MATH 251: Topology II

[website]

[syllabus]

[exam dates]

[participation]

[plagiarism]

[Jan 30 add/drop deadline]

[course objectives]

Exam 1: CLOSED BOOK, in-class, Mon 1/26

know BY HEART defs, examples, non-examples:

bases (§13), continuous maps (18)
$X \times Y$ (15), subspaces (16), quotients (22)
interiors & closures (17)
Hausdorff spaces (17), convergence (17)

separations (23–24), connectedness (25)
paths (23), path-connectedness (25)
compact spaces (26–27)

homotopies (51), path homotopies (51)
homotopy equivalences (58)

Euclidean metric on $\mathbb{R}^n$ [?]

$$d(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$$

Euclidean balls in $\mathbb{R}^n$ [?]

$$B_d(x, \delta) = \{y \in \mathbb{R}^n \mid d(x, y) < \delta\}$$

analytic topology on $\mathbb{R}^n$ [?]

topology gen by d
= topology gen by basis $\{B_d(x, \delta)\}$
= $\{U \subset \mathbb{R}^n \mid U$ is a union of balls $B_d(x, \delta)\}$
= $\{U \subset \mathbb{R}^n \mid$ for all x in U, have $\delta > 0$ s.t. $B_d(x, \delta) \subset U\}$

Ex — if $n = 1$, then $B_d(x, \delta) = (x - \delta, x + \delta)$

Prob — show that via the identity

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R},$$

the analytic and product topologies match

[next:] the circle $S^1 \subset \mathbb{R}^2$

$$S^1 = \{(x, y) \mid x^2 + y^2 = 1\}$$

subspace topology on S^1 [?]

$$\{V \cap S^1 \mid V \text{ is open in } \mathbb{R}^2\}$$

[note:] surjective map p : R to S^1 given by

p(t) = (cos 2πt, sin 2πt)

quotient topology on S^1 [?]

{U sub S^1 | p^{-1}(U) is open in R}

Prob the collection of open arcs {p(t) | a < t < b}, for a, b in R, satisfies the definition of a basis

Prob the following all match:

1) the topology generated by open arcs
2) the subspace top on S^1 from R^2
3) the quotient top on S^1 from R

[recall:] a homeomorphism from X onto Y is a bijection f : X to Y s.t. f and f^{-1} are both cts

in this case X and Y are called homeomorphic

Later no two of R, R^2, S^1 are homeomorphic

but R and R^2 are homotopy equivalent