Last time $Cl\_X(A) = X - Int\_X(X - A)$

Q what is the closure of R^∞ in R^ω
in the box top? in the product top?

a useful formula:

$Cl\_X(A)$
$= X - \{x \mid \text{have } V \text{ open in } X \text{ s.t. } x \text{ in } V \text{ sub } X - A\}$
$= \{x \mid \text{no } V \text{ open in } X \text{ s.t. } x \text{ in } V \text{ and } V \text{ sub } X - A\}$
$= \{x \mid \text{if } V \text{ is open in } X \text{ and contains } x,$
$\text{then } V \text{ cap } A \neq \varnothing\}$

Q let $x = (1, 1/2, 1/3, 1/4, \ldots)$
is x in $Cl\_\{R^\omega\}(R^\infty)$…

…wrt the box topology? [draw]

no: x in $(0, 2) \times (0, 1) \times (0, 2/3) \times \ldots$

…wrt the product topology?

yes: suppose V open in R^ω and x in V

pick basis elt B s.t. x in B sub V
$B = prod\_i\ B\_i$, where $B\_i \neq R$ for only fin many i
so B contains elts of R^∞
so V cap $R^\infty \neq \varnothing$

[other ways to describe Cl_X(A)?]
[limits and convergence?]

Df a sequence $x_1, x_2, \ldots$ of points in X
converges to x
iff, for all open V containing x
have N s.t. $x_N, x_{N + 1}, \ldots$ in V

thus: if some seq in A converges to x in X
then x in $Cl_X(A)$

[Munkres Lem 21.2:
if the topology on X comes from a metric
then the converse holds]

Q can a seq converge to multiple pts?

Ex give X the indiscrete topology:
every seq converges to every pt at once!

Df X is Hausdorff iff, for all $x \neq y$ in X,
there are disjoint open U and V
s.t. x in U and y in V

Thm if X is Hausdorff
then any sequence in X converges to
at most one pt

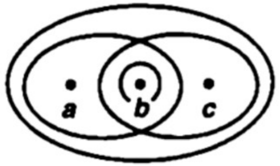
Pf suppose $(x_n)_n$ converges to x and y

suppose $x \neq y$:
then have disj open U, V s.t. x in U and y in V
if $x_N, x_{N + 1}, \ldots$ in U, then notin V
contradiction

Separation Conditions

T_2 = Hausdorff	for all $x \neq y$, disjoint open U, V s.t. x in U and y in V
T_1	for all $x \neq y$, have open U s.t. x in U and y notin U
T_0	for all $x \neq y$, have open U s.t. either x in U and y notin U, or vice versa

Ex T_0 but not T_1:



{b} is open
a notin {b} and b in {b}
but no open U s.t. a in U and b notin U

(Munkres §22)

subset of X	=	set A with injective map A to X
quotient set of X	=	set Y with surjective map X to Y

given a topology on X:

Df   the quotient topology on Y is
$\{V \text{ sub } Y \mid f^{-1}(V) \text{ is open in } X\}$

subspace top on A — coarsest top s.t. A to X is cts

quotient top on Y — finest top s.t. X to Y is cts

i.e.: if T is a top on Y s.t. X to Y is cts then T contains the quotient top on Y

[if V in T, then $f^{-1}(V)$ is open in X so V is also open in the quotient top on Y]

Ex $X = [0, 1]$ and $Y = S^1 := \{x^2 + y^2 = 1\}$

define $f : [0, 1]$ to S^1 by $f(t) = (\cos(2\pi t), \sin(2\pi t))$

Q give $[0, 1]$ the analytic top, i.e., the subsp. top inherited from R_{an}

what is the quotient top on S^1?

suppose $\{B_i\}_i$ is a basis for the top of X

[recall:]

Thm if A is a subset of X then $\{A \cap B_i\}_i$ is a basis for the subspace top on A

Thm if Y is a quotient set of X then $\{C \mid f^{-1}(C) = B_i \text{ for some } i\}$ is a basis for the quotient top on Y