MATH 340: ADVANCED LINEAR ALGEBRA
MIDTERM GUIDE

SPRING 2025

The midterm exam will be held in-class on **Wednesday, February 26**. It will start 1–2 minutes after the start of class time (2:30 pm). It will be a closed-book exam, designed to take < 70 minutes.

What Could Appear.

$\approx$ *Chapter 1.*

- definition of a vector space
- definition of F^S for a set S, and of $F[x]$
- definitions of (linear) subspaces and their sums
- what it means for a sum to be a *direct* sum
- how to check that $U + W$ is a direct sum, for any subspaces $U, W \subseteq V$

$\approx$ *Chapter 2.*

- how to check that a set of vectors is linearly independent
- definition of the span of a set of vectors
- how to check that a set of vectors is a basis for V
- definition of dimension
- the formula relating the dimensions of U, W, $U \cap W$, and $U + W$

$\approx$ *Chapter 3.*

- how to check that a map between vector spaces is linear
- definition of the kernel (= nullspace) and image (= range) of a linear map
- how kernel and image relate to injectivity and surjectivity
- the formula relating $\dim \ker(T)$, $\dim \operatorname{im}(T)$, and $\dim V$ for a linear map $T : V \to W$
- how to describe linear $T : V \to W$ using (ordered) bases for V and W
- matrix-vector and matrix-matrix multiplication
- addition and scalar multiplication of matrices
- what it means for a linear map to be invertible, or for vector spaces to be (linearly) isomorphic
- how to express the matrix of T in a basis $(e_i)_i$, given the matrix of T in a basis $(v_i)_i$ and the expansions $v_i = \sum_k a_{k,i} e_k$
- definition of conjugacy for square matrices
- the formulas for trace and determinant, for 2×2 matrices

$\approx$ *Chapters 5, 8.*

- how to check that a subspace of V is T-stable (= T-invariant), given a linear operator $T : V \to V$
- definitions of eigenlines, eigenvalues, eigenvectors, eigenspaces
- how working over $\mathbf{R}$ versus over $\mathbf{C}$ affects existence of eigenvalues

- distinction between *an* eigenline with eigenvalue λ, *the* eigenspace for λ, and *the* generalized eigenspace for λ
- definitions of diagonalizable, nilpotent, and unipotent operators
- meaning of polynomials evaluated on linear operators (*e.g.*, $(T - \lambda)(T - \mu)$)
- how the minimal polynomial is related to eigenvalues
- the statement of the Jordan canonical form theorem

What We'll Have Covered by Then, But Will Not Appear.

- definitions of (or exotic examples of) fields and rings
- definition of $\mathrm{Hom}(V, W)$
- definitions of projections, involutions, rotations, shears
- definition of trace and determinant in general
- definition of the characteristic polynomial
- various hard proofs (of Steinitz exchange, the dimension formulas, the Jordan canonical form theorem, *etc.*)
- statement of the Cayley–Hamilton theorem

What We Won't Have Covered by Then.

Chapter 3. Products and quotients of vector spaces

Chapter 5. Existence/nonexistence of a minimal polynomial with real coefficients, for a real linear operator

Chapter 8. Properties of trace, *etc.* beyond conjugacy invariance