Last time p : E to X is a covering map iff,

for all x in X, have open U ni x s.t.

1) p^{-1}(U) is homeo to a union of disjoint copies of U
2) p restricts to a homeo from each copy onto U

here E is called a covering space or cover of X

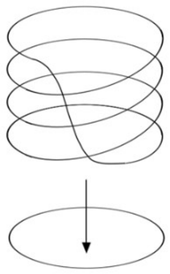
Ex consider $p(x) = (\cos(2\pi x), \sin(2\pi x))$

p : (–1, 1) to S^1 is not a covering map [why?]
but p : R to S^1 is a covering map

Ex identity maps Id : X → X are always covering maps

Ex more generally:

for any n > 0, a covering p_n : S^1 to S^1 s.t. the fiber p_n^{-1}(x) has cardinality n for all x



https://ahilado.wordpress.com/2017/04/14/covering-spaces/

we say that the covering is <u>of degree n</u>, or <u>n-fold</u>

<u>Ex</u> $p_m \times p_n : S^1 \times S^1$ to $S^1 \times S^1$ is a covering:

of what degree? [mn]

more generally:

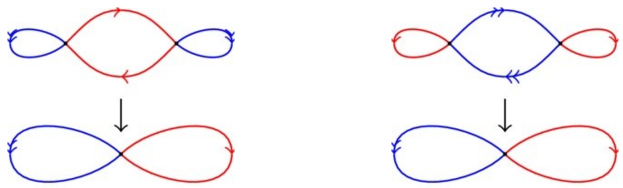
if $p : E$ to X and $p' : E'$ to X' are covering maps, then so is $p \times p'$

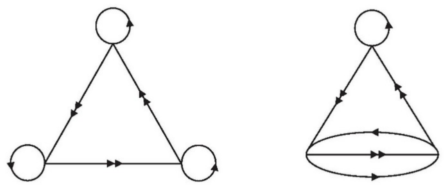
<u>Ex</u> consider the relation ~ on S^2 given by $(x, y, z) \sim (-x, -y, -z)$

$S^2/\sim$ is called the <u>real projective plane</u>, or RP^2

S^2 to RP^2 is a 2-fold covering map

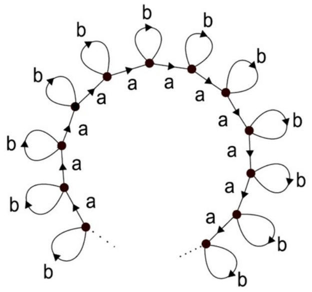
<u>Ex</u> coverings of the figure-eight:



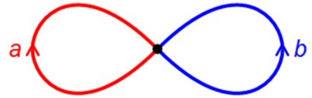


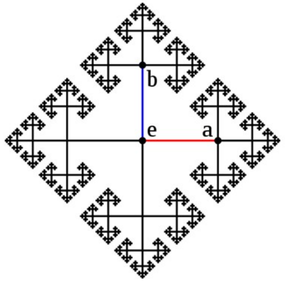
(1) https://www.homepages.ucl.ac.uk/~ucahjde/tg/html/cov-01.html
(2) https://groupoids.org.uk/images/fig10-3.jpg

weirder:



https://www.math.cmu.edu/~nkomarov/NK-NormalSubFreeGrp.pdf





https://math.stackexchange.com/a/3762676