Last time    A sub X

the subspace topology on A induced by X is
{A cap V | V is an open set in X}

if U sub A sub X
then (U open in X) implies (U open in A) [why?]
but converse can fail

Ex    give R the analytic topology

| | {0} open | {1} open |
|---|---|---|
| A = {0, 1} | y | y |
| A = Q | n | n |
| A = {0} cup {1/n \| n in Z_+} | n | y |

Rem    [how do subsp's interact with cts maps?]

1. if A sub X, then the inclusion map i : A to X def by i(a) = a is cts
2. compositions of cts maps are cts
3. so if f : X to Y is cts, then f|_A : A to Y is cts (compare to PS2, #6)

Rem    if A is open in X itself
then (U open in A) iff (U open in X)
(see PS2, #3)

[ Thm    if {B_i}_i is a basis for the top on X,
then {A cap B_i}_i is a basis for
the subspace top on A

Pf by earlier criterion, just need to show:
for all U open in A and x in U,
have i s.t. x in A cap B_i sub U

indeed, U = A cap V for some V open in X
then x in V, so have i s.t. x in B_i sub V
so x in A cap B_i, and also, A cap B_i sub U ]

(Munkres §20) recall from real analysis:

Df a metric on X is a fn d : X × X to [0, ∞)

s.t., for all x, y, z in X,

1) d(x, y) = 0 if and only if x = y
2) d(x, y) = d(y, x)
3) d(x, y) + d(y, z) ≥ d(x, z)

[why is 3) called the “triangle inequality”?]

Ex Euclidean metric e on R^n:

e(x, y) = sqrt((x_1 – y_1)^2 + … + (x_n – y_n)^2)
for all x = (x_1, …, x_n) and y = (y_1, …, y_n)

Df any metric d on X defines the balls

B_d(x, δ) = {y in X | d(x, y) < δ}
for all x in X and δ > 0

Thm the collection {B_d(x, δ) | x in X, δ > 0}
forms a basis on X

Df the metric topology on $X$ generated by $d$ is the one generated by the basis

metric ↦ basis of balls ↦ metric topology

just as different bases can give the same topology so different metrics can give the same topology

Pf of Thm must check axioms for basis

easy to see that the balls cover $X$

enough to show that for all $x, x'$ in $X$ and $\varepsilon, \varepsilon' > 0$
and $z$ in $B_d(x, \varepsilon)$ cap $B_d(x', \varepsilon')$,
have $\delta > 0$ s.t.
$B_d(z, \delta)$ sub $B_d(x, \varepsilon)$ cap $B_d(x', \varepsilon')$

[draw picture]

rephrased:
given that $d(x, z) < \varepsilon$ and $d(x', z) < \varepsilon'$
want $\delta > 0$ s.t.
$d(z, y) < \delta$ implies $d(x, y) < \varepsilon$ and $d(x', y) < \varepsilon'$

notice $0 < \varepsilon - d(x, z)$
so we can find $\alpha$ s.t. $0 < \alpha < \varepsilon - d(x, z)$
then $d(x, z) + \alpha < \varepsilon$
then for all $y$ s.t. $d(z, y) < \alpha$,
get $d(x, y) \leq d(x, z) + d(z, y) < d(x, z) + \alpha < \varepsilon$

similarly, can find $\alpha'$ s.t. $0 < \alpha' < \varepsilon' - d(x', z)$
then for all $y$ s.t. $d(z, y) < \alpha'$
get $d(x', y) \leq d(x', z) + \alpha < \varepsilon'$

now take $\delta = \min(\alpha, \alpha')$
then for all $y$ s.t. $d(z, y) < \delta$,
    get $d(x, y) < \varepsilon$ and $d(x', y) < \varepsilon'$

<u>Ex</u>    square metric on R^n:

$\rho(x, y) = \max(|x_1 - y_1|, \ldots, |x_n - y_n|)$
for all x and y

<u>Q</u>    what are the balls in this metric?

<u>Q</u>    what topology does $\rho$ generate on R^n?