PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

5. Symplectic quotient singularities

Problem 5.1. *Consider the representation space* $\mathrm{Rep}(Q, \delta)$ *for cyclic quiver* Q *with* $r+1$ *vertices and the corresponding morphism* $\mu : \mathrm{Rep}(Q, \delta) \to \mathfrak{gl}(\delta)$*. Describe the irreducible components of* $\mu^{-1}(0)$*. Show that the codimension of each is* r*, and that each contains a free* $\mathrm{GL}(\delta)$*-orbit.*

Exercise 5.1. *Let* A *be a commutative associative unital algebra.*

(1) *Let* A *be equipped with a bracket* $\{\cdot, \cdot\}$*. Show that* $\{1, a\} = 0$ *for all* $a \in A$*.*
(2) *Show that if* $a_1, \dots, a_k$ *are generators of* A*, then there is at most one bracket* $\{\cdot, \cdot\}$ *with given* $\{a_i, a_j\}$*. Show that this bracket satisfies the Jacobi identity for all* a, b, c*, if it does so for all* a_i, a_j, a_k*.*
(3) *Finally, prove that if* $A = \mathbb{C}[a_1, \dots, a_k]$*, then a bracket exists for any values of* $\{a_i, a_j\}$ *as long as* $\{a_i, a_j\} = -\{a_j, a_i\}$*.*

Exercise 5.2. *We can choose a basis* $x_1, \dots, x_n, y_1, \dots, y_n$ *in* V *so that* $\omega(x_i, x_j) = \omega(y_i, y_j) = 0, \omega(y_i, x_j) = \delta_{ij}$*. Let us identify* $S(V)$ *with* $\mathbb{C}[x_1, \dots, x_n, y_1, \dots, y_n]$*. Then the bracket* $\{\cdot, \cdot\}$ *induced by* ω *is given by the formula*

$$\{f, g\} = \sum_{i=1}^{n} \frac{\partial f}{\partial y_i}\frac{\partial g}{\partial x_i} - \frac{\partial f}{\partial x_i}\frac{\partial g}{\partial y_i}.$$

Exercise 5.3. *Check that* $\{\cdot, \cdot\}$ *on* $A = \mathrm{gr}\, \mathcal{A}$ *is well-defined and is indeed a Poisson bracket.*