Warmup give R the analytic top,
[0, 1] the subspace top

recall that {(a, b) | a < b} is a basis for R

Q what can [0, 1] cap (a, b) look like?

Thm {[0, 1] cap (a, b) | a < b} is a basis for the subspace top on [0, 1]

Pf subspace top on [0, 1] is:

{[0, 1] cap V | V open in R}

V is a union of sets (a, b)
so [0, 1] cap V is a union of sets [0, 1] cap (a, b)

in general:

Thm if A sub X, and {B_i}_i is a basis for a given topology on X,
then {A cap B_i}_i is a basis for the subspace topology on A

(Munkres §22)

subset of X = set A with injective map A to X

quotient set of X = set Y with surjective map X to Y given a topology on X:

<u>Df</u> the quotient topology on Y is
$\{U \text{ sub } Y \mid f^{-1}(U) \text{ is open in } X\}$

subspace top on A — coarsest top s.t. A to X is cts

i.e.: if T is a top on A s.t. A to X is cts
then T <u>contains</u> the subspace top

[if U in T, then U = A cap V for some V open in X, meaning $U = i^{-1}(V)$ for i the inclusion map]

quotient top on Y — finest top s.t. X to Y is cts

i.e.: if T is a top on Y s.t. X to Y is cts
then T <u>is contained in</u> the quotient top

[if U in T, then $f^{-1}(U)$ is open in X]

<u>Ex</u> $X = [0, 1]$ and $Y = S^1 := \{x^2 + y^2 = 1\}$

$f : [0, 1]$ to S^1 defined by
$f(t) = (\cos(2\pi t), \sin(2\pi t))$

f is continuous and surjective w.r.t.
the subspace top that [0, 1] inherits from R_{an}

<u>Q</u> what is the quotient top on S^1?

[recall example from start:]

[0, 1] cap (a, b) can look like

- $\varnothing$,
- [0, b) with $0 < b$,
- (a, b) with $0 \leq a < b \leq 1$,
- (a, 1] with $a < 1$.

so the quotient top on S^1 is

{U sub S^1 | f^{-1}(U) is a union of some of the sets above}

[draw]

[turns out to match subsp. top on S^1 from R^2]

<u>Ex</u> $X = R \times \{a, b\}$ = union of two copies of R

$Y = X/{\sim}$ where ~ is this equiv. relation:

$(x, a) \sim (y, a)$ iff $x = y$
$(x, b) \sim (y, b)$ iff $x = y$
$(x, a) \sim (y, b)$ iff $x = y \neq 0$

[draw]

Y is not Hausdorff in the quotient top