Exercise 2.2

Q1: $x+2y-z=6$

$y+z=5$

$z=4$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Row echelon Form

$\boxed{z=4}$

$y+z=5$

$y+4=5$

$\boxed{y=9}$

$x+2-4=6$

$x-2=6$

$\boxed{x=8}$

$x-3y+4z+w=0$

$z-w=4$

$w=1$

$$\left[\begin{array}{cccc|c} 1 & -3 & 4 & 1 & 0 \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$\boxed{w = 1}$

$z - w = 4$

$z - 1 = 4$

$\boxed{z = 5}$

$x - 3y + 20 + 1 = 0$

$x - 3y = -21$

let $\boxed{y = t}$

$x - 3t = -21$

$\boxed{x = -21 + 3t}$

Q#02.

$x + y - z + 2w = 4$

$w = 5$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$\boxed{w = 5}$

$x + y - z + 10 = 4$

$x + y - z = -6$

$y = t \quad z = s$

$x = -6 - t + s.$

$$x - y + z = 0$$

$$x + 2z = 0$$

$$z = 1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

$$\boxed{z = 1}$$

$$x + 2(1) = 0$$

$$\boxed{x = -2}$$

$$x - y + z = 0$$

$$-2 - y + 1 = 0$$

$$-y - 1 = 0$$

$$-y = 1$$

$$\boxed{y = -1}.$$

Q#03 RREF

$$x + y = 2$$

$$z + w = 3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right].$$

In this $z+w=-3$ if we let

$w=t$ then $z+t=-3$, $z=-3-t$

and $x+y=2$ and

let $y=a$ then

$x=2-a$

$x=3$

$y=0$

$z=1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

From row 1 $x=3$, From row 2

$y=0$ and from row 3 $z=1$

Q#04 RREF

$x-2z=5$

$y+z=2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & 2 \end{array}\right].$$

From row 1 $x-2z=5$ let $z=a$

then $x-2a=5$, $x=5+2a$.

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and in row 2 $y+z=2$ let $z=b$

then $y=2-b$.

$x=1$

$y=2$

$z-w=4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -1 & 4 \end{array}\right].$$

from row 1 $x=1$, from row 2 $x=2$

from row 3 $z-w=4$ let $w=a \in R$ then

$z-a=4$ where $z=4+a$.

Q#05

consider the linear system

$x+y+2z=-1$

$x-2y+z=-5$

$3x+y+z=3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right].$$

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a) Gaussian elimination method:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right].$$

$R_2 - R_1,\ R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 0 & -2 & -5 & 6 \end{array}\right].$$

$R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -1 & 4 & -10 \\ 0 & -2 & -5 & 6 \end{array}\right].$$

$-R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & -2 & -5 & 6 \end{array}\right].$$

$R_3 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & 0 & -13 & 26 \end{array}\right].$$

$R_3/_{-13}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

In this row3 $z=-2$ in row2 ; $y-4z=10$

$y+8=10$

$\boxed{y=2}$

and in row 1

$x+2-4=-1$

$x-2=-1$

$\boxed{x=1}$

b) Gaussion Jordan reduction method.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & 0 & 1 & -2 \end{array}\right].$$

R_2+4R_3

$10+4(-2)$
$10-8$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right].$$

R_1-R_2

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

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$R_1 - 2R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$-3-2(-2)$
$-3+4$

In this $x=1$, $y=2$, $z=-2$.

Q#06

$$x+y+2z+3w=13$$
$$x-2y+z+w=8$$
$$3x+y+z-w=1$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right].$$

$R_2 - R_1$, $R_3 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right]$$

$R_2 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -1 & 4 & 8 & 33 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right].$$

$-R_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right].$$

$R_3 + 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & 0 & -13 & -26 & -104 \end{array}\right].$$

$R_3 / -13$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right]$$

From row 3 $\quad z + 2w = 8 \quad$ let $w = r$

$z = 8 - 2r$

$y - 4z - 8w = -3$ $\quad|\quad$ $y = 5 + 6r$

$y - 8 + 2r - 8r = -3$

$y - 6r = 5$

$x+y+2z+3w=13$

$x+5+6r+2(8-2r)+3r=13$

$x+5+6r+16-4r+3r=13$

$x-5r+21=13$

$x-5r=13-21$

$x-5r=8$

$x=8+5r.$

where $r\in\mathbb{R}$.

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right].$$

R_1-R_2

$$\left[\begin{array}{cccc|c} 1 & 0 & 6 & 11 & 46 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right].$$

R_1-6R_3, R_2+4R_3

$11-6(2)$

$11-12$

$-8+8.$

$46-6(8)$

$-33+4(8)$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right].$$

$$x - w = -2$$
$$y = -1$$
$$z + 2w = 8$$

let $w = r$, $r \in \mathbb{R}$

$$x = -2 + r$$
$$y = -1$$
$$z = 8 - 2r.$$

$$x + y + z = 1$$
$$x + y - 2z = 3$$
$$2x + y + z = 2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right].$$

$R_2 - R_1$, $R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & -1 & -1 & 0 \end{array}\right].$$

$\cancel{R_2 - 2R_1}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 5 \\ 0 & -1 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & -1 & -1 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -3 & 2 \end{array}\right]$$

$-R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 2 \end{array}\right].$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 2 \end{array}\right]$$

$R_3/3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \end{array}\right.$$

$3z = 2$

$\boxed{z = 2/3}$

$y + 2/3 = 0$

$\boxed{y = -2/3}$

$x - 2/3 + 2/3 = 0$

$\boxed{x = 0}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 2 \end{array}\right].$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 2 \end{array}\right].$$

$R_3/3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2/3 \end{array}\right].$$

$R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 2/3 \end{array}\right]$$

$$\boxed{\begin{array}{l} x = 1 \\ y = -2/3 \\ z = 2/3 \end{array}}$$

c: $2x+y+z-2w=1$

$3x-2y+z-6w=-2$

$x+y-z-w=-1$

$6x+z-9w=-2$

$5x-y+2z-8w=3$

$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 3 & -2 & 1 & -6 & -2 \\ 2 & 1 & 1 & -2 & +1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right]$$

R_2-3R_1, R_3-2R_1, R_4-6R_1, R_5-5R_1

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right]$$

$R_4 \leftrightarrow R_5$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right].$$

No solution.

$$2x+y+z-2w=1$$
$$3x-2y+z-6w=-2$$
$$x+y-z-w=-1$$
$$6x+z-9w=-2$$
$$5x-y+2z-8w=-1$$

$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & -1 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 3 & -2 & 1 & -6 & -2 \\ 2 & 1 & 1 & -2 & 1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & -1 \end{array}\right]$$

$R_2 - 3R_1, \quad R_3 - 2R_1, \quad R_4 - 6R_1, \quad R_5 - 5R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & -6 & 7 & -3 & 4 \end{array}\right]$$

$R_5 - R_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$-R_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 + 5R_2$

$R_4 + 6R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & -11 & -3 & -14 \\ 0 & 0 & -11 & -3 & -14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & -11 & 3 & -14 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3/-11$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & 1 & -3/11 & 14/11 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

$$z + \frac{-3}{11} w = \frac{14}{11}$$

$$w = r \in \mathbb{R}.$$

$$z = \frac{14}{11} + \frac{3}{11} r.$$

$$y - 3z = -3$$

$$y - \frac{42}{11} - \frac{9}{11} r = -3$$

$$y = -3 + \frac{42}{11} + \frac{9}{11}r.$$

$$y = \frac{-33+42}{11} + \frac{9}{11}r.$$

$$y = \frac{9}{11} + \frac{9}{11}r.$$

$$x + \frac{9}{11} + \frac{9}{11}r - \frac{14}{11} - \frac{3}{11}r - r = -1$$

$$x = -\frac{6}{11} + \frac{17}{11}r.$$

7(a)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

$R_2 - R_1$,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right].$$

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & 1 \end{array}\right].$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & 1 \end{array}\right].$$

$R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -1 & 1 \end{array}\right].$$

$-R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right].$$

$x = -3$

$y = 4$

$z = -1.$

b: $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \end{array}\right]$$

$R_2 - R_1, \; R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right].$$

$-R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right].$$

$R_3 + R_2$, $R_1 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 + R_3$, $R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right].$$

$x = 0$
$y = 0$
$z = 0$.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 5 & 7 & 9 & 0 \end{array}\right]$$

$R_2 - R_1$, $R_3 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right].$$

$-R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right]$$

$R_1 - 2R_2$, $R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$x - z = 0$

$y - 2z = 0.$ $\quad z = r \in \mathbb{R}.$

$x = r$

$y = 2r$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right]$$

$R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right].$$

$R_2 / -2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right].$$

$z = 0.$

$x + 2y + 3z = 0.$

$x + 2y = 0$

$x = -2y.$ $\qquad y = r$

$x = -2r.$

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Q #08

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 1 & 3 & 0 & 1 & 3 \\ 1 & 0 & 2 & 1 & -1 \end{array}\right]$$

$R_2 - R_1$, $R_3 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & -2 & -1 & 0 & -1 \end{array}\right].$$

$R_1 - 2R_2$, $R_3 + 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 9 & 1 & -6 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & -7 & 0 & 5 \end{array}\right].$$

$R_3 / -7$

$$\left[\begin{array}{cccc|c} 1 & 0 & 9 & 1 & -6 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & 1 & 0 & -5/7 \end{array}\right].$$

$R_1 - 9R_3$, $R_2 + 3R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3/7 \\ 0 & 1 & 0 & 0 & 6/7 \\ 0 & 0 & 1 & 0 & -5/7 \end{array}\right]$$

$$y = 6/7$$

$$z = -5/7$$

$$x + w = 3/7$$

$$w = r \in \mathbb{R}$$

$$x = 3/7 - r$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{array}\right]$$

$R_3 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & 1 \end{array}\right]$$

$R_2 + R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & -1 & -1 & 2 & 1 \end{array}\right]$$

$R_1 - R_2$, $R_3 + R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & -2 & -4 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & -1 & 1 & 5 \end{array}\right]$$

$-R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & -2 & -4 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & -1 & -5 \end{array}\right].$$

$R_1 - 3R_3$

$-4+15$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 11 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & -1 & -5 \end{array}\right].$$

$x + w = 11$

$y - w = 4$

$z - w = -5$

$\underset{}{w} \overset{=r}{\in} \mathbb{R}$

$x = 11 - r$

$y = 4 + r$

$z = -5 + r.$

9: $$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 2 & 0 & 1 & 4 \\ 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{array}\right]$$

$R_2-2R_1, R_3-R_1, R_4-R_1, R_5-2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -4 & -1 & -10 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right]$$

R_2-R_5

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -1 & -3 & 12 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right].$$

$-R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & 12 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & 12 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right]$$

$R_1 + R_3$, $R_5 + 3R_2$, $R_3 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 3 & 12 \\ 0 & 0 & 7 & 22 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 11 & 34 \end{array}\right]$$

$R_3/7$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 3 & 12 \\ 0 & 0 & 1 & 22/7 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 11 & 34 \end{array}\right]$$

} mistake in this question

$5 - 2(22/7)$

$\frac{35-44}{7}$

$12 - \frac{66}{7}$

$\frac{74-66}{7}$

$4 - 44/7$ -38

$\frac{28-44}{7}$

$\frac{-24}{7}$

$34 - 11(22/7)$

$R_1 - 2R_3$, $R_2 - 3R_3$, $R_4 - 2R_3$, $R_5 - 11R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -9/7 \\ 0 & 1 & 0 & 8/7 \\ 0 & 0 & 1 & 22/7 \\ 0 & 0 & 0 & -24/7 \\ 0 & 0 & 0 & 4/7 \end{array}\right]$$

No solution

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 1 & 4 & 0 \end{array}\right]$$

$R_2 - 2R_1, \quad R_4 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & 2 & 0 \end{array}\right]$$

$-R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & 2 & 0 \end{array}\right]$$

$R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & 2 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_4$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -3 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$R_1 - 2R_2$, $R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3/8$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 + 3R_3$, $R_2 - 2R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

10: Find a 2×1 matrix x with entries not all zero such that

$Ax = 4x$ where $A = \begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix}$

$Ax = 4x$

where $x = \begin{bmatrix} x \\ y \end{bmatrix}$

consider $AX = 4x \Rightarrow AX - 4X = O_{2\times1}$

$x(A-4I) = O_{2\times1}$

$$\begin{bmatrix} x \\ y \end{bmatrix} \left(\begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \left(\begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$\left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & -2 & 0 \end{array}\right]$$

$y = 1$?

$y = 2$

11: 2×1 matrix x with entries not all zero such that

$$Ax = 3x$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$Ax = 3x$$

$$Ax - 3x = 0_{2\times1}$$

$$(A - 3I)x = 0_{2\times1}$$

$$\left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$\left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right]$$

$$-R_1 = \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right]$$

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$$x - y = 0$$

$$x - y = 0 \qquad y \in \mathbb{R}$$

$$x = r.$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}.$$

12: $Ax = 3x \qquad 3\times 1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

$$Ax = 3x$$

$$Ax - 3x = O_{3\times 1}$$

$$(A - 3I)\,x = O_{3\times 1}$$

$$\left(\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c} -2 & 2 & -1 & 0 \\ 1 & -3 & 1 & 0 \\ 4 & -4 & 2 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ -2 & 2 & -1 & 0 \\ 4 & -4 & 2 & 0 \end{array}\right].$$

$R_2 + 2R_1$, $R_3 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & -4 & 1 & 0 \\ 0 & 8 & -2 & 0 \end{array}\right].$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & -4 & 1 & 0 \\ 0 & 4 & -1 & 0 \end{array}\right]$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2/-4$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & -1/4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$1+3(-1/4)$

$1-3/4$

$\frac{4-3}{4}$

$1/4$

R_1+3R_2

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/4 & 0 \\ 0 & 1 & -1/4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$x+\frac{1}{4}z=0$

$x+1/4z=0$

$x=-1/4z \quad \Rightarrow x=-1/4r$

$y-1/4z=0$

$y=1/4z \qquad z=r\in\mathbb{R}$

$\Rightarrow y=\frac{1}{4}r.$

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13: 3×1

$Ax = 1x$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

$Ax = 1x$

$Ax - 1x = O_{3\times 1}$

$(A - 1I)x = O_{3\times 1}$

$$\left(\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 4 & -4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ 1 & -1 & 1 & 0 \\ 4 & -4 & 4 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 4 & -4 & 4 & 0 \end{array}\right].$$

$R_3 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$R_2/2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$1 - 1/2$

$\frac{2-1}{2}$

$1/2$

$R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 1/2 z = 0$

$x = -1/2 z.$

$y - 1/2 z = 0$

$y = 1/2 z \qquad z = r \in \mathbb{R}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/2 r \\ 1/2 r \\ r \end{bmatrix}$$

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14:-

In following ~~real~~ linear system, determine all values of a for

a) no solution

b) unique solution

c) infinity many solution.

$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + (a^2 - 5)z = a$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right]$$

$R_2 - R_1$, $R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

a) if $a^2 - 4 = 0$ $\quad a - 2 \neq 0$ then it has no solution

$a^2 = 4$

$a = \pm 2$

when $a=-2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & (-2^2+4 & -2-2 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -4 \end{array}\right].$$

NO solution. at $a=-2$.

$a=2$.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$0x+0y+0z=0$

This show that It has infinitely many solution.

b) In Infinite many solution

$a^2-4=0$ and $a-2=0$ at same time

$a=\pm 2$ $\quad a=2$

For $a=2$ it has infinite many solution

c) UNIQUE solution

all value $\in \mathbb{R}$ other then ± 2 system has unique solution.

15: $\quad x+y+z=2$

$2x+3y+2z=5$

$2x+3y+(a^2-1)z=a+1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & (a^2-1) & a+1 \end{array}\right].$$

R_3-R_2

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right]$$

$R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right].$$

when $a^2-3=0$ $a-4 \neq 0$ then

It has no solution

$a^2=3$

$a = \pm\sqrt{3}$

at $a = -\sqrt{3}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & -\sqrt{3}-4 \end{array}\right].$$

$(-\sqrt{3})^2$

No solution.

at $a = \sqrt{3}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & \sqrt{3}-4 \end{array}\right]$$

No solution.

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b) ~~In~~ infinity many solution

$a^2-3=0$, $a-4=0$ at same time

$a=\pm\sqrt{3}$, $a=4$

There is no value for a which give system infinite many solution.

c) unique ~~No~~ solution.

For all value $a \in \mathbb{R}$ other then $a=\pm\sqrt{3}$ system has unique solution.

16: $x+y+z=2$

$x+2y+z=3$

$x+y+(a^2-5)z=a$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & (a^2-5) & a \end{array}\right]$$

R_2-R_1 , R_3-R_1

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right].$$

$a^2-6=0$, $a-2 \neq 0$ has no solution

$a = \pm\sqrt{6}$

$a = -\sqrt{6}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -\sqrt{6}-2 \end{array}\right].$$

No solution

$a = \sqrt{6}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & \sqrt{6}-2 \end{array}\right]$$

No solution.

b) When $a^2-6=0$ and $a-2=0$ at same time system has infinitely many solution

but $a=\pm\sqrt{6}$ and $a=2$

There is no value of a which have infinite many solution.

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c) For all value of $a \in R$ other then $a = \pm\sqrt{6}$ system has unique solution..

17: $x + y = 3$

$x + (a^2 - 8)y = a$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & a^2-8 & a \end{array}\right]$$

$R_2 - R_1$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right]$$

a) at $a^2 - 9 = 0$ $a - 3 \neq 0$ system has unique solution

$a^2 = 9$

$a = \pm 3$

$a = -3$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 0 & -9 \end{array}\right]$$

No solution.

$a = 3$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

system has infinatly many solution

b) when $a^2 - 9 = 0$ and $a - 3 = 0$ at same time system has infinatly many solution

at $a = 3$ system give infinatly many solution.

a) For all value $a \in \mathbb{R}$ other then $a = \pm 3$ system give unique ~~infinatly many~~ solution

20: Solution:

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 4 \\ 2 & -1 & 3 & 5 \\ 2 & 2 & 0 & -1 \end{array}\right].$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & -1 \\ 2 & -1 & 3 & 5 \\ 2 & 2 & 0 & -1 \end{array}\right]$$

$R_3 - R_1$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & -1 \\ 2 & -1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$R_2 - R_1$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & -1 \\ 0 & -3 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$R_1/2$, $R_2/-3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + y = -1/2$

$y - z = -2$

$x = -1/2\, y$

$-z = -2 - y$

$z = 2 + y$

$y = r \Rightarrow y \in \mathbb{R}$.

$x = -1/2\, r$

$z = 2 + r.$

$y = r.$

21: Solution:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ -3 & -2 & -1 & 2 \\ -2 & 0 & 2 & 4 \end{array}\right].$$

$R_2 + 3R_1$, $R_3 + 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 4 & 8 & 8 \\ 0 & 4 & 8 & 8 \end{array}\right].$$

$R_3 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 4 & 8 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$x + 2y + 3z = 2$

$y + 2z = 2$

$z = r \quad r \in \mathbb{R}$

$y = 2 - 2r.$

$x + 2(2 - 2r) + 3r = 2$

$x + 4 - 4r + 3r = 2$

$\cancel{x = 2 - 4 - r}$ $\qquad x + 4 - r = 2$

$\cancel{x = -r}$ $\qquad x = 2 + r - 4$

$x = -2 + r$

$x = -2 + r.$

$y = 2 - 2r$

$z = r$

22 Solution:

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ 2 & 2 & 0 & c \end{array}\right].$$

$R_3 - R_1$

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ -2 & 1 & -3 & c-a \end{array}\right],$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ 0 & 0 & 0 & c-a+b \end{array}\right].$$

22 solution:

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ 2 & 2 & 0 & c \end{array}\right].$$

$R_1 \Leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right].$$

$R1/2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right]$$

$R_2 - 2R_1$

$R_3 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & -3 & 3 & b-c \\ 0 & -3 & 3 & a-2c \end{array}\right].$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & -3 & 3 & b-c \\ 0 & 0 & 0 & a-2c-b+c \end{array}\right]$$

system has ~~no~~ unique solution if expression $a - b - c$ in REF must be $= 0$.

23:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ -3 & -2 & -1 & b \\ -2 & 0 & 2 & c \end{array}\right]$$

$R_2 + 3R_1$, $R_3 + 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 4 & 8 & c+2a \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 0 & 0 & c+2a-b-3a \end{array}\right]$$

unique solution if expression $c + b - a$ appeare in REF must be zero

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$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right].$$

$R_2 - 2R_1$, $R_3 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right]$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & 0 & 0 & c-5a-b+2a \end{array}\right]$$

system has unique solution if our expression $c - b + 3a$ appeared in REF must be zero.

27:

$$\left[\begin{array}{ccc|c} 2 & 2 & 3 & a \\ 3 & -1 & 5 & b \\ 1 & -3 & 2 & c \end{array}\right].$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & c \\ 3 & -1 & 5 & b \\ 2 & 2 & 3 & a \end{array}\right].$$

$R_2 - 3R_1$, $R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & c \\ 0 & 8 & -1 & b-3c \\ 0 & 8 & -1 & a-2c \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & c \\ 0 & 8 & -1 & b-3c \\ 0 & 0 & 0 & a-2c-b+3c \end{array}\right]$$

System has unique ~~S~~F

expression $a-b+c$ appears

in REF must $= 0$.

28.

$(a-r)x + dy = 0$

$cx + (b-r)y = 0$

$$\left[\begin{array}{cc|c} a-r & d & 0 \\ c & b-r & 0 \end{array}\right]$$

$$R_1/a-r \Rightarrow \left[\begin{array}{cc|c} 1 & \frac{d}{a-r} & 0 \\ c & b-r & 0 \end{array}\right]$$

$$R_2 - cR_1 = \left[\begin{array}{cc|c} 1 & \frac{d}{a-r} & 0 \\ 0 & (b-r) - \frac{cd}{a-r} & 0 \end{array}\right]$$

$\because b - r - \frac{cd}{a-r} = (b-r)(a-r) - cd$ which is equal to 0 in Q. so

$$\left[\begin{array}{cc|c} 1 & \frac{d}{a-r} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x + \frac{d}{a-r} y = 0$ $\quad y = t \in \mathbb{R}$

$x = \frac{-dt}{a-r}$ which show non trivial solution.

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