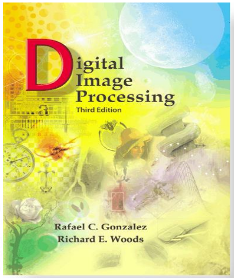
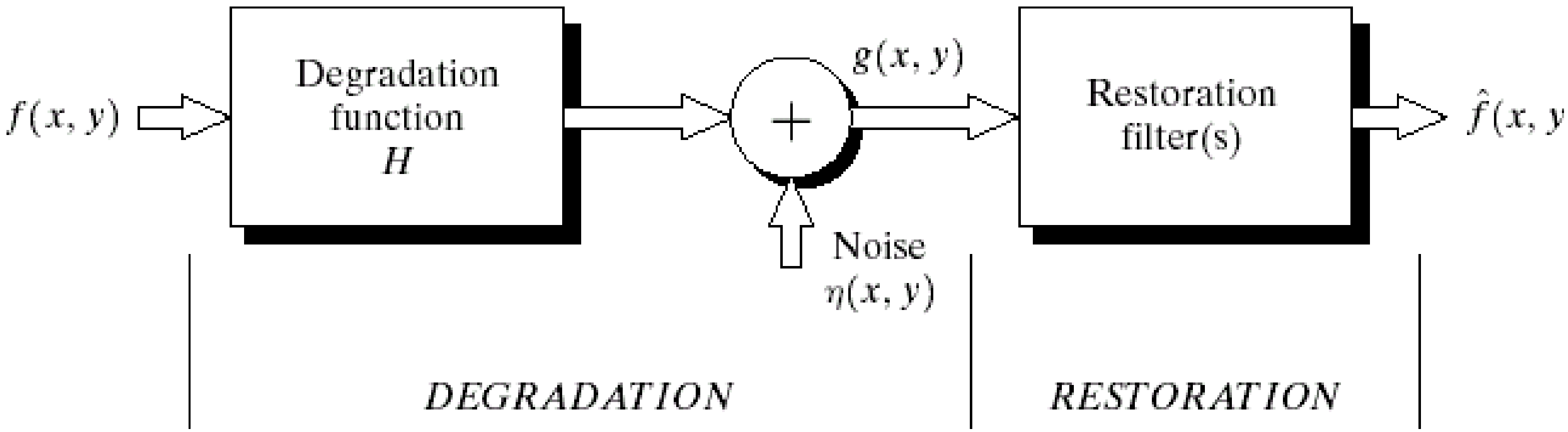


Image Restoration and ~~Reconstruction~~

- Image Degradation Model (Linear/Additive)



$$g(x,y) = f(x,y) \star h(x,y) + \eta(x,y)$$

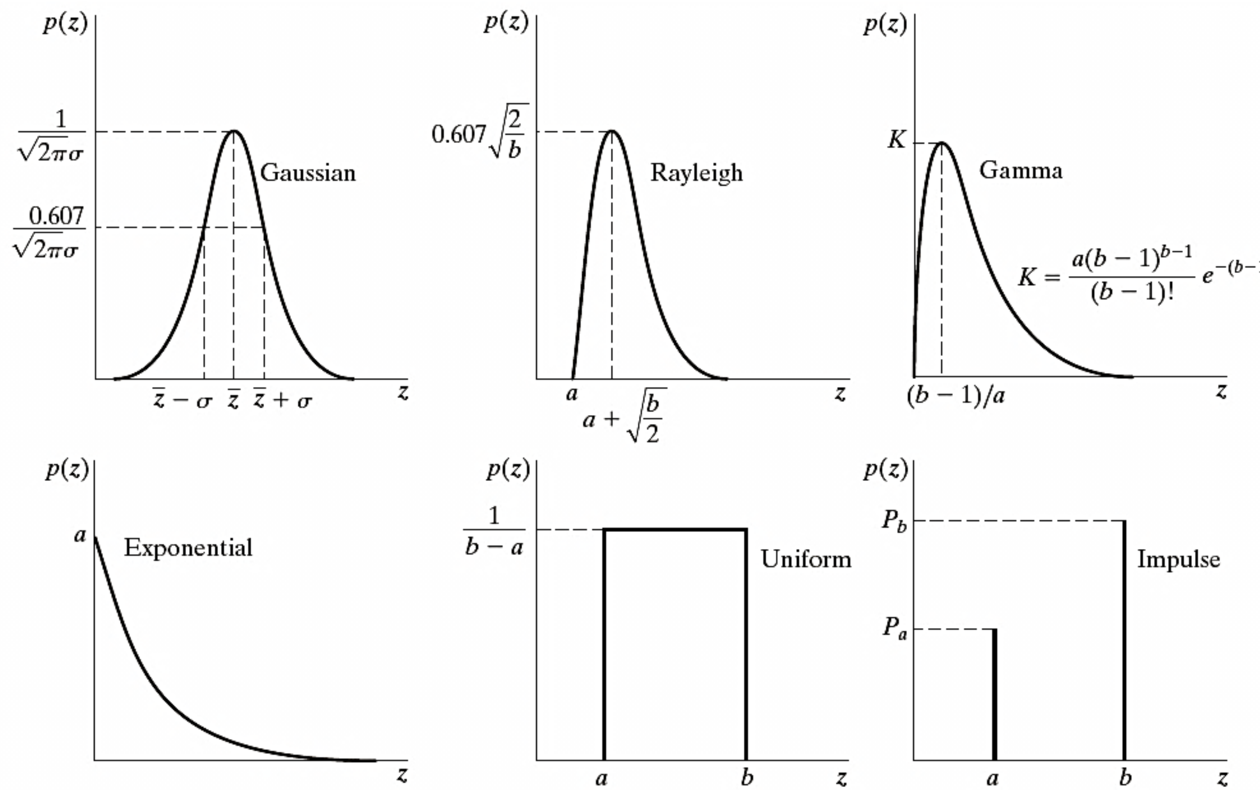
$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Image Restoration and ~~Reconstruction~~

- Source of noise
  - Objects Impurities
  - Image acquisition (digitization)
  - Image transmission
- Spatial properties of noise
  - Statistical behavior of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum
    - White noise (a constant Fourier spectrum)

Image Restoration and ~~Reconstruction~~

- Some Noises Distribution:

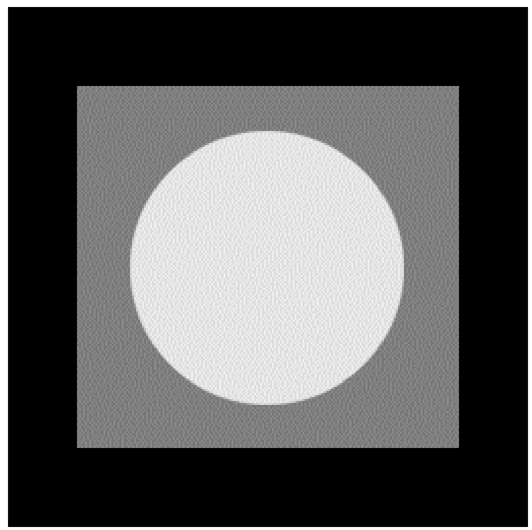


a b c
d e f

FIGURE 5.2 Some important probability density functions.

Image Restoration and ~~Reconstruction~~

- Test Pattern
  - Histogram has three spikes (Impulse)



  - For two independent random variables (x, y)

$$z = x + y \Rightarrow p_Z(z) = p_X(x) * p_Y(y)$$

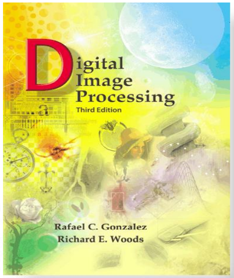
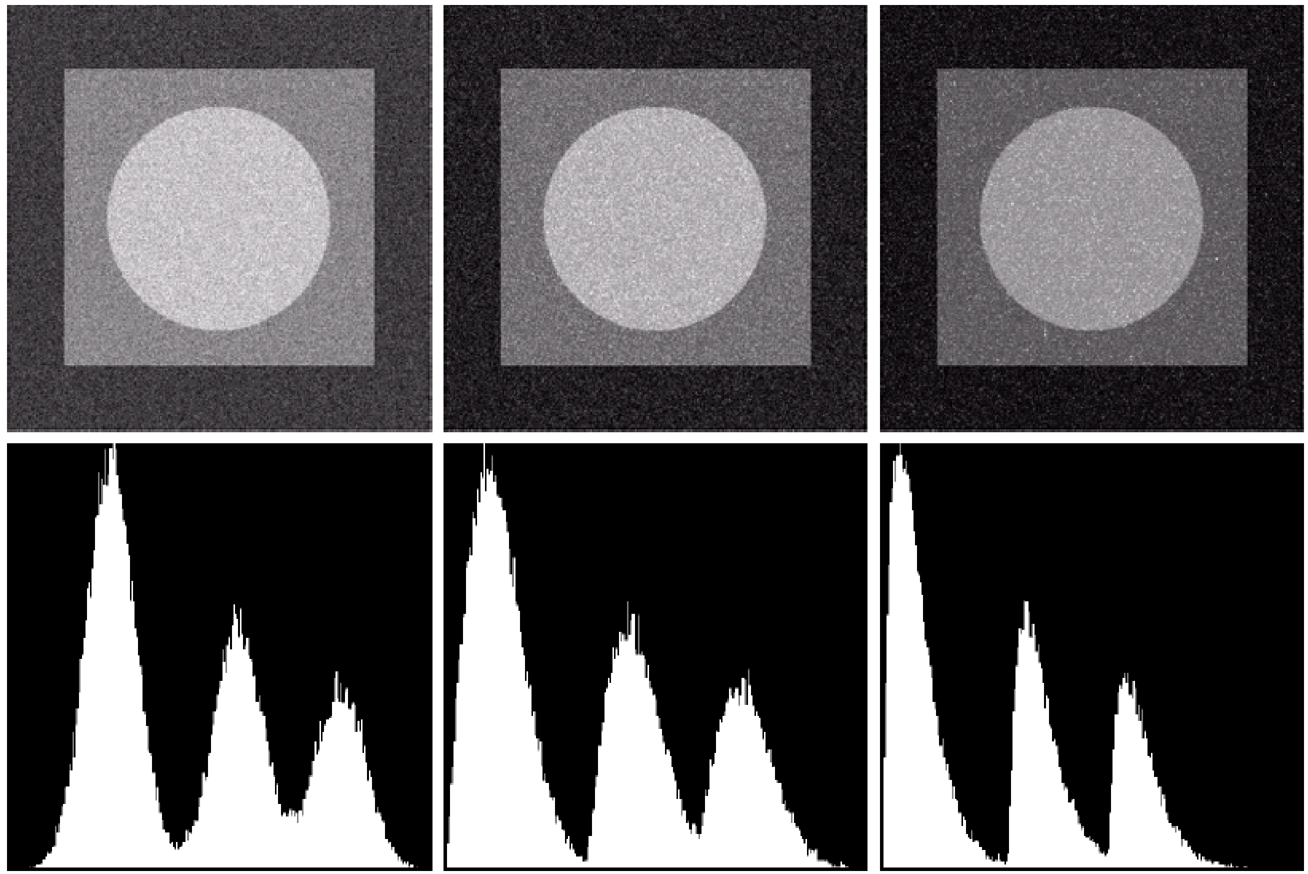


Image Restoration and ~~Reconstruction~~

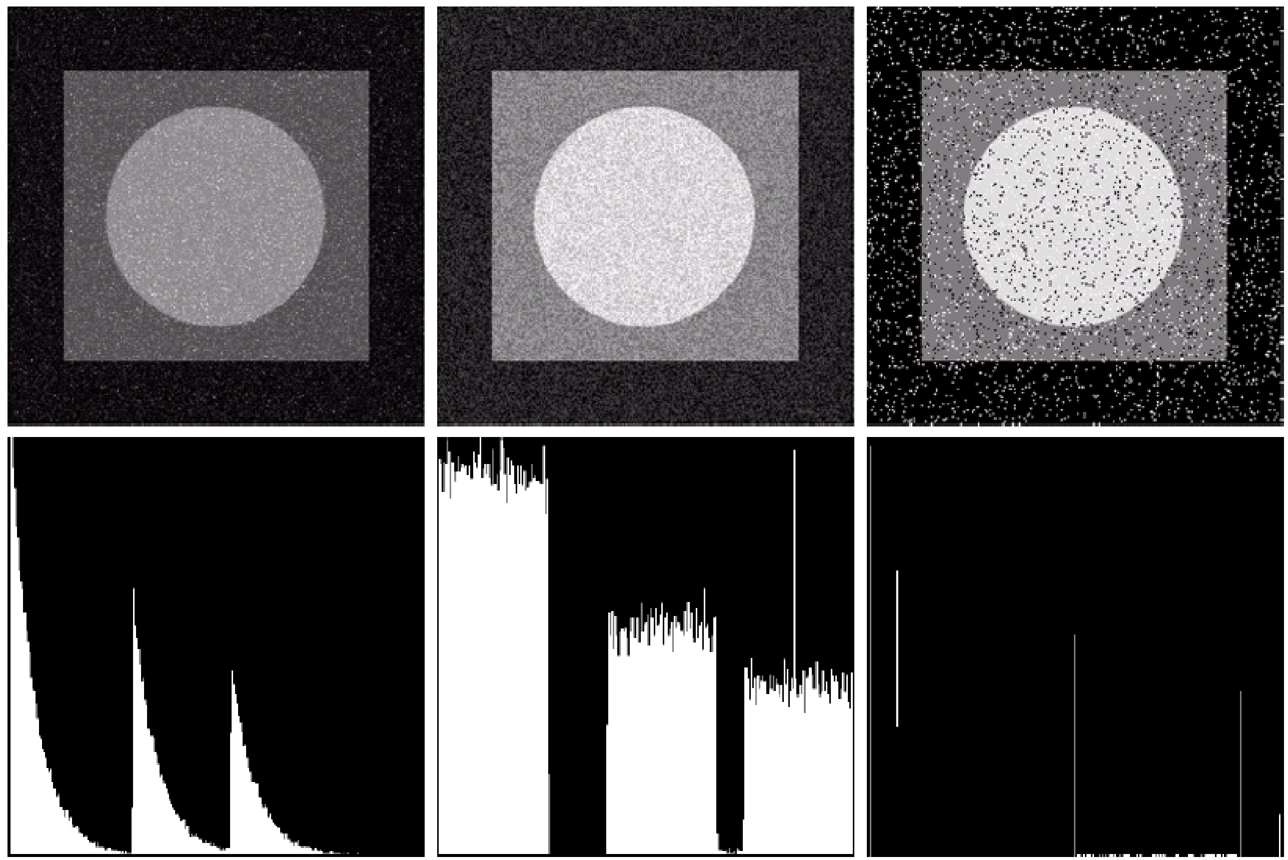
- Noisy Images(1):



Gaussian Rayleigh Gamma

Image Restoration and ~~Reconstruction~~

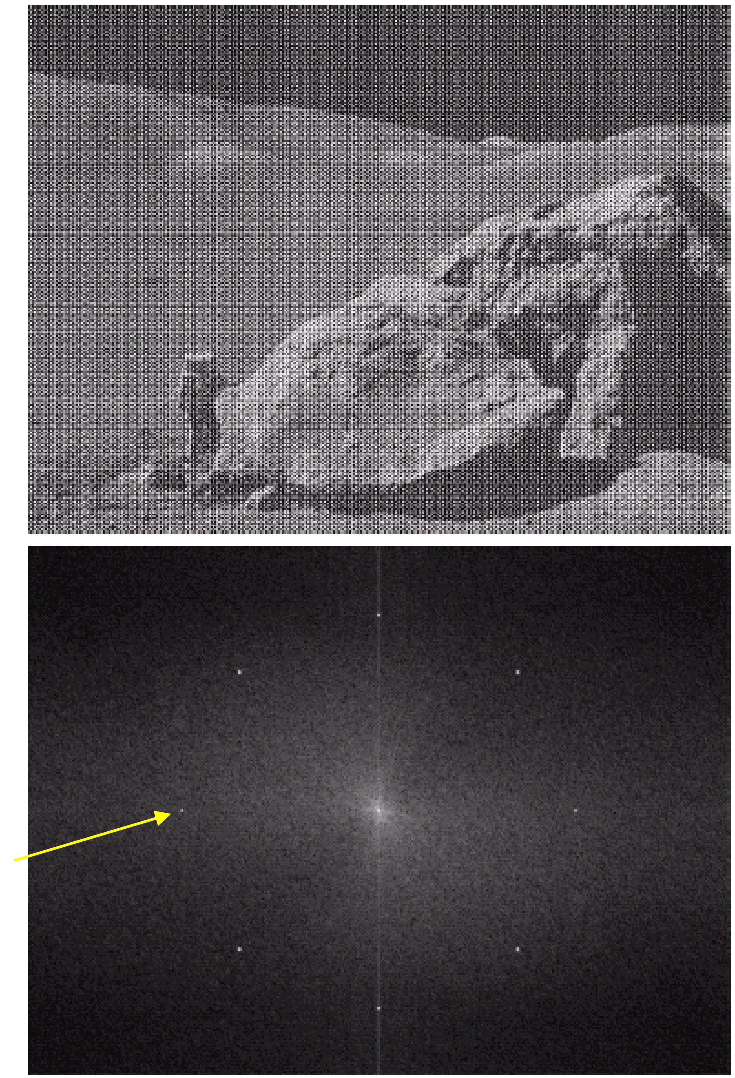
- Noisy Images (2):



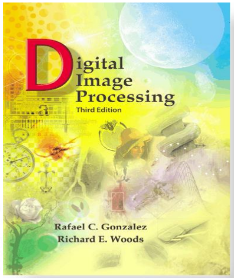
Exponential | Uniform | Salt & Pepper

Image Restoration and ~~Reconstruction~~

- Periodic[1] Noise:
  - Electronic Devices



[1] Disturbance



- Estimation of Noise Parameters
  - Periodic noise
    - Observe the frequency spectrum
  - Random noise with PDFs
    - Case 1: Imaging system is available
      - Capture image of “flat” environment
    - Case 2: Single noisy image is available
      - Take a strip from constant area
      - Draw the histogram and observe it
      - Estimate PDF parameters or measure the mean and variance

Image Restoration and ~~Reconstruction~~

- Noise Estimation:
  - Shape: Histogram of a subimage (Background)

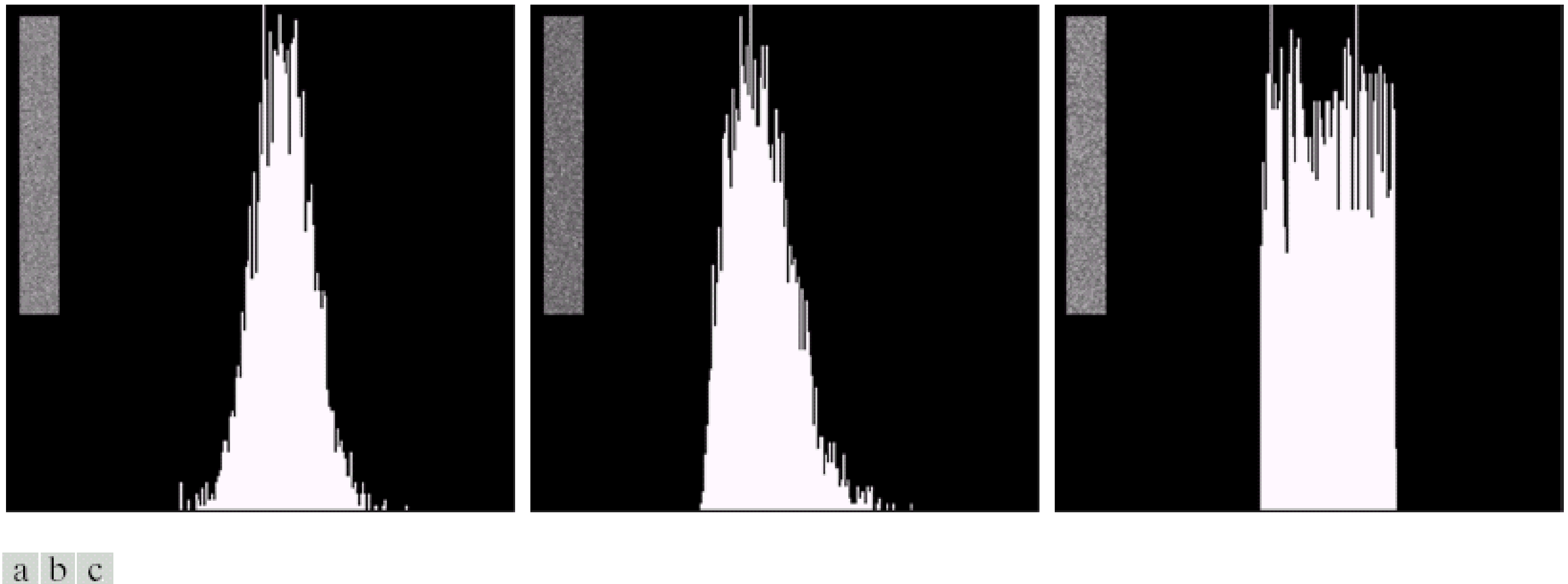


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

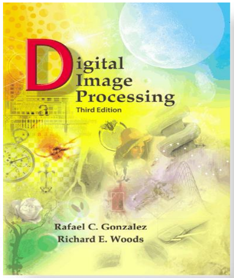


Image Restoration and ~~Reconstruction~~

- Noise Estimation:
  - Parameters:

    Maximum Likelihood method: $\{z_i\}_{i=1}^{N} \Rightarrow \underline{\theta} = \arg\max\left\{\prod_{i=1}^{N_S} p\left(z_i, \underline{\theta}\right)\right\}$
  - Momentum

$$\left.\begin{aligned}\mu &= \sum_{z_i \in S} z_i\, p\left(z_i\right)\\ \sigma^2 &= \sum_{z_i \in S}\left(z_i-\mu\right)^2 p\left(z_i\right)\end{aligned}\right\} \xrightarrow{\text{Solve}} \left\{\underline{\theta}\right\} \xrightarrow{?} \text{PDF Parameters}$$

  - Power Spectrum Estimation:

$$\left|\tilde{N}\left(u,v\right)\right|^2 = E\left\{\left|\Im\left\{\eta\left(x,y\right)\right\}\right|^2\right\} = \frac{1}{N}\sum_{i=1}^{N}\left|\Im\left\{\eta_i\left(x,y\right)\right\}\right|^2$$

$\eta_i\left(x,y\right)$: Flat (No object) Subimages

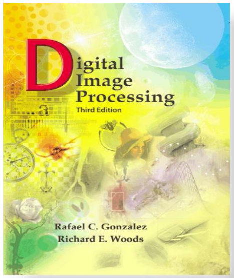


Image Restoration and ~~Reconstruction~~

- Noise-only spatial filter:

$$g(x,y) = f(x,y) + \eta(x,y) \Leftrightarrow G(u,v) = F(u,v) + N(u,v)$$

- Mean Filters:
  - A $m \times n$ mask centered at (x, y): S_{xy}
  - An algebraic operation on the mask pixels!

Image Restoration and ~~Reconstruction~~

- Mean Filter:
  - Arithmetic mean:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

    - Noise Reduction (uncorrelated, zero mean) vs. Blurring
  - Geometric mean:

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} \left(g(s,t) \right) \right]^{\frac{1}{mn}}$$

    - Same smoothing with less detail degradation:

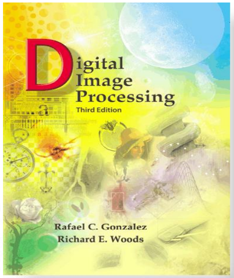


Image Restoration and ~~Reconstruction~~

- Mean Filter:
  - Harmonic mean:

$$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$$

    - Salt Reduction (Pepper will increase)/ Good for Gaussian like
  - Contra-harmonic mean:

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}}\left(g(s,t)\right)^{Q+1}}{\sum_{(s,t)\in S_{xy}}g(s,t)^{Q}}$$

    - Q>0 for pepper and Q<0 for salt noise

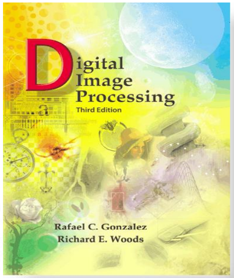


Image Restoration and ~~Reconstruction~~

- Example (1):
  - Additive Noise
  - Mean
    - Arithmetic
    - Geometric

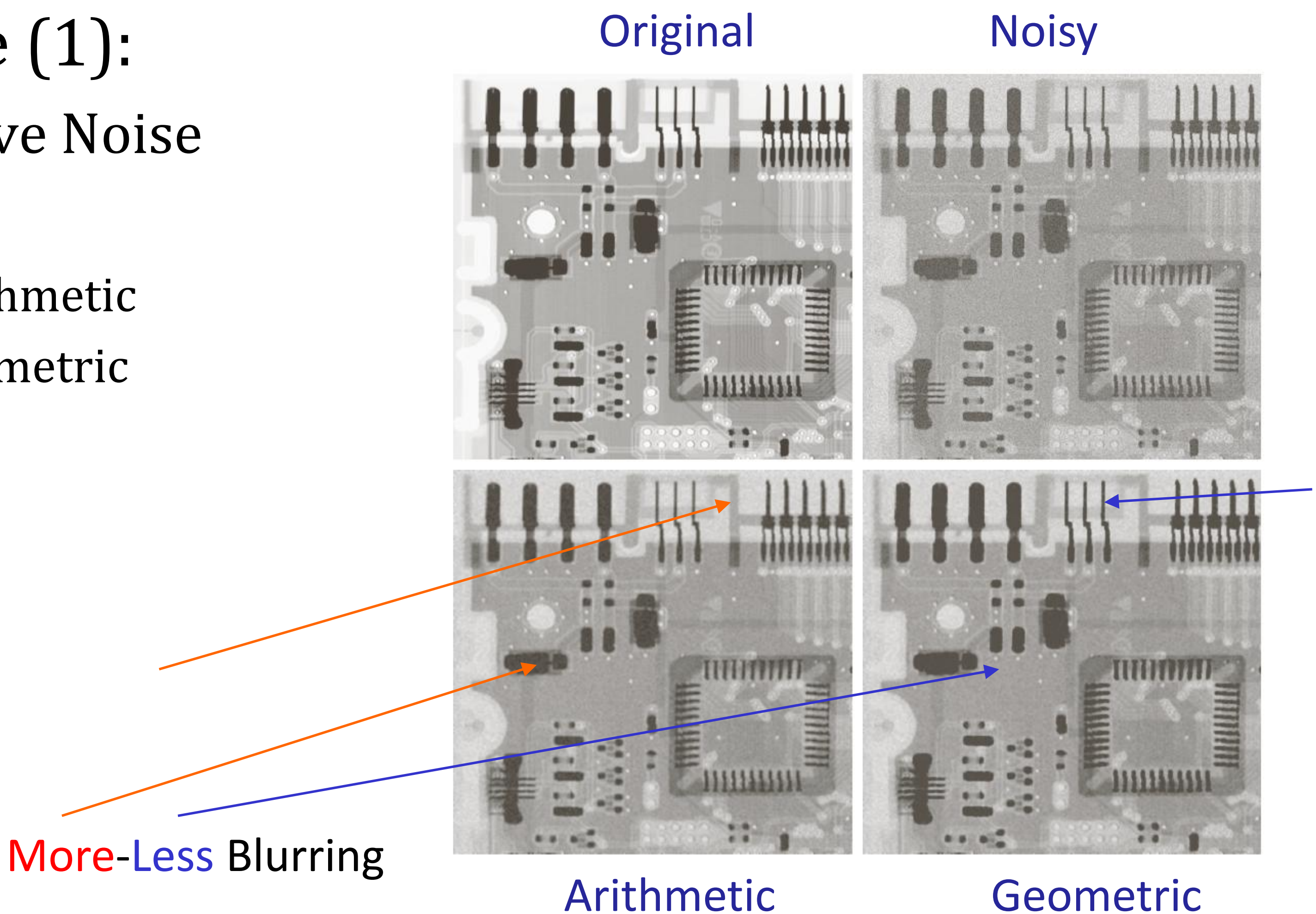
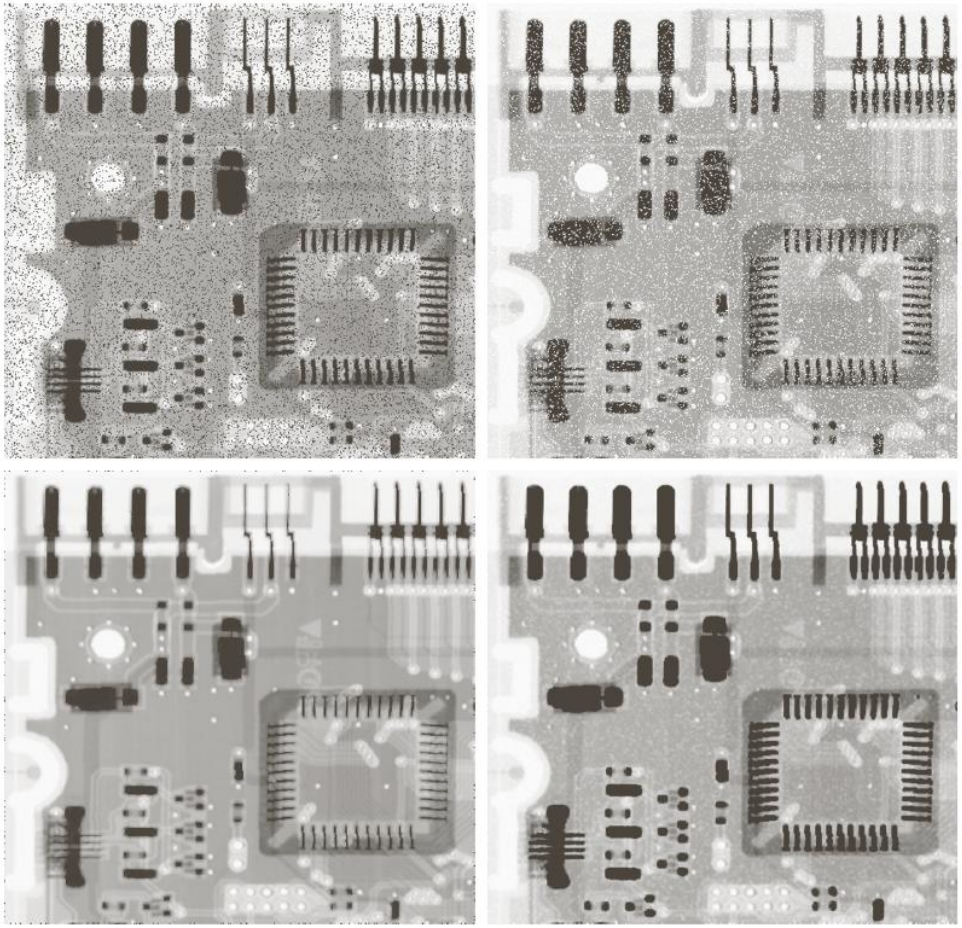


Image Restoration and ~~Reconstruction~~

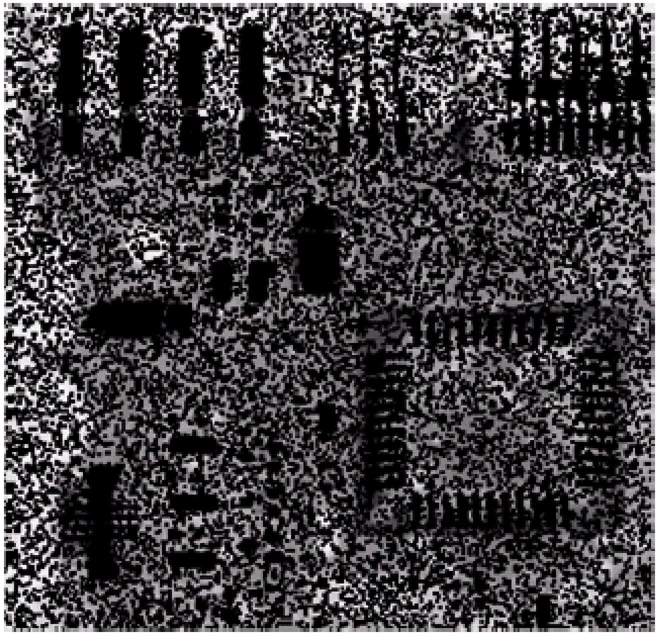
- Example (2):
  - Impulsive
  - Contra-harmonic
  - Correct use

Noisy (Pepper) Noisy (Salt)

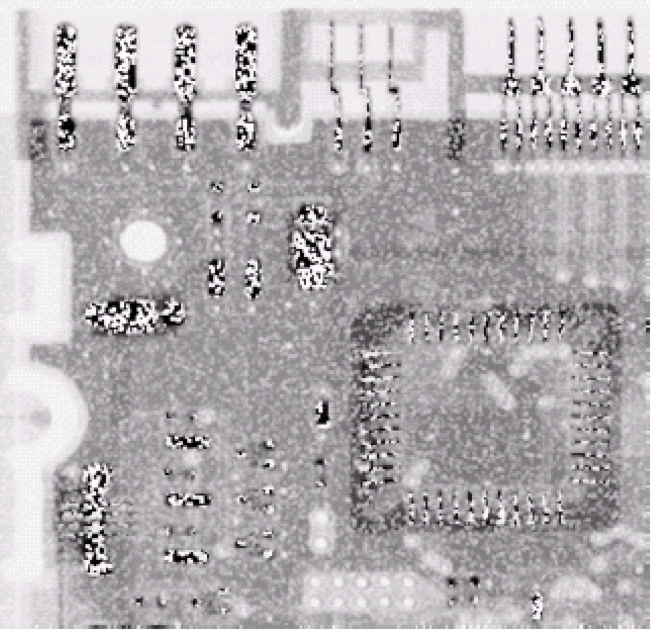


Q=+1.5 Q=-1.5

- Example (3):
  - Impulsive Noise
  - Conrtra-harmonic
  - Wrong use



Q=-1.5



Q=+1.5

Image Restoration and ~~Reconstruction~~

- Noise-only spatial filter:

$$g(x,y) = f(x,y) + \eta(x,y)$$

- Mean Filters:
  - A $m \times n$ mask centered at (x, y): S_{xy}
  - An Ordering/Ranking operation on mask members!

Image Restoration and ~~Reconstruction~~

- Median: Most Familiar OS filter:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

  – Little Blurring/Single/Double valued noises

- Max/min:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{Max}\{g(s,t)\}, \quad \hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}$$

  – Find Brightest/Darkest Points (Pepper/Salt Reducer)

Image Restoration and ~~Reconstruction~~

- Midpoint:
  - OS+Mean Properties (Gaussian, Uniform)

$$\hat{f}(x,y)=\frac{1}{2}\left[\underset{(s,t)\in S_{xy}}{Max}\{g(s,t)\}+\underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}\right]$$

Image Restoration and ~~Reconstruction~~

- Alpha Trimmed mean:
  - Delete ($d/2$) lowest and highest samples

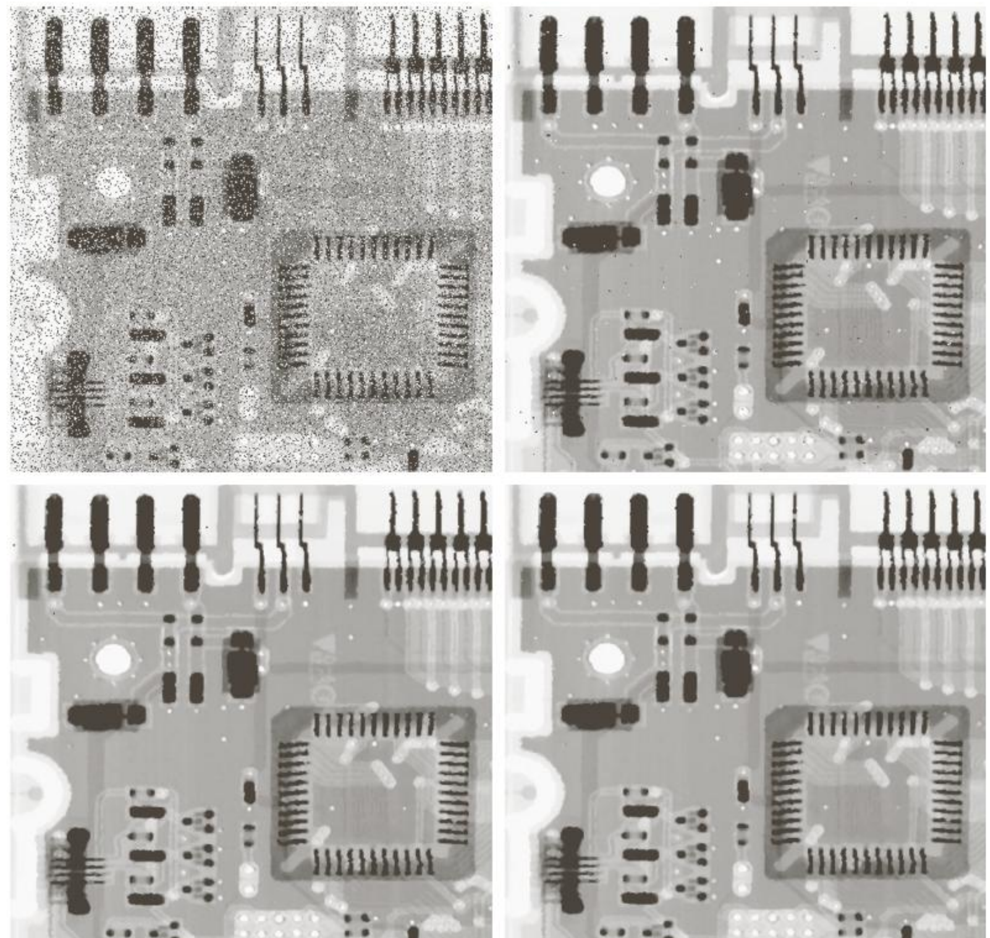
$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

  - $d = 0 \rightarrow$ Mean Filter
  - $d = mn\text{-}1 \rightarrow$ Median Filter
  - Others: Combinational Properties

Image Restoration and ~~Reconstruction~~

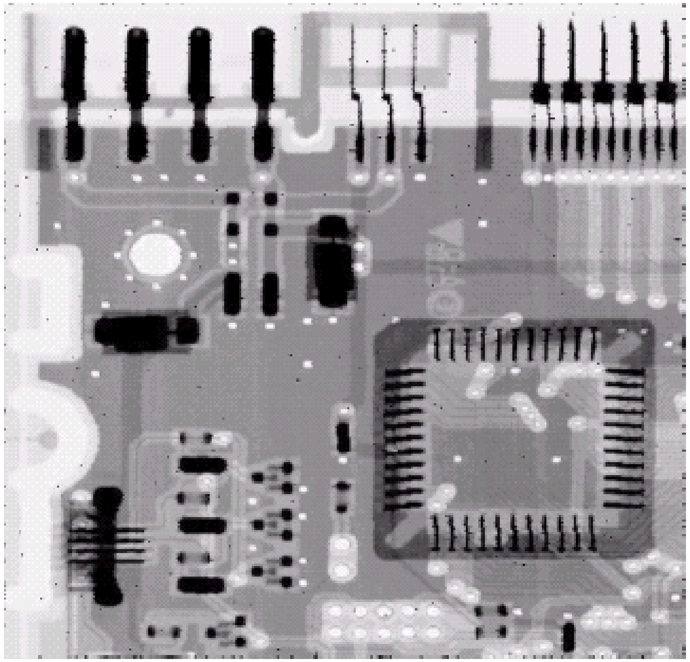
- Example (1):
  - Impulsive Noise
  - Median Filter
  - 1-2-3- passes
  - Less Blurring

Noisy (Salt & Pepper) One Pass

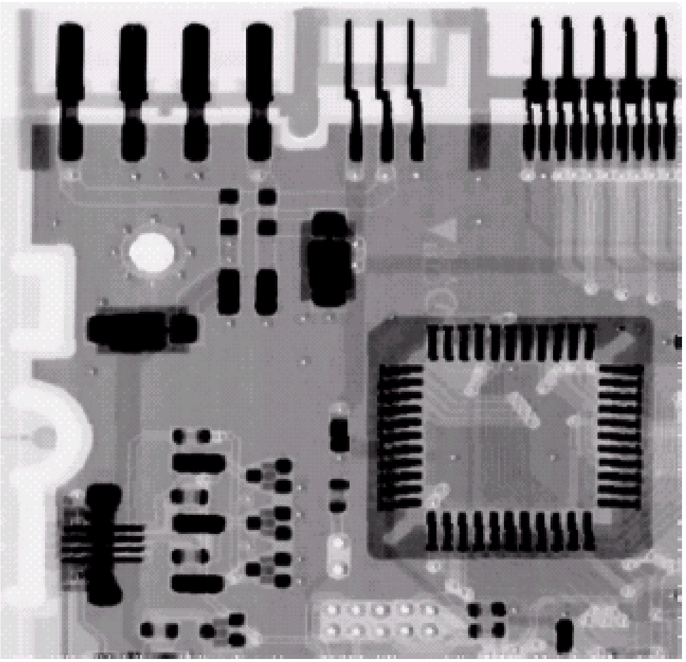


Two Pass Three Pass

- Example (2):
  - Max and min Filters:



3×3 Max Filter



3×3 min Filter

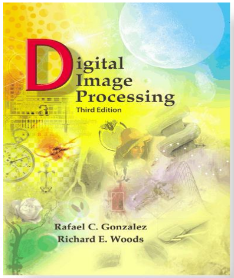


Image Restoration and ~~Reconstruction~~

- Example (3):
  - Noise:
    - Uniform (a) and Impulsive (b)
  - Mask size:
    - 5×5
  - Filters
    - Arithmetic mean (c)
    - Geometric mean (d)
    - Median (e)
    - Alpha Trimmed (f)

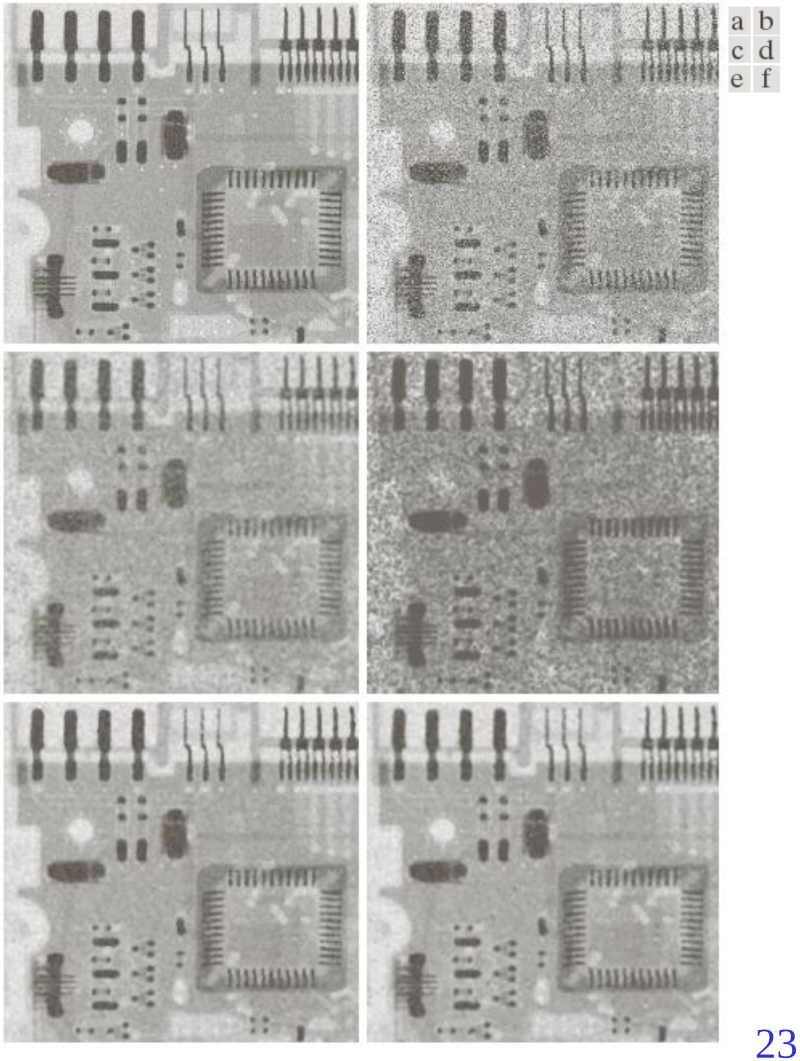


Image Restoration and ~~Reconstruction~~

- Adaptive, local noise reduction:
  - If σ_η^2 is small, return $g(x,y)$
  - If $\sigma_L^2 > \sigma_\eta^2$,return value close to $g(x,y)$
  - If $\sigma_L^2 \approx \sigma_\eta^2$,return the arithmetic mean m_L

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2(x,y)}\left[g(x,y) - m_L(x,y)\right]$$

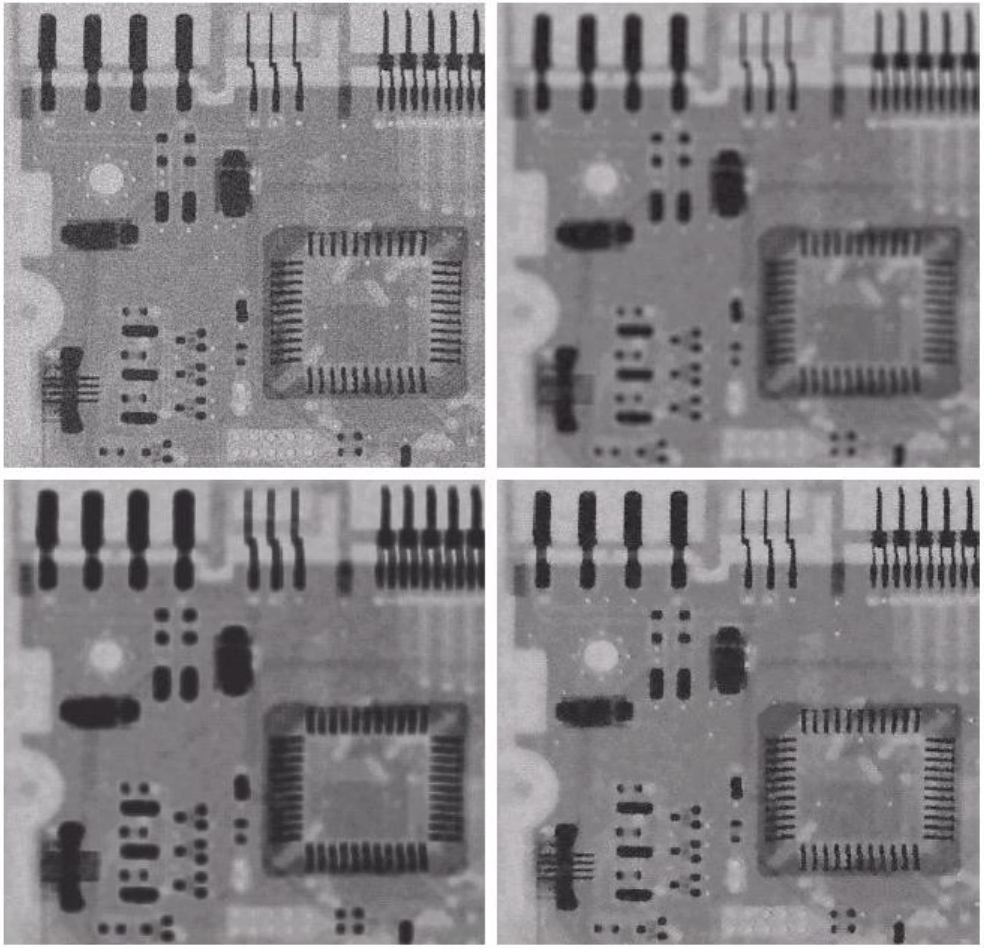
  - Non-stationary noise or poor estimation:

$$\max\left[\frac{\sigma_\eta^2}{\sigma_L^2(x,y)}\right] = 1$$

Image Restoration and ~~Reconstruction~~

- Example:
  - $N(0,100)$
- Filters:
  - Arithmetic Mean
  - Geometric Mean
  - Local

Original Noisy
A. Mean



G- Mean
Local

Image Restoration and ~~Reconstruction~~

- Adaptive Median:

$z_{\min}$ = Minimum intensity in S_{xy}

$z_{\max}$ = Maximum intensity in S_{xy}

z_{med} = Median of intensity in S_{xy}

z_{xy} = Intensity value at (x, y)

S_{Max} = Maximum allowed size of S_{xy}

$$A_1 = Z_{med} - Z_{\min},\ A_2 = Z_{med} - Z_{\max}$$

$$\text{if } \left(A_1 > 0 \text{ and } A_2 < 0\right) \text{ then}$$

$$\left\{\begin{array}{l} B_1 = Z_{xy} - Z_{\min},\ B_2 = Z_{xy} - Z_{\max} \\ \text{if } \left(B_1 > 0 \text{ and } B_2 < 0\right) \text{ then } \hat{Z}_{xy} = Z_{xy} \text{ else } \hat{Z}_{xy} = Z_{med} \end{array}\right\}$$

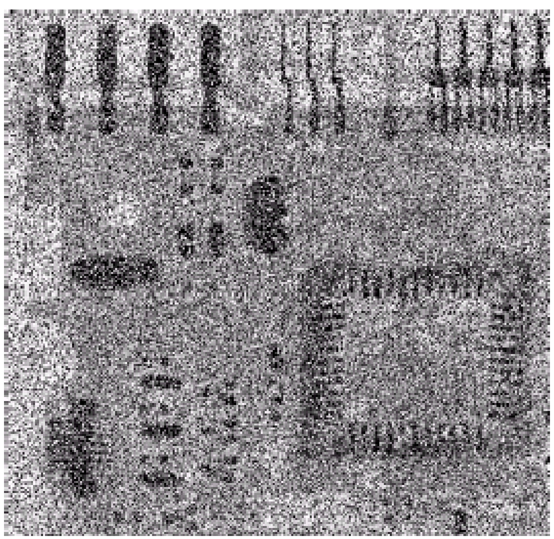
$$\text{else}$$

$$\left\{\begin{array}{l} \text{increase Mask size:} \\ \quad (\le S_{\max}): \text{Check "}A\text{" condition.} \\ \quad (> S_{\max}): \hat{Z}_{xy} = Z_{med} \end{array}\right\}$$

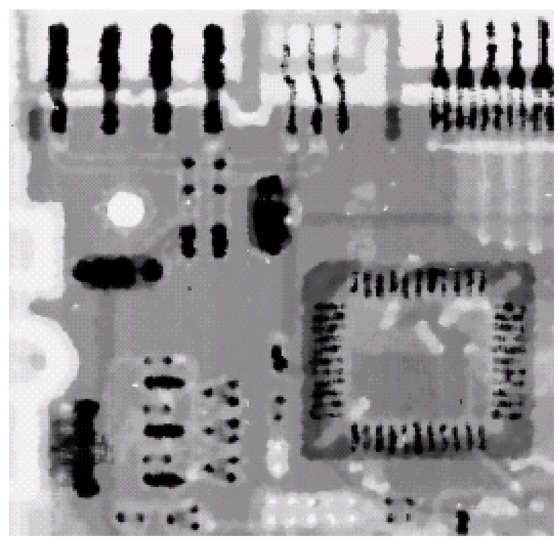
Image Restoration and ~~Reconstruction~~

- Example:
  - Noise: Salt and Pepper (Pa=Pb=0.25)

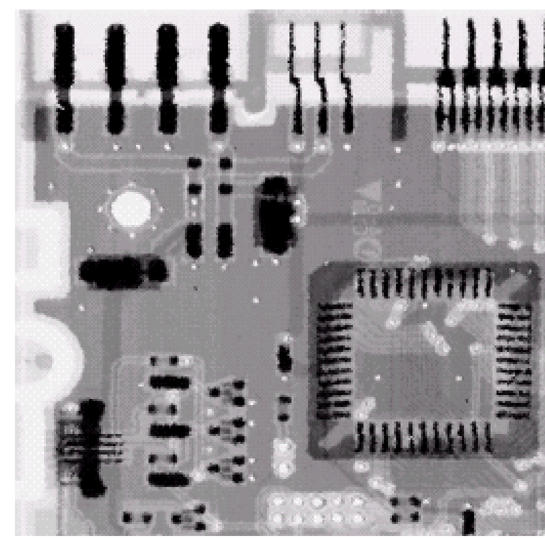
Noisy



Median (7×7)

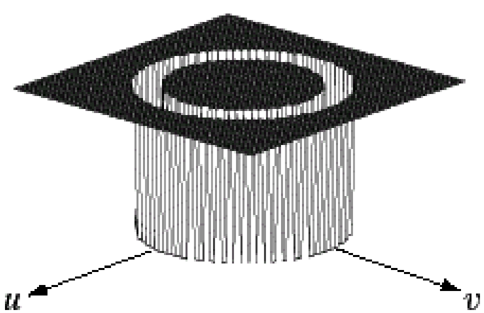


Adaptive Median (Smax=7)

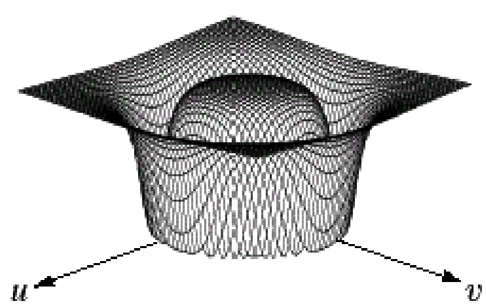


- Band Reject Filter (Periodic Noises)
  - Ideal, Butterworth, and Gaussian

IBRF



BBRF(1)



GBRF

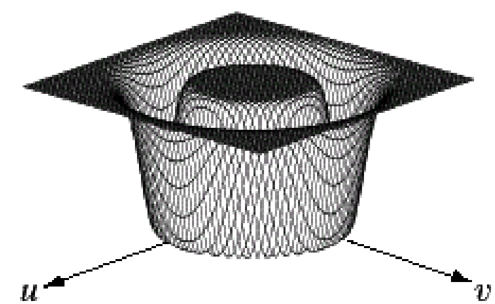


Image Restoration and ~~Reconstruction~~

- Band Reject Filter (Periodic Noises)

$$H_I(u,v)=\begin{cases}1 & D(u,v)<D_0-\dfrac{W}{2}\\ 0 & D_0-\dfrac{W}{2}\le D(u,v)\le D_0+\dfrac{W}{2}\\ 1 & D(u,v)>D_0+\dfrac{W}{2}\end{cases}$$

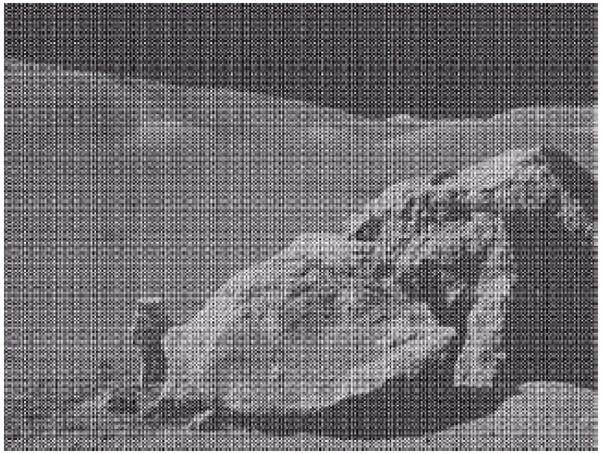
$$H_{B_n}(u,v)=\frac{1}{1+\left[\dfrac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

$$H_G(u,v)=1-\exp\left(-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2\right)$$

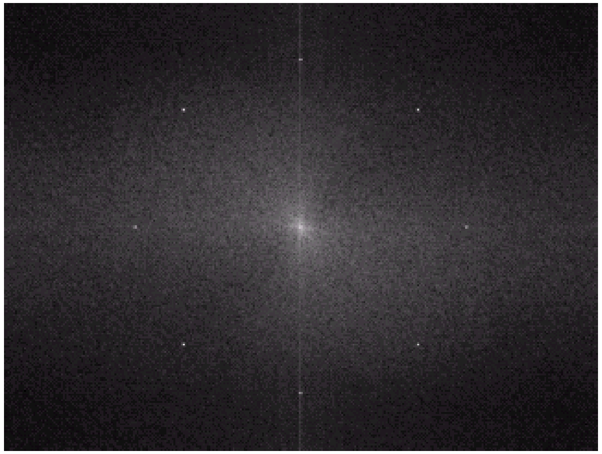
Image Restoration and ~~Reconstruction~~

- Example:

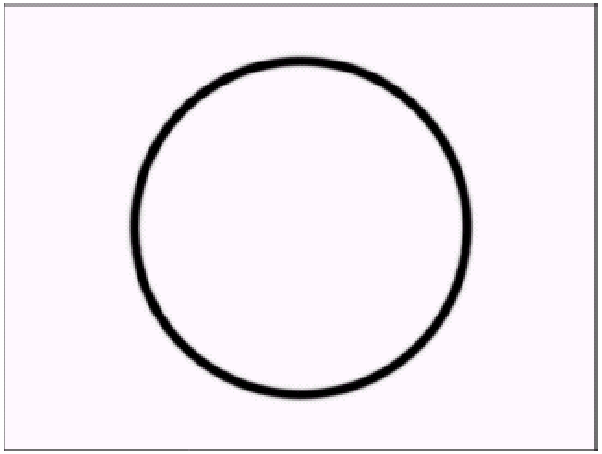
Noisy



Spectrum



BBRF(1)

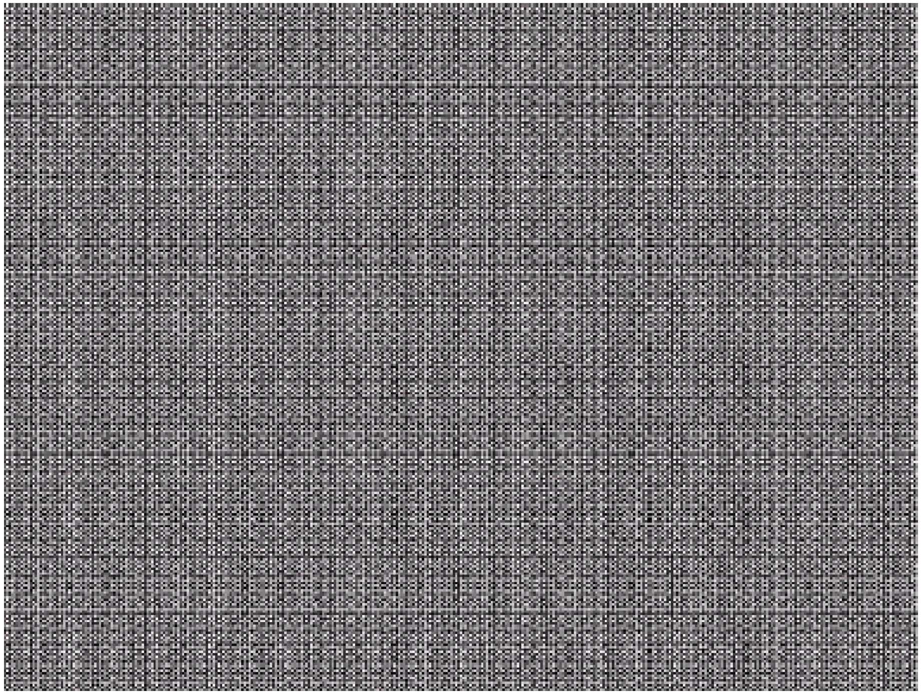


DeNoised

- Band Pass Filter (Extract Periodic Noises)
  - Analysis of Spectrum in different bands

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

  - Noise Pattern:



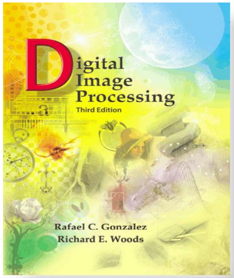
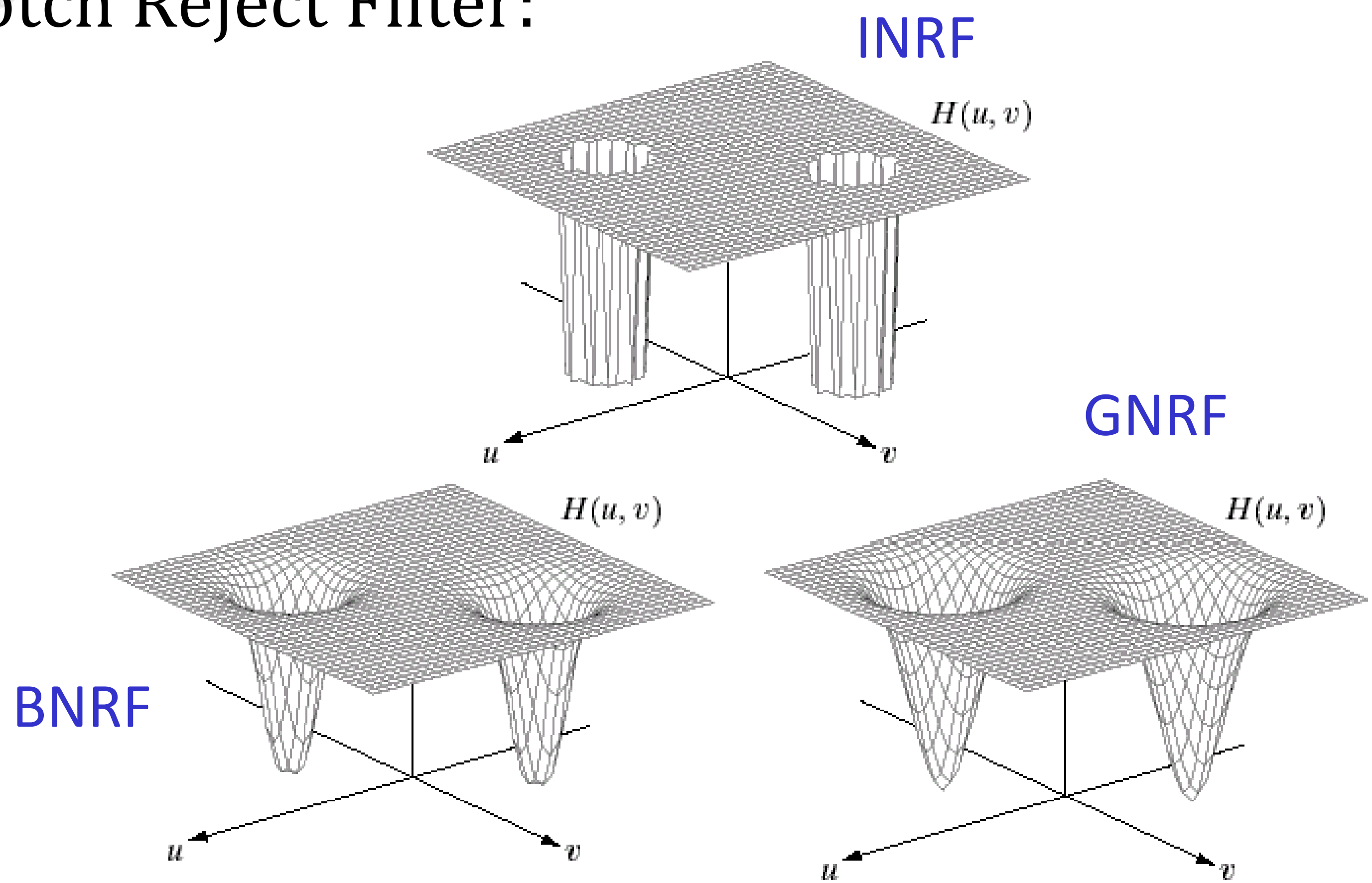


Image Restoration and ~~Reconstruction~~

- Notch Reject Filter:



NPF=1-NRF

Image Restoration and ~~Reconstruction~~

- Notch Filter (Single Frequency Removal):

$$D_1(u,v)=\left[\left(u-M/2-u_0\right)^2+\left(v-N/2-v_0\right)^2\right]^{1/2}$$

$$D_2(u,v)=\left[\left(u-M/2+u_0\right)^2+\left(v-N/2+v_0\right)^2\right]^{1/2}$$

$$H_I(u,v)=\begin{cases}0 & D_1(u,v)\le D_0 \text{ and } D_2(u,v)\le D_0\\ 1 & \text{O.W.}\end{cases}$$

$$H_B(u,v)=\frac{1}{1+\left[\dfrac{D_0^2}{D_1(u,v)D_2(u,v)}\right]}$$

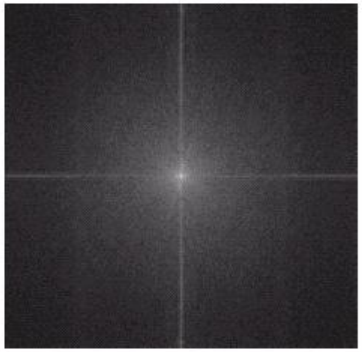
$$H_G(u,v)=1-\exp\left(-\frac{1}{2}\left(\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right)\right)$$

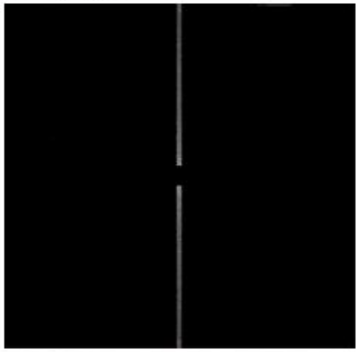
Image Restoration and ~~Reconstruction~~

- Example

- Horizontal Pattern in space
- Vertical Pattern in Frequency

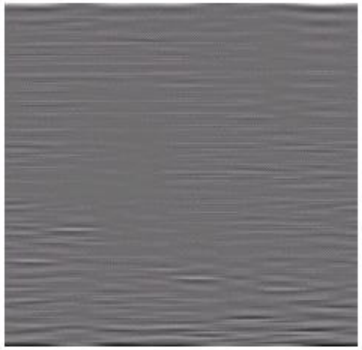
Spectrum

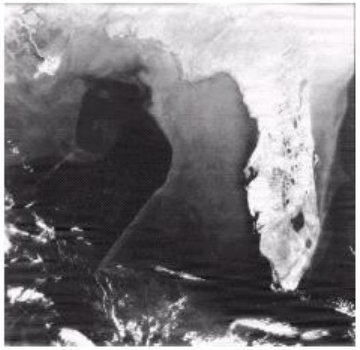




Notch in Vertical lines

Notch Passed Spectrum Vertical





Notch Rejected Spectrum

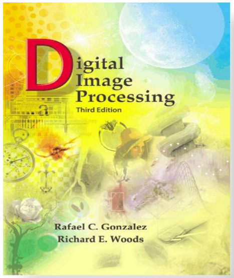
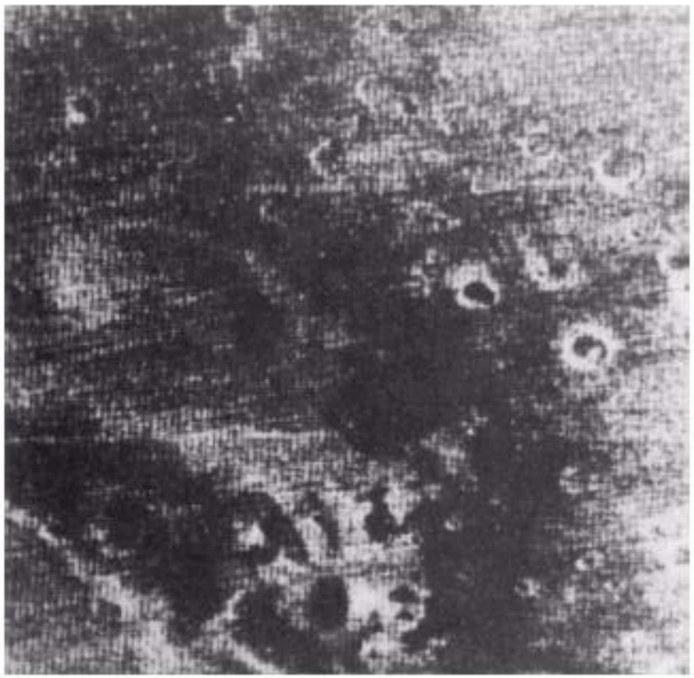
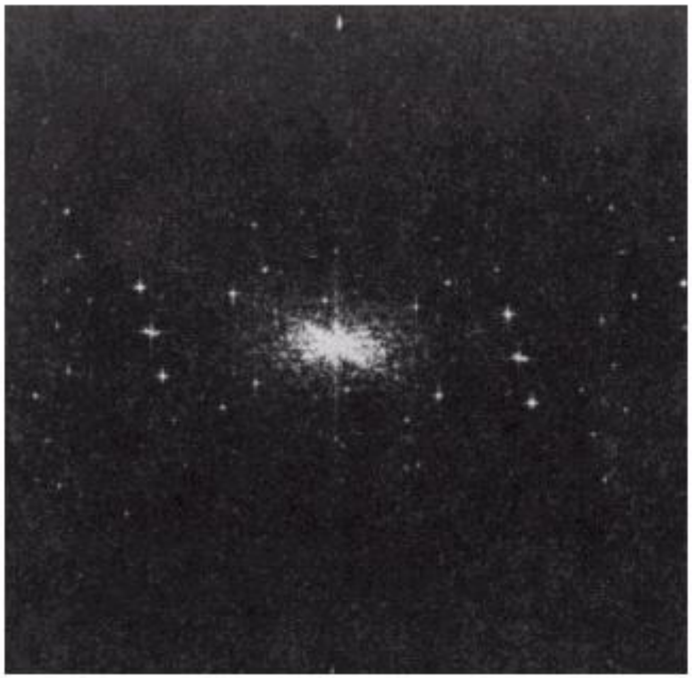


Image Restoration and ~~Reconstruction~~

- Several Single Frequency Noises:



Images



Spectrum

Image Restoration and ~~Reconstruction~~

- Multiple Notch will disturb the image:
  - Extract interference pattern using Notch Pass

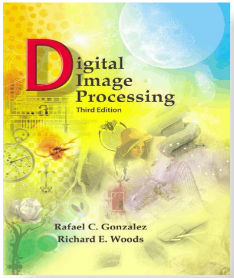
$$N(u,v) = H_{NP}(u,v)G(u,v) \Rightarrow \eta(x,y) = \mathfrak{I}^{-1}\{H_{NP}(u,v)G(u,v)\}$$

  - Now Perform Filter Function in Spatial Domain as An Adaptive-Local Point Process:

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$

  - Optimization Criteria: minimum variance of filtered images

$$\min\left\{\sigma_{\hat{f}}^{2}\right\}$$



- Optimum Notch Filter:

$$\sigma_{\hat{f}}^2(x,y)=\frac{1}{(2d+1)^2}\sum_{s=-d}^{+d}\sum_{t=-d}^{+d}\left[\hat{f}(x+s,y+t)-\bar{\hat{f}}(x,y)\right]^2$$

$$\bar{\hat{f}}(x,y)=\frac{1}{(2d+1)^2}\sum_{s=-d}^{+d}\sum_{t=-d}^{+d}\hat{f}(x+s,y+t)$$

$$\therefore$$

$$\sigma_{\hat{f}}^2(x,y)=\frac{1}{(2d+1)^2}\sum_{s=-d}^{+d}\sum_{t=-d}^{+d}\Big\{\big[g(x+s,y+t)-w(x+s,y+t)\eta(x+s,y+t)\big]-\big[\bar{g}(x,y)-\overline{w(x,y)\eta(x,y)}\big]\Big\}^2$$

Smoothness of $w(x, y)$:

$$w(x+s,y+t)\approx w(x,y),\ \overline{w(x,y)\eta(x,y)}\approx w(x,y)\bar{\eta}(x,y)$$

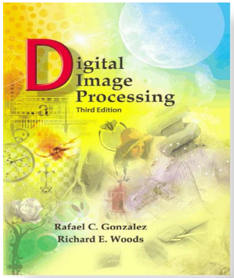


Image Restoration and ~~Reconstruction~~

- Optimum Notch Filter:

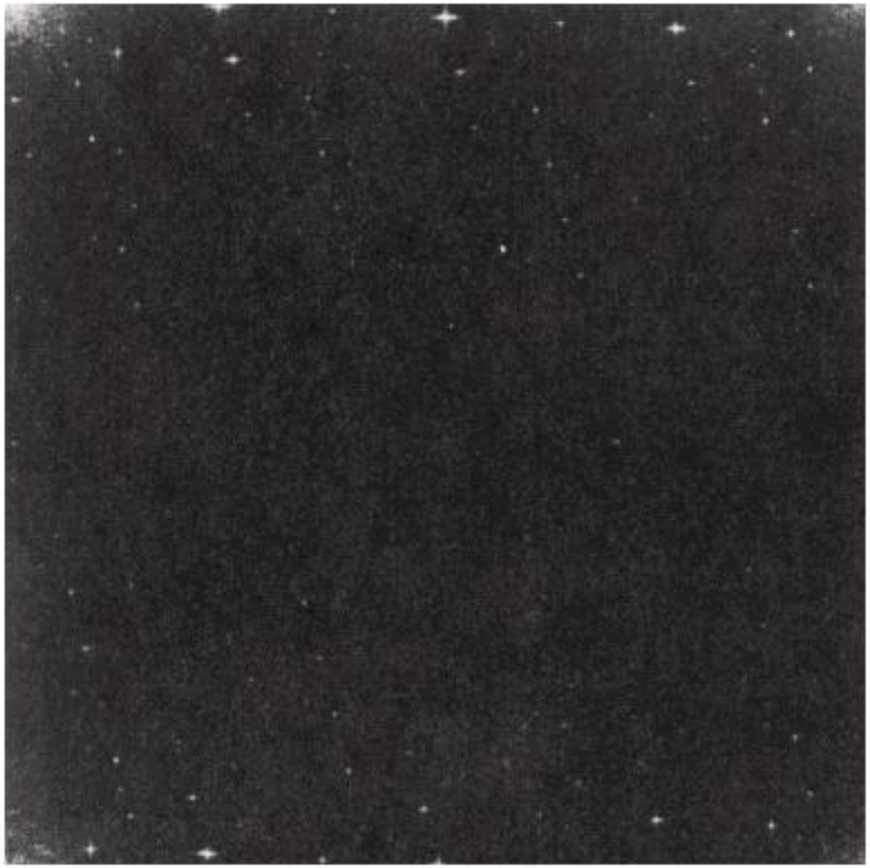
$$\sigma_{\hat{f}}^2(x,y) \simeq \frac{1}{(2d+1)^2} \sum_{s=-d}^{+d} \sum_{t=-d}^{+d} \Big\{ \big[g(x+s,y+t) - w(x,y)\eta(x+s,y+t) - \big]$$

$$\cdots \big[\bar{g}(x,y) - w(x,y)\bar{\eta}(x,y) \big] \Big\}^2$$

$$\bullet \frac{\partial \sigma_{\hat{f}}^2(x,y)}{\partial w(x,y)} = 0 \Rightarrow w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

- Computed for each pixel using surrounded mask

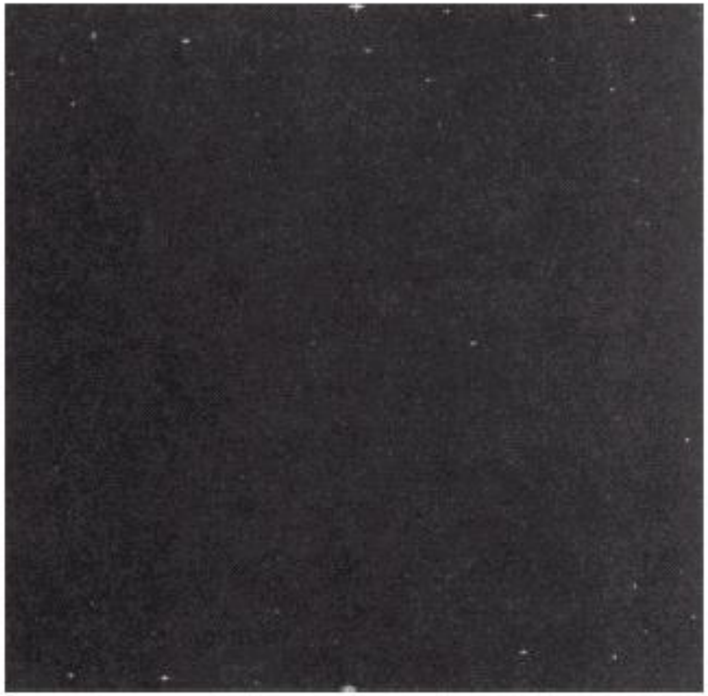
- Result:



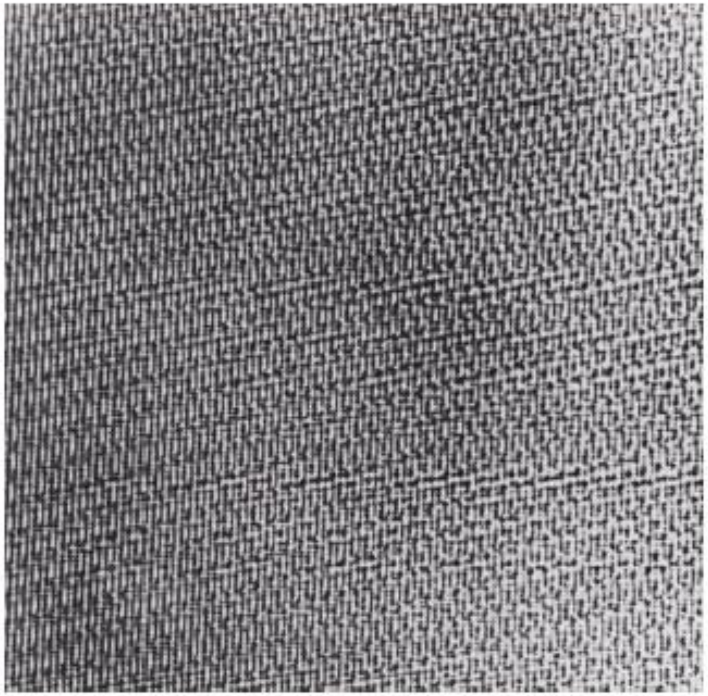
Former images Spectrum (without fftshift)

- Result:

$N(u,v)$



$\eta(x,y)$



- Result:

Filtered Image

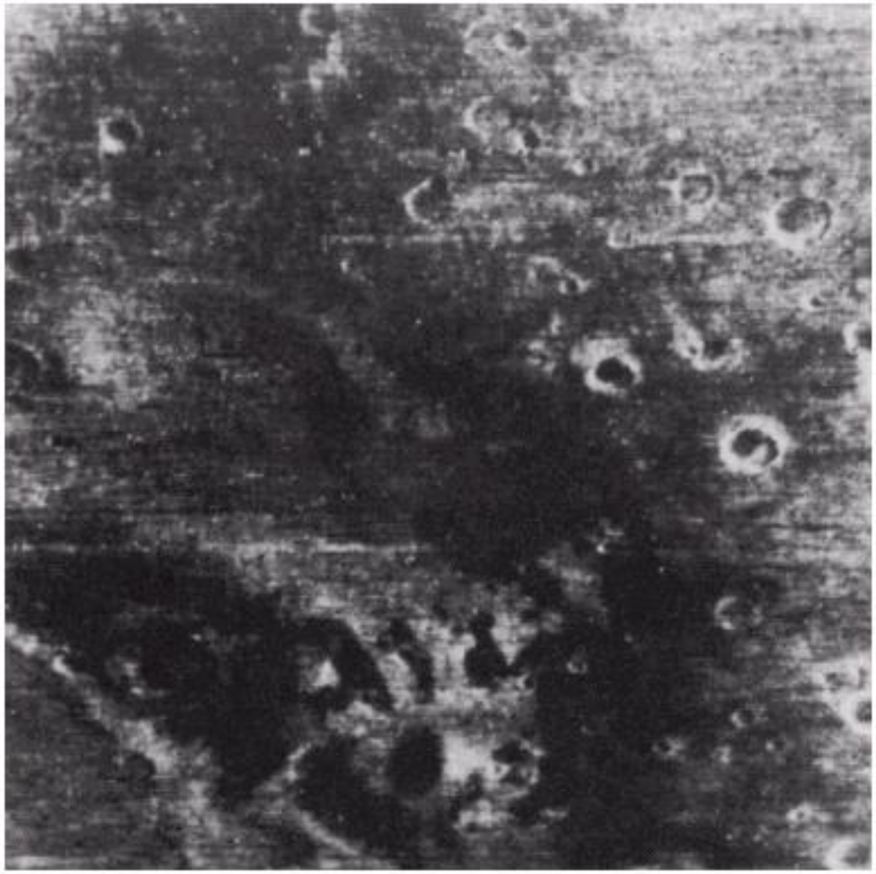


Image Restoration and ~~Reconstruction~~

- Linear Degradation:

L-System: $$\begin{cases} g(x,y) = H\{f(x,y)\} + \eta(x,y) \\ H\{f(x,y)\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta) H\{\delta(x-\alpha, y-\beta)\} d\alpha d\beta \\ h(x,y,\alpha,\beta) = H\{\delta(x-\alpha, y-\beta)\} \end{cases}$$

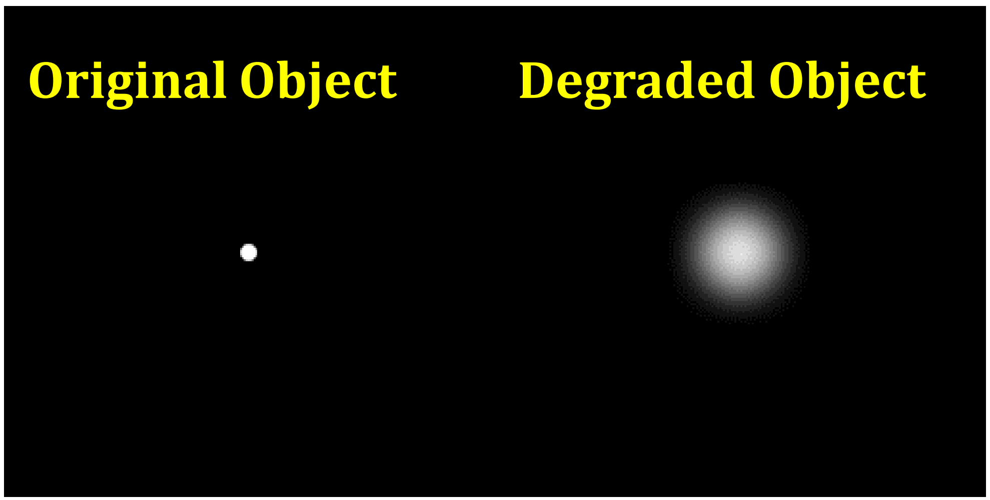
LSI-System: $$\begin{cases} H\{f(x,y)\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta) h(x-\alpha, y-\beta) d\alpha d\beta \\ g(x,y) = f(x,y) \bigstar h(x,y) + \eta(x,y) \\ G(u,v) = F(u,v) H(u,v) + N(u,v) \end{cases}$$

Image Restoration and ~~Reconstruction~~

- Degradation Estimation:
  - Image Observation:
    - Look at the image and …
  - Experiments:
    - Acquire image using well defined object (pinhole)
  - Modeling:
    - Introduce certain model for certain degradation using physical knowledge.

Image Restoration and ~~Reconstruction~~

- Degradation (Using Experimental PSF)



$$H_s(u,v) = \frac{G(u,v)}{A_{PSF}}$$

Image Restoration and ~~Reconstruction~~

- Atmospheric Turbulence:

NASA

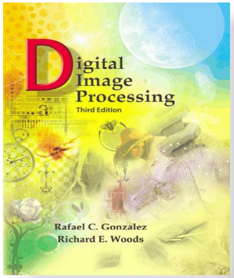


Image Restoration and ~~Reconstruction~~

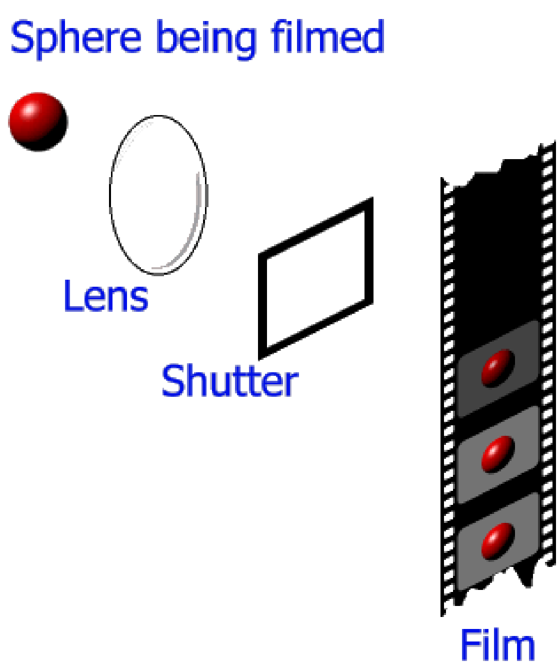
- Modeling of turbulence in atmospheric images:

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

$$H(u,v) = \exp\left(-k\left(u^2+v^2\right)^{5/6}\right)$$

- Motion Blurring:
  - Camera Limitation;
  - Object movement.



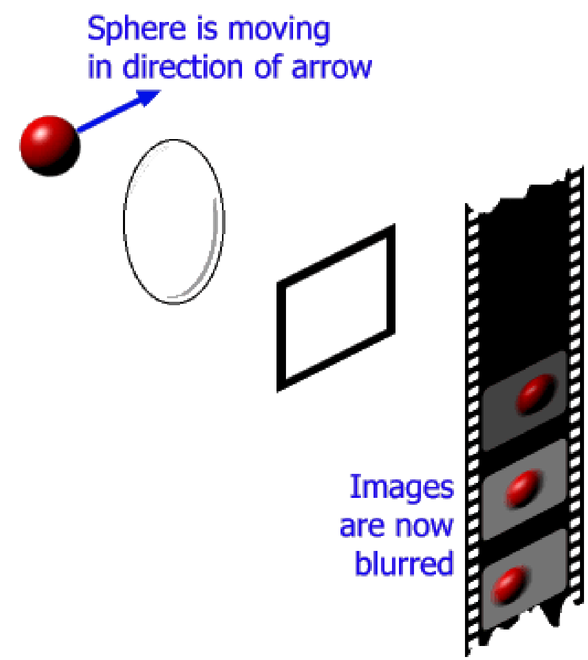


Image Restoration and ~~Reconstruction~~

- Motion Blurring Modeling:

$$g(x,y)=\int_0^T f\big(x-x_0(t),y-y_0(t)\big)dt$$

$$G(u,v)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\int_0^T f\big(x-x_0(t),y-y_0(t)\big)dt\right)e^{-j2\pi(ux+vy)}dxdy$$

$$G(u,v)=F(u,v)\int_0^T e^{-j2\pi\left(ux_0(t)+vy_0(t)\right)}dt=F(u,v)H(u,v)$$

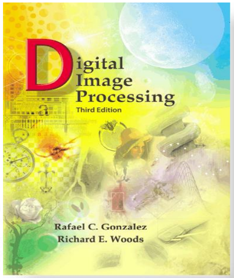


Image Restoration and ~~Reconstruction~~

- Linear one/Two dimensional motion blurring:

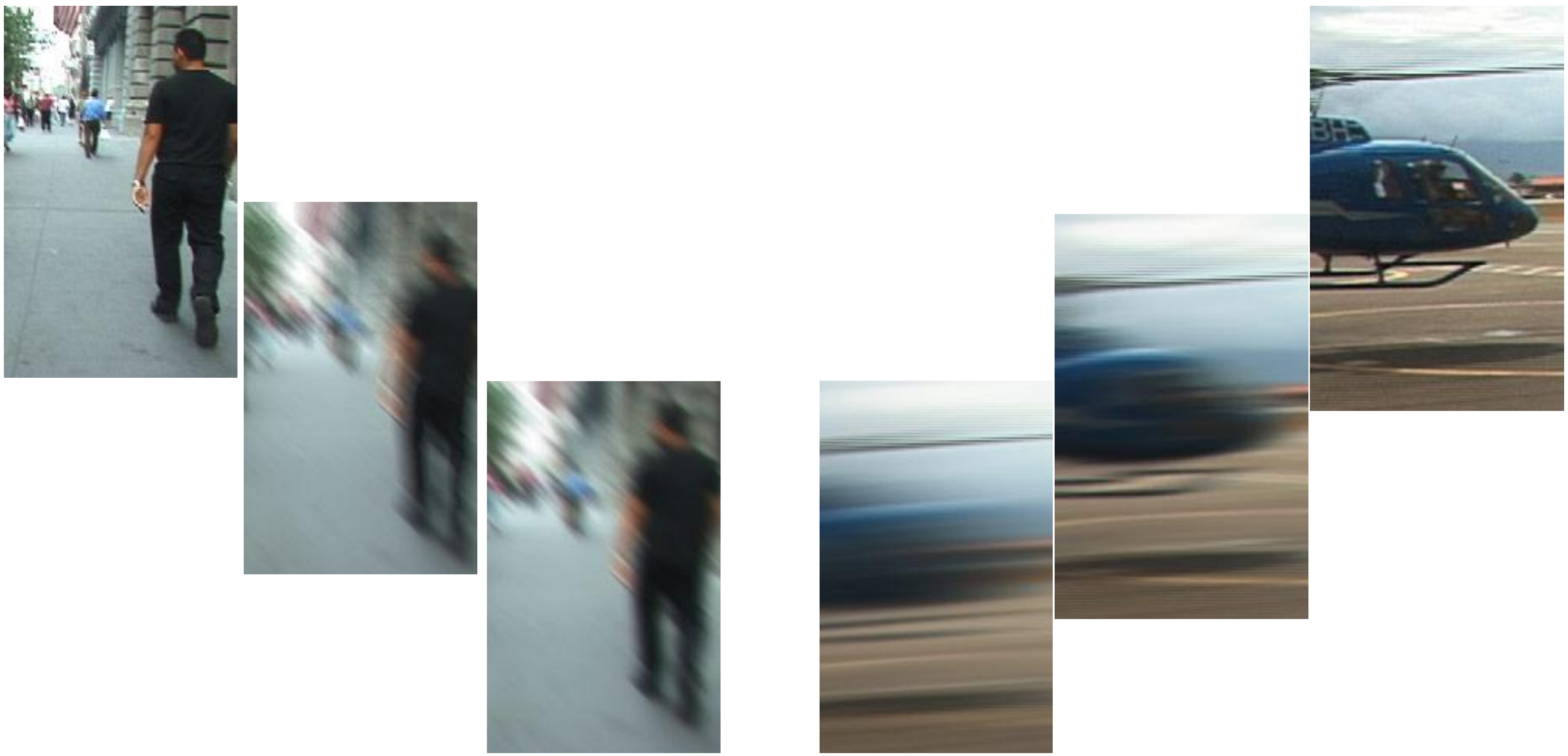
$$x_0(t)=\frac{at}{T},\quad t_{Max}=T\Rightarrow H(u,v)=\frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$

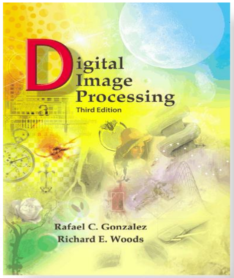
$$x_0(t)=\frac{at}{T},\quad y_0(t)=\frac{bt}{T}\Rightarrow H(u,v)=\frac{T}{\pi(ua+vb)}\sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$

- Motion Blurring Example (1):
  - a=b=0.1 and T=1
  - Original (Left) and Notion Blurred (Right)

Image Restoration and ~~Reconstruction~~

- Motion Blurring Example (2):





- Inverse Filtering:
  - Without Noise:

$$\hat{F}(u,v)=\frac{G(u,v)}{\hat{H}(u,v)}=\frac{F(u,v)H(u,v)}{\hat{H}(u,v)}\approx F(u,v)$$

$\Rightarrow$ Problem of division by zero or NaN!

- Inverse Filtering:
  - With Noise:

$$\hat{F}(u,v) = \frac{G(u,v)}{\hat{H}(u,v)} = \frac{F(u,v)H(u,v)+N(u,v)}{\hat{H}(u,v)} \approx F(u,v) + \frac{N(u,v)}{\hat{H}(u,v)}$$

$\Rightarrow$ Problem of division by zero!

$\Rightarrow$ Impossible to recover even if H(.,.) is known!!

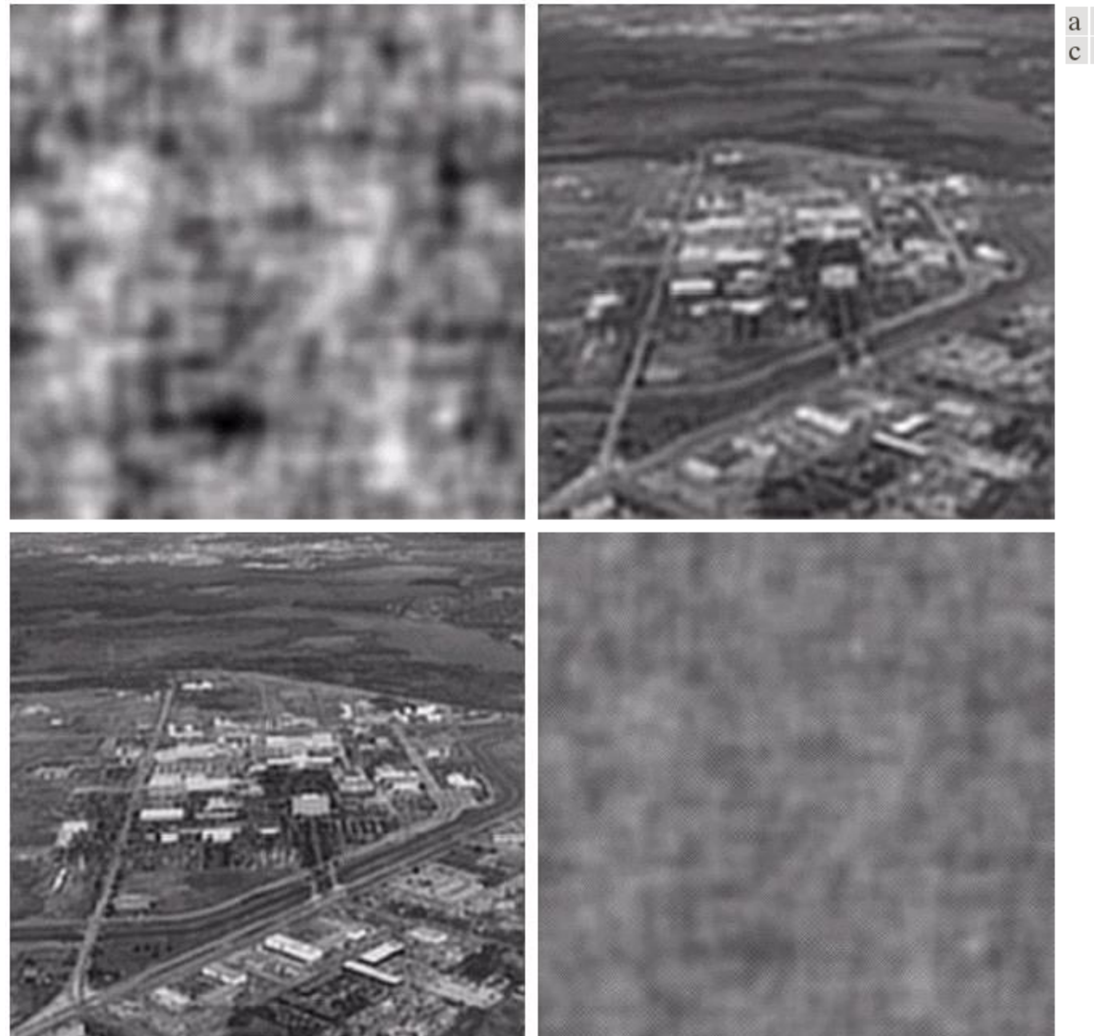
Image Restoration and ~~Reconstruction~~

- Pseudo Inverse (Constrained) Filtering:
  - Set infinite (large) value to zero;

$$\hat{F}(u,v) = \begin{cases} \dfrac{G(u,v)}{\hat{H}(u,v)} & \left|\hat{H}(u,v)\right| \geq H_{THR} \\ 0 & \left|\hat{H}(u,v)\right| < H_{THR} \end{cases}$$

Image Restoration and ~~Reconstruction~~

- Example:
  - Full Band (a)
  - Radius 50 (b)
  - Radius 70 (c)
  - Radius 85 (d)



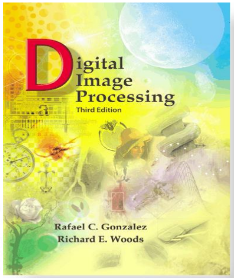


Image Restoration and ~~Reconstruction~~

- Degradation plus Noise:

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Uncorrelated Noise and Image:

$$W(u,v) = \frac{P_{FF}H^*}{P_{FF}|H|^2 + P_{NN}} = \frac{H^*}{|H|^2 + \dfrac{P_{NN}}{P_{FF}}}$$

$$= \frac{1}{H}\frac{|H|^2}{|H|^2 + \dfrac{P_{NN}}{P_{FF}}} = \frac{1}{H}\frac{|H|^2}{|H|^2 + \underbrace{\left(\dfrac{P_{FF}}{P_{NN}}\right)}_{SNR}^{-1}}$$

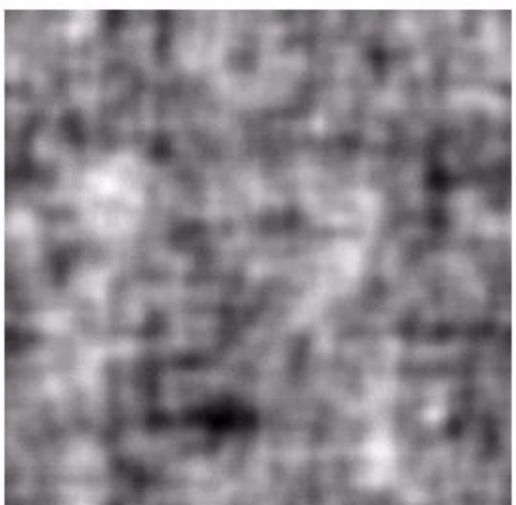
Image Restoration and ~~Reconstruction~~

- Degradation plus Noise:
  - White Noise

$$W(u,v)=\frac{1}{H}\frac{|H|^2}{|H|^2+\underbrace{K}_{\text{Select Interavtively}}}$$

Image Restoration and ~~Reconstruction~~

- Example:



Full Inverse

Pseudo Inverse

Wiener

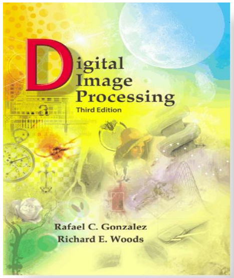


Image Restoration and ~~Reconstruction~~

- Example:
  - Motion Blurring+Noise

Degraded | Inverse | Wiener

Noise Decrease