<u>Thm</u>　　if p : E to X is a covering, then:

I)　for any path γ : [0, 1] to X and e in E s.t.
　　p(e) = γ(0),

　　a unique Γ : [0, 1] to R s.t.
　　Γ(0) = e and γ = p ∘ Γ, called a <u>lift</u> of γ

II)　for any path homotopy h : [0, 1]^2 to X s.t.
　　h(−, 0) = γ, and lift Γ of γ,

　　a <u>unique</u> path-homotopy H : [0, 1]^2 to R s.t.
　　H(−, 0) = Γ and h = p ∘ H, called a <u>lift</u> of h

<u>Pf</u>　　　Lemmas 54.1 and 54.2 in Munkres

recall　　o = (1, 0) in S^1
　　　　ω_n(s) = (cos(2πns), sin(2πns))　(n in Z)

<u>Cor</u>　　Φ : Z to π_1(S^1, o) def by Φ(n) = [ω_n]
　　　is bijective

[previously: Φ is a homomorphism]

<u>Φ surjective</u>　for any loop γ at o
　　　　　　　want n s.t. [γ] = [ω_n]

observe: p^{-1}(o) = Z
so for any a in Z, I) gives Γ s.t.
　　Γ(0) = a and γ = p ∘ Γ
let b = Γ(1)
　　then b in p^{-1}(o) = Z

now, by PS6, #2, have $[\Gamma] = [\dot{\omega}_{a, b}]$
so by PS5, #1, have $[\gamma] = [\omega_{b - a}]$

Φ injective   suppose that $[\omega_m] = [\omega_n]$

observe: $\dot{\omega}_{0, m}$ lifts ω_m
so for any h from ω_m to ω_n, II) gives H s.t.
    $H(-, 0) = \dot{\omega}_{0, m}$ and $h = p \circ H$
let $\Gamma = H(-, 1)$
    then Γ lifts $\omega_n = h(-, 1)$

but by I), there is a unique lift of ω_n starting at 0
so $\Gamma = \dot{\omega}_{0, n}$
so H goes from $\dot{\omega}_{0, m}$ to $\dot{\omega}_{0, n}$
so $m = n$

proof that Φ is injective shows in general:

Cor   if   p : E to X is a covering,
        $\gamma_0 \sim_p \gamma_1$ in X,
        e in E s.t. $p(e) = \gamma_0(0) = \gamma_1(0)$,

then the unique lifts Γ_0, Γ_1 starting at e satisfy
$\Gamma_0 \sim_p \Gamma_1$

Cor   if   p : E to X is a covering,
        $p(e) = x$,

then $p_* : \pi_1(E, e)$ to $\pi_1(X, x)$ is injective

Pf   let Γ_0, Γ_1 be loops in E based at e s.t.
        $p_*([\Gamma_0]) = p_*([\Gamma_1])$

since $p_*(\Gamma_i) = [p \circ \Gamma_i]$ and Γ_i lifts $p \circ \Gamma_i$,
we require $[\Gamma_0] = [\Gamma_1]$

Ex   recall the covering $p_n : S^1$ to S^1

under $\pi_1(S^1) \simeq \mathbb{Z}$, we have $\mathrm{im}((p_n)_*) \simeq n\mathbb{Z}$

(Munkres §79)

Df   a pointed covering of X is a pair (p, e)
          s.t. $p : E$ to X is a path-conn. covering,
                 e in E

a pointed equivalence from (p, e) to (p', e') is
a homeo $f : (E, e)$ to (E', e') s.t. $p = p' \circ f$ [draw]

Thm (Galois Correspondence)

if X is conn. & locally simply-conn., then

$$(p : E \to X, e) \mapsto p_*(\pi_1(E, e))$$

is a bijection

     {pointed coverings of X}/~
     to {subgroups of $\pi_1(X, x)$}

leads to topological analogue of Galois theory

Rem   if $E = E'$ but $e \neq e'$
         then (p, e), (p', e') may not be equivalent