<u>Last time</u> $\pi_1(X, x) = \{[\gamma] \mid \text{loops } \gamma \text{ in } X \text{ at } x\}$
under the operation
$[\beta] * [\gamma] = [\beta * \gamma]$ [what's the id elt?]

<u>Q</u> how much does it depend on X, x?

$$[0, 1] \xrightarrow{\gamma} X \xrightarrow{f} Y$$

if γ is a path in X, then $f \circ \gamma$ is a path in Y

$$[0, 1] \times [0, 1] \xrightarrow{h} X \xrightarrow{f} Y$$

if h is a path homotopy from γ to γ'
then $f \circ h$ is a path homotopy from $f \circ \gamma$ to $f \circ \gamma'$

$$[0, 1] \xrightarrow{\beta * \gamma} X \xrightarrow{f} Y$$

$$[f \circ (\beta * \gamma)](s) = f((\beta * \gamma)(s))$$
$$=$$
$$[(f \circ \beta) * (f \circ \gamma)](s) = \begin{cases} f(\beta(2s)) & s \leq 1/2 \\ f(\gamma(2s)) & s \geq 1/2 \end{cases}$$

<u>Thm</u> suppose $f : X \to Y$ is cts

1) if γ, γ' are paths in X s.t. $\gamma \sim_p \gamma'$
then $f \circ \gamma, f \circ \gamma'$ are paths in Y s.t. $f \circ \gamma \sim_p f \circ \gamma'$

2) if β, γ are paths in X s.t. $\beta(1) = \gamma(0)$,
then $f \circ (\beta * \gamma) = (f \circ \beta) * (f \circ \gamma)$

<u>Cor</u> suppose $f : X$ to Y is cts and $f(x) = y$

1) well-def map $f_* : \pi_1(X, x)$ to $\pi_1(Y, y)$ s.t.

$$f_*([\gamma]) = [f \circ \gamma]$$

[if $[\gamma] = [\gamma']$, then $[f \circ \gamma] = [f \circ \gamma']$]

2) f_* is a group homomorphism:

$$f_*([\beta] * [\gamma]) = f_*([\beta]) * f_*([\gamma])$$

[LHS $= f_*([\beta * \gamma])$
$= [f \circ (\beta * \gamma)]$
$= [(f \circ \beta) * (f \circ \gamma)]$
$= [f \circ \beta] * [f \circ \gamma]$ = RHS]

space & point		group
(X, x)	$\mapsto$	$\pi_1(X, x)$
cts map		homomorphism
$f : X$ to Y	$\mapsto$	$f_* : \pi_1(X, x)$ to $\pi_1(Y, y)$

Q what about a composition of cts maps?

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

<u>Thm</u> $(g \circ f)_* = g_* \circ f_*$

<u>Pf</u> $(g \circ f)_*([\gamma]) = [(g \circ f) \circ \gamma]$
$=$
$g_*(f_*([\gamma])) = g_*([f \circ \gamma]) = [g \circ (f \circ \gamma)]$

analogies:

sets	bijections
spaces	homeomorphisms
groups	isomorphisms

<u>Cor</u> if $f : X \to Y$ is a homeo
then f_* is an isomorphism

<u>Ex</u> if $f = id_X$, then $f_* = id_{\pi_1(X, x)}$

<u>Pf</u> know f_* is always a homomorphism
need f_* bijective

show that $(f^{-1})_*$ is the two-sided inverse to f_*