

10-414/714 – Deep Learning Systems: Algorithms and Implementation

Hardware Acceleration

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Outline

General acceleration techniques

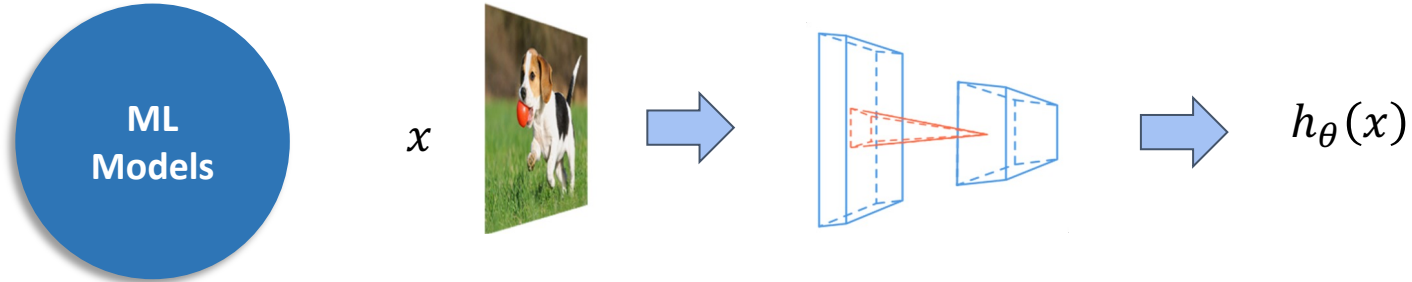
Case study: matrix multiplication

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General acceleration techniques

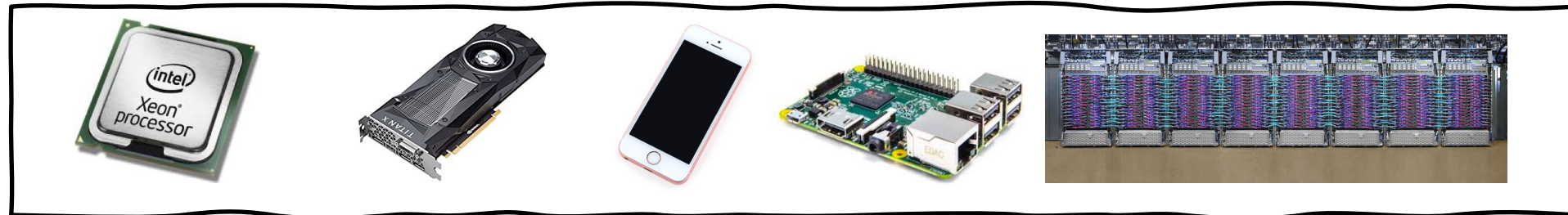
Case study: matrix multiplication

Layers in machine learning frameworks



Computational graph

Tensor linear algebra libraries



Vectorization

Adding two arrays of length 256

```
void vecadd(float* A, float *B, float* C) {  
    for (int i = 0; i < 64; ++i) {  
        float4 a = load_float4(A + i*4);  
        float4 b = load_float4(B + i*4);  
        float4 c = add_float4(a, b);  
        store_float4(C + i* 4, c);  
    }  
}
```

Additional requirements: memory (A, B, C) needs to be aligned to 128 bits

Data layout and strides

Question: how to store a matrix in memory

Row major: $A[i, j] \Rightarrow \text{Adata}[i * A.\text{shape}[1] + j]$

Column major: $A[i, j] \Rightarrow \text{Adata}[j * A.\text{shape}[0] + i]$

Strides format: $A[i, j] \Rightarrow \text{Adata}[i * A.\text{strides}[0] + j * A.\text{strides}[1]]$

Discussion about strides

Advantages: can perform transformation/slicing in zero copy way

- Slice: change the begin offset and shape
- Transpose: swap the strides
- Broadcast: insert a stride equals 0

Disadvantages: memory access becomes not continuous

- Makes vectorization harder
- Many linear algebra operations may require compact the array first

Parallelization

```
void vecadd(float* A, float *B, float* C) {  
    #pragma omp parallel for  
    for (int i = 0; i < 64; ++i) {  
        float4 a = load_float4(A + i*4);  
        float4 b = load_float4(B + i*4);  
        float4 c = add_float4(a, b);  
        store_float4(C * 4, c);  
    }  
}
```

Executes the computation on multiple threads

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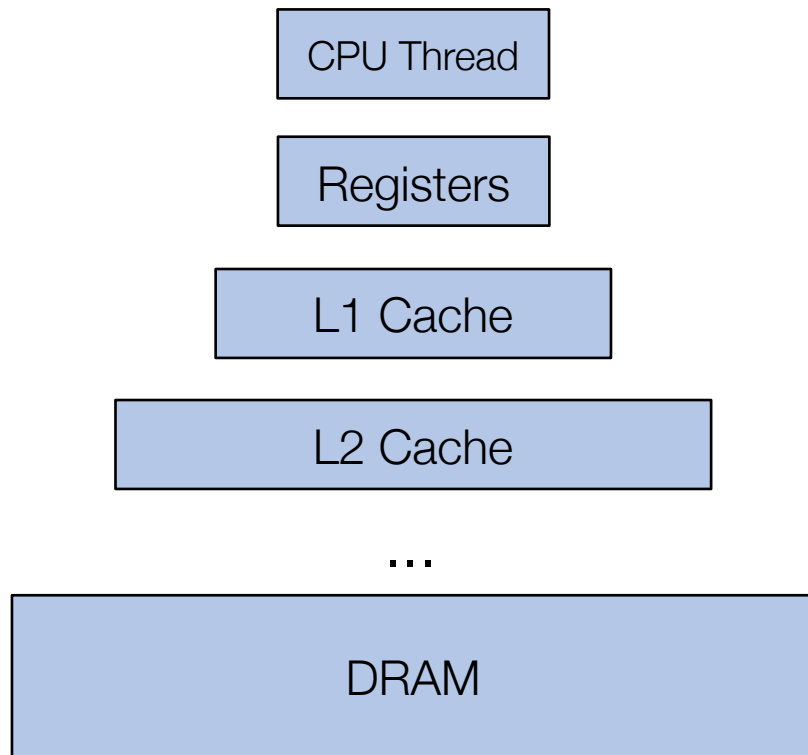
Vanilla matrix multiplication

Compute $C = \text{dot}(A, B.T)$

```
float A[n][n], B[n][n], C[n][n];  
  
for (int i = 0; i < n; ++i)  
    for (int j = 0; j < n; ++j) {  
        C[i][j] = 0;  
        for (int k = 0; k < n; ++k) {  
            C[i][j] += A[i][k] * B[j][k];  
        }  
    }
```

$O(n^3)$

Memory hierarchy on modern CPUs



Latency

Source: Latency numbers every programmer should know

0.5 ns

7ns 14x L1 cache

200ns 20x L2 cache, 200x L1 cache

Architecture aware analysis

```
dram float A[n][n], B[n][n], C[n][n];
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
        register float c = 0;
        for (int k = 0; k < n; ++k) {
            register float a = A[i][k];
            register float b = B[j][k];
            c += a * b;
        }
        C[i][j] = c;
    }
}
```

A's dram->register time cost: n^3

B's dram->register time cost: n^3

A's register memory cost : 1

B's register memory cost : 1

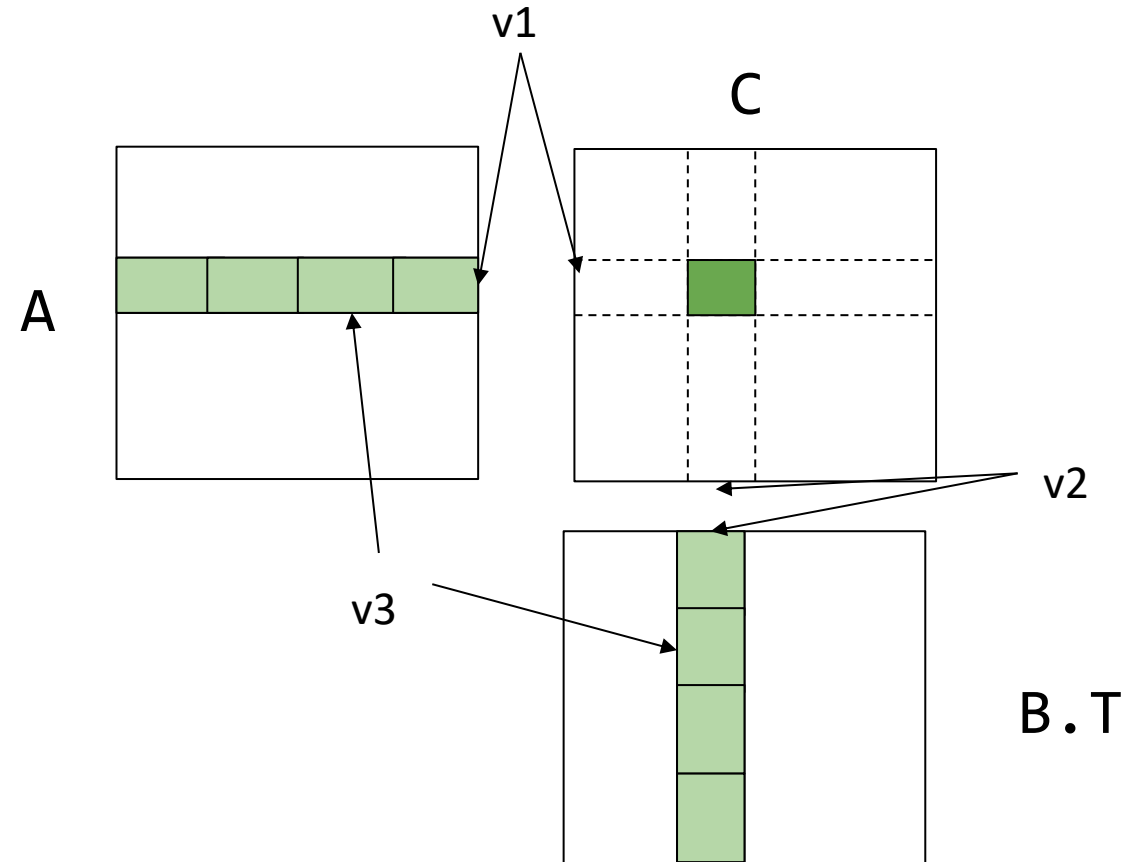
C's register memory cost : 1

Load cost: $2 * \text{dramspeed} * n^3$

Register cost: 3

Register tiled matrix multiplication

```
dram float A[n/v1][n/v3][v1][v3];  
dram float B[n/v2][n/v3][v2][v3];  
dram float C[n/v1][n/v2][v1][v2];  
  
for (int i = 0; i < n/v1; ++i) {  
    for (int j = 0; j < n/v2; ++j) {  
        register float c[v1][v2] = 0;  
        for (int k = 0; k < n/v3; ++k) {  
            register float a[v1][v3] = A[i][k];  
            register float b[v2][v3] = B[j][k];  
            c += dot(a, b.T);  
        }  
        C[i][j] = c;  
    }  
}
```



Register tiled matrix multiplication

```
dram float A[n/v1][n/v3][v1][v3];
dram float B[n/v2][n/v3][v2][v3];
dram float C[n/v1][n/v2][v1][v2];

for (int i = 0; i < n/v1; ++i) {
    for (int j = 0; j < n/v2; ++j) {
        register float c[v1][v2] = 0;
        for (int k = 0; k < n/v3; ++k) {
            register float a[v1][v3] = A[i][k];
            register float b[v2][v3] = B[j][k];
            c += dot(a, b.T);
        }
        C[i][j] = c;
    }
}
```

A's dram->register time cost: $n^3/v2$

B's dram->register time cost: $n^3/v1$

A's register memory cost: $v1*v3$

B's register memory cost: $v2*v3$

C's register memory cost: $v1*v2$

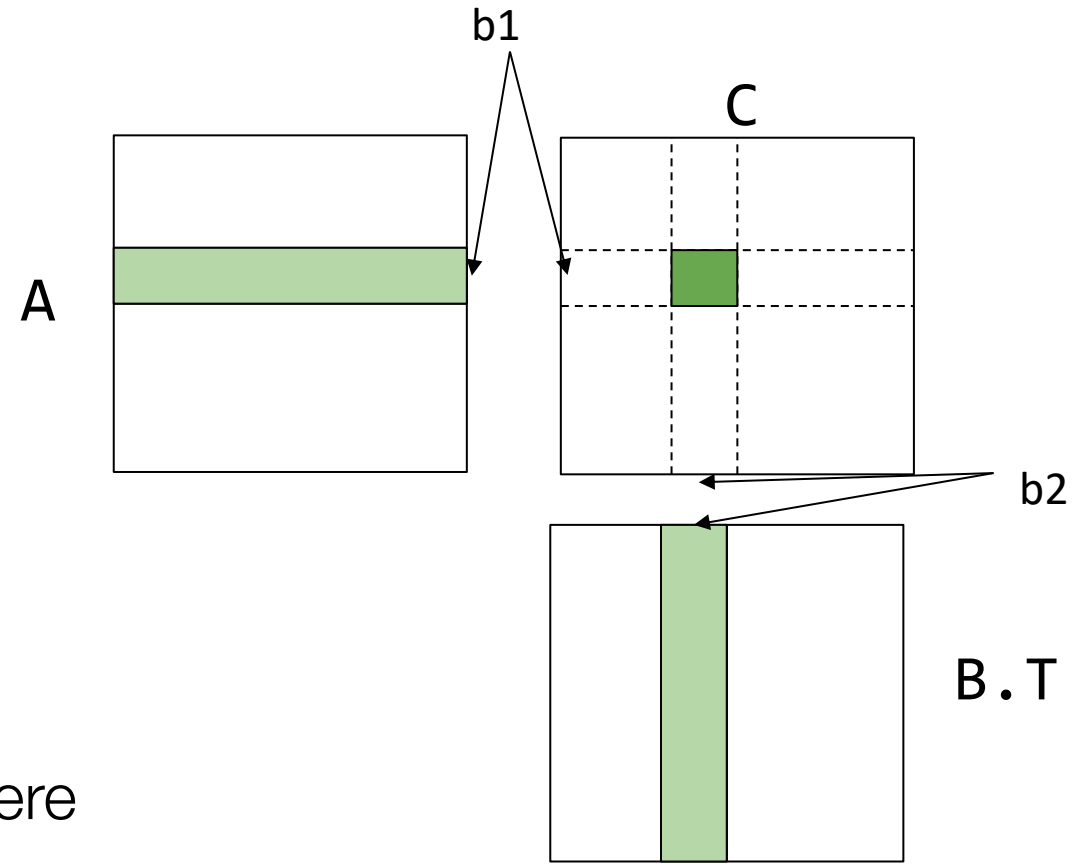
load cost: $\text{dramspeed} * (n^3/v2 + n^3/v1)$

Register cost: $v1*v3 + v2*v3 + v1*v2$

Cache line aware tiling

```
dram float A[n/b1][b1][n];  
dram float B[n/b2][b2][n];  
dram float C[n/b1][n/b2][b1][b2];  
for (int i = 0; i < n/b1; ++i) {  
    l1cache float a[b1][n] = A[i];  
    for (int j = 0; j < n/b2; ++j) {  
        l1cache b[b2][n] = B[j];  
  
        C[i][j] = dot(a, b.T);  
    }  
}
```

Sub-procedure, can apply register tiling here



Cache line aware tiling

```
dram float A[n/b1][b1][n];
dram float B[n/b2][b2][n];
dram float C[n/b1][n/b2][b1][b2];
for (int i = 0; i < n/b1; ++i) {
    l1cache float a[b1][n] = A[i];
    for (int j = 0; j < n/b2; ++j) {
        l1cache b[b2][n] = B[j];

        C[i][j] = dot(a, b.T);
    }
}
```

A's dram->l1 time cost: n^2

B's dram->l1 time cost: $n^3 / b1$

Constraints:

- $b1 * n + b2 * n < \text{l1 cache size}$
- To still apply register blocking on dot
 - $b1 \% v1 == 0$
 - $b2 \% v2 == 0$

Putting it together

```
dram float A[n/b1][b1/v1][n][v1];
dram float B[n/b2][b2/v2][n][v2];

for (int i = 0; i < n/b1; ++i) {
    l1cache float a[b1/v1][n][v1] = A[i];
    for (int j = 0; j < n/b2; ++j) {
        l1cache b[b2/v2][n][v2] = B[j];
        for (int x = 0; x < b1/v1; ++x)
            for (int y = 0; y < b2/v2; ++y) {
                register float c[v1][v2] = 0;
                for (int k = 0; k < n; ++k) {
                    register float ar[v1] = a[x][k][:];
                    register float br[v2] = b[y][k][:];
                    C += dot(ar, br.T)
                }
            }
    }
}
```

load cost:

$\text{l1speed} * (n^3/v2 + n^3/v1) +$
 $\text{dramspeed} * (n^2 + n^3/b1)$

Key insight: memory load reuse

```
dram float A[n/v1][n/v3][v1][v3];  
dram float B[n/v2][n/v3][v2][v3];  
dram float C[n/v1][n/v2][v1][v2];
```

a get reused v_2 times
b get reused v_1 times

```
for (int i = 0; i < n/v1; ++i) {  
    for (int j = 0; j < n/v2; ++j) {  
        register float c[v1][v2] = 0;  
        for (int k = 0; k < n/v3; ++k) {  
            register float a[v1][v3] = A[i][k];  
            register float b[v2][v3] = B[j][k];  
            c += dot(a, b.T);  
        }  
        C[i][j] = c;  
    }  
}
```

A's dram->register time cost: n^3/v_2
B's dram->register time cost: n^3/v_1

Common reuse patterns

```
float A[n][n];  
float B[n][n];  
float C[n][n];
```

```
C[i][j] = sum(A[i][k] * B[j][k], axis=k)
```

Access of A is independent of j,
tile the j dimension by v enables reuse of A for v times.

Discuss: possible reuse pattern in convolution

```
float Input[n][ci][h][w];  
float Weight[co][ci][K][K];  
float Output[n][co][h][w];
```

```
Output[b][co][y][x] =  
    sum(Input[b][k][y+ry][x+rx] *  
        Weight[co][k][ry][rx], axis=[k, ry, rx])
```

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