# Analytical Approach for Estimating Weld Bead Width in Welding

## 1 Introduction

Welding is a process where molten metal solidifies to form a joint. The shape and dimensions of the weld pool play a crucial role in determining the weld quality. In this study, we estimate the **weld bead width** based on **surface tension** effects and a fixed contact angle of the molten pool.

We use the **surface tension equation** and the **Young-Dupré relation** for solid-liquid interactions to establish a theoretical model for the weld bead width.

## 2 Governing Equations

### 2.1 Surface Tension as a Function of Temperature

The surface tension of the molten metal is given by:

$$\sigma^P\,(\mathrm{mN/m}) = (1925 \pm 65) - (0.455 \pm 0.034) \times (T - 1808) \tag{1}$$

where:

- $\sigma^P$ is the surface tension (in mN/m),
- $T$ is the operating temperature (K),
- The equation accounts for **temperature dependence** and experimental uncertainty.
- The equation is taken from ISIJ International, Vol. 51 (2011), No. 10, pp. 1580–1586

For conservative estimation, we use the **minimum** possible value:

$$\sigma^P_{\text{min}} = (1925 - 65) - (0.455 + 0.034) \times (T - 1808) \tag{2}$$

We convert mN/m to N/m:

$$\sigma^P(\text{N/m}) = \sigma^P(\text{mN/m}) \times 10^{-3} \tag{3}$$

## 2.2 Contact Angle Relation and Force Balance

The **Young-Dupré equation** relates the surface tension to the contact angle:

$$\gamma_s = \sigma \cos\theta + \gamma_{sl} \tag{4}$$

where:

- $\gamma_s$ is the surface energy of the solid,
- $\gamma_{sl}$ is the solid-liquid interface energy (often negligible),
- $\theta$ is the **fixed contact angle** during welding.
- This equation is taken from the text provided i.e. Diss. ETH No. 25555

Assuming the weld pool forms a near-semicircular shape, the width can be approximated using **force balance in surface tension and gravity**: Now we imagine the weld pool as a cap-like shape (like a liquid drop on a flat surface). For this drop:

- The surface tension provides a horizontal pulling force that tries to expand the pool.
- The metal's own weight (gravitational pull) opposes spreading.

This leads to a balance:

Capillary Force = Hydrostatic Resistance Which results in

$$w = \frac{2\sigma}{\rho g \cos\theta} \tag{5}$$

where:

- $w$ is the **weld bead width** (m),

- $\rho$ is the **density** of molten iron ($7.17 \times 10^3$ µN m$^{-3}$ or $7.17 \times 10^3$ kg m$^{-3}$ in SI units),
- $g$ is the gravitational acceleration ($9.81$ m s$^{-2}$),
- $\theta$ is the **fixed contact angle** ($15^\circ$ to $20^\circ$).

# 3 Analytical Calculation

Using the given range:

- **Temperature:** $1300 \leq T \leq 2500$ K
- **Contact Angle:** $15^\circ \leq \theta \leq 20^\circ$

1. Compute **minimum surface tension** using Equation (2).
2. Convert units to SI.
3. Compute width $w$ for the temperature range.

**Unit Check:**

$$w = \frac{2 \times (\mathrm{N/m})}{(\mathrm{kg/m}^3) \times (\mathrm{m/s}^2) \times \cos(\theta)} \tag{6}$$

$$= \frac{\mathrm{m}}{\mathrm{s}^2} \tag{7}$$

This ensures that the result is consistent with a length unit (meters).

# 4 Results and Interpretation

- The width is plotted against temperature to observe trends.
- If the estimated width is **too large**, additional forces (such as capillary effects and flow dynamics) must be considered.
- If needed, refining input parameters (contact angle variations, surface energy corrections) improves realism.

## 5 Conclusion

This analytical model provides a **first-order approximation** of weld bead width. The derived formula captures the **influence of surface tension and gravity**, with the **contact angle as a key control parameter**. Further refinements could involve computational fluid dynamics (CFD) simulations or experimental validations.

```
% Define constants
rho = 7170; % Density in kg/m^3
g = 9.81; % Gravity in m/s^2
theta_deg = [8, 12]; % Contact angles in degrees
theta_rad = deg2rad(theta_deg); % Convert to radians

% Define temperature range
T = linspace(1300, 2500, 100); % Temperature range in Kelvin

% Compute minimum surface tension using the adjusted formula
sigma_min = 1860 - 0.489 * (T - 1808);

% Compute capillary length in meters
L_cap_min = sqrt(sigma_min ./ (rho * g));

% Convert capillary length to mm
L_cap_mm_min = L_cap_min * 1000; % Convert meters to mm

% Compute weld bead width for both angles in mm
width_8_min = 2 * L_cap_mm_min * tan(theta_rad(1)); % Width for 8 degrees
width_12_min = 2 * L_cap_mm_min * tan(theta_rad(2)); % Width for 12 degrees

% Plot results
figure;
plot(T, width_8_min, 'r-', 'LineWidth', 2);
hold on;
plot(T, width_12_min, 'b--', 'LineWidth', 2);
xlabel('Temperature (K)');
ylabel('Weld Bead Width (mm)');
title('Weld Bead Width (Min Surface Tension Case)');
legend('\theta = 8^\circ', '\theta = 12^\circ');
grid on;
```

Figure 1: MATLAB Code

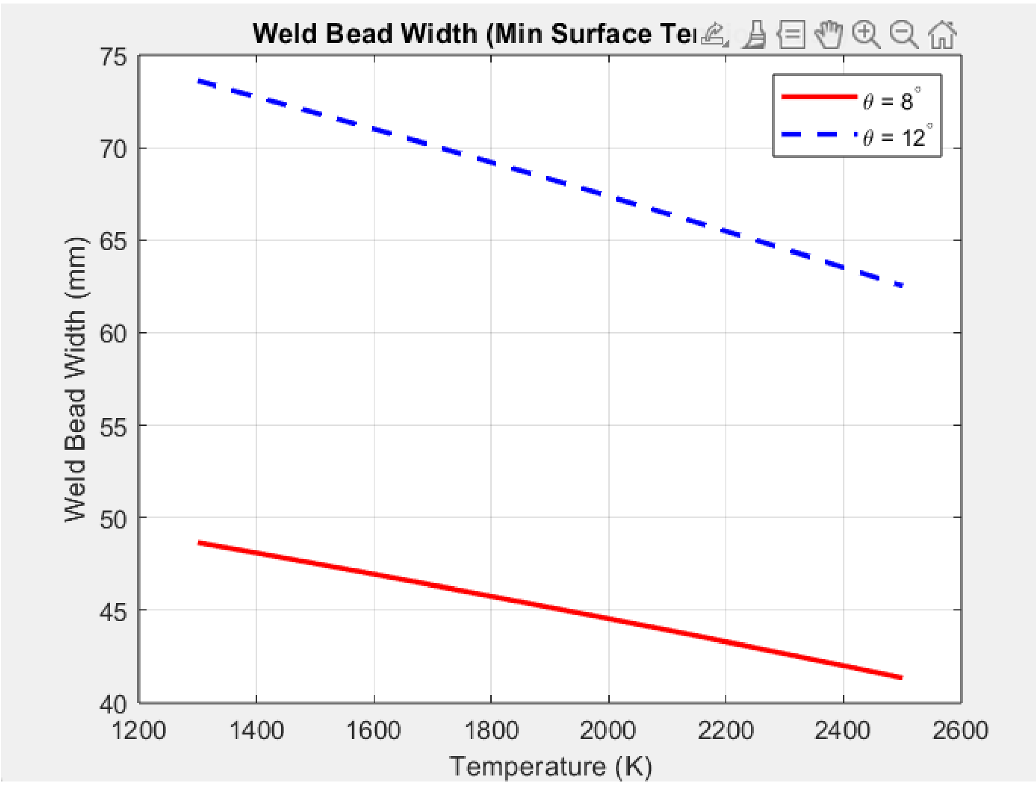


Figure 2: Plot