Last time $\quad Cl\_X(A) = X – Int\_X(X – A)$

Q $\quad$ what is the closure of $R^\infty$ in $R^\omega$
in the box top? in the product top?

a useful formula:

$Cl\_X(A)$
$= X – \{x \mid$ have V open in X s.t. x in V sub $X – A\}$
$= \{x \mid$ no V open in X s.t. x in V and V sub $X – A\}$
$= \{x \mid$ if V is open in X and contains x,
then V cap $A \neq \varnothing\}$

Q $\quad$ let $x = (1, 1/2, 1/3, 1/4, \ldots)$
is x in $Cl\_\{R^\omega\}(R^\infty)$…

…wrt the box topology? [draw]

no: x in $(0, 2) \times (0, 1) \times (0, 2/3) \times \ldots$

…wrt the product topology?

yes: suppose V open in $R^\omega$ and x in V

pick basis elt B s.t. x in B sub V
$B = prod\_i\ B\_i$, where $B\_i \neq R$ for only fin many i
so B contains elts of $R^\infty$
so V cap $R^\infty \neq \varnothing$

[other ways to describe Cl_X(A)?]
[limits and convergence?]

Df a sequence $x_1, x_2, \ldots$ of points in X
<u>converges</u> to x
iff, for all open V containing x
have N s.t. $x_N, x_{N+1}, \ldots$ in V

thus: if some seq in A converges to x in X
then x in $Cl_X(A)$

[ Munkres Lem 21.2:
if the topology on X comes from a metric
then the converse holds ]

Q can a seq converge to multiple pts?

Ex give X the indiscrete topology:
every seq converges to every pt at once!

Df X is <u>Hausdorff</u> iff, for all $x \neq y$ in X,
there are disjoint open U and V
s.t. x in U and y in V

Thm if X is Hausdorff
then any sequence in X converges to at most one pt

Pf suppose $(x_n)_n$ converges to x and y

suppose $x \neq y$:
then have disj open U, V s.t. x in U and y in V
if $x_N, x_{N+1}, \ldots$ in U, then notin V
contradiction

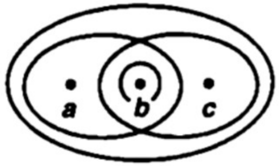
[ Separation Conditions

T_2 = Hausdorff — for all $x \neq y$, disjoint open U, V s.t. x in U and y in V

T_1 — for all $x \neq y$, have open U s.t. x in U and y notin U

T_0 — for all $x \neq y$, have open U s.t. either x in U and y notin U, or vice versa

Ex — T_0 but not T_1:




{b} is open
a notin {b} and b in {b}
but no open U s.t. a in U and b notin U ]