STAT3100 2024F
Assignment 2

Due: 11:59pm Friday, October 11, 2024

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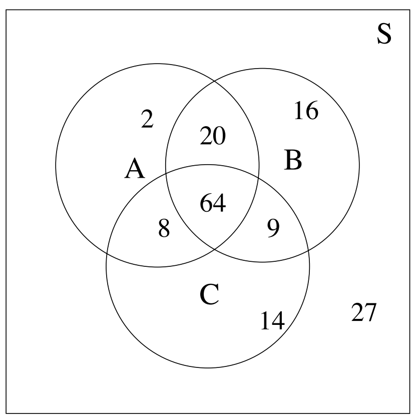
1. A lottery sells tickets numbered from 00001 through 50000. What is the probability of drawing a number that is divisible by 200.
   Solution: Let A be the event that the number drawn is divisible by 200. There are $N(A) = \frac{50000}{200} = 250$ numbers divisible by 200. With equal probabiliy of drawing any one ticket, we have

$$P(A) = \frac{N(A)}{N} = \frac{250}{50000} = 0.005.$$

2. Among 160 persons interviewed as part of an urban mass transportation study, some live more than 3 miles away from the city centre (A), some regularly drive to work (B), and some would like to switch to public mass transportation if it was available (C). Use the information provided by the below Venn diagram (the numbers are the counts of persons that fall into each given region) to find probability and interpret the probability in the following sub-questions.



   (a) $P(A)$ (hint: interpreted as the probability that a person randomly chosen from the 160 interviewed persons lives more than 3 miles away from the city centre).
   Solution:

$$N = 160, \quad N(A) = 2 + 20 + 64 + 8 = 94, \quad P(A) = \frac{94}{160} = 0.5875$$

   (b) $P(A \cap B)$,
   Solution: $P(A \cap B) = \frac{N(A \cap B)}{N} = \frac{20+64+8}{160} = 0.525$ is the probability that a person randomly chosen from the 160 interviewed persons lives more than 3 milesaway from the city centre and regularly drive to work.

(c) $P(B \cup C)$,
Solution: $P(B \cup C) = \frac{N(B \cup C)}{N} = \frac{16+20+64+9+8+14}{160} = 0.81875$ is the probability that a person randomly chosen from the 160 interviewed persons would regularly drive to work or would like to switch to public mass transportation if itwas available.

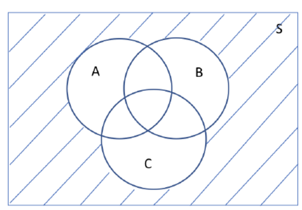
(d) $P(\overline{A} \cup \overline{B} \cup C)$;
Solution: $P(\overline{A} \cup \overline{B} \cup C) = \frac{N(\overline{A} \cup \overline{B} \cup C)}{N} = \frac{160-20}{160} = 0.875$, is the probability that a person randomly chosen from the 160 interviewed persons would be anyone except those who would not want to switch to public mass transportation if is is available, live 3 miles away from the city centre and prefere regularly drive to work.

(e) $P(B \cap (A \cup C))$.
Solution: $P(B \cap (A \cup C)) = \frac{N(B \cap (A \cup C))}{N} = \frac{20+64+9}{160} = 0.58125$ is the probability that a person randomly chosen from the 160 interviewed persons regularly drive to work, and live 3 mile away or wants to switch to public transportation if it is available.

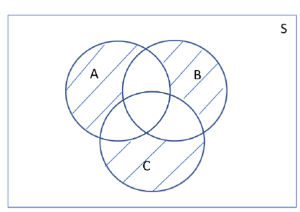
3. Consider events A, B, and C in a sample space S.

(a) Use set notation for union, intersection and complement to express the following events:

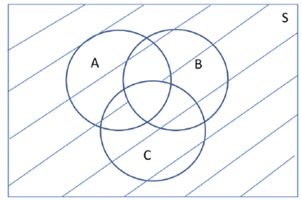
i none of A , B or C occurs.
Solution: $\overline{A \cup B \cup C}$. Please see the shaped area in the vien diagram for the event.



ii the event that exactly one of the events A, B or C occurs.
Solution: $(A \cap \overline{B \cup C}) \cup (B \cap \overline{A \cup C}) \cup (C \cap \overline{A \cup B})$.



iii at most two of the events A, B, C occur.
Solution: $\overline{A \cap B \cap C}$.



(b) Express the probabilities of events in (a) only in terms of $P(A)$, $P(B)$, $P(C)$, $P(A \cap B)$, $P(A \cap C)$, $P(B \cap C)$ and $P(A \cap B \cap C)$ as needed.

i. **Solution:**

$$\begin{aligned} & P(\overline{A \cup B \cup C}) \\ = \; & 1 - P(A \cup B \cup C) \\ = \; & 1 - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap C). \end{aligned}$$

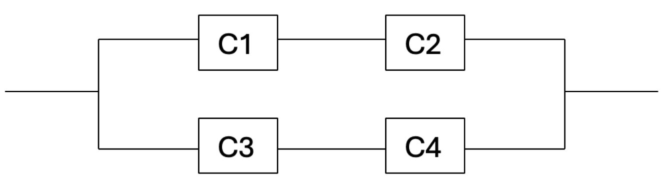
ii. **Solution:**

$$\begin{aligned} & P[(A \cap \overline{B \cup C}) \cup (B \cap \overline{A \cup C}) \cup (C \cap \overline{A \cup B})] \\ = \; & P(A \cap \overline{B \cup C}) + P(B \cap \overline{A \cup C}) + P(C \cap \overline{A \cup B}) \\ = \; & P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \\ & \quad + P(B) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C) \\ & \quad + P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \\ = \; & P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(A \cap C) - 2P(B \cap C) + 3P(A \cap B \cap C) \end{aligned}$$

iii. **Solution:**

$$P(\overline{A \cap B \cap C}) = 1 - P(A \cap B \cap C).$$

4. Suppose a system consists of four components C_1, C_2, C_3 and C_4 structured as in the figure below. The system will remain working if any one of the top or bottom line works. Each of the top and bottom lines consists of two components, each of which must remain operative for each line to function. If each component has the probability of 0.05 of failing, what is the reliability of the whole system.



Solution: Let W_1, W_2, W_3, and W_4 denote the events that C_1, C_2, C_3, and C_4 are working respectively. For the top line remains working, both C_1 and C_2 must remain working. The probability of both C_1 and C_2 are working is

$$P(W_1 \cap W_2) = P(W_1)P(W_2) = 0.95 \cdot 0.95 = 0.9025$$

Similarly, for the bottom line remains working, both C_3 and C_4 must reamin working. The probability of both C_3 and C_4 are working is

$$P(W_3 \cap W_4) = P(W_3)P(W_4) = 0.95 \cdot 0.95 = 0.9025.$$

For the system remain working, at least one of the top or bottom line has to remain working. So the probability of the system remains working is

$$\begin{aligned}
& 1 - P(\text{both top and bottom lines are not working}) \\
= & 1 - P(\text{top line is not working})P(\text{bottom line is not working}) \\
= & 1 - (1 - P(\text{top line is working}))(1 - P(\text{bottom line is not working})) \\
= & 1 - (1 - P(W_1 \cap W_2))(1 - P(W_3 \cap W_4)) \\
= & 1 - (1 - 0.9025)(1 - 0.9025) \\
= & 0.99049375
\end{aligned}$$

5. Only two firms V and W consider bidding on a road-building job, which may or may not be awarded depending on the amount of dollar bids and some other criteria. Firm V submits a bid. The probability of firm V being awarded is $\frac{3}{4}$ provided that firm W does not bid. The probability is $\frac{3}{4}$ that W will bid, and if it does, the probability that V will get the job is only $\frac{1}{3}$.

   (a) What is the probability that V will get the job?
   Solution: Let V be the event that V gets the job and B_W be the event that W bids.

   $$\begin{aligned}
   P(V) &= P(V|B_W)P(B_W) + P(V|\overline{B_W})P(\overline{B_W}) \\
   &= (\tfrac{1}{3})(\tfrac{3}{4}) + (\tfrac{3}{4})(\tfrac{1}{4}) \\
   &= \tfrac{7}{16}
   \end{aligned}$$

   (b) If V gets the job, what is the probability that W did not bid?
   Solution:

   $$\begin{aligned}
   P(\overline{B_W}|V) &= \tfrac{P(\overline{B_W} \cap V)}{P(V)} \\
   &= \tfrac{P(V|\overline{B_W})P(\overline{B_W})}{P(V)} \\
   &= \tfrac{(3/4)(1/4)}{7/16} \\
   &= \tfrac{3}{7}
   \end{aligned}$$

6. At an electronics plant, it is known from past experience that the probability is 0.86 that a new worker who has attended the company's training program will meet the production quota and that the corresponding probability is 0.35 for the new worker has never attended the training program. If 80% of all new workers attend the training program, what is the probability that the new worker will meet the production quota?

Solution: We let A be the event that a new worker attended the training program and B be event that a new worker will meet the quota. So we have

$$P(A) = 0.8, \quad P(B|A) = 0.86, \quad P(B|\bar{A}) = 0.35,$$

and what is P(B)?

$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|\bar{A})P(\bar{A}) \\ &= (0.86)(0.8) + (0.35)(0.2) \\ &= 0.688 + 0.07 \\ &= 0.758 \end{aligned}$$