MATH 251: Topology II

[website]

[syllabus]

- [exam dates]
- [participation]
- [plagiarism]
- [Jan 30 add/drop deadline]
- [course objectives]

Exam 1: CLOSED BOOK, in-class, Mon 1/26

know BY HEART defs, examples, non-examples:

bases (§13), metrics (20–21)
subspaces (16), quotients (22), products (19)
interiors & closures (17)
Hausdorff spaces (17), convergence (17)

separations (23–24)
connected components (25)
paths (23), path components (25)
compact spaces (26–27)

homotopies (51), path homotopies (51)
homotopy equivalences (58)

Euclidean metric on R^n [?]

$d(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$

Euclidean balls in R^n [?]

$B_d(x, \delta) = \{y \in R^n \mid d(x, y) < \delta\}$

Euclidean / analytic topology on R^n [?]

topology gen by d
= topology gen by basis $\{B_d(x, \delta)\}$
= $\{U \subset R^n \mid U$ is a union of balls $B_d(x, \delta)\}$
= $\{U \subset R^n \mid$ for all x in U, have $\delta > 0$
s.t. $B_d(x, \delta) \subset U\}$

<u>Ex</u> if $n = 1$, then $B_d(x, \delta) = (x - \delta, x + \delta)$
so open balls in R are open intervals

<u>Prob</u> metric topologies are Hausdorff

[note:] $R^n = R \times R \times \ldots \times R$, with n copies

product topology on $R \times R \times \ldots \times R$ [?]

= box topology on $R \times R \times \ldots \times R$ [for n finite]
= topology gen by $U_1 \times \ldots \times U_n$
where $U_i \subset R$ is open for each i
= $\{U \mid U$ is a union of sets of the form
$U_1 \times \ldots \times U_n$ with U_i's open$\}$
= $\{U \mid U$ is a union of sets of the form
$(a_1, b_1) \times \ldots \times (a_n, b_n)\}$

<u>Ex</u> $(-1, 0) \times (-1, 0)$ cup $(0, 1) \times (0, 1)$
is open in the product top on $R \times R$
but not of the form $U_1 \times U_2$

<u>Prob</u> via $R^n = R \times R \times \ldots \times R$,

analytic top on R^n
=
product top on $R \times R \times \ldots \times R$

[next:] circle $S^1 = \{(x, y) \text{ in } R^2 \mid x^2 + y^2 = 1\}$

subspace topology on S^1 [?]

$\{V \text{ cap } S^1 \mid V \text{ is open in } R^2\}$

[note:] surjective map $p : R$ to S^1 given by

$$p(t) = (\cos 2\pi t, \sin 2\pi t)$$

quotient topology on S^1 [?]

$\{U \text{ sub } S^1 \mid p^{-1}(U) \text{ is open in } R\}$

<u>Prob</u> the collection of <u>open arcs</u>
$\{p(t) \mid a < t < b\}$, for a, b in R,
satisfies the definition of a basis

<u>Prob</u> the following all match:
1) the topology generated by open arcs
2) the subspace top on S^1 from R^2
3) the quotient top on S^1 from R

[recall:] a <u>homeomorphism</u> from X onto Y is a bijection f : X to Y s.t. f and f^{-1} are both cts

in this case X and Y are called <u>homeomorphic</u>

[recall:] R and S^1 are not homeomorphic

<u>Q</u> how many ways to show that?