Question 1: Normal Distribution

A)

```
# NORMAL.PY

import numpy as np
import pandas as pd
from tqdm import tqdm
import matplotlib.pyplot as plt
import seaborn as sns

#imput parameters
x_mean=1.
standard_deviation=2.
df = pd.DataFrame({'sample size':[], '<x>':[], '|<x> - x_mean|^2':[]})

for sample_size in tqdm(range(10000, 100000000, 100000)):

    # numpy function to sample the normal distribution
    x_norm=np.random.normal(x_mean,standard_deviation,sample_size)
    new_df = pd.DataFrame({'sample size':[sample_size], '<x>':[x_norm.mean()], '|<x> - x_mean|^2':[(abs(x_norm.mean()-x_mean))**2]})
    df = pd.concat([df, new_df], ignore_index=True)
```
```
41%|████████████████████████████████████████             | 406/1000 [02:20<06:48,  1.46it/s]
```

To leave no doubt in the readers mind of the relationship between the sample size and the value of |<x> - x_mean|^2, I've ran 1000 iterations where the sample size increases from 10000 to about 100000000 (step size of 100000). From this, we can plot the relationship between these two things.

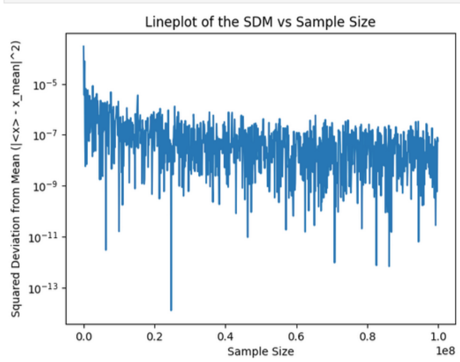
```
[17]: df
```

[17]:

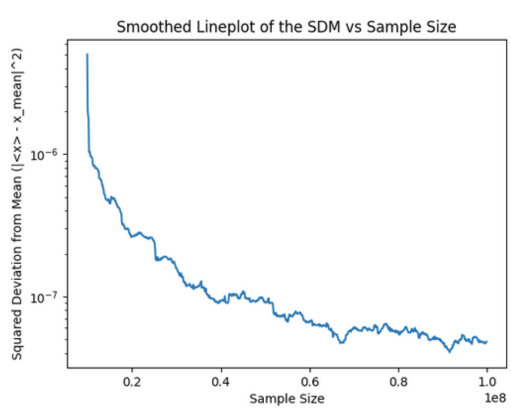
	sample size	<x>	|<x> - x_mean|^2
0	10000.0	0.982739	2.979302e-04
1	110000.0	0.995486	2.037501e-05
2	210000.0	1.001889	3.567434e-06
3	310000.0	0.991169	7.798536e-05
4	410000.0	1.000346	1.196980e-07
...	...	...	...
995	99510000.0	0.999929	5.063471e-09
996	99610000.0	1.000237	5.611406e-08
997	99710000.0	1.000024	5.879312e-10
998	99810000.0	0.999720	7.827286e-08
999	99910000.0	0.999763	5.627128e-08

1000 rows × 3 columns

```
[29]: sns.lineplot(data=df, x='sample size', y='|<x> - x_mean|^2')
plt.yscale('log')
plt.ylabel('Squared Deviation from Mean (|<x> - x_mean|^2)')
plt.xlabel('Sample Size')
plt.title('Lineplot of the SDM vs Sample Size')
plt.show()
```



In case this relationship is still unclear, I'll smooth out the pattern with a sliding window (rolling function). The following is with a window of 100 data points:



The code used to generate the smoothed plot on the left is below. It's clear that increasing the sample size decreases the SDM.

```
[39]: # Applying a rolling window for smoothing
df['smoothed_deviation'] = df['|<x> - x_mean|^2'].rolling(window=100).mean()

# Plot the smoothed data
sns.lineplot(data=df, x='sample size', y='smoothed_deviation')
plt.yscale('log')
plt.ylabel('Squared Deviation from Mean (|<x> - x_mean|^2)')
plt.xlabel('Sample Size')
plt.title('Smoothed Lineplot of the SDM vs Sample Size')
plt.show()
```

B)

I used the input parameters shown below (x_mean = 1, standard_deviation = 2, sample_size = 1000000). I then calculated the variance and the standard deviation. The calculated standard deviation is really close to 2, as expected.

[52]:
```python
# Reported Input Parameters
x_mean = 1.
standard_deviation = 2.
sample_size = 1000000

# Numpy function to sample the normal distribution
x_norm=np.random.normal(x_mean,standard_deviation,sample_size)

# Answers for variance and stddev (calculated with Numpy)
print(f'The variance of this sample is {np.var(x_norm)}')
print(f'The standard deviation of this sample is {math.sqrt(np.var(x_norm))}')
```

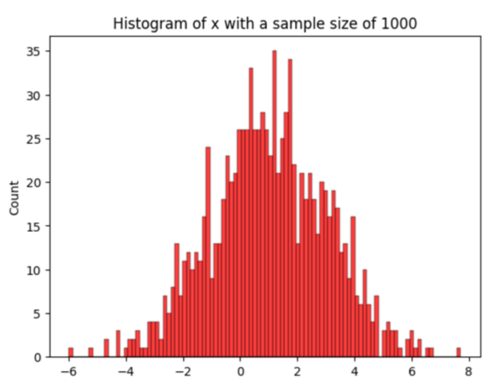
```
The variance of this sample is 4.002788226897621
The standard deviation of this sample is 2.000696935294704
```

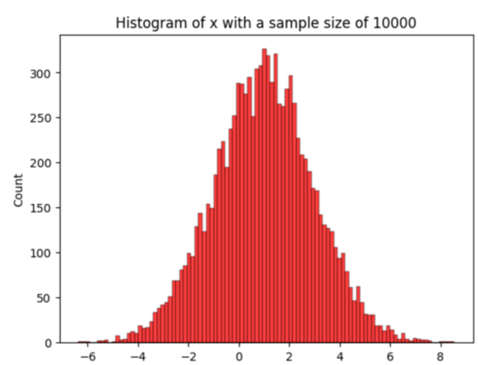
C)

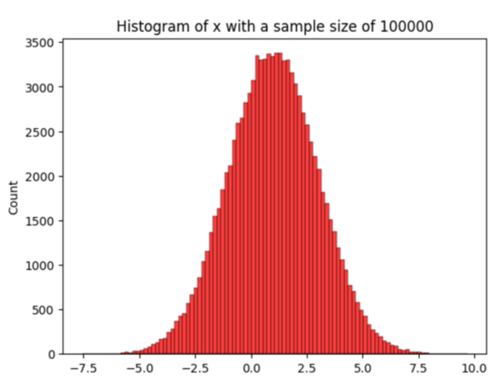
I didn't bother changing the bin size because the plots looked fine with a consistent bin size (n=100). All code and plots shown below. It's always cool to see the normal distribution arise from chaos.

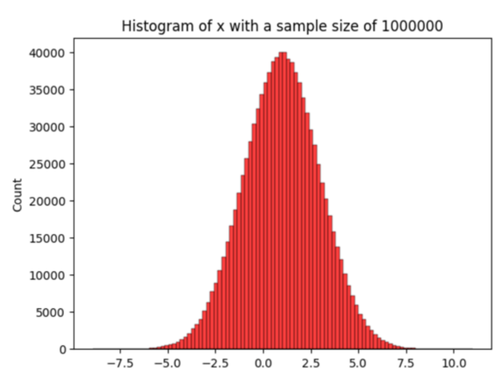
[63]:
```python
# Input Parameters
x_mean = 1.
standard_deviation = 2.
sample_size_1000=np.random.normal(x_mean,standard_deviation,1000)
sample_size_10000=np.random.normal(x_mean,standard_deviation,10000)
sample_size_100000=np.random.normal(x_mean,standard_deviation,100000)
sample_size_x_norm_1000000=np.random.normal(x_mean,standard_deviation,1000000)
```

[68]:
```python
for data in [[x_norm_1000, '1000'], [x_norm_10000, '10000'], [x_norm_100000, '100000'], [x_norm_1000000, '1000000']]:
    sns.histplot(data[0], bins=100, color='red')
    plt.title(f'Histogram of x with a sample size of {data[1]}')
    plt.show()
```









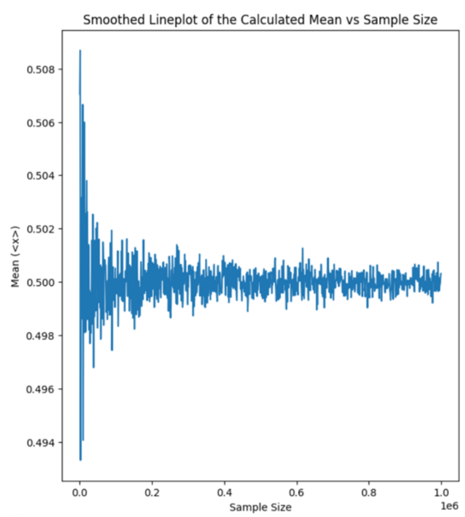
Question 2: Uniform Distribution

A)

I would expect <x> to be 0.5 as it's the mean, since (0+1)/2 = 0.5.

B)

Similar to Q1, to leave no doubt in the readers mind of the relationship between the sample size and <x>, I've ran 1000 iterations where the sample size increases from 1000 to about 1000000 (step size of 1000). From this, we can plot the relationship between these two things.



The code used to generate the plot on the left is below. It's clear that <x> approaches 0.5 as the sample size increases.

```
[72]: # UNIFORM.PY
import numpy as np
sample_size=10000
x_uniform=np.random.random(sample_size)
```

```
[112]: df = pd.DataFrame({'sample size':[], '<x>':[]})

for sample_size in tqdm(range(1000, 1000000, 1000)):
    x_uniform=np.random.random(sample_size)
    new_df = pd.DataFrame({'sample size':[sample_size], '<x>':[x_uniform.mean()]})
    df = pd.concat([df, new_df], ignore_index=True)
```

```
100%|
```

```
[114]: plt.figure(figsize=(7, 8))
sns.lineplot(data=df, x='sample size', y='<x>')

plt.ylabel('Mean (<x>)')
plt.xlabel('Sample Size')
plt.title('Smoothed Lineplot of the Calculated Mean vs Sample Size')
plt.show()
```

C)

This is just a matter of changing the range of the random numbers generated.

```
[117]: import numpy as np

# Define the sample size
sample_size = 10000

# Generate random values between -1 and 3
# To do this, use the formula: a + (b - a) * np.random.random(size)
# where a is the lower bound and b is the upper bound
x_uniform = -1 + (3 - (-1)) * np.random.random(sample_size)

# Open the file to write the generated values
with open('x_uniform.dat', 'w') as f:
    for x in x_uniform:
        f.write(f"{x}\n")  # Writing each random number to the file

print("Random numbers between -1 and 3 have been generated and written to 'x_uniform.dat'.")
```

```
Random numbers between -1 and 3 have been generated and written to 'x_uniform.dat'.
```

I verified this was correct by looking at a histogram and some summary statistics.
As requested in the pdf, I've attached this script to the submission as "question_2c.py".

Question 3: Can you calculate Pi?

A)

This is a classic way to estimate π with randomness. Basically, the ratio of points inside the circle to the total number of points multiplied by 4 will approximate the value of π. The reason this works is because the area of a quarter circle is π/4, compared to the area of the unit square, which is 1.

B)

I wrote a script to estimate the value of π given a bunch of different sample sizes. It can be seen clearly that increasing the sample size leads to better estimations (see print statements).

```
[138]: import numpy as np

for sample_size in [100, 1000, 10000, 100000, 1000000, 10000000, 100000000]:
    x=np.random.random(sample_size)
    y=np.random.random(sample_size)
    in_the_circle = 0
    for i in range(sample_size):
        if x[i]**2 + y[i]**2 <= 1:
            in_the_circle += 1

    # Approximate π using the method I described in part A
    print(f'Estimate for a sample size of {sample_size} is {in_the_circle/sample_size * 4}')
```

```
Estimate for a sample size of 100 is 3.16
Estimate for a sample size of 1000 is 3.14
Estimate for a sample size of 10000 is 3.1348
Estimate for a sample size of 100000 is 3.1458
Estimate for a sample size of 1000000 is 3.13956
Estimate for a sample size of 10000000 is 3.1411036
Estimate for a sample size of 100000000 is 3.14146944
```

C)

I made two functions to make the code more modular. All code is attached below, and the standard errors can again be found in the print statements

```
# Function to compute standard error
def compute_standard_error(sample_size, num_trials=100):
    pi_estimates = []

    # Perform multiple trials to calculate π estimates
    for _ in range(num_trials):
        pi_estimates.append(estimate_pi(sample_size))

    # Compute mean and standard deviation of π estimates
    mean_pi = np.mean(pi_estimates)
    std_dev_pi = np.std(pi_estimates)

    # Calculate standard error
    standard_error = std_dev_pi / np.sqrt(num_trials)

    print(f"Sample size: {sample_size}, Mean π: {mean_pi}, Standard Error: {standard_error}")
```

```
[140]: def estimate_pi(sample_size):
    x = np.random.random(sample_size)
    y = np.random.random(sample_size)
    in_the_circle = 0
    for i in range(sample_size):
        if x[i]**2 + y[i]**2 <= 1:
            in_the_circle += 1
    return in_the_circle / sample_size * 4
```

```
# Run the standard error calculation for different sample sizes
for sample_size in [100, 1000, 10000, 100000, 1000000, 10000000]:
    compute_standard_error(sample_size)
```

```
Sample size: 100, Mean π: 3.1515999999999997, Standard Error: 0.01584217156831727
Sample size: 1000, Mean π: 3.14424, Standard Error: 0.004102611851004189
Sample size: 10000, Mean π: 3.140952000000001, Standard Error: 0.0016702936747769836
Sample size: 100000, Mean π: 3.1413108, Standard Error: 0.000580923913778731
Sample size: 1000000, Mean π: 3.1416828399999996, Standard Error: 0.00015503054971198344
Sample size: 10000000, Mean π: 3.1415515960000007, Standard Error: 4.8724243327526635e-05
```

**** NOTE: I MADE ALL THE PLOTS WITH SEABORN/MATPLOTLIB, ALL CODE IS ATTACHED ****