CHAPTER #01

NON-HOMOGENEOUS LINEAR SYSTEM OF EQUATION

Linear equation $\Rightarrow \underbrace{ax+by}_{\text{called unknown}} = k$

(NOT variable)

→ In linear algebra we use $x_1, x_2, x_3 \dots x_n$ as unknown.

- Suppose we have two linear equation

$$a_1x_1 + b_1x_2 = k_1$$

$$a_2x_2 + b_2x_2 = k_2$$

- In case of NON-Homogeneous ↗ $k_1 = 0, k_2 \neq 0$; $k_1 \neq 0, k_2 = 0$

→ k_1 and k_2 ⇒ Both never be zero.

- In case of Homogeneous → ($ax_1 + bx_2 + cx_3 = 0$)

k_1 and k_2 is zero.

NOTES:-

Product of unknown is not allowed.

$$2x_1 + \boxed{x_1 x_2} + x_3 = 5$$

any root of unknown is not allowed

$$2x_1 + \boxed{\sqrt{x_2}} + x_3 = 1$$

any power of unknown is not allowed

$$2x_1 + \boxed{(x_2)^n} + x_3 = 1$$

But: $2x_1+\sqrt{3}x_2+x_3=(5)^{1/3}$

In this No variable product, root, power

so it is linear.

NON-HOMOGenIOUS:-

→ UNIQUE Solution

→ NO Solution

→ Infinity Many Solution

UNIQUE SOlUTION

$x_1-3x_2=-7$

$2x_1+x_2=7$

In solution $x_1=2$; $x_2=3$

So its means its Give a unique

Solution; (NOT (Many or NO Solution))

NO Solution

$x_1-3x_2=-7$

$2x_1-6x_2=14$

In this case $0=0-14$

(NO Solution)

Infinite many Solution

$$x_1 - 3x_2 = -7$$

$$3x_1 - 9x_2 = -21$$

In this case $0 = 0$.

Means eqs. are overlapping so its give many infinite solution.

$$x_1 - 3x_2 = -7$$

$$x_1 = -7 + 3x_2$$

x_2 is dependent we say it is a free unknown or arbitrary unknown.

x_2 is dependent so $x_2 \in \mathbb{R}$

Suppose we take

$x_1 = 2$ so $x_2 = 3$

$x_1 = 143$ so $x_2 = 50$

(Infinite Many Solutions)

HOMOGENIOUS:-

→ Trivial Solution

→ NON-Trivial Solution

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Trivial Solution:

In HOMOGENEOUS

$$a_1x_1 + a_2x_2 + a_3x_3 = 0$$

$$a_4x_1 + a_5x_2 + a_6x_3 = 0$$

$$a_7x_1 + a_8x_2 + a_9x_3 = 0.$$

$$x_1 = 0,\ x_2 = 0,\ x_3 = 0.$$

This system also called consistent.

The solution $(0,0,0)$ of the above Homogeneous equation is called Trivial solution.

(BTW)

$$x_1 - 3x_2 = 0$$

$$2x_1 + x_2 = 0$$

$x_1 = 0;\ x_2 = 0.$

called ⟺ (Trivial solution) HOMOGeneous

(unique solution) NON Hom

NON

0 =

e.g

NO

NON-Trivial Solution

$$x_1 - 3x_2 = 0$$
$$3x_1 - 9x_2 = 0$$

$0 = 0$ (overlapping)

$x_1 - 3x_2 = 0$

$x_1 = 3x_2$ (x_2 is Independent)

so $x_2 \in \mathbb{R}$

e.g $x_2 = 1$; $x_1 = 3$

$x_2 = 5$; $x_1 = 15$

Its give infinite solution but incase of Homogeneous we called the infinite Solution as Non-Trivial Solution).

NOTE:-

In NON-HOMOGeneous, There are unique, NO solution, infinite many solution.

In Homogeneous

Trivial , Non-Trivial

↳ NOT have NO solution.

↳ Give many infinite solution which called N.T.S

C)

17:

a)

QUESTION: NO: 16

Given the Linear System

$3x + 4y = s$

$6x + 8y = t.$

a) Determine practicle values for s and t So that the system is consistent.

$2(1) - 2$

$$6\cancel{x} + 8\cancel{y} = 2s$$
$$-\cancel{6x} \pm \cancel{8y} = t$$
$$0 = 2s - t$$

It has only one solution
It is consistent

$2s - t = 0$

$2s = t$

But the system is dependent It has infinite many solution.

when $s = 1$ then $t = 2$

$s = 3$ then $t = 6$

$s = 10$ then $t = 20$

$s = 5$ then $t = 10$

b) Inconsistent

when $t = 2s$ system is always consistent (but dependent).

$t - 2s \neq 0$

$t \neq 2s$

$s = 1$, $t \neq 2$

$s = 3$, $t \neq 6$

$s = 10$, $t \neq 20$

c) Consistent relation

$t = 2s.$

It guarantee that system is consisten

17. Given the linear system

$x + 2y = 10$

$3x + (6+\tau)y = 30$

a) $x + 2y = 10$

$3x + (6+\tau)y = 30.$

$3(1) - 2$

$\Rightarrow \quad \cancel{3x} + 6y = 30$

$\cancel{3x} + (6+\tau)y = 30$

$6y - (6+\tau)y = 0.$

$\cancel{6y} - \cancel{6y} - \tau y = 0$

$-\tau y = 0$

$\tau y = 0.$

infinite Solution

when $\tau = 0$

$0 = 0$ (overlapping)

Given infinite solution

at $x + 2y = 10$

$x = 10 - 2y \qquad y \in \mathbb{R}$

$y = 0, \; x = 10$

$y = 1, \; x = 8$

System has unique solution if $t \neq 0$ if $t \neq 0$ then $ty=0$ has only one solution $y=0$ then where $x+2(0)=10$.
$x=10$

b) When t (is) any no. then 0 it give unique solution.

$y=0$ then $x=10$

unique solution

c) infinite value of t that is different from 0 is solution for part (b)

Properties of Matrices

Commutative property

→ $A+B = B+A$

→ $AB \neq BA$ (AB may or may not equal to BA)

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \end{bmatrix}$

First matric column and second matric rows are same so multiply hold

~~AB~~ $AB = \begin{bmatrix} 1+4 & 2+6 & 1+2 \\ 3+8 & 6+12 & 3-4 \end{bmatrix}$

$\begin{bmatrix} 5 & 8 & -1 \\ 11 & 18 & -1 \end{bmatrix}$

· But BA multiplication not hold

Cancellation law

(For real no. $ac = ad$ implies $c = d$)

For matrics

$AB = AC$ is $\neq$ $B = C$

→ if A is non-singular ($|A| \neq 0$) then we can cancel A and $B = C$

→ if A is singular ($|A| = 0$) then we can't cancel A and $B \neq C$

Zero with zero matrics,

(For real no. $ab = 0$ implies $a = 0$ or $b = 0$).

Solution of Simultaneous OF matrices

$$a_1x_1 + b_1x_2 = C_1$$

$$a_2x_1 + b_2x_2 = C_2$$

$$\underset{A}{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{B}{\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}}$$

$$AX = B$$

$$X = A^{-1}B.$$

(This is inverse).

$$\frac{1}{|A|} \times \text{Adj}\, A$$

$$X = \frac{1}{|A|}(\text{Adj}\, A)\, B$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

trices

Singular | **NON - Singular**

$|A| = 0$ | $|A| \neq 0$.

AdJoint

~~Let~~ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$Adj A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Determinant

2×2

$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$|A| = ad - bc$

cofactor of an element

$A_{ij} = (-1)^{i+j} \times M_{ij}$ → minor of an element.

~~rows~~ ~~column~~

1 2

$\boxed{A_{12}}$

1.1

$x+2y=8$

$3x-4y=4$

$x+2y=8 \quad -(1)$

$3x-4y=4 \quad -(2)$

$3(1) - 2$

$$\begin{array}{r} \cancel{3x}+6y=24 \\ \cancel{3x}-4y=4 \\ -\quad+\quad- \\ \hline 10y=20 \end{array}$$

$\boxed{y=2}$

$x+2(2)=8$

$x=8-4$

$\boxed{x=4}$

$x=4,\ y=2.$

$2x - 3y + 4z = -12$ — (1)

$x - 2y + z = -5$ — (2)

$1 \quad -4 \quad -3 \quad -5$

$3x + y + 2z = 1$ — (3)

eq(i) − 2(2)

$\cancel{2x} - 3y + 4z = -12$

$\cancel{2x} - 4y + 2z = -10$

$- \quad + \quad - \quad +$

$y + 2z = -2$ — (4)

3(2) − 3

$\cancel{3x} - 6y + 3z = -15$

$\cancel{3x} + y + 2z = 1$

$- \quad - \quad - \quad -$

$-7y + z = -16$ — (5)

−7(4) − 5

$\cancel{-7y} - 14z = 14$

$\cancel{-7y} + z = -16$

$+ \quad - \quad +$

$-15z = 30$

$\boxed{z = -2}$

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$y + 2z = -2$

$y + 2(-2) = -2$

$y - 4 = -2$

$y = -2 + 4$

$\boxed{y = 2}$

$x - 4 - 2 = -5$

$x - 6 = -5$

$x = -5 + 6$

$\boxed{x = 1}$

$3x + 2y + z = 2 \quad — (1)$

$4x + 2y + 2z = 8 \quad — (2)$

$x - y + z = 4 \quad — (3)$

(1) − (2).

$$\begin{array}{r} 3x + \cancel{2y} + z = 2 \\ 4x + \cancel{2y} + 2z = 8 \\ - \quad - \quad - \quad - \\ \hline -x - z = -6 \end{array}$$

$x + z = 6 \quad — (4).$

$2 + 2(3)$

$$4x + \cancel{2y} + 2z = 8$$
$$2x \cancel{-2y} + 2z = 8$$

$$6x + 4z = 16$$

$$2(3x + 2z) = 16$$

$$3x + 2z = 8 \quad (5)$$

$3(4) - 5$

$$\cancel{3x} + 3z = 18$$
$$\cancel{3x} + 2z = 8$$

$$\boxed{z = 10}$$

$$x + 10 = 6$$

$$\boxed{x = -4}$$

$$-4 - y + 10 = 4$$

$$-y + 6 = 4$$

$$-y = 4 - 6$$

$$\boxed{y = 2}$$

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$2x + 4y + 6z = -12$ — (1)

$2x - 3y - 4z = 15$ — (2)

$3x + 4y + 5z = -8$ — (3)

(1) − (2)

$$\cancel{2x} + 4y + 6z = -12$$
$$\cancel{2x} - 3y - 4z = 15$$
$$- \quad + \quad + \quad -$$

$7y + 10z = -27$ — (4)

(1) − (3)

$$\cancel{2x + 4y + 6z = -12}$$
$$\cancel{3x + 4y + 8z = -8}$$
$$\cancel{- \quad - \quad - \quad +}$$

$\cancel{-x + z = -4}$ — (5)

3(2) − 2(3)

$$\cancel{6x} - 9y - 12z = 45$$
$$\cancel{6x} + 8y + 10z = -16$$
$$- \quad - \quad - \quad +$$

$-17y - 22z = 61$ — (6)

$17(4)+7(5)$

$$\begin{array}{r} \cancel{119y}+170z = -459 \\ -\cancel{119y}-154z = 427 \\ \hline 16z = -32 \end{array}$$

$$\boxed{z=-2}$$

$+7y+10(-2)=-27$

$+7y-20=-27$

$+7y=-27+20$

$+7y=-7$

$$\boxed{y=-1}$$

$2x+3+8=15$

$2x+11=15$

$2x=4$

$$\boxed{x=2}$$

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$x+y=5$

$3x+3y=10$

$3(1)-2$

$\cancel{3x}+\cancel{3y}=15$

$\cancel{3x}+\cancel{3y}=10$

$0=-5$

No Solution ~~Infinite many Solution~~

$x+y=5$

$x=5-y$

$y\in\mathbb{R}$.

$x=4$; $y=1$

$x=3$; $y=2$

(b). $x+y-2z=5$

$2x+3y+4z=2$

$-2(1)+2$

$-\cancel{2x}-2y+4z=-10$

$\cancel{2x}+3y+4z=2$

$y+8z=-8$

$y=-8-8z$

$3(1)-2$

$\cancel{3x}+\cancel{3y}-6z=15$

$\cancel{2x}+\cancel{3y}+4z=2$

$x-10z=13$

$x=13+10z$

$x + -8 - 8z - 2z = 5$

$x - 8 - 10z = 5$

$x - 10z = 13$

$x = 13 + 10z$ z is any real no.

$2(1) - (2)$

$$2x + 2y - 4z = 10$$
$$2x + 3y + 4z = 2$$

$$4x + 6y = 12$$

$$x = \frac{1}{4}(12 - 6y) \Rightarrow x = 3 - 3/2\, y$$

$$13 + 10z = 3 - 3/2\, y.$$
$$10z = -10 - 3/2\, y.$$
$$z = -1 - \frac{3}{20} y.$$

(4)

$$x + 4y - z = 12$$
$$3x + 8y - 2z = 4$$

$2(1) - 2$

$$2x + 8y - 2z = 24$$
$$3x + 8y - 2z = 4$$
$$- \quad - \quad + \quad -$$

$$-x = 20$$

$$\boxed{x = -20}$$

$$-20 + 4y - z = 12$$

$$4y - z = 32$$

$$4y = 32 + z$$

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8. $3x + 4y - z = 8$

$6x + 8y - 2z = 3$

$2(1) - 2$

$\cancel{6x} + \cancel{8y} - \cancel{2z} = 16$

$\cancel{6x} + \cancel{8y} - \cancel{2z} = 3$

$- \quad - \quad + \quad -$

$0 = +13$

~~infinite many solution~~

~~$3x+4y-z=8$~~ NO solution.

9. $x + y + 3z = 12$

$2x + 2y + 6z = 6$

$2(1) - 2$

$\cancel{2x} + \cancel{2y} + \cancel{6z} = 24$

$\cancel{2x} + \cancel{2y} + 6z = 6$

$- \quad - \quad - \quad -$

$0 = 18$

NO solution

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ion.

(10) $x+y=1$

$2x-y=5$

$3x+4y=2$

(1)+(2)

$$\begin{array}{r} x+\cancel{y}=1 \\ 2x-\cancel{y}=5 \\ \hline 3x=6 \end{array}$$

$\boxed{x=2}$

$2+y=1$

$y=1-2$

$\boxed{y=-1}$

(11) $2x+3y=13$

$x-2y=3$

$5x+2y=27$

$$\begin{array}{r} x-\cancel{2y}=3 \\ 5x+\cancel{2y}=27 \\ \hline 6x=30 \end{array}$$

$x=30/6$ $\boxed{x=5}$

(13)

$5 - 2y = 3$

$-2y = 3 - 5$

$-2y = -2$

$\boxed{y = 1}$

(17)

$x - 5y = 6$

$3x + 2y = 1$

$5x + 2y = 1$

$3x + \cancel{2y} = \cancel{1}$

$5x + \cancel{2y} = \cancel{1}$

$-2x = 0$

$\boxed{x = 0}$

$3x + 2y = 1$

$2y = 1$

$y = 1/2$

$x - 5(y) = 6$

$0 - 5y = 6$

$-5y = 6$

$y = -6/5$

y = 1 is not solution

of 1st eqn so

It has no-solution.

(14)

(13).

$$x+3y=-4$$
$$2x+5y=-8$$
$$x+3y=-5$$

$$\cancel{x}+\cancel{3y}=-4$$
$$\cancel{x}+\cancel{3y}=-5$$
$$\underline{-\quad\;-\qquad\;+}$$

$$0=-1$$

No solution.

(14).

$$2x+3y-z=6$$
$$2x-y+2z=-8$$
$$3x-y+z=-7$$

(1) - (2).

$$\cancel{2x}+3y-z=6$$
$$\cancel{2x}-y+2z=-8$$
$$\underline{-\quad+\quad-\quad+}$$

$$4y-3z=14 \quad \text{—(4)}$$

(2) - (3).

$$2x-\cancel{y}+2z=-8$$
$$3x-\cancel{y}+z=-7$$
$$\underline{-\quad+\quad-\quad+}$$

$$-x+z=-1 \quad \text{—(5)}$$

① + ③ ②

$2x + \cancel{3y} - z = 6$

$6x - \cancel{3y} + 6z = -24$

$8x + 5z = -18$ — ⑥

8 ⑤ + 6

$\cancel{-8x} + 8z = -8$

$\cancel{8x} + 5z = -18$

$13z = -26$

$\boxed{z = -2}$

$-x - 2 = -1$

$-x = -1 + 2$

$-x = 1$

$\boxed{x = -1}$

$4y - 3(-2) = 14$

$4y + 6 = 14$

$4y = 14 - 6$

$4y = 8$

$\boxed{y = 2}$

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QUESTION: 15:

$$2x - y = 5$$
$$4x - 2y = t$$

a) System is consistent ⇒ value of t

$2(1) - 2$

$$\cancel{4x} - \cancel{2y} = 10$$
$$4x \underset{+}{-} 2y = t$$

$$0 = 10 - t$$

$$\boxed{t = 10}$$

System is consistent if it has at least one solution so the variable are cut outs and the L.H.S=0 so system is consistent.

b) ⇒

$$4x - 2y = t$$
$$4x = t + 2y$$

System is inconsistent when they have no solution

It is only happen when take false eq. if we let $t \neq 10$.

All values (each value) of $t = 10$,

For example $t = 4$

c). Infinitely many diffrent solution value can be choosen in part b) each t that is diffrent from 10.

Question 18:

Is every homogeneous linear system always consistent explain

let we considerd that

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0.$$

is always satisfy $x_1 = 0, x_2 = 0$ $x_3 = 0$ Such a system is always consistent solution $(0,0,0)$ called trivial solution

So every homogeneous linear system is consistent since it has trivial solution.

$2x+3y-z=0$

$x-4y+5z=0$

a) $x_1=1 \quad y_1=-1 \quad z_1=-1$ is solution.

$2(1)+3(-1)-(-1)=0$

$2-3+1=0$

$0=0$ (True).

$1-4(-1)+5(-1)=0$

$1+4-5=0$

$0=0$ (True)

Hence $x_1=1 \quad y_1=-1 \quad z_1=-1$ is a solution system.

b) $x_2=-2,\ y_2=2,\ z_2=2$ is a Solution

$2(-2)+3(2)-2=0$

$-4+6-2=0$

$0=0$ (True).

$-2-4(2)+5(2)=0$

$-2-8+10=0$

$0=0$ (True)

Hence it is a solution system.

c) $x = x_1 + x_2 = -1$

$y = y_1 + y_2 = 1$

$z = z_1 + z_2 = 1$

a solution of linear system.

$2(-1) + 3(1) - 1 = 0$

$-2 + 3 - 1 = 0$

$0 = 0$ (True)

$-1 - 4(1) + 5(1) = 0$

$-1 - 4 + 5 = 0$

$-5 + 5 = 0$

$0 = 0$ (True)

Hence it is a solution system.

d) is $3x, 3y, 3z$ where x, y, and z as in part c a solution of linear system,

$x = -3 \quad y = 3 \quad z = 3$

$2(-3) + 3(3) - 3 = 0$

$-6 + 9 - 3 = 0$

$0 = 0$ (True)

$-3 - 4(3) + 5(3) = 0$

$-3 - 12 + 15 = 0$

$0 = 0$ (True)

Solution.

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20:

$$2x+y-2z=-5$$
$$3y+z=7$$
$$z=4$$

$$3y+4=7$$
$$3y=7-4$$
$$3y=3$$
$$\boxed{y=1}$$

$$2x+1-2(4)=-5$$
$$2x+1-8=-5$$
$$2x-7=-5$$
$$2x=-5+7$$
$$2x=2$$
$$\boxed{x=1}$$

21:

$$4x=8$$
$$-2x+3y=-1$$
$$3x+5y-2z=11$$

$$4x=8$$
$$\boxed{x=2}$$

$$-4+3y=-1$$

$3y = 3$

$\boxed{y = 1}$.

$9(2) + 5(1) - 2z = 11$

$6 + 5 - 2z = 11$

$11 - 2z = 11$

$-2z = 0$

$\boxed{z = 0}$.

22:

$x = 1, y = 2, z = r$

then solve

$2x + 3y - z = 11$ (1)

$x - y + 2z = -7$ (2)

$4x + y - 2z = 12$ (3).

(1) $\rightarrow$ $2(1) + 3(2) - r = 11$

$2 + 6 - r = 11$

$8 - r = 11$

$-r = 11 - 8$

$-r = 3$

$\boxed{r = -3}$

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(2) $x - y + 2z = -7$

$1 - 2 + 2r = -7$

$-1 + 2r = -7$

$2r = -7 + 1$

$2r = -6$

$\boxed{r = -3}$

(3). $4x + y - 2z = 12$

$4(1) + 2 - 2(r) = 12$

$4 + 2 - 2r = 12$

$6 - 2r = 12$

$-2r = 6$

$\boxed{r = -3}$

system has solution

$x = 1$ $y = 2$ $z = r$ when

$\boxed{r = -3}$

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23: $x = r, y = 2, z = 1$

$3x - 2z = 4$ ①

$x - 4y + z = -5$ ②

$-2x + 3y + 2z = 9$ ③

① $3r - 2(1) = 4$

$3r - 2 = 4$

$3r = 6$

$\boxed{r = 2}$

② $r - 8 + 1 = -5$

$x - 7 = -5$

$\boxed{x = 2}$

③ $-2r + 6 + 2 = 9$

$-2r + 8 = 9$

$-2r = 1$

$r = -1/2$ NOT satisfy.

No solution.

Exercise 1.2

Let

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 3 & 2 \\ -4 & 3 & 5 \\ 6 & 1 & -1 \end{bmatrix}$$

What is a_{12}, a_{22}, a_{23}?

$a_{12} = -3$

$a_{22} = -5$

$a_{23} = 4$.

What is c_{13}, c_{31}, c_{33}?

$c_{13} = 2$

$c_{31} = 6$

$c_{33} = -1$.

What is b_{11}, b_{31}?

$b_{11} = 4$

$b_{31} = 5$

4Q $\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$

$a+b=4$ --- (1)

$c+d=6$ --- (2)

$c-d=10$ --- (3)

$a-b=2$ --- (4)

(1) + (4)

$a+\cancel{b}=4$

$a-\cancel{b}=2$

$2a=6$

$\boxed{a=3}$

(1) => $3+b=4$

$\boxed{b=1}$

(2) + (3)

$c+\cancel{d}=6$

$c-\cancel{d}=10$

$2c=16$

$\boxed{c=8}$

$8+d=6$

$\boxed{d=-2}$

$$\begin{bmatrix} a+2b & 2a-b \\ 2c+d & c-2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix}$$

$a+2b=4$ — (1) $\quad$ $2a-b=-2$ — (2)

$2c+d=4$ — (3) $\quad$ $c-2d=-3$ — (4)

$1 \rightarrow \quad a+2b=4$

$2(1) \Rightarrow \quad 2a+4b=8$ — (5)

(5) − (2)

$$\begin{array}{r} \cancel{2a}+4b=8 \\ \cancel{2a}-b=-2 \\ \hline 5b=10 \end{array}$$

$\boxed{b=2}$

(1) → $\quad a+4=4$

$\boxed{a=0}$

2(4)

$$\begin{array}{r} \cancel{2c}+d=4 \\ \cancel{2c}-4d=-6 \\ \hline 5d=10 \end{array}$$

$\boxed{d=2}$

$2c+2=4$

$2c=2$

$\boxed{c=1}$

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$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$.

$D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$, $E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$, $F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$

$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

QUESTION: NO:06.

C+E and E+C

$$C+E = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}.$$

E+C is also same

A+B

$$A+B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

Matrices are only add when these are in same order.

D-F

$$D-F = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -7 \\ 0 & 1 \end{bmatrix}.$$

-3C+5O

$$-3C+5O = -3\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + 5\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & +3 & -9 \\ -12 & -3 & -15 \\ -6 & -3 & -9 \end{bmatrix}.$$

2C-3E

$$2C-3E = 2\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - 3\begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 6 \\ 8 & 2 & 10 \\ 4 & 2 & 6 \end{bmatrix} - \begin{bmatrix} 6 & -12 & 15 \\ 0 & 3 & 12 \\ 9 & 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 10 & -9 \\ 8 & -1 & -2 \\ -5 & -4 & 3 \end{bmatrix}.$$

2B+F

$$2B+F = 2\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}.$$

$$= \begin{bmatrix} 2 & 0 \\ 4 & 2 \\ 6 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

~~Bo~~ Not add bcz order are not Same.

Question:17:

3D+2F

$$3D+2F = 3\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} + 2\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}.$$

$$= \begin{bmatrix} 9 & -6 \\ 6 & 12 \end{bmatrix} + \begin{bmatrix} -8 & 10 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 10 & 18 \end{bmatrix}$$

3(2A) and 6A

$$3(2A) = 3\left(2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}\right), \quad 6A = 6\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$3(2A) = 3\begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}, \quad 6A = \begin{bmatrix} 6 & 12 & 18 \\ 12 & 6 & 24 \end{bmatrix}$$

$$3(2A) = \begin{bmatrix} 6 & 12 & 18 \\ 12 & 6 & 24 \end{bmatrix}, \quad 6A = \begin{bmatrix} 6 & 12 & 18 \\ 12 & 6 & 24 \end{bmatrix}.$$

3A + 2A and 5A

$$3A + 2A = 5A = 5\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 & 15 \\ 10 & 5 & 20 \end{bmatrix}$$

$2(D+F)$ and $2D+2F$

$$2(D+F) = 2D+2F = 2\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} + 2\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} -8 & 10 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 6 \\ 8 & 14 \end{bmatrix}$$

$(2+3)D$ and $2D+3D$

$$(2+3)D = 2D+3D = 5D = 5\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & -10 \\ 10 & 20 \end{bmatrix}$$

$3(B+D)$

$$3(B+D) = 3\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}\right)$$

Not add bcz order are not same

Question :No:08

A^T and $(A^T)^T$

$$A^T = \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$(A^T)^T = \left(\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}.$$

$(C+E)^T$ and $C^T + E^T$

$$(C+E)^T = \left(\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \right)^T$$

$$= \left(\begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 3 & 4 & 2 \\ -1 & 1 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

$$E^T = \begin{bmatrix} 2 & 0 & 3 \\ -4 & 1 & 2 \\ 5 & 4 & 1 \end{bmatrix}$$

$$C^T + F^T = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}.$$

Same

$(2D+3F)^T$

$$2D+3F = 2\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} + 3\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} -12 & 15 \\ 6 & 9 \end{bmatrix}.$$

$$= \begin{bmatrix} -6 & 11 \\ 10 & 17 \end{bmatrix}$$

$$(2D+3F)^T = \begin{bmatrix} -6 & 10 \\ 11 & 17 \end{bmatrix}$$

$D-D^T$

$$D-D^T = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}.$$

$2A^T+B$

$$2A^T+B = 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 6 & 3 \\ 9 & 10 \end{bmatrix}$$

$(3D-2F)^T$

$$3D-2F = 3\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} - 2\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -6 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} -8 & 10 \\ 4 & 6 \end{bmatrix}.$$

$$= \begin{bmatrix} 17 & -16 \\ 2 & 6 \end{bmatrix}$$

$$(3D-2F)^T = \begin{bmatrix} 17 & 2 \\ -16 & 6 \end{bmatrix}.$$

Question 9

$(2A)^T$

$$2A = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$(2A)^T = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix}.$$

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$(A-B)^t$

$$A-B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}.$$

Not possible

$(3B^t - 2A)^t$

$$3B^t - 2A = 3\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}\right)^t - 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$3B^t - 2A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 1 & -2 \end{bmatrix}.$$

$$(3B^t - 2A)^t = \begin{bmatrix} 1 & -4 \\ 2 & 1 \\ 3 & -2 \end{bmatrix}.$$

$(3A^T - 5B^T)^T$

$$(3A^T - 5B^T) = 3\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}\right)^T - 5\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}\right)^T$$

$$= 3\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 \\ 6 & 3 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 10 & 15 \\ 0 & 5 & 10 \end{bmatrix}$$

Not possible.

$(-A)^T$ and $-(A)^T$

$$(-A)^T = -(A)^T = -\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}\right)^T$$

$$= -\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix}$$

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$(C+E+F^T)^T$

$$C+E+F^T = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} + \left(\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}\right)^T$$

- Not possible

Question: 10.

Matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ a linear combination of a matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Justify answer.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

$$2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}.$$ Yes

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Question 11

$\begin{bmatrix} 4 & 1 \\ \cdot & -3 \end{bmatrix}$ a linear combination of matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$?

$$-3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -3 \end{bmatrix}.$$

NO

12.

$$\lambda I_3 - A = \lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 6 & 2 & 3 \\ 5 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 6 & 2 & 3 \\ 5 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \lambda-1 & -2 & -3 \\ -6 & \lambda-2 & -3 \\ -5 & -2 & \lambda-4 \end{bmatrix}$$

Exercise

$a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} \ \\ \ \end{bmatrix}$

$\begin{bmatrix} 1 \times 4 + 2 \times -1 \end{bmatrix} = \begin{bmatrix} 4 - 2 \end{bmatrix} = \begin{bmatrix} 2 \end{bmatrix}$

$a = \begin{bmatrix} -3 \\ -2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} -3 \times 1 + -2 \times -2 \end{bmatrix}$

$= \begin{bmatrix} -3 + 4 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$

$a = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 4 \times 1 + 2 \times 3 + -1 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 4 + 6 - 6 \end{bmatrix} = \begin{bmatrix} 4 \end{bmatrix}$

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$a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 1\times1 + 1\times0 + 0\times1 \end{bmatrix}$

$= \begin{bmatrix} 1 + 0 + 0 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$

$a = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 2\times3 + 1\times2 \end{bmatrix}$

$= \begin{bmatrix} 6 + 2 \end{bmatrix} = \begin{bmatrix} 8 \end{bmatrix}$

$a = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times3 + -1\times2 \end{bmatrix} = \begin{bmatrix} 3 - 2 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$

$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 1\times-2 + 2\times0 + 3\times1 \end{bmatrix} = \begin{bmatrix} -2 + 0 + 3 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$

$$a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$a \cdot b = [1 \times 1 + 0 \times 0 + 0 \times 0] = 1.$$

3:- let $a = b = \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$ if $a \cdot b = 17$ find x

$$a \cdot b = \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$$

$$17 = [-3 \times -3 + 2 \times 2 + x \times x].$$

$$17 = 9 + 4 + x^2$$

$$17 - 13 = x^2$$

$$x^2 = 4$$

$$x = \pm 2.$$

4: $v = \begin{bmatrix} 1 \\ -3 \\ 4 \\ x \end{bmatrix}$ $w = \begin{bmatrix} x \\ 2 \\ -1 \\ 1 \end{bmatrix}$

$$v \cdot w = 0$$

$v \cdot w = 0$

$$v \cdot w = \begin{bmatrix} 1 \\ -3 \\ 4 \\ x \end{bmatrix} \cdot \begin{bmatrix} x \\ 2 \\ -1 \\ 1 \end{bmatrix}.$$

$0 = 1 \times x + -3 \times 2 + 4 \times -1 + x \times 1$

$0 = x - 6 - 4 + x$

$0 = 2x - 10$

$2x = 10$

$\boxed{x = 5}$

5: $v \cdot w = 0$ and $v \cdot u = 0$.

$$v = \begin{bmatrix} x \\ 1 \\ y \end{bmatrix}, \quad w = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \text{ and } u = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$v \cdot w = 0$

$$0 = \begin{bmatrix} x \\ 1 \\ y \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$0 = 2x - 2 + y$

$2x + y = 2$ — 1 .

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$v \cdot u = 0$

$$0 = \begin{bmatrix} x \\ 1 \\ y \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$$

$0 = x + 8 + 2y$

$x + 2y = -8$ — (2).

(1) - 2(y)

$\cancel{2x} + y = 2$

$\cancel{2x} + 4y = -16$

$- \quad - \quad +$

$-3y = 18$

$\boxed{y = -6}$

$x + 2y = -8$

$x + 2(-6) = -8$

$x - 12 = -8$

$x = -8 + 12$

$\boxed{x = 4}$.

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6. $v \cdot w = 0$ $\quad v \cdot u = 0$

$$v = \begin{bmatrix} x \\ 1 \\ y \end{bmatrix} \quad w = \begin{bmatrix} x \\ -2 \\ 0 \end{bmatrix} \quad u = \begin{bmatrix} 0 \\ -9 \\ y \end{bmatrix}$$

$v \cdot w = 0$

$$\begin{bmatrix} x \\ 1 \\ y \end{bmatrix} \cdot \begin{bmatrix} x \\ -2 \\ 0 \end{bmatrix} = 0.$$

$$x^2 - 2 + 0 = 0.$$

$$x^2 - 2 = 0$$

$$x^2 = 2.$$

$$\boxed{x = \pm\sqrt{2}.}$$

$v \cdot u = 0.$

$$\begin{bmatrix} x \\ 1 \\ y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -9 \\ y \end{bmatrix} = 0$$

$$0 + 9 + y^2 = 0.$$

$$y^2 = 9$$

$$\boxed{y = \pm 3}.$$

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7: let $w = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$ w.w.

$$w \cdot w = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1$$

8: Find all values of x so that $u \cdot u = 50$ where $u = \begin{bmatrix} x \\ 3 \\ 4 \end{bmatrix}$

$$u \cdot u = 50$$

$$\begin{bmatrix} x \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ 3 \\ 4 \end{bmatrix} = 50$$

$$x^2 + 9 + 16 = 50$$

$$x^2 + 25 = 50$$

$$x^2 = 25$$

$$x = \pm 5.$$

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9: $v \cdot v = 1$ $\quad v = \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix}$.

$$\begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix} = 1$$

$$(1/2)(1/2) + (-1/2)(-1/2) + (x)(x) = 1.$$

$$1/4 + 1/4 + x^2 = 1$$

$$1/2 + x^2 = 1$$

$$x^2 = 1 - 1/2$$

$$x^2 = 1/2$$

$$x = \pm\sqrt{1/2}.$$

10: $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ find x and y

$$A \cdot B = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 \times y + 2 \times x + x \times 1 \\ 3 \times y + -1 \times x + 2 \times 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} y + 2x + x \\ 3y - x + 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} y + 3x \\ 3y - x + 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$y + 3x = 6$ – (1)

$3y - x + 2 = 8$ – (2)

$3y - x = 6$ – (3).

3(1) – 3

$\cancel{3y} + 9x = 18$

$\cancel{3y} - x = 6$

$\quad +\quad -$

$10x = 12$

$x = 6/5$

$y + 18/5 = 6$.

$y = 6 - 18/5$

$y = 12/5$.

11: AB

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \times 1 + 2 \times 2 + 3 \times 3 & 1 \times 0 + 2 \times 1 + 3 \times 2 \\ 2 \times 1 + 1 \times 2 + 4 \times 3 & 2 \times 0 + 1 \times 1 + 4 \times 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 + 4 + 9 & 0 + 2 + 6 \\ 2 + 2 + 12 & 0 + 1 + 8 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}.$$

BA

$$BA = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}.$$

~~Not Possible.~~

$$= \begin{bmatrix} 1 \times 1 + 0 \times 2 & 1 \times 2 + 0 \times 1 & 1 \times 3 + 0 \times \\ 2 \times 1 + 1 \times 2 & 2 \times 2 + 1 \times 1 & 2 \times 3 + 1 \\ 3 \times 1 + 2 \times 2 & 3 \times 2 + 2 \times 1 & 3 \times 3 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 10 \\ 7 & 8 & 17 \end{bmatrix}$$

F^cE

$$F^cE = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} -2+0+9 & 4+0+6 & -5+0+3 \\ 4+0+15 & -8+4+10 & 10+16+5 \end{bmatrix}.$$

$$= \begin{bmatrix} 7 & 10 & -2 \\ 19 & 6 & 31 \end{bmatrix}.$$

$CB+D$

$$CB+D = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 - 2 + 9 & 0-1+6 \\ 4+2+15 & 0+1+10 \\ 2+2+9 & 0+1+6 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}.$$

$$\begin{bmatrix} 10 & 5 \\ 21 & 11 \\ 13 & 7 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

Impossible.

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$AB+D^2$

$$AB+D\cdot D = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix} + \begin{bmatrix} 9-4 & -6+10 \\ 6+10 & -4+25 \end{bmatrix}.$$

$$\begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix} + \begin{bmatrix} 5 & -16 \\ 16 & 21 \end{bmatrix}.$$

$$\begin{bmatrix} 19 & -8 \\ 32 & 30 \end{bmatrix}$$

12: $DA+B$

$$DA+B = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3-4 & 6-2 & 9-8 \\ 2+10 & 2+5 & 12+20 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 1 \\ 12 & 7 & 32 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}.$$

EC

$$EC = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6-16+10 & -2-4+5 & 6-20+15 \\ 0+4+8 & 0+1+4 & 0+5+12 \\ 9+4+2 & -3+2+1 & 9+10+3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ 12 & 5 & 17 \\ 15 & 0 & 22 \end{bmatrix}$$

CE

$$\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6+0+9 & -12-1+6 & 15-4+3 \\ 8+0+15 & -16+1+10 & 20+4+5 \\ 4+0+9 & -8+1+6 & 10+4+3 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -7 & 14 \\ 23 & -5 & 29 \\ 13 & -1 & 17 \end{bmatrix}.$$

EB + F

$$EB + F = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 - 8 + 15 & 0 - 4 + 10 \\ 0 + 2 + 12 & 0 + 1 + 8 \\ 3 + 6 + 3 & 0 + 2 + 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 6 \\ 14 & 9 \\ 12 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}.$$

$$\begin{bmatrix} 8 & 8 \\ 14 & 13 \\ 15 & 9 \end{bmatrix}.$$

FC+D

$$\begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}.$$

Not possible.

FD-3B

$$FD-3B = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -3+4 & 2+10 \\ 0+8 & 0+20 \\ 9+10 & -6+25 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 6 & 3 \\ 9 & 6 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 12 \\ 8 & 20 \\ 19 & 19 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 6 & 3 \\ 9 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 12 \\ 2 & 17 \\ 10 & 13 \end{bmatrix}$$

(AB)-D

$$AB-2D = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix} - 2\begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}.$$

$$\begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix} - \begin{bmatrix} 6 & -4 \\ 4 & 10 \end{bmatrix}.$$

$$\begin{bmatrix} 8 & 12 \\ 12 & -1 \end{bmatrix}$$

$F^T B + D$

$$F^T B + D = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -1+0+9 & 0+0+6 \\ 2+8+15 & 0+4+10 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}.$$

$$\begin{bmatrix} 8 & 6 \\ 25 & 14 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}.$$

$$\begin{bmatrix} 11 & 4 \\ 27 & 19 \end{bmatrix}$$

Scanned with CamScanner

$2F - 3(AE)$

$$2\begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix} - 3\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}\right)$$

impossible subtraction

$BD + AE$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -4 & 5 \\ 6 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

impossible

bcz sum are not possible

14: $A(BD)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}\right)$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}\left(\begin{bmatrix} 3+0 & -2+0 \\ 6+2 & -4+5 \\ 9+4 & -6+10 \end{bmatrix}\right)$$

Scanned with CamScanner

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 8 & 1 \\ 13 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3+16+39 & -2+2+12 \\ 6+8+52 & 4+1+16 \end{bmatrix}$$

$$\begin{bmatrix} 58 & 12 \\ 66 & 21 \end{bmatrix}$$

$A(C+E)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \left(\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \right)$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 5 & 4 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 5+8+15 & 4+4+9 & 8+18+9 \\ 10+4+20 & 8+2+12 & 16+9+12 \end{bmatrix}$$

$$\begin{bmatrix} 28 & 17 & 35 \\ 34 & 22 & 37 \end{bmatrix}$$

Scanned with CamScanner

(AB)D

$$\begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 42+16 & -28+40 \\ 48+18 & -32+45 \end{bmatrix}$$

$$\begin{bmatrix} 58 & 12 \\ 66 & 13 \end{bmatrix}$$

AC+AE

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3+8+6 & -1+2+3 & 3+15+9 \\ 6+4+8 & 2+1+4 & 6+20+12 \end{bmatrix} +$$

$$\begin{bmatrix} 2+0+9 & -4+2+6 & 5+8+3 \\ 4+0+12 & -8+1+8 & 10+4+4 \end{bmatrix}$$

$$\begin{bmatrix} 17 & 4 & 27 \\ 18 & 7 & 38 \end{bmatrix} + \begin{bmatrix} 11 & 4 & 16 \\ 16 & 1 & 18 \end{bmatrix}.$$

$$\begin{bmatrix} 28 & 8 & 43 \\ 34 & 8 & 56 \end{bmatrix}.$$

$(2AB)^T$ and $2(AB)^T$

$$AB = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 14 & 16 \\ 8 & 9 \end{bmatrix}.$$

$$2(AB)^T \quad 2\begin{bmatrix} 14 & 16 \\ 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 32 \\ 16 & 18 \end{bmatrix}.$$

A(C - 3E)

$$A\left(\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - 3\begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}\right)$$

$$A\left(\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -12 & 15 \\ 0 & 3 & 12 \\ 9 & 6 & 3 \end{bmatrix}\right)$$

$$A\begin{bmatrix} -3 & 11 & -12 \\ 4 & -2 & -7 \\ -7 & -5 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}\begin{bmatrix} -3 & 11 & -12 \\ 4 & -2 & -7 \\ -7 & -5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -3+8+21 & 11-4-15 & -12-14+18 \\ -6+4-28 & 22-2-20 & -24-7+24 \end{bmatrix}$$

$$\begin{bmatrix} -16 & -8 & -8 \\ -30 & 0 & -7 \end{bmatrix}$$

A^T or $(A^T)^T$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}.$$

$(AB)^T$ or $B^T A^T \Rightarrow B^T A^T = (AB)^T$

$$AB = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 14 & 16 \\ 8 & 9 \end{bmatrix}.$$

$(C+E)^T B$ and $C^T B + E^T B$

$(C+E)^T B$

$$C+E = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

$$(C+E)^T = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

$$(C+E)^T B = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5+8+15 & 0+4+10 \\ -5+4+9 & 0+2+6 \\ 8+18+12 & 0+9+8 \end{bmatrix}$$

$$\begin{bmatrix} 28 & 14 \\ 8 & 8 \\ 38 & 17 \end{bmatrix}.$$

A(2B) and 2(AB)

$$AB = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}.$$

$$2(AB) = \begin{bmatrix} 28 & 16 \\ 32 & 18 \end{bmatrix}.$$

Question: 16

$$A = \begin{bmatrix} 1 & 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 4 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 0 & 1 \end{bmatrix}$$

AB^T

$$B^T = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$$

$$AB^T = \begin{bmatrix} 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8-6 \end{bmatrix}.$$

$$= \begin{bmatrix} 3 \end{bmatrix}.$$

CA^T

$$\begin{bmatrix} -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} -3+0-3 \end{bmatrix} = \begin{bmatrix} -6 \end{bmatrix}$$

$(BA^T)C$

$$\left(\begin{bmatrix} -1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \right) \begin{bmatrix} -3 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1+8-6 \end{bmatrix} \begin{bmatrix} -3 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} -3 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} -3 & 0 & 1 \end{bmatrix}.$$

$A^T B$

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} -1 & 4 & 2 \end{bmatrix}.$$

$$\begin{bmatrix} -1 & 4 & 2 \\ -2 & 8 & 4 \\ 3 & -12 & -6 \end{bmatrix}.$$

CC^T

$$\begin{bmatrix} -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} +9+0+1 \end{bmatrix} = \begin{bmatrix} 10 \end{bmatrix}.$$

$C^T C$

$$\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -3 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 9 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 1 \end{bmatrix} =$$

17:

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

a) (1,2) entry

$$\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2\times -1 + 3\times 2 \end{bmatrix}$$

$$\begin{bmatrix} -2+6 \end{bmatrix} = \begin{bmatrix} 4 \end{bmatrix}$$

b) (2,3) entry

$$\begin{bmatrix} -1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -1\times 3 + 4\times 4 \end{bmatrix}$$

$$\begin{bmatrix} -3+16 \end{bmatrix} = 13$$

c) (3,1) entry

$$\begin{bmatrix} 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0 + 3 = 3$$

d) (3,3) entry

$$\begin{bmatrix} 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 0+12 = 12$$

18: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

DI_2 and I_2D

$$DI_2 = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 0+3 \\ -1+0 & 0-2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}.$$

$$I_2D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 3+0 \\ 0+1 & 0-2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}.$$

19: $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$.

$AB \neq BA$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & -1+8 \\ 6-6 & -3+8 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix}.$$

$$BA = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 4-2 \\ -3+12 & -6+8 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}.$$

21:

a) the First column

$$A\,col_1 B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + -3 + 8 \\ 3 + 6 + 16 \\ 4 + -6 + 12 \\ 2 + 3 + 20 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 25 \\ 10 \\ 25 \end{bmatrix}.$$

b) the third column

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} -1 + 3 + 10 \\ -3 - 6 + 20 \\ -4 + 6 + 15 \\ -2 - 3 + 25 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \\ 17 \\ 20 \end{bmatrix}.$$

20: the Second column

$$= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-3+4 \\ 0+6+8 \\ 0-6+6 \\ 0+3+10 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 0 \\ 13 \end{bmatrix}.$$

the Forth colu mn

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 2-4+2 \\ 6+8+4 \\ 8-8+3 \\ 4+4+5 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 3 \\ 13 \end{bmatrix}.$$

20. $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix}$ O is 3x2

compute AO

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AO = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2 x 3
3 x 2.

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

23. $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 2 & 3 \\ 5 & -1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

$$AC = \begin{bmatrix} 2 & -3 & 4 \\ -1 & 2 & 3 \\ 5 & -1 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 - 3 + 16 \\ -2 + 2 + 12 \\ 10 - 2 - 8 \end{bmatrix} = \begin{bmatrix} 17 \\ 12 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 17 \\ 12 \\ 1 \end{bmatrix} = 2 \cdot \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} + 1 \cdot \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix} + 4 \cdot \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}$$

24: $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 4 & 3 \\ 3 & 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 2 & 4 \end{bmatrix}$

AB is linear combination of column A

$$AB = \begin{bmatrix} 1-6-2 & -1-4-4 \\ 2+12+6 & -2+8+12 \\ 3+0-4 & -3+0-8 \end{bmatrix}.$$

$$= \begin{bmatrix} -7 & -9 \\ 20 & 18 \\ -1 & -11 \end{bmatrix}.$$

express the column of matrix in linear combination

$$\begin{bmatrix} -7 \\ 20 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}.$$

$$\begin{bmatrix} -9 \\ 18 \\ -11 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}.$$

25:-

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}.$$

a) $AB = 3a_1 + 5a_2 + 2a_3$

where a_j is the jth column of A $j = 1, 2, 3$

$$AB = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}.$$

$$\begin{bmatrix} 6 - 15 + 2 \\ 3 + 10 + 8 \end{bmatrix} = \begin{bmatrix} -7 \\ 21 \end{bmatrix}.$$

$3a_1 + 5a_2 + 2a_3$

$$3\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 5\begin{bmatrix} -3 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} -15 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 8 \end{bmatrix}.$$

$$\begin{bmatrix} 6 - 15 + 2 \\ 3 + 10 + 8 \end{bmatrix} = \begin{bmatrix} -7 \\ 21 \end{bmatrix}.$$

$AB = 3a_1 + 5a_2 + 2a_3.$

b) $AB = \begin{bmatrix} (\text{row}_1 (A)) B \\ (\text{row}_2 (A)) B \end{bmatrix}$.

$$AB = \begin{bmatrix} \begin{bmatrix} 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 15 + 2 \\ 3 + 10 + 8 \end{bmatrix} = \begin{bmatrix} -7 \\ 21 \end{bmatrix} = AB.$$

$\Downarrow$ find part (a).

26:

Find a value of r so that $AB^T = 0$ where

$A = \begin{bmatrix} r & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & -1 \end{bmatrix}$.

$$B^T = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$$AB^T = \begin{bmatrix} r & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}.$$

$$= \begin{bmatrix} r + 3 + 2 \end{bmatrix} = \begin{bmatrix} r + 5 \end{bmatrix}$$

$r+5=0$

$r=-5$

b) alternative of AB^t is BA^t

$$\begin{bmatrix} 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} r \\ 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} r+3+2 \end{bmatrix}$$

$$= r+5$$

$r+5$ is also equal to AB^t.

27: $A = \begin{bmatrix} 1 & r & 1 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 2 & 5 \end{bmatrix}$.

$AB^t = 0$.

$$AB^t = \begin{bmatrix} 1 & r & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 5 \end{bmatrix}$$

$$= -2+2r+5$$

$2r+5=2$

$2r=2-5$

if we take $s=2$ $r=0$

this means infinite many solution.