Algebraic Topology

Edition: 1

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1 Definitions

Prove that $f^{-1}(A) = f_{|C}^{-1}(A) \cup f_{|K}^{-1}(A)$.
Suppose $x \in f^{-1}(A)$ then $f(x) \in A$. If $x \in C$, then $f(x) = f_{|C}(x) \in A$ so $x \in f_{|C}^{-1}(A)$. Likewise for $x \in K$. Suppose $x \in f_{|C}^{-1}(A) \cup f_{|K}^{-1}(A)$. If $x \in f_{|C}^{-1}(A)$, $f_{|C}(x) = f(x) \in A$. Likewise for $x \in f_{|K}^{-1}(A)$.

In a compact metric space, we can associate a Lebesgue number for every open covering. We can assume for every δ, there exists $x \in X$, for any $\alpha \in A$, $B_\delta(x) \not\subset U_\alpha$. So we choose $\delta = 1/n$ and x as x_n such that $B_{1/n}(x_n)$ is not contained in any U_α.

The term "space obtained by attaching an n-cell to X along f" sees no motiviation. Importantly, I don't see what is an n-cell. Since the notation is $X \cup_f D^n$, can I trivially consider D^n i.e. the n dimensional unit disk, as the so-called n-cell?

The understanding of this definition means an equivalence class $(X \sqcup D^n)\backslash \sim$. First why X is disjoint with D^n? Next how should I understand the equivalence relation generated by $x \sim f(x)$ for all $x \in S^{n-1} \subset D^n$? In this argument, I see two problems: first please prove that $S^{n-1} \subset D^n$, second if $x \sim f(x)$ what about $f(x)$ and $f(f(x))$? Note that $f : S^{n-1} \longrightarrow X$, so $f(x) \in X$ and $f(x) \notin D^n$ as is S^{n-1}. Thus $f(x) \not\sim f(f(x))$. Show me some typical elements in $(X \sqcup D^n)\backslash \sim$? For example, if $x \in X\backslash f(S^{n-1}) \sqcup x \in D^n\backslash S^{n-1}$, then $\{x\}$ is an element in it; if $x \in S^{n-1}$, then $\{x, f(x)\}$ is an element in it. Suppose without the part S^{n-1}, we are actually not touch with a part of X, say $X\backslash f(S^{n-1})$ and the disk without boundry, say $D^n\backslash S^{n-1}$. But we when it comes to the boundry, we stick the sphere, under the guide of f, to the subset of X, say $f(S^{n-1})$.

How can I understand the process of obtaining $X^{(n)}$ from $X^{(n-1)}$:

$$X^{(n)} = \left(X^{(n-1)} \sqcup \bigsqcup_\alpha D^n_\alpha \right) / \{x \sim f_\alpha(x)\}$$

The first key question is that what are D^n_α? Why do they not intersect with each other? Well I think that these n-cells just centered at different center such that they can keep away from each other.

A 1-dimensional ball is an interval, while a 1-dimensional sphere is the sets of two end points.

Hawaiian Earring is an example of a non-cell complex.

The real question is, for example, how can we ensure that when we attach a 1-cell to a single point, we get a circle? Since there is no mechanism during the gluing that gurantees the "form" of the final space. A place that seems to be safe is to implicitly assume that we can achieve this control, at worse, up to some continous transformations.

References