PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

16. Symplectic resolutions and their deformations

Exercise 16.1. *Consider the G-action on $X \times \mathbb{C}$ given by $g(x,z) = (gx, \theta(g)z)$. Show that $x \in X^{\theta-ss}$ iff $\overline{G(x,1)}$ doesn't intersect $X \times \{0\}$.*

Exercise 16.2. *Prove that for $\theta = (-1)_{i \in Q_0}$, the subset R^{ss} consists of all elements*

$$(x_a, x_{a^*}, y_i, z_i)_{a \in \underline{Q}_1, i \in \underline{Q}_0}$$

(here $x_a \in \operatorname{Hom}(V_{t(a)}, V_{h(a)}), x_{a^*} \in \operatorname{Hom}(V_{h(a)}, V_{t(a)}), y_i \in \operatorname{Hom}(W_i, V_i), z_i \in \operatorname{Hom}(V_i, W_i)$) such that there are no proper subspaces $V_i' \subset V_i$ that are stable under all x_a, x_{a^*} and such that $\operatorname{im} y_i \subset V_i'$. Deduce that the action of $\mathrm{GL}(v)$ on $\mathrm{Rep}(Q, v, w)^{\theta-ss}$ is free.

Problem 16.1. *Prove that a generic fiber $\mu^{-1}(\lambda), \lambda \in \mathfrak{g}^{*G}$, is smooth and connected. You may use the following strategy:*

1. *Show that all G-orbits in $\mu^{-1}(\lambda)$ are free. Deduce that $\mu^{-1}(\lambda)$ is smooth.*
2. *Show that $\mu^{-1}(\mathbb{C}\lambda)$ is normal.*
3. *The affine version of Zariski's main theorem says that the morphism $\mu^{-1}(\mathbb{C}\lambda) \to \mathbb{C}\lambda$ decomposes into a composition of a morphism $\mu^{-1}(\mathbb{C}\lambda) \to X$ with connected general fiber and a finite morphism $X \to \mathbb{C}\lambda$. Use this to prove that $\mu^{-1}(\lambda)$ is connected.*

Problem 16.2. *Show that the algebra $W_\hbar(V)[f^{-1}]$ is Noetherian. Deduce from here that any left ideal is closed in the $\hbar$-adic topology.*

Exercise 16.3. *Reducing modulo $\hbar^k$ for all k, show that the data of $W_\hbar(V)[f^{-1}]///G$ constitutes a sheaf with respect to the covering $V_f///G$ (where $f \in \mathbb{C}[V]^{G,n\theta}$ for some $n > 0$, and V_f stands for the principal open subset defined by f) of $V///^\theta G := \mu^{-1}(\mathfrak{z}^*)//^\theta G$. Recall that this means the following: if we have a covering of $V_f///G$ by $V_{f_i}///G, i = 1, \ldots, n$ and sections $a_1, \ldots, a_n$ of $W_\hbar(V)[f_i^{-1}]///G$ that agree on intersections, then they glue together to a unique element $W_\hbar(V)[f^{-1}]///G$.*

Exercise 16.4. *Let V_1, V_2 are $\mathbb{C}[[\mathfrak{z}^*, \hbar]]$-modules that are flat, complete and separated. Let $\iota : V_1 \to V_2$ be a $\mathbb{C}[[\mathfrak{z}^*, \hbar]]$-module homomorphism that is an isomorphism modulo $(\mathfrak{z}, \hbar)$. Show that ι is an isomorphism.*