Numerical Analysis – Fall 2023

Assignment #1

Issued: Sept. 27, 2023 Due: Oct. 14, 2023

Please hand in the C or Matlab or Python or others, graphics, and a brief description of your reasoning as well as comments if any. You should pack all of your files into a .rar or .zip file, titled as "xxxxxxx(your student ID)_Homework_1", and then submit it by uploading to server before 11:59pm of the due day.

Problem 1:

Suppose $f \in C[a, b]$, that x_1 and x_2 are in $[a, b]$.

a. Show that a number ξ exists between x_1 and x_2 with

$$f(\xi) = \frac{f(x_1) + f(x_2)}{2} = \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2).$$

b. Suppose that c_1 and c_2 are positive constants. Show that a number ξ exists between x_1 and x_2 with

$$f(\xi) = \frac{c_1 f(x_1) + c_2 f(x_2)}{c_1 + c_2}.$$

c. Give an example to show that the result in part **b.** does not necessarily hold when c_1 and c_2 have opposite signs with $c_1 \neq -c_2$.

Problem 2:

Let $f \in C[a, b]$ be a function whose derivative exists on (a, b). Suppose f is to be evaluated at x_0 in (a, b), but instead of computing the actual value $f(x_0)$, the approximate value, $\tilde{f}(x_0)$, is the actual value of f at $x_0 + \epsilon$, that is, $\tilde{f}(x_0) = f(x_0 + \epsilon)$.

a. Use the Mean Value Theorem 1.8 to estimate the absolute error $|f(x_0) - \tilde{f}(x_0)|$ and the relative error $|f(x_0) - \tilde{f}(x_0)|/|f(x_0)|$, assuming $f(x_0) \neq 0$.

b. If $\epsilon = 5 \times 10^{-6}$ and $x_0 = 1$, find bounds for the absolute and relative errors for

i. $f(x) = e^x$

ii. $f(x) = \sin x$

c. Repeat part (b) with $\epsilon = (5 \times 10^{-6})x_0$ and $x_0 = 10$.

Problem 3:

Perform the following computations (i) exactly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. (iv) Compute the relative errors in parts (ii) and (iii).

a. $\dfrac{4}{5} + \dfrac{1}{3}$

b. $\left(\dfrac{1}{3} + \dfrac{3}{11}\right) - \dfrac{3}{20}$

Problem 4 （证明题）:

Suppose that as x approaches zero,

$$F_1(x) = L_1 + O(x^\alpha) \quad \text{and} \quad F_2(x) = L_2 + O(x^\beta).$$

Let c_1 and c_2 be nonzero constants, and define

$$F(x) = c_1F_1(x) + c_2F_2(x) \quad \text{and}$$
$$G(x) = F_1(c_1x) + F_2(c_2x).$$

Show that if $\gamma = \text{minimum } \{\alpha, \beta\}$, then as x approaches zero,

a. $F(x) = c_1L_1 + c_2L_2 + O(x^\gamma)$

b. $G(x) = L_1 + L_2 + O(x^\gamma)$.

Problem 5:

Implement the Bisection method in C or matlab and find solutions accurate to within 10^{-5} for the following problems. **(List the midpoints in each iteration as well) .** **【请在作业中上交程序代码】**

a. $e^x - x^2 + 3x - 2 = 0$ for $0 \le x \le 1$

b. $x\cos x - 2x^2 + 3x - 1 = 0$ for $0.2 \le x \le 0.3$ and $1.2 \le x \le 1.3$

Problem 6:

Implement the fixed-point iteration method and find solutions accurate to within 10^{-2} for the following problems. **(List pn in each iteration as well)**. **【请在作业中上交程序代码】**

a. $2\sin \pi x + x = 0$ on [1, 2], use $p_0 = 1$

b. $3x^2 - e^x = 0$

Problem 7 （证明题）:

Let $g \in C^1[a,b]$ and p be in (a, b) with g(p) = p and $|g'(p)| > 1$. Show that there exists a $\delta > 0$ such that if $0 < |p_0 - p| < \delta$, then $|p_0 - p| < |p_1 - p|$. Thus, no matter how close the initial approximation p_0 is to p, the next iterate p_1 is father away, so the fixed-point iteration does not converge if $p_0 \ne p$.