<u>Last time</u> if X, Q are sets,
$p : X \to Q$ is a surjective map,
X has a topology,

then the resulting quotient topology on Q is
$\{U \subseteq Q \mid p^{-1}(U) \text{ is open in } X\}$

a <u>quotient space</u> of X is a topological space formed this way [how do they appear in practice?]

<u>Df</u> an equivalence relation on a set X is a rule $\sim$ on elts of X^2 s.t., for all x, y, z,

I) $x \sim x$, for all x in X
II) if $x \sim y$, then $y \sim x$
III) if $x \sim y$ and $y \sim z$, then $x \sim z$

<u>Fact</u> if $p : X \to Q$ is a surjective map, then the rule $\sim$ on X^2 def by

$$x \sim y \text{ iff } p(x) = p(y)$$

is an equivalence relation

conversely, every equiv relation arises from a surjective map this way

<u>Ex</u> $X = \mathbb{R} \times \{a, b\}$ = union of two copies of $\mathbb{R}$
$Y = X/\sim$ where $\sim$ is this equiv. relation:

$(x, a) \sim (y, a)$ iff $x = y$
$(x, b) \sim (y, b)$ iff $x = y$
$(x, a) \sim (y, b)$ iff $x = y \neq 0$

[draw]

Y is not Hausdorff in the quotient top

[draw]

(Munkres §23)

<u>Df</u> a separation of X is
a pair of disjoint <u>nonempty</u> opens U, V
s.t. X = U cup V

we say that X is <u>connected</u> iff it has no separation
i.e.
whenever X = U cup V for disjoint open U, V,
we require either U = Ø or V = Ø

<u>Rem</u> if U, V form a separation
then U, V are not just open, but closed

“clopen” = “both open and closed”

<u>Ex</u> X = [0, 1/2) cup (1/2, 1] in the subsp top
inherited from analytic R

U = [0, 1/2) and V = (1/2, 1] is a separation

<u>Ex</u> Q in the subsp top inherited from R

for α irrational, U = {x in Q | x < α},
V = {x in Q | x > α}
is a separation

Thm R in the analytic topology is connected

[but the proof is hard]

Moral subspaces of connected spaces
need not be connected

[but upshot:]

Thm if A is a connected subspace of X
and U, V form a separation of X
then either A sub U or A sub V

Pf if A cap U and A cap V both nonempty,
then they form a separation of A
so either A cap U = ∅ or A cap V = ∅

Df the maximal connected subspaces of X
are called its connected components

Thm
1) if X, X′ are each connected, then so is X × X′
2) if f : X to Y is cts and X is connected,
then f(X) is connected as a subspace of Y
3) if A, A′ sub X are connected
and A cap A′ ≠ ∅,
then A cup A′ is connected

Ex observe: only one topology exists on ∅
is ∅ connected? yes!

Q is R^ω connected in the box topology?
in the product topology?