Last time a path in X from x to y is a cts map

$\gamma : [0, 1]$ to X s.t. $\gamma(0) = x$,
$\gamma(1) = y$,

where [0, 1] has the analytic top

[sometimes replace [0, 1] with [a, b], where a < b]

Df X is path-connected iff, for all x, y in X, there is a path from x to y

[stronger or weaker than "connected"?]

Thm path-connected implies connected

will need these facts:

1) [0, 1] is connected
2) images of connected spaces under cts maps are connected [as a subspace of the range]
3) if U, V form a separation of X, and A sub X is a connected subspace, then A sub U or A sub V

Pf of Thm suppose X is not connected
pick a separation U, V
pick x in U and y in V
pick a path γ from x to y

[0, 1] is connected
so $\gamma([0, 1])$ is a connected subspace of X
so either $\gamma([0, 1])$ sub U or $\gamma([0, 1])$ sub V: uh-oh

Rem a connected space need not be path-connected!

in analytic R^2, consider the subspaces:

$A = \{(0, y) \mid -1 \le y \le 1\}$

$S = \{(x, \sin(1/x)) \mid 0 \le x \le 1\}$

[draw]

A cup S is called the topologist's sine curve

there is no path between any point of A and any point of S [hard, but intuitive], but:

Fact A cup S is a connected subspace of R^2

Pf sketch closures of connected subspaces remain connected [as subspaces]

show: S is connected and A cup S = $Cl_{R^2}(S)$

(Munkres §25)

Df X is locally connected, resp. locally path-connected,

iff, for all x in X and open U containing x, there is some connected, resp. path-connected V s.t. x in V sub U

Thm locally path-connected implies locally connected

Ex	loc. path.-conn.?	loc. conn.?
[0, 1) cup (1, 2]	yes	yes
{0} cup {1/n}_n	no	no
Q	no	no
top. sine curve	no	yes [hard]

Df the connected components, resp. path components, of X are the maximal connected, resp. path-connected, subspaces

Ex	path comp.?	conn. comp.?
[0, 1) cup (1, 2]	[0, 1), (1, 2]	same
{0} cup {1/n}_n	singletons	same
Q	singletons	same
top. sine curve	A, S	whole space

Thm [Munkres Thm 25.5]

each path component of X is contained in some connected component of X

if X is locally path-connected, then path components = connected components

(Munkres §26)

Df an open cover of X is a collection of open sets of X whose union is X

a subcover is a subcollection that remains a cover

<u>Df</u> X is <u>compact</u> iff every open cover of X contains a finite subcover

<u>Facts</u>

1) (Heine–Borel) $[0, 1]$ is compact
2) images of a compact space under cts maps are compact
3) if X, Y are compact, then $X \times Y$ is compact

[compare to corresponding statements about connectedness]

next time: compactness of subspaces