

Numerical Analysis – Fall 2023

Assignment #5

Issued: Nov. 8, 2023

Due: Nov. 11, 2023

Problem 1:

证明下列积分计算公式具有三次代数精度。

$$\int_a^b f(x)dx \approx \frac{b-a}{b} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

Problem 2:

假设 $N(h)$ 对于任意 $h > 0$ ，都是 M 的近似，其中 $M = N(h) + K_1 h^2 + K_2 h^4 + K_3 h^6 + \dots$,

K_1, K_2, K_3, \dots 是常数。请使用 $N(h), N\left(\frac{h}{3}\right), N\left(\frac{h}{9}\right)$ 来产生 M 的 $O(h^6)$ 近似。

Problem 3:

Approximate the following integrals using the Trapezoidal rule and Simpson's rule, respectively.

a. $\int_{-0.25}^{0.25} (\cos x)^2 dx$

b. $\int_{-0.5}^0 x \ln(x+1) dx$

c. $\int_{0.75}^{1.3} ((\sin x)^2 - 2x \sin x + 1) dx$

d. $\int_e^{e+1} \frac{1}{x \ln x} dx$

Problem 4:

Use Romberg integration to compute $R_{3,3}$ for the following integrals.

a. $\int_{-1}^1 (\cos x)^2 dx$

b. $\int_{-0.75}^{0.75} x \ln(x+1) dx$

c. $\int_1^4 ((\sin x)^2 - 2x \sin x + 1) dx$

d. $\int_e^{2e} \frac{1}{x \ln x} dx$

Problem 5:

Use Euler's method to approximate the solutions for each of the following initial-value problems.

a. $y' = y/t - (y/t)^2, \quad 1 \leq t \leq 2, \quad y(1) = 1, \text{ with } h = 0.1$

b. $y' = 1 + y/t + (y/t)^2, \quad 1 \leq t \leq 3, \quad y(1) = 0, \text{ with } h = 0.2$