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A theory of visual control of braking based on information about time-to-collision

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Abstract. A theory is presented of how a driver might visually control his braking. A mathematical analysis of the changing optic array at the driver's eye indicates that the simplest type of visual information, which would be sufficient for controlling braking and would also be likely to be easily picked up by the driver, is information about time-to-collision, rather than information about distance, speed, or acceleration/deceleration. It is shown how the driver could, in principle, use visual information about time-to-collision in registering when he is on a collision course, in judging when to start braking, and in controlling his ongoing braking. Implications of the theory for safe speeds and safe following distances are discussed, taking into account visual angular velocity detection thresholds, and some suggestions are made as to how safety on the roads might be improved.

1 Introduction

In order to avoid colliding with stationary obstacles, other moving vehicles, and pedestrians, a driver has to register when he is on a collision course, start braking early enough, and adjust his braking to an appropriate level. In addition, when following another vehicle, he needs to keep a safe distance behind it to allow for the possibility that it might brake.

In controlling his braking and vehicle-following behaviour, the driver has to monitor visually, more or less continuously, his motion relative to the vehicle or other obstacle in his path, in relation to his distance from it. An understanding of how he does this, what visual information he uses, and what visual factors affect his performance is clearly important for improving road safety.

There has been some experimental work done on the problem, but the picture is far from complete. In vehicle-following situations, drivers' braking response times to the braking of the lead vehicle have been measured (e.g. Rockwell and Safford 1966; Rutley and Mace 1969). Even for an alert driver in an experimental situation the braking response time is about 1 s. However, if the lead vehicle is without brake lights, the time can be longer than 2 s, since the driver first has to register that he is closing on the lead vehicle.

The braking response—moving the foot to the brake pedal—is, however, only the first stage. The *braking adjustment* stage is just as critical, but it does not appear to have been investigated in any detail. A driver does not normally initiate full-power braking as soon as he sees that a lead vehicle is braking—among other things this would risk being run into from behind. Rather, the driver most likely adjusts his braking on the basis of his assessment of the urgency of the situation. Conventional brake lights do not help him in this, since they do not indicate how hard the lead vehicle is braking, and so the driver has to rely on direct (noncoded) visual information about how rapidly he is closing on the lead vehicle. If, therefore, the information is poor, there is the danger that the driver will not adjust his braking to a high enough level in time. For instance suppose he is following another vehicle at 80 km h^{-1} (50 miles h^{-1}) with a headway of 46 m (150 ft)—i.e. he is following the broad guideline of 3 ft per mile h^{-1} (0·56 m per km h^{-1}) given in the *British Highway Code* (1974)—and the lead vehicle suddenly brakes severely at 0·7*g*. Even if the driver had a fast braking response time of 1 s, if he were initially to brake mildly at $0 \cdot 1g$, he would have less than $1 \cdot 2$ s in which to increase his braking to $0 \cdot 7g$ to avoid collision.

The reason why it is important, and sometimes critical, to adjust braking to an appropriate level early enough is that the earlier a deceleration is applied, the more effective it is; for a given average deceleration, the higher the deceleration at the beginning, the shorter the stopping distance.

Adjusting braking to an appropriate level is, of course, equally important when stopping for a stationary obstacle or slowing down behind a more slowly moving vehicle. In these cases the driver also has to judge *when* to start braking. He does not normally start to brake as soon as he sees an obstacle in his path (e.g. a roundabout), nor does he normally brake at full power. Rather, he most likely controls his braking on the basis of visual information about his speed of approach in relation to his distance away from the obstacle. Spurr (1969) obtained deceleration profiles for skilled drivers stopping at nominated points from different speeds. The data are valuable and will be discussed later. Unfortunately, however, they give only a partial picture of braking performance; since only deceleration was monitored, accurate data were lacking on speed and distance from the stopping point during braking. No more-detailed investigations have apparently been carried out.

In short, little is known about how a driver visually controls his braking. The paucity of empirical data is probably due, in part at least, to the lack of a comprehensive theoretical formulation of the problem to guide research. [Mathematical models of vehicle-following behaviour have, however, been formulated and tested—see, e.g., Herman and Potts (1961).]

The present paper sets out to formulate a theory of visual control of braking. It is based on a mathematical analysis of the changing optic array at the driver's eye, aimed at discovering simple visual variables which afford the driver sufficient information for controlling his braking. The conclusion reached is that the simplest type of visual information which is sufficient for controlling braking and is also likely to be picked up easily by the driver, is information about time-to-collision, rather than information about distance, speed, or acceleration/deceleration, as might be supposed. It will be shown how the driver could, in principle, use visual information about time-to-collision in registering when he is on a collision course, in judging when to start braking, and in controlling his ongoing braking. Implications of the theory for safe speeds and safe following distances will be discussed, taking into account visual angular-velocity detection thresholds, and some suggestions will be made as to how safety on the roads might be improved.

2 The locomotor optic flow field

The visual stimulus for a driver—as, indeed, for anyone whose head is moving at all relative to the environment—is not a static optic array but an optic flow field, a continuously changing optic array. Therefore, it is to the optic flow field that we must look to discover what visual information is potentially available to the driver.

Gibson (1966) has argued that the optic flow field affords information both about the layout of the environment and about the movement of the observer relative to it. There is mathematical support for this argument; there are properties of the optic flow field which specify aspects of the environment and others which specify aspects of the observer's movement (Purdy 1958; Gordon 1966; Lee 1974).

The visual system is very adept at picking up both types of information. The first type, which is classically referred to as motion parallax, was recognised by Helmholtz (1925), and experiments have shown how sensitive the visual system is to this information (see e.g. Tschermak-Seysenegg 1939; Johansson 1973). The second type of information was first explicitly recognised by Gibson (1950), who termed it visual kinaesthesis. The visual system is highly sensitive to this information too, as is shown for example by experiments on visual control of balance (Lee and Lishman 1975). Indeed, the experiments indicate that vision affords the most sensitive kinaesthetic information.

Let us analyse the optic flow field at the driver's eye to see what information it affords him for controlling braking. We shall assume that the eye is moving along a linear path through an environment consisting of surfaces which are made up of spatially contiguous surface elements. A surface element may be thought of as a small patch which is optically differentiable from its neighbours by virtue of there being a discontinuity across its boundary in the intensity and/or spectral composition of the light reflected from the surface.

We shall consider a simple model of the eye, comprising a pin hole and a hemispherical 'retina' (figure 1). The model retains the essential optics of the human eye, while ignoring the lens system. Small cones of light reflected from the surface elements in the environment pass through the pin hole and strike the hemispherical retina, forming a changing 'patchwork quilt' of *optic images*—an optic flow field.

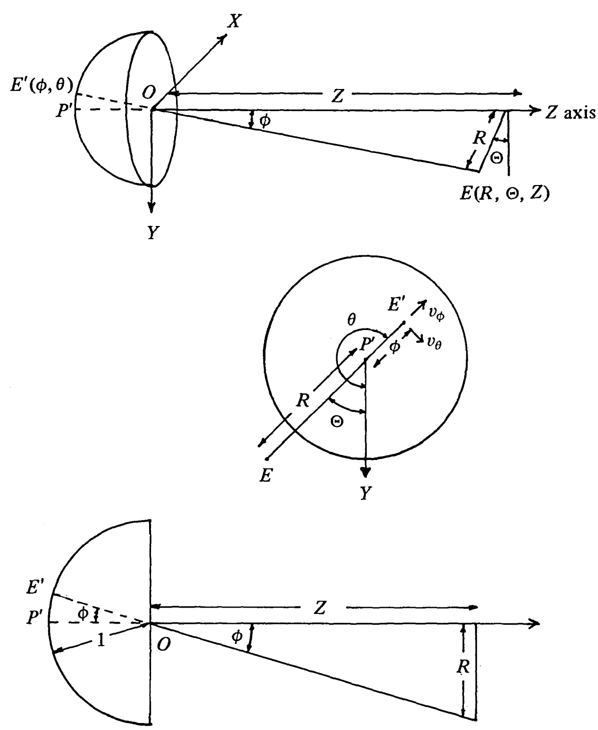

Figure 1. The locomotor optic flow field. A simplified eye, comprising a pin-hole O at the centre of a hemispherical 'retina' of unit radius, is at time t moving along the Z-axis with velocity $V(t)$. The eye carries with it a rectangular Cartesian reference frame $OXYZ$, with the Y-axis vertical. The optic image of the surface element to which the eye is heading is P', the 'locomotor pole'. E is an arbitrary surface element, whose position relative to the eye at time t is given by the cylindrical polar coordinates (R, Θ, Z). E' is the optic image of E; its position on the retina is given by the spherical coordinates (ϕ, θ), and it has components of velocity (v_ϕ, v_θ) along, respectively, lines of longitude and latitude, with respect to the locomotor pole P'. If E is stationary in the environment, or is moving in the same or the opposite direction as the eye, then E' moves along a line of longitude (i.e. $v_\theta = 0$).

Since we are primarily concerned with the *locomotor* optic flow field, which is determined only by the *translatory* movement of the eye relative to the environment, we shall consider the hemispherical retina to be fixed in the head. To take into account rotations of the human eye in its orbit, the pin hole may be considered to correspond to the nodal point of the lens system of the eye and the hemispherical retina to comprise a 'virtual retina'. As the eye rotates, approximately about its nodal point, the anatomical retina scans the virtual retina. Thus, the optic flow field, taken relative to the anatomical retina, has two components: a locomotor component and a uniform rotary component due to eye rotation. Since these two components are, in principle, separable, we shall ignore eye rotations in the analysis.

We want to determine relationships between *physical* variables describing the geometrical layout of the environment and the driver's movement, and *optical* variables of the locomotor optic flow field. These relationships will tell us what visual information is potentially available to the driver.

In figure 1 the optic geometry is shown and a notation defined. We shall first derive some basic equations. Suppose that the surface element, $E(R, \Theta, Z)$, is part of the fixed environment, (i.e. R and Θ are constant and Z varies over time). Then, from figure 1,

$$\Theta = \theta - \pi \text{ (radians)}; \tag{1}$$

differentiating with respect to time we obtain

$$v_\theta(\phi, \theta, t) = 0. \tag{2}$$

This means that the optic flow from stationary surface elements (and, in fact, also from elements which are moving parallel to the locomotion path of the eye) is along a set of great circles through a fixed point on the hemispherical retina. The fixed point is the locomotor pole, which is therefore optically defined. It specifies the point to which the eye is heading.

Further, for small values of ϕ,

$$\frac{Z(t)}{R} = \frac{1}{\phi}; \tag{3}$$

differentiating with respect to time we have

$$\frac{V(t)}{R} = \frac{v_\phi(\phi, \theta, t)}{\phi^2}. \tag{4}$$

From equations (3) and (4) we obtain

$$\frac{Z(t)}{V(t)} = \frac{\phi}{v_\phi(\phi, \theta, t)}. \tag{5}$$

Equations (4) and (5) imply that neither the distance coordinates (R, Z) of a surface element, nor the velocity of locomotion $V(t)$ is optically specified in *absolute* terms. Indeed, this could not be the case, since distance and velocity are in principle only definable *relative* to some standard distance. It can be shown, in fact, that distance and velocity are optically specified in terms of some fixed visible distance, e.g. eye-height above the ground (Lee 1974). However, as will be shown, visual information about distance and velocity as such is not necessary for controlling braking.

3 Visual specification of time-to-collision

Consider a driver closing on a more slowly moving lead vehicle, or a stationary obstacle. When and how hard he has to brake does not depend simply on his spatial proximity to the vehicle nor simply on his closing velocity and acceleration/deceleration, but on some relationship between these variables. One simple relationship is his *temporal* proximity to the vehicle—the time-to-collision.

Equation (5), which applies to the case both of a stationary obstacle and of a more slowly moving lead vehicle, shows that the time-to-collison if the closing velocity were maintained [i.e. $Z(t)/V(t)$] is optically specified. Designating the time-to-collision at time t by $T_c(t)$, we have, for small visual angles ϕ,

$$T_c(t) = \frac{\phi}{v_\phi(\phi,\theta,t)} \,. \tag{6}$$

In other words, for small visual angles, the time-to-collision is specified by the ratio of the values of the optic variables ϕ and v_ϕ at any point in the sector of the optic flow field corresponding to the near face of the obstacle.

How might a driver pick up the time-to-collision information? The problem is that the optic variables ϕ and v_ϕ are defined relative to the *optic array*, which is assumed to be *fixed* relative to the head, whereas the retina *moves* relative to the head. It might therefore seem that additional information is necessary about the position and movement of the eye in its orbit in order to determine the 'true' values of the optic variables, relative to the optic array, from their values relative to the retina.

However, while such additional information about eye movement and position might, in fact, be used, it is not necessary. A more efficient solution to the pick-up problem would seem to be afforded by the simple fact that when the visual angle subtended at the eye by the obstacle, or part of it, is small, then from equation (6) it follows that *every* image element of the obstacle in the optic array moves away from the locomotor pole at a rate v_ϕ which is proportional to its angular distance ϕ from the pole. In other words, the image of the obstacle undergoes a *pure dilation*. The inverse of the rate of dilation, which we shall designate by the optic variable $\tau(t)$, specifies the time-to-collision. That is

$$T_c(t) = \tau(t). \tag{7}$$

It will be noted that the value of the optic variable $\tau(t)$ on the retina is *independent* of how the eye is moving. In other words, it is a purely *relative* variable, that is, the kind on which the visual system seems very likely to operate (Gibson 1968). The driver could, in principle, register the value of $\tau(t)$ simply from the inverse of the proportionate rate of separation of the retinal images of any two points on the obstacle—e.g. the tail lights of a vehicle. That is

$$\begin{aligned}\tau(t) &= \frac{1}{\text{(rate of dilation of the retinal image of the obstacle)}} \\ &= \frac{\text{(angular separation of any two image points of the obstacle)}}{\text{(rate of separation of the image points)}}\end{aligned} \tag{8}$$

It has been shown experimentally how a dilating visual image, in specifying impending collision, can give rise to avoidant behaviour, both in human adults and animals (Schiff 1965) and in human infants (Bower et al 1970). It is also clear from everyday experience that a person can pick up time-to-collision (or time-to-contact) information visually. Consider, for example, catching a ball. You not only have to position your hand on the trajectory of the ball, but you also have to start closing your hand just before the ball reaches it. If the ball is moving fast, this requires precise timing. Playing a tennis shot similarly involves precise timing. Jumping down from a height is another clear example. The impact-absorbing muscular adjustments of the body have to be initiated at the right time *before* contacting the ground

(Melvill Jones and Watt 1971). Jumping with eyes shut from an unknown height, even a small one, can deliver a nasty jolt.

4 Visual specification of temporal headway

When following another vehicle the driver has to keep a safe distance away, and the faster he is travelling the greater the safe distance. One way he could do this would be to control his speed so that his *temporal headway* (i.e. the time gap) was never less than a certain margin value. Temporal headway could, in principle, be registered as easily as time-to-collision with a vehicle, providing the stretch of road behind the lead vehicle were clearly visible, since it is simply the time-to-contact with that part of the road which is currently right behind the lead vehicle. The question of safe temporal headways will be discussed in section 9.

5 Visual specification of collision courses

Broadly speaking, a driver is on a collision course with a lead vehicle or stationary obstacle if maintenance of the current dynamic state of his vehicle would result in collision. The relevant dynamic variable is the acceleration/deceleration of the vehicle

Table 1. The different types of collision course with a lead vehicle and their specifications. Notation: Z = vehicle separation; V = closing velocity; A = closing acceleration; D = closing deceleration; τ = visual variable specifying time-to-collision if V were kept constant; $\dot{\tau}$ = rate of increase of τ.

Type of collision course	Specification	
	physical	visual
Closing at a constant or accelerating rate	$V>0$; $A \geqslant 0$	$\tau>0$; $\dot{\tau} \leqslant -1$
Closing at an inadequate decelerating rate	$V>0$; $D<V^2/2Z$	$\tau>0$; $\dot{\tau}<-\frac{1}{2}$
Receding at a decelerating rate	$V<0$; $A>0$	$\tau<0$; $\dot{\tau}<-1$

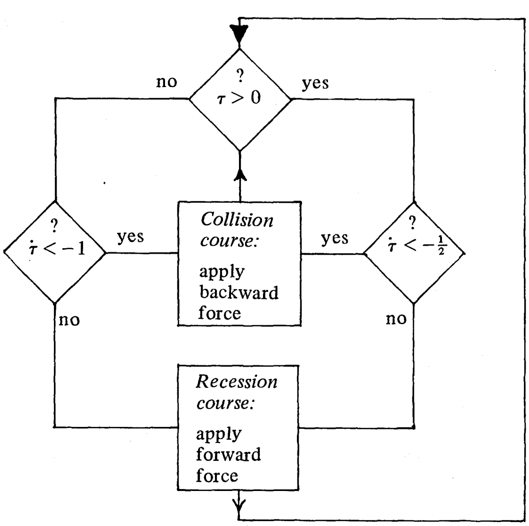

Figure 2. How the visual variable τ, which specifies the time-to-collision if the closing velocity were maintained, and its time derivative $\dot{\tau}$ could be used by the driver in determining the type of course he is on and hence the action he needs to take: whether to apply a backward force to his vehicle (or reduce the forward force on it), by braking harder or accelerating less, or a forward force; by doing the opposite.

rather than its velocity, since the latter is a second-order quantity which varies with the acceleration/deceleration. Furthermore, it is the acceleration/deceleration of his vehicle over which the driver has direct control; he can maintain it or change it almost instantaneously.

Therefore, it may be said that *a driver is on a collision course with a lead vehicle or stationary obstacle if, and only if, maintenance of his current acceleration/deceleration would result in collision* (assuming the acceleration/deceleration of the lead vehicle does not change). Thus, for example, a driver is not necessarily on a collision course with a lead vehicle simply if he is closing on it; if his deceleration is adequate, he is not on a collision course.

The three types of collision course are listed in table 1, together with their specifications. Further details are given in the Appendix.

Figure 2 illustrates how a driver could use the value of the visual variable τ and its time derivative $\dot{\tau}$ in determining whether he is on a collision or a non-collision (recession) course. The latter would be relevant to attempting to keep up with the traffic flow.

6 Judging when to start braking

A driver does not normally start to brake the moment he realises that he is on a collision course, for to do so would often unnecessarily slow his progress (consider, for example, approaching a roundabout or gaining on another vehicle). What seems most likely is that the driver bases his judgment on when to start braking on his assessment of the *urgency* of the situation.

The *objective* urgency depends on various factors: his rate of closing and separation from the obstacle; its speed and deceleration if it is another vehicle; and his own braking power. It is unlikely that the driver would be able to take all these factors into account, even if he could pick up all the relevant information accurately. Instead, he probably uses a *heuristic* to assess the urgency.

We have already shown how the driver could use the visual variable τ, which specifies the time-to-collision under constant closing velocity, to determine whether or not he is on a collision course. Could he use the same visual variable as a heuristic in judging when to start braking?

6.1 *Braking for a stationary obstacle*

Let us first consider approaching a stationary vehicle or other obstacle at a steady speed V. Suppose that the driver were to start his braking reaction when τ, the time-to-collision, reached a certain margin value τ_m. Suppose, further, that he required a free time t_f in which to achieve an appropriate deceleration (i.e. to move his foot to the brake and adjust his deceleration to an adequate level). Then, when this point was reached, his distance from the obstacle would be approximately $(\tau_m - t_f)V$ and so the constant deceleration required to avoid collision would be $V/2(\tau_m - t_f)$.

Assuming that the driver would require a free time of 2 s, figure 3 indicates that it would be safe to use a margin value τ_m of 5 s for speeds up to 110 km h^{-1} (68 miles h^{-1}), providing the brakes and traction were such that a deceleration of $0{\cdot}5g$ could be maintained. However, to brake comfortably at less than $0{\cdot}3g$ at these speeds, the appropriate margin value is about 8 s.

It seems possible that a driver might tend to start his braking reaction when the time-to-collision reaches a certain margin value, *irrespective* of his speed. This would mean that his braking would be harder the higher his speed. Spurr (1969), in fact, found this to be the case for test drivers who were required to stop at a designated point from various speeds up to 100 km h^{-1} (62 miles h^{-1}), but unfortunately it is not possible to determine from his deceleration data the margin values used by the drivers. The same phenomenon appears to be common on the roads and could account for some of the accidents which occur. The obvious danger in using a constant margin value is that the driver would most likely set the value on the basis of his normal experience of braking from moderate speeds. Thus, for example, if he habitually braked moderately hard at about $0 \cdot 35g$ at 50 km h^{-1} (31 miles h^{-1}), then for the same margin value of 4 s for all speeds this would mean that above 100 km h^{-1} (62 miles h^{-1}) his deceleration would need to be greater than $0 \cdot 7g$, which would normally be impossible.

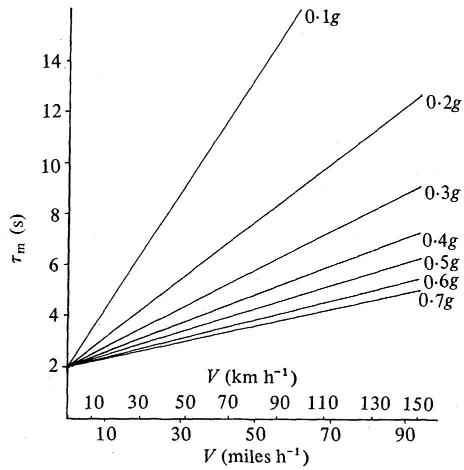

Figure 3. The time-to-collision margin values τ_m at which the driver would have to start his braking reaction when approaching a stationary obstacle at a speed V if he were to be able to stop with the constant decelerations indicated. It is assumed that he would require a free time of 2 s in which to attain the appropriate deceleration.

6.2 *Braking behind a moving vehicle*

Now let us consider closing on a moving lead vehicle. If it were maintaining a steady speed, then the margin time-to-collision heuristic would work as well as if it were stationary. However, if it were braking, then the driver would usually have to brake harder than if it were stationary. The reason is that, though his stopping distance would be increased by the stopping distance of the lead vehicle, if his closing velocity were low he would get very close to the vehicle before the time-to-collision reached the margin value. For example, consider a driver travelling at 110 km h^{-1} (68 miles h^{-1}) and closing on a lead vehicle. Suppose that he were to start his braking reaction early (for a stationary obstacle), when the time-to-collision reached a margin value of 8 s, and that he required 2 s free time in which to achieve appropriate braking. If when the margin value was reached the lead vehicle's speed was 90 km h^{-1} (56 miles h^{-1}) and it was braking hard at $0 \cdot 7g$, then the driver would be 44 m away when he started his braking reaction, 19 m away when he actually started braking hard, and the constant deceleration required would be impossibly high ($1 \cdot 7g$)—as compared with 245 m, 184 m and $0 \cdot 26g$ if the lead vehicle had been stationary.

It is clear, therefore, that it would not be safe to use the same margin time-to-collision when closing on a *moving* vehicle as when closing on a stationary vehicle. This is not to say, however, that a driver might not adopt the practice. Many drivers do, in fact, get too close to the vehicle in front before braking.

Is there an additional heuristic the driver might use to improve his safety? One possibility is a *margin temporal headway*. As pointed out in section 4, the temporal headway is simply the time-to-collision with that part of the road currently directly behind the lead vehicle, and so it could in principle be registered as easily as the time-to-collision with the vehicle itself, providing the stretch of road behind the lead vehicle were clearly visible.

Suppose the driver were to use the two heuristics in combination. That is, suppose he were to start his braking reaction when either the time-to-collision with the lead vehicle or the temporal headway reached a margin value. Table 2 shows the maximum constant decelerations which could be required using various margin values. The method of calculation is given in the Appendix.

It is clear from table 2 that a safe heuristic, when closing on a moving vehicle at speeds up to 110 km h^{-1} (68 miles h^{-1}), would be to start the braking reaction when, irrespective of the time-to-collision, the *temporal headway* reached a margin value of about 5 s, if a $0{\cdot}5g$ deceleration could be sustained, or a value of about 8 s if only a moderate deceleration of about $0{\cdot}3g$ were desired or possible.

However, in poor visibility, when the only thing clearly visible some distance ahead might be the tail lights of a vehicle, it might not be possible to judge the temporal headway accurately. In this case, time-to-collision, which would be likely to be more easily detected, could be used. However, as the table indicates, the time-to-collision margin value should be as large as possible. In fact, in poor visibility the only relatively safe rule would seem to be for the driver to start his braking reaction as soon as he registers that he is on a collision course.

Table 2. The maximum constant decelerations (in units of g) which could be required when closing on a lead vehicle braking at $0{\cdot}7g$, if the driver were to start his braking reaction when either the time-to-collision reached a margin value τ_m or the temporal headway reached a margin value Δt_m, and if he required 2 s free time in which to achieve an appropriate deceleration. If, when the driver started his braking reaction, the lead vehicle were travelling at a speed other than the critical speed V_L^*, the constant deceleration required would be less. The decelerations are derived from equations (17) to (19), which are given in the Appendix.

Δt_m(s)	τ_m(s)	$\frac{V_L^*}{V}$	Speed, $V\left(\frac{\text{km h}^{-1}}{\text{miles h}^{-1}}\right)$				
			30 19	50 31	80 50	110 68	150 93
0	$\geqslant 0$	1·00	∞	∞	∞	∞	∞
2	$\leqslant \Delta t_m$	0	∞	∞	∞	∞	∞
2	8	0·75	1·24	1·24	1·24	1·24	1·24
2	30	0·93	0·80	0·80	0·80	0·80	0·80
2	∞	1·00	0·70	0·70	0·70	0·70	0·70
4	$\leqslant \Delta t_m$	0	0·21	0·35	0·57	0·78	1·06
4	8	0·50	0·20	0·31	0·47	0·61	0·77
4	30	0·87	0·17	0·26	0·35	0·42	0·49
4	∞	1·00	0·16	0·23	0·31	0·37	0·42
6	$\leqslant \Delta t_m$	0	0·11	0·18	0·28	0·39	0·53
6	8	0·25	0·10	0·17	0·28	0·38	0·51
6	30	0·80	0·10	0·15	0·22	0·29	0·36
6	∞	1·00	0·09	0·14	0·20	0·25	0·30
8	$\leqslant \Delta t_m$	0	0·07	0·12	0·19	0·26	0·35
8	8	0	0·07	0·12	0·19	0·26	0·35
8	30	0·73	0·07	0·11	0·16	0·22	0·28
8	∞	1·00	0·06	0·10	0·15	0·19	0·24

7 Controlling braking

Judging *when* to start braking is only the first stage. The driver then has to *control* his braking force, and to do this he needs information about the adequacy of his current deceleration. Without this information the driver is effectively braking blind. Emergency braking is blind braking, its purpose being to stop in the shortest possible distance. Normal braking, however, is smooth and controlled.

What visual information does the driver have for controlling his braking? In section 5 (see also figure 2), it was shown that the adequacy of the current deceleration is specified in the optic flow field at the driver's eye by the value of the time derivative, $\dot{\tau}$, of the visual variable τ, which specifies the time-to-collision if the closing velocity were kept constant. The current deceleration is adequate if and only if $\dot{\tau} \geqslant -0{\cdot}5$. The driver could, therefore, control his braking by monitoring the value of $\dot{\tau}$ and adjusting his deceleration so that $\dot{\tau}$ is always greater than $-0{\cdot}5$. In particular, he could attempt to maintain $\dot{\tau}$ at a safe margin value $\dot{\tau}_m$. Let us examine the consequences of his achieving this.

7.1 *Stopping at a stationary obstacle*

Figure 4 shows normalised plots of deceleration against time and velocity against separation for a hypothetical driver, who in stopping at a stationary obstacle is maintaining the visual variable $\dot{\tau}$ at a safe margin value $\dot{\tau}_m$. (The method of calculation is given in the Appendix.) It will be seen that, for all values of $\dot{\tau}_m$ other than $-0{\cdot}5$, deceleration *decreases* monotonically with time, and the higher the value of $\dot{\tau}_m$, the higher the initial deceleration, the greater the separation when the vehicle reaches a particular closing velocity, and the longer the stopping time. In other words, at any time during the braking the deceleration is higher than it strictly needs to be (i.e., if it were maintained, the vehicle would stop short), and this is more so the higher the value of $\dot{\tau}_m$. The hypothetical driver is, in effect, allowing himself a safety margin, which is greater the higher the value of $\dot{\tau}_m$.

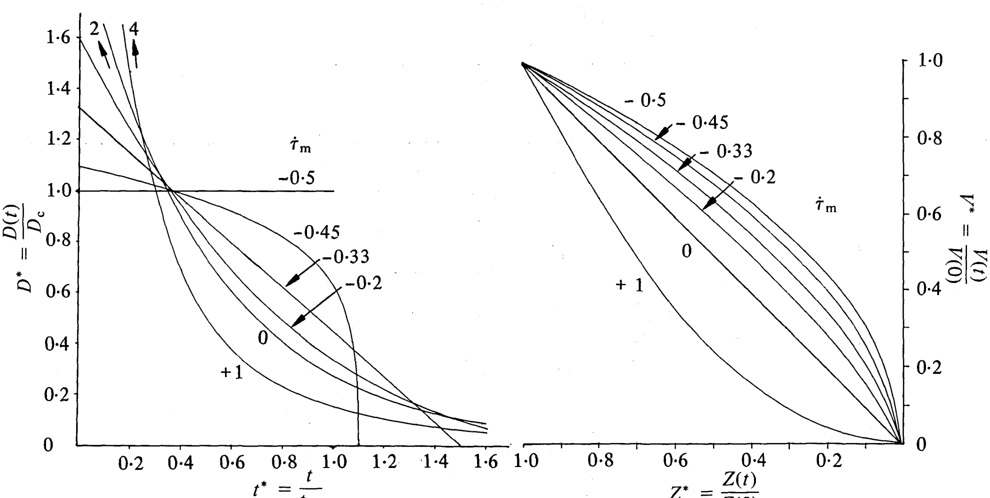

Figure 4. Normalised deceleration versus time and velocity versus separation curves for a hypothetical driver who is controlling his braking, to stop at a stationary obstacle, by maintaining the visual variable $\dot{\tau}$ at different safe margin values $\dot{\tau}_m$. The visual variable $\dot{\tau}$ is the time derivative of the visual variable τ which specifies the time-to-collision if the current closing velocity were kept constant. $D(t)$, $V(t)$, and $Z(t)$ are, respectively, the closing deceleration, closing velocity, and separation at time t. D_c is the constant deceleration which would be required. t_{bc} is the braking time under a constant deceleration. $V(0)$ and $Z(0)$ are the initial closing velocity and separation.

It seems likely that a driver does, in fact, control his braking by attempting to maintain a safe margin value of $\dot{\tau}$. Spurr (1969) found that the deceleration curves of test drivers stopping at a nominated point tended to conform to the pattern of an initial steep linear increase in deceleration followed by a gradual decrease. Furthermore, he found that many of the drivers were remarkably consistent in their braking from different speeds. For each driver he plotted on the same graph the deceleration curves for different speeds in a dimensionless form (i.e. deceleration in units of the mean deceleration during the stop, and time in units of the braking time). The data in figure 5 are for one driver and are taken from figure 2 of Spurr's paper; the curve is what would have obtained if the driver had maintained a margin value $\dot{\tau}_m$ of $-0{\cdot}425$. As will be seen, the agreement is quite close, from the initial peak deceleration onwards (the initial build up of deceleration is not taken into account in the theory, it being assumed that the driver attains the peak deceleration instantaneously).

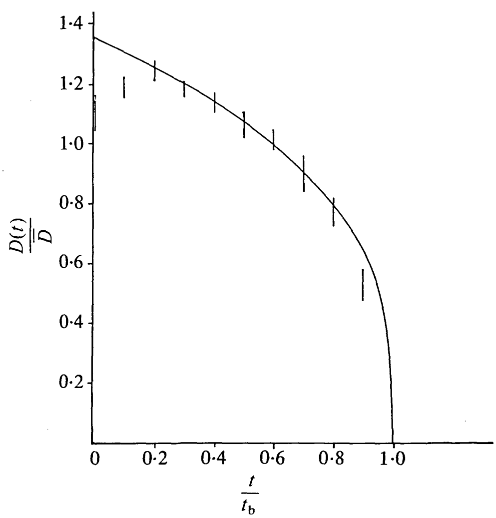

Figure 5. The data, taken from Spurr (1969, figure 2), are for a driver stopping at a nominated point from various speeds up to 100 km h^{-1} (62 miles h^{-1}). They are plotted in a dimensionless form: $D(t)$ is the deceleration at time t after starting to brake; $\bar{D}$ is the mean deceleration during the stop; t_b is the stopping time. The curve is derived from the theory given in the text. It is what would have obtained if the driver had been controlling his braking by maintaining the visual variable $\dot{\tau}$ at a margin value of $-0{\cdot}425$.

7.2 *Braking behind a moving vehicle*

Since the theory of visual control of braking described above is expressed in terms of closing velocities and decelerations (i.e. velocities and decelerations *relative* to the obstacle), it applies equally to braking behind a *moving* vehicle. Thus, if a lead vehicle were *stopping,* the driver could in principle stop safely right behind it by maintaining the visual variable $\dot{\tau}$ at *any* safe margin value $\dot{\tau}_m$ greater than or equal to $-0{\cdot}5$. (His deceleration curve would, of course, be different from those shown in figure 4, since it would be influenced by the deceleration of the lead vehicle.)

However, if he were braking to *follow* another vehicle, his aim would not be to reduce the closing velocity to zero just when he reached the vehicle, but rather when he was a safe following distance away from it. One way he might do this is to control his braking by maintaining $\dot{\tau}$ at a *positive* margin value. This would mean that the time-to-collision, if the current closing velocity were maintained, was progressively increasing, and so he would never reach the lead vehicle. Furthermore, as shown in figure 4, the more positive $\dot{\tau}_m$ is, the quicker is the closing velocity reduced. Therefore, if the driver were to brake using a relatively high positive margin value of $\dot{\tau}$, he would quickly reduce his closing velocity to near zero, and he could then make finer adjustments to his speed based on his assessment of his temporal headway.

8 Safe speeds when approaching a stationary obstacle

Consider a driver approaching a lead vehicle or stationary obstacle of width W at a constant closing velocity V. Suppose that, at time t, his distance from the obstacle is $Z(t)$ and the visual angle subtended at his eye by the obstacle is $\alpha(t)$. Then, if $Z(t)$ is large compared with W,

$$\alpha(t) = \frac{W}{Z(t)}; \tag{9}$$

differentiating with respect to time, we obtain

$$\dot{\alpha}(t) = \frac{WV}{[Z(t)]^2}, \tag{10}$$

where $\dot{\alpha}(t)$ denotes the rate of increase of the visual angle α at time t.

From equations (9) and (10), the time-to-collision at time t, $T_c(t) = Z(t)/V$, is therefore visually specified by

$$T_c(t) = \tau(t) = \frac{\alpha(t)}{\dot{\alpha}(t)}. \tag{11}$$

Thus the driver's ability to register the value of the visual variable $\tau(t)$, and consequently his ability to obtain information for judging when to start braking and for controlling his braking, will depend on his ability to register the visual angular velocity $\dot{\alpha}(t)$. If a threshold visual angular velocity $\dot{\alpha}_{th}$ is assumed which is independent of the visual angle[(1)], it follows from equations (9), (10), and (11) that the driver will not be able to detect that he is closing on the obstacle until he is at a threshold time-to-collision τ_{th}, where

$$\tau_{th} = \left(\frac{W}{V\dot{\alpha}_{th}}\right)^{1/2}. \tag{12}$$

Thus τ_{th} will be shorter the faster the closing velocity V, and longer the greater the width W of the obstacle. The implications of this are illustrated in figure 6. Curves are shown for obstacles of various widths and for two values, $\frac{1}{12}$ and 1 deg s^{-1}, of the threshold visual angular velocity. These values probably represent a fair estimate of the range normally encountered in driving in all but dense fog. The

[(1)] Hartmann (1972) has obtained data which indicate that the visual angular velocity detection threshold $\dot{\alpha}_{th}$ can depend to some extent on the visual angle α (at least for small angles in the range 0·03–1 deg). The empirical formula he proposed, translated into the present notation, was

$$\dot{\alpha}_{th} = k\alpha^{\lambda},$$

where λ is zero for high-contrast visual displays and increases up to about 1·5 for very low contrast, and k depends on the observation time. Janssen (1974) has proposed a similar empirical formula, with a value of λ equal to 0·75, to account for the threshold data obtained by Harvey and Michon (1974). The implication of the formula is that at threshold the value of the visual variable λ specifying the time to collision will be

$$\tau_{th} = k^{-1/(2-\lambda)}\left(\frac{W}{V}\right)^{(1-\lambda)/(2-\lambda)}.$$

This would yield similar-shaped curves to those shown in figure 6, for $0 \leqslant \lambda < 1$.

straight lines in the figure show the times-to-collision, for different closing velocities, at which the driver would need to start his braking reaction if he were to be able to stop at the constant decelerations indicated, assuming that he would need a free time of 2 s in which to attain the appropriate deceleration.

In general, it will be seen that the *faster* the closing velocity, the *shorter* will be the time-to-collision when the driver is first able to detect it, and, at the same time, the *longer* does the time-to-collision need to be at which he should start his braking reaction. In other words, danger increases as the closing velocity increases. Maximum safe closing velocities correspond to the points where the straight lines cut the curves of figure 6. The implication is that if the driver is closing at a higher velocity, he should initially brake 'blind' until he can pick up time-to-collision information for controlling his braking.

The danger is also greater the narrower the visible width of the obstacle and the lower its visibility. This clearly has implications for the design of rear lights on vehicles (Hartmann 1972; Roszbach 1972), as well as lighting on stationary obstacles such as road works and tollbooths.

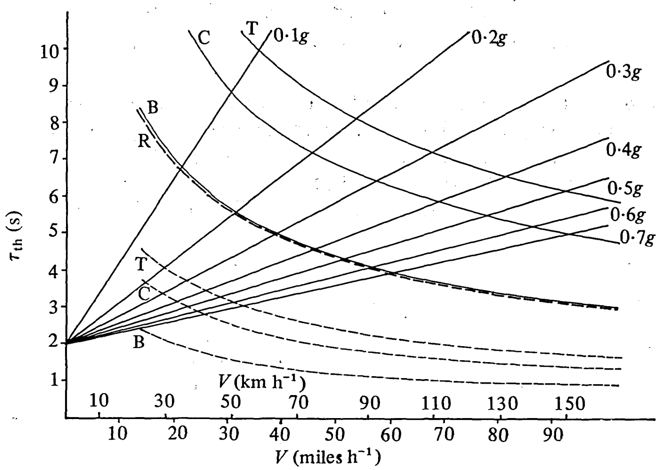

Figure 6. The theoretical curves show how the maximum time-to-collision τ_{th} which a driver could register decreases (a) as the closing velocity V increases, (b) as the width of the obstacle decreases (R = end of road, width 7·6 m; T = truck, width 2·2 m; C = car, width 1·5 m; B = motor bike rider, width 0·6 m), and (c) as the visual angular velocity threshold $\dot{\alpha}_{th}$ increases (solid lines are for good visibility, $\dot{\alpha}_{th} = \frac{1}{12}$ deg s^{-1}; broken lines are for poor visibility, $\dot{\alpha}_{th}$ = 1 deg s^{-1}). The straight lines show the times-to-collision at which the driver would have to start his braking reaction in order to stop safely at the constant decelerations indicated, assuming that he would need a free time of 2 s in which to attain the appropriate deceleration. The points where the straight lines cut the curves correspond to maximum safe closing velocities.

9 Safe following headways and speeds

A driver, following another vehicle at a constant speed, may be said to be maintaining a safe following distance if and only if he can, by braking, avoid colliding with the lead vehicle, no matter how hard it may brake.

In order to determine safe following distances for different speeds, we need to consider the nature of the braking task which confronts the driver when the lead vehicle brakes. First, he has to register that he is closing on the lead vehicle. Second, he has to judge when to start braking. And finally, he has to control his deceleration appropriately.

If we are concerned with safe following distances for any eventuality, we may assume that the lead vehicle does not show brake lights. Then, for each stage of the braking task, the driver is dependent on visual closing information, about how he is moving relative to the lead vehicle. Even if the lead vehicle does show brake lights, the driver is still dependent on visual closing information for the second and third stages of the braking task—and very likely for the first stage too, since present-day brake lights do not indicate the severity of braking and so only serve to inform the driver that he *may* need to brake.

Thus it may be said that a driver is maintaining a safe following distance if and only if he can register early enough that he is closing on the lead vehicle. There are two critical times: the *closing registration time,* that is the time that elapses before the driver can register that he is closing on the lead vehicle; and the *critical braking initiation time,* that is the maximum time, from when the lead vehicle starts braking, that the driver can allow to elapse before he has to brake at his full power. The difference between these times may be referred to as the driver's *free time.* During this time he has to move his foot to the brake and attain an adequate deceleration.

Let us assume, as in section 8, that the driver would not be able to register that he was closing until the rate of increase of the visual angle subtended at his eye by the lead vehicle exceeded a certain threshold value, which, among other things,

Table 3. Minimum safe temporal headways (to the nearest second) which would allow the driver 2 s free time in which to attain adequate braking if the lead vehicle were to stop at $0 \cdot 7g$. Blank cells mean that there is no safe temporal headway at that speed. The values are based on assumptions given in the text. *British Highway Code* (1974) guidelines: *H.C.* (a)—3 ft per mile h^{-1} ($0 \cdot 56$ m per km h^{-1}); *H.C.* (b)—"overall shortest stopping distance" (with $0 \cdot 7g$ braking capacity).

Braking capacity	Visibility	Lead vehicle	Speed $\left(\frac{\text{km h}^{-1}}{\text{miles h}^{-1}}\right)$				
			30 19	50 31	80 50	110 68	150 93
$0 \cdot 3g$	good	truck	3	4	5	8	
		car	3	4	6		
		motor bike	3	5			
	poor	truck	4				
		car	4				
		motor bike					
$0 \cdot 5g$	good	truck	3	3	4	5	10
		car	3	3	4	6	
		motor bike	3	4	6		
	poor	truck	3				
		car	3				
		motor bike					
$0 \cdot 7g$	good	truck	2	2	3	3	6
		car	2	2	3	4	8
		motor bike	2	3	4		
	poor	truck	3	4			
		car	3				
		motor bike					
H.C.(a)			2	2	2	2	2
H.C.(b)			2	2	3	3	4

would depend on the visibility. Then the closing registration time would be longer the higher the driver's visual angular velocity threshold, and also, as shown in section 8, the narrower the width of the lead vehicle. The closing registration time would also depend on the deceleration of the lead vehicle, the initial speed, and the following distance (see Appendix), as would the critical braking initiation time, which would depend also on the driver's braking capacity.

A computer analysis was run, using the formulae given in the Appendix, to determine minimum safe temporal headways under a variety of conditions. The results are given in table 3. The deceleration of the lead vehicle was taken as $0 \cdot 7g$. The width of the truck, car, and motor bike rider were taken as $2 \cdot 2$ m, $1 \cdot 5$ m, and $0 \cdot 6$ m respectively. It was assumed that the driver would need a minimum 2 s free time, and that in good visibility his visual angular velocity threshold would be about $\frac{1}{12}$ deg s^{-1} and in poor visibility about 1 deg s^{-1}.

Table 3 indicates that, in general, the temporal headway needs to be longer the higher the speed, the lower the visibility, and the narrower the lead vehicle. The table also indicates that there are *maximum safe following speeds,* which depend on the conditions. At higher speeds than these there is no safe temporal headway, as is indicated by the blank cells in the table. The reason for this is that as the speed increases so does the safe temporal headway, and so, beyond a certain speed, the temporal headway would be so long for the conditions that the lead vehicle would have stopped before the driver had registered that he was closing on it. The maximum safe following speeds, therefore, correspond to the maximum safe speeds when approaching a *stationary* vehicle, which are indicated in figure 6 (the calculations for figure 6 and table 3 were based on the same assumptions about visual angular velocity thresholds).

10 Improving road safety

It has been shown that there is potentially available to the driver visual information about time-to-collision and temporal headway, with respect to a lead vehicle or stationary obstacle, and that this information would be sufficient for controlling all aspects of braking: registering when he is on a collision course, judging when to start braking, and controlling his deceleration. It has also been shown how the driver's ability to pick up the visual information would be affected by factors such as his visual angular velocity detection threshold, which would vary with the visibility, and the visible width of the obstacle.

What are the implications for improving road safety? One serious accident problem is rear-end collisions. They are alarmingly frequent. For example, in Britain in 1974 over 23000 vehicles involved in injury accidents were struck from behind as they were travelling along the road, and 29% of all accidents involving two or more vehicles occurred when they were all travelling in the same direction[(2)]. In addition, there were large numbers of stationary vehicles involved in accidents: over 20000 parked vehicles, over 12000 vehicles going ahead but held up, over 43000 vehicles turning right or waiting to, and over 10000 vehicles turning left or waiting to[(3)]. (While the latter two categories do not specify whether the vehicle was moving or stationary, it is reasonable to assume that a fair proportion of the vehicles were in fact stopped and struck from behind.)

The problem appears to be more acute during the hours of darkness. In 1974 the fatal and serious injury rear-end accident rate (per vehicle km) during hours of darkness was about twice that during daylight hours[(4)].

[(2)] *Road Accidents in Great Britain 1974* (London: HMSO) p xiii.

[(3)] *Road Accidents in Great Britain 1974* (London: HMSO) p 53.

[(4)] Computed from statistical data provided by the Transport and Road Research Laboratory.

How might the frequency of rear-end collisions, particularly in poor visibility, be reduced? In the first place, it is very likely that if the rear of a vehicle were better delineated this would improve a driver's ability to detect how he was closing on the vehicle, and hence improve his ability to judge when to start braking and how hard to brake, as well as his ability to maintain a safe headway. Present-day tail lights most likely give the driver rather poor visual information about closing, since the only relevant visual variable is the rate of increase of the visual angle subtended at his eye by two lights. [Janssen (1972) has shown that this variable is a much more powerful source of information about closing than any change in the apparent brightness of the lights.] Various proposals have been made for improving rear lighting on vehicles by, for example, spatially separating the tail, brake, and turn lights (see, e.g., Mortimer 1970), but none of the proposals has been implemented on the roads.

A possibility which does not seem to have received much attention is to give the rear of a vehicle at night more visual texture, as it has in daylight. The visual image of the vehicle would then comprise an optic flow pattern in which the relative motions between closely adjacent visual image elements might be more readily detected. One way of achieving this might be to have a light panel, with alternating clear and opaque vertical bars, across the back of the vehicle. Another way might be to have a striped reflector strip on the back bumper, similar to that which has been mandatory on heavy goods vehicles in Britain since 1971 and which has apparently helped reduce the number of rear-end collisions with such vehicles, especially when they are parked at night(5).

Another possibility is a strip of lenticular material, with either a reflector strip or a light behind it, which is so constructed that the driver sees a light band across the back of the vehicle which gets progressively wider the closer he gets to the vehicle. The device would act like a visual velocity amplifier. When closing, the visual image of the light band would expand faster than the visual image of the vehicle, thus making closing more detectable. Furthermore, if there were a striped marker strip directly above the light band, then (a) the expansion of the visual image of the band relative to the marker strip should be readily detectable, since dynamic vernier acuity would be operative, and (b) the width of the band relative to a calibrated marker strip could indicate the distance between the vehicles and so facilitate control of headway.

Another improvement that could be made is in brake lights. The problem with conventional brake lights is that they do not give the driver information relevant to judging when to brake, how hard to brake, or, indeed, whether it is necessary to brake at all. For such judgments he has to rely on direct (noncoded) visual information about how he is closing on the vehicle. While in most circumstances this information may be adequate, in more extreme situations it might not be.

One situation where a driver could benefit from additional coded information is when the vehicle that he is following brakes hard and/or brakes to stop. An 'imperative' brake light signal would be useful here in informing the driver that he should quickly start to brake and raise his deceleration to a high level as soon as possible(6).

Another situation is where the driver is closing on a lead vehicle which is stopping, is stopped, or is travelling at an unusually slow speed on a fast road. In each of those situations, the driver has to brake (unless he can overtake the vehicle), and so

(5) *Road Accidents in Great Britain 1974* (London: HMSO) p xi.

(6) Rutley and Mace (1969) found that a multiple brake-light system, which indicated three levels of deceleration, led to a small reduction in braking response time in vehicle following. They concluded that a two-level system might be more justified.

some signal informing him of this fact should be helpful. Such a signal should be especially helpful if he were closing at a fast rate, particularly in the dark or in poor daytime visibility, since, as was pointed out in section 8, he could be only a short time away from collision with the vehicle before he is able to pick up visual closing information, and so he might well start braking too late.

The same imperative brake light signal could be used in both the following and closing situations since its meaning would be the same, namely: "Start braking now and brake as hard as the circumstances allow until you can detect how you are closing and so can adjust your deceleration appropriately".

The imperative brake lights could, for instance, consist of two L-shaped bars of red light, one on each side of the vehicle. The vertical segment would aid detection of the signal down a line of traffic. Conventional, mild-warning brake lights could be placed at the corners of the L. The imperative brake lights could be activated by a simple transducer in the brake line, which responded only when the pressure reached a certain level, and they would remain on until cancelled by pressure on the accelerator pedal. Thus, in emergency braking the imperative brake lights would go on automatically, and in normal stopping the driver could activate them by brief hard pressure on the brake pedal. In both cases the lights would remain on when the vehicle came to rest, and would be turned off only when it started up again. When travelling at an unusually slow speed, the imperative brake lights could be turned on by a dash board switch.

Another important safety problem is maintaining a safe headway when following another vehicle. Most drivers apparently find it difficult to judge headways, or choose to drive dangerously[7]. It is an easy matter for a driver to *calculate* his temporal headway with reasonable accuracy. He simply has to estimate, e.g. by counting, how long it takes him to reach that point, say at the side of the road, which the lead vehicle is now passing. However, this is a distracting and time-consuming task which could reasonably be carried out only in relatively relaxed driving conditions. It would be much better if a driver could be trained to directly *perceive* the temporal headway. As pointed out in section 4, the temporal headway is visually specified by the inverse of the rate of dilation of the visual image of the section of road directly behind the lead vehicle. If it could be shown that a driver can be readily trained to pick up this information, there would be a case for incorporating the training in driving schools. A way of improving a driver's ability to pick up information about his temporal headway at night might be to have lights hidden from view behind the back bumper, which illuminate the road directly behind a vehicle.

Finally, there is the problem of fog on motorways. Why do drivers often drive too fast for the conditions? It would seem that one basic problem is that, if there is no vehicle visible ahead, the driver will be uncertain as to the range of his visibility. A possible visual aid might therefore be to have small configurations of lights on posts down the side of the motorway (e.g. the 100 m posts on British motorways). The driver could then assess how far ahead in time he can clearly see by, for example, starting to count when he first detects that he is closing on the lights and stopping counting when he reaches them. If he were to adjust his speed until the time was such that he could, in principle, stop at the lights (about 5 s, say), he should be able to stop if he came upon a stationary vehicle.

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[7] *Road Accidents in Great Britain 1974* (London: HMSO) p xiv.

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APPENDIX

A1 Notation

For convenience, the principal symbols used in the text are listed below. Where a symbol is followed by (t), it means that its value is a function of time.

Physical variables

t	time
$Z(t)$	distance from an obstacle
$V(t)$	closing velocity on the obstacle
$D(t)$	closing deceleration on the obstacle
$A(t)$	closing acceleration on the obstacle
W	width of the obstacle
$T_c(t)$	time-to-collision with the obstacle if the closing velocity were kept constant
t_f	free time required by the driver to move his foot to the brake pedal and attain an adequate deceleration

Visual variables

$\alpha(t)$	visual angle subtended by an obstacle (or part of it)
$\dot{\alpha}(t)$	rate of increase over time of $\alpha(t)$
$\tau(t)$	visual variable specifying $T_c(t)$, the time-to-collision with the obstacle if the closing velocity were kept constant
$\dot{\tau}(t)$	rate of increase over time of $\tau(t)$
τ_m	margin value of $\tau(t)$
$\dot{\tau}_m$	margin value of $\dot{\tau}(t)$
$\Delta t(t)$	visual variable specifying the temporal headway (the time-gap) with respect to a lead vehicle or stationary obstacle
Δt_m	margin value of $\Delta t(t)$

A2 Types of collision course with a lead vehicle and their specifications (see table 1)

As a preliminary, we note from equation (7) that

$$\tau = \frac{Z}{V}, \tag{13}$$

and so, differentiating with respect to time, we obtain

$$\dot{\tau} = -\left(1+\frac{ZA}{V^2}\right) = -\left(1-\frac{ZD}{V^2}\right). \tag{14}$$

There are three types of collision course:

(i) *Closing at a constant or accelerating rate.* The collision course is defined in physical terms by

$$V > 0; \qquad A \geqslant 0,$$

and so, from equations (13) and (14), it is visually specified by

$$\tau > 0; \qquad \dot{\tau} \leqslant -1.$$

The driver is on this type of collision course in the following situations:
(a) when he is cruising along and comes upon a slower vehicle which is either cruising, decelerating, or stopped;
(b) when he is slowing down behind a slower vehicle which has the same or a higher deceleration;
(c) when he is accelerating towards a slower vehicle which has the same or a lower acceleration, or is cruising, decelerating, or stopped—e.g. when both vehicles are accelerating to a cruising speed.

(ii) *Closing at an inadequate decelerating rate.* A closing deceleration is adequate if and only if the closing velocity would be reduced to zero before the separation reached zero: i.e. if $V^2/2D < Z$. Therefore, the collision course is defined in physical terms by

$$V > 0; \qquad D < \frac{V^2}{2Z},$$

and so, from equations (13) and (14), is visually specified by

$$\tau > 0; \qquad \dot{\tau} < -\tfrac{1}{2}.$$

The driver is on this type of collision course in the following situations:
(a) when he is cruising along and comes upon a slower vehicle which is accelerating but not enough—e.g. a vehicle which has recently entered the traffic stream;
(b) when he is braking inadequately behind a slower vehicle which may be either stopped, cruising, decelerating less, or even accelerating;
(c) when he is accelerating towards a slower vehicle which has a higher acceleration but not high enough—e.g. when both vehicles are accelerating to a cruising speed.

(iii) *Receding at a decelerating rate.* The collision course is defined in physical terms by

$$V < 0; \qquad A > 0,$$

and so, from equations (13) and (14), is visually specified by

$$\tau < 0; \qquad \dot{\tau} < -1.$$

The driver is on this type of collision course in the following situations:
(a) when he is cruising along and comes upon a faster vehicle which is decelerating—e.g. a vehicle which has just overtaken the driver;
(b) when he is slowing down behind a faster vehicle which has a higher deceleration;
(c) when he is accelerating towards a faster vehicle which is decelerating, cruising or accelerating less.

A3 The consequences of starting the braking reaction when either the temporal headway or the time-to-collision with a decelerating lead vehicle reaches a margin value (formulae for table 2)

Consider the driver travelling at a steady speed V and closing on a lead vehicle which is decelerating at a constant rate D_{L}. If the driver were to start his braking reaction when the temporal headway reached a margin value Δt_{m}, he would do so at a separation of $V\Delta t_{\mathrm{m}}$, irrespective of the lead vehicle's speed. If the speed of the lead vehicle at that time were V_{L}, and if the driver required a free time t_{f} in which to attain appropriate braking, then the constant deceleration, D_{c}, required would be given by

$$Vt_{\mathrm{f}} + \frac{V^2}{2D_{\mathrm{c}}} = V\Delta t_{\mathrm{m}} + \frac{V_{\mathrm{L}}^2}{2D_{\mathrm{L}}},$$

i.e.

$$D_{\mathrm{c}} = \frac{D_{\mathrm{L}}V^2}{2D_{\mathrm{L}}V(\Delta t_{\mathrm{m}} - t_{\mathrm{f}}) + V_{\mathrm{L}}^2}. \tag{15}$$

Therefore, the slower the speed of the lead vehicle V_{L}, the harder would the driver have to brake.

If on the other hand, the driver were to start his braking reaction when the time-to-collision reached a margin value τ_{m}, and if, at that time, the lead vehicle's speed were V_{L}, he would start his braking reaction at a separation of $(V-V_{\mathrm{L}})\tau_{\mathrm{m}}$. Therefore, the constant deceleration D_{c} required would be given by

$$Vt_{\mathrm{f}}+\frac{V^2}{2D_{\mathrm{c}}} = (V-V_{\mathrm{L}})\tau_{\mathrm{m}}+\frac{V_{\mathrm{L}}^2}{2D_{\mathrm{L}}}\,,$$

i.e.

$$D_{\mathrm{c}} = \frac{D_{\mathrm{L}}V^2}{(D_{\mathrm{L}}\tau_{\mathrm{m}}-V_{\mathrm{L}})^2+2D_{\mathrm{L}}V(\tau_{\mathrm{m}}-t_{\mathrm{f}})-D_{\mathrm{L}}^2\tau_{\mathrm{m}}^2}. \tag{16}$$

Therefore, the greater the speed of the lead vehicle V_{L} (below a speed equal to $D_{\mathrm{L}}\tau_{\mathrm{m}}$), the harder would the driver have to brake.

Now suppose the driver were to start his braking reaction when *either* the temporal headway reached a margin value Δt_{m} *or* the time-to-collision reached a margin value $\tau_{\mathrm{m}}(\geqslant \Delta t_{\mathrm{m}})$. Then the combinations of separation Z, and the speed of the lead vehicle V_{L}, when the driver would start his braking reaction, would correspond to the points on the graph of figure 7. For points on the sloping line [equation: $Z=(V-V_{\mathrm{L}})\tau_{\mathrm{m}}$], braking would be initiated by time-to-collision. For points on the horizontal line (equation: $Z=V\Delta t_{\mathrm{m}}$), braking would be initiated by temporal headway. But, from equations (15) and (16), the nearer a point on either line is to the point P of intersection of the lines, the harder would the driver have to brake. Therefore, the maximum constant deceleration, $D_{\max}$, would be required at P, where V has the critical value V^* given by

$$V^* = \left(1-\frac{\Delta t_{\mathrm{m}}}{\tau_{\mathrm{m}}}\right)V\,. \tag{17}$$

Hence, from equations (15) and (16), for $\tau_{\mathrm{m}} \geqslant \Delta t_{\mathrm{m}} > 0$, we have

$$D_{\max} = \frac{D_{\mathrm{L}}V}{V(1-\Delta t_{\mathrm{m}}/\tau_{\mathrm{m}})^2+2D_{\mathrm{L}}(\Delta t_{\mathrm{m}}-t_{\mathrm{f}})}\,. \tag{18}$$

If $\Delta t_{\mathrm{m}} > \tau_{\mathrm{m}} \geqslant 0$, then braking will always be initiated by temporal headway. In this case $D_{\max}$, which will occur when $V=0$, is according to equation (15) given by

$$D_{\max} = \frac{V}{2(\Delta t_{\mathrm{m}}-t_{\mathrm{f}})}\,. \tag{19}$$

Finally, if $\Delta t_{\mathrm{m}} = 0$, then if V_{L} were equal to V, the vehicles would be touching before the driver started his braking reaction, and so $D_{\max}$ would be infinite.

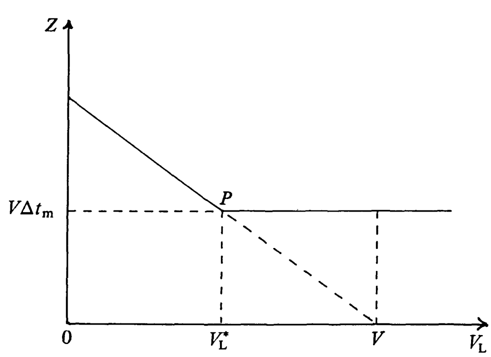

Figure 7. See text for explanation.

A4 Controlling braking for a stationary obstacle (formulae for figure 4)

Suppose the driver starts to brake at time $t = 0$. If he is so adjusting his deceleration that at any time t during his braking $\dot{\tau} = \dot{\tau}_m$, then, we have from equation (14)

$$\frac{D(t)Z(t)}{[V(t)]^2} = 1+\dot{\tau}_m \, . \tag{20}$$

Since $V(t) = -dZ(t)/dt$ and $D(t) = d^2Z(t)/dt^2$, we have

$$Z(t)\frac{d^2Z(t)}{dt^2} = (1+\dot{\tau}_m)\left[\frac{dZ(t)}{dt}\right]^2 . \tag{21}$$

Solving this differential equation, we obtain the equations of motion of the vehicle:

$$\begin{aligned} Z(t) &= Z(0)\left[1+\frac{\dot{\tau}_m V(0)t}{Z(0)}\right]^{-1/\dot{\tau}_m} , \\ V(t) &= V(0)\left[1+\frac{\dot{\tau}_m V(0)t}{Z(0)}\right]^{-(1+1/\dot{\tau}_m)} , \\ D(t) &= \frac{(1+\dot{\tau}_m)V(0)^2}{Z(0)}\left[1+\frac{\dot{\tau}_m V(0)t}{Z(0)}\right]^{-(2+1/\dot{\tau}_m)} . \end{aligned} \tag{22}$$

In order to compare the braking profiles for different values of $\dot{\tau}_m$ it is convenient to normalise the equations of motion by writing

$$t^* = \frac{t}{t_{bc}} ,$$

$$Z^*(t^*) = \frac{V(t)}{V(0)} ,$$

$$D^*(t^*) = \frac{D(t)}{D_c} ;$$

that is, expressing time in units of t_{bc}—the time to brake at a constant deceleration; deceleration in units of D_c—the constant deceleration required; and separation and velocity in units of their initial values $Z(0)$ and $V(0)$.

The normalised equations of motion of the vehicle, for $\dot{\tau}_m \neq 0$, are then

$$\begin{aligned} Z^*(t^*) &= (1+2\dot{\tau}_m t^*)^{-1/\dot{\tau}_m} \\ V^*(t^*) &= (1+2\dot{\tau}_m t^*)^{-(1+1/\dot{\tau}_m)} \\ D^*(t^*) &= 2(1+\dot{\tau}_m)(1+2\dot{\tau}_m t^*)^{-(2+1/\dot{\tau}_m)} \end{aligned} \tag{23}$$

A5 Safe following headways (formulae for table 3)

Consider a driver following another vehicle at a distance Z_0, with both vehicles travelling at the same constant speed V_0. Suppose that at time $t = 0$ the lead vehicle brakes with a constant deceleration D. Then, at a later time t, *before* the lead vehicle has stopped and *before* the driver of the following vehicle has started to brake, the vehicle separation $Z(t)$ and the closing velocity $V(t)$ will be

$$Z(t) = Z_0 - \tfrac{1}{2}D_L t^2 , \tag{24}$$

$$V(t) = D_L t \, . \tag{25}$$

The visual angle $\alpha(t)$ subtended at the driver's eye by the lead vehicle of width W will be therefore

$$\alpha(t) = \frac{W}{Z(t)}\,, \tag{26}$$

and so, on differentiating with respect to time, the rate of increase of the visual angle $\alpha(t)$ will be

$$\dot{\alpha}(t) = \frac{WD_{\mathrm{L}}t}{(Z_0 - \frac{1}{2}D_{\mathrm{L}}t^2)^2} \tag{27}$$

Clearly, $\dot{\alpha}(t)$ increases over time and is smaller the greater the initial vehicle separation Z_0. Therefore, assuming that $\dot{\alpha}(t)$—and hence closing—will not be detectable until it reaches a threshold value $\dot{\alpha}_{\mathrm{th}}$, then, if the following distance Z_0 were greater than a certain margin value Z_{m}, the driver would not register that he was closing until *after* the lead vehicle had stopped (at time $t = V_0/D_{\mathrm{L}}$). From equation (27) Z_{m} is given by

$$\dot{\alpha}_{\mathrm{th}} = \frac{WV_0}{(Z_{\mathrm{m}} - V_0^2/2D_{\mathrm{L}})^2}\,,$$

i.e.

$$Z_{\mathrm{m}} = \frac{V_0^2}{2D_{\mathrm{L}}} + \left(\frac{WV_0}{\dot{\alpha}_{\mathrm{th}}}\right)^{1/2}. \tag{28}$$

If the following distance were greater than Z_{m}, then, from equation (12), the vehicle separation when the driver first registered that he was closing would be $(WV_0/\dot{\alpha}_{\mathrm{th}})^{1/2}$. Therefore, t_{f}—the free time the driver would need in order to move his foot to the brake and attain an adequate deceleration D—would be given by

$$V_0 t_{\mathrm{f}} + \frac{V_0^2}{2D} = \left(\frac{WV_0}{\dot{\alpha}_{\mathrm{th}}}\right)^{1/2}$$

i.e.

$$t_{\mathrm{f}} = \left(\frac{W}{V_0\dot{\alpha}_{\mathrm{th}}}\right)^{1/2} - \frac{V_0}{2D}\,. \tag{29}$$

If, on the other hand, the following distance were *less* than Z_{m}, then the driver would register that he was closing after a time t_{r}, *before* the lead vehice had stopped. From equation (27) t_{r} is given by the lowest positive real root of the equation

$$\dot{\alpha}_{\mathrm{th}} = \frac{WD_{\mathrm{L}}t_{\mathrm{r}}}{(Z_0 - \frac{1}{2}D_{\mathrm{L}}t_{\mathrm{r}}^2)^2}\,. \tag{30}$$

The time t_{b} at which he would need to have started braking with deceleration D is given by

$$V_0 t_{\mathrm{b}} + \frac{V_0^2}{2D} = Z_0 + \frac{V_0^2}{2D_{\mathrm{L}}}$$

i.e.

$$t_{\mathrm{b}} = \frac{Z_0}{V_0} - \frac{V_0}{2D} + \frac{V_0}{2D_{\mathrm{L}}}\,. \tag{31}$$

The free time t_{f} he would need in order to move his foot to the brake and attain the deceleration D is

$$t_{\mathrm{f}} = t_{\mathrm{b}} - t_{\mathrm{r}}\,. \tag{32}$$

The above formulae were used in a computer program to calculate the minimum safe temporal headways given in table 3.