MATH 430
Introduction to Topology

mqtrinh.github.io/math/teaching/yale/math-430/

9 psets	36%
1 midterm Mon 2/24	24%
1 final	40%

Munkres, Topology, 2nd Ed.

[late hw policy]
[schedule]
[intros]

I) algebraic topology:

how to tell shapes apart?

Euler: Königsberg (1736), “V – E + F = 2” (1750)
Poincaré: Analysis situs (1890s–1900s)

II) point-set topology:

what IS shape?

Weyl: rigorous defn of “surface” (1913)
Hausdorff: topological spaces (1914)

[we first do (II), then do (I)]

(Munkres §12) fix a set X

<u>Def</u> a topology on X is
a collection T of subsets of X

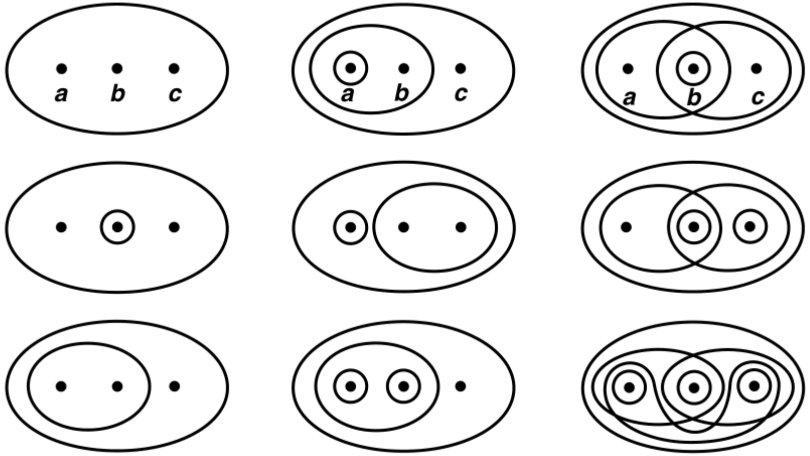
s.t. 1) Ø and X are in T

2) if {U_i}_i is a subcollection of T, then the union of the U_i in X is an element of T

3) if {U_i}_i is a <u>finite</u> subcollection of T, then the intersection of the U_i in X is an element of T

we say that T is a <u>topology</u> on X
the elements of T are its <u>open sets</u>

<u>Ex</u> X = {a, b, c}



[in each case, Ø is not depicted, of course]

[give a collection of subsets that isn’t a topology?]

Ex $X = R^n$ with $|x| = \sqrt{\sum_i x_i^2}$

write $B(x, \delta) = \{y \in R^n \mid |y - x| < \delta\}$

we say $U \subseteq R^n$ is analytically open iff
for all x in U,
there is some $\delta > 0$ s.t. $B(x, \delta) \subseteq U$

Thm {analytically open sets} is a topology on R^n

1) easy
2) suppose U_i anlytc opens, $U = \bigcup_i U_i$
pick x in U
x belongs to x in U_j for some j
pick $\delta > 0$ s.t. $B(x, \delta) \subseteq U_j \subseteq U$

3) suppose finitely many i,
U_i analytic opens, $V = \bigcap_i U_i$
pick x in V
for all i, pick $\delta_i > 0$ s.t. $B(x, \delta_i) \subseteq U_i$

[what next?] let $\delta = \min_i \delta_i$
then $B(x, \delta) \subseteq B(x, \delta_i) \subseteq U_i$ for all i
therefore $B(x, \delta) \subseteq V$

[observe: 3) wouldn't work for infinite $\delta_i \to 0$]

we call this the analytic topology T_{an} on R^n

Rem far from the only topology possible!
always have discrete and indiscrete topologies

Ex X arbitrary

$T_f = \{\varnothing\} \cup \{U \subset X \text{ s.t. } X - U \text{ is finite}\}$

Prop T_f is a topology on X

1) easy
2) easy
3) suppose $X - U_i$ finite for all i,
   $Y = \bigcap_i U_i$
   then $X - Y = \bigcup_i X - U_i$
   unions of finite sets are finite, so $Y \in T_f$

we call this the finite complement topology

Df given topologies T, T′ on the same X

when T is a subcollection of T′, we say that
T is coarser than T′
and T′ is finer than T

[T′ is more refined: it sees more open sets]

Ex [how do the topologies on R^n compare?
analytic, discrete, indiscrete, finite-comp]

$T_{indisc} \subset T_f \subset T_{an} \subset T_{disc}$

Rem topologies can be incomparable:
think about $X = \{a, b, c\}$

Ex $\quad X = Z$, the set of integers

we say U sub Z is evenly spaced iff

U is a union of sets of the form $aZ + b$

with a, b in Z and $a \neq 0$

$T = \{\varnothing\}$ cup {evenly spaced sets}

Prop $\quad$ T is a topology on Z

[assume this for now]

Cor $\quad$ there are infinitely many prime numbers

[deduction observed by Furstenberg in 1955]

Proof $\quad$ assume finitely many primes p

then $Z - \bigcap_p pZ$ is open

because $Z - pZ$ is open for all p

but $Z - \bigcup_p pZ = \{\pm 1\}$

because if $|a| > 1$, then some prime divides a

so $\{\pm 1\}$ is open, but not evenly spaced $\square$