

Analytical Approach for Estimating Weld Bead Width in Welding

1 Introduction

Welding is a process where molten metal solidifies to form a joint. The shape and dimensions of the weld pool play a crucial role in determining the weld quality. In this study, we estimate the **weld bead width** based on **surface tension** effects and a fixed contact angle of the molten pool.

We use the **surface tension equation** and the **Young-Dupré relation** for solid-liquid interactions to establish a theoretical model for the weld bead width.

2 Governing Equations

2.1 Surface Tension as a Function of Temperature

The surface tension of the molten metal is given by:

$$\sigma^P \text{ (mN/m)} = (1925 \pm 65) - (0.455 \pm 0.034) \times (T - 1808) \quad (1)$$

where:

- σ^P is the surface tension (in mN/m),
- T is the operating temperature (K),
- The equation accounts for **temperature dependence** and experimental uncertainty.

For conservative estimation, we use the **minimum** possible value:

$$\sigma_{\min}^P = (1925 - 65) - (0.455 + 0.034) \times (T - 1808) \quad (2)$$

We convert mN/m to N/m:

$$\sigma^P \text{ (N/m)} = \sigma^P \text{ (mN/m)} \times 10^{-3} \quad (3)$$

2.2 Contact Angle Relation and Force Balance

The **Young-Dupré equation** relates the surface tension to the contact angle:

$$\gamma_s = \sigma \cos \theta + \gamma_{sl} \quad (4)$$

where:

- γ_s is the surface energy of the solid,
- γ_{sl} is the solid-liquid interface energy (often negligible),
- θ is the **fixed contact angle** during welding.

Assuming the weld pool forms a near-semicircular shape, the width can be approximated using **force balance in surface tension and gravity**:

$$w = \frac{2\sigma}{\rho g \cos \theta} \quad (5)$$

where:

- w is the **weld bead width** (m),
- ρ is the **density** of molten iron ($7.17 \times 10^3 \text{ } \mu\text{N m}^{-3}$ or $7.17 \times 10^3 \text{ kg m}^{-3}$ in SI units),
- g is the gravitational acceleration (9.81 m s^{-2}),
- θ is the **fixed contact angle** (15° to 20°).

3 Analytical Calculation

Using the given range:

- **Temperature:** $1300 \leq T \leq 2500 \text{ K}$
- **Contact Angle:** $15^\circ \leq \theta \leq 20^\circ$

1. Compute **minimum surface tension** using Equation (2).
2. Convert units to SI.
3. Compute width w for the temperature range.

Unit Check:

$$w = \frac{2 \times (\text{N/m})}{(\text{kg/m}^3) \times (\text{m/s}^2) \times \cos(\theta)} \quad (6)$$

$$= \frac{\text{m}}{\text{s}^2} \quad (7)$$

This ensures that the result is consistent with a length unit (meters).

4 Results and Interpretation

- The width is plotted against temperature to observe trends.
- If the estimated width is **too large**, additional forces (such as capillary effects and flow dynamics) must be considered.
- If needed, refining input parameters (contact angle variations, surface energy corrections) improves realism.

5 Conclusion

This analytical model provides a **first-order approximation** of weld bead width. The derived formula captures the **influence of surface tension and gravity**, with the **contact angle as a key control parameter**. Further refinements could involve computational fluid dynamics (CFD) simulations or experimental validations.

```

1  % Define constants
2  rho = 7170; % Density in kg/m^3
3  g = 9.81; % Gravity in m/s^2
4  theta_deg = [8, 12]; % Contact angles in degrees
5  theta_rad = deg2rad(theta_deg); % Convert to radians
6
7  % Define temperature range
8  T = linspace(1300, 2500, 100); % Temperature range in Kelvin
9
10 % Compute minimum surface tension using the adjusted formula
11 sigma_min = 1860 - 0.489 * (T - 1808);
12
13 % Compute capillary length in meters
14 L_cap_min = sqrt(sigma_min ./ (rho * g));
15
16 % Convert capillary length to mm
17 L_cap_mm_min = L_cap_min * 1000; % Convert meters to mm
18
19 % Compute weld bead width for both angles in mm
20 width_8_min = 2 * L_cap_mm_min * tan(theta_rad(1)); % Width for 8 degrees
21 width_12_min = 2 * L_cap_mm_min * tan(theta_rad(2)); % Width for 12 degrees
22
23 % Plot results
24 figure;
25 plot(T, width_8_min, 'r-', 'LineWidth', 2);
26 hold on;
27 plot(T, width_12_min, 'b--', 'LineWidth', 2);
28 xlabel('Temperature (K)');
29 ylabel('Weld Bead Width (mm)');
30 title('Weld Bead Width (Min Surface Tension Case)');
31 legend('\theta = 8^\circ', '\theta = 12^\circ');
32 grid on;

```

Figure 1: MATLAB Code

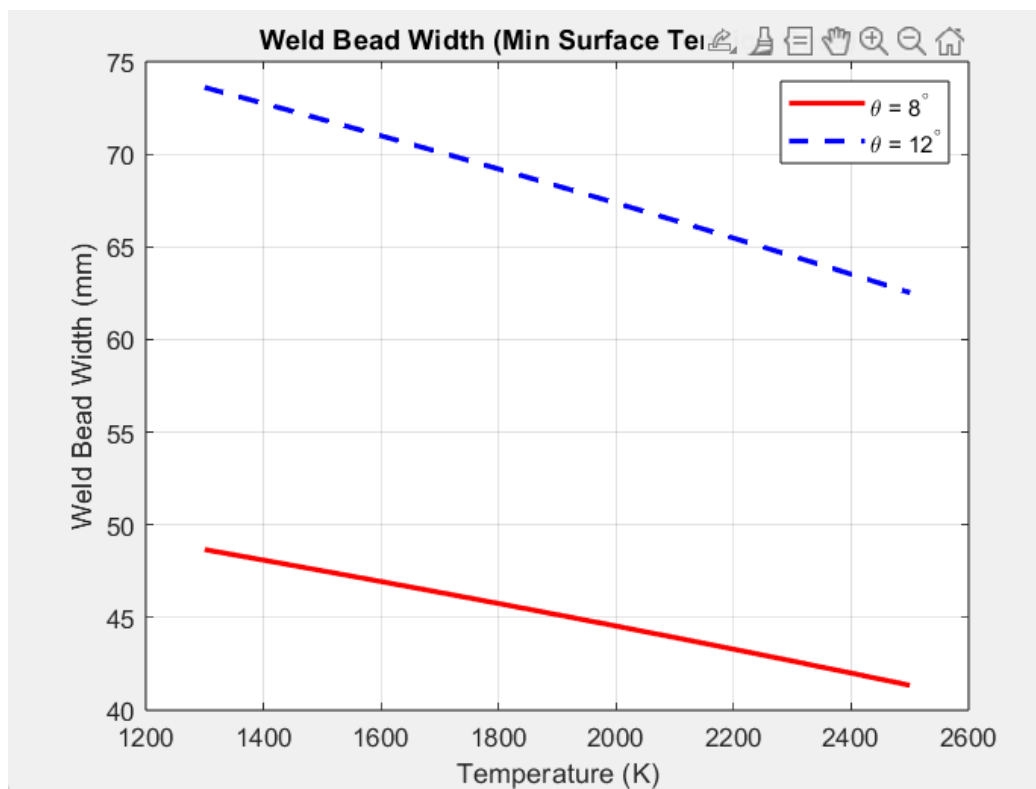


Figure 2: Plot