PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

18. Category $\mathcal{O}$ for Cherednik algebras

Exercise 18.1. *Use* $\operatorname{gr} H_c = S(\mathfrak{h} \oplus \mathfrak{h}^*)\#W$ *to show that* H_c *is Noetherian.*

Exercise 18.2. *We have* $[h, x] = x, [h, w] = 0, [h, y] = -y$ *for all* $x \in \mathfrak{h}^*, w \in W, y \in \mathfrak{h}$.

Problem 18.1. *Write an action of* y *on* $\Delta(E)$ *via a 1st order differential operator with poles.*

Exercise 18.3. *Show that* $\operatorname{Hom}_{\mathcal{O}}(\Delta(E), M) = \operatorname{Hom}_W(E, M)$, *where* $M^{\mathfrak{h}} = \{m \in M | \mathfrak{h}m = 0\}$.

Exercise 18.4. *Show that each* $\Delta(E)$ *has a unique irreducible quotient, denoted* $L(E)$. *Show that the natural inclusion* $E \hookrightarrow \Delta(E)$ *gives rise to an inclusion* $E \hookrightarrow L(E)$. *Further, show that the objects* $L(E)$ *form a complete list of irreducible objects in* $\mathcal{O}$.

Exercise 18.5. *Prove that* h *acts locally finitely on any object in* $\mathcal{O}$ *and that any object in* $\mathcal{O}$ *has finite length. Deduce that all generalized subspaces for* h *are finite dimensional and that any module in* $\mathcal{O}$ *is finitely generated over* $\mathbb{C}[\mathfrak{h}]$.

Exercise 18.6. *Show that (HW1) and (HW2) hold for* $\mathcal{O}_c$.

Exercise 18.7. *Show that if an exact sequence* $0 \to \Delta(E) \to M \to \Delta(E') \to 0$ *does not split, then* $E' < E$. *Deduce that if* $M_1 \oplus M_2$ *is* Δ*-filtered, then* M_1 *and* M_2 *are* Δ*-filtered.*