Numerical Characteristics

Expectation

$$
\begin{align}
\mathrm{E}[X+Y] &= \mathrm{E}[X]+\mathrm{E}[Y] \tag{1}\\
\mathrm{E}[cX] &= c\,\mathrm{E}[X] \tag{2}\\
\mathrm{E}[X-c] &= \mathrm{E}[X]-c \tag{3}
\end{align}
$$

Variance

$$
\begin{align}
\mathrm{Var}[X] \triangleq \mathrm{E}[(X-\mathrm{E}[X])^2] &= \mathrm{E}[X^2]-(\mathrm{E}[X])^2 \tag{4}\\
\mathrm{Var}[X+Y] &= \mathrm{Var}[X]+\mathrm{Var}[Y]+2\,\mathrm{Cov}(X,Y) \tag{5}\\
\mathrm{Var}[cX] &= c^2\,\mathrm{Var}[X] \tag{6}\\
\mathrm{Var}[X-c] &= \mathrm{Var}[X] \tag{7}
\end{align}
$$

Covariance

$$
\begin{align}
\mathrm{Cov}[X,Y] &\triangleq \mathrm{E}[(X-\mu_X)(Y-\mu_Y)] \tag{8}\\
\mathrm{Cov}[X,Y] &= \mathrm{Cov}[Y,X] \tag{9}\\
\mathrm{Cov}[X,X] &= \mathrm{E}[(X-\mu_X)(X-\mu_X)] = \mathrm{Var}[X] \tag{10}
\end{align}
$$

Correlation Coefficient

$$
\begin{gathered}
X^* \triangleq \frac{X-\mu_X}{\sigma_X}\\
Y^* \triangleq \frac{Y-\mu_Y}{\sigma_Y}\\
\mathrm{E}[X^*]=0, \mathrm{Var}[X^*]=1\\
\mathrm{E}[Y^*]=0, \mathrm{Var}[Y^*]=1
\end{gathered}
$$

$$
\rho(X,Y)=\mathrm{Cov}[X^*,Y^*]=\mathrm{E}[X^*Y^*]\in[-1,1]
$$

Normal Distribution

$$
N(x;\mu,\sigma^2)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}
$$

From 1-d to n-d, will normal distribution become

$$N(\mathrm{x}; \vec{\mu}, \vec{\sigma^2}) = \ldots, \text{ where}$$
$$\mathrm{x} \in \mathbb{R}^n, \mathrm{x} = [x_1, \ldots, x_n]$$
$$\vec{\mu} \in \mathbb{R}^n, \vec{\mu} = [\mu_1, \ldots, \mu_n]$$
$$\vec{\sigma^2} \in \mathbb{R}^n, \vec{\sigma^2} = [\sigma_1^2, \ldots, \sigma_n^2]??$$

In fact,

$$N(\mathrm{x}; \mu, \Sigma) = \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} e^{-\frac{(\mathrm{x}-\mu)^T \Sigma^{-1} (\mathrm{x}-\mu)}{2}}, \text{ where}$$
$$\Sigma = \begin{bmatrix} \mathrm{Cov}[x_1, x_1] & \ldots & \mathrm{Cov}[x_1, x_n] \\ \vdots & \mathrm{Cov}[x_i, y_j] & \vdots \\ \mathrm{Cov}[x_n, x_1] & \ldots & \mathrm{Cov}[x_n, x_n] \end{bmatrix}$$