ via WOLS of Y on covariates in blip and treatment-free models, with weights
 $w = |A - P(A = 1|X; \hat{\alpha})| = |A - \pi(X)|$.



Identifying the best treatment regime

- Suppose the true outcome model is:

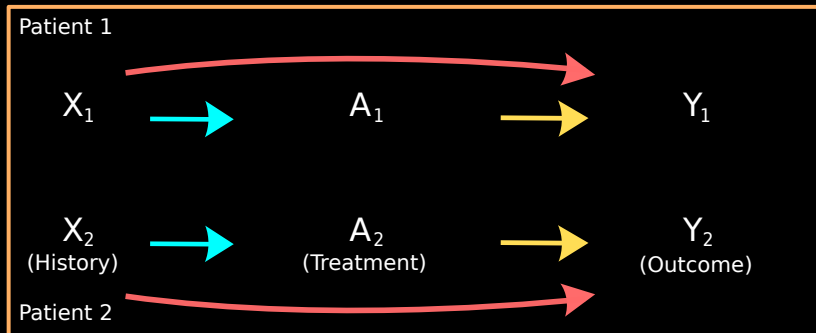
$$E[Y|X, A; \beta, \psi] = \beta_0 + \beta_1 X + \beta_2 X^2 + A(\psi_0 + \psi_1 X)$$

- But we propose:

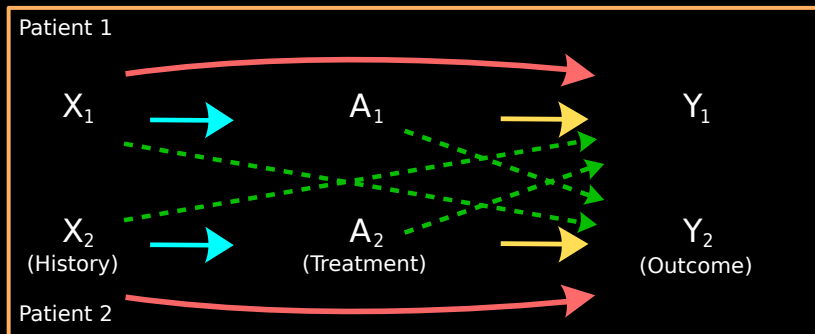
$$E[Y|X, A; \beta, \psi] = \beta_0 + \beta_1 X + A(\psi_0 + \psi_1 X)$$

- WOLS with weights $w = |A - P(A = 1|X; \hat{\alpha})| = |A - \pi(X)|$ will still yield consistent estimators of ψ_0, ψ_1 .
- Estimators are “doubly robust”: consistent if at least one of **treatment-free** or **treatment** components correctly specified.
- The **blip** must always be correct.

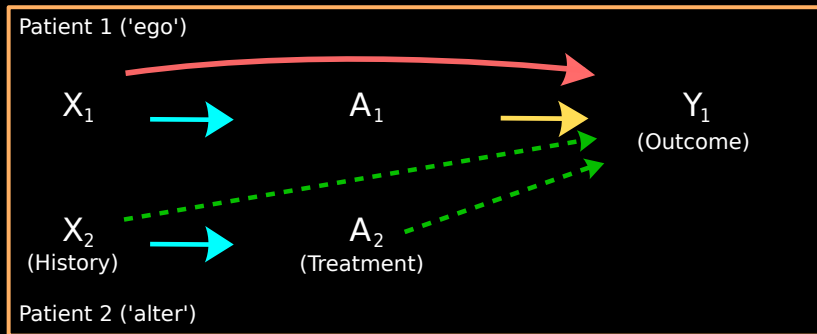




Challenge: Account for others.



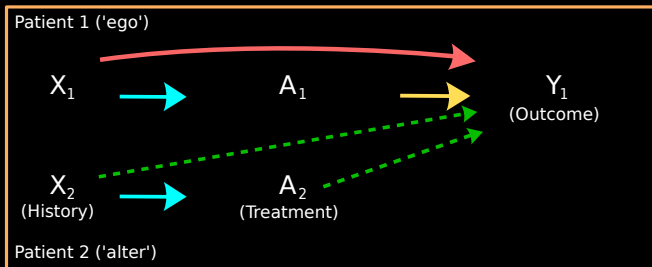
Challenge: Interference between neighbours.



Approach: Identify study unit ('ego') and neighbours ('alters').

- We might propose the following model

$$E[Y_1|X_1, X_2, A_1, A_2; \beta, \psi] = \beta_0 + \beta_1 X_1 + \beta_2 A_2 + A_1(\psi_0 + \psi_1 X_1 + \psi_1 A_2)$$



- More generally, let $t(A_{\mathcal{N}_i})$ = some function of neighbours' treatments, e.g.:
 - The number or proportion of treated neighbours.
 - The existence of a treated neighbour.
- Then can generalize outcome model to:

$$E[Y_i|\cdot] = \beta_0 + \beta_1 X_i + \beta_2 t(A_{\mathcal{N}_i}) + A_i(\psi_0 + \psi_1 X_i + \psi_1 t(A_{\mathcal{N}_i}))$$

Network propensity function for individual i with neighbours \mathcal{N}_i and treated neighbours $S_{i,A}$:

$$\pi_{i,A_i,S_{i,A}}(X_i, \mathcal{N}_i, X_i) = P(A_i \cap S_{i,A} | X_i, \mathcal{N}_i, X_i)$$

$$= \overbrace{\pi_i(X_i)^{A_i} (1 - \pi_i(X_i))^{1-A_i}}^{\text{Individual } i} \cdot \overbrace{\prod_{j \in S_{j,A}} \pi_j(X_j)}^{\text{Treated neighbours}} \cdot \overbrace{\prod_{j \in \mathcal{N}_i \setminus S_{i,a}} (1 - \pi_j(X_j))}^{\text{Untreated neighbours}}$$

dWOLS may be extended using the network propensity function, for example, WOLS for the outcome model

$$E[Y_1|X_1, X_2, A_1, A_2; \beta, \psi] = \beta_0 + \beta_1 X_1 + \beta_2 A_2 + A_1(\psi_0 + \psi_1 X_1 + \psi_1 A_2)$$

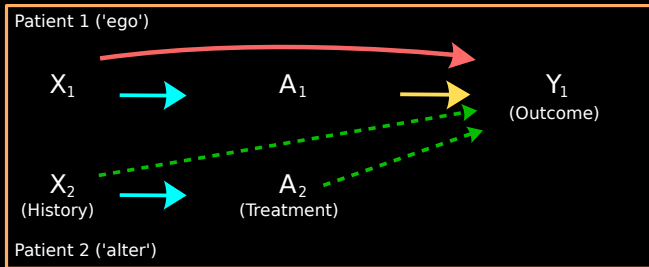
with weights

$$w_i = |A_i - P(A_i = 1|X_i = x_i)| \prod_j |A_j - P(A_j = 1|X_j)|$$

which retains the double robustness property.

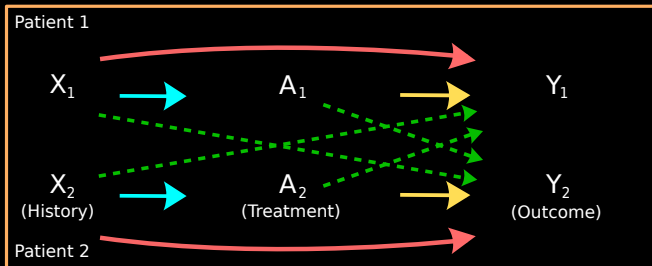
Extension: Simultaneous Optimization

- Limitation: Assumes an 'ego' setup:



Extension: Simultaneous Optimization

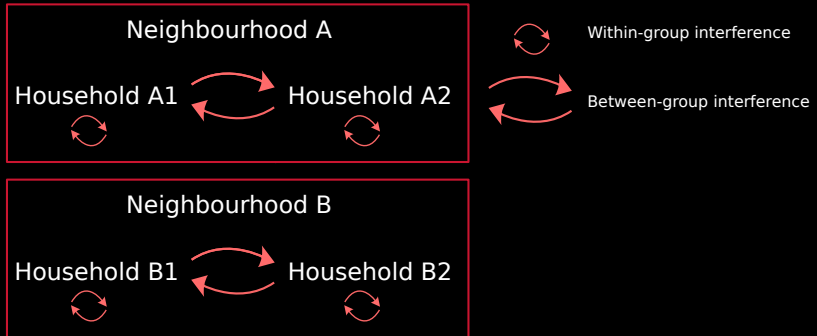
- Extension in a dyadic structure: identify and optimize a *dyad-health function*.



- Limitation: Assumes a continuous outcome.
- dWOLS: Extended to numerous other outcome types in the absence of interference.
- dWPOM: A dWOLS extension for ordinal outcomes with interference, via proportional odds model.

Extension: Hierarchical Models

Hierarchical structures of interference can evolve.



- **Interference** an important challenge for precision medicine.
- Progress in addressing interference for **continuous, ordinal, and utility-based** outcomes.
- Methods have been applied to the **Population Assessment of Tobacco Health (PATH) Study**.
- Upcoming work to address **hierarchical structures**.
- Future work concerns logistical challenges such as **implementation of treatment regimes, and cost constraints**.

Acknowledgments



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dWOLS extensions



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Hierarchical models

- **dWOLS**: M. P. Wallace and E. E. M. Moodie (2015). Doubly-robust dynamic treatment regimen estimation via weighted least squares. *Biometrics* **71(3)** 636-644.
- **Network Propensity Weights (main result presented)**: C. Jiang, M. P. Wallace and M. E. Thompson (2023). Dynamic treatment regimes with interference. *Canadian Journal of Statistics* **51(2)** 469-502.
- **Ordinal outcomes**: C. Jiang, M. E. Thompson, and , M. P. Wallace (2024). Estimating dynamic treatment regimes for ordinal outcomes with household interference: Application in household smoking cessation. *arXiv:2306.12865*, *Statistical Methods in Medical Research* (in revision).
- **Dyadic Networks**: M. Mussavi Rizi, J. A. Dubin, and M. P. Wallace (2023). Dynamic Treatment Regimes on Dyadic Networks. *Statistics in Medicine* (in review, please contact me!).

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