

# MATH 251: Topology II

[website]

[syllabus]

[exam dates]

[participation]

[plagiarism]

[Jan 30 add/drop deadline]

[course objectives]

$\mathbb{R}$  real line

what topologies can we put on  $\mathbb{R}$ ?

analytic

discrete

indiscrete

finite complement (aka cofinite)

[generated by  $\mathbb{R} - S$  for finite  $S$ ]

lower limit (aka Sorgenfrey)

[generated by  $[a, b)$  for  $a < b$ ]

Q which are most similar to analytic?

e.g.,        connected  
             Hausdorff  
             locally compact

Df a top space  $X$  is connected iff

there is no pair of disjoint nonempty open  $U, V$   
sub  $X$  s.t.  $X = U \cup V$  [draw]

e.g.,        analytic  
             indiscrete  
             cofinite

Df  $X$  is Hausdorff [séparé] iff

for any  $x \neq y$  in  $X$ , have disjoint open  $U, V$  sub  $X$   
s.t.  $x$  in  $U$  and  $y$  in  $V$  [draw]

e.g.,        analytic  
             discrete  
             lower limit

Df  $X$  is locally compact iff

for any  $x$  in  $X$ , have open  $U$  and compact  $K$  sub  $X$   
s.t.  $x$  in  $U$  sub  $K$  [draw]

[which is stronger, compact or locally compact?]

locally compact but not compact:

analytic  $[x \in (x - 1, x + 1) \text{ sub } [x - 1, x + 1]]$   
discrete  $[\{x\} \text{ is both open and compact}]$

compact:

indiscrete  
cofinite  $[\text{if } \{U_i\}_i \text{ is an open cover of } R, \text{ then}$   
 $\text{some } U_i \text{ is nonempty}]$

Thm the lower limit top is not locally compact

$[\text{will do proof later}]$

focus of this course:

locally compact Hausdorff spaces  
 $[\text{usually but not always connected}]$

next two weeks:

more on Hausdorffness,  
connectedness,  
compactness