[a long digression on inverse limits and the p-adic numbers]

<u>Last time</u> (X, x) gives $\pi_1(X, x)$
f gives f_*

if f is a homeo, then f_* is an iso

Ex if $f = id_X$, then $f_* = id_{\pi_1(X, x)}$

<u>Q</u> is the converse true?

<u>Ex</u> R and $\{0\}$ are not homeomorphic [why?]
but $\pi_1(R, 0) = \pi_1(\{0\}, 0)$

<u>Thm</u> if X is a convex subspace of R^n
then for any x and loop γ based at x,
have $\gamma \sim_p e_x$
thus $\pi_1(X, x) = \{[e_x]\}$

<u>Pf</u> recall the homotopy
$h(s, t) = (1 - t)*x + t*\gamma(s)$

is it a path homotopy?

$h(0, t) = (1 - t)*x + t*\gamma(0) = (1 - t)*x + t*x = x$
$h(1, t) = (1 - t)*x + t*\gamma(1) = (1 - t)*x + t*x = x$

[further discussion of star convexity]

Moral convex subspaces of R^n need not be homeomorphic
but they are all simply connected:
their π_1's are all trivial

but recall:
any nonempty convex X is contractible:
there is x_0 in X s.t. $id_X \sim$ (constant map at x_0)

(Munkres §58)

Df a homotopy equivalence btw X and Y is a pair of cts $f : X$ to Y and $g : Y$ to X

s.t. $g \circ f \sim id_X$ and $f \circ g \sim id_Y$

here we say X and Y are homotopy equivalent

Ex $X = R^2 - \{(0, 0)\}$ and $Y = S^1$

$r : X$ to Y $r(x, y) = (x, y)/|(x, y)|$
$i : Y$ to X $i(x, y) = (x, y)$

then $(i \circ r)(x, y) = (x, y)/|(x, y)|$, so $i \circ r \sim id_X$
via $h((x, y), t) = ((1 - t)/|(x, y)| + t)*(x, y)$
while $(r \circ i) = id_Y$

so $R^2 - \{(0, 0)\}$ is homotopy equivalent to S^1