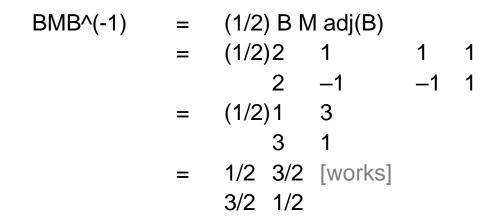
Warmup suppose T: F^2 to F^2 sends
$$v1 = 1 \quad to \quad 2 \qquad v2 = -1 \quad to \quad 1$$

$$1 \quad 2 \quad 1 \quad -1$$
matrix of T wrt std basis (e1, e2) of F^2?

1) matrix wrt (v1, v2): 
$$M = 2$$
 0

2) 
$$v1 = e1 + e2$$
  $B = 1$   $-1$   $v2 = -e1 + e2$  1 1

3) 
$$det(B) = 2$$
  $[B^{(-1)} = 1/2 \ 1/2]$   $adj(B) = 1 \ 1 \ -1/2 \ 1/2]$ 



<u>Df</u> a subspace W sub V is T-stable iff

(Axler §3A) given a linear op T : V to V:

w in W implies Tw in W

for any linear op T : V to V,
{0} and V are are [trivially] T-stable

when V = sum\_i W\_i for T-stable W\_i, studying T reduces to studying T|\_{W\_i} for all i

Ex suppose that wrt some basis, T: F^3 to F^3 has the matrix

best situation: each W\_i is a line

[what means?]

\* \* 0

in this case: there is a basis (w\_i)\_i s.t.

\* \* 0

 $W_i = Fw_i$ 

0 0 \*

Tw\_i = a\_iw\_i for some i matrix of T wrt w\_i is diagonal

nontrivial T-stable subspaces?  $\{(x, y, 0) \mid x, y\}$  $\{(0, 0, z) \mid z\}$  <u>Df</u> for any linear op T : V to V

when a matrix for T has nontrivial <u>diagonal blocks</u>, the blocks indicate nontrivial T-stable subspaces [why care?] an <u>eigenline</u> of T is a T-stable line [dim-1 sub.]
an <u>eigenvector</u> of T is v in V s.t. Fv is an eigenline
(which forces v ≠ **0**)
if Tv = λv, then we call λ the <u>eigenvalue</u> of the line

<u>Ex</u>	T : F^2 to F^2 1/2 3/2 3/2 1/2	given by the matrix wrt the std basis	λx λy	=	0	0	х у	=	0 y
{(x is an eigenline with eigenvalue 2 x)}			_	[take $\lambda = 0$ :] Fe1 is an eigenline with eigenvalue 0 [take $\lambda \neq 0$ :] Fe2 is an eigenline with eigenvalue 1					
{(x is a -x)}	ın eigenline with eig	genvalue –1	<u>Ex</u>		N: 0 1	F^2 to F^ 0 0	2	•	en by the matrix the std basis
Ex	P : F^2 to F^2	given by the matrix	λx	=	0	0	X	=	0
	0 0 0 1	wrt the std basis	λy		1	0	У		X
[does P have any eigenvectors?]			[regardless of λ:] x = 0 Fe2 is the only eigenline with eigenvalue 0 [same regardless of whether F = R or F = C]						

<u>Ex</u>	H: R^2 to R^2 1 –1 1 1	given by the matrix wrt the std basis	compare to H' : C^2 to C^2 given by 1 -1 1 1
λx =	1 –1 x	= x - y	claim: 1 + i and 1 – i are eigenvalues
λy	1 1 y	x + y	
messy to	o solve		(1 + i)x = x - y imply $ix = -y$ and $iy = x$
_			(1+i)y = x + y
notice:	$(1/\sqrt{2}) H = 1/\sqrt{2}$	$\sqrt{2}$ $-1/\sqrt{2}$	
	1/	√2 1/√2	so {(x is an eigenline with eigenvalue 1 + i
	= cc	$s(\pi/4)$ –sin( $\pi/4$ )	ix)}
		$n(\pi/4)$ cos $(\pi/4)$	<b>,,</b>
			similarly
so H is the composition of: rotate by $\pi/4$			$\{(x) \text{ is an eigenline with eigenvalue } 1 - i\}$
	•	scale by √2	-ix)}
no H-sta	able lines through	•	/J

Rem	C^2 is iso to	the comp	lex'n of R^2	Summary				
	H_C (R^2)_C to (R^2)_C =			if W sub V is stable under T : V to V then we can study T in terms of smaller op's T _W				
	C^2	to H'	C^2	nicer when nicest when	W is a line V is a sum of eigenlines			
<u>Moral</u>	choice of R	vs C affec	ts eigenstuff	over R over C	T may have no eigenlines T will have some eigenline [later],			
			has no eigenvals, have eigenvals		but V need not be sum(eigenlines)			
	[later: we wi	ll show it h	nas at least onel	Q for next time				
	[later: we will show it has at least one]			let D : F[x] to F[x] be D(p) = dp/dx what are the D-stable linear subspaces?				