Last time   suppose $X = \prod_{i \in I} X_i$

the box topology on X is gen by
$\{\prod_i U_i \mid U_i \text{ open in } X_i \text{ for all } i\}$

the product topology on X is gen by
$\{\prod_i U_i \mid U_i \text{ open in } X_i \text{ for all } i,$
$U_i \neq X_i \text{ for only fin many } i\}$

if $I = \{1, \dots, n\}$ and $X_i = \mathbb{R}$ for all i, then $X = \mathbb{R}^n$

here, box = product

Q1   is it the same as the analytic top? [yes]
[square balls are open in box top]
[prod's of anlytc opens are anlytc open]

Q2   any open set in $\mathbb{R}^n$ not of the form
$U_1 \times U_2 \times \dots \times U_n$?

[draw]

if $I = \mathbb{Z}_+$ and $X_i = \mathbb{R}$ for all i, then $X = \mathbb{R}^\omega$

Q3   let $V = (-1, 1) \times (-1, 1) \times \dots$ in $\mathbb{R}^\omega$

open in box topology? [yes]
open in product topology? [no]

why not? [$V \neq \varnothing$, but no product basis elt sub V]

in general: prod top can be coarser than box top

Df for all i' in I

let $pr_{i'} : \prod_i X_i \to X_{i'}$ be

the projection map $pr_{i'}((x_i)_i) = x_{i'}$

Thm the product topology on $\prod_i X_i$ is the coarsest s.t. $pr_{i'}$ is cts for all i'

Pf suppose that T is a top on $\prod_i X_i$ in which $pr_{i'}$ is cts for all i'

then $pr_{i'}^{-1}(U_{i'})$ in T

for all i' and $U_{i'}$ open in $X_{i'}$

so for any finite J sub I,

$\bigcap_{i' \in J} pr_{i'}^{-1}(U_{i'})$ is open

but $\bigcap_{i' \in J} pr_{i'}^{-1}(U_{i'})$

$= \{(x_i) \mid x_{i'} \in U_{i'} \text{ for all } i' \in J\}$

$= \{\prod_i U_i \mid U_i = X_i \text{ for all } i \notin J\}$

so T contains the basis for the product top

so T contains the product top

similarly,

Thm the subspace top on A sub X is the coarsest s.t. the inclusion of A is cts

	subspace topology	product topology
makes	inclusion cts	projections cts

(Munkres §17)   suppose A sub X

interior Int_X(A)
= {a in X | have U open in X s.t. a in U sub A}
= union of open sets of X that are subsets of A

closure Cl_X(A)
= X – Int_X(X – A)
= X – bigcup_{V sub X – A and open in X} V
= bigcap_{V sub X – A and open in X} (X – V)
= bigcap_{Z sup A and closed in X} Z
= intersection of closed sets of X containing A

[draw]

Q   what is the closure of R^∞ in R^ω
in the box top? in the product top?