MATH 250: TOPOLOGY I MIDTERM GUIDE

FALL 2025

The midterm exam will be held in-class on **Wednesday, October 8, 2025**. It will start at 12:30 pm and end at 2:00 pm.

You will be allowed to look at any notes on paper that you wrote prior to the exam, and at the textbook (Munkres, *Topology*, 2nd Ed.). However, you will not be allowed to use electronic devices of any kind—including phones, computers, tablets, or other visual/audio devices—or any software.

What Could Appear.

§12.

- definition of a topology
- definitions of the discrete, indiscrete, and finite-complement topologies on any set, and why these are topologies
- definition of the analytic topology on $\mathbf{R}^n$
- which of the above topologies on $\mathbf{R}$ are finer or coarser than others

§13.

- what it means for a collection of subsets of X to be a basis
- what it means for a basis to generate a given topology on X
- examples where different bases generate the same topology on X

§18.

- what it means for a map between topological spaces to be continuous
- how to check continuity of $f\colon X \to Y$ using a basis for the topology on Y
- what it means for topological spaces to be homeomorphic
- examples of homeomorphisms between distinct sets (*e.g.*, $\mathbf{R}$ and $(0, 1)$)
- examples of continuous bijections that are not homeomorphisms

§16.

- definition of the subspace topology on $A \subseteq X$, given a topology on X
- how the subspace topology on A is related to continuity of the inclusion map from A into X
- examples where some subset of A is open in A, but not in X

§15, 19.

- definition of a direct product of sets $\prod_{i \in I} X_i$
- why $\mathbf{R}^n$ and $\mathbf{R}^\omega$ are examples of direct products of sets
- definitions of the box and product topologies on $\prod_{i \in I} X_i$, given a topology on X_i for each i

- which of the box or product topologies is finer than the other
- how the product topology on $\prod_{i \in I} X_i$ is related to continuity of the various projection maps $\mathrm{pr}_j \colon \prod_{i \in I} X_i \to X_j$
- the closures of $\mathbf{R}^\infty$ in the box and product topologies on $\mathbf{R}^\omega$ (PS 3, #3)

§20.

- definition of a metric
- definition of d-balls for a metric d, and of the topology they generate
- definitions of the euclidean and square metrics on $\mathbf{R}^n$ and on $\mathbf{R}^\infty$
- examples of different metrics on $\mathbf{R}^n$ that generate the same topology
- definition of the uniform topology on $\mathbf{R}^\omega$ (PS 2, #11), and how it compares to the box topology (#12)

§17.

- definitions of the interior and closure of a subset A of a topological space X
- how to check that a point in X belongs to the closure of a subset A
- what it means for a sequence of points to converge to a given point
- definition of the Hausdorff property
- what the Hausdorff property implies for convergence of sequences

§22.

- definition of the quotient topology on a set Q, given a topology on X and a surjective map $f \colon X \to Q$
- how the quotient topology on Q is related to continuity of f
- examples of quotient spaces (*e.g.*, constructed using equivalence relations)

§23–25.

- what it means for a topological space X to be connected, or for subsets $U, V \subseteq X$ to form a separation of X
- how connected subspaces of X interact with separations of X
- how connectedness interacts with continuous maps and finite products
- why $\mathbf{Q}$ is totally disconnected as a subspace of (analytic) $\mathbf{R}$, but not discrete
- definition of path-connectedness
- definitions of connected components and path components
- what it means to be locally connected or locally path connected
- examples of spaces that are disconnected but locally connected

$\approx$ *§26–27.*

- open-covering definition of compactness
- statement of Heine–Borel for $[0, 1]$

What We'll Have Covered by Then, But Will Not Appear.

- the evenly-spaced topology on $\mathbf{Z}$
- the countable-complement topology
- the axiom of choice
- equivalence/inequivalence of metrics (Problem Set 2, #10)
- the proof of the connectedness of $\mathbf{R}$
- the intermediate value theorem
- the tube lemma (Munkres Lem. 26.8)
- the topologist's sine curve
- how compactness interacts with continuous maps and finite products
- the proof of Heine–Borel for $[0, 1]$
- the countability and separation axioms