Last time if X, Q are sets,
p : X to Q is a surjective map,
X has a topology,

then the resulting quotient topology on Q is
{U sub Q | p^{-1}(U) is open in X}

a quotient space of X is a topological space formed this way [how do they appear in practice?]

Df an equivalence relation on a set X is a rule ~ on elts of X^2 s.t., for all x, y, z,

I) x ~ x, for all x in X
II) if x ~ y, then y ~ x
III) if x ~ y and y ~ z, then x ~ z

Fact if p : X to Q is a surjective map, then the rule ~ on X^2 def by

x ~ y iff p(x) = p(y)

is an equivalence relation

conversely, every equiv relation arises from a surjective map this way

Ex X = R × {a, b} = union of two copies of R
Y = X/~ where ~ is this equiv. relation:

(x, a) ~ (y, a) iff x = y
(x, b) ~ (y, b) iff x = y
(x, a) ~ (y, b) iff x = y ≠ 0

[draw]

Y is not Hausdorff in the quotient top

[draw]

(Munkres §23)

<u>Df</u> a separation of X is
a pair of disjoint <u>nonempty</u> opens U, V
s.t. X = U cup V

we say that X is <u>connected</u> iff it has no separation
i.e.
whenever X = U cup V for disjoint open U, V,
we require either U = Ø or V = Ø

<u>Rem</u> if U, V form a separation
then U, V are not just open, but closed

"clopen" = "both open and closed"

<u>Ex</u> X = [0, 1/2) cup (1/2, 1] in the subsp top
inherited from analytic R

U = [0, 1/2) and V = (1/2, 1] is a separation

<u>Ex</u> Q in the subsp top inherited from R

for α irrational, U = {x in Q | x < α},
V = {x in Q | x > α}
is a separation

__Thm__ R in the analytic topology is connected

[but the proof is hard]

__Moral__ subspaces of connected spaces
need not be connected

[but upshot:]

__Thm__ if A is a connected subspace of X
and U, V form a separation of X
then either A sub U or A sub V

__Pf__ if A cap U and A cap V both nonempty,
then they form a separation of A
so either A cap U = ∅ or A cap V = ∅

__Df__ the maximal connected subspaces of X
are called its __connected components__

__Thm__

1) if X, X′ are each connected, then so is X × X′
2) if f : X to Y is cts and X is connected,
then f(X) is connected as a subspace of Y
3) if A, A′ sub X are connected
and A cap A′ ≠ ∅,
then A cup A′ is connected