MATH 251: Topology II

[website]

[syllabus]

- [exam dates]
- [participation]
- [plagiarism]
- [Jan 30 add/drop deadline]
- [course objectives]

Exam 1: CLOSED BOOK, in-class, Mon 1/26

know BY HEART definitions and examples of:

topologies, bases, metrics
subspaces, quotient spaces, product spaces
interior, closure, convergence
Hausdorff spaces, non-Hausdorff spaces

connectedness
path-connectedness
compactness

homotopies, homotopy equivalences
path homotopies
fundamental groups

Euclidean metric on R^n [?]

$d(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$

Euclidean balls in R^n [?]

$B_d(x, \delta) = \{y \in R^n \mid d(x, y) < \delta\}$

Euclidean / analytic topology on R^n [?]

topology gen by d
= topology gen by basis $\{B_d(x, \delta)\}$
= $\{U \subseteq R^n \mid U$ is a union of balls $B_d(x, \delta)\}$
= $\{U \subseteq R^n \mid$ for all x in U, have $\delta > 0$
s.t. $B_d(x, \delta) \subseteq U\}$

<u>Ex</u> if $n = 1$, then $B_d(x, \delta) = (x - \delta, x + \delta)$
so open balls in R are open intervals

<u>XC</u> metric topologies are Hausdorff

[note:] $R^n = R \times R \times \ldots \times R$, with n copies

product topology on $R \times R \times \ldots \times R$ [?]

= box topology on $R \times R \times \ldots \times R$ [for n finite]
= topology gen by $U_1 \times \ldots \times U_n$
where $U_i \subseteq R$ is open for each i
= $\{U \mid U$ is a union of sets of the form
$U_1 \times \ldots \times U_n$ with U_i's open$\}$
= $\{U \mid U$ is a union of sets of the form
$(a_1, b_1) \times \ldots \times (a_n, b_n)\}$

<u>Ex</u> $(-1, 0) \times (-1, 0)$ cup $(0, 1) \times (0, 1)$
is open in the product top on $R \times R$
but not of the form $U_1 \times U_2$

<u>XC</u> via $R^n = R \times R \times \ldots \times R$,

analytic top on R^n
=
product top on $R \times R \times \ldots \times R$

[next:] circle $S^1 = \{(x, y) \text{ in } R^2 \mid x^2 + y^2 = 1\}$

subspace topology on S^1 [?]

$\{V \cap S^1 \mid V \text{ is open in } R^2\}$

[note:] surjective map $p : R \to S^1$ given by

$$p(t) = (\cos 2\pi t, \sin 2\pi t)$$

quotient topology on S^1 [?]

$\{U \subseteq S^1 \mid p^{-1}(U) \text{ is open in } R\}$

<u>XC</u> the collection of <u>open arcs</u>
$\{p(t) \mid a < t < b\}$, for a, b in R,
satisfies the definition of a basis

<u>XC</u> the following all match:

1) the topology generated by open arcs
2) the subspace top on S^1 from R^2
3) the quotient top on S^1 from R

[recall:] a homeomorphism from X onto Y is a bijection $f : X \to Y$ s.t. f and f^{-1} are both cts

in this case X and Y are called homeomorphic

[recall:] $\mathbb{R}$ and S^1 are not homeomorphic

Q how many ways to show that?