Exercise 4.2

Example:03

Show that set of all polynomial of degree less or equal to n is vector space.

we know $V = P_n(t) = \{a_n t^n + a_{n-1} t^{n-1} + \dots a_1 t + a_0 \;\because a_i \in \mathbb{R}\}$

V is set of all polynomial of degree less or equal to n

let $p(t), q(t), r(t) \in V$ then | only cofficent are change variable same.

$p(t) = a_n t^n + a_{n-1} t^{n-1} + \dots + a_1 t + a_0$

$q(t) = b_n t^n + b_{n-1} t^{n-1} + \dots\ b_1 t + b_0$

$r(t) = c_n t^n + c_{n-1} t^{n-1} + \dots\ c_1 t + c_0.$

- ~~commutative~~ clouser.

$p(t) + q(t)$ as

$p(t) + q(t) = (a_n + b_n) t^n + (a_{n-1} + b_{n-1}) t^{n-1} \dots$
$(a_1 + b_1) t + (a_0 + b_0).$

if c is scalor we also define

$c \cdot p(t)$ as

$= (ca_n) t^n + (ca_{n-1}) t^{n-1} + \dots + (ca_1) t + (ca_0)$

property 1 (commutative law)

$q(t) + p(t) = (b_n + a_n) t^n + (b_{n-1} + a_{n-1}) t^{n-1}$
$+ \dots\dots + (b_1 + a_1) t + (b_0 + a_0) t^0$

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P(t) + q(t) already solved.

Property 2 (Associative law)

let p(t), q(t) and r(t) ∈ V Then It is evident/obvious that

$$p(t) + (q(t) + r(t)) = (p(t) + q(t)) r(t)$$

we don't need to solve furthur.

P:3) zero polynomial (Additive identity)

in paper write this

$$\underline{0} = 0 = 0t^n + 0t^{n-1} + 0t^{n-2} + \ldots\ldots + 0t + 0t^0$$

zero polynomial element.

Property 04: Additive inverse

constant let

$$s(t) = 2t^n - 5t^{n-1} + t^{n-2} + - - - - - 6t + \sqrt{7}$$

$$-s(t) = -2t^n + 5t^{n-1} - t^{n-2} + \cdots + 6t - \sqrt{7}$$

or in General

$$-a_n t^n - a_{n-1} t^{n-1} - - - - - a_1 t - a_0.$$

Scalar properties:

$c, d \in \mathbb{R}$.

Distribution of multiplication or addition

Normal multiplication and addition

$c \odot (P(t) \oplus q(t)) = c \odot P(t) \oplus c \odot q(t)$

normal + write also.

L.H.S $c \odot \underbrace{(P(t) \oplus q(t))}_{\text{already solved}}$

$c \odot ((a_n + b_n) t^n + (a_{n-1} + b_{n-1}) t^{n-1} + \text{——}$
$+ (a_1 + b_1) t + (a_0 + b_0))$

also write simple.

$c \odot (a_n + b_n) t^n + c \odot (a_{n-1} + b_{n-1}) t^{n-1} + \cdots +$
$c \odot (a_1 + b_1) t + c \odot (a_0 + b_0)$

$(c \cdot a_n t^n + c \cdot a_{n-1} t^{n-1} + \cdots + c a_1 t + c a_0) +$
$(c \cdot b_n t^n + c \cdot b_{n-1} t^{n-1} + \cdots + c b_1 t + c b_0)$

$= c \cdot (a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0) +$
$c \cdot (b_n t^n + b_{n-1} t^{n-1} + \cdots + b_1 t + b_0)$

$c \cdot P(t) + c \cdot q(t) = R.H.S$

Scalar multiplication Distribute.

2) $(c+d)\cdot p(t) = (c+d)a_n t^n + (c+d)a_{n-1}t^{n-1} + \cdots$
$+ (c+d)a_1 t + (c+d)a_0$

$= c.a_n t^n + d.a_n t^n + c a_{n-1} t^{n-1} + d a_{n-1} t^{n-1} +$
$\cdots + c a_1 t + d a_1 t + c.a_0 + d.a_0$

$= c.(a_n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0)$
$+ d(a_n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0)$
$= c \odot p(t) \oplus d \odot p(t).$

addition distribute

$1.p(t) = p(t).$

Remark

We show later that the vector space P_n behave algebraically in exactly same ~~matter~~ manners as $\mathbb{R}^{n+1}$

$V = P_n(t) = \{a_n t^n + a_{n-1}t^{n-1} \cdots a_1 t^1 + a_0\}$
$a_i \in \mathbb{R}$

$V = P_n(t) = \{(a_n, a_{n-1}, a_{n-2} \cdots a_1, a_0);$
$a_i \in \mathbb{R}\} = \mathbb{R}^{n+1}$

degree

For $n=1$

$p(t) = \{a_0 + a_1 t : a_i \in \mathbb{R}\}$

Straight line

$n=2$

$P_2(t) = \{a_0 + a_1 t + a_2 t^2, a_i \in \mathbb{R}\}$ degree 2 or less

Parabola and straight line

$n=3$

$P_3(t) = \{a_0 + a_1 t + a_2 t^2 + a_3 t^3, a_i \in \mathbb{R}\}$ degree 3 or less.

Cubic function, parabolas and straight lines.

if $P_n(t)$ is a vector space so are the $p(t)$, $P_2(t)$ and $P_3(t)$.

Question: 01

let v be the set of all polynomials (exactly) of degree 2 with the definations of addition and scalor multiplication as in ex 6.

a) Show that v is not closed under addition

b) is v closed under scalor multiplication? explain.

exactly degree 2 means
we don't take $a_2 = 0$.
Its only have all Parabola

a) $V = P_2(t) = \{a_0 + a_1 t + a_2 t^2 : a_i \in \mathbb{R}, a_2 \neq 0\}$

$p(t) = 4t + 2t^2 \in W$
$q(t) = 1 + t - 2t^2 \in V$
$p(t) + q(t) = 1 + 5t$ does not
belong to V
We need parabola t^2
Not closed under addition.

b) $\overbrace{c = 0}^{\text{~~scalar we take anything~~}} \in \mathbb{R}$
$c \cdot p(t) = 0 \cdot (4t + 2t^2) = 0 \rightarrow$ we don't tak $a_2 = 0$
does not belong to V
~~so there~~
Not closed under multiplication. no zero is come

Question:02

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$abcd = 0$

(a) is v closed under addition:

$$A_1 + A_2 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}.$$

$$(a_1+a_2)(b_1+b_2)(c_1+c_2)(d_1+d_2) = 0.$$

$$(a_1b_1 + a_2b_2 + a_1b_2 + a_2b_1)(c_1d_1 + c_2d_2 + c_1d_2 + c_2d$$

$$= 0.$$

$a_1b_1c_1d_1 + a_1b_1c_2d_2 + a_1b_1c_2d_1 + a_1b_1c_1d_2 +$
$a_1b_2c_1d_1 + a_1b_2c_2d_1 + a_1b_2c_2d_2 + a_1b_2c_1d_2 +$
$a_2b_1c_1d_1 + a_2b_1c_2d_1 + a_2b_1c_2d_2 + a_2b_1c_1d_2$
$+ a_2b_2c_1d_1 + a_2b_2c_2d_1 + a_2b_2c_2d_2 +$
$a_2b_2c_1d_1 = 0.$

$a_1b_1c_1d_1 = 0$

$a_2b_2d_2c_2 = 0$

Rest of other are not 0 always then we can say v is not under closed addition.

(b) is v closed under scalor multiplication

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

$$kA = k\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$k(a)k(b)k(c)k(d) = 0$$

$$k^4(abcd) = 0$$

$$\because abcd = 0$$

$$k^4(0) = 0$$

$$0 = 0$$

v closed under multiplication.

(c) What is the zero vector in set v?

yes $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a set ino vector.

if we multiply o with A Answer is A.

(d) Does every matrix A in V have a negative that is in V explain?

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{then} \quad -A = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}.$$

$$(-a)(-b)(-c)(-d) = 0$$

$abcd = 0$ then yes.

(e) is V a vector space? explain

NO .bcz It is not closed under scalar multiplication

Question: 03

$$A = \begin{bmatrix} a & b \\ 2b & d \end{bmatrix}.$$

(a) V closed under addition?

$$A_1 + A_2 = \begin{bmatrix} a_1 & b_1 \\ 2b_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ 2b_2 & d_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ 2b_1+2b_2 & d_1+d_2 \end{bmatrix} \in V$$

Yes.

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(b) is v closed under scalor multiplication?

$$A = \begin{bmatrix} a & b \\ 2b & d \end{bmatrix}.$$

$$c.A = c\begin{bmatrix} a & b \\ 2b & d \end{bmatrix}$$

$$= \begin{bmatrix} ca & cb \\ 2cb & cd \end{bmatrix} \in V \quad \text{yes.}$$

(c) what is the zero vector in set v?

$$0.A = \begin{bmatrix} 0 & 0 \\ 2(0) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in V$$

yes.

(d) does every matrix A in V have a negative that is in V? explain.

$$A = \begin{bmatrix} a & b \\ 2b & d \end{bmatrix}$$

$$-A = \begin{bmatrix} -a & -b \\ -2b & -d \end{bmatrix} \quad \text{obviously} \in V$$

e) is v a vector space ? explain

$0 \cdot u = 0$ for any $u \in v$

$c \cdot 0 = 0$ for any scalar c

if $c \cdot u = 0$ then $c = 0$ or $u = 0$.

$(-1) \cdot u = -u$ for any vector.

1) $0 \begin{bmatrix} a & b \\ 2b & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in v$

2) $c \cdot \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in v$

3) $c \cdot A = 0$ assume $c \neq 0$ $u \neq 0$

$c \cdot \begin{bmatrix} a & b \\ 2b & d \end{bmatrix} = \begin{bmatrix} c \cdot a & c \cdot b \\ c \cdot 2b & c \cdot d \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence either $c = 0$ or $u = 0$.

4) $(-1) \begin{bmatrix} a & b \\ 2b & d \end{bmatrix} = \begin{bmatrix} -a & -b \\ -2b & -d \end{bmatrix} \in v$

It is vector space.

also check

$A_1 + A_2, kA, \vec{0}, -(A),$

Question: 04

V is set of 2×1

$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ with integer entries

$[v_1 + v_2]$ is even

$\begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} = 5+9=14$ is even.

It is closed under addition.

$k \cdot v = \frac{1}{2}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1+2=3$ not even

It is not closed under multiplication

It is not a vector space.

Question:05

prove in detail that R^n is a vector space?

let $U = (U_1, U_2, \cdots U_n)$, $V = (V_1, V_2, \cdots V_n)$

(i) Clouser property?

$U + V \in R^n$

$U = (U_1, U_2, \cdots U_n)$ and $V (V_1, V_2, \cdots V_n)$

$U + V = (U_1 + V_1, U_2 + V_2, \cdots U_n + V_n) \in R^n$

It is closed under addition

(ii) Communtative property.

$U + V = (U_1 + V_1, U_2 + V_2 \cdots U_n + V_n)$

$V + U = (V_1 + U_1, V_2 + U_2, V_3 + U_3 \cdots V_n + U_n)$

It is communtative under addition.

(iii) Associative property.

$(U + V) + W = U + (V + W)$

$(U + V) + W = [(U_1, U_2, U_3 \cdots U_n) + (V_1, V_2 \cdots V_n)] + [(W_1, W_2 \cdots W_n)] \in R^n$

$= [U_1 + V_1, U_2 + V_2, U_3 + V_3 \cdots U_n + V_n] + [W_1, W_2 \cdots W_n]$

$$\left[(u_1+v_1)+w_1,\ (u_2+v_2)+w_2,\ (u_3+v_3)+w_3 \ldots\ldots (u_n+v_n)+w_n\right]$$

$$u+(v+w) = \left[(u_1, u_2 \ldots u_n)+\left[(v_1, v_2 \ldots v_n)+(w_1, w_2, \ldots w_n)\right]\right]$$

$$= (u_1, u_2 \ldots u_n) + (v_1+w_1, v_2+w_2 \ldots\ldots v_n+w_n)$$

$$= u_1+(v_1+w_1),\ u_2+(v_2+w_2) \ldots\ldots u_n+(v_n+w_n)$$

$$= (u_1+v_1)+w_1,\ (u_2+v_2)+w_2 \ldots (u_n+v_n)w_3$$

Hence proved.

Associative law closed under addition.

(iv) Additive Identity.

$u = (u_1, u_2, u_3 \cdots u_n)$ and $0 = (0, 0, \cdots 0)$

$$0+u = (0, 0, \cdots 0) + (u_1, u_2, u_3 \cdots u_n)$$

$$= (u_1, u_2, \cdots u_n) = u$$

Same

$$u+0 = (u_1, u_2 \cdots u_n) + (0, 0, 0 \cdots 0)$$

$$= (u_1, u_2 \cdots u_n) = u$$

Hence proved.

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(v) Additive inverse

$U = (U_1, U_2, U_3 \dots U_n)$ and $-U = (-U_1, -U_2 \dots -U_n) \in R_n$

$U + (-U) = (U_1, U_2, U_3 \dots U_n) + (-U_1, -U_2 \dots U_n)$

$= (U_1 - U_1, U_2 - U_2, U_3 - U_3 \dots U_n - U_n)$

$= 0$

$U + (-U) = (-U) + U = 0$

(vi) Scalor multiplication

$U = (U_1, U_2, U_3 \dots U_n)$

$KU = K(U_1, U_2, U_3 \dots U_n)$

$= (KU_1, KU_2, KU_3 \dots KU_n) \in R^n$

vii) Distributive law

$K(U+V) = K[(U_1, U_2 \dots U_n) + (V_1, V_2 \dots V_n)]$

$= K[(U_1 + V_1, U_2 + V_2, U_3 + V_3 \dots U_n + V_n)]$

$= K(U_1 + V_1), K(U_2 + V_2), K(U_3 + V_3) \dots K(U_n + V_n)$

$= ku_1 + kv_2,\ ku_2 + kv_2 \cdots\cdots ku_n + kv_n$

$(ku_1, ku_2, ku_3 \cdots ku_n) + (kv_1, kv_2, kv_3 \cdots kv_n)$

$= k(u_1, u_2 \cdots\cdots u_n) + k(v_1, v_2 \cdots\cdots v_n)$

$= ku + kv$

Distributive law.

(viii) ~~kx~~ $(c+d)u$

$(c+d)(u_1, u_2, u_3 \cdots\cdots u_n)$

$(c+d)u_1, (c+d)u_2, (c+d)u_3 \cdots\cdots (c+d)u_n$

$(cu_1 + du_1, cu_2 + du_2 \cdots\cdots cu_n + du_n)$

$(cu_1, cu_2, \cdots cu_n) + (du_1, du_2 \cdots\cdots du_n)$

$c(u_1, u_2, u_3 \cdots u_n) + d(u_1, u_2 \cdots u_n)$

$cu + du$ Proved

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(ix) $c(du)$

$\cancel{u=(}$

$c[d(u_1, u_2, u_3 \cdots u_n)]$

$c[du_1, du_2, du_3 \cdots du_n]$

$[c(du_1), c(du_2), c(du_3) \cdots c(du_n)]$

using associative Property

$[(cd)u_1, (cd)u_2, (cd)u_3 \cdots (cd)u_n]$

$cd[u_1, u_2, u_3 \cdots u_n]$

$cd(u)$

(x) $1 \cdot u$

$1(u_1, u_2, u_3 \cdots u_n)$

$(1u_1, 1u_2, 1u_3 \cdots 1u_n)$

$(u_1, u_2, u_3 \cdots u_n) = u$

10 property hold for vector space

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6) Show that P, the set of all polynomial, is a vector space

practice for power less or equal to 3

$V = P_3(t) = a_3t^3 + a_2t^2 + a_1t + a_0 \in R$

V is the set of polynomial degree less or equal to 3

let $P(t), q(t), r(t) \in V$ then

$P(t) = a_3t^3 + a_2t^2 + a_1t + a_0$

$q(t) = b_3t^3 + b_2t^2 + b_1t + b_0$

$r(t) = c_3t^3 + c_2t^2 + c_1t + c_0$

(i) closure property

$P(t) + q(t) = (a_3t^3 + a_2t^2 + a_1t + a_0) + (b_3t^3 + b_2t^2 + b_1t + b_0)$

$= (a_3t^3 + b_3t^3) + (a_2t^2 + b_2t^2) + (a_1t + b_1t) + (a_0 + b_0)$

$= (a_3 + b_3)t^3 + (a_2 + b_2)t^2 + (a_1 + b_1)t + (a_0 + b_0)$

(ii) Commutative law

$p(t)+q(t) =$ (i) law

$$q(t)+p(t) = (b_3 t^3 + b_2 t^2 + b_1 t + b_0) + (a_3 t^3 + a_2 t^2 + a_1 t + a_0)$$

$$= (b_3+a_3)t^3 + (b_2+a_2)t^2 + (b_1+a_1)t + (b_0+a_0)$$

also $$= (a_3+b_3)t^3 + (a_2+b_2)t^2 + (a_1+b_1)t + (a_0+b_0)$$

$$= p(t)+q(t).$$

iii) Associative law

$$(p(t)+q(t))+r(t)$$

$$\left[(a_3 t^3 + a_2 t^2 + a_1 t + a_0) + (b_3 t^3 + b_2 t^2 + b_1 t + b_0)\right] + [c_3 t^3 + c_2 t^2 + c_1 t + c_0]$$

$$[(a_3+b_3)t^3 + (a_2+b_2)t^2 + (a_1+b_1)t + (a_0+b_0)] + (c_3 t^3 + c_2 t^2 + c_1 t + c_0)$$

$$[(a_3+b_3)+c_3]t^3 + [(a_2+b_2)+c_2]t^2 + [(a_1+b_1)+c_1]t + [(a_0+b_0)+c_0].$$

$$p(t)+[q(t)+r(t)]$$

$$[a_3+(b_3+c_3)]t^3 + [a_2+(b_2+c_2)]t^2 + [a_1+(b_1+c_1)]t + [a_0+(b_0+c_0)]$$

Associative law

(iv) Additive identity

$p(t) = a_3 t^3 + a_2 t^2 + a_1 t^1 + a_0$

$0 = 0t^3 + 0t^2 + 0t + 0.$

$0 + p(t) = 0 + (a_3 t^3 + a_2 t^2 + a_1 t + a_0)$

$= a_3 t^3 + a_2 t^2 + a_1 t + a_0$

(v) Additive inverse

$p(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$

then

$-p(t) = -a_3 t^3 - a_2 t^2 - a_1 t - a_0.$

$p(t) + (-p(t)) = a_3 t^3 - a_3 t^3 + a_2 t^2 - a_2 t^2 + a_1 t - a_1 t + a_0 - a_0.$

$= 0$

(vi) Scalar multiplication

$k \cdot p(t) = k(\cancel{p(t)}\ a_3 t^3 + a_2 t^2 + a_1 t + a_0)$

$= k a_3 t^3 + k a_2 t^2 + k a_1 t + k a_0$

(vi) distributive law

$k(p(t) + q(t))$

$= k\left[(a_3 t^3 + a_2 t^2 + a_1 t + a_0) + (b_3 t^3 + b_2 t^2 + b_1 t$

$= k\left[(a_3+b_3) t^3 + (a_2+b_2) t^2 + (a_1+b_1) t + (a_0 +$

$= k\cdot(a_3+b_3) t^3 + k\cdot(a_2+b_2) t^2 + k\cdot(a_1+b_1)$
$k\cdot(a_0+b_0)$

$= (ka_3+kb_3) t^3 + (ka_2+kb_2) t^2 + (ka_1+kb_1) t +$
$(ka_0 + kb_0)$

$= \left[ka_3 t^3 + ka_2 t^2 + ka_1 t^1 + ka_0\right] + \left[kb_3 t^3 + kb_2 t\right.$
$\left. + kb_1 t + kb_0\right]$

$= k\left[a_3 t^3 + a_2 t^2 + a_1 t + a_0\right] + k\left[b_3 t^3 + b_2 t^2 + b_1 t + b_0\right.$

$= kp(t) + k(q(t))$

(viii) $(c+d)\,p(t)$

$(c+d)\left[a_3 t^3 + a_2 t^2 + a_1 t + a_0\right]$

$= (c+d)a_3 t^3 + (c+d)a_2 t^2 + (c+d)a_1 t + (c+d)a_0$

$= ca_3 t^3 + da_3 t^3 + ca_2 t^2 + da_2 t^2 + ca_1 t + da_1 t$
$+ ca_0 + da_0$

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$[ca_3t^3 + ca_2t^2 + ca_1t + ca_0] + [da_3t^3 + da_2t^2 + da_1t +$
$da_0]$

$c[a_3t^3 + a_2t^2 + a_1t + a_0] + d[a_3t^3 + a_2t^2 + a_1t$
$+ a_0]$

$c\,p(t) + d\,p(t)$

(xi) $c(dp(t))$

$c(d[a_3t^3 + a_2t^2 + a_1t + a_0])$

$= c[da_3t^3 + da_2t^2 + da_1t + da_0]$

$= [cda_3t^3 + cda_2t^2 + cda_1t + cda_0]$

$= [(cd)a_3t^3 + (cd)a_2t^2 + (cd)a_1t + (cd)a_0]$

$= cd[a_3t^3 + a_2t^2 + a_1t + a_0]$

$(cd)\,p(t)$

(x) $1 \cdot p(t)$

$1 \cdot [a_3t^3 + a_2t^2 + a_1t + a_0]$

$[1 \cdot a_3t^3 + 1 \cdot a_2t^2 + 1 \cdot a_1t + 1 \cdot a_0]$

$[a_3t^3 + a_2t^2 + a_1t + a_0]$

$p(t)$

It is a vector space

Question: 07:

The set of all positive real numbers with the operations of ⊕ as a ordinary addition and ⊙ as ordinary multiplication.?

→ Set of all positive real number then property failed is when v vector is multiply with scalar c and let c is -1 then we have -1v which would be negative which is not true.

Question: 08

The set of all order pair of real number with the operation

$$(x, y) \oplus (x', y') = (x + x', y + y')$$

and

$$r \odot (x, y) = (x, ry)$$

Solution

i) $U \oplus V = V \oplus U$

$U = (x, y)$, $V = (x', y')$

$U \oplus V = (x, y) \oplus (x', y')$

$= [(x + x') (y + y')]$

$= [x' + x, y' + y]$

$= V \oplus U$

It is hold.

ii) A $U \oplus (V \oplus W) = (U \oplus V) \oplus W$

$U = (x, y)$, $V = (x', y')$, $W = (x^c, y^c)$

$U \oplus (V \oplus W)$

$(x, y) \oplus ((x', y') \oplus (x^c, y^c))$

$(x, y) \oplus (x' + x^c, y' + y^c)$

$(x + x' + x^c, y + y' + y^c)$

$(x + x', y + y') \oplus (x^c, y^c)$

$(U \oplus V) \oplus W$.

It is hold.

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iii) $0 \oplus v = v \oplus 0 = u$

$u = (x, y) \qquad 0 = (0a, 0b)$

$0 \oplus u = (0a, 0b) \oplus (x, y)$

$= ((0a + x), (0b + y))$

$= (x, y) = u$

It is hold.

iv) $u \oplus (-u) = (-u) \oplus u = 0$

$u = (x, y) \qquad -u = (-x, -y)$

$u \oplus (-u) = (x, y) \oplus (-x, -y)$

$= (x + (-x), y + (-y))$

$= ((-x + x), (-y + y))$

$= -u \oplus u$

$= (0, 0) = 0$

It is hold.

(v) $c \odot (u \oplus v) = c \odot u \oplus c \odot v$

$u = (x, y) \quad v = (x', y')$

$c \odot (u \oplus v)$

$c \odot ((x, y) \oplus (x', y'))$

$c \odot (x + x', y + y')$

$(x + x', cy + cy')$

$(x, cy) \oplus (x', cy')$

$c \odot u \oplus c \odot v$

It is hold.

(vi) $(c+d) \odot u = c \odot u \oplus d \odot u$

$(c+d) \odot u$

$(c+d) \odot (x, y)$

$(x, (c+d) y)$

$(x, cy + dy)$

$c \odot u \oplus d \odot u$

$c \odot (x, y) \oplus d \odot (x, y)$

$(x, cy) \oplus (x, dy)$

$(x + x, cy + dy)$

$(2x, cy + dy)$

It is not hold.

viii) $c \odot (d \odot u) = cd \odot u$

$c \odot (d \odot (x, y))$

$c \odot (x, dy)$

(x, cdy)

$cd \odot u$

It is hold.

v(ii)

$1 \odot u = u$

$1 \odot (x, y)$

$(x, 1y)$

$(x, y) = u$

It is hold

$(c+d) \odot u = c \odot u \oplus d \odot u$ is not hold

Question: 09

The set of all ~~(polynomial)~~ order pair triples of real number with the operation

$(x, y, z) \oplus (x', y', z') = (x+x', y+y', z+z')$

and

$r \odot (x, y, z) = (x, 1, z)$

(in this case we easily understand that only some multiplication property dont hold bcz change occur due to condition).

(i) $c \odot (u \oplus v) = c \odot u \oplus c \odot v$

$u = (x, y, z) \quad v = (x', y', z')$

$c \odot (u \oplus v)$

$c \odot ((x, y, z) \oplus (x', y', z'))$

~~$c \odot (x \oplus x', y \oplus y', z \oplus z)$~~

$c \odot (x + x', y + y', z + z')$

$(x + x', 1, z + z')$

$c \odot u \oplus c \odot v$

$c \odot (x, y, z) \oplus c \odot (x', y', z')$

$(x, 1, z) \oplus (x', 1, z')$

$(x + x', 1 + 1, z + z')$

$(x + x', 2, z + z')$

that's not hold

(ii) $(c + d) \odot u = c \odot u \oplus d \odot u$

$(c + d) \odot u$

$(c + d) \odot (x, y, z)$

$(x, 1, z)$

$c \odot u \oplus d \odot u = c \odot (x, y, z) \oplus d \odot (x, y, z)$

$= (x, 1, z) \oplus (x, 1, z)$

$= (2x, 2, 2z)$

NOT hold

* property 8 hold

(iii) $1 \odot u = u$

$1 \odot (x, y, z)$

$(x, 1, z) \neq (x, y, z)$

not hold

Question 10:-

The set of all 2×1 matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ where $x \leq 0$ with the usuall operations in R^2

(i) $u \oplus v = v \oplus u$

$u = \begin{bmatrix} x \\ y \end{bmatrix} \quad v' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

$u \oplus v = \begin{bmatrix} x \\ y \end{bmatrix} \oplus \begin{bmatrix} x' \\ y' \end{bmatrix}$

$= \begin{bmatrix} x + x' \\ y + y' \end{bmatrix}$

$= \begin{bmatrix} x' + x \\ y' + y \end{bmatrix}$

$= v \oplus u$ it is hold

ii) $u \oplus (v \oplus w) = (u \oplus v) \oplus w$

$u \oplus (v \oplus w)$

$$\begin{bmatrix} x \\ y \end{bmatrix} \oplus \left[\begin{bmatrix} x' \\ y' \end{bmatrix} \oplus \begin{bmatrix} x'' \\ y'' \end{bmatrix}\right]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \oplus \begin{bmatrix} x'+x'' \\ y'+y'' \end{bmatrix}$$

$$\begin{bmatrix} x+x'+x'' \\ y+y'+y'' \end{bmatrix}$$

$$\begin{bmatrix} x+x' \\ y+y' \end{bmatrix} \oplus \begin{bmatrix} x'' \\ y'' \end{bmatrix}$$

$(u \oplus v) \oplus w$

iii) $0 \oplus u = u \oplus 0 = u$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 0+x \\ 0+y \end{bmatrix} = \begin{bmatrix} x+0 \\ y+0 \end{bmatrix} = u \oplus 0 = 0$$

$u \oplus (-u) = (-u) \oplus u = 0$

$u = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad -u = \begin{bmatrix} -x \\ -y \end{bmatrix} \notin V$

bcz $x \leq 0$ then $-x > 0$

and $-u \notin V$ this property

does not hold.

$c \odot (u \oplus v) = c \odot u \oplus c \odot v$

~~it is only~~ ~~fails when~~ ~~...~~

This property is not hold

When $c < 0$ and $v \in V$

so $cv \notin V$

$(c+d) \odot u = c \odot u \oplus d \odot u$

$c \odot (d \odot u) = cd \odot u$

for any $c < 0$ and $cv \notin V$

$1 \odot u = u$

$1 \odot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = u$

hold.

Question: 11

The set of all polynomial order pair of real no. with the operation

$$(x, y) \oplus (x', y') = (x + x', y + y')$$

and

$$r \odot (x, y) = (0, 0)$$

we notice that addition property almost hold but multiplication property ....

$1 \odot u = u$

$u = (x, y)$

$$1 \odot u = 1 \odot (x, y)$$
$$= (0, 0) \neq (x, y)$$

It is not hold property

$$(c+d) \odot u = c \odot u \oplus d \odot u$$
$$(c+d) \odot (x, y) = c \odot (x, y) \oplus d \odot (x, y)$$
$$(0, 0) = (0, 0) \oplus (0, 0)$$

It is hold

but property $1 \odot u = u$ not hold so It is not vector space

12: Let V be the set of all +ve real number define $\oplus$ by

$$u \oplus v = uv$$

$\oplus$ ordinary multiplication.

and define $\odot$ by

$$c \odot v = v^c$$

prove a vector space.

→ $u \oplus v = v \oplus u$

let $u = (x, y)$ $v = (x', y')$

$(x, y) \oplus (x', y') = (x, y)(x', y')$

or

$u \oplus v = uv$ closed under addition

$v \oplus u = vu$

~~closed~~ commutative law hold

→ $(u \oplus v) \oplus w = u \oplus (v \oplus w)$

$(u \oplus v) \oplus w = u \oplus (v \oplus w)$

$(uv) \oplus w = u \oplus (vw)$

$uvw = uvw$

hold

$\rightarrow \quad u \oplus 0 = 0 \oplus u = u$

$$u \oplus 0 = 0 \oplus u$$
$$u(0) = 0(u)$$
$$0 = 0$$

$\rightarrow \quad u \oplus u^{-1} = u \oplus u^{-1}$

$$u \oplus u^{-1} = uu^{-1} = 1$$

$\rightarrow \quad c \odot (u \oplus v) = c \odot u \oplus c \odot v$

$$c \odot (uv) = u^c \oplus v^c$$
$$(uv)^c = u^c v^c$$
$$u^c v^c = u^c v^c$$

hold.

$\rightarrow \quad (c+d) \odot u = c \odot u \oplus d \odot u$

$$(c+d) \odot u = u^c \oplus u^d$$
$$u^{c+d} = u^c u^d$$
$$u^c \cdot u^d = u^c \cdot u^d$$

hold.

$\rightarrow \quad c \odot (d \odot u) = (cd) \odot u$

$$c \odot u^d = u^{cd}$$
$$(u^d)^c = u^{cd}$$

hold.

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$\rightarrow 1 \odot u = u$

$u^1 = u$

hold.

It is a vector space.

Q#16
let V be the set of all +ve real no.
define $\oplus$ by $u \oplus v = uv - 1$ and
$\odot$ by $c \odot v = v$. is V a vector space.

$\rightarrow \quad u \oplus v = v \oplus u$

$u \oplus v = uv - 1$

$v \oplus u = vu - 1$ hold.

$\rightarrow \quad u \oplus (v \oplus w) = (u \oplus v) \oplus w$

$u \oplus (v \oplus w) = u \oplus (vw - 1)$

$= u(vw) - 1$

$(u \oplus v) \oplus w = (uv - 1) \oplus w$

$= (uv)w - 1$

hold.

$\rightarrow \quad c \odot (u \oplus w) = c \odot u \oplus c \odot w$

$c \odot (u \oplus w) = c \odot (uw - 1)$

$= uw - 1$

$c \odot u \oplus c \odot w = c \odot (u) \oplus c \odot w)$

$= u \oplus w$

Not hold / No vector space

17: let V be the set of all real no. define $\oplus$ by $u \oplus v = uv$ and $\odot$ by $c \odot u = c + u$ is V vector space.

→ $u \oplus w = w \oplus u$

$u \oplus w = uw$

$w \oplus u = wu$ hold.

→ $(u \oplus v) \oplus w = u \oplus (v \oplus w)$

$(uv) \oplus w = u \oplus (vw)$

$uvw = uvw$

hold.

→ $c \odot (u \oplus w) = c \odot u \oplus c \odot w$

$c \odot (uw) = (c+u) \oplus (c+w)$

$c + uw \neq (c+u)(c+w)$

Not hold.

Not a vector space.

18: Let V be the set of all real no.

define $\oplus$ by $u \oplus v = 2u - v$

$\odot$ by $c \odot u = cu$

is vector space

→ $u \oplus w = w \oplus u$

$2u - w \neq 2w - u$

NOT hold

→ $(u \oplus v) \oplus w = u \oplus (v \oplus w)$

$(2u - v) \oplus w = u \oplus (2v - w)$

$2(2u - v) - w = 2u - 2v + w$

$4u - 2v - w \neq 2u - 2v + w$

NOT hold.

→ $u \oplus 0 = 0 \oplus u = u$

$2u - 0 = 2(0) - u$

$2u \neq -u$

NOT hold

It is not a vector space.

Exercise: 4.7

Non Homogenous → NOT passed from origin ($Ax = b$)

Homogenous → passed from origin

($Ax = 0$)

Homogenous system for finding a basis and dimension of Solution space.

Step 1 Solve the given homogenous system by Gauss Jordan reduction.

→ if no arbitrary then solution space is 0 which has no dimension and basis.

Step 2: → if Solution x contain arbitrary constant write x as a linear combination of vector $x_1, x_2 \cdots x_p$ with $s_1, s_2 \cdots s_p$ as coefficients

$$x = s_1 x_1 + s_2 x_2 + \cdots + s_p x_p$$

Step 3 → The set of vector $\{x_1, x_2 \cdots x_p\}$ is a basis for a solution space of $Ax = 0$ the dimension of solution space is p.

Question:03

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$2x_1 + x_2 - x_3 + x_4 = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Gauss- Jordan reduction method.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & 1 & 0 \end{array}\right]$$

$R_2 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -3 & -1 & 0 \end{array}\right].$$

$R_1 + R_2$ $$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -1 & -3 & -1 & 0 \end{array}\right].$$

$-R_2$ $$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 \end{array}\right]$$

$$x_1 - 2x_3 = 0.$$

$$x_2 + 3x_3 + x_4 = 0$$

$x_3 = s$, $x_4 = t$

$x_1 = 2s$

$x_2 = -3s - t$

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$$x = \begin{bmatrix} 2s \\ -3s-t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Basis of null space or solution space (w) is

$$\left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(w) dimension = 2.

Q#04

$$\begin{bmatrix} 1 & -1 & 1 & -2 & 1 \\ 3 & -3 & 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & -2 & 1 & 0 \\ 3 & -3 & 2 & 0 & 2 & 0 \end{array}\right]$$

$$R_2 - 3R_1 \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 6 & -1 & 0 \end{array}\right]$$

$$R_1 + R_2 \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 4 & 0 & 0 \\ 0 & 0 & -1 & 6 & -1 & 0 \end{array}\right]$$

$$-R_2 \quad \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & -6 & 1 & 0 \end{array}\right]$$

$$x_1 - x_2 + 4x_4 = 0.$$

$$x_3 - 6x_4 + x_5 = 0.$$

$$x_2 = s \qquad x_4 = t \qquad x_5 = r$$

$$x_1 = +s - 4t$$

$$x_3 = +6t - r$$

we have

$$x = \begin{bmatrix} s-4t \\ s \\ 6t-r \\ t \\ r \end{bmatrix} = s\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -4 \\ 0 \\ 6 \\ 1 \\ 0 \end{bmatrix} + r\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Base of null space

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Dimension = 3

Question: 05

$$x_1+2x_2-x_3+3x_4=0$$
$$2x_1+2x_2-x_3+2x_4=0$$
$$x_1+3x_3+3x_4=0$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 2 & 2 & -1 & 2 & 0 \\ 1 & 0 & 3 & 3 & 0 \end{array}\right]$$

R_2-2R_1
R_3-R_1
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 0 & -2 & 1 & -4 & 0 \\ 0 & -2 & 4 & 0 & 0 \end{array}\right]$$

R_3-R_2
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 0 & -2 & 1 & -4 & 0 \\ 0 & 0 & 3 & 4 & 0 \end{array}\right]$$

$R_2/-2$
$R_3/3$
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 0 & 1 & -1/2 & 2 & 0 \\ 0 & 0 & 1 & 4/3 & 0 \end{array}\right]$$

$-1-2(-1/2)$

R_1-2R_2
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1/2 & 2 & 0 \\ 0 & 0 & 1 & 4/3 & 0 \end{array}\right]$$

$R_2 + \frac{1}{2} R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 8/3 & 0 \\ 0 & 0 & 1 & 4/3 & 0 \end{array}\right].$$

$2 - \frac{1}{2}(4/3)$

$\frac{6-8}{3} =$

$2 + \frac{1}{2}(4/3$

$x_1 - x_4 = 0.$

$x_2 + 8/3\, x_4 = 0$

$x_3 + 4/3\, x_4 = 0.$

$x_4 = s$

$x_1 = s$

$x_2 = -8/3\, s$

$x_3 = -4/3\, s$

$x_4 = s.$

$$x = \begin{bmatrix} s \\ -8/3\, s \\ -4/3\, s \\ s \end{bmatrix} = s \begin{bmatrix} 1 \\ -8/3 \\ -4/3 \\ 1 \end{bmatrix}.$$

Basis of null space

$$\left\{ \begin{bmatrix} 1 \\ -8/3 \\ -4/3 \\ 1 \end{bmatrix} \right\}.$$

dimension 1

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Q# 06.

$$x_1 - x_2 + 2x_3 + 3x_4 + 4x_5 = 0$$
$$-x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 0$$
$$x_1 - x_2 + 3x_3 + 5x_4 + 6x_5 = 0.$$
$$3x_1 - 4x_2 + x_3 + 2x_4 + 3x_5 = 0.$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 2 & 3 & 4 & 0 \\ -1 & 2 & 3 & 4 & 5 & 0 \\ 1 & -1 & 3 & 5 & 6 & 0 \\ 3 & -4 & 1 & 2 & 3 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2+R_1 \\ R_3-R_1 \\ R_4-3R_1 \\ \end{array} \left[\begin{array}{ccccc|c} 1 & -1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 5 & 7 & 9 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \\ 0 & -1 & -5 & -7 & -9 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1+R_2 \\ R_4+R_2 \\ \\ \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 7 & 10 & 13 & 0 \\ 0 & 1 & 5 & 7 & 9 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1-7R_3 \\ R_2-5R_3 \\ \\ \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & -4 & -1 & 0 \\ 0 & 1 & 0 & -3 & -1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 - 4x_4 - x_5 = 0$.

$x_2 - 3x_4 - x_5 = 0$.

$x_3 + 2x_4 + 2x_5 = 0$.

$x_4 = s \qquad x_5 = t$

$x_1 = 4s + t$

$x_2 = +3s + t$

$x_3 = -2s - 2t$

$x_4 = s$

$x_5 = t$

we know

$$x = \begin{bmatrix} 4s+t \\ 3s+t \\ -2s-2t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} 4 \\ 3 \\ -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}.$$

$$\text{Span} = \left\{ \begin{bmatrix} 4 \\ 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Dimension 2.

Q# 07

$$\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 4 & 3 & 3 & 3 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 0 \\ 1 & 2 & 2 & 1 & 2 & 0 \\ 2 & 4 & 3 & 3 & 3 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array}\right]$$

$R_2 - R_1$
$R_3 - 2R_1$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array}\right]$$

$R_4 - R_2$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \end{array}\right]$$

$(-1)(R_4) \leftrightarrow R_3$ $\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$.

$R_3/2$

$R_1 - R_2$ $\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$.

$x_1 + 2x_2 + 3x_4 = 0.$

$x_3 - x_4 + x_5 = 0.$

$x_5 = 0.$

$x_2 = s \quad x_4 = t$

$x_1 = -2s + 3t$

$x_3 = t$

$x_2 = s$

$x_4 = t$

$x_5 = 0.$

$$x = \begin{bmatrix} -2s-3t \\ s \\ t \\ t \\ 0 \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{Base } \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Dimension = 2.

Question:08

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 1 & 2 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right]$$

Scanned with CamScanner

$$R_3 - R_2 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

$$\frac{R_3}{-3} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2 + R_3 \\ R_1 - 2R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right].$$

$x_1 = 0$

$x_2 = 0$ 0 span 0 dimension.

$x_3 = 0$

No span and dimension.

Q# 09

$$\begin{bmatrix} 1 & 2 & 2 & -1 & 1 \\ 0 & 2 & 2 & -2 & -1 \\ 2 & 6 & 2 & -4 & 1 \\ 1 & 4 & 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Scanned with CamScanner

$$\left[\begin{array}{ccccc|c} 1 & 2 & 2 & -1 & 1 & 0 \\ 0 & 2 & 2 & -2 & -1 & 0 \\ 2 & 6 & 2 & -4 & 1 & 0 \\ 1 & 4 & 0 & -3 & 0 & 0 \end{array}\right]$$

$R_3 - 2R_1$
$R_4 - R_1$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 2 & -1 & 1 & 0 \\ 0 & 2 & 2 & -2 & -1 & 0 \\ 0 & 2 & -2 & -2 & -1 & 0 \\ 0 & 2 & -2 & -2 & -1 & 0 \end{array}\right]$$

$R_4 - R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 2 & -1 & 1 & 0 \\ 0 & 2 & 2 & -2 & -1 & 0 \\ 0 & 2 & -2 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 - R_2$
$R_1 - R_2$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 2 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{R_2}{2}, \frac{R_3}{-4}$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & -1 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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$$R_2 - R_3 \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & -1 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_4 + 2x_5 = 0.$

$x_2 - x_4 - 1/2\, x_5 = 0.$

$x_3 = 0.$

$x_4 = s \quad x_5 = t$

$x_1 = -s - 2t$

$x_2 = s + 1/2\, t$

$x_3 = 0$

$x_4 = s$

$x_5 = t$

$$\text{Span} = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1/2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

dimension = 2

Q # 10

$$\begin{bmatrix} 1 & 2 & -3 & -2 & 1 & 3 \\ 1 & 2 & -4 & 3 & 3 & 4 \\ -2 & -4 & 6 & 4 & -3 & 2 \\ 0 & 0 & -1 & 5 & 1 & 9 \\ 1 & 2 & -3 & -2 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccccc|c} 1 & 2 & -3 & -2 & 1 & 3 & 0 \\ 1 & 2 & -4 & 3 & 3 & 4 & 0 \\ -2 & -4 & 6 & 4 & -3 & 2 & 0 \\ 0 & 0 & -1 & 5 & 1 & 9 & 0 \\ 1 & 2 & -3 & -2 & 0 & 7 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2-R_1 \\ R_3+2R_1 \\ R_5-R_1 \end{array} \left[\begin{array}{cccccc|c} 1 & 2 & -3 & -2 & 1 & 3 & 0 \\ 0 & 0 & -1 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 8 & 0 \\ 0 & 0 & -1 & 5 & 1 & 9 & 0 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 \end{array}\right]$$

$$\begin{array}{l} R_4-R_2 \\ R_5-R_3 \end{array} \left[\begin{array}{cccccc|c} 1 & 2 & -3 & -2 & 1 & 3 & 0 \\ 0 & 0 & -1 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 8 & 0 \\ 0 & 0 & 0 & 0 & -1 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 \end{array}\right]$$

$$\begin{array}{l} R_4 - R_3 \\ R_5/-4 \\ -R_2 \end{array} \quad \left[\begin{array}{cccccc|c} 1 & 2 & -3 & -2 & 1 & 3 & 0 \\ 0 & 0 & 1 & -5 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 + 3R_2 \\ R_5 \leftrightarrow R_4 \\ -R_3 \end{array} \quad \left[\begin{array}{cccccc|c} 1 & 2 & 0 & -17 & -5 & 0 & 0 \\ 0 & 0 & 1 & -5 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & +1 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0. & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 + 5R_3 \\ R_2 + 2R_3 \end{array} \quad \left[\begin{array}{cccccc|c} 1 & 2 & 0 & -17 & 0 & -40 & 0 \\ 0 & 0 & 1 & -5 & 0 & -17 & 0 \\ 0 & 0 & 0 & 0 & 1 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 + 40R_4 \\ R_2 + 17R_4 \\ R_3 + 8R_4 \end{array} \quad \left[\begin{array}{cccccc|c} 1 & 2 & 0 & -17 & 0 & 0 & 0 \\ 0 & 0 & 1 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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$x_1 + 2x_2 - 17x_4 = 0$ $\Rightarrow$ $x_1 = -2\tau + 17s$

$x_3 - 5x_4 = 0$ $\qquad$ $x_3 = 5s$

$x_5 = 0$

$x_6 = 0$

$x_4 = s \quad x_2 = \tau$

$$\text{Span} \Rightarrow x = \left\{ \begin{bmatrix} -2\tau + 17s \\ \tau \\ 5s \\ s \\ 0 \\ 0 \end{bmatrix} = \tau \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 17 \\ 0 \\ 5 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$\text{span} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 17 \\ 0 \\ 5 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Dimension = 2

$V=$ [illegible]

non homogenous

$a_1, a_2 \cdots a_n = \mathbb{R}$.

Homogeno

$a_1 = a_2 = a_3 \cdots =$

Exercise 4.4

Question 01

Spanning Set

$S = \left\{ e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

R^3 $\quad T = \left\{ v_1 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \right\}$

$V =$ arbitrary

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

M_{22}

$S = \left\{ e_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, e_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$\rightarrow \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 3 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 4 \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 5 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\rightarrow S = \left\{ M_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}, M_3 = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \right\}$

P_2

All polynomial degree less then or equal to 2.

$\{1, x, x^2\}$ and $\{2, 2x, 2x^2\}$

$S = \{ p_1^{(i)} = i^2, p_2^{(i)} = i, p_3 = 1 \}$

$S = \{ e_1 = 2x^2, e_2 = 2x, e_3 = 2 \}$

Question: 2

$S = \{t^3, t^2, t\}$ $V = P_3$
↓
All polynomial degree less then equal to 3.

$S = \{e_1 = t^3, e_2 = t^2, e_3 = t, e_4 \neq 1\}$

Not spannig set.

(b) $S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ $V = R^2$
↓
two dimensional space vector

$S = \left\{ e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ (1,0) ← x y ↙ (0,1)

there are $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ missing

so not spanning set.

(C) $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ $V = M_{22}$
↓
Matrix

No, M_{22} span 4 Matrix

Not Spanning set.

Question :03

in each part determine whether the given vector $P(t)$ in P_2 belongs to span $\{P_1(t), P_2(t), P_3(t)\}$ where

$P_1(t) = t^2 + 2t + 1$

$P_2(t) = t^2 + 3$

$P_3(t) = t - 1$

a) $P(t) = t^2 + t + 2$

LC $\quad v = a_1 v_1 + a_2 v_2 + a_3 v_3$

in $P(t)$ form

$P(t) = a_1 P_1(t) + a_2 P_2(t) + a_3 P_3(t)$

Find a_1, a_2, a_3

$P(t) = a_1 P_1(t) + a_2 P_2(t) + a_3 P_3(t)$

$\{t^2 + t + 2 = a_1 \{t^2 + 2t + 1) + a_2 (t^2 + 3) + a_3 (t - 1)\}$

$t^2 + t + 2 = (a_1 + a_2) t^2 + (2a_1 + a_3) t + (a_1 + 3a_2 - a_3)$

$$\left.\begin{aligned} a_1 + a_2 &= 1 \\ 2a_1 + a_3 &= 1 \\ a_1 + 3a_2 - a_3 &= 2 \end{aligned}\right\} \rightarrow \text{Non homogeneous system.}$$

Scanned with CamScanner

$\cancel{a+b=1}$

$2a+$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & -1 & 2 \end{array}\right].$$

$R_2 - 2R_1$
$R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & -1 \\ 0 & 2 & -1 & 1 \end{array}\right].$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$a+b=1$

$-2b+c=-1$

to simply this solution set the variable is 0

let $c=0$

$a+b=1$

$-2b=-1$

$\boxed{b=1/2}$

$a+1/2=1$

$a=1-1/2$

$\boxed{a=1/2}$

Scanned with CamScanner

$(P(t) = t^2+t+2 = \frac{1}{2} P1 + \frac{1}{2} P_2$

(b) $P(t) = 2t^2+t+3$

$P(t) = a_1 P_1(t) + a_2 P_2(t) + a_3 P_3(t)$

$2t^2+t+3 = a_1(t^2+2t+1) + a_2(t^2+3) + a_3(t-1)$

$a_1 + a_2 = 2$

$2a_1 + a_3 = 1$

$a_1 + 3a_2 - a_3 = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & -1 & 3 \end{array}\right].$$

$R_2 - 2R_1$

$R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 1 & -3 \\ 0 & 2 & -1 & 1 \end{array}\right].$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

$P(t) \notin$ spans

there is no solution.

(c) $= -t^2 + t - 4$

$P(t) = -t^2 + t - 4$

$-t^2 + t - 4 = a_1(t^2 + 2t + 1) + a_2(t^2 + 3) + a_3(t - 1)$

$a_1 + a_2 = -1$

$2a_1 + a_3 = 1$

$a_1 + 3a_2 - a_3 = -4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & -1 & -4 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 - R_1$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & +3 \\ 0 & 2 & -1 & -3 \end{array}\right]$$

$R_3 + R_2$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$a + b = 1$

$-2a + c = 3$ let: $c = 0$

$-2a = 3$

$\boxed{a = -3/2}$

$b = 1 + 3/2$

$\boxed{b = 1/2}$

d) $p(t) = -2t^2+3t+1$

$-2t^2+3t+1 = a(t^2+2t+1)+b(t^2+3)+c(t-1)$

$a+b=-2$

$2a+c=3$

$a+3b-c=1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 2 & 0 & 1 & 3 \\ 1 & 3 & -1 & 1 \end{array}\right].$$

R_2-2R_1
R_3-R_1
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & -2 & 1 & 7 \\ 0 & 2 & -1 & 3 \end{array}\right].$$

R_3+R_2
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & -2 & 1 & 7 \\ 0 & 0 & 0 & 3 \end{array}\right].$$

No solution.

$p(t) \notin$ spans

Question: 04

In each part determine wheather the given vector A in M_{22} belongs to span $\{A_1, A_2, A_3\}$, where

$$A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

NOTE:
M_{22} needs 4

(a) $A = \begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$

$$A = a_1 A_1 + a_2 A_2 + a_3 A_3$$

$$\begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix} = a_1 \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} + a_2 \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} +$$

$$a_3 \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & -a_1 \\ 0 & 3a_1 \end{bmatrix} + \begin{bmatrix} a_2 & a_2 \\ 0 & 2a_2 \end{bmatrix} + \begin{bmatrix} 2a_3 & 2a_3 \\ -a_3 & a_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1+a_2+2a_3 & -a_1+a_2+2a_3 \\ -a_3 & 3a_1+2a_2+a_3 \end{bmatrix}$$

$$a_1+a_2+2a_3=5$$
$$-a_1+a_2+2a_3=1$$
$$-a_3=-1$$
$$3a_1+2a_2+a_3=9$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ -1 & 1 & 2 & 1 \\ 0 & 0 & -1 & -1 \\ 3 & 2 & 1 & 9 \end{array}\right]$$

R_2+R_1
R_4-3R_1

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & -5 & -6 \end{array}\right]$$

R_2+R_4

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & -5 & -6 \end{array}\right]$$

$$R_4 + R_3 = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -6 & -6 \end{array}\right].$$

$$R_4 \cancel{+} -6R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$$a_1 + a_2 + 2a_3 = 5$$

$$a_2 - a_3 = 0$$

$$\boxed{a_3 = 1}$$

$$\boxed{a_2 = 1}$$

$$a_1 + 1 + 2 = 5$$

$$a_1 + 3 = 5$$

$$\boxed{a_1 = 2}$$

A $\in$ spans

Question: 04

b) $A = \begin{bmatrix} -3 & -1 \\ 3 & 2 \end{bmatrix}$

Solve in Q(1) R.H.S

$$\begin{bmatrix} -3 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} a_1+a_2+2a_3 & -a_1+a_2+2a_3 \\ -a_3 & 3a_1+2a_2+a_3 \end{bmatrix}$$

$a_1+a_2+2a_3=-3$

$-a_1+a_2+2a_3=-1$

$-a_3=3$

$3a_1+2a_2+a_3=2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -3 \\ -1 & 1 & 2 & -1 \\ 0 & 0 & -1 & 3 \\ 3 & 2 & 1 & 2 \end{array}\right]$$

R_2+R_1

R_4-3R_1

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -3 \\ 0 & 2 & 4 & -4 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & -5 & +11 \end{array}\right]$$

$$R_2 + R_4 = \left[\begin{array}{ccc|c} 1 & 1 & 2 & -3 \\ 0 & 1 & -1 & +7 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & 5 & +11 \end{array}\right]$$

$$R_4 + R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & -3 \\ 0 & 1 & -1 & +7 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & -6 & +18 \end{array}\right]$$

$$R_4 - 6R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & -3 \\ 0 & 1 & -1 & +7 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$a_1 + a_2 + 2a_3 = -3$$

$$a_2 - a_3 = 7$$

$$-a_3 = 3$$

$$\boxed{a_3 = -3}$$

$$a_2 + 3 = 7$$

$$\boxed{a_2 = 4}$$

$$a_1 + 4 - 6 = -3$$

$$a_1 - 2 = -3$$

$$a_1 = -3 + 2$$

$$\boxed{a_1 = -1}$$

c) $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}$

$$\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} a_1+a_2+2a_3 & -a_1+a_2+2a_3 \\ -a_3 & 3a_1+2a_2+a_3 \end{bmatrix}$$

$$a_1+a_2+2a_3 = 3$$
$$-a_1+a_2+2a_3 = -2$$
$$-a_3 = +3$$
$$3a_1+2a_2+a_3 = 2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ -1 & 1 & 2 & -2 \\ 0 & 0 & -1 & 3 \\ 3 & 2 & 1 & 2 \end{array}\right]$$

R_2+R_1
R_4-3R_1

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 2 & 4 & +1 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & -5 & -7 \end{array}\right]$$

R_2+R_4

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & 5 & -7 \end{array}\right]$$

$$R_4 + R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & -6 & -13 \end{array}\right].$$

$$R_4 - 6R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & -31 \end{array}\right].$$

$$\begin{array}{r} -13 \\ -18 \\ \hline -31 \end{array}$$

No solution.

d) $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 + 2a_3 & -a_1 + a_2 + 2a_3 \\ -a_3 & 3a_1 + 2a_2 + a_3 \end{bmatrix}.$$

$a_1 + a_2 + 2a_3 = 1$

$-a_1 + a_2 + 2a_3 = 0$

$-a_3 = 2$

$3a_1 + 2a_2 + a_3 = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 2 \\ 3 & 2 & 1 & 1 \end{array}\right]$$

$R_2 + R_1$
$R_4 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & -1 & -5 & -2 \end{array}\right]$$

$R_2 + R_4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & -1 & 5 & -2 \end{array}\right]$$

$R_4 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & -6 & -3 \end{array}\right]$$

$R_4 - 6R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -15 \end{array}\right]$$

There is no solution.

Question : NO:07

$$V = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$ in span

V is arbitrary

a) $V_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}$

$V_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$

$V_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

$V_4 = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 1 & 0 & 1 & 0 & d \end{array}\right].$$

$R_4 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 0 & 0 & 0 & -1 & d-a \end{array}\right]$$

$-R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 0 & 0 & 0 & 1 & a-d \end{array}\right]$$

$\cancel{a+c+d=}$

$x_1 + x_3 + x_4 = a$

$x_2 + x_3 + x_4 = b$

$x_3 + x_4 = c$

$x_4 = a - d$

$x_4 = a - d$

$x_3 = c - a + d$

$x_2 = b - c$

$x_1 = -c + a$

our set spans R_4.

(b) $\begin{bmatrix} 1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$

Span R_4

Dimension of R_4 is 4

we have only 3 vectors

So they dont span R_4.

c) $V_1 = \begin{bmatrix} 6 & 4 & -2 & 4 \end{bmatrix}$ $V_2 = \begin{bmatrix} 2 & 0 & 0 & 1 \end{bmatrix}$

$V_3 = \begin{bmatrix} 3 & 2 & -1 & 2 \end{bmatrix}$ $V_4 = \begin{bmatrix} 5 & 6 & -3 & 2 \end{bmatrix}$

$V_5 = \begin{bmatrix} 0 & 4 & -2 & -1 \end{bmatrix}$

$V = \begin{bmatrix} a, b & c & d \end{bmatrix}$

$$\left[\begin{array}{ccccc|c} 6 & 2 & 3 & 5 & 0 & a \\ 4 & 0 & 2 & 6 & 4 & b \\ -2 & 0 & -1 & -3 & -2 & c \\ 4 & 1 & 2 & 2 & -1 & d \end{array}\right]$$

$$R_1+3R_3 \quad R_2+2R_3 \quad R_4+2R_3 \quad \left[\begin{array}{ccccc|c} 0 & 2 & 0 & -4 & -6 & a+3c \\ 0 & 0 & 0 & 0 & 0 & b+2c \\ -2 & 0 & -1 & -3 & -2 & c \\ 0 & 1 & 0 & -4 & -5 & d+2c \end{array}\right]$$

No solution

No span

d) $v_1\,[1 \;\; 1 \;\; 0 \;\; 0]$; $v_2\,[1 \;\; 2 \;\; -1 \;\; 1]$; $v_3\,[0 \;\; 0 \;\; 1 \;\; 1]$

$v_4 - [2 \;\; 1 \;\; 2 \;\; 1]$ $\quad v = [a \;\; b \;\; c \;\; d]$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 2 & a \\ 1 & 2 & 0 & 1 & b \\ 0 & -1 & 1 & 2 & c \\ 0 & 1 & 1 & 1 & d \end{array}\right]$$

$$R_4+R_3 \quad R_2-R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 2 & a \\ 0 & 1 & 0 & -1 & b-a \\ 0 & -1 & 1 & 2 & c \\ 0 & 0 & 2 & 3 & d+c \end{array}\right]$$

$$R_3+R_2 \quad R_1-R_2 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 3 & a-b+c \\ 0 & 1 & 0 & -1 & b-a \\ 0 & 0 & 1 & 1 & c+b-a \\ 0 & 0 & 2 & 3 & d+c \end{array}\right]$$

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$$R_4 - 2R_3 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 2a-b \\ 0 & 1 & 0 & -1 & b-a \\ 0 & 0 & 1 & 1 & c+b-a \\ 0 & 0 & 0 & 1 & d+c-2(c+b-a) \end{array}\right].$$

$$\begin{array}{l} R_1 - 3R_3 \\ R_2 + R_3 \\ R_3 - R_4 \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2a-b-3(d-c-2b+a) \\ 0 & 1 & 0 & 0 & b-a+d-c-2b+a \\ 0 & 0 & 1 & 0 & c+b-a-(d-c-2b+a) \\ 0 & 0 & 0 & 1 & d-c-2b+a \end{array}\right]$$

$a_1 = 2a-b-3(d-c-2b+a)$

$a_2 = b-a+d-c-2b+a$

$a_3 = c+b-a-(d-c-2b+a)$

$a_4 = d-c-2b+a$

$a_1 = -a+5b+3c-3d$

$a_2 = -b-c+d$

$a_3 = -2a+3b+2c-d$

$a_4 = d-c-2b+a$

Set span R_4.

Question 09

Do the polynomial t^3+2t+1, t^2-t+2, t^3+2, $-t^3+t^2-5t+2$ span P_3?

$$V = \{ e_1(t^3+2t+1) + e_2(t^2-t+2) + e_3(t^3+2) \\ e_4(-t^3+t^2-5t+2) \}$$

$$V = [a \ b \ c \ d]$$

$$at^3+bt^2+ct+d = (e_1+e_3-e_4)t^3 + (e_2+e_1)t^2 \\ + (2e_1-e_2-5e_4)t + (e_1+2e_2+2e_3+2e_4)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & a \\ 0 & 1 & 0 & 1 & b \\ 2 & -1 & 0 & -5 & c \\ 1 & 2 & 2 & 2 & d \end{array}\right]$$

$R_3 - 2R_1$

$R_4 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & a \\ 0 & 1 & 0 & 1 & b \\ 0 & -1 & -2 & -3 & c-2a \\ 0 & 2 & 1 & 3 & d-a \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & a \\ 0 & 1 & 0 & 1 & b \\ 0 & -1 & -2 & -3 & c-2a \\ 0 & 2 & 1 & 3 & d-a \end{array}\right].$$

$R_3 + R_2 \quad R_4 - 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & a \\ 0 & 1 & 0 & 1 & b \\ 0 & 0 & -2 & -2 & c-2a+b \\ 0 & 0 & 1 & 1 & d-a-2b \end{array}\right]$$

$2R_4 + R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & a \\ 0 & 1 & 0 & 1 & b \\ 0 & 0 & -2 & -2 & +c+2a+b \\ 0 & 0 & 0 & 0 & 2d-2a-2b+c \\ & & & & -2a+b \end{array}\right]$$

$-R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & a \\ 0 & 1 & 0 & 1 & b \\ 0 & 0 & 2 & 2 & 2a-c-b \\ 0 & 0 & 0 & 0 & 2d-4a+c-b \end{array}\right]$$

No Solution.

does not span P_3

10: Does the set

$$S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

Span M_{22}?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a_1 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + a_3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + a_4 \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a_1 + a_3 & a_1 + a_4 \\ a_2 + a_4 & a_2 + a_3 + a_4 \end{bmatrix}.$$

$a - a_1 + a_3$

$b = a_1 + a_4$

$c = a_2 + a_4$

$d = a_2 + a_3 + a_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & a \\ 1 & 0 & 0 & 1 & b \\ 0 & 1 & 0 & 1 & c \\ 0 & 1 & 1 & 1 & d \end{array}\right]$$

$R_2 - R_1$ $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & a \\ 0 & 0 & -1 & 1 & -a+b \\ 0 & 1 & 0 & 1 & c \\ 0 & 1 & 1 & 1 & d \end{array}\right]$.

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & a \\ 0 & 1 & 0 & 1 & c \\ 0 & 0 & -1 & 1 & b-a \\ 0 & 1 & 1 & 1 & d \end{array}\right].$$

$R_4 - R_2$ $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & a \\ 0 & 1 & 0 & 1 & c \\ 0 & 0 & -1 & 1 & b-a \\ 0 & 0 & 1 & 0 & d-c \end{array}\right]$

$R + R_3$ $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & a \\ 0 & 1 & 0 & 1 & c \\ 0 & 0 & -1 & 1 & -a+b \\ 0 & 0 & 0 & 1 & -a+b-c+d \end{array}\right]$

$a_1 + a_3 = a$

$a_2 + a_4 = c$

$-a_3 + a_4 = -a + b$

$a_4 = -a + b - c + d$

$-a_3 = -a + b - a_4$

$= -a + b + a - b + c - d = c - d$

$a_3 = d - c$

$x_2 = c - x_4$

$= c + a - b + c - d = a - b + 2c - d$

$x_1 = a - x_3 = a - (-c + d) = a + c - d$

span M_{22}

Exercise 4.5

Linear Independent

let $S = \{v_1, v_2, v_3 \ldots v_k\}$ be the set of vector in vector space V. Then the vector in set S are said to be linearly indepenedent if

$a_1 v_1 + a_2 v_2 \ldots a_k v_k = 0$

$a_1 = a_2 = \ldots = a_k = 0.$

let something not $= 0$

$2v_1 + 4v_2 = 0$

$2v_1 = -4v_2$

$v_1 = -2v_2$

linear dependent.

v_1 is the multiple of v_2 x

x axis not is the multiple of y-axis.

Example.

let subset w of R^3 where $w = \left\{ \begin{bmatrix} a \\ b \\ a+b \end{bmatrix} \right\}$

Consider $\begin{bmatrix} a+0b \\ 0a+b \\ a+b \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

efficent and linear independent

The spanning set of w is $S_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$, Here are some other spanning set of w

$$S_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$$ not L.I → Third are depend on 1st two.

$$S_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} \right\}$$ NOT L.I

$$S_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$ NOT L.I

efficent spanning set is S_1 as it is Smallest and vector are linear independent.

Example : 06

$v_1 = t^2 + t + 2$

$v_2 = 2t^2 + t$

$v_3 = 3t^2 + 2t + 2$

in P_2 LinearD or L.ID

consider $a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$ — $\boxed{\text{2 Marks}}$

0 are vector.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 1 & 2 & 0 \\ 2 & 0 & 2 & 0 \end{array}\right] \rightarrow \text{0 are entries.}$$

$R_2 - R_1$

$R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -4 & -4 & 0 \end{array}\right]$$

$R_3 - 4R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$a_2 + a_3 = 0$, $a_1 + 2a_2 + 3a_3 = 0$.

a_3 is arbitrary

so given set of vectors are (L.D).

Question: 03

Determine whether

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix} \right\}$$

is a linearly independent set in R^4

$$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$$

$$a_1 \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix} + a_2 \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix} + a_3 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix} = 0.$$

$$a_1 + 4a_2 + 2a_3 = 0$$

$$2a_1 + 3a_2 = 0$$

$$a_1 + a_2 + a_3 = 0$$

$$-a_1 + 3a_3 = 0.$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ -1 & 0 & 3 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2-2R_1 \\ R_3-R_1 \\ R_4+R_1 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -5 & -4 & 0 \\ 0 & -3 & -1 & 0 \\ 0 & 4 & 5 & 0 \end{array}\right]$$

$$R_2+R_4 \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -3 & -1 & 0 \\ 0 & 4 & 5 & 0 \end{array}\right]$$

$$-R_2 \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -3 & -1 & 0 \\ 0 & 4 & 5 & 0 \end{array}\right]$$

$$\begin{array}{l} R_3+3R_2 \\ R_4-4R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 9 & 0 \end{array}\right]$$

$$\frac{a_3}{-4} \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 9 & 0 \end{array}\right]$$

$R_4 - 9R_3$
$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$a_1 = a_2 = a_3 = 0$

So It is Linear independent.

Question : 05

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 2 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 \\ 5 & 1 & 8 & 5 & 0 \end{array}\right].$$

is Linear independent.

$R_3 \leftrightarrow R_1$
$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 3 & 2 & 0 \\ 5 & 1 & 8 & 5 & 0 \end{array}\right]$$

$R_2 + R_1$
$R_3 - 2R_1$
$R_4 - 5R_1$
$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 2 & 0 \\ 0 & 3 & -1 & 0 & 0 \\ 0 & 6 & -2 & 0 & 0 \end{array}\right]$$

~~2R3~~

$R_4 - 2R_3$
$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 2 & 0 \\ 0 & 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

$R_3 \leftrightarrow R_2$
$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ 0 & 3 & -1 & 0 & 0 \\ 0 & -1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

$R_2/3$
$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1/3 & 0 & 0 \\ 0 & -1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

system are not Linear independent.

Q #06

- we have found non-trivial Soloution s is not Linearly independent.

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Q#07

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right].$$

$2x_3 = 0$

$x_3 = 0$

$-x_2 = 0$

$x_2 = 0$

$x_1 + 2x_2 + x_3 = 0$

$x_1 = 0$

S is Linear independent.

Q# 10:

Let $x_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 1 \\ 6 \\ 2 \\ 0 \end{bmatrix}$

belong to null space of A. is $\{x_1, x_2, x_3\}$ Linear independent?

$a_1v_1 + a_2v_2 + a_3v_3 = 0.$

$$a_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + a_3 \begin{bmatrix} 1 \\ 6 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{Bmatrix} a_1 + a_2 + a_3 \\ 2a_1 + 6a_3 \\ -a_2 + 2a_3 \\ a_1 + a_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 0 & 6 & 0 \\ 0 & -1 & 2 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right].$$

$R_2 - 2R_1$
$R_4 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right].$$

$2R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right].$$

$R_3 \leftrightarrow R_4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$a_1 = a_2 = a_3 = 0$

Linearly independent.

Question: 11

(a) $[1 \quad 1 \quad 0]$, $[0 \quad 2 \quad 3]$, $[1 \quad 2 \quad 3]$
$[3 \quad 6 \quad 6]$ vector R^3

lineary independent

$$a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = 0.$$

$$a_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} + a_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + a_4 \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$a_1 + a_3 + 3a_4 = 0$$
$$a_1 + 2a_2 + 2a_3 + 6a_4 = 0$$
$$3a_2 + 3a_3 + 6a_4 = 0$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 1 & 2 & 2 & 6 & 0 \\ 0 & 3 & 3 & 6 & 0 \end{array}\right]$$

$$R_2 - R_1 \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 0 & 2 & 1 & 3 & 0 \\ 0 & 3 & 3 & 6 & 0 \end{array}\right]$$

$$R_2 - R_3 \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 0 & -1 & -2 & -3 & 0 \\ 0 & 3 & 3 & 6 & 0 \end{array}\right]$$

$$-R_2 \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & 6 & 0 \end{array}\right].$$

$$R_3 - 3R_2 \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & -3 & 0 \end{array}\right].$$

Linear dependent.

Q11(b)

$$[1 \quad 1 \quad 0], [3 \quad 4 \quad 2]$$

$$a_1 v_1 + a_2 v_2 + \cancel{a_3 v_3} = 0.$$

$$a_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} = 0.$$

$$\begin{bmatrix} a_1 + 3a_2 \\ a_1 + 4a_2 \\ 2a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 1 & 4 & 0 \\ 0 & 2 & 0 \end{array}\right]$$

Linear independent.

c) $[1 \; 1 \; 0], [0 \; 2 \; 3], [1 \; 2 \; 3], [0 \; 0 \; 0]$

$$a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = 0.$$

$$a_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} + a_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + a_4 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a + a_3 \\ a_1 + 2a_2 + 2a_3 \\ 3a_2 + 3a_3 \end{bmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 0 \\ 0 & 3 & 3 & 0 & 0 \end{array}\right]$$

$a_4 \neq 0$

Linearly dependent

Q#12

a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 6 \\ 8 & 6 \end{bmatrix}$

$a_1v_1 + a_2v_2 + a_3v_3 + a_4v_4 = 0.$

$$a_1\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} + a_2\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + a_3\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix} + a_4\begin{bmatrix} 4 & 6 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1+a_2+4a_4 & a_1+3a_3+6a_4 \\ 2a_1+2a_3+8a_4 & a_1+2a_2+a_3+6a_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a_1+a_2+4a_4 = 0$

$a_1+3a_3+6a_4 = 0.$

$2a_1+2a_3+8a_4 = 0.$

$a_1+2a_2+a_3+6a_4 = 0.$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 4 & 0 \\ 1 & 0 & 3 & 6 & 0 \\ 2 & 0 & 2 & 8 & 0 \\ 1 & 2 & 1 & 6 & 0 \end{array}\right]$$

$$R_2 - R_1, \quad R_3 - 2R_1, \quad R_4 - R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 4 & 0 \\ 0 & -1 & 3 & 2 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{array}\right]$$

$$-R_2 \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{array}\right].$$

$$R_3 + 2R_2, \quad R_4 - R_2 \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 0 & -4 & -4 & 0 \\ 0 & 0 & 4 & 4 & 0 \end{array}\right].$$

$$R_4 + R_3 \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 0 & -4 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Linear dependent.

(b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$

$a_1v_1 + a_2v_2 + a_3v_3 = 0.$

$$a_1 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + a_3 \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 + a_2 & a_1 + a_3 \\ a_1 & a_1 + 2a_2 + 2a_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

$a_1 + a_2 = 0$

$a_1 + a_3 = 0.$

$a_1 = 0$

$a_1 + 2a_2 + 2a_3 = 0.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 0 \end{array}\right].$$

$R_3 \leftrightarrow R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 \end{array}\right].$$

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$$\begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right].$$

Linear independent.

c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix},$

$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

$$a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = 0.$$

$$a_1 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + a_3 \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + a_4 \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} a_1 + 2a_2 + 3a_3 + 2a_4 & a_1 + 3a_2 + a_3 + 2a_4 \\ a_1 + a_2 + 2a_3 + a_4 & a_1 + 2a_2 + a_3 + a_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a_1 + 2a_2 + 3a_3 + 2a_4 = 0$

$a_1 + 3a_2 + a_3 + 2a_4 = 0.$

$a_1 + a_2 + 2a_3 + a_4 = 0.$

$a_1 + 2a_2 + a_3 + a_4 = 0.$

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$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 1 & 3 & 1 & 2 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \end{array}\right]$$

$R_2 - R_1$
$R_3 - R_1$
$R_4 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right]$$

$R_3 + R_2$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & -3 & -1 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right]$$

$R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & -3 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

$a_1 = a_2 = a_3 = a_4 = 0$.

Linear independent

Q #13

a) $t^2+1,\ t-2,\ t+3$

$a_1v_1+a_2v_2+a_3v_3=0.$

$a_1[t^2+1]+a_2[t-2]+a_3[t+3]=[0t^2+0t+0]$

$(a_1)t^2+(a_2+a_3)t+(a_1-2a_2+3a_3)=[0t^2+0t+0]$

$a_1=0$

$a_2+a_3=0$

$a_1-2a_2+3a_3=0.$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -2 & 3 & 0 \end{array}\right].$$

$$R_3-R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 3 & 0 \end{array}\right]$$

$$R_3+2R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right].$$

Linear independent.

b) $2t^2+t,\ t^2+3,\ t$

$a_1v_1+a_2v_2+a_3v_3=0.$

$0=a_1(2t^2+t)+a_2(t^2+3)+a_3(t)$

$(2a_1+a_2)t^2+(a_1+a_3)t+3a_2=0t^2+0t+0$

$2a_1+a_2=0$

$a_1+a_3=0$

$3a_2=0.$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right]$$

linearly independent.

(c) $2t^2+t+1,\ 3t^2+t-5,\ t+13$

$a_1v_1+a_2v_2+a_3v_3=0$

$0t^2+0t+0$

$a_1(2t^2+t+1)+a_2(3t^2+t-5)+a_3(t+13)=0$

$(2a_1+3a_2)t^2+(a_1+a_2+a_3)t+(a_1-5a_2+13a_3)=0$

t^2+0t+0

$2a_1+3a_2=0$

$a_1+a_2+a_3=0$

$a_1-5a_2+13a_3=0.$

$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & -5 & 13 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & -5 & 13 & 0 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -6 & 12 & 0 \end{array}\right]$$

$R_3 + 6R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Linearly dependent.

Exercise 4.6.

Basis and dimension

Basis

The vectors $v_1, v_2, v_3 \ldots v_k$ in a vector space V are said to form a basis for V if

$v_1, v_2 \ldots v_k$ Span v and

$v_1, v_2 \ldots v_k$ are linearly independent.

dimension

The dimension of a nonzero vector space v is the number of vector in a basis for v.

example

let $v = R^3$. The vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

form a basis for R^3, called the natural basis or Standard basis for R^3.

Similarly $[1\ 0\ 0], [0\ 1\ 0], [0\ 0\ 1]$ is the natural basis of R_3.

The natural basis of R^n is denoted by $\{e_1, e_2 \dots e_n\}$ where

$$e_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow \text{ith row.}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}, \quad e_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \text{ so} \dots \text{on.}$$

Example: 2

→ The set of vector $\{t^2, t, 1\}$ forms a standard or natural basis for the vector space P_2 (all polynomial of degree less or equal to 2.

∴ Dimension of P_2 is

→ The set of vector $\{t^3, t^2, t, 1\}$ forms a standard or natural basis for vector space P_3 (all polynomial of degree less or equal to 3)

Scanned with CamScanner

∴ Dimension of P_3 is 4.

→ The set of vector $\{t^n, t^{n-1}, \dots t, 1\}$ forms a basis for the vector space P_n called the natural or standard basis for P_n

∴ Dimension of P_n is $n+1$

example: 3

The set of vector $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ forms a standard or natural basis for vector space M_{22}

∴ Dimension of M_{22} is 4.

∴ Dimension of M_{23} is 6.

Q# 15

Finall all values of a

$$\left\{ \begin{bmatrix} a^2 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & a & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \right\}$$

is a basis for R_3

Set of vector L.I →

Matrix must have identity in REF

→ Matrix is invertible → $|A| \neq 0$.

$$\begin{vmatrix} a^2 & 0 & 1 \\ 0 & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

second row

$0 + a(a^2-1) + 0 = 0$

$a(a^2-1) = 0$

$a=0 \quad a=\pm 1$ given vectors

and L.D.

Given set will be L.I for all values of $a \in \mathbb{R}$ other than $0, 1, -1$

if vector are not invertible

No L.I.

Whose matrix are in identity form

in RREF are invertible

Q # 24 Dimension of given sub-space of R4

a) All vector of form $[a \; b \; c \; d]$ where $a=b$

(b) All vector of form

$$[a+c \quad a-b \quad b+c \quad -a+b]$$

R4 have subspace one w.

$$w = \{[a+c \quad a-b \quad b+c \quad -a+b] : a, b, c \in R\}$$

$$[a+c \quad a-b \quad b+c \quad -a+b] = a[1 \quad 1 \quad 0 \quad -1]$$
$$+ b[0 \quad -1 \quad 1 \quad 1] + c[1 \quad 0 \quad 1 \quad 0]$$

cofficent of a, b, c in w.

$v_1 = [1 \quad 1 \quad 0 \quad -1]$; $v_2 = [0 \quad -1 \quad 1 \quad 1]$

$v_3 = [1 \quad 0 \quad 1 \quad 0]$.

$$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$$