(Munkres §17) X top space

Df X is T1 iff

for any $x \neq y$ in X, have open U sub X s.t. x in U and y notin U [draw] [differs from Munkres!]

[recall:] X is Hausdorff (aka T2) iff

for any $x \neq y$ in X, have disjoint open U, V sub X s.t. x in U and y in V [draw]

so T2 implies T1

[is there a T1 space that is not T2?]

Ex R in the cofinite top

not T2:

given $x \neq y$ and open U, V s.t. x in U and y in V,
know U, V are nonempty
so R – U and R – V are finite
so U and V are infinite
so cannot have V sub R – U or vice versa

but T1:

given $x \neq y$, take U = R – {y}
then U is open and x in U and y notin U

in general:

<u>Prob</u> the following are equivalent:

1) X is T1
2) {x} is closed for all x in X
3) all finite subsets of X are closed

<u>Ex</u> R in the indiscrete top

not T1:

for any x in R, only open containing x is R

in general:

if |X| > 1, then the indiscrete top on X is not T1

<u>Df</u> given a sequence (x_1, x_2, …) in X

we say it <u>converges</u> to x in X iff, for any open nbd U ni x, we have N s.t. x_n in U for all n ≥ N

<u>Thm</u> if X is Hausdorff,

then a sequence in X converges to at most one point of X

<u>Prob</u> suppose that X is cofinite R

show that every sequence in X converges to every point of X at the same time

[so T1 is not enough!]

Thm subspaces and products of Hausdorff spaces are Hausdorff

but quotients need not be

Prob subspaces and products of T1 spaces are T1

Prob quotients of T1 spaces need not be T1

(Munkres §23) [recall:] X is connected iff

there are no disjoint nonempty open U, V sub X s.t. X = U cup V [draw]

Thm products and quotients of connected spaces are connected

but subspaces need not be

in general:

Thm (Intermediate Value)

images of connected spaces under cts maps are connected

Thm suppose A sub X

if A is connected, and A sub B sub Cl_X(A), then B is connected [recall: Cl_X(A) is the intersection of all closed sets in X containing A]

<u>Pf</u> suppose B = C cup D is a separation

since A is connected, either A sub C or A sub D

[by Munkres Lemma 23.2]

suppose A sub C

then B sub Cl_X(A) sub Cl_X(C)

but D sub Int_X(X – C) = X – Cl_X(C)

[recall: Int_X(X – C) is the union of all open sets in X contained inside X – C]