MATH 250: Topology I

[intros]

[recap: website
LaTeX
textbook
late hw policy
exam dates]

[initial reading]

I) algebraic topology:

how to tell shapes apart?

Euler: Königsberg (1736), “V – E + F = 2” (1750)
Poincaré: Analysis situs (1890s–1900s)

II) point-set topology:

what IS shape?

Weyl: rigorous defn of “surface” (1913)
Hausdorff: topological spaces (1914)

[we first do (II), then do (I)]

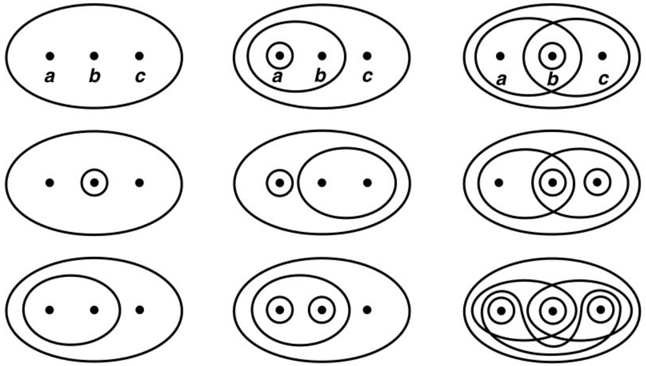
(Munkres §12) fix a set X

Def a topology on X is
a collection T of subsets of X s.t.

1) Ø, X in T
2) if {U_i}_i is a subcollection of T, then the union of the U_i in X is in T
3) if {U_i}_i is a finite subcollection of T, then the intersection of the U_i in X is in T

we say that
(X, T) is a topological space
the elements of T are its open sets

Ex X = {a, b, c}




[in each case, Ø is not depicted]

[example collection of subsets that isn't a topology?]

Ex X = R^n with |x| = sqrt(sum_i x_i^2)

write B(x, δ) = {y in R^n | |y – x| < δ}

we say U sub R^n is analytically open iff
 for all x in U,
 there is some δ > 0 s.t. B(x, δ) sub U

Thm {analytically open sets} is a topology on R^n

1) easy
2) suppose U_i anlytc opens, U = bigcup_i U_i
 pick x in U
 x belongs to x in U_j for some j
 have δ > 0 s.t. B(x, δ) sub U_j sub U
3) suppose finitely many i,
 U_i analytic opens, V = bigcap_i U_i
 pick x in V
 for all i, pick δ_i > 0 s.t. B(x, δ_i) sub U_i

 [what next?] let δ = min_i δ_i
 then B(x, δ) sub B(x, δ_i) sub U_i for all i
 therefore B(x, δ) sub V

[observe: 3) wouldn't work for infinite δ_i → 0]

we call this the analytic topology T_{an} on R^n

Rem not the only topology possible:
 also discrete and indiscrete topologies

Ex X arbitrary we call this the finite complement topology

T_f = {Ø} cup {U sub X s.t. X – U is finite}

Thm T_f is a topology on X

1) easy
2) suppose {U_i}_i is a subcollection of T_f,
   U = bigcup_i U_i
   [what happens if U_i = Ø for all i?]
   [what if U_i ≠ Ø for some i?]
3) suppose finitely many i,
   U_i subcollection of T_f
   V = bigcap_i U_i
   [what happens if U_i = Ø for some i?]
   [what if U_i ≠ Ø for all i?]