

# CS531 Programming Assignment 2: Towers of Corvallis

Michael Lam, Xu Hu  
EECS, Oregon State University

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## **Abstract**

In this assignment we design, implement and discuss two different informed search algorithms and heuristics to solve the Towers of Corvallis, which is a variation of Towers of Hanoi.

## **1 Introduction**

The Towers of Corvallis puzzle is a variation on the Towers of Hanoi puzzle. While similarly consisting of 3 pegs and  $n$  disks, the Corvallis variation allows any disk to go on top of any other disk. The goal is to find the smallest number of moves in getting from an initial state to the goal state, which is defined as the order 9876543210 on peg A for 10 disks and similarly for fewer disks.

We implement two informed search algorithms: A\* and RBFS (recursive best-first search). As informed searches, we also implement two heuristics, one admissible and one non-admissible. For each algorithm and heuristic function, we evaluate the performance by testing across different number of disks and different initial states.

## **2 A\* Search**

We implement the A\* search algorithm for finding the paths between initial states to a given goal state in the problem of Tower of Corvallis. It summarized as follow:

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**Algorithm 1** A\* Search

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```
exploredSet =  $\emptyset$ 
frontier = [initialPath]
while number(explored) < NMAX do
  if frontier ==  $\emptyset$  then

    return FALSE
  end if
  path = frontier.pop()
  state = path[0]
  if state == goalState then

    return path
  end if
  for action in state.validActions() do
    for newState in action.results() do
      newPath = path + newState
      if ismember(frontier,newPath) == FALSE then
        frontier.push(newPath)
      end if
    end for
  end for
end while
```

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Here, for implementing the frontier, we use the priority queue, which is actually a heap data structure. We use a callback function  $f(state) = g(state) + h(state)$  as the priority, the  $g(state)$  is the length of the path and  $h(state)$  is the heuristic for estimating the distance between current state to goal state. We will analyze several admissible and non-admissible heuristics in section ??.

### 3 Recursive Best-First Search

Recursive best-first search or RBFS works by storing an f-limit for each node. The algorithm uses the f-limit to decide which subtree of the problem tree to explore by considering the best and 2nd best (alternative) f-limits. In order to keep the search functional, RBFS also updates the f-values of each node

during the search.

The advantage of RBFS over A\* is that RBFS uses less memory. Whereas A\* stores all of its explored nodes, RBFS will only keep relevant nodes in memory. However, the disadvantage of RBFS over A\* is that RBFS could expand more nodes than A\* due to redundancy. Since RBFS does not store all nodes explored, it can re-expand the same nodes and thereby increasing computation time.

The following is the pseudocode:

```
RBFS(state, f-limit)
  if state is the goal state
    return solution
  successors := all children of state
  if successors is empty
    return failure
  else
    for each s in successors
      s.f := max(s.g + s.h, state f)
    loop do
      best := lowest f-value node in successors
      if best.f > f-limit
        return failure, best.f
      alternative := second best f-value of any node in successors
      result, best.f := RBFS(best, min(f-limit, alternative))
      if result is not failure
        return result
```

## 4 Experiments

## 5 Discussion

We discuss our results and answer the questions in this section.

1. Show an example solution sequence for each algorithm for the largest size you tested

For size  $n = 10$  and A\* on problem "7126049853":

For size  $n = 10$  and RBFS on problem "7126049853":

[7126049853,-,-], [126049853,7,-], [26049853,17,-], [6049853,217,-], [049853,6217,-], [49853,06217,-], [9853,406217,-], [853,9406217,-], [53,89406217,-], [3,589406217,-], [-,3589406217,-], [3,589406217,-], [-,3589406217,-], [3,589406217,-], [-,3589406217,-], [3,589406217,-], [3,589406217,-], [53,89406217,-], [53,9406217,8], [53,406217,98], [3,5406217,98], [3,406217,598], [3,06217,4598], [-,06217,34598], [0,6217,34598], [0,217,634598], [0,17,2634598], [10,7,2634598], [210,7,634598], [210,67,34598], [3210,67,4598], [43210,67,598], [543210,67,98], [6543210,7,98], [76543210,-,98], [76543210,9,8], [876543210,9,-], [9876543210,-,-]

2. Is there a clear preference ordering among the heuristics you tested considering the number of nodes searched and the total CPU time taken to solve the problems for the two algorithms?

3. Can a small sacrifice in optimality give a large reduction in the number of nodes expanded? What about CPU time?

For RBFS, one can sacrifice optimality by pruning parts of the search tree. While it may not necessarily yield an optimal solution, this may yield a large reduction in the number of expanded nodes if the RBFS search considers one path down the search tree without having to backtrack and expand nodes redundantly. If one is lucky, RBFS may yield an optimal solution if parts of the search tree were pruned properly.

4. How did you come up with your heuristic evaluation functions?

5. How do the two algorithms compare in the amount of search involved and the cpu-time?

6. Do you think that either of these algorithms scale to even larger problems? What is the largest problem you could solve with the best algorithm+heuristic combination? Report the wall-clock time, CPU-time, and the number of nodes searched.

7. Is there any tradeoff between how good a heuristic is in cutting down the number of nodes and how long it took to compute? Can you quantify it?

8. Is there anything else you found that is of interest?