

## MATH 665: TOPICS IN QUANTUM ALGEBRA

FALL 2024 SYLLABUS

We discuss the relationship between representations of linear groups over finite and  $p$ -adic fields, a part of Lie theory, and isotopy invariants of knot and links, a part of geometric topology. The bridge is the theory of Hecke algebras and their cocenters.

INSTRUCTOR Minh-Tâm Trinh ([minh-tam.trinh@yale.edu](mailto:minh-tam.trinh@yale.edu))

TIME TTh 2:30–3:45 PM

PLACE 17 Hillhouse Ave, Room 03 (basement) (**NEW**)

WEBPAGE <https://mqtrinh.github.io/math/teaching/yale/math-665/>

In place of a textbook, I will typeset course notes and post them to the webpage as we go along. See also the bibliography at the end of this syllabus.

### SCHEDULE

8/29	Introduction	
9/3–9/5	1. Finite Reductive Groups	Set 0 ( <i>due</i> 10/10) Set 1 ( <i>due</i> 9/19)
9/10–9/12		
9/17–9/19		
9/24–9/26	2. Hecke Algebras and Link Invariants	Set 2 ( <i>due</i> 10/17)
10/1–10/3		
10/8–10/10		
10/15–10/17	3. Categorification, <i>October Recess</i>	
10/22–10/24		Set 3 ( <i>due</i> 11/14)
10/29–10/31		
11/5–11/7	4. Cocenters and Symmetric Functions	
11/12–11/14		
11/19–11/21		Set 4 ( <i>due</i> 12/5)
11/26–11/28	<i>November Recess</i>	
12/3–12/5	5. Special Topics	

### LOGISTICS

**Emails.** If you need to email me about the course, please put “MATH 665” in the email subject. That helps me keep everything organized. You may address me as “Minh-Tam” or as “Dr. Trinh”.

**Grades.** Problem Set 0 is only assigned to the undergraduates enrolled in the course. If you are taking the course for a grade (in any role), then:

- To pass the course, you must earn points on at least one problem set.
- To get a B-range grade or higher, you must earn more than one-third of the total possible points on all problem sets combined (including Problem Set 0 if applicable).
- To get an A-range grade, you must earn more than half of the total possible points (including Problem Set 0 if applicable).

You should write your homework in complete sentences. The grammar does not have to be perfect. You do not have to rewrite the problem statements.

There is no attendance grade. If you get sick, please don't come to class! Stay at home and take care of yourself.

**Office Hours.** By appointment.

## REFERENCES

- [BBD] A. Beilinson, J. Bernstein, P. Deligne. Faisceaux pervers. *Astérisque*, **100** (1983).
- [B] M. Broué. Reflection Groups, Braid Groups, Hecke Algebras, Finite Reductive Groups. *Current Developments in Mathematics* (2000), 1–107.
- [C1] R. W. Carter. *Finite Groups of Lie Type*. John Wiley & Sons (1993).
- [C2] R. W. Carter. I. On the Representation Theory of the Finite Groups of Lie Type over an Algebraically Closed Field of Characteristic 0. In *Algebra IX*. Ed. A. I. Kostrikin & I. R. Shafarevich. Encyclopaedia of Mathematical Sciences, Vol. 77 (1996), 1–120.
- [EMTW+] B. Elias, S. Makisumi, U. Thiel, G. Williamson *et al.* *Introduction to Soergel Bimodules*. RSME Springer Series, Vol. 5. Springer (2020).
- [G] M. Geck. *An Introduction to Algebraic Geometry and Algebraic Groups*. Oxford Graduate Texts in Mathematics. Oxford University Press (2003).
- [GP] M. Geck & G. Pfeiffer. *Characters of Finite Coxeter Groups and Iwahori–Hecke Algebras*. Clarendon Press, Oxford (2000).
- [J] V. F. R. Jones. Hecke Algebra Representations of Braid Groups and Link Polynomials. *Ann. of Math.* (2), **126**(2) (Sep., 1987), 335–388.
- [Kh] M. Khovanov. Link Homology and Categorification. *Proceedings of the International Congress of Mathematicians, Madrid, Spain, 2006*. European Mathematical Society.
- [KTr] O. Kivinen & M. Q. Trinh. The Hilb-vs-Quot Conjecture. Preprint (2023). [arXiv:2310.19633](https://arxiv.org/abs/2310.19633)
- [KTs] O. Kivinen & C. Tsai. Shalika Germs for Tamely Ramified Elements in  $GL_n$ . Preprint (2023). [arXiv:2209.02509](https://arxiv.org/abs/2209.02509)
- [K] R. E. Kottwitz. Harmonic Analysis on Reductive  $p$ -adic Groups and Lie Algebras. In *Harmonic Analysis, the Trace Formula, and Shimura Varieties*. Ed. J. Arthur, D. Ellwood, R. Kottwitz. Clay Mathematics Proceedings, Vol. 4 (2003), 393–522.
- [L] I. Losev. MATH 7313: Representation Theory. Northeastern University. Unpublished course notes (2015). <https://github.com/mqtrinh/mqtrinh.github.io/tree/main/math/resources/losev>
- [Lu] G. Lusztig. *Characters of Reductive Groups over a Finite Field*. Princeton University Press (1984).
- [M] I. G. Macdonald. *Symmetric Functions and Hall Polynomials*. 2nd Ed. Oxford University Press (1995).
- [Mau] D. Maulik. Stable Pairs and the HOMFLY Polynomial. *Invent. math.*, **204** (2016), 787–831.
- [Mi] J. S. Milne. *Algebraic Groups: The Theory of Group Schemes of Finite Type over a Field*. Cambridge University Press (2017).
- [S] O. Schiffmann. Lectures on Hall Algebras. Preprint (2009). [arXiv:0611617](https://arxiv.org/abs/0611617)

- [T] M. Q. Trinh. From the Hecke Category to the Unipotent Locus. Preprint (2021). [arXiv:2106.07444](#)
- [Y] Z. Yun. Lectures on Springer Theories and Orbital Integrals. *Geometry of Moduli Spaces and Representation Theory*. Ed. R. Bezrukavnikov, A. Braverman, Z. Yun. IAS/Park City Mathematics Series, **24** (2017), 155–215.