MATH 251: TOPOLOGY II

SPRING 2026 PRACTICE PROBLEMS

Contents

NOTE: All citations are to Munkres's textbook, *Topology*, 2nd Edition. When a problem statement has a proof in Munkres, try your best to find your own proof, before comparing with his.

0. Review of Topology I

Problem 1. Show that the following conditions on a topological space X are equivalent:

(1) For any pair of distinct points $x, y \in X$, we can find open $U \subseteq X$ containing x but not y.
(2) For any $x \in X$, the singleton set $\{x\}$ is closed.
(3) All finite subsets of X are closed.

In this situation, we say that X is a *T1 space*.

Problem 2 (Theorem 17.10). Let X be Hausdorff, *a.k.a.* a *T2 space*. Show that if a sequence of points in X converges to points x and y at the same time, then $x = y$.

Problem 3. Let X be the real line $\mathbf{R}$ in its finite complement, or *cofinite*, topology. Show that every sequence of points in X converges to every point of X simultaneously.

Problem 4. Show that:

(1) Subspaces and products of T1 spaces are T1.
(2) (**§17, #11–12**) Subspaces and products of T2 spaces are T2.

Problem 5 (Lemma 23.2). Suppose that $U, V \subseteq X$ form a *separation* of X, meaning U and V are disjoint nonempty open sets whose union is X.

Let $A \subseteq X$ be a subspace. Show that if A is *connected*, meaning there is no separation of A, then either $A \subseteq U$ or $A \subseteq V$.

1. Locally Compact Hausdorff Spaces