Warmup give R the analytic top,
[0, 1] the subspace top

recall that {(a, b) | a < b} is a basis for R

Q what can [0, 1] cap (a, b) look like?

Thm {[0, 1] cap (a, b) | a < b} is a basis for the subspace top on [0, 1]

Pf subspace top on [0, 1] is:

{[0, 1] cap V | V open in R}

V is a union of sets (a, b)
so [0, 1] cap V is a union of sets [0, 1] cap (a, b)

in general:

Thm if A sub X, and {B_i}_i is a basis for a given topology on X,
then {A cap B_i}_i is a basis for the subspace topology on A

(Munkres §22)

subset of X = set A with injective map A to X

quotient set of X = set Y with surjective map X to Y given a topology on X:

<u>Df</u> the quotient topology on Y is
{U sub Y | f^{-1}(U) is open in X}

subspace top on A — coarsest top s.t. A to X is cts

i.e.: if T is a top on A s.t. A to X is cts
then T <u>contains</u> the subspace top

[if U in T, then U = A cap V for some V open in X,
meaning U = i^{-1}(V) for i the inclusion map]

quotient top on Y — finest top s.t. X to Y is cts

i.e.: if T is a top on Y s.t. X to Y is cts
then T <u>is contained in</u> the quotient top

[if U in T, then f^{-1}(U) is open in X]

<u>Ex</u> X = [0, 1] and Y = S^1 := {x^2 + y^2 = 1}

f : [0, 1] to S^1 defined by
f(t) = (cos(2πt), sin(2πt))

f is continuous and surjective w.r.t.
the subspace top that [0, 1] inherits from R_{an}

<u>Q</u> what is the quotient top on S^1?

[recall example from start:]

[0, 1] cap (a, b) can look like

∅,

[0, b) with $0 < b$,

(a, b) with $0 \le a < b \le 1$,

(a, 1] with $a < 1$.

so the quotient top on S^1 is

{U sub S^1 | f^{-1}(U) is a union of some
of the sets above}

[draw]

[turns out to match subsp. top on S^1 from R^2]