MATH 250: TOPOLOGY I

FALL 2025 INITIAL READING

Note: You do not need to submit anything for this assignment.

1. Sets and Logic

Next week, we will start Chapter 2 (Topological Spaces and Continuous Functions) of Munkres's textbook *Topology*, 2nd Edition. Please check Chapter 1 (Set Theory and Logic) to make sure that you are familiar with the language introduced there, especially

> set, element, subset, union, intersection, complement, disjoint union, product, power set, image, inverse image, injective (map), surjective, bijective,

and the related notations

$$\{\quad\},\ \in,\ \subseteq,\ X \cup Y,\ X \cap Y,\ X - Y,\ X \sqcup Y,\ X \times Y,\ \textit{etc.}$$

We will write $\mathbf{Z}$ for the set of integers, $\mathbf{Q}$ for the set of rational numbers, and $\mathbf{R}$ for the set of real numbers.

2. Groups

2.1. Soon after the midterm exam, we will begin to discuss the fundamental group of a topological space. For this, we need several notions from group theory.

If you haven't encountered them before: A *group* consists of a set G and a *law* or *operation* $*$ that turns two elements $x, y \in G$ into a new element $x * y \in G$ called their *composition*, such that the following axioms hold:

(1) *Associativity.* For any $x, y, z \in G$, we have $(x * y) * z = x * (y * z)$. This informally means that we can ignore parentheses when performing the group operation, as long as we preserve the order of the elements.
(2) *Identity.* There is an *identity* element $e \in G$ such that for any $x \in G$, we have $e * x = x * e = x$.
(3) *Inverses.* For any $x \in G$, there is some *inverse* element $y \in G$ such that $x * y = y * x = e$. It turns out that this element is unique to x, so it might be denoted by a notation like x^{-1} or $-x$.

Sometimes, the symbol $*$ is omitted, so that the composition of x with y is merely denoted xy.

Example 1. Taking G to be any one of $\mathbf{Z}$, $\mathbf{Q}$, or $\mathbf{R}$, and $*$ to be the addition law $+$, creates a group. (Why?) Keeping the same G, but taking $*$ to be the multiplication law $\times$, does not create a group. (Why?)

Example 2. Fix a set X. Taking G to be the set of bijections from X onto itself, and $*$ to be the composition-of-maps operation $\circ$, yields a group.

2.2. You have time up through early October to learn or review the following terms:

abelian/commutative (group), subgroup, left/right coset, normal subgroup, quotient group, homomorphism, kernel, isomorphism.

Try to come up with examples for each. We will also review most of them in class, but rather quickly.

2.3. Professor Davenport might suggest various nice expositions about groups. Some informal expositions that I like:

- Keith Conrad's notes. The most basic articles are in the section from "Why groups?" through "Isomorphisms", but you may enjoy reading many others.
- Pavel Etingof's "Groups around us".

If you prefer a formal exposition, I might recommend Chapter 2 of Artin's *Algebra*, 2nd Edition. This chapter (not the full text) is available here.

3. Analysis Situs

This course is about topology, the study of abstract shape. While many cultures and individuals contributed ideas to this subject before the 20th century, our concept of topology as a branch of mathematics—especially algebraic topology—owes the most to an 1895 paper of Henri Poincaré titled *Analysis situs*.

There is an English translation of the paper by John Stillwell, freely available online (*e.g.*, here). Try to read:

§1. First definition of manifold
§2. Homeomorphism

(If you prefer French, then you can even try Poincaré's original.) Notice that the writing style of the time was much more informal than it is now, and that Poincaré does not always use jargon in the same way that we do.

Look at his first example of a manifold, given by a polynomial equation in x_1 and x_2. Graph the set of (real) solutions, and using the graph, try to express for yourself the ideas that Poincaré is communicating.