



## Engineering Economics Chap 1-3

Engineering Economics (COMSATS University Islamabad)



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# INTRODUCTION

This chapter discusses the elements of economics and the interaction between its various components. This is followed by an analysis of the need and scope of engineering economics. Later, elements of cost and break-even analysis are presented.

## 1.1 ECONOMICS

Economics is the science that deals with the production and consumption of goods and services and the distribution and rendering of these for human welfare.

The following are the economic goals.

- A high level of employment ✓
- Price stability ✓
- Efficiency ✓
- An equitable distribution of income ✓
- Growth ✓

Some of the above goals are interdependent. The economic goals are not always complementary; in many cases they are in conflict. For example, any move to have a significant reduction in unemployment will lead to an increase in inflation.)

### 1.1.1 Flow in an Economy

The flow of goods, services, resources and money payments in a simple economy are shown in Fig. 1.1. Households and businesses are the two major entities in a simple economy. Business organizations use various economic resources like land, labour and capital which are provided by households to produce consumer goods and services which will be used by them. Business organizations make payment of money to the households for receiving various resources. The households in turn make payment of money to business organizations for receiving consumer goods and services. This cycle shows the interdependence between the two major entities in a simple economy.

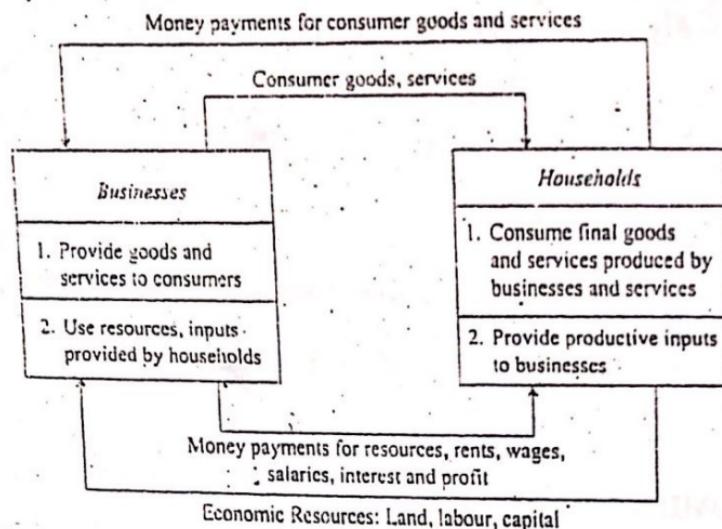


Fig. 1.1 Flow of goods, services, resources and money payments in a simple economy.

### 1.1.2 Law of Supply and Demand

An interesting aspect of the economy is that the demand and supply of a product are interdependent and they are sensitive with respect to the price of that product. The interrelationships between them are shown in Fig. 1.2.

From Fig. 1.2 it is clear that when there is a decrease in the price of a product, the demand for the product increases and its supply decreases. Also, the product is more in demand and hence the demand of the product increases. At the same time, lowering of the price of the product makes the producers restrain from releasing more quantities of the product in the market. Hence, the supply of the product is decreased. The point of intersection of the supply curve and the demand curve is known as the equilibrium point. At the price corresponding to this point, the quantity of supply is equal to the quantity of demand. Hence, this point is called the equilibrium point.

#### Factors influencing demand

The shape of the demand curve is influenced by the following factors:

- Income of the people
- Prices of related goods
- Tastes of consumers

If the income level of the people increases significantly, then their purchasing power will naturally improve. This would definitely shift the demand curve to the north-east direction of Fig. 1.2. A converse situation will shift the demand curve to the south-west direction.

If, for instance, the price of television sets is lowered drastically its demand would naturally go up. As a result, the demand for its associated product,

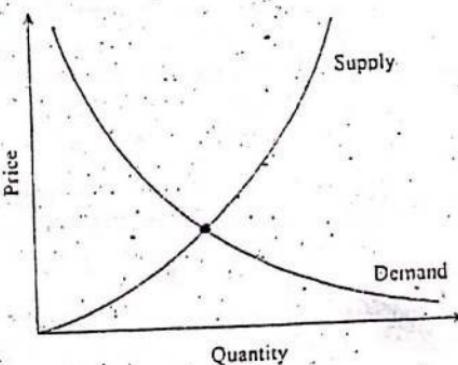


Fig. 1.2 Demand and supply curve.

namely VCDs would also increase. Hence, the prices of related goods influences the demand of a product.

Over a period of time, the preference of the people for a particular product may increase, which in turn, will affect its demand. For instance, diabetic people prefer to have sugar-free products. If the incidence of diabetes rises naturally there will be increased demand for sugar-free products.

#### Factors influencing supply

The shape of the supply curve is affected by the following factors:

- Cost of the inputs
- Technology
- Weather
- Prices of related goods

If the cost of inputs increases, then naturally, the cost of the product will go up. In such a situation, at the prevailing price of the product the profit margin per unit will be less. The producers will then reduce the production quantity, which in turn will affect the supply of the product. For instance, if the prices of fertilizers and cost of labour are increased significantly, in agriculture, the profit margin per bag of paddy will be reduced. So, the farmers will reduce the area of cultivation, and hence the quantity of supply of paddy will be reduced at the prevailing prices of the paddy.

If there is an advancement in technology used in the manufacture of the product in the long run, there will be a reduction in the production cost per unit. This will enable the manufacturer to have a greater profit margin per unit at the prevailing price of the product. Hence, the producer will be tempted to supply more quantity to the market.

Weather also has a direct bearing on the supply of products. For example, demand for woollen products will increase during winter. This means the prices of woollen goods will be increased in winter. So, naturally, manufacturers will supply more volume of woollen goods during winter.

Again, take the case of television sets. If the price of TV sets is lowered significantly, then its demand would naturally go up. As a result, the demand for associated products like VCDs would also go up. Over a period of time, this will lead to an increase in the price of VCDs, which would result in more supply of VCDs.

## 1.2 CONCEPT OF ENGINEERING ECONOMICS

Science is a field of study where the basic principles of different physical systems are formulated and tested. Engineering is the application of science. It establishes varied application systems based on different scientific principles.

From the discussions in the previous section, it is clear that price has a major role in deciding the demand and supply of a product. Hence, from the organization's point of view, efficient and effective functioning of the organization would certainly help it to provide goods/services at a lower cost which in turn will enable it to fix a lower price for its goods or services.

The following section discusses the different types of efficiency and their impact on the operation of businesses and the definition and scope of engineering economics.

### 1.2.1 Types of Efficiency

Efficiency of a system is generally defined as the ratio of its output to input. The efficiency can be classified into *technical efficiency* and *economic efficiency*.

#### Technical efficiency

It is the ratio of the output to input of a physical system. The physical system may be a diesel engine, a machine working in a shop floor, a furnace, etc.

$$\text{Technical efficiency (\%)} = \frac{\text{Output}}{\text{Input}} \times 100$$

The technical efficiency of a diesel engine is as follows:

$$\text{Technical efficiency (\%)} = \frac{\text{Heat equivalent of mechanical energy produced}}{\text{Heat equivalent of fuel used}} \times 100$$

In practice, technical efficiency can never be more than 100%. This is mainly due to frictional loss and incomplete combustion of fuel, which are considered to be unavoidable phenomena in the working of a diesel engine.

#### Economic efficiency

Economic efficiency is the ratio of output to input of a business system.

$$\text{Economic efficiency (\%)} = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{\text{Worth}}{\text{Cost}} \times 100$$

'Worth' is the annual revenue generated by way of operating the business and 'cost' is the total annual expenses incurred in carrying out the business. For the survival and growth of any business, the economic efficiency should be more than 100%.

Economic efficiency is also called 'productivity'. There are several ways of improving productivity.

- Increased output for the same input
- Decreased input for the same output
- By a proportionate increase in the output which is more than the proportionate increase in the input
- By a proportionate decrease in the input which is more than the proportionate decrease in the output
- Through simultaneous increase in the output with decrease in the input.

Increased output for the same input. In this strategy, the output is increased while keeping the input constant. Let us assume that in a steel plant, the layout of the existing facilities is not proper. By slightly altering the location of the billet-making section, and bringing it closer to the furnace which produces hot metal, the scale formation at the top of ladles will be considerably reduced. The molten metal is usually carried in ladles to the billet-making section. In the long run, this would give more yield in terms of tonnes of billet produced. In this exercise, there is no extra cost involved. The only task is the relocation of the billet-making facility which involves an insignificant cost.

Decreased input for the same output. In this strategy, the input is decreased to produce the same output. Let us assume that there exists a substitute raw material to manufacture a product and it is available at a lower price. If we can identify such a material and use it for manufacturing the product, then certainly it will reduce the input. In this exercise, the job of the purchase department is to identify an alternate substitute material. The process of identification does not involve any extra cost. So, the productivity ratio will increase because of the decreased input by way of using cheaper raw materials to produce the same output.

Less proportionate increase in output is more than that of the input. Consider the example of introducing a new product into the existing product mix of an organization. Let us assume that the existing facilities are not fully utilized and the R&D wing of the company has identified a new product which has a very good market and which can be manufactured with the surplus facilities of the organization. If the new product is taken up for production, it will lead to—

- an increase in the revenue of the organization by way of selling the new product in addition to the existing product mix and

- an increase in the material cost and operation and maintenance cost of machineries because of producing the new product.

If we examine these two increases closely, the proportionate increase in the revenue will be more than the proportionate increase in the input cost. Hence, there will be a net increase in the productivity ratio.

When proportionate decrease in input is more than that of the output. Let us consider the converse of the previous example, i.e. dropping an uneconomical product from the existing product mix. This will result in the following:

- A decrease in the revenue of the organization
- A decrease in the material cost, and operation and maintenance cost of machinery

If we closely examine these two decreases, we will see that the proportionate decrease in the input cost will be more than the proportionate decrease in the revenue. Hence, there will be a net increase in the productivity ratio.

Simultaneous increase in output and decrease in input. Let us assume that there are advanced automated technologies like robots and automated guided vehicle system (AGVS), available in the market which can be employed in the organization we are interested in. If we employ these modern tools, then:

- There will be a drastic reduction in the operation cost. Initially, the cost on equipment would be very high. But, in the long run, the reduction in the operation cost would break-even the high initial investment and offer more savings on the input.
- These advanced facilities would help in producing more products because they do not experience fatigue. The increased production will yield more revenue.

In this example, in the long run, there is an increase in the revenue and a decrease in the input. Hence, the productivity ratio will increase at a faster rate.

### 1.2.2 Definition and Scope of Engineering Economics

As stated earlier, efficient functioning of any business organization would enable it to provide goods/services at a lower price. In the process of managing organizations, the managers at different levels should take appropriate economic decisions which will help in minimizing investment, operating and maintenance expenditures besides increasing the revenue, savings and other related gains of the organization.

#### Definition

Engineering economics deals with the methods that enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

**Scope**

The issues that are covered in this book are elementary economic analysis, interest formulae, bases for comparing alternatives, present worth method, future worth method, annual equivalent method, rate of return method, replacement analysis, depreciation, evaluation of public alternatives, inflation adjusted investment decisions, make or buy decisions, inventory control, project management, value engineering, and linear programming.

### 1.3 ELEMENTS OF COSTS

Cost can be broadly classified into *variable cost* and *overhead cost*. Variable cost varies with the volume of production while overhead cost is fixed, irrespective of the production volume.

Variable cost can be further classified into direct material cost, direct labour cost, and direct expenses. The overhead cost can be classified into factory overhead, administration overhead, selling overhead, and distribution overhead!

Direct material costs are those costs of materials that are used to produce the product. Direct labour cost is the amount of wages paid to the direct labour involved in the production activities. Direct expenses are those expenses that vary in relation to the production volume, other than the direct material costs and direct labour costs.

Overhead cost is the aggregate of indirect material costs, indirect labour costs and indirect expenses. Administration overhead includes all the costs that are incurred in administering the business. Selling overhead is the total expense that is incurred in the promotional activities and the expenses relating to sales force. Distribution overhead is the total cost of shipping the items from the factory site to the customer sites.

The selling price of a product is derived as shown below:

- Direct material costs + Direct labour costs + Direct expenses = Prime cost
- Prime cost + Factory overhead = Factory cost
- Factory cost + Office and administrative overhead = Costs of production
- Cost of production + Opening finished stock - Closing finished stock = Cost of goods sold.
- Cost of goods sold + Selling and distribution overhead = Cost of sales
- Cost of sales + Profit = Sales
- Sales/Quantity sold = Selling price per unit

In the above calculations, if the opening finished stock is equal to the closing finished stock, then the cost of production is equal to the cost of goods sold.

## 1.4 OTHER COSTS/REVENUES

The following are the costs/revenues other than the costs which are presented in the previous section:

- Marginal cost
- Marginal revenue
- Sunk cost
- Opportunity cost

### 1.4.1 Marginal Cost

Marginal cost of a product is the cost of producing an additional unit of that product. Let the cost of producing 20 units of a product be Rs. 10,000, and the cost of producing 21 units of the same product be Rs. 10,045. Then the marginal cost of producing the 21st unit is Rs. 45.

### 1.4.2 Marginal Revenue

Marginal revenue of a product is the incremental revenue of selling an additional unit of that product. Let, the revenue of selling 20 units of a product be Rs. 15,000 and the revenue of selling 21 units of the same product be Rs. 15,085. Then, the marginal revenue of selling the 21st unit is Rs. 85.

### 1.4.3 Sunk Cost

This is known as the past cost of an equipment/asset. Let us assume that an equipment has been purchased for Rs. 1,00,000 about three years back. If it is considered for replacement, then its present value is not Rs. 1,00,000. Instead, its present market value should be taken as the present value of the equipment for further analysis. So, the purchase value of the equipment in the past is known as its sunk cost. The sunk cost should not be considered for any analysis done from nowonwards.

### 1.4.4 Opportunity Cost

In practice, if an alternative ( $X$ ) is selected from a set of competing alternatives ( $X, Y$ ), then the corresponding investment in the selected alternative is not available for any other purpose. If the same money is invested in some other alternative ( $Y$ ), it may fetch some return. Since the money is invested in the selected alternative ( $X$ ), one has to forego the return from the other alternative ( $Y$ ). The amount that is foregone by not investing in the other alternative ( $Y$ ) is known as the opportunity cost of the selected alternative ( $X$ ). So the opportunity cost of an alternative is the return that will be foregone by not investing the same money in another alternative.

Consider that a person has invested a sum of Rs. 50,000 in shares. Let the expected annual return by this alternative be Rs. 7,500. If the same amount is

invested in a fixed deposit, a bank will pay a return of 18%. Then, the corresponding total return per year for the investment in the bank is Rs. 9,000. This return is greater than the return from shares. The foregone excess return of Rs. 1,500 by way of not investing in the bank is the opportunity cost of investing in shares.

### 1.5 BREAK-EVEN ANALYSIS

The main objective of break-even analysis is to find the cut-off production volume from where a firm will make profit. Let

$s$  = selling price per unit

$v$  = variable cost per unit

$FC$  = fixed cost per period

$Q$  = volume of production

The total sales revenue ( $S$ ) of the firm is given by the following formula:

$$S = s \times Q$$

The total cost of the firm for a given production volume is given as

$$TC = \text{Total variable cost} + \text{Fixed cost}$$

$$= v \times Q + FC$$

The linear plots of the above two equations are shown in Fig. 1.3. The intersection point of the total sales revenue line and the total cost line is called

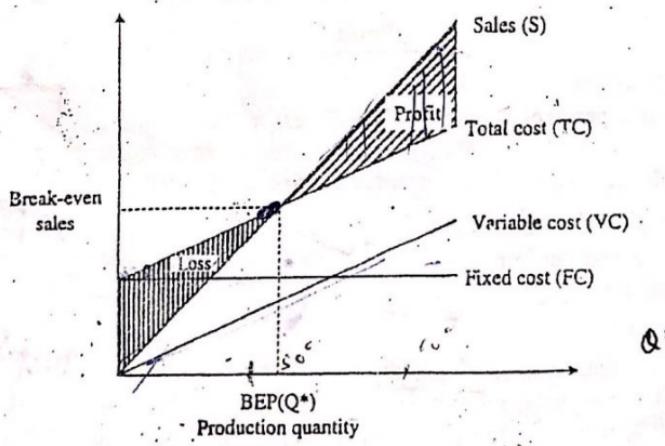


Fig. 1.3 Break-even chart.

the break-even point. The corresponding volume of production on the  $X$ -axis is known as the break-even sales quantity. At the intersection point, the total cost is equal to the total revenue. This point is also called the no-loss or no-gain situation. For any production quantity which is less than the break-even quantity, the total cost is more than the total revenue. Hence, the firm will be making loss.

For any production quantity which is more than the break-even quantity, the total revenue will be more than the total cost. Hence, the firm will be making profit.

$$\begin{aligned} \text{Profit} &= \text{Sales} - (\text{Fixed cost} + \text{Variable costs}) \\ &= s \times Q - (FC + v \times Q) \end{aligned}$$

The formulae to find the break-even quantity and break-even sales quantity

$$\text{Break-even quantity} = \frac{\text{Fixed cost}}{\text{Selling price/unit} - \text{Variable cost/unit}}$$

(Q)

$$Q = \frac{FC}{s - v} \text{ (in units)}$$

$$\begin{aligned} \text{Break-even sales} &= \left[ \frac{\text{Fixed cost}}{\text{Selling price/unit} - \text{Variable cost/unit}} \right] \times \text{Selling price/unit} \\ &= \frac{FC}{s - v} \times s \text{ (Rs.)} \end{aligned}$$

The contribution is the difference between the sales and the variable costs. The margin of safety (M.S.) is the sales over and above the break-even sales. The formulae to compute these values are

$$\text{Contribution} = \text{Sales} - \text{Variable costs}$$

$$\text{Contribution/unit} = \text{Selling price/unit} - \text{Variable cost/unit}$$

$$\text{Margin of Safety} \quad M.S. = \text{Actual sales} - \text{Break-even sales}$$

$$= \frac{\text{Profit}}{\text{Contribution}} \times \text{sales}$$

$$\text{M.S. as a per cent of sales} = (M.S./\text{Sales}) \times 100$$

**EXAMPLE 1.1** Alpha Associates has the following details:

$$\checkmark \text{ Fixed cost} = \text{Rs. } 20,00,000$$

$$\checkmark \text{ Variable cost per unit} = \text{Rs. } 100$$

$$\checkmark \text{ Selling price per unit} = \text{Rs. } 200$$

Find

(a) The break-even sales quantity,

(b) The break-even sales

(c) If the actual production quantity is 60,000, find (i) contribution; and

(ii) margin of safety by all methods.

**Solution**

$$\checkmark \text{ Fixed cost (FC)} = \text{Rs. } 20,00,000$$

$$\checkmark \text{ Variable cost per unit (v)} = \text{Rs. } 100$$

$$\checkmark \text{ Selling price per unit (s)} = \text{Rs. } 200$$

$$(a) \text{ Break-even quantity} = \frac{FC}{s - v} = \frac{20,00,000}{200 - 100} \\ = 20,00,000 / 100 = 20,000 \text{ units}$$

$$(b) \text{ Break-even sales} = \frac{FC}{s - v} \times s (\text{Rs.}) \quad \begin{matrix} VP \\ \diagdown Q \end{matrix} \\ = \frac{20,00,000}{200 - 100} \times 200 \\ = \frac{20,00,000}{100} \times 200 = \text{Rs. } 40,00,000$$

$$(c) \text{ (i) Contribution} = \text{Sales} - \text{Variable cost} \\ = s \times Q - v \times Q \\ = 200 \times 60,000 - 100 \times 60,000 \\ = 1,20,00,000 - 60,00,000 \\ = \text{Rs. } 60,00,000 \quad \checkmark$$

(ii) Margin of safety

METHOD I  $\rightarrow (Q \times P)$

$$\text{M.S.} = (\text{Sales}) - \text{Break-even sales} \quad 1200 \\ = 50,000 \times 200 - 40,00,000 \\ = 1,20,00,000 - 40,00,000 = \text{Rs. } 80,00,000$$

METHOD II

$$\text{M.S.} = \frac{\text{Profit}}{\text{Contribution}} \times \text{Sales} \quad \begin{matrix} \text{Profit} \\ \text{Contribution} \end{matrix}$$

$$\text{Profit} = \text{Sales} - (FC + v \times Q) \\ = 60,000 \times 200 - (20,00,000 + 100 \times 60,000) \\ = 1,20,00,000 - 80,00,000 \\ = \text{Rs. } 40,00,000 \quad \checkmark$$

$$\text{M.S.} = \frac{40,00,000}{60,00,000} \times 1,20,00,000 = \text{Rs. } 80,00,000$$

$$\text{M.S. as a per cent of sales} = \frac{80,00,000}{1,20,00,000} \times 100 = 67\%$$

## 1.6 PROFIT/VOLUME RATIO (P/V RATIO)

P/V ratio is a valid ratio which is useful for further analysis. The different formulae for the P/V ratio are as follows:

$$P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}} = \frac{\text{Sales} - \text{Variable costs}}{\text{Sales}}$$

The relationship between BEP and P/V ratio is as follows:

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}}$$

The following formula helps us find the M.S. using the P/V ratio:

$$\text{M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}}$$

**EXAMPLE 1.2** Consider the following data of a company for the year 1997:

Sales = Rs. 1,20,000.

Fixed cost = Rs. 25,000

Variable cost = Rs. 45,000

Find the following:

(a) Contribution

$S - V.C$

(b) Profit

(c) BEP

(d) M.S.

**Solution**

$$(a) \text{Contribution} = \text{Sales} - \text{Variable costs}$$

$$= \text{Rs. } 1,20,000 - \text{Rs. } 45,000$$

$$= \text{Rs. } 75,000$$

$$(b) \text{Profit} = \text{Contribution} - \text{Fixed cost}$$

$$= \text{Rs. } 75,000 - \text{Rs. } 25,000$$

$$= \text{Rs. } 50,000$$

(c) BEP

$$P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}}$$

$$= \frac{75,000}{1,20,000} \times 100 = 62.50\%$$

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}} = \frac{25,000}{62.50} \times 100 = \text{Rs. } 40,000$$

(C)  $\text{M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}} = \frac{50,000}{62.50} \times 100 = \text{Rs. } 80,000$

**EXAMPLE 1.3** Consider the following data of a company for the year 1998:

Sales = Rs. 80,000

Fixed cost = Rs. 15,000

Variable cost = 35,000

Find the following:

- (a) Contribution ✓
- (b) Profit ✓
- (c) BEP ✓
- (d) M.S.

**Solution**

Per x Sale  
Cost

$$\begin{aligned}\text{(a) Contribution} &= \text{Sales} - \text{Variable costs} \\ &= \text{Rs. } 80,000 - \text{Rs. } 35,000 \\ &= \text{Rs. } 45,000\end{aligned}$$

$$\begin{aligned}\text{(b) Profit} &= \text{Contribution} - \text{Fixed cost} \\ &= \text{Rs. } 45,000 - \text{Rs. } 15,000 \\ &= \text{Rs. } 30,000\end{aligned}$$

F.C  
56.25

(c) BEP

$$\text{P/V ratio} = \frac{\text{Contribution}}{\text{Sales}} = \frac{45,000}{80,000} \times 100 = 56.25\%$$

$$\text{BEP} = \frac{\text{Fixed cost}}{\text{P/V ratio}} = \frac{15,000}{56.25} \times 100 = \text{Rs. } 26,667$$

$$\text{(d) M.S.} = \frac{\text{Profit}}{\text{P/V ratio}} = \frac{30,000}{56.25} \times 100 = \text{Rs. } 53,333.33$$

## QUESTIONS

- Define economics. Also discuss the flow of goods, services, resources and money payments in a simple economy with the help of a suitable diagram.
- Illustrate the effect of price on demand and supply; illustrate with the help of a diagram.
- Discuss the factors which influence demand and supply.
- Distinguish between technical efficiency and economic efficiency by giving examples.

5. What are the ways by which the economic efficiency can be improved?
6. Give the definition and scope of engineering economics.
7. Clearly explain the method of deriving the selling price of a product.
8. Define the following costs with examples:
  - (a) Sunk cost
  - (b) Opportunity cost
  - (c) Marginal cost
  - (d) Marginal revenue.
9. Define break-even point. Draw a break-even chart and explain its components.
10. Krishna Company Ltd. has the following details:

Fixed cost = Rs. 40,00,000

Variable cost per unit = Rs. 300

Selling price per unit = Rs. 500

Find

- (a) The break-even sales quantity
- (b) The break-even sales
- (c) If the actual production quantity is 1,20,000, find the following:
  - (i) Contribution
  - (ii) Margin of safety by all methods

11. Consider the following data of a company for the year 1998.

Sales = Rs. 2,40,000

Fixed cost = Rs. 50,000

Variable cost = Rs. 75,000

Find the following:

- (a) Contribution
- (b) Profit
- (c) BEP
- (d) Margin of safety



# 2

## PREVENTATIVE ECONOMIC ANALYSIS

### 2.1. INTRODUCTION

Whether it is a business situation or a day-to-day event in somebody's personal life, there are a large number of economic decision making involved. One can manage many of these decision problems by using simple economic analysis. For example, an industry can source its raw materials from a nearby place or from a far-off place. In this problem, the following factors will affect the decision:

- Price of the raw material
- Transportation cost of the raw material
- Availability of the raw material
- Quality of the raw material

Consider the alternative of sourcing raw materials from a nearby place with the following characteristics:

- The raw material is more costly in the nearby area.
- The availability of the raw material is not sufficient enough to support the operation of the industry throughout the year.
- The raw material requires pre-processing before it is used in the production process. This would certainly add cost to the product.
- The cost of transportation is minimal under this alternative.

On the other hand, consider another alternative of sourcing the raw materials from a far-off place with the following characteristics:

- The raw material is less costly at the far off place.
- The cost of transportation is very high.
- The availability of the raw material at this site is abundant and it can support the plant throughout the year.
- The raw material from this site does not require any pre-processing before using it for production.

Under such a situation, the procurement of the raw material should be decided in such a way that the overall cost is minimized.

The above example clearly highlights the various components of cost that are involved in each of the alternatives of the decision-making process as well as a method of taking a suitable decision.

## 2.2 EXAMPLES FOR SIMPLE ECONOMIC ANALYSIS

In this section, the concept of simple economic analysis is illustrated using suitable examples in the following areas:

- Material selection for a product
- Design selection for a product
- Design selection for a process industry
- Building material selection for construction activities
- Process planning/Process modification

### 2.2.1 Material Selection for a Product/Substitution of Raw Material

The cost of a product can be reduced greatly by substitution of the raw materials. Among various elements of cost, raw material cost is most significant and it forms a major portion of the total cost of any product. So, any attempt to find a suitable raw material will bring a reduction in the total cost in any one or combinations of the following ways:

- Cheaper raw material price
- Reduced machining/process time
- Enhanced durability of the product

Therefore, the process of raw material selection/substitution will result in finding an alternate raw material which will provide the necessary functions that are provided by the raw material that is presently used. In this process, if the new raw material provides any additional benefit, then it should be treated as its welcoming feature. This concept is demonstrated with two numerical problems.

**EXAMPLE 2.1** In the design of a jet engine part, the designer has a choice of specifying either an aluminium alloy casting or a steel casting. Either material will provide equal service, but the aluminium casting will weigh 1.2 kg as compared with 1.35 kg for the steel casting.

The aluminium can be cast for Rs. 80.00 per kg. and the steel one for Rs. 35.00 per kg. The cost of machining per unit is Rs. 150.00 for aluminium and Rs. 170.00 for steel. Every kilogram of excess weight is associated with a penalty of Rs. 1,300 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?

**Solution** (a) Cost of using aluminium metal for the jet engine part:

Weight of aluminium casting/unit = 1.2 kg

Cost of making aluminium casting = Rs. 80.00 per kg

Cost of machining aluminium casting per unit = Rs. 150.00

Total cost of jet engine part made of aluminium/unit

= Cost of making aluminium casting/unit

+ Cost of machining aluminium casting/unit

$$= 80 \times 1.2 + 150 = 96 + 150$$

$$= \text{Rs. } 246$$

(b) Cost of jet engine part made of steel/unit:

Weight of steel casting/unit = 1.35 kg

Cost of making steel casting = Rs. 35.00 per kg

Cost of machining steel casting per unit = Rs. 170.00

Penalty of excess weight of steel casting = Rs. 1,300 per kg

Total cost of jet engine part made of steel/unit

= Cost of making steel casting/unit

+ Cost of machining steel casting/unit

+ Penalty for excess weight of steel casting

$$= 35 \times 1.35 + 170 + 1,300(1.35 - 1.2)$$

$$= \text{Rs. } 412.25$$

**DECISION** The total cost/unit of a jet engine part made of aluminium is less than that for an engine made of steel. Hence, aluminium is suggested for making the jet engine part. The economic advantage of using aluminium over steel/unit is  $\text{Rs. } 412.25 - \text{Rs. } 246 = \text{Rs. } 166.25$

**EXAMPLE 2.2** A company manufactures dining tables which mainly consist of a wooden frame and a table top. The different materials used to manufacture the tables and their costs are given in Table 2.1.

Table 2.1 Data for Example 2.2

Description of item	Quantity	Cost
Wood for frame and legs	0.1 m <sup>3</sup>	Rs. 12,000/m <sup>3</sup>
Table top with sunmica finish	1	Rs. 3,000
Leg bushes	4	Rs. 10/bush
Nails	100 g	Rs. 300/kg
Total labour	15 hr.	Rs. 50/hr

In view of the growing awareness towards deforestation and environmental conservation, the company feels that the use of wood should be minimal. The wooden top therefore could be replaced with a granite top. This would require additional wood for the frame and legs to take the extra weight of the granite top. The materials and labour requirements along with cost details to manufacture a table with granite top are given in Table 2.2.

Table 2.2 Data for Example 2.2

Description of item	Quantity	Cost
Wood for frame and legs	0.15 m <sup>3</sup>	Rs. 12,000/m <sup>3</sup>
Granite table top	1.62 m <sup>2</sup>	Rs. 800/m <sup>2</sup>
Leg bushes	4	Rs. 25/bush
Nails	50 g	Rs. 300/kg
Total labour	8 hr	Rs. 50/hr

If the cost of the dining table with a granite top works out to be lesser than that of the table with wooden top, the company is willing to manufacture dining tables with granite tops. Compute the cost of manufacture of the table under each of the alternatives described above and suggest the best alternative. Also, find the economic advantage of the best alternative.

#### Solution (a) Cost of table with wooden top

Cost of wood for frame and legs	= 12,000 × 0.1	= Rs. 1,200
Cost of wooden top		= Rs. 3,000
Cost of bushes	= 10 × 4	= Rs. 40
Cost of nails	= 300 × (100/1,000)	= Rs. 30
Cost of labour	= 50 × 15	= Rs. 750
	Total	= Rs. 5,020

#### (b) Cost of table with granite top

Cost of wood for frame and legs	= 12,000 × 0.15	= Rs. 1,800
Cost of granite top	= 800 × 1.62	= Rs. 1,296
Cost of bushes	= 25 × 4	= Rs. 100
Cost of nails	= 300 × (50/1,000)	= Rs. 15
Cost of labour	= 50 × 8	= Rs. 400
	Total	= Rs. 3,611

The cost of a table with granite top works out to be less than that of a table with a wooden top. Hence, the table with granite top should be selected by the manufacturer.

#### (c) Economic advantage

Cost of a table with wooden top	= Rs. 5,020
Cost of a table with granite top	= Rs. 3,611
Economic advantage of table with granite top	= Rs. 1,409

#### 2.2.2 Design Selection for a Product

The design modification of a product may result in reduce raw material

requirements, increased machinability of the materials and reduced labour. Design is an important factor which decides the cost of the product for a specified level of performance of that product.

The elementary economic analysis applied to the selection of design for a product is illustrated with two example problems.

**EXAMPLE 2.3** Two alternatives are under consideration for a tapered fastening pin. Either design will serve the purpose and will involve the same material and manufacturing cost except for the lathe and grinder operations.

Design A will require 16 hours of lathe time and 4.5 hours of grinder time per 1,000 units. Design B will require 7 hours of lathe time and 12 hours of grinder time per 1,000 units. The operating cost of the lathe including labour is Rs. 200 per hour. The operating cost of the grinder including labour is Rs. 150 per hour. Which design should be adopted if 1,00,000 units are required per year and what is the economic advantage of the best alternative?

### Solution

Operating cost of lathe including labour = Rs. 200 per hr

Operating cost of grinder including labour = Rs. 150 per hr

#### (a) Cost of design A

No. of hours of lathe time per 1,000 units = 16 hr

No. of hours of grinder time per 1,000 units = 4.5 hr

Total cost of design A/1,000 units

$$\begin{aligned} &= \text{Cost of lathe operation per 1,000 units} \\ &\quad + \text{Cost of grinder operation per 1,000 units} \\ &= 16 \times 200 + 4.5 \times 150 \\ &= \text{Rs. } 3,875 \end{aligned}$$

Total cost of design A/1,00,000 units =  $3,875 \times 1,00,000 / 1,000$

$$= \text{Rs. } 3,87,500$$

#### (b) Cost of design B

No. of hours of lathe time per 1,000 units = 7 hr

No. of hours of grinder time per 1,000 units = 12 hr

Total cost of design B/1,000 units

$$\begin{aligned} &= \text{Cost of lathe operation/1,000 units} \\ &\quad + \text{Cost of grinder operation/1,000 units} \\ &= 7 \times 200 + 12 \times 150 \\ &= \text{Rs. } 3,200 \end{aligned}$$

Total cost of design B/1,00,000 units =  $3,200 \times 1,00,000 / 1,000$

$$= \text{Rs. } 3,20,000$$

**DECISION** The total cost/1,00,000 units of design B is less than that of design A. Hence, design B is recommended for making the tapered fastening pin.

Economic advantage of the design B over design A per 1,00,000 units  
 $= \text{Rs. } 3,87,500 - \text{Rs. } 3,20,000$   
 $= \text{Rs. } 67,500$

**EXAMPLE 2.4 (Design selection for a process industry).** The chief engineer of refinery operations is not satisfied with the preliminary design for storage tanks to be used as part of a plant expansion programme. The engineer who submitted the design was called in and asked to reconsider the overall dimensions in the light of an article in the *Chemical Engineer*, entitled "How to size future process vessels?"

The original design submitted called for 4 tanks 5.2 m in diameter and 7 m in height. From a graph of the article, the engineer found that the present ratio of height to diameter of 1.35 is 111% of the minimum cost and that the minimum cost for a tank was when the ratio of height to diameter was 4 : 1. The cost for the tank design as originally submitted was estimated to be Rs. 9,00,000. What are the optimum tank dimensions if the volume remains the same as for the original design? What total savings may be expected through the redesign?

#### *Solution (a) Original design*

Number of tanks = 4

Diameter of the tank = 5.2 m

Radius of the tank = 2.6 m

Height of the tank = 7 m

Ratio of height to diameter =  $7/5.2 = 1.35$

$$\begin{aligned} \text{Volume/tank} &= (22/7)r^2h = (22/7)(2.6)^2 \times 7 \\ &= 148.72 \text{ m}^3 \end{aligned}$$

#### *(b) New design*

Cost of the old design = 111% of the cost of the new design (optimal design)

Optimal ratio of the height to diameter = 4:1

$$h : d = 4 : 1$$

$$4d = h$$

$$d = h/4$$

$$r = h/8$$

$$\begin{aligned} \text{Volume} &= (22/7)r^2h = 148.72 \text{ (since, the volume remains the same)} \\ (22/7)(h/8)^2h &= 148.72 \end{aligned}$$

$$h^3 = \frac{148.72}{(22/7)} \times 64 = 3,028.48$$

$$h = 14.47 \text{ m}$$

$$r = h/8 = 14.47/8 = 1.81 \text{ m}$$

Therefore,

$$\begin{aligned}\text{Diameter of the new design} &= 1.81 \times 2 \\ &= 3.62 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Cost of the new design} &= 9,00,000 \times (100/111) \\ &= \text{Rs. } 8,10,810.81\end{aligned}$$

$$\begin{aligned}\text{Expected savings by the redesign} &= \text{Rs. } 9,00,000 - \text{Rs. } 8,10,810.81 \\ &= \text{Rs. } 89,189.19\end{aligned}$$

### 2.2.3 Building Material Selection

As discussed in the introduction to this chapter, the sourcing of raw materials will have a significant effect on the cost of any product. Hence, it is assumed that the price of raw material is location dependent. While sourcing a raw material, the cost of transportation is to be considered in conjunction with the price of the raw material. This concept is demonstrated with a numerical example.

**EXAMPLE 2.5** In the design of buildings to be constructed in Alpha State, the designer is considering the type of window frame to specify. Either steel or aluminium window frames will satisfy the design criteria. Because of the remote location of the building site and lack of building materials in Alpha State, the window frames will be purchased in Beta State and transported for a distance of 2,500 km to the site. The price of window frames of the type required is Rs. 1,000 each for steel frames and Rs. 1,500 each for aluminium frames. The weight of steel window frames is 75 kg each and that of aluminium window frame is 28 kg each. The shipping rate is Re 1 per kg per 100 km. Which design should be specified and what is the economic advantage of the selection?

#### Solution

Distance between Alpha State and Beta State = 2,500 km

Transportation cost = Re 1/kg/100 km

##### (a) Steel window frame

Price of steel window frame/unit = Rs 1,000

Weight of steel window frame/unit = 75 kg

Total cost of steel window frame/unit

$$\begin{aligned}&= \text{Price of steel window frame/unit} \\ &\quad + \text{Transportation cost of steel window frame/unit} \\ &= 1,000 + (75 \times 2,500 \times 1)/100 \\ &= \text{Rs. } 2,875\end{aligned}$$

##### (b) Aluminium window frame

Price of aluminium window frame/unit = Rs. 1,500

Weight of aluminium window frame/unit = 28 kg

Total cost of aluminium window frame/unit

$$\begin{aligned}
 &= \text{Price of aluminium window frame/unit} \\
 &\quad + \text{Transportation cost of aluminium window frame/unit} \\
 &= 1,500 + (28 \times 2,500 \times 1)/100 \\
 &= \text{Rs. } 2,200
 \end{aligned}$$

**DECISION** The total cost/unit of the aluminium window frame is less than that of steel window-frame. Hence, aluminium window frame is recommended.

The economic advantage/unit of the aluminium window frame over the steel window frame = Rs. 2,375 - 2,200  
= Rs. 675

#### 2.2.4 Process Planning/Process Modification

While planning for a new component, a feasible sequence of operations with the least cost of processing is to be considered. The process sequence of a component which has been planned in the past is not static. It is always subject to modification with a view to minimize the cost of manufacturing the component. So, the objective of process planning/process modification is to identify the most economical sequence of operations to produce a component.

The steps in process planning are as follows:

1. Analyze the part drawing to get an overall picture of what is required.
2. Make recommendations to or consult with product engineers on product design changes.
3. List the basic operations required to produce the part to the drawing or specifications.
4. Determine the most practical and economical manufacturing method and the form or tooling required for each operation.
5. Devise the best way to combine the operations and put them in sequence.
6. Specify the gauging required for the process.

Steps 3-5 aim to determine the most practical and economical sequence of operations to produce a component. This concept is demonstrated with a numerical problem.

**EXAMPLE 2.6** The process planning engineer of a firm listed the sequences of operations as shown in Table 2.3 to produce a component.

Table 2.3 Data for Example 2.6

Sequence	Process sequence
1	Turning - Milling - Shaping - Drilling
2	Turning - Milling - Drilling
3	All operations are performed with CNC machine

The details of processing times of the component for various operations and their machine hour rates are summarized in Table 2.4.

Table 2.4 Machine Hour Rates and Processing Times (minutes) for Example 2.6

Operation	Machine hour rate (Rs.)	Process sequence		
		1	2	3
Turning	200	5	5	-
Milling	400	8	14	-
Shaping	350	10	7	-
Drilling	300	3	3	-
CNC operations	1,000	-	-	8

Find the most economical sequence of operations to manufacture the component.

*Solution (a) Cost of component using process sequence 1.* The process sequence 1 of the component is as follows:

Turning – Milling – Shaping – Drilling

The calculations for the cost of the above process sequence are summarized in Table 2.5.

Table 2.5 Workings for Process Sequence 1

Operation No.	Operation	Time		Machine hour rate (Rs.)	Cost (Rs.)
		(min)	(hr)		
1	Turning	5	0.083	200	16.60
2	Milling	8	0.133	400	53.20
3	Shaping	10	0.167	350	58.45
4	Drilling	3	0.050	300	15.00
					Total: 143.25

*(b) Cost of component using process sequence 2.* The process sequence 2 of the component is as follows:

Turning – Milling – Drilling

The calculations for the cost of the above process sequence are given in Table 2.6.

Table 2.6 Workings for Process Sequence 2

Operation No.	Operation	Time		Machine hour rate (Rs.)	Cost (Rs.)
		(min)	(hr)		
1	Turning	5	0.083	200	16.60
2	Milling	14	0.233	400	93.20
3	Drilling	3	0.050	300	15.00
					Total: 124.80

(c) Cost of component using process sequence 3. The process sequence 3 of the component is as follows:

Only CNC operations

The calculations for the cost of the above process sequence are summarized in Table 2.7

Table 2.7 Workings for Process Sequence 3

Operation No.	Operation	Time		Machine hour rate (Rs.)	Cost (Rs.)
		(min)	(hr)		
1	CNC operations	8	0.133	1,000	133

The process sequence 2 has the least cost. Therefore, it should be selected for manufacturing the component.

## QUESTIONS

1. List and explain the different situations deserving elementary economic analysis.
2. Explain the steps in the process planning.
3. In the design of an aircraft jet engine part, the designer has a choice of specifying either an aluminium alloy casting or a steel casting. Either material will provide equal service, but the aluminium casting will weigh 5 kg as compared with 7 kg for the steel casting.  
The aluminium part can be cast for Rs. 125 per kg and the steel part can be cast for Rs. 60 per kg. The cost of machining per unit is Rs. 200 for the aluminium part and Rs. 250 for the steel part. Every kilogram of excess weight is associated with a penalty of Rs. 2,500 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?
4. Two alternatives are under consideration for a hexagonal bolt fastening pin. Either design will serve equally well and will involve the same material and manufacturing cost except for the lathe and grinder operations.

Design A will require 20 hours of lathe time and 3 hours of grinder time per 10,000 units. Design B will require 10 hours of lathe time and 22 hours of grinder time per 10,000 units. The operating cost of the lathe including labour is Rs. 400 per hour. The operating cost of the grinder including labour is Rs. 300 per hour. Which design should be adopted if 10,00,000 units are required per year and what is the economic advantage of the best alternative?

5. A building contractor can source door frames from either a nearby shop or a far-off forest area. The cost details are as summarized in the following table. The total requirement of wood for the construction work is 75 tons.

Items	Nearby shop	Far-off forest area
Distance to site	Negligible	900 km
Transportation cost per ton per km	Negligible	Rs. 100
Material cost/ton	Rs. 2,000	Rs. 1,250

Find the best alternative for buying the wooden frames. Also find the economic advantage of the best decision.

6. Consider Example 2.4. Rework this example if the ratio of the height to diameter corresponding to the minimum cost is 6:1 instead of 4:1.
7. The process planning engineer of a firm listed down the sequences of operations, as shown in the following table to produce a component:

Sequence	Process sequence
1	Turning – Milling – Shaping – Drilling
2	Turning – Milling – Drilling
3	All operations are performed with CNC machine

The details of process time for the components for various operations and their machine hour rates are tabulated now.

Operation	Machine hour rate (Rs.)	Process sequence		
		1	2	3
Turning	300	8	8	
Milling	350	10	14	
Shaping	380	12		
Drilling	350	5	5	
CNC operation	1,200			10

Find the most economical sequence of operations to manufacture the component.

## 3 INTEREST FORMULAE THEIR APPLICATION

*Compound interest = interest on interest*

### 3.1 INTRODUCTION

McQ Interest rate is the rental value of money. It represents the growth of capital per unit period. The period may be a month, a quarter, semiannual or a year. An interest rate 15% compounded annually means that for every hundred rupees invested now, an amount of Rs. 15 will be added to the account at the end of the first year. So, the total amount at the end of the first year will be Rs. 115. At the end of the second year, again 15% of Rs. 115, i.e. Rs. 17.25 will be added to the account. Hence the total amount at the end of the second year will be Rs. 132.25. The process will continue thus till the specified number of years.

### 3.2 TIME VALUE OF MONEY

If an investor invests a sum of Rs. 100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in Table 3.1.

Table 3.1 Compound Amounts

(amount of deposit = Rs. 100.00)

Year end	Interest (Rs.)	Compound amount (Rs.)
0		100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14

The formula to find the future worth in the third column is

$$F = P \times (1 + i)^n$$

where

$P$  = principal amount invested at time 0,

$F$  = future amount,

= original Amount - Interest accrued / time period

Interest rate

$i$  = interest rate compounded annually,

$n$  = period of deposit.

The maturity value at the end of the fifth year is Rs. 201.14. This means that the amount Rs. 201.14 at the end of the fifth year is equivalent to Rs. 100.00 at time 0 (i.e. at present). This is diagrammatically shown in Fig. 3.1. This explanation assumes that the inflation is at zero percentage.

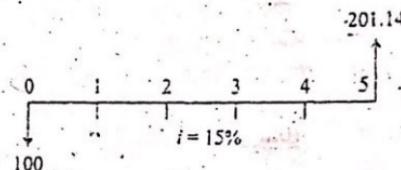


Fig. 3.1 Time value of money.

Alternatively, the above concept may be discussed as follows: If we want Rs. 100.00 at the end of the  $n$ th year, what is the amount that we should deposit now at a given interest rate, say 15%? A detailed working is shown in Table 3.2.

Table 3.2 Present Worth Amounts

(rate of interest = 15%)

End of year (n)	Present worth	Compound amount after $n$ year(s)
0		100
1	86.96	100
2	75.61	100
3	65.75	100
4	57.18	100
5	49.72	100
6	43.29	100
7	37.59	100
8	32.69	100
9	28.43	100
10	24.72	100

The formula to find the present worth in the second column is

$$P = \frac{F}{(1+i)^n}$$

From Table 3.2, it is clear that if we want Rs. 100 at the end of the fifth year, we should now deposit an amount of Rs. 49.72. Similarly, if we want Rs. 100.00 at the end of the 10th year, we should now deposit an amount of Rs. 24.72.

Also, this concept can be stated as follows:

A person has received a prize from a finance company during the recent

$$F_1 = P(1+i)$$

$$F_2 = F_1 + F_1(i) \stackrel{\text{Engineering Economics}}{=} P(1+i^2) + P(1+i)$$

$$= P(1+i)(1+i) = P(1+i)^2$$

1. Spot payment of Rs. 24.72 or

2. Rs. 100 after 10 years from now (this is based on 15% interest rate compounded annually).

If the prize winner has no better choice that can yield more than 15% interest rate compounded annually, and if 15% compounded annually is the common interest rate paid in all the finance companies, then it makes no difference whether he receives Rs. 24.72 now or Rs. 100 after 10 years.

On the other hand, let us assume that the prize winner has his own business wherein he can get a yield of 24% interest rate (more than 15%) compounded annually, it is better for him to receive the prize money of Rs. 24.72 at present and utilize it in his business. If this option is followed, the equivalent amount for Rs. 24.72 at the end of the 10th year is Rs. 212.45. This example clearly demonstrates the time value of money.

### 3.3 INTEREST FORMULAS

While making investment decisions, computations will be done in many ways. To simplify all these computations, it is extremely important to know how to use interest formulas more effectively. Before discussing the effective application of the interest formulas for investment-decision making, the various interest formulas are presented first.

$$F = P + I$$

mca Interest rate can be classified into simple interest rate and compound interest rate.

In simple interest, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable. In compound interest, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period.

The notations which are used in various interest formulae are as follows:

$P$  = principal amount

$n$  = No. of interest periods

$i$  = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

$F$  = future amount at the end of year  $n$

$A$  = equal amount deposited at the end of every interest period

$G$  = uniform amount which will be added/subtracted period after period to/ from the amount of deposit  $A$  at the end of period  $1$

#### 3.3.1 Single-Payment Compound Amount

Here, the objective is to find the single future sum ( $F$ ) of the initial payment ( $P$ ) made at time  $0$  after  $n$  periods at an interest rate  $i$  compounded every period. The cash flow diagram of this situation is shown in Fig. 3.2.

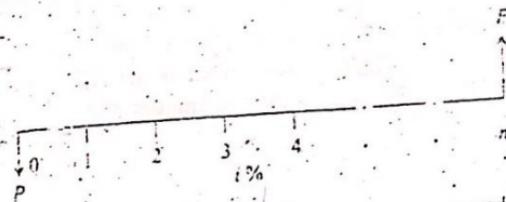


Fig. 3.2 Cash flow diagram of single-payment compound amount.

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

where  $(F/P, i, n)$  is called as single-payment compound amount factor.

EXAMPLE 3.1 A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

$$P = \text{Rs. } 20,000$$

$i = 18\%$  compounded annually

$n = 10$  years

$$F = P(1 + i)^n = P(F/P, i, n)$$

$$= 20,000 (F/P, 18\%, 10)$$

$$= 20,000 \times 5.234 = \text{Rs. } 1,04,680$$

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

### 3.3.2 Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount ( $P$ ) of a single future sum ( $F$ ) which will be received after  $n$  periods at an interest rate of  $i$  compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.

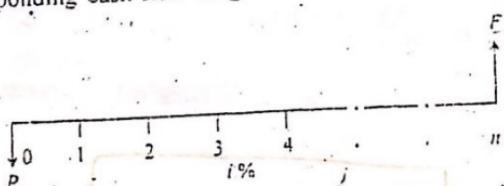


Fig. 3.3 Cash flow diagram of single-payment present worth amount.

The formula to obtain the present worth is

$$P = \frac{F}{(1 + i)^n} = F(P/F, i, n)$$

where

$(P/F, i, n)$  is termed as single-payment present worth factor.

**EXAMPLE 3.2** A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

**Solution**

$$F = \text{Rs. } 1,00,000$$

$i = 15\%$ , compounded annually

$n = 10$  years

$$\begin{aligned} P &= F/(1+i)^n = F(P/F, i, n) \\ &= 1,00,000 (P/F, 15\%, 10) \\ &= 1,00,000 \times 0.2472 \\ &= \text{Rs. } 24,720 \end{aligned}$$

$$P = F \cdot V / (1+i)^n$$

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

### 3.3.3 Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of  $n$  equal payments which are made at the end of every interest period till the end of the  $n$ th interest period at an interest rate of  $i$ , compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 3.4.

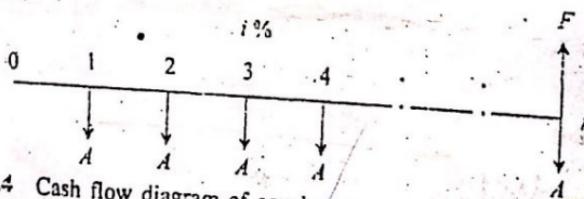


Fig. 3.4 Cash flow diagram of equal-payment series compound amount.

In Fig. 3.4,

$A$  = equal amount deposited at the end of each interest period

$n$  = No. of interest periods

$i$  = rate of interest

$F$  = single future amount

The formula to get  $F$  is

$$F = A \frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

where

$(F/A, i, n)$  is termed as *equal-payment series compound amount factor*.

**EXAMPLE 3.3** A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for

the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

**Solution**

$$A = \text{Rs. } 10,000$$

$$n = 25 \text{ years}$$

$$i = 20\%$$

$$F = ?$$

$$\text{Diagram: } F = PMD \frac{(1+i)^n - 1}{i}$$

The corresponding cash flow diagram is shown in Fig. 3.5.

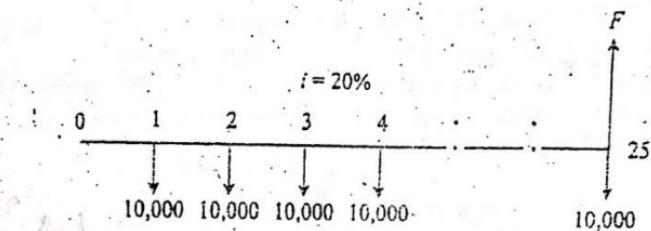


Fig. 3.5 Cash flow diagram of equal-payment series compound amount.

$$\begin{aligned} F &= A \frac{(1+i)^n - 1}{i} \Rightarrow F \\ &= A(F/A, i, n) \\ &= 10,000(F/A, 20\%, 25) \\ &= 10,000 \times 471.981 \\ &= \text{Rs. } 47,19,810 \end{aligned}$$

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

### 3.3.4 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount ( $A$ ) that should be deposited at the end of every interest period for  $n$  interest periods to realize a future sum ( $F$ ) at the end of the  $n$ th interest period at an interest rate of  $i$ .

The corresponding cash flow diagram is shown in Fig. 3.6.

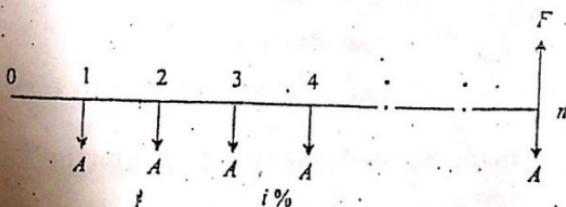


Fig. 3.6 Cash flow diagram of equal-payment series sinking fund

In Fig. 3.6,

$A$  = equal amount to be deposited at the end of each interest period

$n$  = No. of interest periods

$i$  = rate of interest

$F$  = single future amount at the end of the  $n$ th period

The formula to get  $A$  is

$$A = F \cdot \frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

where

$(A/F, i, n)$  is called as *equal-payment series sinking fund factor*.

**EXAMPLE 3.4** A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

*Solution*

$$F = \text{Rs. } 5,00,000$$

$$n = 15 \text{ years}$$

$$i = 18\%$$

$$A = ?$$

The corresponding cash flow diagram is shown in Fig. 3.7.

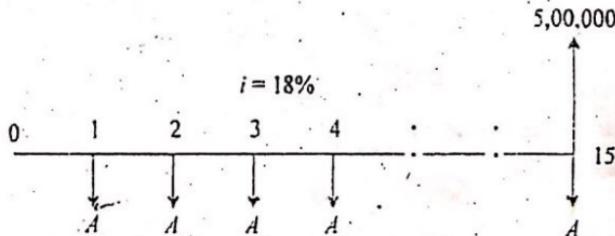


Fig. 3.7 Cash flow diagram of equal-payment series sinking fund.

$$\begin{aligned} A &= F \cdot \frac{i}{(1+i)^n - 1} = F(A/F, i, n) \\ &= 5,00,000(A/F, 18\%, 15) \\ &= 5,00,000 \times 0.0164 \\ &= \text{Rs. } 8,200 \end{aligned}$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

### 3.3.5 Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal

payment made at the end of every interest period for  $n$  interest periods at an interest rate of  $i$  compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.8. Here,

$P$  = present worth

$A$  = annual equivalent payment

$i$  = interest rate

$n$  = No. of interest periods

$$P = PMT \frac{1 - (1+i)^{-n}}{i}$$

The formula to compute  $P$  is

$$P = A \frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i; n)$$

where

$(P/A, i, n)$  is called *equal-payment series present worth factor*.

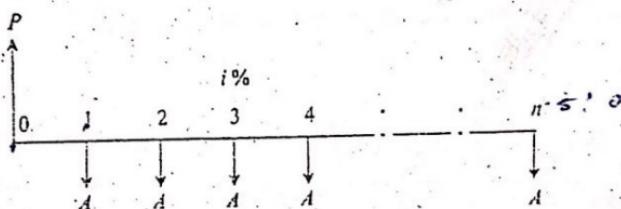


Fig. 3.8 Cash flow diagram of equal-payment series present worth amount.

**EXAMPLE 3.5** A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

### Solution

$$A = \text{Rs. } 10,00,000$$

$$i = 15\%$$

$$n = 20 \text{ years}$$

$$P = ?$$

The corresponding cash flow diagram is illustrated in Fig. 3.9.

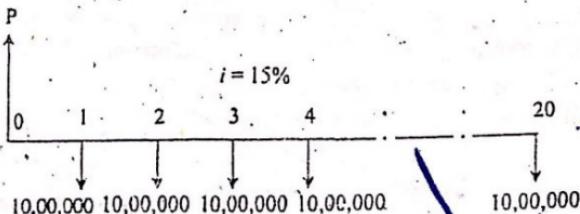


Fig. 3.9 Cash flow diagram of equal-payment series present worth amount.

$$\begin{aligned}
 P &= A \frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n) \\
 &= 10,00,000 \times (P/A, 15\%, 20) \\
 &= 10,00,000 \times 6.2593 \\
 &= \text{Rs. } 62,59,300
 \end{aligned}$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

### 3.3.6 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount ( $A$ ) which is to be recovered at the end of every interest period for  $n$  interest periods for a loan ( $P$ ) which is sanctioned now at an interest rate of  $i$  compounded at the end of every interest period (see Fig. 3.10).

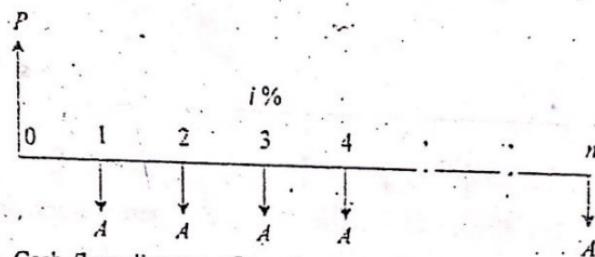


Fig. 3.10 Cash flow diagram of equal-payment series capital recovery amount.

In Fig. 3.10,

$P$  = present worth (loan amount)

$A$  = annual equivalent payment (recovery amount)

$i$  = interest rate

$n$  = No. of interest periods

The formula to compute  $A$  is as follows:

$$A = P \frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

where,

$(A/P, i, n)$  is called *equal-payment series capital recovery factor*.

**EXAMPLE 3.6** A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

**Solution**

$$P = \text{Rs. } 10,00,000$$

$$i = 18\%$$

$$n = 15 \text{ years}$$

The corresponding cash flow diagram is shown in Fig. 3.11.

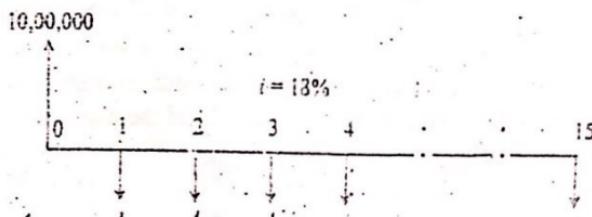


Fig. 3.11 Cash flow diagram of equal-payment series capital recovery amount.

$$\begin{aligned}
 A &= P \frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n) \\
 &= 10,00,000 \times (A/P, 18\%, 15) \\
 &= 10,00,000 \times (0.1964) \\
 &= \text{Rs. } 1,96,400
 \end{aligned}$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

### 3.3.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount  $A_1$  at the end of the first year and with an equal increment ( $G$ ) at the end of each of the following  $n - 1$  years with an interest rate  $i$  compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12.

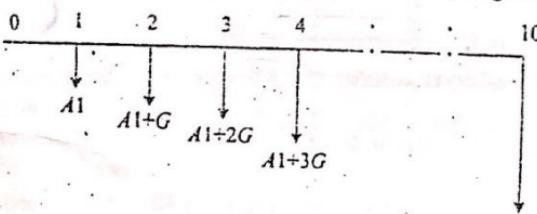


Fig. 3.12 Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute  $A$  under this situation is .

$$\begin{aligned}
 A &= A_1 + G \frac{(1+i)^n - 1}{i(1+i)^n - i} \\
 &= A_1 + G (A/G, i, n)
 \end{aligned}$$

where

$(A/G, i, n)$  is called *uniform gradient series factor*.

**EXAMPLE 3.7** A person is planning for his retired life. He has 10 more years

of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

**Solution** Here,

$$A_1 = \text{Rs. } 4,000$$

$$G = \text{Rs. } 500$$

$$i = 15\%$$

$$n = 10 \text{ years}$$

$$A = ? \text{ & } F = ?$$

The cash flow diagram is shown in Fig. 3.13.

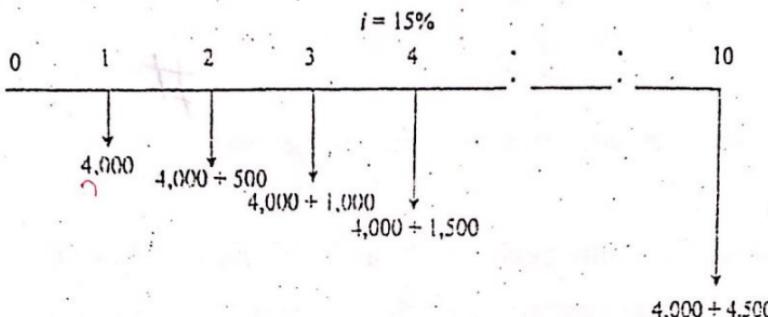


Fig. 3.13 Cash flow diagram of uniform gradient series annual equivalent amount.

$$\begin{aligned}
 A &= A_1 + G \frac{(1+i)^n - i(n-1)}{i(1+i)^n - 1} \\
 &= A_1 + G(A/G, i, n) \\
 &= 4,000 + 500(A/G, 15\%, 10) \\
 &= 4,000 + 500 \times 3.3832 \\
 &= \text{Rs. } 5,691.60
 \end{aligned}$$

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$\begin{aligned}
 F &= A(F/A, i, n) \\
 &= A(F/A, 15\%, 10) \\
 &= 5,691.60(20.304) \\
 &= \text{Rs. } 1,15,562.25
 \end{aligned}$$

At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,562.25.

**EXAMPLE 3.8** A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and

thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

**Solution** Here,

$$A_1 = \text{Rs. } 8,500$$

$$G = -\text{Rs. } 500$$

$$i = 15\%$$

$$n = 10 \text{ years}$$

$$A = ? \text{ & } F = ?$$

The cash flow diagram is shown in Fig. 3.14.

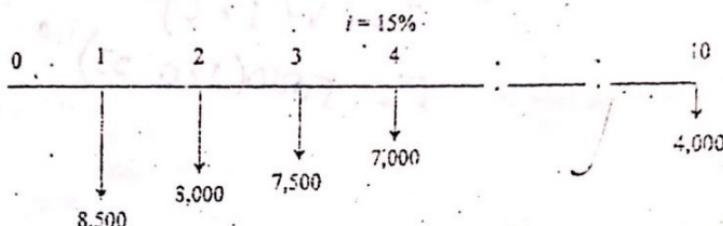


Fig. 3.14 Cash flow diagram of uniform gradient series annual equivalent amount.

$$\begin{aligned} A &= A_1 - G \frac{(1+i)^n - i(n-1)}{i(1+i)^n - i} \\ &= A_1 - G (A/G, i, n) \\ &= 8,500 - 500(A/G, 15\%, 10) \\ &= 8,500 - 500 \times 3.3832 \\ &= \text{Rs. } 6,808.40 \end{aligned}$$

This is equivalent to paying an equivalent amount of Rs. 6,808.40 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$\begin{aligned} F &= A(F/A, i, n) \\ &= A(F/A, 15\%, 10) \\ &= 6,808.40(20.304) \\ &= \text{Rs. } 1,38,237.75 \end{aligned}$$

At the end of the 10th year, the compound amount of all his payments is Rs. 1,38,237.75.

### 3.3.8 Effective Interest Rate

Let  $i$  be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in

a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is

$$\text{Effective interest rate, } R = (1 + i/C)^C - 1$$

where,

$i$  = the nominal interest rate

$C$  = the number of interest periods in a year.

**EXAMPLE 3.9** A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

**Solution**

$$P = \text{Rs. } 5,000$$

$$n = 10 \text{ years}$$

$$i = 12\% \text{ (Nominal interest rate)}$$

$$F = ?$$

$$F = PV(1+i)^n$$

$$F = 5000(1+0.12)^{10}$$

**METHOD 1**

$$\text{No. of interest periods per year} = 4$$

$$\text{No. of interest periods in 10 years} = 10 \times 4 = 40$$

$$\text{Revised No. of periods (No. of quarters), } N = 40$$

$$\text{Interest rate per quarter, } r = 12\%/4$$

= 3%, compounded quarterly.

$$F = P(1+r)^N = 5,000(1+0.03)^{40}$$

$$= \text{Rs. } 16,310.19$$

**METHOD 2**

$$\text{No. of interest periods per year, } C = 4$$

$$\text{Effective interest rate, } R = (1 + i/C)^C - 1$$

$$= (1 + 12\%/4)^4 - 1$$

= 12.55%, compounded annually.

$$F = P(1+R)^n = 5,000(1+0.1255)^{10}$$

$$= \text{Rs. } 16,308.91$$

### 3.4 BASES FOR COMPARISON OF ALTERNATIVES

In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives. Let us assume that an organization has a huge sum of money for potential investment and there are three different projects whose initial outlay and annual revenues during their lives are known. The executive has to select the best alternative among these three competing projects.

There are several bases for comparing the worthiness of the projects. These bases are:

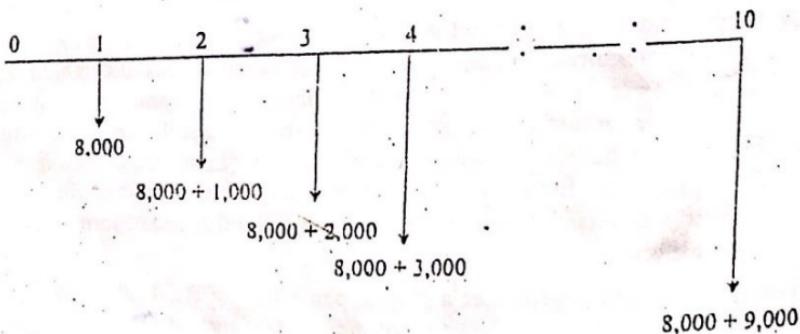
1. Present worth method
2. Future worth method
3. Annual equivalent method
4. Rate of return method

These methods are discussed in detail in Chapters 4-7.

## QUESTIONS

1. Explain the time value of money.
2. Give practical applications of various interest formulas.
3. A person deposits a sum of Rs. 1,00,000 in a bank for his son's education who will be admitted to a professional course after 6 years. The bank pays 15% interest rate, compounded annually. Find the future amount of the deposited money at the time of admitting his son in the professional course. 
$$P(1+i)^n$$
4. A person needs a sum of Rs. 2,00,000 for his daughter's marriage which will take place 15 years from now. Find the amount of money that he should deposit now in a bank if the bank gives 18% interest, compounded annually.
5. A person who is just 30 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 30 years starting from the end of next year. The bank gives 15% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old. F.A
6. A company is planning to expand its business after 5 years from now. The expected money required for the expansion programme is Rs. 5,00,00,000. The company can invest Rs. 50,00,000 at the end of every year for the next five years. If the assured rate of return of investment is 18% for the company, check whether the accumulated sum in the account would be sufficient to meet the fund for the expansion programme. If not, find the difference in amounts for which the company should make some other arrangement after 5 years.
7. A financial institution introduces a plan to pay a sum of Rs. 15,00,000 after 10 years at the rate of 18%, compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10 years to receive Rs. 15,00,000 after 10 years from the institution.
8. A company is planning to expand its business after 5 years from now. The money required for the expansion programme is Rs. 4,00,00,000. What annual equivalent amount should the company deposit at the end of every year at an interest rate of 15% compounded annually to get Rs. 4,00,00,000 after 5 years from now?

- ✓ 9. A company wants to set-up a reserve which will help it to have an annual equivalent amount of Rs. 15,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made as the reserve amount now.
10. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually. Suggest the best alternative to the customers.
11. A company takes a loan of Rs. 20,00,000 to modernize its boiler section. The loan is to be repaid in 20 equal installments at 12% interest rate, compounded annually. Find the equal installment amount that should be paid for the next 20 years.
12. A bank gives loan to a company to purchase an equipment which is worth of Rs. 5,00,000, at an interest rate of 18% compounded annually. This amount should be repaid in 25 yearly equal installments. Find the installment amount that the company has to pay to the bank.
13. A working woman is planning for her retired life. She has 20 more years of service. She would like to deposit 10% of her salary which is Rs. 5,000 at the end of the first year and thereafter she wishes to deposit the same amount (Rs. 5,000) with an annual increase of Rs. 1,000 for the next 14 years with an interest rate of 18%. Find the total amount at the end of the 15th year of the above series.
14. Consider the following cash flow diagram. Find the total amount at the end of the 10th year at an interest rate of 12%, compounded annually.



15. A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 10,000, at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 10,000) with an annual increase of Rs. 2,000 for the next 9 years with an interest rate of 20%. Find the total amount at the end of the 10th year of the above series.

16. A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 30,000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 30,000) with an annual decrease of Rs. 2,000 for the next 9 years with an interest rate of 18%. Find the total amount at the end of the 10th year of the above series.
17. A person invests a sum of Rs. 50,000 in a bank at a nominal interest rate of 18% for 15 years. The compounding is monthly. Find the maturity amount of the deposit after 15 years.