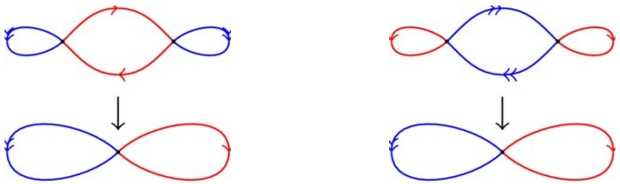
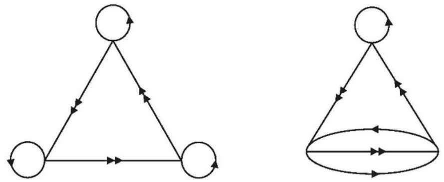
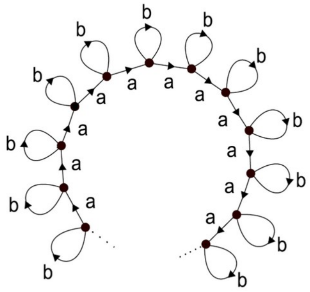
<u>Ex</u> coverings of the figure-eight X:





(1) https://www.homepages.ucl.ac.uk/~ucahjde/tg/html/cov-01.html
(2) https://groupoids.org.uk/images/fig10-3.jpg

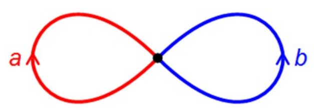
pick basepts: what are the hom's of π_1's?

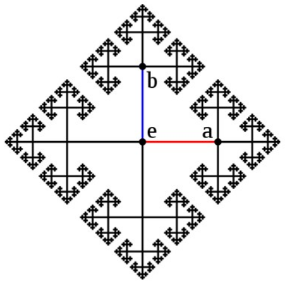


https://www.math.cmu.edu/~nkomarov/NK-NormalSubFreeGrp.pdf

let e = left-hand vertex in E
b = blue loop in $\pi_1(X, x)$
r = red loop in $\pi_1(X, x)$

LHS: $\pi_1(E, e) = \langle b, rbr^{-1} \rangle$
RHS: $\pi_1(E, e) = \langle r, brb^{-1} \rangle$





https://math.stackexchange.com/a/3762676

Ex consider the relation ~ on S^2 given by (x, y, z) ~ (–x, –y, –z)

S^2/~ is called the real projective plane, or RP^2

S^2 to RP^2 is a 2-fold covering map

recall: E is simply connected iff it is path connected and its π_1 is trivial

by thm from last time: if X is “nice”, then there is a path-conn covering of X corr to {id} sub π_1(X)

Df a universal cover of X is a path-conn covering E to X corr to {id} sub π_1(X)

note: E is necessarily simply connected and unique up to equiv of (pointed) coverings

Ex S^2 is the universal cover of RP^2

Ex the infinite 4-valent tree (above) is the universal cover of the figure-eight

(Munkres §61) separation theorems:
easy to state, hard to prove

Df a simple closed curve in R^2 (or S^2) is
a subspace homeomorphic to S^1

Jordan Separation Thm

if C sub R^2 is a simple closed curve,
then R^2 – C is disconnected

[stronger:] Jordan Curve Thm

if C sub R^2 is a simple closed curve
then R^2 – C has exactly two connected comp's

next time:

Thm if C sub S^2 is a simple closed curve
then S^2 – C is not path connected

Rem this implies Jordan Separation:

- S^2 – C is open in S^2, so it remains locally path connected (§25); thus its path comp's are its conn comp's (Theorem 25.5)
- pick p notin C, homeo f : S^2 – {p} to R^2; then bijection between comp's of S^2 – C and comp's of R^2 – f(C) (Lemma 61.1)