MATH 430
Introduction to Topology

mqtrinh.github.io/math/teaching/yale/math-430/

9 psets	36%
1 midterm Mon 2/24	24%
1 final	40%

Munkres, Topology, 2nd Ed.

[late hw policy]
[schedule]
[intros]

I) algebraic topology:

how to tell shapes apart?

Euler: Königsberg (1736), “V – E + F = 2” (1750)
Poincaré: Analysis situs (1890s–1900s)

II) point-set topology:

what IS shape?

Weyl: rigorous defn of “surface” (1913)
Hausdorff: topological spaces (1914)

[we first do (II), then do (I)]

(Munkres §12) fix a set X

Def a topology on X is
a collection T of subsets of X

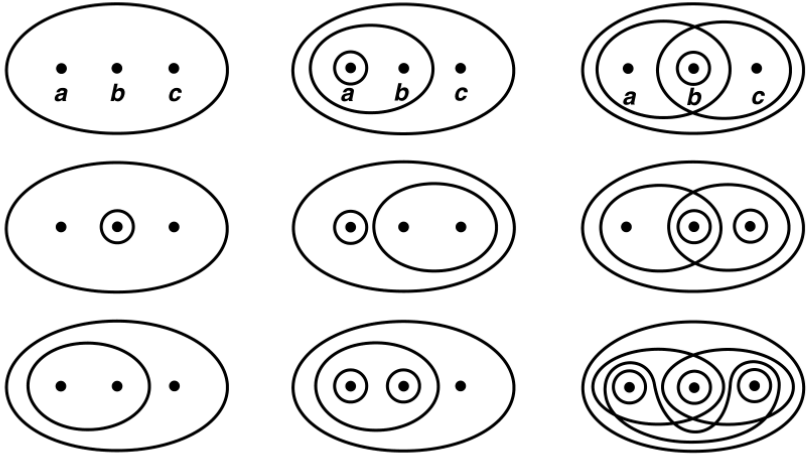
s.t. 1) Ø and X are in T

2) if {U_i}_i is a subcollection of T, then the union of the U_i in X is an element of T

3) if {U_i}_i is a finite subcollection of T, then the intersection of the U_i in X is an element of T

we say that (X, T) is a topological space
the elements of T are its open sets

Ex X = {a, b, c}



[in each case, Ø is not depicted, of course]

[give a collection of subsets that isn't a topology?]

Ex $X = R^n$ with $|x| = \text{sqrt}(\text{sum}_i x_i^2)$

write $B(x, \delta) = \{y \text{ in } R^n \mid |y - x| < \delta\}$

we say U sub R^n is analytically open iff
for all x in U,
there is some $\delta > 0$ s.t. $B(x, \delta)$ sub U

Thm {analytically open sets} is a topology on R^n

1) easy
2) suppose U_i anlytc opens, $U = \text{bigcup}_i U_i$
pick x in U
x belongs to x in U_j for some j
pick $\delta > 0$ s.t. $B(x, \delta)$ sub U_j sub U

3) suppose finitely many i,
U_i analytic opens, $V = \text{bigcap}_i U_i$
pick x in V
for all i, pick $\delta_i > 0$ s.t. $B(x, \delta_i)$ sub U_i

[what next?] let $\delta = \min_i \delta_i$
then $B(x, \delta)$ sub $B(x, \delta_i)$ sub U_i for all i
therefore $B(x, \delta)$ sub V

[observe: 3) wouldn't work for infinite $\delta_i \to 0$]

we call this the analytic topology T_{an} on R^n

Rem far from the only topology possible!
always have discrete and indiscrete topologies

Ex X arbitrary

T_f = {Ø} cup {U sub X s.t. X – U is finite}

Prop T_f is a topology on X

1) easy
2) suppose X – U_i finite for all i

   X – bigcup_i U_i = bigcap_i U_i

   [what if instead U_i = Ø for some i?]
3) suppose X – U_i finite for all i

   X – bigcap_i U_i = bigcup_i X – U_i

   unions of finite sets are finite

   [what if instead U_i = Ø for some i?]

we call this the *finite complement* topology

Df given topologies T, T′ on the same X

when T is a subcollection of T′, we say that

T is *coarser* than T′

and T′ is *finer* than T

[T′ is more refined: it sees more open sets]

Ex [how do the topologies on R^n compare?
analytic, discrete, indiscrete, finite-comp]

T_{indisc} sub T_f sub T_{an} sub T_{disc}

Rem topologies can be incomparable:
think about X = {a, b, c}

<u>Ex</u> X = Z, the set of integers

we say U sub Z is <u>evenly spaced</u> iff
U is a union of sets of the form aZ + b
with a, b in Z and a ≠ 0

<u>Prop</u> {evenly spaced sets} is a topology on Z

[assume this for now]

<u>Cor</u> there are infinitely many prime numbers

[deduction observed by Furstenberg in 1955]

<u>Proof</u> assume finitely many primes p

then Z – bigcup_p pZ = bigcap_p (Z – pZ) is open
because Z – pZ is open for all p

but Z – bigcup_p pZ = {±1}
because if |a| > 1, then some prime divides a

so {±1} is open, but not evenly spaced □