Last time $\mathrm{Cl}_X(A) = \mathrm{Int}_X(A)$

Q what is the closure of $\mathbb{R}^\infty$ in $\mathbb{R}^\omega$
in the box top? in the product top?

a useful formula:

$\mathrm{Cl}_X(A)$
$= X - \{x \mid \text{have } V \text{ open in } X \text{ s.t. } x \in V \subseteq X - A\}$
$= \{x \mid \text{no } V \text{ open in } X \text{ s.t. } x \in V \text{ and } V \subseteq X - A\}$
$= \{x \mid \text{if } V \text{ is open in } X \text{ and contains } x,$
$\text{then } V \cap A \neq \varnothing\}$

Q let $x = (1, 1/2, 1/3, 1/4, \ldots)$
is x in $\mathrm{Cl}_{\mathbb{R}^\omega}(\mathbb{R}^\infty)$…

…wrt the box topology? [draw]

no: x in $(0, 2) \times (0, 1) \times (0, 2/3) \times \ldots$

…wrt the product topology?

yes: suppose V open in $\mathbb{R}^\omega$ and x in V

pick basis elt B s.t. $x \in B \subseteq V$
$B = \prod_i B_i$, where $B_i \neq \mathbb{R}$ for only fin many i
so B contains elts of $\mathbb{R}^\infty$
so $V \cap \mathbb{R}^\infty \neq \varnothing$

[other ways to describe $\mathrm{Cl}_X(A)$?]
[limits and convergence?]

Df a sequence $x_1, x_2, \ldots$ of points in X
converges to x
iff, for all open V containing x
have N s.t. $x_N, x_{N+1}, \ldots$ in V

thus: if some seq in A converges to x in X
then x in $Cl_X(A)$

[Munkres Lem 21.2:
if the topology on X comes from a metric
then the converse holds]

Q can a seq converge to multiple pts?

Ex give X the indiscrete topology:
every seq converges to every pt at once!

Df X is Hausdorff iff, for all $x \neq y$ in X,
there are disjoint open U and V
s.t. x in U and y in V

Thm if X is Hausdorff
then any sequence in X converges to
at most one pt

Pf suppose $(x_n)_n$ converges to x and y

suppose $x \neq y$:
then have disj open U, V s.t. x in U and y in V
if $x_N, x_{N+1}, \ldots$ in U, then notin V
contradiction

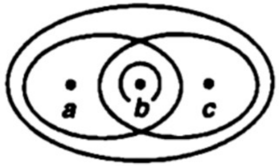
<u>Separation Conditions</u>

T_2 = Hausdorff — for all $x \neq y$, disjoint open U, V
s.t. x in U and y in V

T_1 — for all $x \neq y$, have open U
s.t. x in U and y notin U

T_0 — for all $x \neq y$, have open U
s.t. either x in U and y notin U,
or vice versa

<u>Ex</u> T_0 but not T_1:



{b} is open
a notin {b} and b in {b}
but no open U s.t. a in U and b notin U

(Munkres §22)

subset of X = set A with
injective map A to X

<u>quotient set</u> of X = set B with
surjective map X to B

given a topology on X:

<u>Df</u> the quotient topology on B is
$\{V \text{ sub } B \mid f^{-1}(V) \text{ is open in } X\}$

subspace top on A — coarsest top s.t. A to X is cts

quotient top on B — finest top s.t. X to B is cts

Pf let T be a top on B s.t. X to B is cts

pick V open in T
then f^{-1}(V) open in X
so V also open in the quotient top on B

Ex X = [0, 1] and B = S^1 := {x^2 + y^2 = 1}

define f : [0, 1] to S^1 by f(t) = (cos(2πt), sin(2πt))
then f is surjective

Q give [0, 1] the analytic top, i.e., the subsp. top inherited from R_{an}

what is the quotient top on S^1?