

### 3D Rotation

Lie Group

$$SO(3)$$

$$R \in \mathbb{R}^{3 \times 3}$$

$$RR^T = I$$

$$\det(R) = 1$$

$$\exp(\theta a^\wedge) = \cos \theta I + (1 - \cos \theta) aa^T + \sin \theta a^\wedge \quad \text{Exponential}$$

$$\text{Logarithmic} \quad \theta = \arccos \frac{\text{tr}(R) - 1}{2} \quad Ra = a$$

Lie Algebra

$$\mathfrak{so}(3)$$

$$\phi \in \mathbb{R}^3$$

$$\phi^\wedge = \begin{bmatrix} 0 & -\phi_3 & \phi_2 \\ \phi_3 & 0 & -\phi_1 \\ -\phi_2 & \phi_1 & 0 \end{bmatrix}$$

### 3D Transform

Lie Group

$$SE(3)$$

$$T \in \mathbb{R}^{4 \times 4}$$

$$T = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$$

$$\exp(\xi^\wedge) = \begin{bmatrix} \exp(\phi^\wedge) & J\rho \\ 0^T & 1 \end{bmatrix}$$

$$J = \frac{\sin \theta}{\theta} I + \left(1 - \frac{\sin \theta}{\theta}\right) aa^T + \frac{1 - \cos \theta}{\theta} a^\wedge \quad \text{Exponential}$$

$$\text{Logarithmic} \quad \theta = \arccos \frac{\text{tr}(R) - 1}{2} \quad Ra = a \quad t = J\rho$$

Lie Algebra

$$\mathfrak{se}(3)$$

$$\xi = \begin{bmatrix} \rho \\ \phi \end{bmatrix} \in \mathbb{R}^6$$

$$\xi^\wedge = \begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix}$$