Using MCMC to measure linear lattice with Turn-by-Turn data

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* Assuming an N consecutive TbT data for a BPM. $x_0, x_1, x_2, \cdots x_{N-1}$

* Ignoring damping and decoherence, the TbT data is a pure betatron oscillation.

* Step 1

  extracting the fractional tune with FFT and NAFF : ν

* Step 2

  determining the parameters to be inferred. (par N)

  $\Rightarrow \beta = \beta_0 + \Delta\beta$: β_0 is the model β

  $\Delta\beta$ is the distortion (par 1)

  $\Rightarrow \alpha = \alpha_0 + \Delta\alpha$: α_0 is the model α

  $\Delta\alpha$ is the distortion (Par 2)

  $\Rightarrow \gamma = \dfrac{1+\alpha^2}{\beta}$

  $\Rightarrow$ beam coordinates at 0^{th} turn $\langle 1 \rangle$

  (x_0, p_{x0}) (par 3. par 4)

* Step 3 : One turn Map function

$$\mathcal{M} = \begin{bmatrix} \beta \cos 2\pi\nu + \alpha \sin 2\pi\nu & \beta \sin 2\pi\nu \\ -\gamma \sin 2\pi\nu & \beta \cos 2\pi\nu - \alpha \sin 2\pi\nu \end{bmatrix}$$

$$(x_N, P_{xN}) = \mathcal{M}^N (x_0, P_{x0})$$

⇒ Expection (liklihood)

$\Delta x = x_N - x_{measured\ N}$, should be

a Normal distribution $\mathcal{N}(\mu, \sigma^2)$

↑

~~μ, other~~

μ : closed orbit at this BPM (par 4)

σ^2 : BPM noise level (par 5)

* Step 4

Using MCMC to infer

① $\Delta\beta$, $\Delta\alpha$, x_0, P_{x0}, μ, σ.

Summary :

- $\beta_0 + \Delta\beta$, $\alpha_0 + \Delta\alpha$, μ is ~~b~~ the measured ~~B~~ linear optics. note that α can be measured in this way

- $x_{measured} - x_{COD}(\mu) = \tilde{x}_{measured}$
  then μ can be removed from the list of inferred parameter

- σ^2 is BPM noise

- each BPM can be analyzed independently and Parallelly