# MATH 665: TOPICS IN QUANTUM ALGEBRA

FALL 2024 SYLLABUS

We discuss the relationship between representations of linear groups over finite and $p$-adic fields, a part of Lie theory, and isotopy invariants of knot and links, a part of geometric topology. The bridge is the theory of Hecke algebras and their cocenters.

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| Instructor | Minh-Tâm Trinh (minh-tam.trinh@yale.edu) |
| Time | TTh 2:30–3:45 PM |
| Place | 17 Hillhouse Ave, Room 03 (basement) (**NEW**) |
| Webpage | https://mqtrinh.github.io/math/teaching/yale/math-665/ |

In place of a textbook, I will typeset course notes and post them to the webpage as we go along. See also the bibliography at the end of this syllabus.

## Schedule

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| 8/29 | Introduction | |
| 9/3–9/5 | 1. Finite Reductive Groups | Set 0 (*due* 10/10)<br>Set 1 (*due* 9/19) |
| 9/10–9/12 | | |
| 9/17–9/19 | | |
| 9/24–9/26 | 2. Hecke Algebras and Link Invariants | Set 2 (*due* 10/17) |
| 10/1–10/3 | | |
| 10/8–10/10 | | |
| 10/15–10/17 | 3. Categorification, *October Recess* | |
| 10/22–10/24 | | Set 3 (*due* 11/14) |
| 10/29–10/31 | | |
| 11/5–11/7 | 4. Cocenters and Symmetric Functions | |
| 11/12–11/14 | | |
| 11/19–11/21 | | Set 4 (*due* 12/5) |
| 11/26–11/28 | *November Recess* | |
| 12/3–12/5 | 5. Special Topics | |

## Logistics

**Emails.** If you need to email me about the course, please put "MATH 665" in the email subject. That helps me keep everything organized. You may address me as "Minh-Tam" or as "Dr. Trinh".

**Grades.** Problem Set 0 is only assigned to the undergraduates enrolled in the course. If you are taking the course for a grade (in any role), then:

- To pass the course, you must earn points on at least one problem set.
- To get a B-range grade or higher, you must earn more than one-third of the total possible points on all problem sets combined (including Problem Set 0 if applicable).
- To get an A-range grade, you must earn more than half of the total possible points (including Problem Set 0 if applicable).

You should write your homework in complete sentences. The grammar does not have to be perfect. You do not have to rewrite the problem statements.

There is no attendance grade. If you get sick, please don't come to class! Stay at home and take care of yourself.

**Office Hours.** By appointment.

## References


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[C2] R. W. Carter. I. On the Representation Theory of the Finite Groups of Lie Type over an Algebraically Closed Field of Characteristic 0. In *Algebra IX*. Ed. A. I. Kostrikin & I. R. Shafarevich. Encyclopaedia of Mathematical Sciences, Vol. 77 (1996), 1–120.

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[KTr] O. Kivinen & M. Q. Trinh. The Hilb-vs-Quot Conjecture. Preprint (2023). arXiv:2310.19633

[KTs] O. Kivinen & C. Tsai. Shalika Germs for Tamely Ramified Elements in $\mathrm{GL}_n$. Preprint (2023). arXiv:2209.02509

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[T] M. Q. Trinh. From the Hecke Category to the Unipotent Locus. Preprint (2021). arXiv:2106.07444

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