Numerical Analysis – Fall 2023

Assignment #3

Issued: Oct. 13, 2023 Due: Oct. 23, 2023

Please upload to the 'hw3' directory if you submit your homework in time.

Problem 1:

The following linear systems $A\mathbf{x} = \mathbf{b}$ have $\mathbf{x}$ as the actual solution and $\tilde{\mathbf{x}}$ as an approximate solution. Compute $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty$ and $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

a.
$$x_1 + 2x_2 + 3x_3 = 1,$$
$$2x_1 + 3x_2 + 4x_3 = -1,$$
$$3x_1 + 4x_2 + 6x_3 = 2,$$
$$\mathbf{x} = (0, -7, 5)^t,$$
$$\tilde{\mathbf{x}} = (-0.2, -7.5, 5.4)^t.$$

b.
$$x_1 + 2x_2 + 3x_3 = 1,$$
$$2x_1 + 3x_2 + 4x_3 = -1,$$
$$3x_1 + 4x_2 + 6x_3 = 2,$$
$$\mathbf{x} = (0, -7, 5)^t,$$
$$\tilde{\mathbf{x}} = (-0.33, -7.9, 5.8)^t.$$

Problem 2:

Show that if A is symmetric, then $||A||_2 = \rho(A)$.

Problem 3:

Implement the algorithm of Gaussian elimination with scaled partial pivoting, and solve the following linear systems.

a.
$$0.03x_1 + 58.9x_2 = 59.2,$$
$$5.31x_1 - 6.10x_2 = 47.0.$$
Actual solution [10, 1].

b.
$$3.03x_1 - 12.1x_2 + 14x_3 = -119,$$
$$-3.03x_1 + 12.1x_2 - 7x_3 = 120,$$
$$6.11x_1 - 14.2x_2 + 21x_3 = -139.$$
Actual solution $[0, 10, \frac{1}{7}]$.

Problem 4:

Implement the Jacobi iterative method and list the first three iteration results when solving the following linear systems, using $\mathbf{x}^{(0)} = \mathbf{0}$.

a. $$\begin{aligned} 4x_1 + x_2 - x_3 &= 5, \\ -x_1 + 3x_2 + x_3 &= -4, \\ 2x_1 + 2x_2 + 5x_3 &= 1. \end{aligned}$$

b. $$\begin{aligned} -2x_1 + x_2 + \tfrac{1}{2}x_3 &= 4, \\ x_1 - 2x_2 - \tfrac{1}{2}x_3 &= -4, \\ x_2 + 2x_3 &= 0. \end{aligned}$$

Problem 5:

Use the Jacobi method and Gauss-Seidel method to solve the following linear systems, with TOL = 0.001 in the $L\infty$ norm.

a. $$\begin{aligned} 3x_1 - x_2 + x_3 &= 1, \\ 3x_1 + 6x_2 + 2x_3 &= 0, \\ 3x_1 + 3x_2 + 7x_3 &= 4. \end{aligned}$$

b. $$\begin{aligned} 10x_1 - x_2 \phantom{{}- 2x_3} &= 9, \\ -x_1 + 10x_2 - 2x_3 &= 7, \\ - 2x_2 + 10x_3 &= 6. \end{aligned}$$

Problem 6:

Prove: If **A** is a matrix and $\rho_1, \rho_2, \cdots, \rho_k$ are distinct eigenvalues of **A** with associated eigenvectors $x_1, x_2, \cdots, x_k$, then $\{x_1, x_2, \cdots, x_k\}$ is a linearly independent set.

Problem 7:

Prove that a strictly diagonally dominant matrix is invertible.