

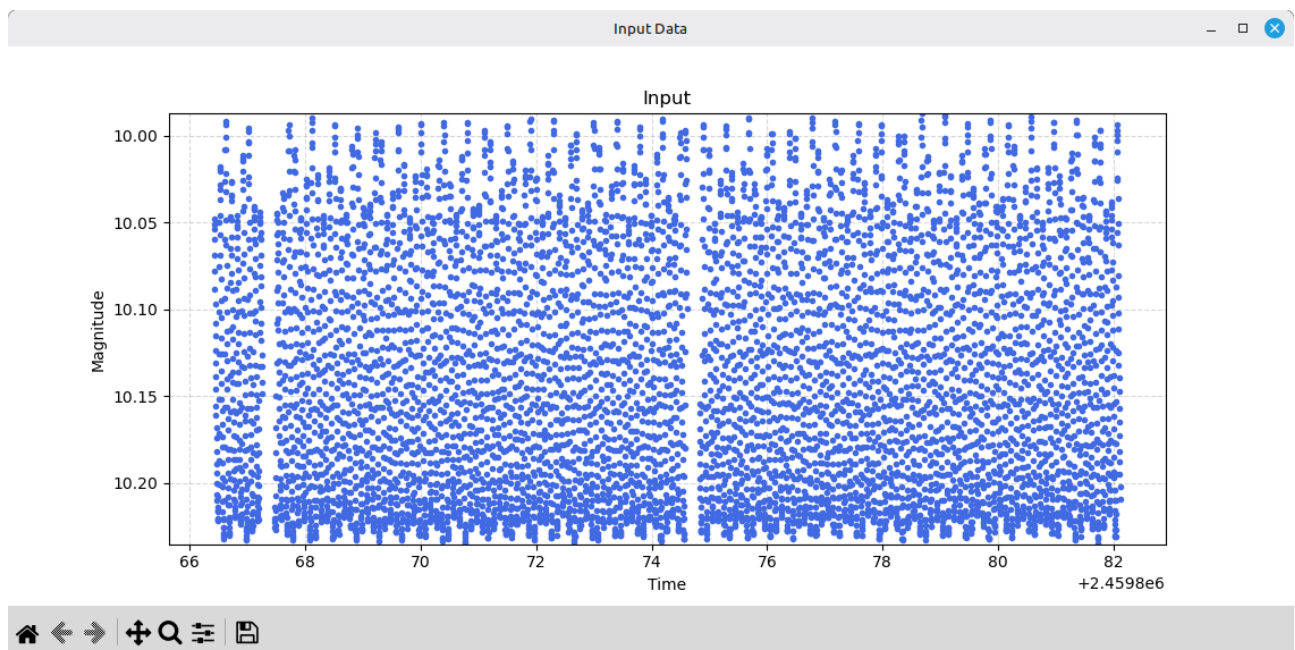
# Multi-periodic star: an example of finding and refining periods

TYC 3637-1152-1 is a high-amplitude delta Scuti star with multiple periods (2019NewA...68...39P). Let's find its periods using TESS data and refine them. Use the "TYC\_3637-1152-1\_Sector57\_TESS-SPOC\_part.txt" example file.

Open "TYC\_3637-1152-1\_Sector57\_TESS-SPOC\_part.txt" using the File → Open Data file... menu command, or with the corresponding tool button:



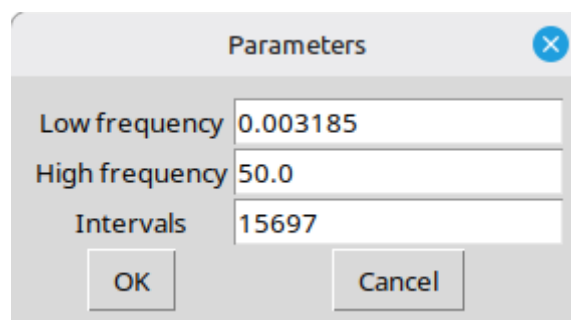
The file is in the "data" folder. The light curve will then be displayed:



Now, calculate a periodogram. Use Operations → DC DFT (Ferraz-Mello)... or the following tool button:

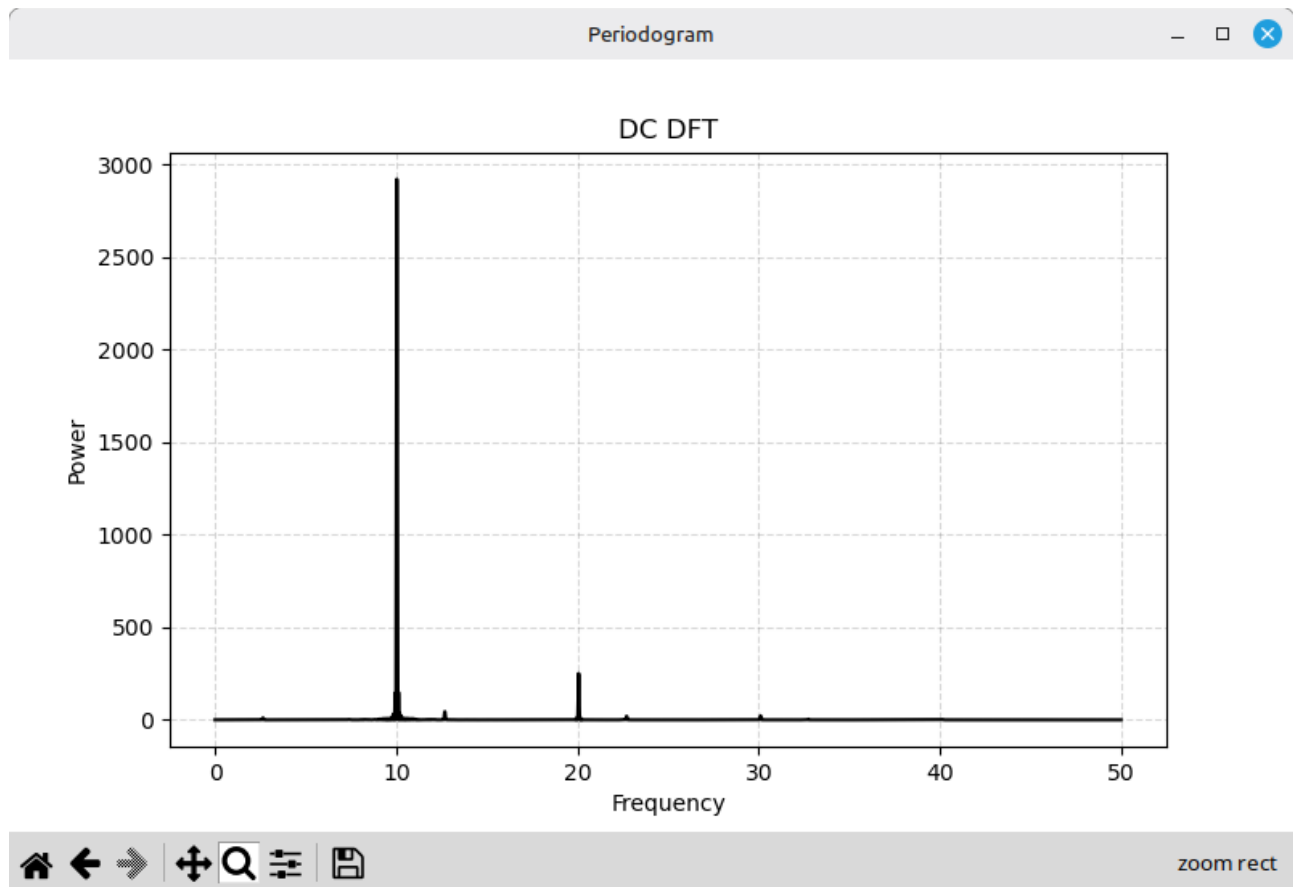


You may use the default parameters:

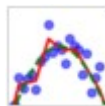
A dialog box titled "Parameters" with a close button (X) in the top right corner. It contains three input fields: "Low frequency" with the value 0.003185, "High frequency" with the value 50.0, and "Intervals" with the value 15697. At the bottom are "OK" and "Cancel" buttons.

Parameter	Value
Low frequency	0.003185
High frequency	50.0
Intervals	15697

This calculation will take some time. Once calculated, the periodogram will display a prominent peak at a frequency of about  $10.033 \text{ d}^{-1}$ , which corresponds to a period of 0.0997 days. The second harmonic is also visible at a frequency of  $20.066 \text{ d}^{-1}$  ( $P=0.0498$  days), and upon zooming in, the third and fourth harmonics become apparent.



Let's create a model using the main period of 0.0997 days and its harmonics. Select Operations → Polynomial Fit... from the menu or click the following button:



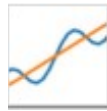
Enter the period, specify the number of harmonics (4, including the primary), check the "Optimize" checkbox, and click OK:

The figure shows a "Parameters" dialog box for configuring a polynomial fit. It features a table-like structure with the following fields and values:

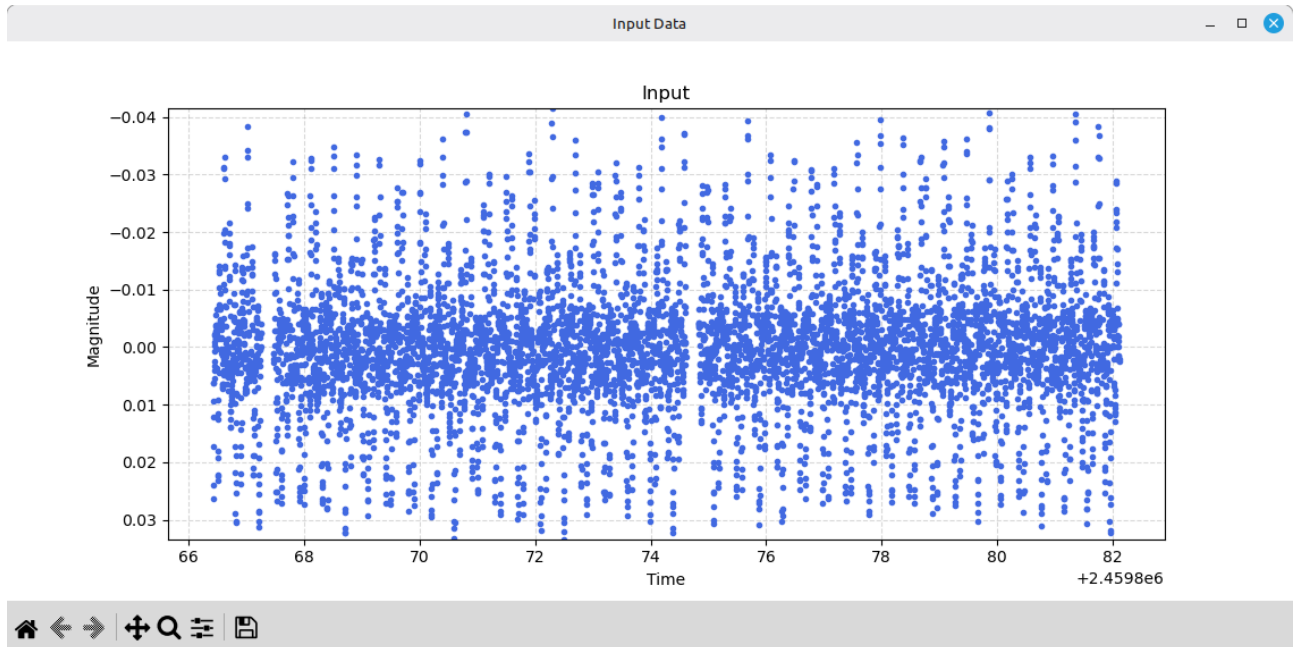
Trig. Polyn. Period	Degree	Optimize
0	4	<input checked="" type="checkbox"/>
0.0997	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>
0	0	<input type="checkbox"/>

At the bottom of the dialog, there are two buttons: "OK" and "Cancel". Additionally, there is a checkbox labeled "Calculate period errors via bootstrap" which is currently unchecked, with a note "May take a while!" next to it.

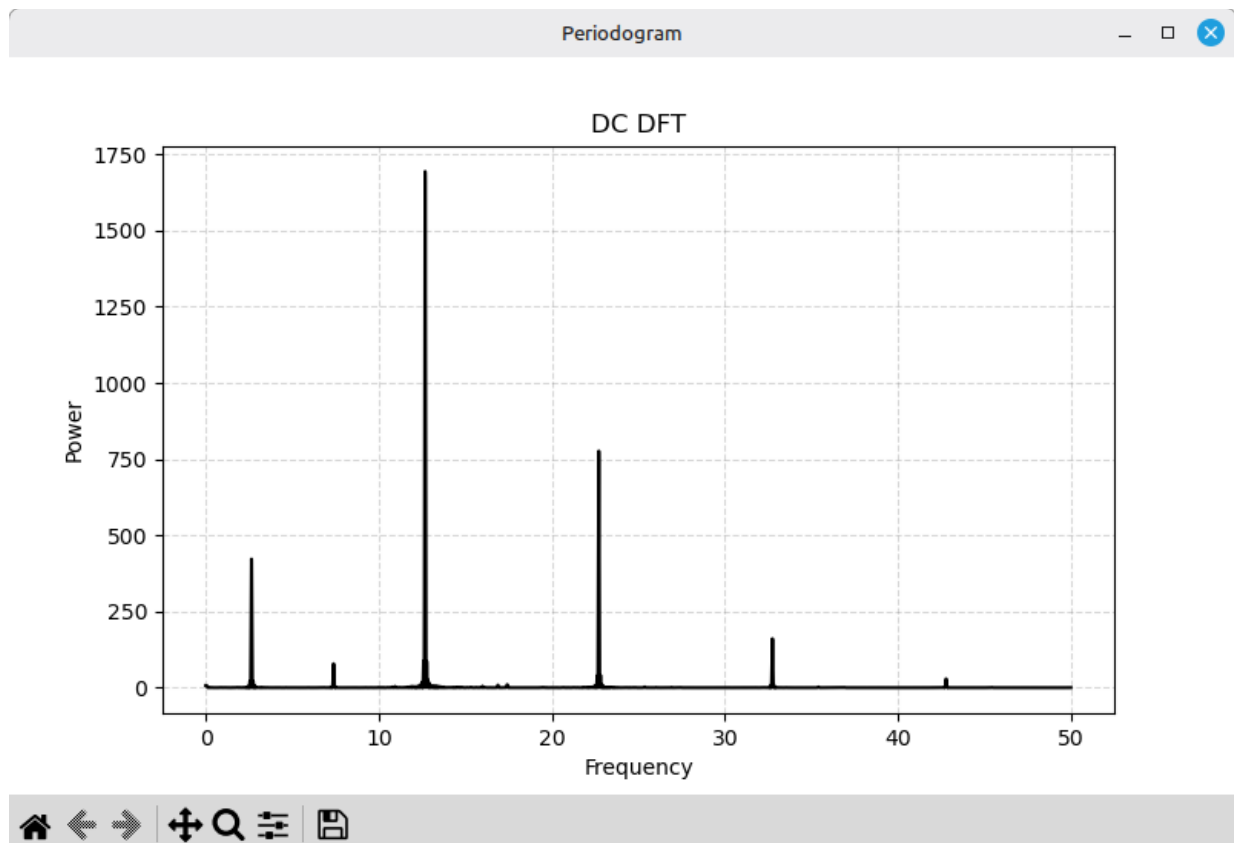
You will see a new window with the approximation. Now select Operations → Detrend from the menu, or click the appropriate button:



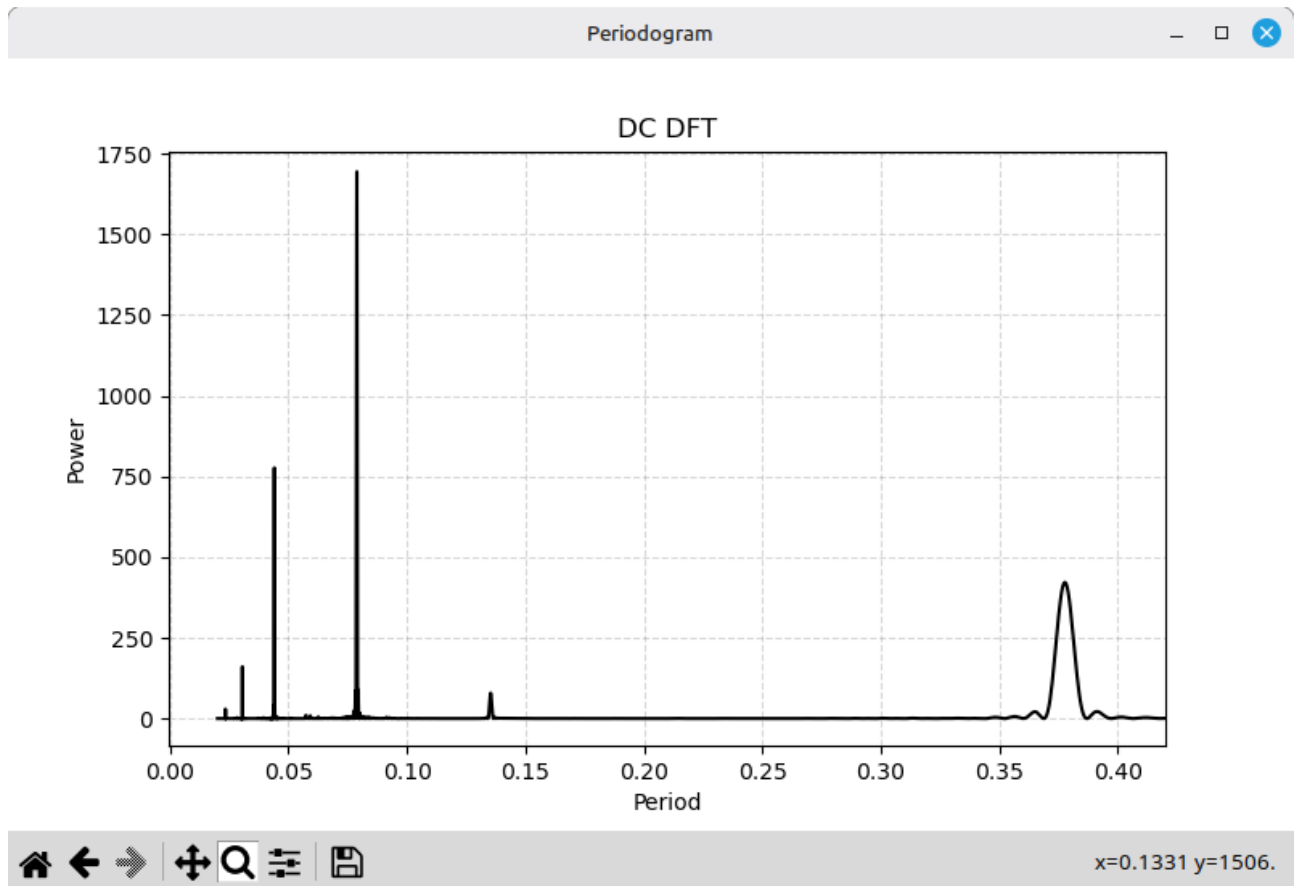
The input data will be replaced with the 'detrended' data, i.e., the residuals:



Repeat the periodogram analysis using the same parameters as before. Now the additional frequencies become clearly visible:



Change the periodogram view mode to “Power vs Period”, using the View → Plot DFT Result → Power(Period). Zoom the plot:



Estimate the values of the period using the zoom and pan buttons, as well as your mouse. The mouse coordinates are displayed in the bottom-right corner.

The table below lists the six periods visible after detrending, along with the primary period:

N	Period	Frequency	Comment
1	0.09966	10.3509	
2	0.07886	12.6807	
3	0.04402	22.7169	$f_1 + f_2$
4	0.37781	2.6468	$f_1 - f_2$
5	0.03054	32.7439	$2 * f_1 + f_2$
6	0.13538	7.3866	$2 * f_1 - f_2$
7	0.02337	42.7899	$3 * f_1 + f_2$

We can now refine the period values and estimate their uncertainties.

Launch the “Polynomial Fit” dialog. Enter the periods, setting the degree to 4 for the first one and to 1 for each of the others. Also, make sure to check the 'Optimize' box for all periods.

Parameters

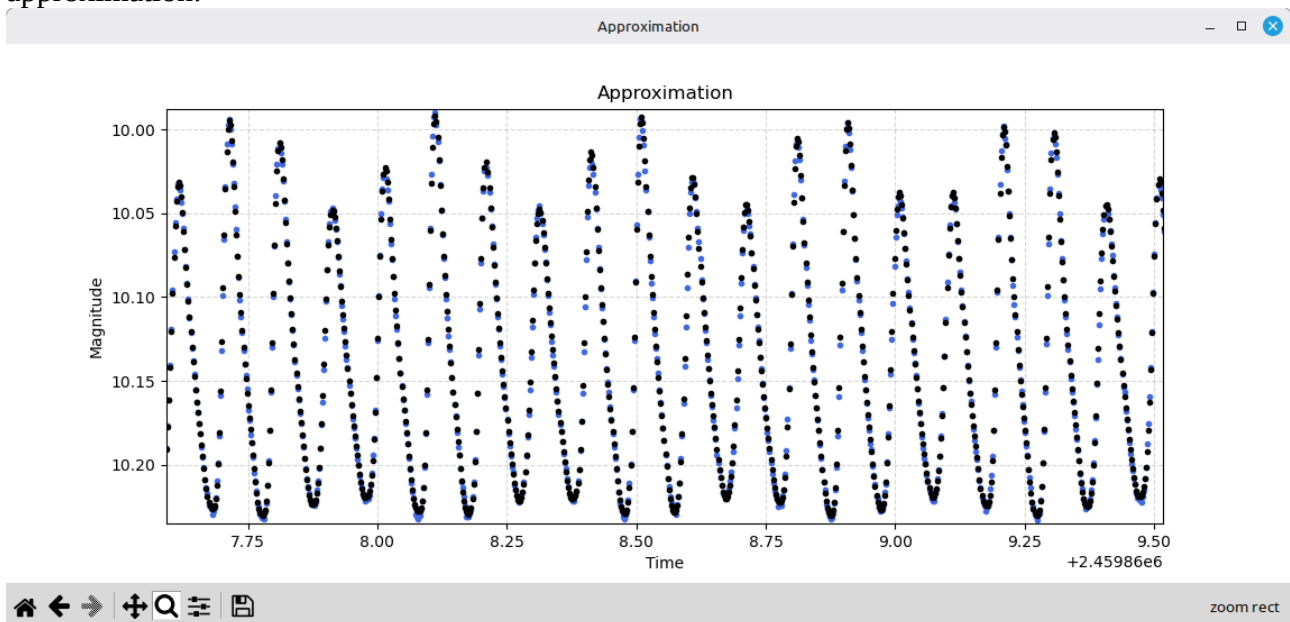
Polynomial Degree	0				
Trig. Polyn. Period 1	0.09966	Degree	4	Optimize	<input checked="" type="checkbox"/>
Trig. Polyn. Period 2	0.07886	Degree	1	Optimize	<input checked="" type="checkbox"/>
Trig. Polyn. Period 3	0.04402	Degree	1	Optimize	<input checked="" type="checkbox"/>
Trig. Polyn. Period 4	0.37781	Degree	1	Optimize	<input checked="" type="checkbox"/>
Trig. Polyn. Period 5	0.03054	Degree	1	Optimize	<input checked="" type="checkbox"/>
Trig. Polyn. Period 6	0.13538	Degree	1	Optimize	<input checked="" type="checkbox"/>
Trig. Polyn. Period 7	0.02337	Degree	1	Optimize	<input checked="" type="checkbox"/>
Trig. Polyn. Period 8	0.0	Degree	0	Optimize	<input type="checkbox"/>
Trig. Polyn. Period 9	0.0	Degree	0	Optimize	<input type="checkbox"/>

Calculate period errors via bootstrap ☐ May take a while!

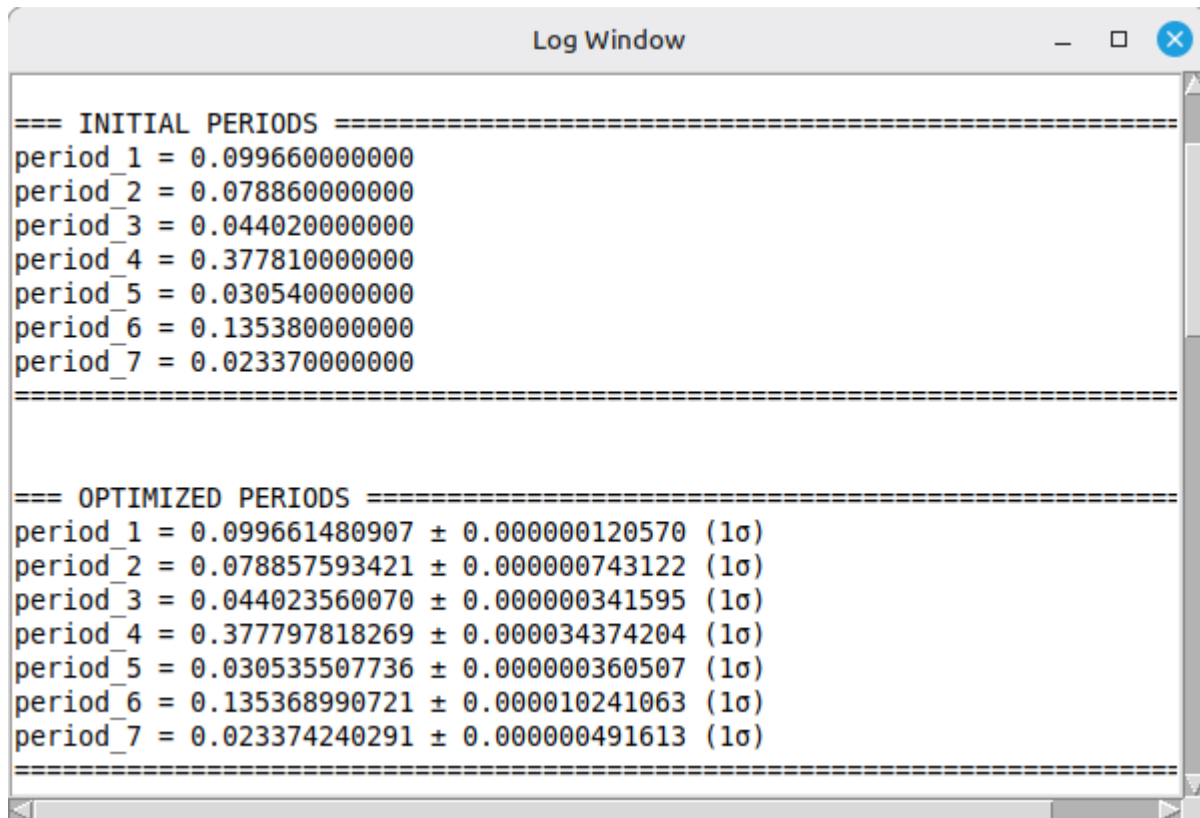
OK Cancel

Then click OK.

Within a few seconds, a plot showing the approximation will appear. Below is a zoomed-in section of the plot, where blue dots represent the original data and black dots indicate the model approximation:



Look at the Log window. There, you will find the refined periods, their estimated errors, and additional useful information, including the polynomial coefficients and their associated uncertainties.



```

Log Window

=== INITIAL PERIODS ===
period_1 = 0.099660000000
period_2 = 0.078860000000
period_3 = 0.044020000000
period_4 = 0.377810000000
period_5 = 0.030540000000
period_6 = 0.135380000000
period_7 = 0.023370000000

=== OPTIMIZED PERIODS ===
period_1 = 0.099661480907 ± 0.000000120570 (1σ)
period_2 = 0.078857593421 ± 0.000000743122 (1σ)
period_3 = 0.044023560070 ± 0.000000341595 (1σ)
period_4 = 0.377797818269 ± 0.000034374204 (1σ)
period_5 = 0.030535507736 ± 0.000000360507 (1σ)
period_6 = 0.135368990721 ± 0.000010241063 (1σ)
period_7 = 0.023374240291 ± 0.000000491613 (1σ)

```

The table below presents the initial period values, the refined period values, and their associated errors.

N	Period	Frequency	Comment	Improved period	Error
1	0.09966	10.3509		<b>0.09966148</b>	<b>0.00000012</b>
2	0.07886	12.6807		<b>0.07885759</b>	<b>0.00000074</b>
3	0.04402	22.7169	f1+f2	<b>0.04402356</b>	<b>0.00000034</b>
4	0.37781	2.6468	f1-f2	<b>0.377798</b>	<b>0.000034</b>
5	0.03054	32.7439	2*f1+f2	<b>0.03053551</b>	<b>0.00000036</b>
6	0.13538	7.3866	2*f1-f2	<b>0.135369</b>	<b>0.000010</b>
7	0.02337	42.7899	3*f1+f2	<b>0.02337424</b>	<b>0.00000049</b>

Note that only the first and second periods are independent. The remaining periods correspond to linear combinations of these fundamental frequencies and should therefore not be optimized independently. However, this constraint is not currently supported in **V\*-fit**.

(End of the document)