PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

8. Spherical SRA

Exercise 8.1. *Use the theorem that* $H_{un} \cong H$ *when* V *is symplectically irreducible to deduce that* H *is a graded deformation of* $S(V)\#\Gamma$ *even if* V *is not symplectically irreducible.*

Exercise 8.2. *Prove that* eHe *is a graded deformation of* $S(V)^\Gamma$ *over* $S(P)$*. Also prove that the specialization of* eHe *at* $t, c_1, \ldots, c_m \in \mathbb{C}$ *coincides with* $eH_{t,c}e$ *so that* $\operatorname{gr} eH_{t,c}e = S(V)^\Gamma$*.*

Exercise 8.3. *Prove that the natural homomorphism* $eH_{t,c}e^{opp} \to \operatorname{End}_{H_{t,c}}(H_{t,c}e)$ *is an isomorphism.*

Exercise 8.4. *Let* $\mathcal{A}$ *be a* $\mathbb{Z}_{\geqslant 0}$*-filtered algebra and* M *be its module. Equip* M *with a filtration compatible with that on* $\mathcal{A}$ *in such a way that* $\operatorname{gr} M$ *is finitely generated* $\operatorname{gr}\mathcal{A}$*-module. We set* $\operatorname{End}_{\mathcal{A}}(M)^{\leqslant n} := \{\psi \in \operatorname{End}_{\mathcal{A}}(M) | \psi(M^{\leqslant m}) \subset M^{\leqslant n+m}, \quad \forall m\}$*.*

1. *Show that this is a* $\mathbb{Z}$*-filtration and that* $\operatorname{End}_{\mathcal{A}}(M)^{\leqslant n} = 0$ *for* $n \ll 0$*.*
2. *Construct a natural homomorphism* $\operatorname{gr}\operatorname{End}_{\mathcal{A}}(M) \to \operatorname{End}_{\operatorname{gr}\mathcal{A}}(\operatorname{gr} M)$ *of graded algebras.*
3. *Show that this homomorphism is injective.*

Exercise 8.5. *Let* M, N *be* $\mathbb{Z}$*-filtered vector spaces such that* $M^{\leqslant n} = 0$ *for* $n \ll 0$*. Let* $\varphi : M \to N$ *be a filtration preserving linear map. Show that if* $\operatorname{gr}\varphi : \operatorname{gr} M \to \operatorname{gr} N$ *is an isomorphism, then* φ *is an isomorphism.*

Problem 8.1. *Show that* H, eHe *also satisfy the double centralizer property.*

Exercise 8.6. *Let* $a \in \mathbb{C}^\times$*. Establish a natural isomorphism between* $H_{t,c}$ *and* $H_{at,ac}$*.*