Differential Equations

CLO 4: Bloom Taxonomy Level < Applying >

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Section: A

Assignment : 04

Task :- Ex: 6.3 (q no. 1,3,5,7,9)

(1)

$$y'' = xy$$

$$y'' - xy = 0 \quad -(1)$$

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

putting values of y, y', y'' in eq(1)

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - x \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - \sum_{n=0}^{\infty} c_n x^{n+1} \quad -(2)$$

$k = n-2$	$k = n+1$
$n = k+2$	$n = k-1$

putting values in eq (2)

$$\sum_{k=0}^{\infty} (k+2)(k+1) c_{k+2} x^k - \sum_{k=1}^{\infty} c_{k-1} x^k = 0$$

$$(0+2)(0+1) c_{0+2} x^0 + \sum_{k=1}^{\infty} (k+2)(k+1) c_{k+2} x^k - \sum_{k=1}^{\infty} c_{k-1} x^k = 0$$

$$2C_2 + \sum_{k=1}^{\infty}\left[(k+2)(k+1)C_{k+2} - C_{k-1}\right]x^k = 0$$

Comparing co-efficients both side

$$2C_2 = 0$$

$$\boxed{C_2 = 0}$$

$$(k+1)(k+2)C_{k+2} - C_{k-1} = 0$$

$$(k+1)(k+2)C_{k+2} = C_{k-1}$$

$$C_{k+2} = \frac{C_{k-1}}{(k+1)(k+2)} \quad \because k = 1, 2, 3, \ldots$$

$k = 1$

$$C_{1+2} = \frac{C_{1-1}}{(1+1)(1+2)} = \frac{C_0}{(3)(2)} = \frac{C_0}{6}$$

$$C_3 = \tfrac{1}{6}C_0$$

$k = 2$

$$C_{2+2} = \frac{C_{2-1}}{(2+1)(2+2)} = \frac{C_1}{(4)(3)}$$

$$C_4 = C_1/12$$

$k = 3$

$$C_{3+2} = \frac{C_{3-1}}{(3+1)(3+2)} = \frac{C_2}{(4)(5)} = \frac{C_2}{20} \quad \because C_2 = 0$$

$$C_5 = 0$$

$k = 4$

$$C_{4+2} = \frac{C_{4-1}}{(4+1)(4+2)} = \frac{C_3}{(5)(6)} = \frac{C_3}{30} \quad \because C_3 = \frac{1}{6}C_0$$

$$= \frac{1}{30}\cdot\frac{1}{6}C_0 = \frac{1}{180}C_0$$

$k=5$

$$C_{5+2} = \frac{C_{5-1}}{(5+2)(5+1)} = \frac{C_4}{(7)(6)} = \frac{C_4}{42} \quad \because C_4 = \frac{1}{12}C_1$$

$$= \frac{1}{42}\cdot\frac{1}{12}C_1 = \frac{1}{504}C_1$$

$k=6$

$$C_{6+2} = \frac{C_{6-1}}{(6+2)(6+1)} = \frac{1}{(8)(7)}C_5 = \frac{1}{56}C_5 \quad \because C_5 = 0$$

$$= \frac{1}{56}\cdot 0$$

$$C_8 = 0$$

putting values in

$$y(x) = \sum_{n=0}^{\infty} C_n x^n$$

$$y(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots$$

$$= C_0 + C_1 x + (0)x^2 + \frac{1}{6}C_0 x^3 + \frac{1}{12}C_1 x^4 + (0)x^5 + \frac{1}{180}C_0 x^6 + \frac{1}{504}C_1 x^7 + (0)x^8 + \cdots$$

$$= C_0 + C_1 x + \frac{1}{6}C_0 x^3 + \frac{1}{12}C_1 x^4 + \frac{1}{180}C_0 x^6 + \frac{1}{504}C_1 x^7 + \cdots$$

$$= C_0\left(1 + \frac{1}{6}x^3 + \frac{1}{180}x^6 + \cdots\right) + C_1\left(x + \frac{1}{12}x^4 + \frac{1}{504}x^7 + \cdots\right)$$

$$y_1(x) = 1 + \frac{1}{6}x^3 + \frac{1}{180}x^6 + \cdots \quad (\text{at } x=0)$$

$$y_2(x) = x + \frac{1}{12}x^4 + \frac{1}{504}x^7 + \cdots$$

$$y'' - 2xy' + y = 0 \quad -(1)$$

$$y = \sum_{n=0}^{\infty} C_n x^n \quad -(2)$$

$$y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$$

putting values in eq (1)

$$\sum_{n=2}^{\infty} n(n-1)C_n x^{n-2} - 2x \sum_{n=1}^{\infty} n C_n x^{n-1} + \sum_{n=0}^{\infty} C_n x^n = 0$$

$k = n-2$ $n = k+2$	$k = n$	$k = n$

$$\sum_{n=2}^{\infty} n(n-1)C_n x^{n-2} - 2\sum_{n=1}^{\infty} n C_n x^n + \sum_{n=0}^{\infty} C_n x^n = 0 \quad -(3)$$

Substituting values in eq (3)

$$\sum_{k=0}^{\infty} (k+2)(k+1) C_{k+2} x^k - 2\sum_{k=1}^{\infty} k C_k x^k + \sum_{k=0}^{\infty} C_k x^k = 0$$

$$(2)(1) C_{0+2} x^0 + \sum_{k=1}^{\infty} (k+1)(k+2) C_{k+2} x^k - 2\sum_{k=1}^{\infty} k C_k x^k + C_0 + \sum_{k=1}^{\infty} C_k x^k = 0$$

$$2C_2 + \sum_{k=1}^{\infty} (k+1)(k+2) C_{k+2} x^k - 2\sum_{k=1}^{\infty} k C_k x^k + C_0 + \sum_{k=1}^{\infty} C_k x^k = 0$$

$$C_0 + 2C_2 + \sum_{k=1}^{\infty} \left[(k+1)(k+2) C_{k+2} - (2k-1) C_k \right] x^k = 0$$

$$2C_2 + C_0 = 0$$

$$2C_2 = -C_0$$

$$\boxed{C_2 = -\frac{1}{2} C_0}$$

$$(k+2)(k+1)c_{k+2} - (2k-1)c_k = 0$$

$$(k+2)(k+1)c_{k+2} = (2k-1)c_k$$

$$c_{k+2} = \frac{(2k-1)}{(k+1)(k+2)} c_k$$

$[C_0 = 0 , C_1 \neq 0]$

Using $c_2 = -\frac{1}{2}c_0$, the recurrance relation

$k=1$

$$c_{k+2} = \frac{2k-1}{(k+1)(k+2)} c_k \quad \because k = 1, 2, 3, \dots$$

$k=0$

$$c_{0+2} = \frac{0 \cdot 2 - 1}{(0+1)(0+2)} c_0$$

$$\boxed{c_2 = 0}$$

$$c_3 = \frac{2 \cdot 1 - 1}{(1+2)(1+1)} c_1$$

$$c_3 = \frac{1}{(3)(2)} c_1 = \frac{1}{6} c_1$$

$k=2$

$$c_4 = \frac{2 \cdot 2 - 1}{(2+1)(2+2)} c_2 = \frac{3}{(3)(4)} c_2 = \frac{1}{4} c_2 = \frac{1}{4} \cdot 0 = 0$$

$k=3$

$$c_5 = \frac{3 \cdot 2 - 1}{(3+1)(3+2)} c_3 = \frac{5}{(4)(5)} c_3 = \frac{1}{4} c_3 = \frac{1}{4} \cdot \frac{1}{6} c_1 = \frac{1}{24} c_1$$

$k=4$

$$c_6 = \frac{4 \cdot 2 - 1}{(4+1)(4+2)} c_4 = \frac{7}{(5)(6)} c_4 = \frac{7}{30} c_4 = \frac{7}{30} \frac{1}{4} c_2$$

$$= \frac{7}{120} c_2 = \frac{7}{120} \cdot 0 = 0 \qquad \because c_2 = 0$$

$k=5$

$$c_7 = \frac{2 \cdot 5 - 1}{(5+1)(5+2)} c_5 = \frac{9}{(6)(7)} c_5 = \frac{9}{42} c_5 \Rightarrow \frac{9}{42} \cdot \frac{1}{24} c_1$$

$$c_7 = \frac{1}{112} c_1$$

putting values in eq(2)

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$= C_0 + C_1 x^1 + C_2 x^2 + C_3 x^3 + \cdots$$

$$= C_0 + C_1 x + 0 x^2 + \frac{1}{6} C_1 x^3 + 0 \cdot x^4 + \frac{1}{24} C_1 x^5 + 0 x^6 + \frac{1}{112} C_1 x^7 + \cdots$$

$$= C_1 x + \frac{1}{6} C_1 x^3 + \frac{1}{24} C_1 x^5 + \frac{1}{112} C_1 x^7 + \cdots$$

$$= C_1 \left(x + \frac{1}{6} x^3 + \frac{1}{24} x^5 + \frac{1}{112} x^7 + \cdots \right)$$

$$y_1(x) = x + \frac{1}{6} x^3 + \frac{1}{24} x^5 + \frac{1}{112} x^7 + \cdots$$

(5)

$$y'' + x^2 y' + x y = 0 \quad -(1)$$

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y' = \sum_{n=1}^{\infty} n C_n x^{n-1}, \quad y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$$

putting values in eq (1)

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + x^2 \sum_{n=1}^{\infty} n C_n x^{n-1} + x \sum_{n=0}^{\infty} C_n x^n = 0$$

$k = n-2$	$k = n+1$	$k = n+1$
$n = k+2$	$n = k-1$	$n = k-1$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + \sum_{n=1}^{\infty} n C_n x^{n+1} + \sum_{n=0}^{\infty} C_n x^{n+1} = 0 \quad -(3)$$

putting values of 'k' in eq (3)

$$\sum_{k=0}^{\infty} (k+2)(k+1) C_{k+2} x^k + \sum_{k=2}^{\infty} (k+1) C_{k+1} x^k + \sum_{k=1}^{\infty} C_{k-1} x^k = 0$$

$$(0+2)(0+1)c_{0+2}x^0 + (1+2)(1+1)c_{1+2}x^1 + \sum_{k=2}^{\infty}(k+2)(k+1)c_{k+2}x^k$$
$$+ \sum_{k=2}^{\infty}(k-1)c_{k-1}x^k + \sum_{k=2}^{\infty}c_{k-1}x^k + c_0x^1 = 0$$

$$2c_2 + 6c_3x + c_0x + \sum_{k=2}^{\infty}(k+2)(k+1)c_{k+2}x^k + \sum_{k=2}^{\infty}(k-1)c_{k-1}x^k + \sum_{k=2}^{\infty}c_{k-1}x^k = 0$$

$$2c_2 + (6c_3 + c_0)x + \sum_{k=2}^{\infty}\left[(k+2)(k+1)c_{k+2} + (k-1)c_{k-1} + c_{k-1}\right]x^k = 0$$

$$2c_2 + (6c_3 + c_0)x + \sum_{k=2}^{\infty}\left[(k+2)(k+1)c_{k+2} + (k-1+1)c_{k-1}\right]x^k = 0$$

$$2c_2 + (6c_3 + c_0)x + \sum_{k=2}^{\infty}\left[(k+2)(k+1)c_{k+2} + kc_{k-1}\right]x^k = 0$$

$$2c_2 = 0$$
$$\boxed{c_2 = 0}$$
$$6c_3 + c_0 = 0$$
$$6c_3 = -c_0$$
$$\boxed{c_3 = -\frac{1}{6}c_0}$$

$$(k+1)(k+2)c_{k+2} = -kc_{k-1}$$

$$c_{k+2} = -\frac{k}{(k+1)(k+2)}c_{k-1}$$

$k=2$

$$c_4 = -\frac{2}{(2+1)(2+2)}c_{2-1} = -\frac{2}{12}c_1 = -\frac{1}{6}c_1$$

$k=3$

$$c_5 = -\frac{5}{(5+1)(5+2)}c_{3-1} = -\frac{5}{42}c_2 = -\frac{5}{42}(0) = 0$$

$k=4$

$$c_6 = -\frac{4}{(4+1)(4+2)} c_{4-1} = -\frac{4}{30} c_3 = -\frac{4}{30}\cdot\left(-\frac{1}{6}c_0\right)$$

$$= \frac{1}{45} c_0$$

$k=5$

$$c_7 = -\frac{5}{(5+1)(5+2)} c_4 = \frac{-5}{42} c_4 = \frac{-5}{42}\left(-\frac{1}{6}c_1\right)$$

$$= \frac{5}{252} c_1$$

$k=6$

$$c_8 = -\frac{6}{(6+1)(6+2)} c_5 = -\frac{6}{56} c_5 = -\frac{6}{56}(0) = 0$$

putting values in eq (2)

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

$$= c_0 + c_1 x + (0)x^2 + \left(-\frac{1}{6}c_0\right)x^3 + \left(-\frac{1}{6}c_1\right)x^4 + (0)x^5 + \frac{1}{45}c_0 x^6 + \frac{5}{252}c_1 x^7 + (0)x^8 + \dots$$

$$= c_0\left(1 - \frac{1}{6}x^3 + \frac{1}{45}x^6 + \dots\right) + c_1\left(x - \frac{1}{6}x^4 + \frac{5}{252}x^7 + \dots\right)$$

$$= c_0 y_1(x) + c_1 y_2(x)$$

power series sol of D.E:

$$y_1(x) = 1 - \frac{1}{6}x^3 + \frac{1}{45}x^6 + \dots$$

$$y_2(x) = x - \frac{1}{6}x^4 + \frac{5}{252}x^7 + \dots$$

$$(x^2-1)y'' + 4xy' + 2y = 0$$

$$y = \sum_{n=0}^{\infty} c_n x^n \quad \text{—(2)}$$

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}, \quad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$(x^2-1)y'' + 4xy' + 2y = 0$$

$$(x^2-1)\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + 4x \sum_{n=1}^{\infty} n c_n x^{n-1} + 2\sum_{n=0}^{\infty} c_n x^n = 0$$

$$x^2 \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1) c_n x^{n-1} + 4x \sum_{n=1}^{\infty} n c_n x^{n-1} + 2\sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^n - \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + 4\sum_{n=1}^{\infty} n c_n x^n + 2\sum_{n=0}^{\infty} c_n x^n = 0$$

$k = n$ | $k = n-2$, $n = k+2$ | $k = n$ | $k = n$

$$\sum_{k=2}^{\infty} k(k-1) c_k x^k - \sum_{k=0}^{\infty} (k+1)\overset{(k+2)}{(\cancel{k})} c_{k+2} x^k + 4\sum_{k=1}^{\infty} k c_k x^k + 2\sum_{k=0}^{\infty} c_k x^k = 0$$

$$\sum_{k=2}^{\infty} k(k-1) c_k x^k - 2c_2 - 6c_3 x - \sum_{k=2}^{\infty} (k+1)(k+2) c_{k+2} x^k + 4c_1 x + 4\sum_{k=2}^{\infty} k c_k x^k + 2c_0 + 2c_1 x + 2\sum_{k=2}^{\infty} c_k x^k = 0$$

$$-2c_2 + 2c_0 + x(-6c_3 + 6c_1) + \sum_{k=2}^{\infty} (k^2-k) c_k x^k - \sum_{k=2}^{\infty} (k^2+3k+$$

$$\sum_{k=2}^{\infty}(k^2+3k+2)\,c_{k+2}x^k + 4\sum_{k=2}^{\infty}k c_k x^k + 2\sum_{k=2}^{\infty}c_k x^k = 0$$

$$-2c_2+2c_0+x(-6c_3+6c_1)+\sum_{k=2}^{\infty}\Big[(k^2+3k+2)c_k - (k^2+3k+2)\,c_{k+2}\Big]x^k = 0$$

Comparing co-efficients both sides

① $-2c_2+2c_0=0$

$\cancel{+2}c_2 = \cancel{+2}c_0$

$\boxed{c_2 = c_0}$

② $-6c_3+6c_1=0$

$\boxed{c_3 = c_1}$

③ $(k^2+3k+2)c_k - (k^2+3k+2)c_{k+2} = 0$

$\cancel{(k^2+3k+2)}c_k = \cancel{(k^2+3k+2)}\,c_{k+2}$

$c_k = c_{k+2}$

$k=1$:

$c_1 = c_3 = c_1$

$k=2$:

$c_2 = c_4 = c_0$

$k=3$:

$c_3 = c_5 = c_1$

$k=4$:

$c_4 = c_6 = c_0$

$k=5$:

$c_5 = c_7 = c_1$

$k=6$:

$c_6 = c_8 = c_0$

$k=7$:

$c_7 = c_9 = c_1$

$\vdots \qquad \vdots$

putting values in eq (2)

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$= C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots$$

$$= C_0 + C_1 x + C_0 x^2 + C_1 x^3 + C_0 x^4 + C_1 x^5 + \cdots$$

$$= C_0 (1 + x^2 + x^4 + \cdots) + C_1 (x + x^3 + x^5 + x^7 + \cdots)$$

power series of D.E:-

$$y_1 = 1 + x^2 + x^4 + \cdots$$

$$y_2 = x + x^3 + x^5 + x^7 + \cdots$$

$$y_1 = \sum_{n=0}^{\infty} x^{2n} \quad , \quad y_2 = \sum_{n=0}^{\infty} x^{2n+1}$$

(7)

$$(x-1) y'' + y' = 0 \quad -(1)$$

$$y = \sum_{n=0}^{\infty} c_n x^n \quad -(2)$$

$$y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$$

putting values in eq (1)

$$(x-1) \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + \sum_{n=1}^{\infty} n C_n x^{n-1} = 0$$

$$x \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + \sum_{n=1}^{\infty} n C_n x^{n-1} = 0$$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-1} - \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + \sum_{n=1}^{\infty} n C_n x^{n-1} =$$

$k = n-1$	$k = n-2$	$k = n-1$
$n = k+1$	$n = k+2$	$n = k+1$

$$\sum_{k=1}^{\infty}(k+1)(k)c_{k+1}x^k - \sum_{k=0}^{\infty}(k+2)(k+1)c_{k+2}x^k +$$

$$\sum_{k=0}^{\infty}(k+1)c_{k+1}x^k = 0$$

$$\sum_{k=1}^{\infty}k(k+1)c_{k+1}x^k - (0+2)(0+1)c_{0+2}x^0 - \sum_{k=1}^{\infty}(k+2)(k+1)c_{k+2}x^k +$$

$$(0+1)c_{0+1}x^0 + \sum_{k=1}^{\infty}(k+1)c_{k+1}x^k = 0$$

$$-2c_2 + c_1 + \sum_{k=1}^{\infty}k(k+1)c_{k+1}x^k - \sum_{k=1}^{\infty}(k+2)(k+1)c_{k+2}x^k +$$

$$\sum_{k=1}^{\infty}(k+1)c_{k+1}x^k = 0$$

$$-2c_2 + c_1 + \sum_{k=1}^{\infty}\left[k(k+1)c_{k+1} - (k+2)(k+1)c_{k+2} + (k+1)c_{k+1}\right]x$$

Comparing co-efficients

① $-2c_2 + c_1 = 0$

$$-2c_2 = -c_1$$

$$c_2 = \frac{1}{2}c_1$$

② $k(k+1)c_{k+1} - (k+2)(k+1)c_{k+2} + (k+1)c_{k+1} = 0$

$$\left[c_{k+1}k - (k+2)c_{k+2} + c_{k+1}\right](k+1) = 0$$

$$kc_{k+1} - (k+2)c_{k+2} + c_{k+1} = 0$$

$$(k+1)c_{k+1} - (k+2)c_{k+2} = 0$$

$$\cancel{-}(k+2)c_{k+2} = \cancel{-}(k+1)c_{k+1}$$

$c_0 \neq 0, c_1 = 0)$

$$c_{k+2} = \frac{(k+1)c_{k+1}}{(k+2)} \quad ; k = 1, 2, 3, \ldots$$

$k = 1:$

$$c_3 = \frac{1+1}{1+2} c_{1+1} = \frac{2}{3} c_2 \quad \because c_2 = 0$$

$$c_3 = 0$$

$k = 2:$

$$c_4 = \frac{2+1}{2+2} c_{2+1} = \frac{3}{4} c_3 \quad \because c_3 = 0$$

$$c_4 = 0$$

$k = 3:$

$$c_5 = \frac{3+1}{3+2} c_{3+1} = \frac{4}{5} c_4 \quad \because c_4 = 0$$

$$c_5 = 0$$

$k = 4:$

$$c_6 = \frac{4+1}{4+2} c_{4+1} = \frac{5}{6} c_5 \quad \because c_5 = 0$$

$$c_6 = 0$$

$\vdots$

putting values in eq (2)

$$y_1 = c_0 x + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$$

$$= c_0 + (0)x + (0)x^2 + (0)x^3 + \cdots$$

$$= c_0 + 0$$

$$\boxed{y_1 = c_0}$$

$c_0 = 0, \; c_1 \neq 0$ $\quad \because c_2 = \frac{1}{2} c_1$

$$c_{k+2} = \frac{(k+1)c_{k+1}}{k+2} \quad \because k = 1, 2, 3, \ldots$$

$k = 1 \rightarrow$

$$c_3 = \frac{1+1}{1+2} c_{1+1} = \frac{2}{3} c_2 = \frac{2}{3}\left(\frac{1}{2} c_1\right) = \frac{1}{3} c_1$$

$k=2$:

$$c_4 = \frac{2+1}{2+2} c_{2+1} = \frac{3}{4} c_3 = \frac{3}{4}\left(\frac{1}{3} c_1\right) = \frac{1}{4} c_1$$

$k=3$:

$$c_5 = \frac{3+1}{3+2} c_{3+1} = \frac{4}{5} c_4 = \frac{4}{5}\left(\frac{1}{4} c_1\right) = \frac{1}{5} c_1$$

$k=4$:

$$c_6 = \frac{4+1}{4+2} c_{4+1} = \frac{5}{6} c_5 = \frac{5}{6}\left(\frac{1}{5} c_1\right) = \frac{1}{6} c_1$$

$k=5$:

$$c_7 = \frac{5+1}{5+2} c_{5+1} = \frac{6}{7} c_6 = \frac{6}{7}\left(\frac{1}{6} c_1\right) = \frac{1}{7} c_1$$

$k=6$:

$$c_8 = \frac{6+1}{6+2} c_{6+1} = \frac{7}{8} c_6 = \frac{7}{8}\left(\frac{1}{7} c_1\right) = \frac{1}{8} c_1$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$c_n = \frac{1}{n} c_1$$

$$y_2 = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$$

$$= 0 + c_1 x + \left(\frac{1}{2} c_1\right) x^2 + \left(\frac{1}{3} c_1\right) x^3 + \left(\frac{1}{4} c_1\right) x^4 + \left(\frac{1}{5} c_1\right) x^5 + \left(\frac{1}{6} c_1\right) x^6 + \left(\frac{1}{7} c_1\right) x^7 + \left(\frac{1}{8} c_1\right) x^8 + \cdots$$

$$= c_1 x + \frac{1}{2} c_1 x^2 + \frac{1}{3} c_1 x^3 + \frac{1}{4} c_1 x^4 + \frac{1}{5} c_1 x^5 + \frac{1}{6} c_1 x^6 + \cdots$$

$$\boxed{y_2 = c_1 \sum_{n=1}^{\infty} \frac{x^n}{n}}$$

QNo:-03 (Remaining)

$c_0 \neq 0, \; c_1 = 0$

As $n \geq 3$ for 'c_n'

$$c_{k+2} = \frac{2k-1}{(k+2)(k+1)} c_k \qquad \because k = 1, 2, 3, \dots$$

$k=1$:

$$c_3 = \frac{2.1-1}{(1+2)(1+1)} c_1 = \frac{1}{6} c_1 = \frac{1}{6}(0) = 0$$

$k=2$:

$$c_4 = \frac{2.2-1}{(2+2)(2+1)} c_2 = \frac{3}{12} c_2 = \frac{1}{4} c_2 = \frac{1}{4}\left(-\frac{1}{2} c_0\right)$$

$$c_4 = -\frac{1}{8} c_0$$

$k=3$:

$$c_5 = \frac{2.3-1}{(3+1)(3+2)} c_3 = \frac{5}{20} c_3 = \frac{1}{4} c_3 \qquad \because c_3 = 0$$

$$c_5 = \frac{1}{4}(0) = 0$$

$k=4$:

$$c_6 = \frac{2.4-1}{(4+2)(4+1)} c_4 = \frac{7}{30} c_4 \qquad \because c_4 = -\frac{1}{8} c_0$$

$$= \frac{7}{30}\left(-\frac{1}{8} c_0\right) = -\frac{7}{240} c_0$$

putting values in eq (2)

$$y_2 = c_0 + 0.x + \left(-\frac{1}{2} c_0\right) x^2 + 0.x^3 + \left(-\frac{1}{8} c_0\right) x^4 + 0.x^5 + \left(-\frac{7}{240} c_0\right) x^6 + 0.x^7$$

$$y_2 = c_0 - \frac{1}{2} c_0 x^2 - \frac{1}{8} c_0 x^4 - \frac{7}{240} c_0 x^6 + \dots \Rightarrow c_0\left(1 - \frac{1}{2!} x^2 - \frac{3}{4!} - \frac{21}{6!} + \dots\right)$$