Exercise 4.9

Find a basis for row space of A consisting of vector that (a) are not necessarily row vector of A; and (b) are row vector of A

Question: 05

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 9 & -1 \\ -3 & 8 & 3 \\ -2 & 3 & 2 \end{bmatrix}$$

(a) are not necessarily row vector of A

Vector A → RREF → leading 1 rows contain that are not row vector of A.

$$\begin{array}{l} R_2 - R_1 \\ R_3 + 3R_1 \\ R_4 + 2R_1 \\ \\ \end{array} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 7 & 0 \\ 0 & 14 & 0 \\ 0 & 6 & 0 \end{bmatrix}$$

$$R_2/7 \quad \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 14 & 0 \\ 0 & 6 & 0 \end{bmatrix}$$

$$\begin{array}{l} R_1-2R_2 \\ R_3-14R_2 \\ R_4-7R_2 \end{array} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis of row Space that are not rows (vector) of A is

$\{w_1 = (1, 0, -1) \; ; \; w_2 = (0, 1, 0)\}$

(b) are row vector of A.

$A \to A^T \to$ argumented matrix $[1|0] \to$ leading 1 in RREF occur in column $\to$ contain from Real matrix A rows.

$$[A^T|0] = \left[\begin{array}{cccc|c} 1 & 1 & -3 & -2 & 0 \\ 2 & 9 & 8 & 3 & 0 \\ -1 & -1 & 3 & 2 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2-2R_1 \\ R_3+R_1 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & -3 & -2 & 0 \\ 0 & 7 & 14 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_2/7 \left[\begin{array}{cccc|c} 1 & 1 & -3 & -2 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 - R_2$
$$\left[\begin{array}{cccc|c} 1 & 0 & -5 & -3 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

or)

leading one occur in column 1 and 2.
we conclude the 1st row and 2nd row
OF A form a basis for row Space OF A.

$\{ w_1 = (1, 2, -1), \; w_2 = (1, 9, -1) \}$
form the basis for row space OF A
Which are row vector of A.

1 in

Question: 06.

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 5 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

(a) are not row vector OF A.

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 5 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

$R_2 - 3R_1$
$R_4 + R_1$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -1 & 5 & -9 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & -3 & 10 \end{bmatrix}.$$

$-R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -5 & 9 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & -3 & 10 \end{bmatrix}.$$

$R_1 - 2R_2$
$R_3 - R_2$
$R_4 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 9 & -15 \\ 0 & 1 & -5 & 9 \\ 0 & 0 & 7 & -8 \\ 0 & 0 & 7 & -8 \end{bmatrix}.$$

$R_4 - R_3$

$$\begin{bmatrix} 1 & 0 & 9 & -15 \\ 0 & 1 & -5 & 9 \\ 0 & 0 & 7 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$R_3/7$

$$\begin{bmatrix} 1 & 0 & 9 & -15 \\ 0 & 1 & -5 & 9 \\ 0 & 0 & 1 & -8/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$R_1 - 9R_3$
$R_2 + 5R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{33}{7} \\ 0 & 1 & 0 & 23/7 \\ 0 & 0 & 1 & -8/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis of row space that are not from rows (vector) of A is

$\{w_1 = (1, 0, 0, -33/7), w_2 = (0, 1, 0, 23/7)$
$w_3 = (0, 0, 1, -8/7)\}$

(b) are row vector of A

$$[A^T | 0] = \left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & 0 \\ 2 & 5 & 1 & 0 & 0 \\ -1 & 2 & 2 & -2 & 0 \\ 3 & 0 & 1 & 7 & 0 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 + R_1$
$R_4 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 5 & 2 & -3 & 0 \\ 0 & -9 & 1 & 10 & 0 \end{array}\right]$$

$$-R_2 \quad \left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 5 & 2 & -3 & 0 \\ 0 & -9 & 1 & 10 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1-3R_2 \\ R_3-5R_2 \\ R_4+9R_2 \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 5 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 7 & 7 & 0 \\ 0 & 0 & -8 & -8 & 0 \end{array}\right].$$

$$\begin{array}{l} R_3/7 \\ R_4/-8 \\ \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 5 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right].$$

$$\begin{array}{l} R_1-3R_3 \\ R_2+R_3 \\ R_4-R_3 \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

~~Proof~~: leading one vector occur in C_1, C_2, C_3 we conclude 1 row 2nd row and 3rd row of real vector A

$w_1 = \{(1,2,-1,3), (3,5,2,0), (0,1,2,1)\}$

is a basis of rous space A.

Find a basis for the column space of A consisting of vector that (a) are not necessarily coloumn vector of A (b) are column vectors of A.

Question: 07

$$A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

(a) are not necessarily column vector of A.

$A \to A^T \to RREF \to$ Taking leading one rows (ANS) in column.

$$A^T = \begin{bmatrix} 1 & 1 & 3 & 2 \\ -2 & -1 & 2 & 1 \\ 7 & 4 & -3 & -1 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$

R_2+2R_1
R_3-7R_1

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 8 & 5 \\ 0 & -3 & -24 & -15 \\ 0 & 0 & 5 & 3 \end{bmatrix}.$$

$R_3/-3$

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 8 & 5 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 5 & 3 \end{bmatrix}.$$

R_3-R_2

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}.$$

R_1-R_2
$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 0 & -5 & -3 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

R_1+R_3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$R_3/5$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 1 & 3/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2 - 8R_3$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/5 \\ 0 & 0 & 1 & 3/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

leading one rows taken in column which are basis of column space of A consist of vector that are not column vector of A

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1/5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3/5 \end{bmatrix} \right\}$$

(b) are column vector of A.

A → [A|B] → RREF → leading occur in column in [A|b] taken column in Real A which are column vector of A.

$$\boxed{A|b} = \left[\begin{array}{cccc|c} 1 & -2 & 7 & 0 & 0 \\ 1 & -1 & 4 & 0 & 0 \\ 3 & 2 & -3 & 5 & 0 \\ 2 & 1 & -1 & 3 & 0 \end{array}\right].$$

$R_2 - R_1$
$R_3 - 3R_1$
$R_4 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & -2 & 7 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 8 & -24 & 5 & 0 \\ 0 & 5 & -15 & 3 & 0 \end{array}\right].$$

$R_1 + 2R_2$
$R_3 - 8R_2$
$R_4 - 5R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right].$$

$R_3/5$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right].$$

$R_4 - 3R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

leading 1 occur in C_1, C_2, C_4 we conclude 1st, 2nd and 4th column of A form a basis of column space of A

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \\ 3 \end{bmatrix} \right\}$$

Question:08

$$A = \begin{bmatrix} -2 & 2 & 3 & 7 & 1 \\ -2 & 2 & 4 & 8 & 0 \\ -3 & 3 & 2 & 8 & 4 \\ 4 & -2 & 1 & -5 & -7 \end{bmatrix}$$

(a) are not column vector of A

$$A^T = \begin{bmatrix} -2 & -2 & -3 & 4 \\ 2 & 2 & 3 & -2 \\ 3 & 4 & 2 & 1 \\ 7 & 8 & 8 & -5 \\ 1 & 0 & 4 & -7 \end{bmatrix}$$

$R_1 \leftrightarrow R_5$
$$\begin{bmatrix} 1 & 0 & 4 & -7 \\ 2 & 2 & 3 & -2 \\ 3 & 4 & 2 & 1 \\ 7 & 8 & 8 & -5 \\ -2 & -2 & -3 & 4 \end{bmatrix}$$

$R_2 - 2R_1$
$R_3 - 3R_1$
$R_4 - 7R_1$
$R_5 + 2R_1$
$$\begin{bmatrix} 1 & 0 & 4 & -7 \\ 0 & 2 & -5 & 12 \\ 0 & 4 & -10 & 22 \\ 0 & 8 & -20 & 44 \\ 0 & -2 & 5 & -10 \end{bmatrix}$$

$R_2/2$
$$\begin{bmatrix} 1 & 0 & 4 & -7 \\ 0 & 1 & -5/2 & 6 \\ 0 & 4 & -10 & 22 \\ 0 & 8 & -20 & 44 \\ 0 & -2 & 5 & -10 \end{bmatrix}.$$

$R_3 - 4R_2$
$R_4 - 8R_2$
$R_5 + 2R_2$
$$\begin{bmatrix} 1 & 0 & 4 & -7 \\ 0 & 1 & -5/2 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$R_1 + 7R_3$
$R_2 - 6R_3$
$R_4 + 4R_3$
$R_5 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Taking leading one rows in column.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -5/2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

(b) are column vector of A.

$$[A|b] = \left[\begin{array}{ccccc|c} -2 & 2 & 3 & 7 & 1 & 0 \\ -2 & 2 & 4 & 8 & 0 & 0 \\ -3 & 3 & 2 & 8 & 4 & 0 \\ 4 & -2 & 1 & -5 & -7 & 0 \end{array}\right].$$

$R_2/-2 \leftrightarrow R_1$

$$\left[\begin{array}{ccccc|c} 1 & -1 & -2 & -4 & 0 & 0 \\ -2 & 2 & 3 & 7 & 1 & 0 \\ -3 & 3 & 2 & 8 & 4 & 0 \\ 4 & -2 & 1 & -5 & -7 & 0 \end{array}\right]$$

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R_2+2R_1
R_3+3R_1
R_4+4R_1

$$\left[\begin{array}{ccccc|c} 1 & -1 & -2 & -4 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & -4 & -4 & 4 & 0 \\ 0 & 2 & 9 & 11 & -7 & 0 \end{array}\right].$$

$R_3/-4$
$R_2 \Leftrightarrow R_4$

$$\left[\begin{array}{ccccc|c} 1 & -1 & -2 & -4 & 0 & 0 \\ 0 & 2 & 9 & 11 & -7 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{array}\right]$$

$R_2/2$
R_4+R_3

$$\left[\begin{array}{ccccc|c} 1 & -1 & -2 & -4 & 0 & 0 \\ 0 & 1 & 9/2 & 11/2 & -7/2 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

R_1+R_2

$$\left[\begin{array}{ccccc|c} 1 & 0 & 5/2 & 3/2 & -7/2 & 0 \\ 0 & 1 & 9/2 & 11/2 & -7/2 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

$R_1-5/2R_3$
$R_2-9/2R_3$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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leading column c_1, c_2, c_3

Taken c_1, c_2, c_3 in vector A

basis of column space of A.

$$\left\{ \begin{bmatrix} -2 \\ -2 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \\ 1 \end{bmatrix} \right\}$$

Note

A are not row vector of A	A are row vector of A	A are not column vector of A	A are column vector of A
vector A → RREF → leading one row that are not contain row vector of A (Answer).	$A \to A^T \to [1 \mid 0]$ → RREF → leading 1 occur in column from real vector A in rows.	$A \to A^T \to$ RREF → Taking leading rows in 1 column in (Answer)	$A \to [A \mid b] \to$ RREF → leading 1 column in $[A \mid b]$ contain column vector in real A.

Row rank and column Rank OF given matrix.

Row rank OF A = column Rank OF A

Rank Row matrix OF A

$A \to RREF \to$ leading 1 number of rows count

column Rank OF matrix A.

$A \to A^T \to RREF \to$ leading 1 column counts

row rank and column rank always be equal in matrix.

9) (a) $\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & -5 & -2 & 1 \\ 7 & 8 & -1 & 2 & 5 \end{bmatrix}$.

Row rank OF A.

$A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & -5 & -2 & 1 \\ 7 & 8 & -1 & 2 & 5 \end{bmatrix}$

R_2-3R_1
R_3-7R_1
$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & -5 & -14 & -8 & -2 \\ 0 & 4 & 6 & 4 & 3 \end{bmatrix}.$$

R_2+R_3
$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & -1 & -8 & -4 & 1 \\ 0 & 4 & 6 & 4 & 3 \end{bmatrix}.$$

$-R_2$
$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 1 & 8 & 4 & -1 \\ 0 & 4 & 6 & 4 & 3 \end{bmatrix}.$$

R_1-2R_2
R_3-4R_2
$$\begin{bmatrix} 1 & 0 & -13 & -6 & 3 \\ 0 & 1 & 8 & 4 & -1 \\ 0 & 0 & -26 & -12 & 7 \end{bmatrix}$$

$R_3/-26$
$$\begin{bmatrix} 1 & 0 & -13 & -6 & 3 \\ 0 & 1 & 8 & 4 & -1 \\ 0 & 0 & 1 & 6/13 & -7/26 \end{bmatrix}$$

R_1+13R_3
R_2+8R_3
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & 4/13 & 15/13 \\ 0 & 0 & 1 & 6/13 & -7/26 \end{bmatrix}$$

leading 1 rows are 3

number of non zero rows of A in RREF = 3.

~~so~~ Row Rank = column Rank = 3.

b) $A = \begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 2 & 1 & -5 & 1 & 2 & 0 \\ 3 & 2 & 5 & 1 & -2 & 1 \\ 5 & 8 & 9 & 1 & -2 & 2 \\ 9 & 9 & 4 & 2 & 0 & 2 \end{bmatrix}$.

$R_2 - 2R_1$
$R_3 - 3R_1$
$R_4 - 5R_1$
$R_5 - 9R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & -5 & -9 & 1 & 2 & -2 \\ 0 & -7 & -1 & 1 & -2 & -2 \\ 0 & -7 & -1 & 1 & -2 & -3 \\ 0 & -18 & -14 & 2 & 0 & -7 \end{bmatrix}.$$

$R_4 - R_3$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & -5 & -9 & 1 & 2 & -2 \\ 0 & -7 & -1 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & -3 & -1 & 0 & -2 \end{bmatrix}$$

$R_2 + 5R_5$
$R_3 + 7R_5$

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 0 & -24 & -4 & 2 & -14 \\ 0 & 0 & -22 & -6 & -2 & -14 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & -3 & -1 & 0 & -2 \end{bmatrix}$$

$R_3 - R_2$
$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 0 & -24 & -4 & 2 & -14 \\ 0 & 0 & 2 & 2 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & -3 & -1 & 0 & -2 \end{bmatrix}$$

$R_2/2$
$R_3/2$
$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 0 & -12 & -2 & 1 & -7 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & -3 & -1 & 0 & -2 \end{bmatrix}$$

$R_2 + 12R_3$
$R_5 + 3R_3$
$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 10 & -23 & -7 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 & -6 & -2 \end{bmatrix}$$

$R_2 \leftrightarrow R_5$
$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -6 & -2 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 10 & -23 & -7 \end{bmatrix}$$

$R_4 \leftrightarrow R_5$
$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -6 & -2 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 10 & -23 & -7 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$R_4/10$
$-R_5$
$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -6 & -2 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -23/10 & -7/10 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

leading 1 rows = 5
Rank row and column = 5.

Question: 10.

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 5 & 4 & 0 & -1 \\ 2 & -1 & 2 & 4 & 3 \end{bmatrix}.$$

$R_3 - 2R_1$
$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 5 & 4 & 0 & -1 \\ 0 & -5 & -4 & 0 & 1 \end{bmatrix}.$$

$$R_3 + R_2 \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 5 & 4 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2/5 \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 1 & 4/5 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Row rank = column rank = 2.

(b) $$\begin{bmatrix} 1 & 1 & -1 & 3 & 0 \\ 2 & -4 & 0 & 1 & 1 \\ 5 & -1 & -3 & 7 & 1 \\ 3 & -9 & 1 & 0 & 2 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 5R_1 \\ R_4 - 3R_1 \end{array} \begin{bmatrix} 1 & 1 & -1 & 3 & 0 \\ 0 & -6 & 2 & -5 & 1 \\ 0 & -6 & 2 & -8 & 1 \\ 0 & -12 & 4 & -9 & 2 \end{bmatrix}$$

$$\begin{array}{l} R_3 - R_2 \\ R_4 - 2R_2 \end{array} \begin{bmatrix} 1 & 1 & -1 & 3 & 0 \\ 0 & -6 & 2 & -5 & 1 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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$$\begin{array}{l} R_2/-6 \\ R_3/-3 \end{array} \begin{bmatrix} 1 & 1 & -1 & 3 & 0 \\ 0 & 1 & -1/3 & 5/6 & -1/6 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_4 - R_3 \begin{bmatrix} 1 & 1 & -1 & 3 & 0 \\ 0 & 1 & -1/3 & 5/6 & -1/6 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Row rank = column rank = 3

Find nullity [$\overset{\text{Basis}}{\text{dimension}}$ of null of solution space]

argument RREF with null vector

$\text{rank } A + \underset{\substack{\downarrow \\ \text{Dimension}}}{\text{nullity } A} = \text{number of columns.}$

13: (a)

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 6 & -8 & 1 \\ 5 & 3 & -2 & 10 \end{bmatrix}$$

$R_2 - 2R_1$
$R_3 - 5R_1$
$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 8 & -12 & -5 \\ 0 & 8 & -12 & -5 \end{bmatrix}$$

$R_3 - R_2$
$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 8 & -12 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2/8$
$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & -3/2 & -5/8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 + R_2$
$$\begin{bmatrix} 1 & 0 & 1/2 & 19/8 \\ 0 & 1 & -3/2 & -5/8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank OF $A = 2$.

$$\left[\begin{array}{cccc|c} 1 & 0 & 1/2 & 19/8 & 0 \\ 0 & 1 & -3/2 & -5/8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 1/2\, x_3 + 19/8\, x_4 = 0.$

$x_2 - 3/2\, x_3 - 5/8\, x_4 = 0.$

$x_3 = s \qquad x_4 = t$

$x_1 = -1/2\, s - 19/8\, t$

$x_2 = 3/2\, s + 5/8\, t$

$x_3 = s$

$x_4 = t$

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Basis of null

$$x = \begin{bmatrix} -1/2s - 19/8t \\ 3/2s + 5/8t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -1/2 \\ 3/2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -19/8 \\ 5/8 \\ 0 \\ 1 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} -1/2 \\ 3/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -19/8 \\ 5/8 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Nullity of A = 2

rank = 2 , Nullity = 2

(b) $$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 3 & 2 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{bmatrix}$$

$R_2 - 3R_1$
$R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -4 & -1 & -9 \\ 0 & -5 & 0 & -5 \end{bmatrix}$$

$R_2/-4$
$R_3/-5$

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1/4 & 9/4 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 - R_2$ $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1/4 & 9/4 \\ 0 & 0 & -1/4 & -5/4 \end{bmatrix}$

$-4R_3$ $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1/4 & 9/4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$R_2 - \frac{1}{4} R_3$ $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

Rank = 3.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 5 & 0 \end{array}\right]$$

$x_1 + x_4 = 0$

$x_2 + x_4 = 0$

$x_3 + 5x_4 = 0$

$x_4 = s$

$x_1 = -s$

$x_2 = -s$

$x_3 = -5s$

$x_4 = s$

Base of null

$$x = \begin{bmatrix} -s \\ -s \\ -5s \\ s \end{bmatrix} = s \cdot \begin{bmatrix} -1 \\ -1 \\ -5 \\ 1 \end{bmatrix}$$

Dimension : 1

Rank = 3 , Nullity = 1

14: (a) $\begin{bmatrix} 1 & 3 & -2 & 4 \\ -1 & 4 & -5 & 10 \\ 3 & 2 & 1 & -2 \\ 3 & -5 & 8 & -16 \end{bmatrix}$

$R_2 + R_1$

$R_3 - 3R_1$

$R_4 - 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 7 & -7 & 14 \\ 0 & -7 & 7 & -14 \\ 0 & -14 & 14 & -28 \end{bmatrix}$$

$R_3 + R_2$
$R_4 + 2R_2$

$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 7 & -7 & 14 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2/7$

$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - 3R_2$

$$\begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 2

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_3 - 2x_4 = 0$

$x_2 - x_3 + 2x_4 = 0$

$x_3 = s$

$x_4 = t$

$x_1 = -s + 2t$

$x_2 = s - 2t$

$x_3 = s$

$x_4 = t$

Basis

$$x = \begin{bmatrix} -s+2t \\ s-2t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}.$$

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Nullity = 2

Rank = 2.

(b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & -1 & 2 \end{bmatrix}$

$$\begin{array}{l} \\ R_2-2R_1 \\ R_4-R_1 \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -3 & -2 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -2 & 1 \end{bmatrix}.$$

$$R_2 \leftrightarrow R_3 \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & -3 & -2 & -2 \\ 0 & 0 & -2 & 1 \end{bmatrix}.$$

$$\begin{array}{l} R_1-R_2 \\ R_3+3R_2 \\ \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -5 & 4 \\ 0 & 0 & -2 & 1 \end{bmatrix}.$$

$$R3/-5 \quad \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -4/5 \\ 0 & 0 & -2 & 1 \end{bmatrix}$$

$$\begin{array}{l} R_1-2R_3 \\ R_2+R_3 \\ R_4+2R_3 \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & 3/5 \\ 0 & 1 & 0 & 6/5 \\ 0 & 0 & 1 & -4/5 \\ 0 & 0 & 0 & 0/5 \end{bmatrix}.$$

$\frac{5}{8}R_3$ $\begin{bmatrix} 1 & 0 & 0 & 3/5 \\ 0 & 1 & 0 & 6/5 \\ 0 & 0 & 1 & -4/5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_1 - 3/5 R_4$
$R_2 - \frac{6}{5} R_4$
$R_3 + \frac{4}{5} R_4$
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Rank $=4$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

$x_4 = 0$

Rank - 4, Nullity - 0

Exercise 4·3

Subspace

3D میں Plan اور جو بھی سے پاس ہوں وہ Subspace ہوتا ہے

Which of the given subset of R^3 are Subspace

5: The set of all vector of the form

(a) $\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$. In this case $c = 1$

set $w_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in R^3 ; c = 1$

$kw = \begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$

$2w = 2 \begin{bmatrix} a \\ b \\ 1 \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \\ 2 \end{bmatrix} \notin w_1$

Not close under multipli cal e.

It is not Subspace of R^3

(b) $\begin{bmatrix} a \\ b \\ a+2b \end{bmatrix}$ where $c = a+2b$

$$\underline{u} = \begin{bmatrix} a_1 \\ b_1 \\ a_1+2b_1 \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} a_2 \\ b_2 \\ a_2+2b_2 \end{bmatrix}.$$

$$\underline{u}+\underline{v} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ (a_1+2b_1)+(a_2+2b_2) \end{bmatrix}.$$

$$= \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ (a_1+a_2)+2(b_1+b_2) \end{bmatrix} \in W$$

let : $\begin{bmatrix} a \\ b \\ a+2b \end{bmatrix} \in W.$

c is a scalor

$$cW = c\begin{bmatrix} a \\ b \\ a+2b \end{bmatrix} = \begin{bmatrix} ca \\ cb \\ ca+2cb \end{bmatrix} \in W$$

It is a subspace.

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(c) $\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$ where $b=0$, $\cancel{c=0}$

let $\underline{u} = \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix}$.

$$u + v = \begin{bmatrix} a_1+a_2 \\ 0 \\ 0 \end{bmatrix} \in W$$

let $\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \in W$

c is scalar

$$cW = c\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ca \\ 0 \\ 0 \end{bmatrix} \in W$$

It is a subspace.

(d) $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ where $a+2b-c=0$.

$$\underline{u} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \in W$$

$$\underline{u}+\underline{v} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ c_1+c_2 \end{bmatrix} \in W.$$

$$a_1x+b_1y+c_1z + a_2x+b_2y+c_1z=0.$$

$$(a_1+a_2)x + (b_1+b_2)y + (c_1+c_2)z = 0.$$

let $c \in \mathbb{R}$

$$c\cdot\underline{u} = c\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$$

$$\begin{bmatrix} ca_1 \\ cb_1 \\ ce_1 \end{bmatrix} \in W$$

$$c(a_1x + b_1y + c_1z_1) = c\cdot 0$$

$$(ca_1)x + (cb_1)y + (ce_1)z) = 0.$$

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Hence w is subspace of $\mathbb{R}^3$

Question:06:

(a) $\begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$; $c=0$

$$\underline{u} = \begin{bmatrix} a_1 \\ b_1 \\ 0 \end{bmatrix}, \underline{v} \begin{bmatrix} a_2 \\ b_2 \\ 0 \end{bmatrix}.$$

$$\underline{u}+\underline{v} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ 0+0 \end{bmatrix} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ 0 \end{bmatrix} \in w.$$

$$c\,\underline{u} = c \begin{bmatrix} a_1 \\ b_1 \\ 0 \end{bmatrix} = \begin{bmatrix} ca_1 \\ cb_1 \\ c.0 \end{bmatrix} = \begin{bmatrix} ca_1 \\ cb_1 \\ 0 \end{bmatrix} \in w$$

It is a subspace of $\mathbb{R}^3$

(b) $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ where $a>0$

$$c\,\underline{u} = c \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = -1 \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -b_1 \\ -c_1 \end{bmatrix} \notin w$$

It is not subspace.

(c) $\begin{bmatrix} a \\ a \\ c \end{bmatrix}$ In this we said that $a=b$

$\underline{u} = \begin{bmatrix} a_1 \\ a_1 \\ c_1 \end{bmatrix}$ $\underline{v} \begin{bmatrix} a_2 \\ a_2 \\ c_2 \end{bmatrix}$.

$\underline{u} + \underline{v} = \begin{bmatrix} a_1+a_2 \\ a_1+a_2 \\ c_1+c_2 \end{bmatrix} \in W$

$c\,\underline{w} = c \cdot \begin{bmatrix} a_1 \\ a_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} ca_1 \\ ca_1 \\ cc_1 \end{bmatrix} \in W$

supspace.

(d) $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ **Where $2a-b+c=1$**

we have that $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \in W$

but $2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \notin W$

It is not closed under multiplication.

Not supspace.

Question: 07

Which of the given (Subspace) subset of R_4 are Subspace

(a) $[a, b \quad c \quad d]$ where $a-b=2$

$\underline{u} = [a_1 \quad b_1 \quad c_1 \quad d_1]$, $\underline{v} = [a_2 \quad b_2 \quad c_2 \quad d_2]$.

$\underline{u}+\underline{v} = [a_1+a_2 \quad b_1+b_2 \quad c_1+c_2 \quad d_1+d_2]$.

let $\underline{u}+\underline{v} = [4 \quad 2 \quad 0 \quad 0]$.

$4-2=2$.

but

$\underline{u} \cdot \underline{v} = [a_1 a_2 \quad b_1 b_2 \quad c_1 c_2 \quad d_1 d_2]$.

$\underline{u} \cdot \underline{v} = [4 \quad 2 \quad 0 \quad 0]$

$2 \cdot (\underline{u} \cdot \underline{v}) = 2[\cancel{8}4 \quad 2 \quad 0 \quad 0]$

$= [8 \quad 4 \quad 0 \quad 0]$

$8-4=4 \neq 2$

Not closed under multiplication

Not Subspace

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(b) $[a \ b \ c \ d]$, where $c = a+2b$ and
$d = a-3b$

let

$\underline{u} = [a_1 \ b_1 \ c_1 \ d_1]$ $\underline{v} = [a_2 \ b_2 \ c_2 \ d_2]$

$\underline{u}+\underline{v} = [a_1+a_2 \ \ b_1+b_2 \ \ c_1+c_2 \ \ d_1+d_2]$

$= [a_1+a_2 \ \ b_1+b_2 \ \ (a_1+2b_1)+(a_2+2b_2) \ \ (a_1-3b_1)+(a_2-3b_2)]$
$\in W$

$c \cdot \underline{u} = c \cdot [a_1 \ b_1 \ c_1 \ d_1]$

$= c \cdot [a_1 \ b_1 \ a_1+2b_1 \ a_1-3b_1]$

$= [ca_1 \ cb_1 \ ca+2cb \ ca-3cb] \in W$.

It is a subspace.

(c) $= [a \ b \ c \ d]$ where $a=0$ and
$b=-d$

$\underline{u}+\underline{v} = [a_1+a_2 \ b_1+b_2 \ c_1+c_2 \ d_1+d_2]$

$= [0+0 \ -d_1-d_1 \ c_1+c_2 \ d_1+d_2]$.

$= [0 \ -d_1-d_2 \ c_1+c_2 \ d_1+d_2] \in W$.

$$c \cdot \underline{u} = c \cdot [a_1 \quad b_1 \quad c_1 \quad d_1]$$
$$= c \cdot [0 \quad -d_1 \quad c_1 \quad d_1]$$

$$= [0 \quad -cd_1 \quad +cc_1 \quad cd_1] \in W.$$

It is subspace of $\mathbb{R}^4$.

Question: 08

$[a \ b \ c \ d]; a = b = 0$

$$\underline{u} + \underline{v} = [a_1 + a_2 \quad b_1 + b_2 \quad c_1 + c_2 \quad d_1 + d_2].$$

$$= [0 \quad 0 \quad c_1 + c_2 \quad d_1 + d_2] \in W,$$

$$c \cdot \underline{u} = c \cdot [a_1 \quad b_1 \quad c_1 \quad d_1]$$
$$= c \cdot [0 \quad 0 \quad c_1 \quad d_1]$$
$$= [0 \quad 0 \quad cc_1 \quad cd_1] \in W.$$

It is subspace of $\mathbb{R}^4$

(b) $[a \ b \ c \ d]$ Where $a = 1$ $b = 0$
$a + d = 1$

We have that

$$\underline{u} = [0 \quad 0 \quad 1 \quad 1]$$

$$c \cdot \underline{u} = c \cdot [0 \ 0 \quad 1 \ 1]$$
$$= 2 \cdot [0 \quad 0 \ 1 \ 1]$$
$$= [0 \quad 0 \quad 1 \quad 2] \notin W \quad \because 2 + 0 = 2 \neq 1.$$

Not subspace.

(c) $[a \quad b \quad c \quad d]$ where $a>0$
$b<0$.

We have $\underline{u} = [a_1 \quad b_1 \quad c_1 \quad d_1]$.

$c \cdot \underline{u} = c \cdot [a_1 \quad b_1 \quad c_1 \quad d_1]$

$(-1) \cdot \underline{u} = (-1)\ [a_1 \quad b_1 \quad c_1 \quad d_1]$

$= [-a_1 \quad -b_1 \quad -c_1 \quad -d_1]$

$a_1 \not> 0$ so it is not a subspace of $\mathbb{R}^4$.

Question: 09

(a) $\begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$ where $b = a+c$

$\underline{u} = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} a_2 & b_2 & c_2 \\ d_2 & 0 & 0 \end{bmatrix}$.

$\underline{u} + \underline{v} = \begin{bmatrix} a_1+a_2 & b_1+b_2 & c_1+c_2 \\ d_1+d_2 & 0 & 0 \end{bmatrix}$.

$= \begin{bmatrix} a_1+a_2 & (a_1+c_1)+(a_2+c_2) & c_1+c_2 \\ d_1+d_2 & 0 & 0 \end{bmatrix}$.

$$\begin{bmatrix} a_1+a_2 & (a_1+a_2)+(c_1+c_2) & c_1+c_2 \\ d_1+d_2 & 0 & 0 \end{bmatrix} \in W_1$$

let $\begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} a & a+c & c \\ d & 0 & 0 \end{bmatrix}$$

$$k \cdot A = \begin{bmatrix} k \cdot a & k(a+c) & kc \\ kd & 0 & 0 \end{bmatrix}.$$

$$= \begin{bmatrix} ka & ka+kc & kc \\ kd & 0 & 0 \end{bmatrix} \in W$$

It is a subspace.

(b) $\underline{v} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$ $c > 0$

$$k \cdot \underline{v} = k \cdot \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$$

$$(-1) \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -a & -b & -c \\ -d & 0 & 0 \end{bmatrix} \notin W$$

It is not a subspace, M_{23}

(c) = $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ where $a = -2c$
$f = 2e + d$

$$u_1 + v_1 = \begin{bmatrix} a_1+a_2 & b_1+b_2 & c_1+c_2 \\ d_1+d_2 & e_1+e_2 & f_1+f_2 \end{bmatrix}$$

$$= \begin{bmatrix} -2c_1 - 2c_2 & +b_1+b_2 & c_1+c_2 \\ d_1+d_2 & e_1+e_2 & 2e_1+d_1+2e_2+d_2 \end{bmatrix}$$

$$= \begin{bmatrix} -2(c_1+c_2) & b_1+b_2 & c_1+c_2 \\ d_1+d_2 & e_1+e_2 & 2(e_1+e_2)+(d_1+d_2) \end{bmatrix}$$

$\in W$.

$$k \cdot u = k \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$= k \cdot \begin{bmatrix} -2c & b & c \\ d & e & 2e+d \end{bmatrix}$$

$$= \begin{bmatrix} -2kc & kb & kc \\ kd & ke & 2ke+kd \end{bmatrix} \in W$$

It is Subspace of M_{23}

Question: 10

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \text{ where } a = 2c+1$$

$$\underline{u}+\underline{v} = \begin{bmatrix} a_1+a_2 & b_1+b_2 & c_1+c_2 \\ d_1+d_2 & e_1+e_2 & f_1+f_2 \end{bmatrix}.$$

$$= \begin{bmatrix} 2c_1+1+2c_2+1 & b_1+b_2 & c_1+c_2 \\ d_1+d_2 & e_1+e_2 & f_1+f_2 \end{bmatrix}.$$

$$= \begin{bmatrix} 2(c_1+c_2)+2 & b_1+b_2 & c_1+c_2 \\ d_1+d_2 & e_1+e_2 & f_1+f_2 \end{bmatrix}.$$

$\notin W.$ $\boxed{2(c_1+c_2)+1}$

$$kU = k\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$= \begin{bmatrix} 2kc+k & kb & kc \\ kd & ke & kf \end{bmatrix} \notin W$$

It is not subspace OF W.

(b) $\begin{bmatrix} 0 & 1 & a \\ b & c & 0 \end{bmatrix}$

let $\begin{bmatrix} 0 & 1 & a \\ b & c & 0 \end{bmatrix}$.

$k \cdot \underline{u} = k \cdot \begin{bmatrix} 0 & 1 & a \\ b & c & 0 \end{bmatrix}$.

$2 \cdot \underline{u} = 2 \cdot \begin{bmatrix} 0 & \underline{2} & 2a \\ 2b & 2c & 0 \end{bmatrix}$.

$\notin W$

It is not subspace of W.

(c) $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, $a+c=0$
$b+d+f=0$

$\underline{u}+\underline{v} = \begin{bmatrix} a_1+a_2 & b_1+b_2 & c_1+c_2 \\ d_1+d_2 & e_1+e_2 & f_1+f_2 \end{bmatrix}$

$(a_1+a_2)+(c_1+c_2)=0$

$a_1+c_1+a_2+c_2=0$

$0+0=0$

$0=0.$

$b+d+f=0$

$(b_1+b_2)+(d_1+d_2)+(f_1+f_2)=0.$

$(b_1+d_1+f_1)+(b_2+d_2+f_2)=0.$

$0+0=0$

$0=0.$

$\underline{u+v} \in W.$

$$k \cdot u = k \cdot \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$= \begin{bmatrix} ke & kb & kc \\ kd & ke & kf \end{bmatrix}$$

$\Rightarrow \quad ka+kc=0$

$k(a+c)=0$

$0=0.$

$\rightarrow \quad b+d+f=0.$

$kb+kd+kf=0$

$k(b+d+f)=0$

$0=0.$

It is subspace of M_{23}

Question 11

let w be the set of all 3x3 matrix whose trace is zero Show that S is subspace of M_{33}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}. \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$A+b = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{bmatrix}.$$

$$a_{11}+b_{11}+a_{12}+b_{12}+a_{13}+b_{13}=0$$

$$(a_{11}+a_{12}+a_{13})+(b_{11}+b_{12}+b_{13})=0$$

$$0+0=0$$

$$0=0.$$

$$k \cdot A = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix}.$$

$$ka_{11}+ka_{12}+ka_{13}=0$$

$$k(a_{11}+a_{12}+a_{13})=0$$

$$k(0)=0$$

$$0=0.$$ Subspace of M_{33}

Question: 12

let w be the set of all 3×3 matrix of the form

$$\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix}$$

Show that w is subspace of M_{33}

$$\underline{u}+\underline{v} = \begin{bmatrix} a_1+a_2 & 0 & b_1+b_2 \\ 0 & c_1+c_2 & 0 \\ d_1+d_2 & 0 & e_1+e_2 \end{bmatrix} \in W.$$

$$k \cdot \underline{u} = k \begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix}$$

$$= \begin{bmatrix} ka & 0 & kb \\ 0 & kc & 0 \\ kd & 0 & ke \end{bmatrix} \in W.$$

It is Subspace of M_{33}.

Question:13

let w be the set of all 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Such that $a+b+c+d=0$ Is w is a subspace of M_{22}?

$$u+v = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$$

$$u+v = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}.$$

$$a_1+a_2+b_1+b_2+c_1+c_2+d_1+d_2=0$$

$$(a_1+b_1+c_1+d_1)+(a_2+b_2+c_2+d_2)=0$$

$$0+0=0$$

$$0=0.$$

$$k.A = k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}.$$

$$ka+kb+kc+kd=0$$

$$k(a+b+c+d)=0$$

$$k(0)=0$$

$$0=0.$$

It is subspace of M_{22}.

Question:14

let w be the set of all 2×2 matrix

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

such that $AZ=0$ where $Z=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Is w is subspace of M_{22}?

let $A, B \in w$ we know that

$AZ=0$ and $BZ=0$.

$AZ+BZ=0$

$Z(A+B)=0$

$Z(0)=0$

$0=0$.

$K\cdot(AZ)=0$.

$K(0)=0$

$0=0$

It is Subspace of M_{22}.

In exercise 15-16 Which of following given subset of vector space P_2 are subspace.

Question: 15

The set of all polynomial of the forms

(a) $a_2 t^2 + a_1 t + a_0$, where $a_0 = 0$

$P_1(t) = a_2 t^2 + a_1 t + a_0$

$P_2(t) = b_2 t^2 + b_1 t + b_0$ $\quad\quad a_0 = 0$

$P_1(t) + P_2(t) = a_2 t^2 + a_1 t + a_0 + b_2 t^2 + b_1 t + b_0$

$= (a_2 + b_2) t^2 + (a_1 + b_1) t + \underbrace{(a_0 + b_0)}_{(0+0)=0}$

$\in W$

$k \cdot p(t) = k(a_2 t^2 + a_1 t + a_0)$

$= k a_2 t^2 + k a_1 t + k a_0 \in W$

$= k a_2 t^2 + k a_1 t \in W$

It is subspace of P_2.

(b) $a_2 t^2 + a_1 t + a_0$ where $a_0 = 2$

$P_1(t) = a_2 t^2 + a_1 t + a_0$
$= a_2 t^2 + a_1 t + 2$

$P_2(t) = b_2 t^2 + b_1 t + b_0$
$= b_2 t^2 + b_1 t + 2$

$P_1(t) + P_2(t) = a_2 t^2 + a_1 t + a_0 + b_2 t^2 + b_1 t + b_0$
$= (a_2 + b_2) t^2 + (a_1 + b_1) t + (a_0 + b_0)$
$= (a_2 + b_2) t^2 + (a_1 + b_1) t + 4 \notin W_2$

It is not subspace of P_2

(c) $a_2 t^2 + a_1 t + a_0$ where $a_2 + a_1 = a_0$

$P_1(t) = a_2 t^2 + a_1 t + a_0$
$= a_2 t^2 + a_1 t + (a_2 + a_1)$

$P_2(t) = b_2 t^2 + b_1 t + b_0$
$= b_2 t^2 + b_1 t + (b_2 + b_1)$

$P_1(t) + P_2(t) = a_2 t^2 + a_1 t + (a_2 + a_1) + b_2 t^2 + b_1 t + (b_2 + b_1)$
$= (a_2 + b_2) t^2 + (a_1 + b_1) t + (a_1 + b_2) + (b_1 + a_1)$
$\in W$

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$$k \cdot p(t) = k(a_2 t^2 + a_1 t + a_0)$$
$$= ka_2 t^2 + ka_1 t + k(a_1 + a_2) \in W$$

It is subspace of P_2.

Question: 16

(a) $a_2 t^2 + a_1 t + a_0$ Where $a_1 = 0$ and $a_0 = 0$

$$P_1(t) = a_2 t^2 + a_1 t + a_0$$
$$= a_2 t^2$$
$$P_2(t) = b_2 t^2 + b_1 t + b_0$$
$$= b_2 t^2$$

$$P_1(t) + P_2(t) = a_2 t^2 + b_2 t^2$$
$$= (a_2 + b_2) t^2 \in W.$$

$$k P(t) = k(a_2 t^2 + a_1 t + a_0)$$
$$= ka_2 t^2 + ka_1 t + ka_0$$
$$= ka_2 t^2 \in W$$

It is subspace of P_2

(b) $a_2 t^2 + a_1 t + a_0$, where $a_1 = 2a_0$

$P_1(t) = a_2 t^2 + a_1 t + a_0$
$= a_2 t^2 + 2a_0 t + a_0$

$P_2(t) = b_2 t^2 + b_1 t + b_0$
$= b_2 t^2 + 2b_0 t + b_0$

$P_1(t) + P_2(t) = a_2 t^2 + 2a_0 t + a_0 + b_2 t^2 + 2b_0 t + b_0$
$= (a_2 + b_2) t^2 + (2a_0 + 2b_0) t + (a_0 + b_0)$
$= (a_2 + b_2) t^2 + 2(a_0 + b_0) t + (a_0 + b_0)$
$\in W.$

$k \cdot P(t) = k \cdot (a_2 t^2 + a_1 t + a_0)$
$= k a_2 t^2 + 2k a_0 t + k a_0$
$\in W$

It is subspace of P_2

(c) $a_2 t^2 + a_1 t + a_0$ where $a_2 + a_1 + a_0 = 2$

$P(t) = a_2 t^2 + a_1 t + a_0.$

we have that $1/2 t^2 + 1/2 t + 1 \in W = P(t).$

$2P(t) = t^2 + t + 2 \notin W$

It is not subspace of P_2

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Q#17

Which of the following subset of vector space of M_{nn} are subspace

(a) The set of all $n \times n$ symmetric matrices.

let $A, B \in W_1$

The we have

$A = A^T$ and B and B^T which give us

that $A + B = A^T + B^T = (A+B)^T$

$A + B \in W_1$

let $A \in W$ and k is scalor

$A = A^T$

$kA = kA^T = (kA)^T$, $kA \in W$.

It is Subspace of M_{nn}.

(b) The set of all $n \times n$ diangnol matrices

using diagnol matric def.

let $A, B \in W$ Then we have

$a_{ij} = 0$ and $b_{ij} = 0$ and $1 \le i \le n$ and $1 \le j \le n$ such that $i \neq j$ which give us that

$a_{ij} + b_{ij} = 0 \Rightarrow 0 + 0 = 0$.

for all $1 < i < n$ and $1 < j \leq n$ such that $i \neq j$.

$(A+B) \in W$.

$k \cdot a_{ij} = 0$

$k(0) = 0$ $\quad \therefore 1 < i < n$

$0 = 0$. $\quad i \neq j$

It is subspace of M_{nn}.

(c) The set of all $n \times n$ nonsingular matrices.

$I_n \in W$

$-I_n \in W$

but $I_n - I_n = 0$ which are singular

It is not subspace of M_{nn}.

Question: 18

Which of the following subsets of vector space M_{nn} are subspace.

(a) The set of all $n \times n$ singular matrices.

$A, B \in W$

$\det A = 0$, $\det B = 0$

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$A+B = I_n$

$\det(A+B) = \det(I_n) = 1$

$A+B \notin W_1$

It is not a subspace of M_{nn}.

ace.

(b) The set of all nxn upper triangular matrices. using defination.

$a_{ij} = 0$ and $b_{ij} = 0$.

For all $1 \leq i \leq n$ and $1 \leq j \leq n$.

Incase $i > j$

$a_{ij} + b_{ij} = 0$

$0 + 0 = 0$

$A+B \in W$

$k a_{ij} = 0$

$k(0) = 0$ For all $1 \leq i \leq n$; $i > j$

It is subspace of M_{nn}.

(c) The set of all nxn skew symmetric matrices.

$A, B \in W$

then we have $A^T = -A$ and $B^T = -B$

$(A+B)^T = A^T + B^T = -A - B = -(A+B)$

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$(A+B) \in W$.

$K \cdot (A^T) = K(-A) = -KA$ and

$KA \in W$.

It is subspace of M_{nn}.