Engineering Economic Analysis

FOURTEENTH EDITION

DONALD G. NEWNAN
TED G. ESCHENBACH
JEROME P. LAVELLE
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Brian Newnan, my chemical engineering nephew, who helped guide this book forward

DN

In memoriam to Richard Corey Eschenbach, for his lifelong example of engineering leadership and working well with others

TE

To my lovely wife Christine and sweet daughters Gabrielle, Veronica, Miriam, Regina, and Magdalen, who all inspire me daily to be my best!

JL

My wife Joan, for her continued support

NL

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PREFACE

Our goal has been, and still is, to provide an easy-to-understand and up-to-date presentation of engineering economic analysis for today's students. That means the book's writing style must promote the reader's understanding. We humbly note that our approach has been well received by engineering professors—and more importantly, by engineering students through multiple editions.

Hallmarks of this Book

Since it was first published, this text has become the market-leading book for the engineering economy course. It has always been characterized by

- **A focus on practical applications.** One way to encourage students to read the book, and to remember and apply what they have learned in this course, is to make the book interesting. And there is no better way to do that than to infuse the book with real-world examples, problems, and vignettes.
- **Accessibility.** We meet students where they are. Most don't have any expertise in accounting or finance. We take the time to explain concepts carefully while helping students apply them to engineering situations.
- **Superior instructor and student support packages.** To make this course easier to understand, learn, and teach, Oxford University Press has produced the best support package available. We offer more for students and instructors than any competing text.

New to the 14th Edition

- Since bonus depreciation has been allowed for all but 4 years since 2001, depreciation for corporate tax purposes in the U.S. is best described and taught as bonus plus MACRS—rather than either system alone. Chapter 11 has been heavily revised to reflect this. The chapter also describes why and how straight-line and declining balance methods are used for valuation.
- Chapter 12 on income taxes includes the changes in depreciation and the

simplification of corporate tax rates from 8 brackets and a top rate of 39% to a lower and flat 21%. State income taxes at up to 10% are now more important in practice. Pedagogically, progressive state tax rates schedules reinforce student understanding of marginal and average rates.
•Chapter 13 is now “Economic Life and Replacement Analysis.”
•Economic analysis in practice must answer the question, how long will an alternative be used? Examples include overhauls and the costs of unplanned replacements, failures in use, and foregoing the value of newer technology.
•Students are introduced to spreadsheets designed for choosing the cost minimizing life. Careful function definitions do the financial arithmetic required so that students can focus on learning and reinforce it by doing more problems.
•Replacement analysis now focuses on an existing, aging asset with increasing costs. This better matches the time constraints and objectives of introductory courses.
•Expansion of online course material to include auto-graded algorithmic variations of new and end-of-chapter problems.
•Other changes include 5 new vignettes. Life-cycle costs have an added figure and discussion in Chapter 2. Chapter 6’s coverage of irregular cash flows shows how midlife cash flows for overhauls and expansions are converted to annual equivalents. The chapter clarifies the difference between capital recovery costs and all annual costs. Inflation data has been updated in Chapter 15. After-tax coverage in later chapters is revised.

Strengths of the 14th Edition

•Factor notation and tabulated factors are a clear way to understand and do engineering economic analysis. This is reinforced by spreadsheet annuity functions, which are presented in a visual 5-BUTTON format. The two approaches are mutually reinforcing for faster and deeper student understanding.
•There is an answer icon next to most even-numbered problems with answers in Appendix E.
•Instructors can easily pick a preferred mix of problems with and without answers.
•Students can do extra problems and check their own answers.

•Solutions in the *Instructor's Manual* were completed and text corrections were made *before* the book was finalized for printing.
•Each chapter opens with a list of *keywords*, which are **boldfaced** when first explained and indexed for later reference.
•This text has an identified path for learning how to use spreadsheets in economic analysis. This supports student learning and later engineering practice. It supports faculty member choices from no coverage to heavy reliance. Students can choose whether to learn more on their own.
•Chapter 1 explains data blocks, what-if analysis, and relative/absolute addresses. Appendix A provides more for spreadsheet novices.
•Spreadsheet annuity functions are introduced beginning with Example 3–5; spreadsheet block functions are covered in Chapter 4 after factor approaches for arithmetic and geometric gradients where annuity functions cannot be used.
•Other spreadsheet functions including XNPV, XIRR, SUMPRODUCT, and GOAL SEEK are presented when they will allow or speed solutions in economic analysis.
•Problems in Chapters 12, 13, and 14 on taxes, replacement analysis, and inflation tend to involve more calculations than other chapters so spreadsheets are particularly useful.
•Chapter appendices on investing, diversification, and personal finance build on the loans, savings, and other personal finance examples that have long been used to motivate students and engage them with engineering economy concepts. Our first goal is force the realization that engineering economy really does matter. Second, personal financial success contributes to success as a student, as an engineer, and in life.
•Green engineering and ethics are in every chapter. Ethics questions continue to be part of the Questions to Consider in the vignettes. Along with coverage of multiple objectives beginning in Chapter 1, this can extend coverage of economic analysis and engineering decision-making into a broader context.

Teaching and Learning Package

This book is supported by a carefully crafted set of ancillary teaching and learning materials. The supplements package for this text has been updated and expanded again for this edition, making it the most extensive support package available for this course.

Dashboard

New to this edition, Dashboard delivers a wealth of study resources and automatically graded problems in an intuitive, web-based learning environment. A built-in color-coded gradebook allows instructors to track student progress. Instructors can save their students money by ordering Dashboard packaged with the print edition. Students can also purchase stand-alone Dashboard access (which includes the interactive eBook) online directly at www.oup.com/us/dashboard. Dashboard includes:

•Auto-graded, algorithmic problems for online homework assignments.
•Interactive tutorial questions written by Paul Schnitzler of the University of South Florida and William Smyer of Mississippi State University.
•Practice problems in the style of the FE exam authored by Karen Thorsett, University of Phoenix.

Student Resources

•A Study Guide by Ed Wheeler of the University of Tennessee at Martin and the text authors contains more than 500 additional problems with detailed solutions. It is available for print purchase and integrated into the eBook edition and the Dashboard course at no extra charge.
•Additional free student resources are available online at www.oup.com/us/newnan.
•A set of tutorials on engineering economy applications of Excel by Julie L. Fortune of the University of Alabama in Huntsville.
•A set of 54 cases provides realistic, complex problems. These cases, written by William Peterson and Ted Eschenbach and 13 contributors, also include three chapters on case analysis and an example case solution.
•Spreadsheet problem modules, written by Thomas Lacksonen of the University of Wisconsin Stout.

Instructor Resources

Instructors will find an updated and expanded set of resources available at www.oup.com/us/newnan. Please contact your Oxford University Press representative for access.

•An exam file written and edited by Meenakshi Sundaram of Tennessee Technological University.
•PowerPoint lecture notes for all chapters by Neal Lewis of Fairfield University.
•The compound interest tables are available online as PDF files for adopting professors who prefer to give closed-book exams.
•An Instructor's Manual by John M. Usher of Mississippi State University with complete solutions to all end-of-chapter problems.

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This book remains the best text on the market in large part because of feedback from users. We would appreciate hearing comments about the book, including being informed of any errors that have snuck in despite our best attempts to eradicate them. We also look forward to adding problems and vignettes in the next edition that adopters have found effective for their students. Please write us c/o the Engineering Editor at Oxford University Press, 198 Madison Avenue, New York, NY 10016, or email us directly. Thanks for using the Newnan book!

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ENGINEERING ECONOMIC ANALYSIS

CHAPTER 1

MAKING ECONOMIC DECISIONS

Delivered Food and Decision Making

Hungry? Will you order food and get it delivered to your dorm room or home? If you do, will you call the restaurant directly or use an online platform like UberEats, DoorDash, GrubHub (Seamless), Postmates, or Eat24? Today many people want their food delivered to home or office, and restaurants don't want to miss out on possible sales. Restaurants are joining with third-party online platforms and are turning delivery from a small segment of the restaurant industry to a booming new source of sales at establishments well beyond fast food. However, decision making is more complex than it appears on the surface.

Many establishments are part of chains where engineers have designed facilities for a mix of "take-out" and "dine-in." What must change when the mix changes? Does it make sense to separate pickup from drive-through? Turning a profit in the food business is tough. Partnering with delivery platforms squeezes margins even tighter. These platforms usually charge 10%–40% of the order's cost. The online platforms maintain that they bring "incremental" revenue to restaurants—that the restaurants would not otherwise receive. The platforms also emphasize that delivery orders are a form of marketing, exposing potential new customers who might become lucrative in-restaurant patrons.

However, what sounds like a boon for restaurants carries unexpected risks, including shrinking profit margins and shifting customer allegiances. Deliveries can risk cannibalizing more profitable dine-in sales by encouraging customers to stay at home. If an order takes longer than expected or if the food arrives cold, customers may blame the restaurant, not the delivery platform. The customers might not return, and a negative review may discourage other people from trying the restaurant. Customer loyalty may shift from the restaurant to the third-party delivery service that is now between the restaurant and the customer. Staff compensation at the restaurant may have to change if the driver is tipped, rather than the staff.

Some restaurants, such as Olive Garden, Texas Roadhouse, and Domino's Pizza, are at least for now avoiding the squeezed margins and other risks of third-party platforms. If you are hungry, it would be much kinder to call the restaurant directly or go to the restaurant's website. There are still delivery charges, but no third-party commission.

Contributed by Kate D. Abel, Stevens Institute of Technology

QUESTIONS TO CONSIDER

1. What sort of operational issues could be created in the restaurants that choose to use third-party platforms? How could these issues affect the financial viability of joining with the platforms?
2. This vignette was about restaurant delivery. Can you think of another venue where similar delivery services might be desired in the future?
3. Before reading this vignette, did you think that *how* you ordered your food

could have an economic impact? Which ordering option do you think you will use moving forward? Why?
4. Develop a list of concerns and questions consumers might have for the restaurants and the third-party platforms. Which are economic and which are noneconomic factors?

After Completing This Chapter…

The student should be able to:

• Distinguish between simple and complex problems.
• Discuss the role and purpose of engineering economic analysis.
• Describe and give examples of the nine steps in the *economic decision-making process*.
• Select appropriate economic criteria for use with different types of problems.
• Describe common ethical issues in engineering economic decision making.
• Solve engineering problems with current costs.
• Solve problems that have multiple objectives.

Key Words

absolute address

benefit

brainstorming

cost

criteria

data block

decision making

fixed input

fixed output

green engineering

maximizing profit

model building

multiple objectives

overhead

resolving consequences

shadow price

societal costs

value engineering

what-if analysis

This book is about making decisions. **Decision making** is a broad topic, for it is a major aspect of everyday human existence. This book develops the tools to properly analyze and solve the economic problems that are commonly faced by engineers. Even very complex situations can be broken down into components from which sensible solutions are produced. If one understands the decision-making process and has tools for obtaining realistic comparisons between alternatives, one can expect to make better decisions.

Our focus is on solving problems that confront firms in the marketplace, but many examples are problems faced in daily life. Let us start by looking at some of these problems.

A SEA OF PROBLEMS

A careful look at the world around us clearly demonstrates that we are surrounded by a sea of problems. There does not seem to be any exact way of classifying them, simply because they are so diverse in complexity and "personality." One approach arranges problems by their *difficulty*.

Simple Problems

Many problems are pretty simple, and good solutions do not require much time or effort.

- Should I pay cash or use my credit card?
- Do I buy a semester parking pass or use the parking meters?
- Shall we replace a burned-out motor?
- If we use three crates of an item a week, how many crates should we buy at a time?

Intermediate Problems

At a higher level of complexity we find problems that are primarily economic.

- Shall I buy or lease my next car?
- Which equipment should be selected for a new assembly line?
- Which materials should be used as roofing, siding, and structural support for a new building?
- Shall I buy a 1- or 2-semester parking pass?
- What size of transformer or air conditioner is most economical?

Some numeric examples of operational economics follow the section on ethics later in this chapter.

Complex Problems

Complex problems are a mixture of *economic*, *political*, and *humanistic* elements.

- Honda Motors in North America illustrates complex problems. In Alliston, Ontario, they employ 4000 workers and manufacture the Acura MDX, ZDX, CSX, and Civic. In Lincoln, Alabama, they employ 4000 workers and

manufacture the Odyssey, Pilot, Ridgeline, and Acura MDX. Any decision allocating production must consider, along with economic aspects: reactions of the American, Canadian, Japanese, and Mexican governments; international trade agreements; labor unions in three countries; and the 2014 opening of a second Mexican plant in Celaya.
• The selection of a dating partner (who may later become a permanent partner) is obviously complex. Economic analysis can be of little or no help.
• A firm's annual budget allocates resources and all projects are economically evaluated. The budget process is also heavily influenced by noneconomic forces such as power struggles, geographical balancing, and impact on individuals, programs, and profits. For multinational corporations there are even national interests to be considered.

The chapter's final section presents one approach to more complex problems.

THE ROLE OF ENGINEERING ECONOMIC ANALYSIS

Engineering economic analysis is most suitable for intermediate problems and the economic aspects of complex problems. They have these qualities:

1. The problem is *important enough* to justify our giving it serious thought and effort.
2. The problem can't be worked in one's head—that is, a careful analysis *requires that we organize* the problem and all the various consequences.
3. The problem has *economic aspects* important in reaching a decision.

Engineers determine how money should be spent now to achieve cost savings and to increase revenues and other benefits that span years and often decades. Thus, when engineers face problems meeting the three criteria listed above, engineering economic analysis is required. Engineering in academia focuses on principles and design, but in engineering practice the focus is on money and value—as determined using engineering economy.

Students can apply many engineering economy tools to their personal lives by understanding time value of money, loans, savings, investments, and tax implications. “Trust Me: You’ll Use This” (on pp. 76–77), Appendices 9A, 10A, and 12A, are focused on personal finance issues.

Engineering economy is applied professionally in for-profit firms, nonprofit organizations, and government agencies. Many examples and problems focus on for-profit firms. These firms must consider depreciation and taxes, as covered in Chapters 11 and 12. Nonprofit organizations (most private universities and many hospitals) and government agencies (school districts, cities, states, and federal) often have benefits that are hard to value (Chapter 16). Most engineering economy topics apply to a wide variety of people and organizations.

Examples of Engineering Economic Analysis

Engineering economic analysis focuses on costs, revenues, and benefits that occur at different times. For example, when a civil engineer designs a road, a dam, or a building, the construction costs occur in the near future; but the benefits to users begin only when construction is finished and then continue for a long time.

In fact nearly everything that engineers design calls for spending money in the design and building stages, and only after completion do revenues or benefits occur—usually for years. Thus the economic analysis of costs, benefits, and revenues occurring over time is called *engineering* economic analysis.

Engineering economic analysis is used by firms and government agencies to answer many different questions.

• *Which engineering projects are worthwhile?* Has the mining or petroleum engineer shown that the mineral or oil deposit is worth developing?
• *Which engineering projects should have a higher priority?* Has the industrial engineer shown which factory improvement projects should be funded with the available dollars?
• *How should the engineering project be designed?* Has the mechanical or electrical engineer chosen the most economical motor size? Has the civil or mechanical engineer chosen the best thickness for insulation? Has the

aeronautical engineer made the best trade-offs between (1) lighter materials that are expensive to buy but cheaper to fly and (2) heavier materials that are cheap to buy and more expensive to fly?

Engineering economic analysis can also be used to answer questions that are personally important.

- *How to achieve long-term financial goals:* How much should you save each month to buy a house, retire, or fund a trip around the world? Is going to graduate school a good investment—will your additional earnings in later years balance the cost of attending and your lost income while in graduate school?
- *How to compare different ways to finance purchases:* Is it better to finance your car purchase by using the dealer's low interest rate loan or by taking an available rebate and borrowing money from your bank or credit union?
- *How to make short- and long-term investment decisions:* Should you buy a 1- or 2-semester parking pass? Is a higher salary better than stock options?

THE DECISION-MAKING PROCESS

Decision making may take place by default; that is, a person may not consciously recognize that an opportunity for decision making exists. This fact leads to our first element in a definition of decision making—there must be at least two alternatives available. If only one course of action is available, there is nothing to decide. The only alternative is to proceed with the single available course of action. (It is rather unusual to find that there are no alternative courses of action. More frequently, alternatives simply are not recognized.)

At this point we might conclude that the decision-making process consists of choosing from among alternative courses of action. But this is an inadequate definition. Consider a bettor at the Kentucky Derby who picks a horse by pointing at the program with closed eyes. Does this racehorse selection represent decision making? Yes (assuming the bettor had already ruled out the "do-nothing" alternative of not betting). But the method of deciding seems inadequate and irrational. We want to deal with rational decision making.

Rational Decision Making

Rational decision making is a complex process that contains nine essential elements, which are shown in Figure 1–1. Although these nine steps are shown sequentially, it is common for a decision maker to repeat steps, take them out of order, and do steps simultaneously. For example, when a new alternative is identified more data will be required. Or when the outcomes are summarized, it may become clear that the problem needs to be redefined or new goals established.

The value of this sequential diagram is to show all the steps that are usually required, and to show them in a logical order. Occasionally we will skip a step entirely. For example, a new alternative may be so clearly superior that it is immediately adopted at Step 4 without further analysis. The following sections describe the elements listed in Figure 1–1.

1. Recognize the Problem

The starting point in rational decision making is recognizing that a problem exists.

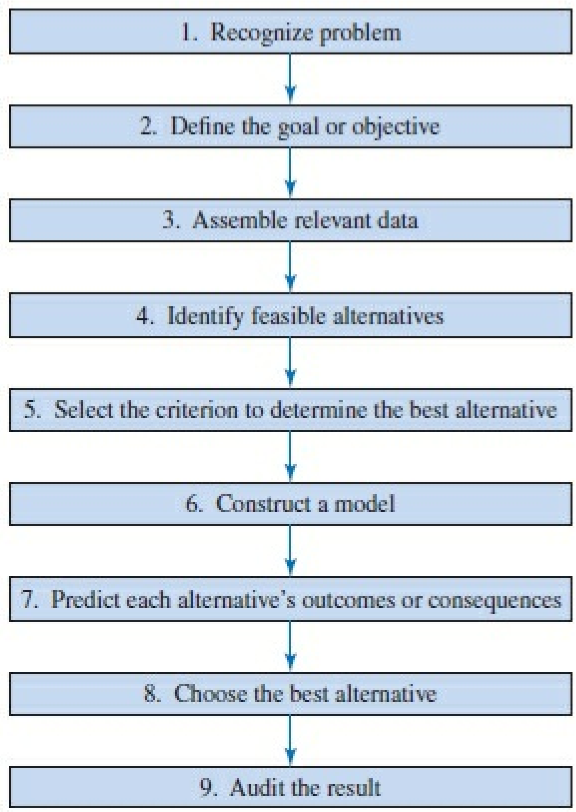


FIGURE 1-1 One possible flowchart of the decision process.

Some years ago, for example, it was discovered that several species of ocean fish contained substantial concentrations of mercury. The decision-making process began with this recognition of a problem, and the rush was on to determine what should be done. Research revealed that fish taken from the ocean decades before and preserved in laboratories also contained similar concentrations of mercury. Thus, the problem had existed for a long time but had not been recognized.

In typical situations, recognition is obvious and immediate. An auto accident, an overdrawn check, a burned-out motor, an exhausted supply of parts all produce the recognition of a problem. Once we are aware of the problem, we can solve it as best we can. Many firms establish programs for total quality management (TQM) or continuous process improvement (CPI) that are designed to identify problems so that they can be solved.

2. Define the Goal or Objective

The goal or objective can be an overall goal of a person or a firm. For example, a personal goal could be to lead a pleasant and meaningful life, and a firm's goal is usually to operate profitably. The presence of multiple, conflicting goals is often the foundation of complex problems.

But an objective need not be an overall goal of a business or an individual. It may be quite narrow and specific: "I want to pay off the loan on my car by May," or "The plant must produce 300 golf carts in the next 2 weeks," are more limited objectives. Thus, defining the objective is the act of exactly describing the task or goal.

3. Assemble Relevant Data

To make a good decision, one must first assemble good information. In addition to all the published information, there is a vast quantity of information that is not written down anywhere but is stored as individuals' knowledge and experience. There is also information that remains ungathered. A question like "How many people in your town would be interested in buying a pair of left-handed scissors?" cannot be answered by examining published data or by asking any one person. Market research or other data gathering would be required to obtain the desired information.

From all this information, what is relevant in a specific decision-making process? Deciding which data are important and which are not may be a complex task. The availability of data further complicates this task. Published data are available immediately at little or no cost; other data are available from specific knowledgeable people; still other data require surveys or research to assemble the information. Some data will be of high quality—that is, precise and accurate, while other data may rely on individual judgment for an estimate.

If there is a published price or a contract, the data may be known exactly. In most cases, the data is uncertain. What will it cost to build the dam? How many vehicles will use the bridge next year and twenty years from now? How fast will a competing firm introduce a competing product? How will demand depend on growth in the economy? Future costs and revenues are uncertain, and the range of likely values should be part of assembling relevant data.

The problem's time horizon is part of the data that must be assembled. How long will the building or equipment last? How long will it be needed? Will it be scrapped, sold, or shifted to another use? In some cases, such as for a road or a tunnel, the life may be centuries with regular maintenance and occasional rebuilding. A shorter time period, such as 50 years, may be chosen as the problem's time horizon, so that decisions can be based on more reliable data.

In engineering decision making, an important source of data is a firm's own accounting system. These data must be examined quite carefully. Accounting data focuses on past information, and engineering judgment must often be applied to estimate current and future values. For example, accounting records can show the past cost of buying computers, but engineering judgment is required to estimate the future cost of buying computers.

Financial and cost accounting are designed to show accounting values and the flow of money—specifically **costs** and **benefits**—in a company's operations. When costs are directly related to specific operations, there is no difficulty; but there are other costs that are not related to specific operations. These indirect costs, or **overhead,** are usually allocated to a company's operations and products by some arbitrary method. The results are generally satisfactory for cost-accounting purposes but may be unreliable for use in economic analysis.

To create a meaningful economic analysis, we must determine the *true* differences between alternatives, which might require some adjustment of cost-accounting data. The following example illustrates this situation.

EXAMPLE 1-1

A firm's printing department charges the other departments for its services to recover its monthly costs. For example, the charge to run 30,000 copies for the shipping department is:

Direct labor	$228
Materials and supplies	294
Overhead costs	271
Cost to shipping department	$793

The shipping department checks with a commercial printer, which would print the same 30,000 copies for $688. The shipping department foreman wants to have the work done externally. The in-house printing department objects to this. The general manager has asked you to recommend what should be done.

SOLUTION

Some of the printing department's output reveals the firm's costs, prices, and other financial information. Thus, the printing department is necessary to prevent disclosing such information to people outside the firm. The firm cannot switch to an outside printer for all needs.

A review of the cost-accounting charges reveals nothing unusual. The charges made by the printing department cover direct labor, materials and supplies, and overhead. The allocation of indirect costs is a customary procedure in cost-accounting systems (see Chapter 17 for more). It can be misleading for decision making, as the following discussion indicates.

The shipping department would reduce its cost by $105 (= $793 – $688) by using the outside printer. In that case, how much would the printing department's costs decline, and which solution is better for the firm?

1. *Direct Labor*. If the printing department had been working overtime, then the overtime could be reduced or eliminated. But, assuming no overtime, how much would the saving be? It seems unlikely that an employee could be fired or even put on less than a 40-hour work week. Thus, although there might be a $228 saving, it is much more likely that there will be no reduction in direct labor.
2. *Materials and Supplies.* There would be a $294 saving in materials and supplies.
3. *Allocated Overhead Costs.* There will be no reduction in the printing department's monthly overhead, and in fact the firm will incur $50 of

additional expenses in purchasing and accounting for processing the purchase order, invoice, and payment.

The firm will save $294 in materials and supplies, will spend $50 in purchasing and accounting, and may or may not save $228 in direct labor if the printing department no longer does the shipping department work. The maximum saving would be $\$294 + 228 - 50 = \$472$. Either value of $294 or $472 is less than the $688 the firm would pay the outside printer. The shipping department should not be allowed to send its printing to the outside printer.

Gathering cost data presents other difficulties. One way to look at the financial consequences—costs and benefits—of various alternatives is as follows.

• *Market Consequences.* These consequences have an established price in the marketplace. We can quickly determine raw material prices, machinery costs, labor costs, and so forth.
• *Extra-Market Consequences.* There are other items that are not directly priced in the marketplace. But by indirect means, a price may be assigned to these items. (Economists call these prices **shadow prices.**) Examples might be the cost of an employee injury or the value to employees of going from a 5-day to a 4-day, 40-hour week.
• *Intangible Consequences.* Numerical economic analysis probably never fully describes the real differences between alternatives. The tendency to leave out consequences that do not have a significant impact on the analysis itself, or on the conversion of the final decision into actual money, is difficult to resolve or eliminate. How does one evaluate the potential loss of workers' jobs due to automation? What is the value of landscaping around a factory? These and a variety of other consequences may be left out of the numerical calculations, but they must be considered in reaching a decision.

4. Identify Feasible Alternatives

One must keep in mind that unless the best alternative is considered, the result will always be suboptimal.[1] Two types of alternatives are sometimes ignored. First, in many situations a do-nothing alternative is feasible. This may be the “Let’s keep doing what we are now doing,” or the “Let’s not spend any money on that problem” alternative. Second, there are often feasible (but unglamorous) alternatives, such as “Patch it up and keep it running for another year before replacing it.”

There is no way to ensure that the best alternative is among the alternatives being considered. One should try to be certain that all conventional alternatives have been listed and then make a serious effort to suggest innovative solutions. Sometimes a group of people considering alternatives in an innovative atmosphere—**brainstorming**—can be helpful. Even impractical alternatives may lead to a better possibility. The payoff from a new, innovative alternative can far exceed the value of carefully selecting between the existing alternatives.

Any good listing of alternatives will produce both practical and impractical alternatives. It would be of little use, however, to seriously consider an alternative that cannot be adopted. An alternative may be infeasible for a variety of reasons. For example, it might violate fundamental laws of science, require resources or materials that cannot be obtained, violate ethics standards, or conflict with the firm’s strategy. Only the feasible alternatives are retained for further analysis.

5. Select the Criterion to Determine the Best Alternative

The central task of decision making is choosing from among alternatives. How is the choice made? Logically, to choose the best alternative, we must define what we mean by *best*. There must be a **criterion,** or set of **criteria,** to judge which alternative is best. Now, we recognize that *best* is on one end of the following relative subjective judgment:

Worst Bad Fair Good Better Best

relative subjective judgment spectrum

Since we are dealing in *relative terms*, rather than *absolute values*, the choice will be the alternative that is relatively the most desirable. Consider a driver found guilty of speeding and given the alternatives of a $475 fine or 3 days in jail. In absolute terms, neither alternative is good. But on a relative basis, one simply makes the best of a bad situation.

There may be an unlimited number of ways that one might judge the various alternatives. Several possible criteria are:

• Create the least disturbance to the environment.
• Improve the distribution of wealth among people.
• Minimize the expenditure of money.
• Ensure that the benefits to those who gain from the decision are greater than the losses of those who are harmed by the decision.[2]
• Minimize the time to accomplish the goal or objective.
• Minimize unemployment.
• Maximize profit.

Selecting the criterion for choosing the best alternative will not be easy if different groups support different criteria and desire different alternatives. The criteria may conflict. For example, minimizing unemployment may require increasing the expenditure of money. Or minimizing environmental disturbance may conflict with minimizing time to complete the project. The disagreement between management and labor in collective bargaining (concerning wages and conditions of employment) reflects a disagreement over the objective and the criterion for selecting the best alternative.

The last criterion—maximize profit—is the one normally selected in engineering decision making. When this criterion is used, all problems fall into one of three categories: neither input nor output fixed, fixed input, or fixed output.

Neither input nor output fixed. The first category is the general and most common situation, in which the amount of money or other inputs is not fixed, nor is the amount of benefits or other outputs. For example:

• A consulting engineering firm has more work available than it can handle. It is considering paying the staff for working evenings to increase the amount of design work it can perform.
• One might wish to invest in the stock market, but the total cost of the investment is not fixed, and neither are the benefits.

• A car battery is needed. Batteries are available at different prices, and although each will provide the energy to start the vehicle, the useful lives of the various products are different.

What should be the criterion in this category? Obviously, to be as economically efficient as possible, we must maximize the difference between the return from the investment (benefits) and the cost of the investment. Since the difference between the benefits and the costs is simply profit, a businessperson would define this criterion as **maximizing profit.**

Fixed input. The amount of money or other input resources (like labor, materials, or equipment) is fixed. The objective is to effectively utilize them. For economic efficiency, the appropriate criterion is to maximize the benefits or other outputs. For example:

• A project engineer has a budget of $350,000 to overhaul a portion of a petroleum refinery.
• You have $300 to buy clothes for the start of school.

Fixed output. There is a fixed task (or other output objectives or results) to be accomplished. The economically efficient criterion for a situation of fixed output is to minimize the costs or other inputs. For example:

• A civil engineering firm has been given the job of surveying a tract of land and preparing a “record of survey” map.
• You must choose the most cost-effective design for a roof, an engine, a circuit, or other component.

For the three categories, the proper economic criteria are:

Category	**Economic Criterion**
Neither input nor output fixed	Maximize profit = value of outputs − cost of inputs.

Fixed input	Maximize the benefits or other outputs.
Fixed output	Minimize the costs or other inputs.

6. Constructing the Model

At some point in the decision-making process, the various elements must be brought together. The *objective, relevant data*, *feasible alternatives*, and *selection criterion* must be merged. For example, if one were considering borrowing money to pay for a car, there is a mathematical relationship between the loan's variables: amount, interest rate, duration, and monthly payment.

Constructing the interrelationships between the decision-making elements is frequently called **model building** or **constructing the model.** To an engineer, modeling may be a scaled *physical representation* of the real thing or system or a *mathematical equation*, or set of equations, describing the desired interrelationships. In economic decision making, the model is usually mathematical.

In modeling, it is helpful to represent only that part of the real system that is important to the problem at hand. Thus, the mathematical model of the student capacity of a classroom might be

$$\text{Capacity} = \frac{lw}{k}$$

where *l* = length of classroom, in meters

w = width of classroom, in meters

k = classroom arrangement factor

The equation for student capacity of a classroom is a very simple model; yet it may be adequate for the problem being solved.

7. Predicting the Outcomes for Each Alternative

A model and the data are used to predict the outcomes for each feasible alternative. As was suggested earlier, each alternative might produce a variety of outcomes. Selecting a motorcycle, rather than a bicycle, for example, may make the fuel supplier happy, the neighbors unhappy, the environment more polluted, and one's savings account smaller. But, to avoid unnecessary complications, we assume that decision making is based on a single criterion for measuring the relative attractiveness of the various alternatives. As will be shown in Example 1–5, one can devise a single composite criterion that is the weighted average of several different choice criteria.

To choose the best alternative, the outcomes for each alternative must be stated in a *comparable* way. Usually the consequences of each alternative are stated in terms of money, that is, in the form of costs and benefits. **Resolving the consequences** is done with all monetary and nonmonetary consequences. The consequences can also be categorized as follows:

Market consequences—where there are established market prices available
Extra-market consequences—no direct market prices, so priced indirectly
Intangible consequences—valued by judgment, not monetary prices.

In the initial problems we will examine, the costs and benefits occur over a short time period and can be considered as occurring at the same time. In other situations the various costs and benefits take place in a longer time period. The result may be costs at one point in time followed by periodic benefits. We will resolve these in the next chapter into a *cash flow diagram* to show the timing of the various costs and benefits.

For these longer-term problems, the most common error is to assume that the current situation will be unchanged for the do-nothing alternative. In reality if a firm does nothing new then current profits will shrink or vanish as a result of the actions of competitors and the expectations of customers. As another example, traffic congestion normally increases over the years as the number of vehicles increases—doing nothing does not imply that the situation will not change.

8. Choosing the Best Alternative

Earlier we said that choosing the best alternative may be simply a matter of determining which alternative best meets the selection criterion. But the solutions to most problems in economics have market consequences, extra-market consequences, and intangible consequences. Since the intangible consequences of possible alternatives are left out of the numerical calculations, they should be introduced into the decision-making process at this point. The alternative to be chosen is the one that best meets the choice criterion after considering both the numerical consequences and the consequences not included in the monetary analysis.

During the decision-making process certain feasible alternatives are eliminated because they are dominated by other, better alternatives. For example, shopping for a computer on-line may allow you to buy a custom-configured computer for less money than a stock computer in a local store. Buying at the local store is feasible, but dominated. While eliminating dominated alternatives makes the decision-making process more efficient, there are dangers.

Having examined the structure of the decision-making process, we can ask, When is a decision made, and who makes it? If one person performs *all* the steps in decision making, then she is the decision maker. *When* she makes the decision is less clear. The selection of the feasible alternatives may be the key item, with the rest of the analysis a methodical process leading to the inevitable decision. We can see that the decision may be drastically affected, or even predetermined, by the way in which the decision-making process is carried out. This is illustrated by the following example.

Liz, a young engineer, was assigned to develop an analysis of additional equipment needed for the machine shop. The single criterion for selection was that the equipment should be the most economical, considering both initial costs and future operating costs. A little investigation by Liz revealed three practical alternatives:

1. A new specialized lathe
2. A new general-purpose lathe

3. A rebuilt lathe available from a used-equipment dealer

A preliminary analysis indicated that the rebuilt lathe would be the most economical. Liz did not like the idea of buying a rebuilt lathe, so she decided to discard that alternative. She prepared a two-alternative analysis that showed that the general-purpose lathe was more economical than the specialized lathe. She presented this completed analysis to her manager. The manager assumed that the two alternatives presented were the best of all feasible alternatives, and he approved Liz's recommendation.

At this point we should ask: Who was the decision maker, Liz or her manager? Although the manager signed his name at the bottom of the economic analysis worksheets to authorize purchasing the general-purpose lathe, he was merely authorizing what already had been made inevitable, and thus he was not the decision maker. Rather Liz had made the key decision when she decided to discard the most economical alternative from further consideration. The result was a decision to buy the better of the two *less economically desirable* alternatives.

9. Audit the Results

An audit of the results is a comparison of what happened against the predictions. Do the results of a decision analysis reasonably agree with its projections? If a new machine tool was purchased to save labor and improve quality, did it? If so, the economic analysis seems to be accurate. If the savings are not being obtained, what was overlooked? The audit may help ensure that projected operating advantages are ultimately obtained. On the other hand, the economic analysis projections may have been unduly optimistic. We want to know this, too, so that the mistakes that led to the inaccurate projection are not repeated. Finally, an effective way to promote *realistic* economic analysis calculations is for all people involved to know that there *will* be an audit of the results!

ETHICS

You must be mindful of the ethical dimensions of engineering economic analysis and of your engineering and personal decisions. This text can only introduce the topic, and we hope that you will explore this subject in greater depth.

Ethics can be described variously; however, a common thread is the concept of distinguishing between right and wrong in decision making. Ethics includes establishing systems of beliefs and moral obligations, defining values and fairness, and determining duty and guidelines for conduct. Ethics and ethical behavior are important because when people behave in ethical ways, individuals and society benefit. Usually the ethical choice is reasonably clear, but there are ethical dilemmas with conflicting moral imperatives. Consider an overloaded and sinking lifeboat. If one or more passengers are thrown into the shark-infested waters, the entire lifeboat can be saved. How is the decision made, how is it implemented, and who if anyone goes into the water? Ethical dilemmas also exist in engineering and business contexts. Ethical decision making requires the understanding of problem context, choices, and associated outcomes.

Ethical Dimensions in Engineering Decision Making

Ethical issues can arise at every stage of the integrated process for engineering decision making described in Figure 1–1. Ethics is such an important part of professional and business decision making that ethical codes or standards of conduct exist for professional engineering societies, small and large organizations, and every individual. Written professional codes are common in the engineering profession, serving as a reference basis for new engineers and a basis for legal action against engineers who violate the code.

One such example is the Code of Ethics of the National Society of Professional Engineers (NSPE). Here is NSPE's fundamental canon of ethical behavior for engineering:

Engineers, in the fulfillment of their professional duties, shall:

• Hold paramount the safety, health and welfare of the public.
• Perform services only in areas of their competence.

• Issue public statements only in an objective and truthful manner.
• Act for each employer or client as faithful agents or trustees.
• Avoid deceptive acts.
• Conduct themselves honorably, responsibly, ethically and lawfully so as to enhance the honor, reputation, and usefulness of the profession.

In addition, NSPE has Rules of Practice and Professional Obligations for its members. Most engineering organizations have similar written standards. For all engineers difficulties arise when they act contrary to these written or internal codes, and opportunities for ethical dilemmas are found throughout the engineering decision-making process. Table 1–1 provides examples of ethical lapses that can occur at each step of the decision-making process.

Lapses = little mistakes

Table 1-1 Example Ethical Lapses by Decision Process Step

Decision Process Step	Example Ethical Lapses
1. Recognize the problem	• "Looking the other way," that is, not to recognize the problem—due to bribes or perhaps fear of retribution for being a "whistle-blower"
2. Define the goal or objective	• Favoring one group of stakeholders by focusing on their objective for a project
3. Assemble relevant data	• Using faulty or inaccurate data
4. Identify feasible alternatives	• Leaving legitimate alternatives out of consideration
5. Select the criterion to determine the best alternative	• Considering only monetary consequences when there are other significant consequences
6. Construct a model	• Using a short horizon that favors one alternative over another
7. Predict each	

alternative's outcomes or consequences	• Using optimistic estimates for one alternative and pessimistic ones for the other alternatives
8. Choose the best alternative	• Choosing an inferior alternative, one that is unsafe, adds unnecessary cost for the end user, harms the environment, etc.
9. Audit the result	• Hiding past mistakes

Ethical dilemmas for engineers may arise in connection with engineering economic analysis in many situations. Following are examples of a few of these.

Gaining Knowledge and Building Trust Versus Favors for Influence

Consider these three situations:

• The salesman for a supplier of HVAC (heating, ventilating, and air conditioning) equipment invites a mechanical engineer and spouse to come along on the company jet for a users' conference at a vacation resort.
• Same salesman and same engineer, but the invitation is for a day of golfing at an exclusive club.
• Same salesman invites the same engineer to lunch.

In each case the salesman is trying to "get the order," and there is likely to be some mix of business—discussing specifications—and pleasure. The first case, which brings up the largest ethical questions, also has the largest business justification. This is the opportunity to meet other users of the products and see displays of the product line. Often, firms and government agencies have strict guidelines that dictate behavior in these situations.

Cost, Quality, and Functionality

One of the most common conflicts in the conceptual and design phase involves the trade-offs between cost, quality, and required functionality. Most modern products entail many thousands of decisions by designers that ultimately affect the cost and quality for the end user.

• A designer in an engineering consulting firm knows that a "gold-plated" solution would be very profitable for his firm (and for his bonus). This solution may also provide excellent reliability and require little maintenance cost.
• Engineers in the consumer durables division of a multinational company know that by using lower-quality connectors, fasteners, and subcomponents they can lower costs and improve the firm's market position. In addition, they know that these design elements have only a limited usable life, and the firm's most profitable business is repairs and extended warranties.

The Environment We Live In

Projects for transportation and power generation typically must consider environmental impacts in their design and in deciding whether the project should be done in any form. Who incurs the costs for the project, and who receives the benefits? Many other engineering products are designed to promote recycling, reduce energy usage, and reduce pollution. Ethical issues can be particularly difficult because there are often stakeholders with opposing viewpoints, and some of the data may be uncertain and hard to quantify.

Green engineering design includes the effects of environmental impacts and gives consideration to life-cycle sustainability issues. In this context, **societal costs** are the negative impacts of a project or product. Reducing these societal costs is the goal of environmental fees and regulation. For the opening vignette on electric vehicles, examples of the social costs of combustion-engine automobiles include tailpipe emissions and the negative environmental impact of mining, refining, and distributing gasoline/diesel fuels. Other examples of difficult choices include:

• Protecting the habitat of an endangered species versus flood control projects that protect people, animals, and structures.
• Meeting the needs for electrical power when all choices have some negative environmental impacts:
• Hydroelectric—reservoir covers land and habitat
• Coal—underground mining can be dangerous, open-pit mining damages habitat, and burning the coal can cause air pollution

- Nuclear—disposal of radioactive waste
- Fuel oil—air pollution and economic dependence
- Wind—visual pollution of wind farms; birds killed by whirling blades
- Determining standards for pollutants: Is 1 part per million OK, or is 1 part per billion needed?

Safety and Cost

Some of the most common and most difficult ethical dilemmas involve trade-offs between safety and cost. If a product is "too safe," then it will be too expensive, and it will not be used. Also sometimes the cost is incurred by one party and the risk by another.

- Should the oil platform be designed for the 100-year, 500-year, or 1000-year hurricane?
- Should the auto manufacturer add run-flat tires, stability control, side-cushion airbags, and rear-seat airbags to every car?
- Should a given product design go through another month of testing?
- Are stainless steel valves required, or is it economically better to use less corrosion-resistant valves and replace them more frequently?

Emerging Issues and "Solutions"

Breaches of the law by corporate leaders of Enron, Tyco, and other firms have led to attempts to prevent, limit, and expose financial wrongdoing within corporations. One part of the solution has been the Sarbanes–Oxley Act of 2002, which imposed requirements on firm executives and auditing accounting firms, as well as penalties for violations.

Globalization is another area of increasing importance for ethical considerations. One reason is that different ethical expectations prevail in the world's various countries and regions. A second reason is that jobs may be moved to another country based on differences in cost, productivity, environmental standards, and so on. What may be viewed as a sweatshop from a U.S. perspective may be viewed as a wonderful opportunity to support many families from the perspective of a less developed nation.

Importance of Ethics in Engineering and Engineering Economy

Many times engineers and firms try to act ethically, but mistakes are made—the data were wrong, the design was changed, or the operating environment was different than expected. In other cases, a choice was made between expediency (profit) and ethics. For example, some engineers and managers within VW chose to manipulate diesel vehicle performance during emission testing. Estimates of international costs to VW exceeded $38B. The firm and management are driven by the need to make a profit, and they expect the engineer to identify when safety will be compromised.

Ethics in engineering economic analysis focuses on how well and how honestly the decision-making process is conducted—the data, method of analysis, recommendations, and follow-up. The first step in avoiding problems is to recognize that ethical issues exist and to make them an explicit part of your decision-making process.

As a student, you've no doubt heard discussions about cheating on exams, plagiarism on written reports, violating university drinking and drug use policies, accepting one job while continuing to interview for others, and selling student sports tickets to nonstudents. You've made your own decisions about your behavior, and you've established patterns of behavior.

You should know that your professors care deeply about the ethical decisions you make at school. Your ethical habits there form a foundation for the character of your work and personal behavior after graduation.

Often recent engineering graduates are asked, "What is the most important thing you want from your supervisor?" The most common response is mentoring and opportunities to learn and progress. When employees with 5, 15, 25, or more years of experience are asked the same question, the most common response at all experience levels is *integrity*. This is what your subordinates, peers, and superiors will expect and value the most from you. Integrity is the foundation for long-term career success.

ENGINEERING DECISION MAKING FOR CURRENT COSTS

Some of the easiest forms of engineering decision making deal with problems of alternative *designs*, *methods*, or *materials*. If results of the decision occur in a very short period of time, one can quickly add up the costs and benefits for each alternative. Then, using the suitable economic criterion, the best alternative can be identified. Three example problems illustrate these situations.

EXAMPLE 1-2

A concrete aggregate mix must contain at least 31% sand by volume for proper batching. One source of material, which has 25% sand and 75% coarse aggregate, sells for $3 per cubic meter ($m^3$). Another source, which has 40% sand and 60% coarse aggregate, sells for $4.40/$m^3$. Determine the least cost per cubic meter of blended aggregates.

SOLUTION

The least cost of blended aggregates results from using just enough higher-cost material to meet the minimum 31% proportion of sand.

$$\text{Let } x = \text{Portion of blended aggregates from \$3.00/m}^3 \text{ source}$$
$$1 - x = \text{Portion of blended aggregates from \$4.40/m}^3 \text{ source}$$

Sand Balance

$$x(0.25) + (1 - x)(0.40) = 0.31$$
$$-.15x = -.09 \Rightarrow x = 0.60$$

The 60%/40% blended aggregate will cost

$$0.60(\$3.00) + 0.40(\$4.40) = 1.80 + 1.76 = \$3.56/\text{m}^3$$

EXAMPLE 1-3

A machine part is manufactured at a unit cost of 40¢ for material and 15¢ for direct labor. An investment of $500,000 in tooling is required. The order calls for 3 million pieces. Halfway through the order, managers learn that a new method of manufacture can be put into effect that will reduce the unit costs to 34¢ for material and 10¢ for direct labor—but it will require $100,000 for additional tooling. This tooling will not be useful for future orders. Other costs are allocated at 2.5 times the direct labor cost. What, if anything, should be done?

SOLUTION

Since there is only one way to handle the first 1.5 million pieces, our problem concerns only the second half of the order. While the arithmetic can easily be done on a calculator, in the real world problems like these are usually done using a spreadsheet. This allows easy substitution of “better” numbers for the initial estimates and supports **what-if analysis**. The first spreadsheet shows the data entry stage of the problem. These values form the problem’s **data block** (see Appendix A). Note that we want a clear, compact table, so columns of these values are alternated with calculation columns for our two alternatives.

	A	B	C	D	E
1	15,00,000	Number of pieces			
2	2.5	Other cost $/direct labor $			
3		A: Present Method		B: New method	
4	Costs	unit	total	unit	total
5	Material	0.4		0.34	
6	Direct labor	0.15		0.1	
7	Other				
8	Added tooling			1,00,000	

The second spreadsheet includes column F to show the formulas for the cells in column E. Note that the formulas in cells E5, E6, and C6 are all copied from cell C5. Because the C5 formula was originally written as =B5*$A$1, the **absolute address** of $A$1 does not change when copied. Note: Appendix A discusses how to efficiently do this and other examples of addressing alternatives that maximize the flexibility of copying formulas.

The most efficient way to create the formulas is to:

- Write the formula for C5 as = “click on B5” * “click on A1” “F4 or Apple T to toggle to $A$1”
- Copy it to C6
- Write the formula for C7 (including an absolute address)
- Copy C5:C7 to E5:E7

Select E5:E9 and click on the “sum” formula button. This can be copied to C9.

	A	B	C	D	E	F
1	15,00,000	Number of pieces				
2	2.5	Other cost $/direct labor $				
3		A: Present Method		B: New method		
4	Costs	unit	total	unit	tool	
5	Material	0.4	$600,000	0.34	$510,000	=D5*$A$1
6	Direct labor	0.15	$225,000	0.1	$150,000	=D6*$A$1
7	Other		$562,500		$375,000	=D6*$A$1
8	Added tooling			1,00,000	$100,000	
9	Total		$1,387,500		$1,135,000	
10			Possible savings		$252,500	

Looking at the results, we can see that much of the total $252,500 in savings comes from the reduced value of other costs. Thus, before making a final decision, one should closely examine the *other costs* to see whether they do, in fact, vary as the *direct labor cost* varies. Assuming they do, the decision would be to change the manufacturing method.

EXAMPLE 1-4

Two different liquid filter systems are being studied to clarify a liquid stream. A traditional filter will operate for one 8-hour shift before being replaced. A special pleated design can last one full week, operating 24 hours a day (3 shifts), 5 days per week. Labor cost to change a filter is estimated to be worth $10.00 for each filter change because a mechanic would work overtime to change the filter. The traditional filters cost $3.50; the special pleated filters cost $90.00. Which filter should be chosen?

SOLUTION

Material costs

Traditional:	$\frac{\$3.50}{\text{filter change}} \times \frac{3 \text{ filter changes}}{\text{day}} \times \frac{5 \text{ days}}{\text{week}} = \$52.50/\text{week}$
Special:	$\frac{\$90.00}{\text{filter change}} \times \frac{1 \text{ filter change}}{\text{week}} = \$90.00/\text{week}$

Labor costs

Traditional:	$\frac{\$10.00}{\text{filter change}} \times \frac{3 \text{ filter changes}}{\text{day}} \times \frac{5 \text{ days}}{\text{week}} = \$150.00/\text{week}$
Special:	$\frac{\$10.00}{\text{filter change}} \times \frac{1 \text{ filter change}}{\text{week}} = \$10.00/\text{week}$

Total costs

Traditional:	$52.50 + $150.00 = $202.50/week
Special:	$90.00 + $10.00 = $100.00/week

The special pleated filter offers a lower total cost alternative. Even though material costs are higher, these are offset by lower labor costs. When comparing alternatives, it is important to include all relevant costs.

WHEN MORE THAN ECONOMICS IS INVOLVED

Consider the moderately complex problem of which job offer to accept. Example 1–5 shows a simple way to address this **multiple-objective** problem. These models should:

• Include all important objectives.
• Weight the relative importance of the objectives.
• Select an objective and rate all alternatives. Then repeat for all objectives.
• Disqualify alternatives that do not meet the minimum performance requirements of one or more objectives.

This example uses simple 0 to 10 rating scales. Since the weights are stated as percentages (or their decimal equivalents), the totals show how close to a perfect 10 each alternative is.

Multi-objective models do much more than calculate a measure of each alternative's attractiveness. Constructing the model enforces a level of clarity about the importance of each objective and how each alternative performs. The model also communicates those assumptions and estimates to others, who may suggest changes. Since there may be multiple iterations in arriving at the final model, spreadsheets are particularly effective here.

Examples in later chapters will show how to convert numeric values to a 0 to 10 point scale. For those who want to search the web for additional examples, this is an *additive* model, because the scores are added together. This is also a *compensatory* model, because strength on one objective can compensate for a weakness on another objective.

Example 1–5 is linked to an individual's financial and life decision making. But this situation can also be viewed from the firm's or government agency's perspective. Which applicant(s) should receive offer(s) of employment? In that case, evaluations from multiple individuals might be combined for the overall total.

EXAMPLE 1-5

A senior undergraduate has received four job offers, but the salary on one is unacceptably low. The other three offers have been rated on three criteria or objectives, with a scale of 0 = barely acceptable and 10 = outstanding! *Job* considers the salary relative to the local cost of housing and the job itself. The latter was hard to estimate because it considered the initial job, growth prospects, the firm, and the industry. *Family* is important to this senior, but the senior wanted to live the right distance away—neither too close nor too far. *Livability* covers the senior's desires on community size, climate, commuting time, and overall political balance. The senior weighted the importance of the three criteria at 50%, 30%, and 20% respectively. Given the following table of ratings, which job offer should the senior accept?

Offer	Job	Family	Livability
A	4	9	5
B	8	5	4
C	6	3	8

SOLUTION

None of the job offers is ideal in any respect, and each has some aspect that is less attractive than the other offers. Comparing the total values, offer B is the most attractive. This table is the result of many hours of thinking, and more model iterations would not be useful. Thus offer B should be accepted.

There are many ways to write the formula, but the easiest uses the function SumProduct. As shown, the function uses a fixed address for the weights, so the formula for offer A can be copied for the other offers.

	A	B	C	D	E
1		Job	Family	Livability	
2	Weight	50%	30%	20%	
3	Offer				Total
4	A	4	9	5	5.7
5	B	8	5	4	6.3
6	C	6	3	8	5.5
7					
8			=SUMPRODUCT($B$2:$D$2,B4:D4)		

SUMMARY

Classifying Problems

Many problems are simple and thus easy to solve. Others are of intermediate difficulty and need considerable thought and/or calculation to properly evaluate. These intermediate problems tend to have a substantial economic component and to require economic analysis. Complex problems, on the other hand, often contain people elements, along with political and economic components. Economic analysis is still very important, but the best alternative must be selected by considering all criteria—not just economics.

The Decision-Making Process

Rational decision making uses a logical method of analysis to select the best alternative from among the feasible alternatives. The following nine steps can be followed sequentially, but decision makers often repeat some steps, undertake some simultaneously, and skip others altogether.

1. Recognize the problem.
2. Define the goal or objective: What is the task?
3. Assemble relevant data: What are the facts? Is more data needed, and is it worth more than the cost to obtain it?
4. Identify feasible alternatives.
5. Select the criterion for choosing the best alternative: possible criteria include political, economic, environmental, and social. The single criterion may be a composite of several different criteria.

6. *Mathematically model* the various interrelationships.
7. Predict the outcomes for each alternative.
8. Choose the best alternative.
9. Audit the results.

Engineering decision making refers to solving substantial engineering problems in which economic aspects dominate and economic efficiency is the criterion for choosing from among possible alternatives. It is a particular case of the general decision-making process. Some of the unusual aspects of engineering decision making are as follows:

1. Cost-accounting systems, while an important source of cost data, contain allocations of indirect costs that may be inappropriate for use in economic analysis.
2. The various consequences—costs and benefits—of an alternative may be of three types:
(a) Market consequences—there are established market prices.
(b) Extra-market consequences—there are no direct market prices, but prices can be assigned by indirect means.
(c) Intangible consequences—valued by judgment, not by monetary prices.
3. The economic criteria for judging alternatives can be reduced to three cases:
(a) When neither input nor output is fixed: maximize profit, which equals the difference between benefits and costs.
(b) For fixed input: maximize benefits or other outputs.
(c) For fixed output: minimize costs or other inputs.
The first case states the general rule from which both the second and third cases may be derived.
4. To choose among the alternatives, the market consequences and extra-market consequences are organized into a cash flow diagram. We will see in Chapter 3 that engineering economic calculations can be used to compare differing cash flows. These outcomes are compared against the selection criterion. From this comparison *plus* the consequences not included in the monetary analysis, the best alternative is selected.
5. An essential part of engineering decision making is the postaudit of results. This step helps to ensure that projected benefits are obtained and to encourage realistic estimates in analyses.

Importance of Ethics in Engineering and Engineering Economy

One of the gravest responsibilities of an engineer is protecting the safety of the public, clients, and/or employees. In addition, the engineer can be responsible for the economic performance of projects and products on which bonuses and jobs depend. Not surprisingly, in this environment one of the most valued personal characteristics is integrity.

Decision Making with Current Costs

When all costs and benefits occur within a brief period of time, the time value of money is not a consideration. We still must use the criteria of maximizing profit, minimizing cost, or maximizing benefits.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

Many engineers earn high salaries for creating profits for their employers and then, at retirement time, find themselves insufficiently prepared financially. This may be because in college courses there is little or no discussion on using engineering economics for the direct personal benefit of the engineer. Among the goals of every engineer should be assuring that adequate funds will be available for anticipated personal needs at retirement.

1-1 A realistic goal of retiring at age 65 with a personal net worth in excess of $2 million can be accomplished by several methods. An independent study ranked the probability of success of the following methods of personal wealth accumulation. Discuss and decide the ranking order of the following five methods.

a. Purchase as many lottery tickets as possible with money saved from salary.
b. Place money saved from salary in a bank savings account.
c. Place all money saved from a salary in a money market account.
d. Invest saved money into rental properties and spend evenings, weekends, and vacations repairing and managing.
e. Invest all money saved into stock market securities, and study investments 10 to 15 hours per week.

SOLUTION

The following letter to Joseph Priestley, the English chemist, was written by his friend Benjamin Franklin. Priestley had been invited to become the librarian for the Earl of Shelburne and had asked for Franklin's advice. What engineering economy principle does Franklin suggest Priestley use to aid in making his decision?

London, September 19, 1772

Dear Sir:

In the affair of so much importance to you wherein you ask my advice, I cannot, for want of sufficient premises, advise you what to determine, but if you please I will tell you how. When these difficult cases occur, they are difficult chiefly because while we have them under consideration, all the reasons Pro and Con are not present to the mind at the same time; but sometimes one set present themselves, and at other times another, the first being out of sight. Hence the various purposes or inclination that alternately prevail, and the uncertainty that perplexes us.

To get over this, my way is to divide a half a sheet of paper by a line into
1-2 two columns; writing over the one PRO and over the other CON. Then during three or four days' consideration I put down under the different heads short hints of the different motives that at different times occur to me, for or against the measure. When I have thus got them all together in one view, I endeavour to estimate their respective weights; and where I find two (one on each side) that seem equal, I strike them both out. If I find a reason Pro equal to some two reasons Con, I strike out the three. If

I judge some two reasons Con equal to three reasons Pro, I strike out the five; and thus proceeding I find at length where the balance lies; and if after a day or two of further consideration, nothing new that is of importance occurs on either side, I come to a determination accordingly. And though the weight of the reasons cannot be taken with the precision of algebraic quantities, yet when each is thus considered separately and comparatively and the whole lies before me, I think I can judge better, and am less likely to make a rash step; and in fact I have found great advantage from this kind of equation in what may be called moral or prudential algebra.

Wishing sincerely that you may determine for the best, I am ever, my dear friend, yours most affectionately. . .

s/Ben Franklin

SOLUTION

1-3 Assume that you are employed as an engineer for Wreckall Engineering, Inc., a firm specializing in the demolition of high-rise buildings. The firm has won a bid to tear down a 30-story building in a heavily developed downtown area. The crane the company owns reaches only to 29 stories. Your boss asks you to perform an economic analysis to determine the feasibility of buying a new crane to complete the job. How would you handle the analysis?

SOLUTION

1-4 By wisely saving and investing, Helen finds she has accumulated $400,000 in savings while her salaried position is providing her with $40,000 per year, including benefits, after income taxes and other deductions.

Helen's salaried position is demanding and allows her little free time, but the desire to pursue other interests has become very strong. What would be your advice to her, if you were asked?

SOLUTION

1-5 Charles belongs to a square dance club that meets twice each month and has quarterly dues of $9 per person. The club moved its meeting place to a more expensive location. To offset the increased cost, members agreed to pay 50 cents apiece each time they attend the meeting. Later the treasurer suggests that the quarterly dues be increased to $12 per person as an alternative to the meeting charge. Discuss the consequences of the proposal. Do you think all the club members will agree to the proposal?

SOLUTION

1-6 A Ph.D. student accepted a full time teaching job in February, with the job starting in June. Two weeks before the job was to start, this person received another job offer at a larger university, paying 10% more. Should they accept this new offer or turn it down?

SOLUTION

1-7 A coal-fired power plant produces electricity for the region. They have been told they need to reduce their emissions of mercury due to the toxic nature of their smoke emissions. The plant says that they cannot afford to reduce mercury, and that people will be laid off if they need to reduce production in order to meet emission standards. What should they do?

SOLUTION

1-8 Car *A* initially costs $500 more than Car *B*, but it consumes 0.04 gallon/mile versus 0.05 gallon/mile for *B*. Both vehicles last 8 years, and *B*'s salvage value (the value when it is traded in after 8 years) is $100 smaller than *A*'s. Fuel costs $2.40 per gallon. Other things being equal, at how many miles of use per year (*X*) is *A* preferred vs. *B*?

SOLUTION

Sam decides to buy a cattle ranch and leave the big-city rat race. He

1-9 locates an attractive 500-acre spread in Montana for $1000 per acre that includes a house, a barn, and other improvements. Sam's studies indicate that he can run 200 cow–calf pairs and be able to market 180 500-pound calves per year. Sam, being rather thorough in his investigation, determines that he will need to purchase an additional $95,000 worth of machinery. He expects that supplemental feeds, medications, and veterinary bills will be about $50 per cow per year. Property taxes are $4000 per year, and machinery upkeep and repairs are expected to run $3000 per year.

If interest is 10% a net salary of $10,000 per year, how much will he have to get for each 500-pound calf?

SOLUTION

1-10 A food processor is considering the development of a new product. Depending on the quality of raw material, he can expect different yields process-wise, and the quality of the final products will also vary considerably. The product development department has identified three alternatives, which it has produced on a pilot scale. The marketing department has used those samples for surveys to estimate potential sales and pricing strategies. The three alternatives, which would use existing equipment, but different process conditions and specifications, are summarized as follows. Indicate which alternative seems to be the best according to the estimated data, if the objective is to maximize total profit per year.

	Alternative		
	1	2	3
Pounds of raw material *A* per unit of product	0.05	0.07	0.075
Pounds of raw material *B* per unit of product	0.19	0.18	0.26
Pounds of raw material *C* per unit of product	0.14	0.12	0.17

Other processing costs ($/unit product)	$0.16	$0.24	$0.23
Expected wholesale price ($/unit product)	0.95	1.05	1.25
Projected volume of sales (units of product)	1,000,000	1,250,000	800,000
Cost of raw material *A* $3.45/lb			
Cost of raw material *B* $1.07/lb			
Cost of raw material *C* $1.88/lb			

SOLUTION

1-11 The total cost of a building (TC) is given by

$$TC = \left(200 + 80X + 2X^2\right)A$$

where X = number of floors and A = floor area (ft^2 per floor)

If the total number of square feet required is 10^6, what is the optimal (minimum cost) number of floors?

SOLUTION

1-12 Two locations are being considered for a new regional office. Factors to consider include the cost of living, utilities and taxes, and quality of life. Each has been rated, as shown:

Factor	Location A	Location B	Factor weight
Cost of living	3	5	25%
Utilities and taxes	4	4	40%
Quality of life	5	3	35%

Given the data shown, which location should be chosen?

SOLUTION

A new warehouse is being planned, and 3 locations are being compared. Factors being considered include local labor cost, taxes, and access to interstate highways. These are summarized in the table as follows:

1-13

Factor	Location 1	Location 2	Location 3	Factor weight
Labor cost	7	6	5	35%
Taxes	8	7	6	25%
Highway access	3	5	8	40%

Which location should be selected?

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Many end-of-chapter problems are primarily numerical, but others require more discussion—especially the case studies and questions linked to ethics. Section C in Chapter 2 of *Cases in Engineering Economy* 2nd on the student website may be helpful for the more discussion-oriented questions.

Decision Making

1-1 Think back over your past academic year and decisions that you made. List a few decisions that you would classify as simple, intermediate, and complex. What did you learn about your decision making by the way you approached these decisions?

Some of the following problems would be suitable for solution by engineering economic analysis. Which ones are they?

(*a*) Would it be better to buy a hybrid car?
(*b*) Should an automatic machine be purchased to replace three workers now doing a task by hand?
(*c*) Would it be wise to enroll for an early morning class to avoid traveling during the morning traffic rush hours and thus improve fuel efficiency?
(*d*) Would you be better off if you changed your major?
(*e*) Should you work more and borrow less even if it delays your graduation?
(*f*) Should a corporate farm build waste mitigation ponds or continue using a contracted service?

1-2 AG

1-3 Which one of the following problems is *most* suitable for analysis by engineering economic analysis?
(*a*) One of your two favorite sandwich shops offers a 10-punch loyalty card and the other does not. Where should you stop today?
(*b*) A woman has $150,000 in a bank checking account that pays no interest. She can either invest it immediately at a desirable interest rate or wait a week and know that she will be able to obtain an interest rate that is 0.15% higher.
(*c*) Joe backed his car into a tree, damaging the fender. He has car insurance that will pay for the fender repair. But if he files a claim for payment, they may charge him more for car insurance in the future.

1-4 If you have $1000 and could make the right decisions, how long would it take you to become a millionaire? Explain briefly what you would do.

1-5 E One can find books on “How I Made My Millions” in any bookstore. In some cases the authors seem to plan to make millions by selling that book. Do you think this is ethical? How would you lay out the factors to analyze this question?

1-6 E The owner of a small machine shop has just lost one of his larger customers. The solution to his problem, he says, is to fire three machinists to balance his workforce with his current level of business. The owner says it is a simple problem with a simple solution.
(*a*) The three machinists disagree. Why?
(*b*) What are the ethical factors from the perspective of the owner and the workers?

1- Designing a chair for use in a classroom seems like a simple task. Make

7 an argument for how this can be considered a complex decision and
G include environmental and ethical factors in your argument.

Toward the end of the twentieth century, the U.S. government wanted to save money by closing a small portion of its domestic military installations. While many people agreed that saving money was a
1-8 desirable goal, people in areas potentially affected by a closing soon reacted negatively. Congress finally selected a panel whose task was to develop a list of installations to close, with the legislation specifying that Congress could not alter the list. Since the goal was to save money, why was this problem so hard to solve?

The college bookstore has put pads of engineering computation paper on
1-9 sale at half price. What is the minimum and maximum number of pads you might buy during the sale? Explain.

Consider the five situations described. Which one situation seems most suitable for solution by economic analysis?

(*a*) John has met two college students that interest him. Beth is a music major who is lots of fun to be with. Alice is a fellow engineering student, but she does not like to party. John wonders what to do.

(*b*) You drive periodically to the post office to send or pick up packages. The parking meters cost $1 for 15 minutes—about the time required for medium length lines. If parking fines cost $20, do you put money in the meter or not?

(*c*) The cost of car insurance varies widely from company to company.
1-10 Should you check with several insurance companies when your policy comes up for renewal?

(*d*) There is a special local sales tax ("sin tax") on a variety of things that the town council would like to remove from local distribution. As a result, a store has opened up just outside the town and offers an abundance of these specific items at prices about 30% less than is charged in town. Should you shop there?

(*e*) One of your professors mentioned that you have a poor attendance record in her class. You wonder whether to drop the course now or wait to see how you do on the first midterm exam. Unfortunately, the course is required for graduation.

A car manufacturer is considering locating an assembly plant in your

1-11G region.
(*a*) List two simple, two intermediate, and two complex problems associated with this proposal.
(*b*) What is NIMBY? Does this come into play for this complex decision?

1-12 Consider the following situations. Which ones appear to represent rational decision making? Explain.
(*a*) Joe's best friend has decided to become a civil engineer, so Joe has decided that he will also become a civil engineer.
(*b*) Jill needs to get to the university from her home. She bought a car and now drives to the university each day. When Jim asks her why she didn't buy a bicycle instead, she replies, "Gee, I never thought of that."
(*c*) Don needed a wrench to replace the spark plugs in his car. He went to the local automobile supply store and bought the cheapest one they had. It broke before he had finished replacing all the spark plugs in his car.

1-13 Identify possible objectives for NASA. For your favorite of these, how should alternative plans to achieve the objective be evaluated?

1-14E Suppose you have just 2 hours to determine how many students would be interested in a highway trash pickup event. Give a step-by-step outline of how you would proceed.

1-15 A college student determines he will have only half of the cost for university housing available for the coming year. List five feasible alternatives.

1-16E Think about the issue of implementing renewable energies in the U.S. Research/find an instance where a decision was made to implement without adequately looking at other potential alternative solutions.

1-17 If there are only two alternatives available and both are unpleasant and undesirable, what should you do?

1-18 The three economic criteria for choosing the best alternative are maximize the difference between output and input, minimize input, and maximize output. For each of the following situations, what is the correct economic criterion?
(*a*) A manufacturer can sell up to two full shifts of production at a fixed price. As production is increased, unit costs increase as a result of overtime pay and so forth. The manufacturer's criterion should be ________.
(*b*) An architectural and engineering firm has been awarded the contract

A to design a wharf with fixed performance specifications for a petroleum company. The engineering firm's criterion for its client should be _______.

(*c*) An off-campus bookstore is choosing its target used/new split for next year. Its criterion should be _______

(*d*) At an auction of antiques, a bidder for a particular porcelain statue would be trying to _______.

1-19 As in Problem 1-18, state the correct economic criterion for each of the following situations.

(*a*) The engineering student club raffled off a donated car; tickets sold for $5 each or three for $10. When the students were selling tickets, they noted that many people had trouble deciding whether to buy one or three tickets. This indicates the buyers' criterion was _______.

(*b*) A student organization bought a soft-drink machine and then had to decide whether to charge 75¢, $1, or $1.25 per drink. The organization recognized that the number of soft drinks sold would depend on the price charged. Eventually the decision was made to charge $1. The criterion was _______.

(*c*) In many cities, grocery stores find that their sales are much greater on days when they advertise special bargains. However, the advertised special prices do not appear to increase the total physical volume of groceries sold by a store. This leads us to conclude that many shoppers' criterion is _______.

(*d*) A recently graduated engineer has decided to return to school in the evenings to obtain a master's degree. He feels it should be accomplished in a manner that will allow him the maximum amount of time for his regular day job plus time for recreation. In working for the degree, he will _______.

1-20 Seven criteria are given in the chapter for judging which is the best alternative. After reviewing the list, devise three additional criteria that might be used.

1- Suppose you are assigned the task of determining the route of a new highway through an older section of town. The highway will require that many older homes be either relocated or torn down. Two possible criteria that might be used in deciding exactly where to locate the highway are:

(*a*) Ensure that there are benefits to those who gain from the decision and

21 that no one is harmed by the decision.
(*b*) Ensure that the benefits to those who gain from the decision are greater than the losses of those who are harmed by the decision.
Which criterion will you select to use in determining the route of the highway? Explain.

1-22 For the project in Problem 1-21, identify the major costs and benefits. Which are market consequences, which are extra-market consequences, and which are intangible consequences?

1-23 You must fly to another city for a Friday meeting. If you stay until Sunday morning your ticket will be \$250, rather than \$800. Hotel costs are \$200 per night. Compare the economics with reasonable assumptions for meal expenses. What intangible consequences may dominate the decision?

1-24 In the fall, Jay Thompson decided to live in a university dormitory. He signed a dorm contract under which he was obligated to pay the room rent for the full college year. One clause stated that if he moved out during the year, he could sell his dorm contract to another student who would move into the dormitory as his replacement. The dorm cost was \$6300 for the two semesters, which Jay had already paid.
A month after he moved into the dorm, he decided he would prefer to live in an apartment. That week, after some searching for a replacement to fulfill his dorm contract, Jay had two offers. One student offered to move in immediately and to pay Jay \$500 per month for the seven remaining months of the school year. A second student offered to move in the second semester and pay \$2800 to Jay.
Jay estimates his food cost per month is \$350 if he lives in the dorm and \$300 if he lives in an apartment with three other students. His share of the apartment rent and utilities will be \$450 per month. Assume each semester is 4½ months long. Disregard the small differences in the timing of the disbursements or receipts.
(*a*) What are the three alternatives available to Jay?
(*b*) Evaluate the cost for each of the alternatives.
(*c*) What do you recommend that Jay do?

1- An electric motor on a conveyor burned out. The foreman told the plant manager that the motor had to be replaced. The foreman said that there were no alternatives and asked for authorization to order the replacement.

1-25 In this situation, is any decision making taking place? If so, who is making the decision(s)?

A farmer must decide what combination of seed, water, fertilizer, and pest control will be most profitable and environmentally conscious for the coming year. The local agricultural college did a study of this farmer's situation and prepared the following table.

1-26 AG

Plan	Direct Cost/Acre	Extra-market Cost/Acre	Income/Acre
A	$750	$150	$1200
B	800	450	1400
C	1000	250	1500
D	1300	200	1650

The last page of the college's study was torn off, and hence the farmer is not sure which plan the agricultural college recommends. Which plan should the farmer adopt considering:
(*a*) only the direct costs,
(*b*) both the direct and extra-market costs?

1-27 Identify the alternatives, outcomes, criteria, and process for the selection of your college major. Did you make the best choice for you?

1-28 Describe a major problem you must address in the next two years. Use the techniques of this chapter to structure the problem and recommend a decision.

1-29 Apply the steps of the decision-making process from this chapter and develop plans to achieve one each of your 5-year, 10-year, and 25-year goals.

One strategy for solving a complex problem is to break it into a group of less complex problems and then find solutions to the smaller problems.

1-30 The result is the solution of the complex problem. Give an example in which this strategy will work. Then give another example in which this strategy will not work.

Ethics

1-31E When you make professional decisions involving investments in engineering projects, what criteria will you use?
Contributed by D. P. Loucks, Cornell University

1-32E What are ethics?
Contributed by D. P. Loucks, Cornell University

1-33E A student accepts a full-time job in November, but a better job comes before graduation in May. What are the ethical dimensions of the student's decision? Would you take the better job? Why or why not?

1-34E Suppose you are an engineer working in a private engineering firm and you are asked to sign documents verifying information that you believe is not true. You like your work and your colleagues in the firm, and your family depends on your income. What criteria can you use to guide your decision regarding this issue?
Contributed by D. P. Loucks, Cornell University

1-35E Find the ethics code for the professional society of your major.
(*a*) Summarize its key points.
(*b*) What are its similarities and differences in comparison to NSPE's ethics code?

1-36 Use a personal example or a published source to analyze what went wrong or right with respect to ethics at the assigned stage(s) of the decision-making process.
(*a*) Recognize problem.
(*b*) Define the goal or objective.
(*c*) Assemble relevant data.
(*d*) Identify feasible alternatives.
(*e*) Select the criterion for determining the best alternative.
(*f*) Construct a model.
(*g*) Predict each alternative's outcomes or consequences.

E (*h*) Choose the best alternative.

(*i*) Audit the result.

For problems 1–37 to 1–49:

(*a*) What ethical issues can arise—personal, business, and/or environmental?

(*b*) Use local, state, national, or international news sources to identify an example situation.

(*c*) Summarize and analyze the ethical issues, including relevant laws, regulations, codes, and processes.

1-37 G Municipal assemblies, school boards, transit boards, and municipal utility boards are responsible for public infrastructure, such as roads and schools. Especially for this responsibility, engineers bring skills, knowledge, and perspectives that can improve public decision making. Often the public role is a part-time one; engineers that fulfill it will also have full-time jobs as employees or owners of engineering firms.

1-38 G Increasing population and congestion often are addressed through road improvement projects. These may pit the interests of homeowners and business owners in the project area against the interests of people traveling through the improvement project and environmental activists.

1-39 E Stadiums for professional sports teams often involve some level of municipal support. Some businesses and home owners benefit, while others do not; some pay more in taxes, while others pay less.

1-40 E Economic development and redevelopment often require significant acreage that is assembled by acquiring smaller parcels. Sometimes this is done through simple purchase, but the property of an “unwilling seller” can be acquired through the process of eminent domain.

1-41 E State governments use a variety of advisory and regulatory bodies. Example responsibilities include oversight of professional engineering licensing and the pricing and operation of regulated utilities. Often the public role is a part-time one, and engineers that fulfill it will also have full-time jobs as employees or owners of engineering firms.

1-42 E Many engineers work in state governments, and some are in high-profile roles as legislators, department commissioners, and so on. Many of these individuals move between working in the private and public sectors.

In the U.S., regulation of payment for overtime hours is done at the state

1-43 E and federal levels. Because most engineering work is accomplished through projects, it is common for engineers to be asked or required to work overtime as projects near deadlines. Sometimes the overtime is paid at time and a half, sometimes as straight time, and sometimes the engineer's salary is treated as a constant even when overtime occurs. In a particular firm, engineering interns, engineers, and partners may be treated the same or differently.

1-44 E At the federal government level, the economic consequences of decisions can be very large. Firms hire lobbyists, legislators may focus on their constituents, and advocacy organizations promote their own agendas. In addition, sometimes some of the players are willing to be unethical.

1-45 E At both state and federal levels, legislators can be involved in "pork barrel" funding of capital projects. These projects may even bypass the economic evaluation using engineering economy that normal projects are subject to.

1-46 G At the international level, a common ethics issue important to engineering and project justification is that of environmental regulation. Often different nations have different environmental standards, and a project or product might be built in either location.

1-47 E At the international level, a common ethics issue important to engineering and project justification is that of worker health and safety. Often different nations have different standards, and a project or product could be built in either location.

1-48 G At the international level, engineering decisions are critical in matters of "sustainable development," a common ethics issue.

1-49 E At the international level, questions arise about whether the U.S. ban on bribery is practical or appropriate. In some countries government workers are very poorly paid, and they can support their families only by accepting money to "grease" a process.

In the 1970s the Ford Motor Company sold its subcompact Pinto model with known design defects. In particular, the gas tank's design and location led to rupture, leaks, and explosion in low-speed, rear-impact collisions. Fifty-nine people burned to death in Pinto accidents. In a cost–benefit analysis weighing the cost of fixing the defects ($11 per vehicle) versus the firm's potential liability for lawsuits on behalf of accident victims, Ford had placed the value of a human life at $200,000. Ford eventually recalled 1.4 million Pintos to fix the gas tank problem for a cost of $30 million to $40 million. In addition the automaker ultimately paid out millions more in liability settlements and incurred substantial damage to its reputation.

1-50

(*a*) Critique Ford's actions from the perspective of the NSPE Code of Ethics.

(*b*) One well-known ethical theory, utilitarianism, suggests that an act is ethically justified if it results in the "greatest good for the greatest number" when all relevant stakeholders are considered. Did Ford's cost–benefit analysis validly apply this theory?

(*c*) What should engineers do when the product they are designing has a known safety defect with an inexpensive remedy?

Contributed by Joseph R. Herkert, North Carolina State University

1-51 The decision-making process used to launch the Challenger shuttle has been extensively analyzed. Briefly summarize the key institutional groups, how the decision was made, and the ethical principles that may have been compromised.

1-52 One of the elements in the flooding of New Orleans during Hurricane Katrina was the failure of some of the levees that protected the city. Outline the role that ethical failures by engineers may have played in this situation. How could society structure decision making to minimize such failures?

1-53G Hurricane Sandy's flooding of New York City highlighted the vulnerability of coastal cities to extreme weather events, which are becoming more common. Strengthening and protecting infrastructure and the environment before the fact can be very expensive—and perhaps never needed. The possible availability of after-the-fact disaster aid can distort economic perspectives. Why is minimizing economic, environmental, and human costs related to extreme weather such a difficult problem for public infrastructure?

Current Costs

A manufacturing firm has received a contract to assemble 1000 units of test equipment in the next year. The firm must decide how to organize its assembly operation. Skilled workers, at $32 per hour each, can individually assemble the test equipment in 2.7 hours per unit. Alternatively, teams of five less-skilled workers (at $22 per hour each) can assemble a unit in 0.8 hours. Which approach is more economical?

1-54 A

1-55 G

Two manufacturing firms, located in cities 90 miles apart, both send their trucks four times a week to the other city full of cargo and return empty. Each driver costs $275 per day with benefits (the round trip takes all day) and each firm has truck operating costs of $1.20 a mile. (*a*) How much could each firm save weekly if each sent its truck twice a week and hauled the other firm's cargo on the return trip? (*b*) What would the savings be if there was a $0.20 per mile emissions tax on all business truck travel?

1-56 A G

An oil company is considering adding a more environmentally friendly grade of fuel at its service stations. To do this, an additional 3600-gallon tank must be buried at each station. Discussions with tank fabricators indicate that the least expensive tank would be cylindrical with minimum surface area. What size tank should be ordered?

Cathy Gwynn for a class project is analyzing a "Quick Shop" grocery store. The store emphasizes quick service, a limited assortment of grocery items, and higher prices. Cathy wants to see if the store hours (currently 0600 to 0100) can be changed to make the store more profitable.

Time Period	Daily Sales in the Time Period
0600–0700	$ 40
0700–0800	70
0800–0900	120

0900–1200	400
1200–1500	450
1500–1800	500
1800–2000	600
2000–2200	200
2200–2300	50
2300–2400	85
2400–0100	40

1-57 The cost of the groceries sold averages 65% of sales. The incremental cost to keep the store open, including the clerk's wage and other operating costs, is $23 per hour. To maximize profit, when should the store be opened, and when should it be closed?

1-58 A Willie Lohmann travels from city to city for business. Every other year he buys a used car for about $18,000. The dealer allows about $8000 as a trade-in allowance, so Willie spends $10,000 every other year for a car. Willie keeps accurate records of his expenses, which total 32.3¢ per mile. Willie's employer has two plans to reimburse car expenses:

A. Actual expenses: Willie will receive all his operating expenses, and $3650 each year for the car's decline in value.

B. Standard mileage rate: Willie will receive 54.5per mile but no

operating expenses and no depreciation allowance.
If Willie travels 15,000 miles per year, which method gives him the larger reimbursement? At what annual mileage do the two methods give the same reimbursement?

1-59 If you rent a car, you can (1) return it with a full gas tank, (2) return it without filling it and pay $5.75/gallon, or (3) accept a fixed price of $60 for gas. The local price is $3.50/gallon for gasoline, and you expect this car to get 25 miles per gallon. The car has a 16-gallon tank. What choice should you make if you expect to drive:
(*a*) 150 miles?
(*b*) 300 miles?
(*c*) 450 miles?
(*d*) How do your answers change if stopping at the filling station takes 20 minutes and your time is worth $15/hr?

1-60 A Your car gets 29 miles per gallon (mpg) at 60 miles per hour (mph) and 25 mpg at 70 mph. At what speed should you make a 525-mile trip:
(*a*) If gas costs $3 per gallon and your time is worth $18/hr?
(*b*) If gas costs $4 per gallon and your time is worth $12/hr?
(*c*) If gas costs $5 per gallon and your time is worth $9/hr?
(*d*) Build a spreadsheet (see Appendix A) to calculate the total trip cost for gas costs of $2, $3, $4, and $5 and values of time of $6, $9, $12, $15, and $18 per hour. Do two tables: one at 60 mph and one at 70 mph.

1-61 G A city needs to choose area rubbish disposal areas.
Area A: A gravel pit has a capacity of 16 million cubic meters. Owing to the possibility of high groundwater the Regional Water Pollution Control Board has restricted the lower 2 million cubic meters of fill to inert material only (earth, concrete, asphalt, paving, brick, etc.). This must be purchased and hauled to this area for the bottom fill.
Area B: Capacity is 14 million cubic meters. For 20% of the city, the haul is the same distance as for Area *A*. The round-trip haul is 5 miles longer for 60% of the city, and 2 miles shorter for 20% of the city.
Assume the following conditions:
• Cost of inert material placed in Area *A* will be $9.40/m^3.
• Average speed of trucks from last pickup to disposal site is 25 miles

per hour.
• The rubbish truck and a two-man crew will cost $210 per hour.
• Truck capacity of 4½ tons per load or 20 m^3.
• Sufficient cover material is available at all areas.

Which of the sites do you recommend?

A firm is planning to manufacture a new product. As the selling price is increased, the quantity that can be sold decreases. Numerically the sales department estimates:

$$P = \$475 - 0.25Q$$

where P = selling price per unit

Q = quantity sold per year

1-62 A On the other hand, management estimates that the average unit cost of manufacturing and selling the product will decrease as the quantity sold increases. They estimate

$$C = \$48Q + \$22{,}500$$

where C = cost to produce and sell Q per year

The firm's management wishes to maximize profit. What quantity should the decision makers plan to produce and sell each year and what profit will be earned?

The vegetable buyer for a group of grocery stores has decided to sell packages of sprouted grain in the vegetable section of the stores. The product is perishable, and any remaining unsold after one week in the store is discarded. The supplier will deliver the packages to the stores, arrange them in the display space, and remove and dispose of any old packages. The price the supplier will charge the stores depends on the size of the total weekly order for all the stores.

Weekly Order	Price per Package

Less than 1000 packages	75¢
1000–1499	63
1500–1999	55
2000 or more	48

1-63 The vegetable buyer estimates the quantity that can be sold per week, at various selling prices, as follows:

Selling Price	Packages Sold per Week
$1.20	300
.90	600
.80	1200
.66	1800
.52	2300

The sprouted grain will be sold at the same price in all the grocery stores.

(*a*) How many packages should be purchased per week, and at which of the five prices listed above should they be sold?

(*b*) Build a spreadsheet (see Appendix A) to calculate the profit for every combination of selling price and weekly order size.

Jim Jones, a motel owner, noticed that just down the street the “Motel 66” advertises a $66-per-night room rental rate on its sign. As a result, this competitor has rented all 80 rooms every day by late afternoon. Jim, on the other hand, does not advertise his rate, which is $85 per night, and he averages only a 72% occupancy of his 50 rooms.

There are a lot of other motels nearby, but only Motel 66 advertises its rate on its sign. (Rates at the other motels vary from $68 to $110 per night.) Jim estimates that his actual incremental cost per night for each room rented, rather than remaining vacant, is $16. This $16 pays for all the cleaning, laundering, maintenance, utilities, and so on. Jim believes his eight alternatives are:

	Alternative		Resulting Occupancy Rate
		Advertise and Charge	
	1	$65 per night	100%
1-64	2	72 per night	93
	3	79 per night	82
	4	86 per night	68
		Do Not Advertise and Charge	

5	$78 per night	72%
6	85 per night	65
7	92 per night	62
8	99 per night	54

What should Jim do? Show how you reached your conclusion.

1-65 A grower estimates that if he picks his apple crop now, he will obtain 1000 boxes of apples, which he can sell at $30 per box. However, he thinks his crop will increase by 120 boxes of apples for each week he delays picking, but that the price will drop at a rate of $1.50 per box per week; in addition, he estimates that approximately 20 boxes per week will spoil for each week he delays picking.

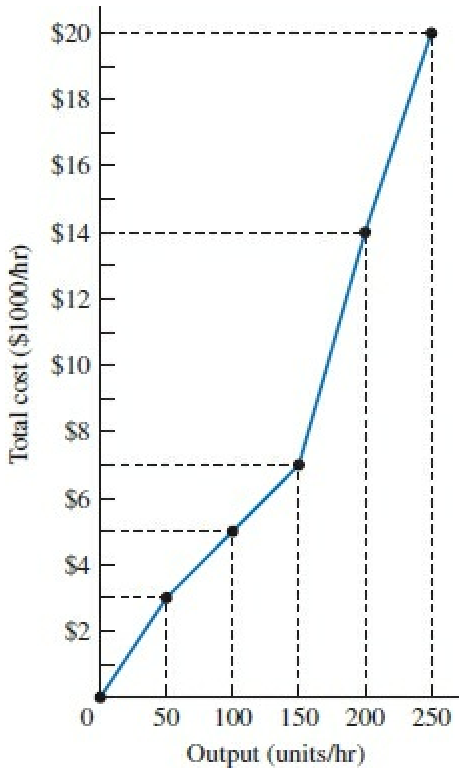
(*a*) When should he pick his crop to obtain the largest total cash return? How much will he receive for his crop at that time?

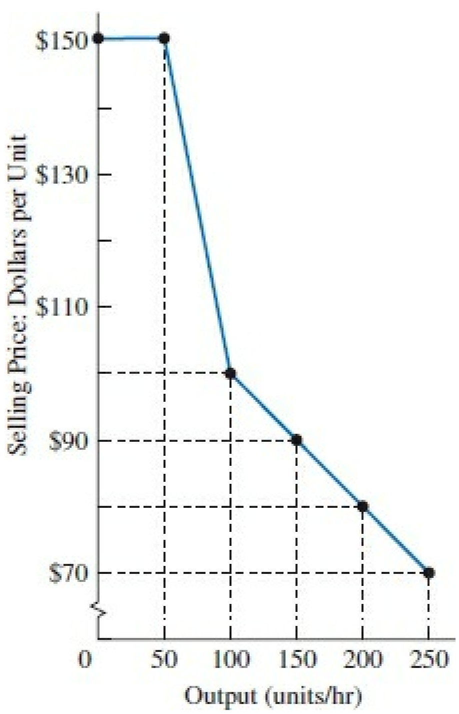
(*b*) Build a spreadsheet (see Appendix A) to calculate the profit for 0, 1, 2, …, 6 weeks.

1-66 On her first engineering job, Joy Hayes was given the responsibility of determining the production rate for a new product. She has assembled the data presented in the graphs. Note that costs are in $1000s.

(*a*) Select an appropriate economic criterion and estimate the production rate based upon it.

(*b*) Joy's boss told Joy: "I want you to maximize output with minimum input." Joy wonders if it is possible to meet her boss's criterion. What would you tell her?





Multiple Objectives

Use the data in Example 1–5.

(*a*) What is the total score for each offer if the three objectives have the same weight?

1-67 (*b*) Holding livability's weight constant, how important does family have to be for offer A to be the best choice? Remember that the weights must sum to 1.

(*c*) Holding family's weight constant, how important does livability have to be for offer C to be the best choice?

1-68 A A graduating senior has been accepted by three universities for an M.S. in engineering. Two criteria have been identified. The first is the program and university's academic ranking. The second is the cost. A third criteria of location was initially considered, but then the student recognized that it is only for about a year, and applications were only made to acceptable schools. The student is currently enrolled in the first university, which is rated as a 5 for academic rank and a 7 for cost. The second is a larger out-of-state public university, which is rated as a 7 for academic rank and a 6 for cost. The third is a prestigious private school, which is rated as a 10 academically and a 3 for its higher cost.

(*a*) What is the total score for each school if the academic rank has a weight of 40%?

(*b*) If academic rank has a weight of 60%, what is the total score for each school?

Minicases

1-69 Pick a decision involving multiple objectives that you must make. Estimate each objective's weighted importance, rate the alternative choices on each objective, and develop the totals for a model like Example 1–5.

1-70 G Green engineering is a design construct that explicitly considers environmental and sustainability factors within the design process. It seeks to promote responsible use of limited resources, and to produce environmentally ethical and safe engineering products, goods, and services. Using the web, find two lists of principles that have been suggested by different groups.

(*a*) Write a short paragraph on each list of principles that describes the who, when, and why of their formation.

(*b*) Compare and contrast the two lists. Which do you think is best and why? Is there anything that you see is missing from both lists?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 1	**New Office Equipment**
	Student develops requirements for new copier machines with little given data. Organizational thinking required.
CASE 2	**Budgeting Issues**
	Strategies for a group’s operating budget request. One focus is ethics.

[1] A group of techniques called value analysis or **value engineering** is used to examine past decisions and current trade-offs in designing alternatives.

[2] This is the Kaldor criterion.

CHAPTER 2

ESTIMATING ENGINEERING COSTS AND BENEFITS

LightTUBe

The Tullahoma Utilities Board (TUB) installed a $3.1 million Automated Metering Information System in its full service area in Tennessee. The process uses special meters, each equipped with a radio transmitter. They collect information on water and electric usage at residences and businesses, and then forward the information to collectors mounted on nearby poles. The usage information is then relayed using LightTUBe, the utility’s fiber optics network, eliminating the need for employees to physically read utility meters.

It provides not only real-time monitoring capability for residential and commercial users but also the ability to monitor the entire system's health and loads.

The change to the automated system allows TUB customers to better understand and actively manage their consumption patterns. Customers are able to access a portal to examine their use habits and reduce their bills by doing household tasks that require more electricity during off-peak hours. The new system also reduces personnel costs, provides better information on leak detection, outage management, and any theft of service.

After the system was economically justified and built for automated metering, TUB found that there was significant unused bandwidth on the fiber optic network. It now operates the LightTUBe network throughout the City of Tullahoma to provide Fiber to the Premise (FTTP), producing high quality video, high speed Internet, and telephone services. LightTUBe provides voice, video, and data services over 250 miles of Fiber Optic cable. TUB serves nearly 10,000 residential, commercial, and industrial customers.

This "hybrid" business model crosses traditional boundaries associated with public utilities, phone companies, and Internet/cable TV providers. Traditionally, public utilities operate with a set profit margin, which allows them to minimize the cost to the customer while maintaining operational and future growth requirements. Most cable TV, Internet, and phone companies operate on a private sector business model in which profits are maximized.

Contributed by James Simonton, University of Tennessee Space Institute

QUESTIONS TO CONSIDER

1. How do you estimate the cost—and ultimately the benefit—of such a "hybrid" operation with the traditional utility segment of the business (water, electricity, and sewage) working to minimize cost, while the LighTUBe segment works to maximize profit?
2. Discuss the ethical issues related to using public funds for an operation that

could be seen as a for-profit venture that is in competition with private sector companies.
3. In general, private sector project cost estimating may seem the same as public sector estimating. The real difference in this case would be the ability of TUB to consider both what it collects and consumer surplus value. How would a project that blurs the line between the two be handled?
4. When performing an economic analysis of "hybrid" projects, what tools would be appropriate to establish the benefit for justification purposes?

After Completing This Chapter…

The student should be able to:

• Define and use costs and benefits of many types including: average, external, fixed, incremental, internal, marginal, opportunity, sunk, and variable costs and benefits.
• Provide specific examples of how and why these concepts are important.
• Define engineering estimating for costs and benefits.
• Explain the three types of engineering estimate, as well as common difficulties encountered in making engineering estimates.
• Use several common mathematical estimating models in estimating costs and benefits.
• Discuss the impact of the *learning curve* on estimates.
• State the relationship between cost estimating and estimating project benefits.
• Draw *cash flow diagrams* to show project costs and benefits.

Key Words

average cost

book cost

breakeven cost

cash cost

cash flow diagram

cost and price index

detailed estimate

estimation by analogy

external cost

fixed cost

incremental cost

internal cost

learning curve

life-cycle cost

marginal cost

nonrecurring cost

opportunity cost

per-unit model

power-sizing model

profit–loss breakeven chart

recurring cost

rough estimate

segmenting model

semidetailed estimate

sunk cost

total cost

triangulation

variable cost

work breakdown structure

Estimating the engineering costs and benefits of proposed decision choices is “where the numbers come from.” In this chapter we describe cost and benefit concepts and methods. These include fixed and variable costs, marginal and average costs, sunk and opportunity costs, recurring and nonrecurring benefits and costs, incremental cash costs, book costs, and life-cycle costs. We then describe the various types of estimates and difficulties sometimes encountered. The models that are described include unit factor, segmenting, cost indexes, power sizing, triangulation, and learning curves. The chapter discusses estimating benefits, developing cash flow diagrams, and drawing these diagrams with spreadsheets.

FIXED, VARIABLE, MARGINAL, AND AVERAGE COSTS

Fixed costs are constant or unchanging regardless of the level of output or activity. In contrast, **variable** costs depend on the level of output or activity. A **marginal** cost is the variable cost for one more unit, while the **average** cost is the total cost divided by the number of units.

In a production environment, for example, fixed costs, such as those for factory floor space and equipment, remain the same even though production quantity, number of employees, and level of work-in-process may vary. Labor costs are classified as a *variable* cost because they depend on the number of employees and the number of hours they work. Thus *fixed* costs are level or constant regardless of output or activity, and *variable* costs are changing and related to the level of output or activity.

As another example, many universities charge full-time students a fixed cost for 12 to 18 hours and a cost per credit hour for each credit hour over 18. Thus for full-time students who are taking an overload (>18 hours), there is a variable cost that depends on the level of activity, but for most full-time students tuition is a fixed cost.

This example can also be used to distinguish between *marginal* and *average* costs. A marginal cost is the cost of one more unit. This will depend on how many credit hours the student is taking. If currently enrolled for 12 to 17 hours, adding one more is free. The marginal cost of an additional credit hour is $0. However, for a student taking 18 or more hours, the marginal cost equals the variable cost of one more hour.

To illustrate average costs, suppose the cost of 12 to 18 hours is $3600 per term and overload credits are $240/hour. If a student takes 12 hours, the *average* cost is $3600/12 = $300 per credit hour. If the student were to take 18 hours, the *average* cost would decrease to $3600/18 = $200 per credit hour. If the student takes 21 hours, the *average* cost is $205.71 per credit hour [$3600 + (3 × $240)]/21. Average cost is thus calculated by dividing the total cost for all units by the total number of units. Decision makers use average cost to attain an overall cost picture of the investment on a per unit basis.

Marginal cost is used to decide whether an additional unit should be made, purchased, or enrolled in. For our example full-time student, the marginal cost of another credit is $0 or $240 depending on how many credits the student has already signed up for.

EXAMPLE 2-1

The Federation of Student Societies of Engineering (FeSSE) wants to offer a one-day training course to help students in job hunting and to raise funds. The organizing committee is sure that they can find alumni, local business people, and faculty to provide the training at no charge. Thus the main costs will be for space, meals, handouts, and advertising.

The organizers have classified the costs for room rental, room setup, and advertising as fixed costs. They also have included the meals for the speakers as a fixed cost. Their total of $225 is pegged to a room that will hold 40 people. So if demand is higher, the fixed costs will also increase.

The variable costs for food and bound handouts will be $20 per student. The organizing committee believes that $35 is about the right price to match value to students with their budgets. Since FeSSE has not offered training courses before, they are unsure how many students will reserve seats.

Develop equations for FeSSE's total cost and total revenue, and determine the number of registrations that would be needed for revenue to equal cost.

SOLUTION

Let *x* equal the number of students who sign up. Then,

$$\begin{aligned}\text{Total cost} &= \$225 + \$20x \\ \text{Total revenue} &= \$35x\end{aligned}$$

To find the number of student registrants for revenue to equal cost, we set the equations equal to each other and solve.

$$\begin{aligned}\text{Total cost} &= \text{Total revenue} \\ \$225 + \$20x &= \$35x \\ \$225 &= (\$35 - \$20)x \\ x &= 225/15 = 15 \text{ students}\end{aligned}$$

While this example has been defined for a student engineering society, we could just as easily have described this as a training course to be sponsored by a local chapter of a professional technical society. The fixed cost and the revenue would increase by a factor of about 10, while the variable cost would probably double or triple.

If a firm were considering an in-house short course, the cost of the in-house course would be compared with the cost per employee (a variable cost) for enrolling employees in external training.

From Example 2–1 we see how it is possible to calculate total fixed and total variable costs. Furthermore, these values can be combined into a single **total** cost equation as follows:

$$\text{Total cost} = \text{Total fixed cost} + \text{Total variable cost} \quad (2\text{-}1)$$

Example 2–1 developed *total cost* and *total revenue* equations to describe a training course proposal. These equations can be used to create what is called a **profit–loss breakeven chart** (see Figure 2–1). A plot of revenues against costs for various levels of output (activity) allows one to illustrate a *breakeven point* (in terms of costs and revenue) and regions of *profit* and *loss*. These terms can be defined as follows.

Breakeven point: The activity level at which total costs are *equal to* the revenue (or savings) generated. This is the level at which one "just breaks even."
Profit region: The variable *x* is greater than the breakeven point and total revenue is *greater* than total costs.
Loss region: The variable *x* is less than the breakeven point and total revenue is *less* than total costs.

Notice in Figure 2–1 that the *breakeven* point for the number of persons in the training course is 15 people. For more than 15 people, FeSSE will make a profit. If fewer than 15 sign up, there will be a net loss.

The fixed costs of our simple model are in reality *fixed* over a range of values for *x*. In Example 2–1, that range was 1 to 40 students. If *zero* students signed up, then the course could be canceled and many of the fixed costs would not be incurred. Some costs such as advertising might already have been spent, and there might be cancellation fees. If more than 40 students signed up, then greater costs for larger rooms or multiple sessions would be incurred. The model is valid only within the range named.

When modeling a specific situation, we often use *linear* variable costs and revenues. However, sometimes the relationship may be nonlinear. For example, employees are often paid at 150% of their hourly rate for overtime hours, so that production levels requiring overtime have higher variable costs. Total cost in Figure 2–2 is a fixed cost of $3000 plus a variable cost of $200 per unit for straight-time production of up to 10 units and $300 per unit for overtime production of up to 5 more units.

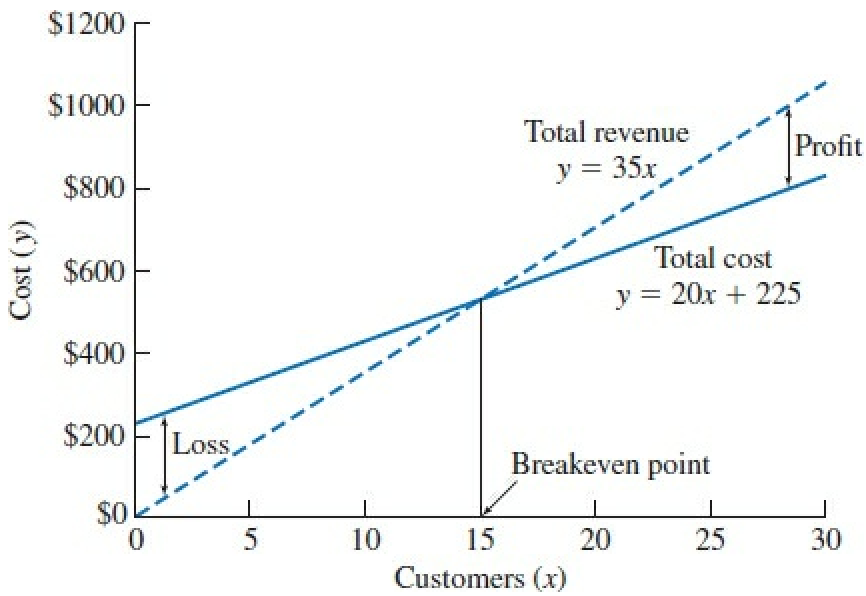


FIGURE 2-1 Profit–loss breakeven chart for Example 2–1.

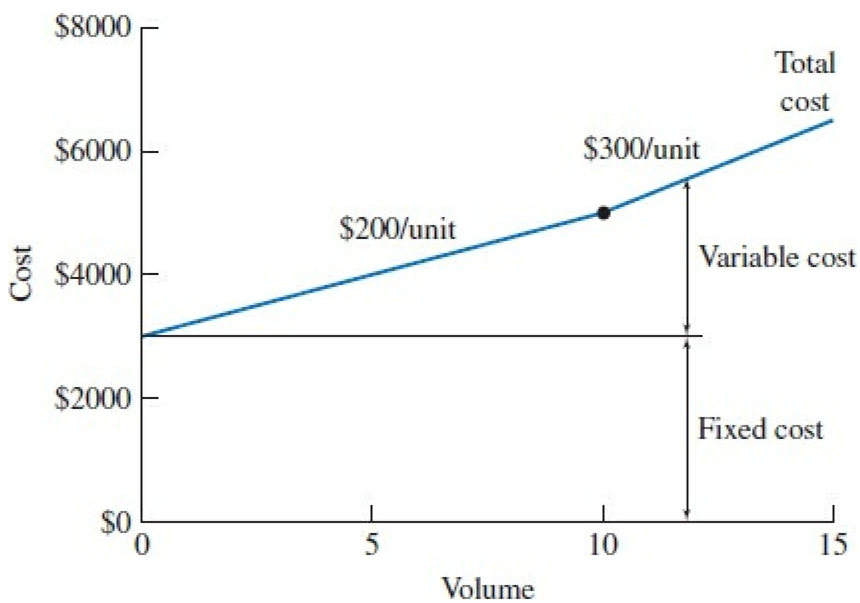


FIGURE 2-2 Nonlinear variable costs.

Figure 2–2 can also be used to illustrate marginal and average costs. At a volume of 5 units the marginal cost is $200 per unit, while at a volume of 12 units the marginal cost is $300 per unit. At 5 units the average cost is $800 per unit, or (3000 + 200 × 5)/5. At 12 units the average cost is $467 per unit, or (3000 + 200 × 10 + 300 × 2)/12.

Sunk Costs

A **sunk cost** is money already spent as a result of a *past* decision. If only 5 students signed up for the training course in Example 2–1, the advertising costs would be a *sunk cost*.

Sunk costs must be ignored in engineering economic analysis because current decisions cannot change the past. For example, dollars spent last year to purchase new production machinery is money that is *sunk:* the money has already been spent—there is nothing that can be done now to change that action. As engineering economists we deal with present and future opportunities.

Many times it is difficult not to be influenced by sunk costs. Consider 100 shares of stock in XYZ, Inc., purchased for $15 per share last year. The share price has steadily declined over the past 12 months to a price of $10 per share today. Current decisions must focus on the $10 per share that could be attained today (as well as future price potential), not the $15 per share that was paid last year. The $15 per share paid last year is a *sunk cost* and has no influence on present opportunities.

As another example, when Regina was a sophomore, she purchased a newest-generation laptop from the college bookstore for $2000. By the time she graduated, the most anyone would pay her for the computer was $400 because the newest models were faster and cheaper and had more capabilities. For Regina, the original purchase price was a *sunk cost* that has no influence on her present opportunity to sell the laptop at its current market value of $400.

When we get to Chapters 11 and 12 on depreciation and income taxes, we will find an exception to the rule of *ignore sunk costs*. When an asset is sold or disposed of, then the sunk cost of what was paid for it is important in figuring out how much is owed in taxes. This exception applies only to the after-tax analysis of capital assets.

Opportunity Costs

An **opportunity cost** is associated with using a resource in one activity instead of another. Every day firms use resources to accomplish various tasks —forklifts transport materials, engineers design products and processes, assembly lines make a product, and parking lots provide parking for employees' vehicles. There are costs for these intended purposes. These are also forgone opportunity costs. For instance, the assembly line could produce a different product, and the parking lot could be rented out, used as a building site, or converted into a small airstrip. Each alternative use would provide some benefit to the firm. These opportunity costs can be included, or they can be addressed by considering that alternative use as another decision-making choice.

As an example, suppose a college student may travel through Europe over the summer break. The student should estimate all the *out-of-pocket* cash costs for air travel, lodging, meals, entertainment, and train passes. Suppose this amounts to $3000 for a 10-week period—which the student can afford. However, the *true* cost includes not only *out-of-pocket* cash costs but also the *opportunity cost*. By taking the trip, the student is giving up the *opportunity* to earn $5000 as a summer intern. The student's total cost is thus $8000.

Remember that opportunity costs are really foregone benefits. When those benefits are not chosen they become costs. The key is to make a choice whereby the *actual* benefits realized outweigh the foregone benefits not chosen. Example 2–2 shows how opportunity costs are part of decisions about idle or under-used assets. What benefit is foregone by keeping the pumps?

EXAMPLE 2-2

A distributor of electric pumps must decide what to do with a "lot" of old electric pumps purchased 3 years ago. Soon after the distributor purchased the lot, technology advances made the old pumps less desirable to customers. The pumps are becoming obsolescent as they sit in inventory. The pricing manager has the following information.

Distributor’s purchase price 3 years ago	$ 7,000
Distributor’s storage costs to date	1,000
Distributor’s list price 3 years ago	9,500
Current list price of the same number of new pumps	12,000
Amount offered for the old pumps from a buyer 2 years ago	5,000
Current price the lot of old pumps would bring	3,000

Looking at the data, the pricing manager has concluded that the price should be set at $8000. This is the money that the firm has “tied up” in the lot of old pumps ($7000 purchase and $1000 storage), and it was reasoned that the company should at least recover this cost. Furthermore, the pricing manager has argued that an $8000 price would be $1500 less than the list price from 3 years ago, and it would be $4000 less than what a lot of new pumps would cost ($12,000 − $8000). What would be your advice on price?

SOLUTION

Let’s look more closely at each of the data items.

Distributor’s purchase price 3 years ago: This is a sunk cost that should not be considered in setting the price today.
Distributor’s storage costs to date: The storage costs for keeping the pumps in inventory are sunk costs; that is, they have been paid. Hence they should not influence the pricing decision.
Distributor’s list price 3 years ago: If there have been no willing buyers in the past 3 years at this price, it is unlikely that a buyer will emerge in the future. This past list price should have no influence on the current pricing decision.
Current list price of newer pumps: Newer pumps now include technology and features that have made the older pumps less valuable. Directly comparing the older pumps to those with new technology is misleading. However, the price of the new pumps and the value of the new features help determine the market value of the old pumps.
Amount offered by a buyer 2 years ago: This once was an opportunity. At the

time of the offer, the company chose to keep the lot and thus the $5000 offered became an opportunity cost for keeping the pumps. This amount should not influence the current pricing.
Current price the lot could bring: The price a willing buyer in the marketplace offers is called the asset's *market value*. This $3000 is the relevant opportunity cost for decision making.

From this analysis, it is easy to see the flaw in the pricing manager's reasoning. In an engineering economist analysis we deal only with *today's* and prospective *future* opportunities. It is impossible to go back in time and change decisions that have been made. Thus, the pricing manager should recommend to the distributor that the price be set at the current value that a buyer assigns to the item: $3000.

Recurring and Nonrecurring Costs

Recurring costs refer to any expense that is known and anticipated and that occurs at regular intervals. **Nonrecurring costs** are one-of-a-kind expenses that occur at irregular intervals and thus are sometimes difficult to plan for or anticipate from a budgeting perspective.

Examples of recurring costs include those for resurfacing a highway and reshingling a roof. Annual expenses for maintenance and operation are also recurring expenses. Examples of nonrecurring costs include the cost of installing a new machine (including any facility modifications required), the cost of augmenting equipment based on older technology to restore its usefulness, emergency maintenance expenses, and the disposal or close-down costs associated with ending operations.

In engineering economic analyses *recurring costs* are modeled as cash flows that occur at regular intervals (such as every year or every 5 years). Their magnitude can be estimated, and they can be included in the overall analysis. *Nonrecurring costs* can be handled easily in our analysis if we are able to anticipate their timing and size. However, this is not always so easy to do.

There are also recurring and nonrecurring benefits. A nonrecurring benefit is a single cash inflow that is anticipated today or in the future, such as the proceeds from selling a house, business, vehicle, or any other asset. Personal examples include a graduation gift, retirement plan lump sum distribution, lottery winnings, or inheritance. Engineering projects are often intended to produce recurring benefits that continue for months, years, or decades. Examples include sales of a new product, faster travel on a safer bridge or highway, attending events at a sports arena or theater, and the services of schools, hospitals, and libraries.

Incremental Costs

One of the fundamental principles in engineering economic analysis is that in choosing between competing alternatives, the focus is on the *differences* between those alternatives. This is the concept of **incremental costs.** For instance, one may be interested in comparing two options to lease a vehicle for personal use. The two lease options may have several specifics for which costs are the same. However, there may be incremental costs associated with one option but not with the other. In comparing the two leases, the focus should be on the differences between the alternatives, not on the costs that are the same.

The principle described above for costs also holds true for the incremental benefits of competing alternatives. Consider the case of lease options for the vehicle. The benefits associated with each option were assumed to be the same, and thus we were only interested in the incremental cost differences. However, what if the benefits of the two options were different? In this case your focus would be on the differences of the costs *and the benefits* associated with each option.

EXAMPLE 2-3

Philip is choosing between model *A* (a budget model) and model *B* (with more features and a higher purchase price). What *incremental costs* would Philip incur if he chose model *B* instead of the less expensive model *A*?

Cost Items	Model *A*	Model *B*

Purchase price	$10,000	$17,500
Installation costs	3,500	5,000
Annual maintenance costs	2,500	750
Annual utility expenses	1,200	2,000
Disposal costs after useful life	700	500

SOLUTION

We are interested in the incremental or *extra* costs that are associated with choosing model *B* instead of model *A*. To obtain these we subtract model *A* costs from model *B* costs for each category (cost item) with the following results.

Cost Items	(Model *B* Cost – *A* Cost)	Incremental Cost of *B*
Purchase price	17,500 − 10,000	$7500
Installation costs	5,000 − 3,500	1500
Annual maintenance costs	750 − 2,500	−1750/yr
Annual utility expenses	2,000 − 1,200	800/yr
Disposal costs after useful life	500 − 700	−200

Notice that for the cost categories given, the incremental costs of model *B* are both positive and negative. Positive incremental costs mean that model *B* costs more than model *A*, and negative incremental costs mean that there would be a *savings* (reduction in cost) if model *B* were chosen instead.

As described in the problem statement, because model *B* has more features, the decision must also include incremental benefits offered by those features rather than focussing only on costs.

Cash Costs Versus Book Costs

A *cash cost* requires the cash transaction of dollars "out of one person's pocket" into "the pocket of someone else." When you buy dinner for your friends or make your monthly car payment you are incurring a **cash cost** or **cash flow.** Cash costs and cash flows are the basis for engineering economic analysis.

Book costs do not require the transaction of dollars "from one pocket to another." Rather, **book costs** are cost effects from past decisions that are recorded "in the books" (accounting books) of a firm. In one common book cost, asset depreciation (which we discuss in Chapter 11), the expense paid for a particular business asset is "written off" on a company's accounting system over a number of periods. Book costs do not ordinarily represent cash flows and thus are not included in engineering economic analysis. One exception to this is the impact of asset depreciation on tax payments—which are cash flows and are included in after-tax analyses.

CONSIDERING ALL COSTS

Life-Cycle Costs

A product's sales volume and revenues follow a life cycle as shown in Figure 2–3. Recognizing that these phases occur for all products and projects can ensure that all costs and benefits are included in the economic analysis. Fabrication plants for making computer chips have very large up-front costs. Mines and power plants can require expensive cleanup or monitoring for a century or more. Energy-saving projects focus on spending now to save more later. Road and building designs balance current needs with uncertain future needs. In each case, good decisions require considering all costs and benefits throughout the life cycle.

Life-cycle costing is the concept of designing products, projects, and services with a full and explicit recognition of the associated costs and benefits over the various phases of their life cycles. Since *all* costs and benefits over the life cycle are considered, this is the correct way to make economic decisions. When first introduced, life-cycle costing was contrasted with making decisions based on initial costs. Today, green engineering adds an emphasis on the environmental costs of final disposal and of the energy used to produce and operate that may be incurred by the public or the planet, rather than the firm or agency.

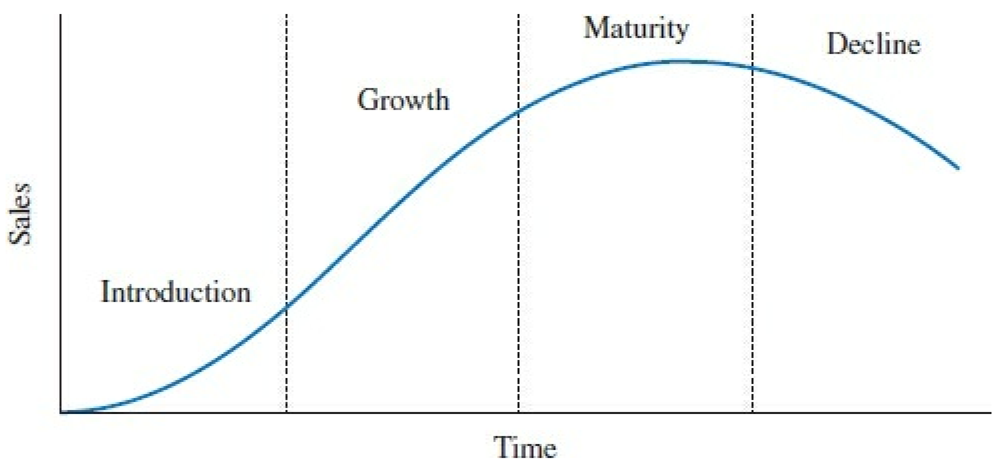


FIGURE 2-3 Typical product life cycle

Beginning —— Time ——→ End

Needs Assessment and Justification Phase	Conceptual or Preliminary Design Phase	Detailed Design Phase	Production or Construction Phase	Operational Use Phase	Decline and Retirement Phase
Requirements	Impact Analysis	Allocation of Resources	Product, Goods, & Services Built	Operational Use	Declining Use
Overall Feasibility	Proof of Concept	Detailed Specifications	All Supporting Facilities Built	Use by Ultimate Customer	Phase Out
Conceptual Design Planning	Prototype/ Breadboard	Component and Supplier Selection	Operational Use Planning	Maintenance and Support	Retirement
	Development and Testing	Production or Construction Phase		Processes, Materials, and Methods Used	Responsible Disposal
	Detailed Design Planning			Decline and Retirement Planning	

FIGURE 2-4 Typical life cycle for products, goods, and services.

Figure 2–4 describes some typical phases for the life cycles of products, projects, and services. The life-cycle curve of costs when added up over time becomes the cumulative life-cycle costs spent curve of Figure 2–5.

Design Changes and Cost Impacts

Two key concepts in life-cycle costing are that the later design changes are made, the higher the costs, and that decisions made early in the life cycle tend to “lock in” costs and benefits that will be incurred later. Figure 2–5 illustrates how costs are committed early in the product life cycle—nearly 70–90% of all costs are set during the design phases. At the same time, as the figure shows, only 10–30% of cumulative life-cycle costs have been spent. In addition, notice that as life-cycle costs are committed, ultimate life-cycle benefits are set. This highlights the fact that early decisions not only commit resources, but also bound the benefits that will be realized by the product, good, or service.

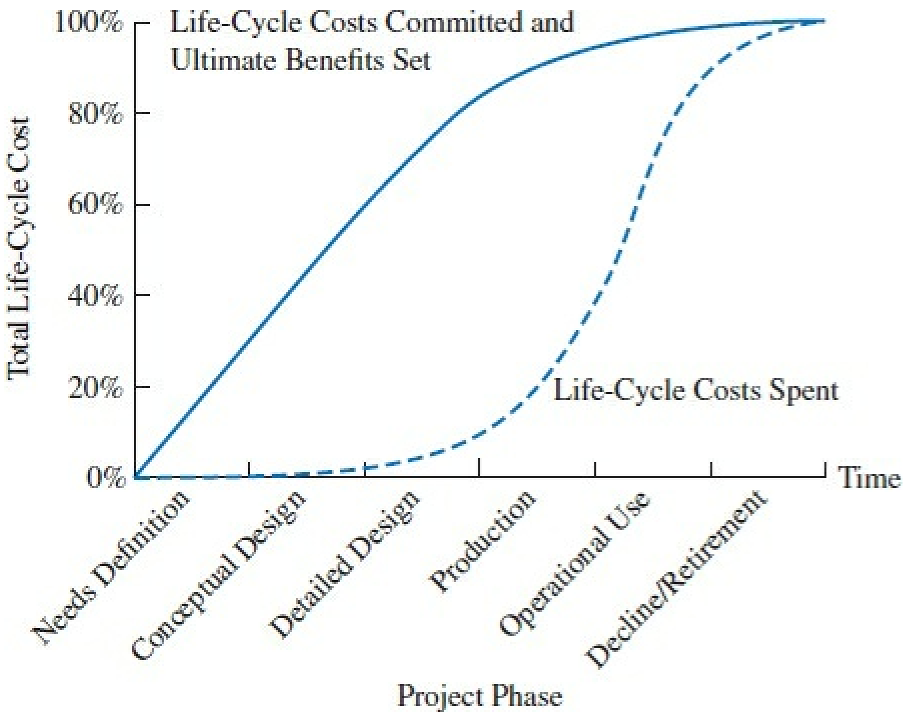


FIGURE 2-5 Cumulative life-cycle costs committed and dollars spent.

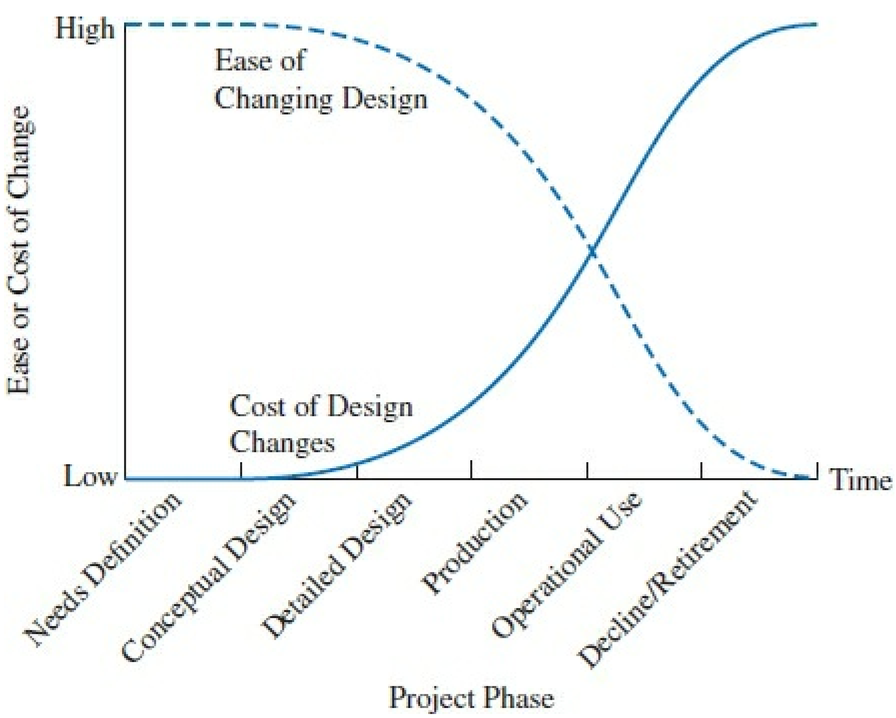


FIGURE 2-6 Ease of life-cycle design change and costs of change.

Figure 2–6 reinforces these concepts by illustrating that later product changes are more costly and that earlier changes are easier (and less costly) to make. When planners try to save money at an early design stage, the result is often a poor design that results in change orders during construction and prototype development. These changes, in turn, are more costly than working out a better design would have been.

From Figures 2–5 and 2–6 we see that the time to consider all life-cycle effects and make design changes is during the needs and conceptual/preliminary design phases—before a lot of dollars are committed. Some of the life-cycle effects that engineers should consider at design time include product costs for liability, production, material, testing and quality assurance, and maintenance and warranty.

Internal and External Costs

An important extension of life-cycle costing by green or sustainable engineering is to consider both internal and external costs in a design, project, or product. **Internal costs** are incurred and paid by the firm and are used to calculate the production cost. This cost is part of decisions on pricing and ultimately profitability. In contrast, **external costs** are outside the firm’s normal cost accounting and thus do not directly affect the price of goods/services. Green engineering focuses on the environmental impacts of inputs, such as electric power, or outputs, such as discarded packaging. Examples include costs of disposal, decommissioning, or landfilling; effects on animals and habitats; degradation of air and/or water quality and quality of life; and managing wastes and pollutants. Often the focus is designing for repair, reuse, or recycling, rather than disposal as landfill.

Figure 2–7 also includes social costs, such as for full-time workers who need Medicaid or food stamps—costs that are paid through taxes rather than by employers. Other examples include poor working conditions and communities and individuals impacted by layoffs, plant closures, and so on.

While trade-offs between external and internal costs are common, the goal of green engineering is shown in Figure 2–7: lower external costs by thoughtful attention; lower internal costs by innovation; and increase profits as buyers respond to a better corporate reputation and *green* choices for consumers.

Internal and external costs are particularly important in public-sector applications of engineering economy. Figure 16-1's smokestack and plant boundary emphasizes the internal/external costs discussed here.

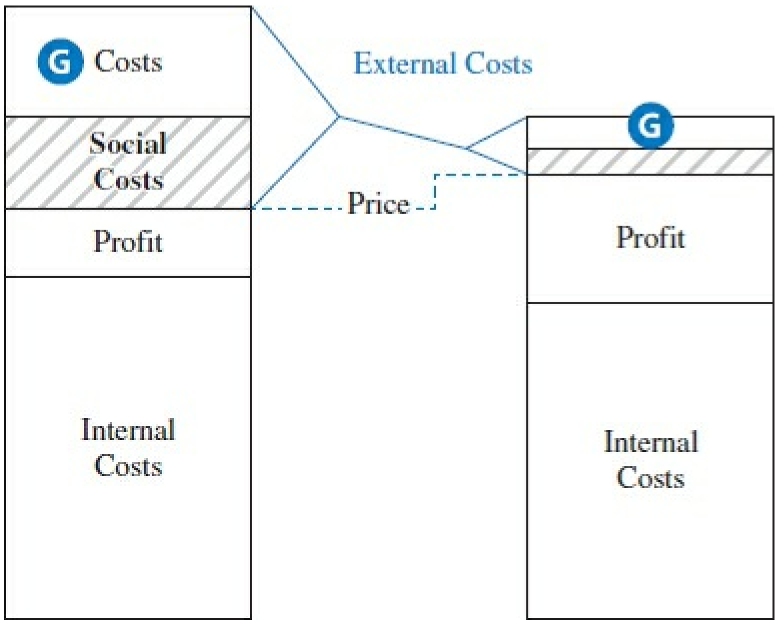


FIGURE 2-7 Green design goal of reducing *all* costs and increasing profit.

EXAMPLE 2-4

JPL Enterprises Co. includes potential environmental costs in their design process. From the following list, identify the internal and external costs associated with a new mountaintop ski resort project.

- Site costs
- Land costs
- Water quality costs
- Legal costs
- Community costs
- Design costs
- Habitat costs

• Viewshed costs
• Recreational costs
• Administrative costs
• Roadway costs
• Labor costs
• Materials costs
• Equipment costs
• Overhead costs
• Construction costs

SOLUTION

Internal costs include the site, land, legal, design, administrative, labor, materials, equipment, overhead, and construction categories.

External (societal) costs include items such as:

Water quality costs: impacts on downstream water sources and aquifers
Community costs: prospective loss of culture, quaintness, community values
Habitat costs: effect on natural habitat for native/migratory animals
Viewshed costs: negative impact on the visual sight lines
Recreational costs: impact on hikers, fishermen, hunters, birdwatchers, and others
Roadway costs: cost to community to build roads and bridges (developers sometimes share these costs with local, state and federal entities)

ESTIMATING BENEFITS

Along with estimating the costs of proposed projects, engineering economists must often also quantify the anticipated benefits. Example benefits include sales from products, revenues from bridge tolls and electric power sales, cost reductions from reduced material or labor costs, less time spent in traffic jams, and reduced risk of flooding. Many engineering projects are undertaken precisely to secure the benefits.

Uncertainty associated with benefit estimates is asymmetric, with broader limits for negative outcomes. Compared to costs, benefits are more likely to be overestimated, thus an example set of limits might be (−50%, +20%). Another important difference between cost and benefit estimation is that many costs of engineering projects occur in the near future (design and construction), but benefits are further into the future—thus more uncertainty is typical.

The estimation of economic benefits is an important step that should not be overlooked. Most of the models, concepts, and issues that apply in estimating costs also apply to estimating economic benefits.

THE ESTIMATING PROCESS

Engineering economic analysis focuses on the future consequences of current decisions. Because these consequences are in the future, usually they must be estimated and cannot be known with certainty. Estimates that may be needed in engineering economic analysis include purchase costs, annual revenue, yearly maintenance, interest rates for investments, annual labor and insurance costs, equipment salvage values, and tax rates.

Estimating is the foundation of economic analysis. As in any analysis procedure, the outcome is only as good as the numbers used to reach the decision. For example, a person who wants to estimate her federal income taxes for a given year could do a very detailed analysis, including social security deductions, retirement savings deductions, itemized personal deductions, exemption calculations, and estimates of likely changes to the tax code. However, this very technical and detailed analysis will be grossly inaccurate if poor data are used to predict the next year's income. Thus, to ensure that an analysis is a reasonable evaluation of future events, it is very important to make careful estimates.

Types of Cost Estimates

We can define three general types of estimate whose purposes, accuracies, and underlying methods are quite different.

Rough estimates: Order-of-magnitude estimates used for high-level planning, for determining project feasibility, and in a project's initial planning and evaluation phases. Rough estimates tend to involve back-of-the-envelope numbers with limited detail or accuracy. The intent is to quantify and consider the order of magnitude of the numbers involved. These estimates require minimum resources to develop, and their accuracy is generally −30 to +60%.

Notice the nonsymmetry in the estimating error. This is because decision makers tend to underestimate the magnitude of costs (negative economic effects). Also, as Murphy's law predicts, there seem to be more ways for results to be worse than expected than there are for the results to be better than expected.

Semidetailed estimates: Used for budgeting purposes at a project's conceptual or preliminary design stages. These estimates are more detailed, and thus require additional time and resources. Greater sophistication is used in developing semidetailed estimates than the rough-order type, and their accuracy is generally −15 to +20%.
Detailed estimates: Used during a project's detailed design and contract bidding phases. These estimates are made from detailed quantitative models, blueprints, product specification sheets, and vendor quotes. Detailed estimates involve the most time and resources to develop and thus are much more accurate than rough or semidetailed estimates. The accuracy of these estimates is generally −3 to +5%.

The upper limits of +60% for rough order, +20% for semidetailed, and +5% for detailed estimates are based on construction data for plants and infrastructure. Final costs for software, research and development, and new military weapons often have much higher corresponding percentages.

Accuracy of Estimate

In considering the three types of estimates it is important to recognize that each has its unique purpose, place, and function in a project's life cycle. Rough estimates are used for general feasibility activities and ranking possible projects; semidetailed estimates support budgeting and preliminary design decisions; and detailed estimates are used for establishing design details and contracts. As one moves from rough to detailed design estimates, one also moves from less to more accuracy. As a result “significant digits” become more important with detailed estimates as opposed to rough estimates. For example, at the feasibility phase of a large construction project one might estimate costs to the nearest million dollars when looking at several design decisions. However, when contracts are signed after detailed design they will be to the dollar.

When both costs and benefits are estimated for a decision situation one should balance the order of accuracy of each. One should not estimate costs to the nearest $100 while estimating benefits to the nearest $1000. Such an imbalance may skew a comparison of the true difference in costs and benefits for a proposed action.

Continuity in perspective in estimating costs and benefits is another important accuracy issue. We had previously mentioned that most people tend to underestimate costs and overestimate benefits in isolation. Care must be taken to balance one’s perspective to ensure a consistent approach to quantifying both costs and benefits.

Differences in degree of accuracy and cost-benefit perspective may result in an inaccurate analysis and thus favor one decision choice over another. Care must be taken from the outset to mitigate or eliminate these effects.

Cost Versus Accuracy Trade-off

Increasing the accuracy of an estimate is not a free thing—it requires added time and resources. Figure 2–8 illustrates the trade-off between accuracy and cost. In general, in engineering economic analysis, the resources spent must be justified by the need for detail in the estimate. From the figure we see that low accuracy estimates should have low costs, and high accuracy estimates will have higher costs. This relationship applies for cost estimates as well as estimating the benefits associated with a prospective choice. As a rule, we should not spend resources developing accuracy that is not warranted by the use and purpose of that estimate. For example, during the project feasibility phase we would not want to use our people, time, and money to develop detailed estimates for infeasible alternatives that will be quickly eliminated from further consideration.

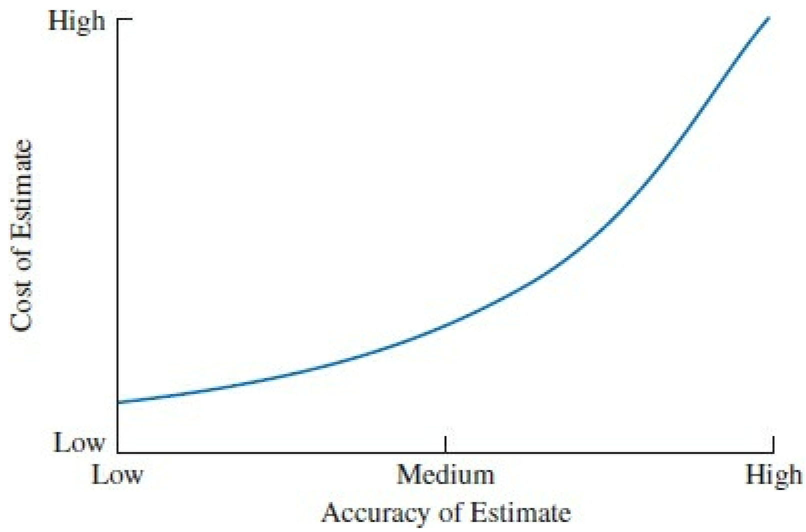


FIGURE 2-8 Accuracy versus cost trade-off in estimating costs and benefits.

Difficulties in Estimation

Estimating is difficult because the future is unknown. With few exceptions (such as with legal contracts) it is difficult to foresee future economic consequences exactly. In this section we discuss several aspects of estimating that make it a difficult task.

One-of-a-Kind Estimates

Estimated parameters can be for one-of-a-kind or first-run projects. The first time something is done, it is difficult to estimate both the costs required to design, produce, and maintain a product over its life cycle as well as the anticipated benefits. Consider the projected cost estimates that were developed for the first NASA missions. The U.S. space program initially had no experience with human flight in outer space; thus the development of the cost estimates for design, production, launch, and recovery of the astronauts, flight hardware, and payloads was a “first-time experience.” In addition, estimating the benefits of space exploration, such as advances in aircraft design, database management, surveying, water preservation, telemetry, and forest management (to name a few) were initially difficult to envision. The same is true for any endeavor lacking local or global historical cost data. New products or processes that are unique and fundamentally different make estimating costs difficult.

The good news is that there are very few one-of-a-kind estimates to be made in engineering design and analysis. Nearly all new technologies, products, and processes have “close cousins” that have led to their development. The concept of **estimation by analogy** allows one to use knowledge about well-understood activities to anticipate costs and benefits for new activities. In the 1950s, at the start of the military missile program, aircraft companies drew on their in-depth knowledge of designing and producing aircraft when they bid on missile contracts. As another example, consider the problem of estimating the production labor requirements for a brand new product, *X*. A company may use its labor knowledge about Product *Y*, a similar type of product, to build up the estimate for *X*. Thus, although “first-run” estimates are difficult to make, estimation by analogy can be an effective tool.

Time and Effort Available

Our ability to develop engineering estimates is constrained by time and person-power availability. In an ideal world, it would *cost nothing* to use *unlimited resources* over an *extended period of time*. However, reality requires the use of limited resources in fixed intervals of time. Thus for a rough estimate only limited effort is used.

Constraints on time and person-power can make the overall estimating task more difficult. If the estimate does not require as much detail (such as when a rough estimate is the goal), then time and personnel constraints may not be a factor. When detail is necessary and critical (such as in legal contracts), however, requirements must be anticipated and resource use planned.

Estimator Expertise

Consider two common expressions: *The past is our greatest teacher*, and *Knowledge is power*. These simple axioms hold true for much of what we encounter during life, and they are true in engineering estimating as well. The more experienced and knowledgeable the engineering estimator is, the less difficult the estimating process will be; the more accurate the estimate will be, the less likely it is that a major error will occur, and the more likely it is that the estimate will be of high quality.

How is experience acquired in industry? One approach is to assign inexperienced engineers relatively small jobs, to create expertise and build familiarity with products and processes. Another strategy used is to pair inexperienced engineers with mentors who have vast technical experience. Technical boards and review meetings conducted to “justify the numbers” also are used to build knowledge and experience. Finally, many firms maintain databases of their past estimates and the costs that were actually incurred.

ESTIMATING MODELS

This section develops several estimating models that can be used at the rough, semidetailed, or detailed design levels. For rough estimates the models are used with rough data; likewise for detailed design estimates they are used with detailed data.

Per-Unit Model

The **per-unit model** uses a “per unit” factor, such as cost per square foot, to develop the estimate desired. This is a very simplistic yet useful technique, and is often used as a first, rough estimate for industrial projects. The per-unit model is commonly used in the construction industry. As an example, you may be interested in a new home that is constructed with a certain type of material and has a specific construction style. Based on this information a contractor may quote a cost of $110 per square foot for your home. If you are interested in a 2000-square-foot floor plan, your cost would thus be: 2000 × 110 = $220,000. Other examples where per-unit factors are used for both costs and benefits include

- Service cost per customer
- Safety cost per employee

- Gasoline cost per mile
- Cost of defects per batch
- Maintenance cost per window
- Mileage cost per vehicle
- Utility cost per square foot of floor space
- Housing cost per student
- Sales per customer region
- Revenue per acre
- Fee per transaction
- Royalty per book
- Revenue per mile
- Quality improvement per training hour
- Rent per square footage
- Sales increase per representative

It is important to note that the per-unit model does not make allowances for economies of scale (the fact that higher quantities usually cost less on a per-unit basis). In most cases, however, the model can be effective at getting the decision maker "in the ballpark" of likely costs and benefits, and it can be very accurate if accurate data are used.

EXAMPLE 2-5

Gaber Land Corp. is evaluating a 4-acre waterfront property for development into rental condominiums. The front 2-acre lot is more expensive to purchase than the rear 2-acre lot, and condo leases closer to the waterfront can be more expensive than those units in the rear. Gaber is considering a design that includes a 32-unit building on each lot. Data includes the following:

Initial Costs

Lot purchase prices: $400,000/acre front lot, $100,000/acre back lot
Legal fees, applications, permits, etc.: $80,000
Site clearing and preparation: $3000/acre
Paving roadways, parking, curbs, and sidewalks: 25% of total lot at $40,000/acre

Construction costs: $3,000,000 per building

Recurring Costs

Taxes and insurance: $5000/month per building
Landscaping: 25% of lot at $1000/acre/month
Security: $1000/building + $1500/month
Other costs: $2000/month

Revenue (assume 90% annual occupancy)

Front lot units: $2500/unit/month
Rear lot units: $1750/unit/month
Other revenue: $5000/month

Answer the following: (1) Use the concept of the per-unit model to estimate the total initial cost, annual cost, and annual revenue of this prospective project, and (2) If you made the simplifying assumption of no changes to costs and revenues for 10 years, estimate the profitability of this prospective investment ignoring the effects of money's value over time.

SOLUTION

(1) Using the per unit model:

Total Initial Cost		
Purchase price: (400,000 × 2) + (100,000 × 2)	=	$1,000,000
Legal costs	=	80,000
Site clearing & preparation: 3000 × 4	=	12,000
Roadways, etc.: (.25 × 4) × 40,000	=	40,000
Construction: 3,000,000 × 2	=	6,000,000
		$7,132,000

Annual Cost		
Taxes and insurance: 10,000 × 12	=	$120,000
Landscaping: (.25 × 4) × 1000 × 12	=	12,000

Security: (1000 × 2) + (1500 × 12)	=	20,000
Other costs: 2000 × 12	=	24,000
		$176,000

Annual Revenue		
Front lot leases: (32 × 2500 × 12) × .90	=	$864,000
Rear lot leases: (32 × 1750 × 12) × .90	=	604,800
Other revenue: 5000 × 12	=	60,000
		$1,528,800

(2) Using the fundamental relationship that Net Profit = Revenue – Costs:

$$\begin{aligned} \text{Net Profit} &= [1{,}528{,}800 \times 10] - [7{,}132{,}000 + (176{,}000 \times 10)] \\ &= \$6{,}396{,}000 \end{aligned}$$

Segmenting Model

The **segmenting model** can be described as "divide and conquer." An estimate is decomposed into its individual components, estimates are made at those lower levels, and then the estimates are aggregated (added) back together. It is much easier to estimate at the lower levels because they are more readily understood. This approach is common in engineering estimating in many applications and for any level of accuracy needed. In estimating costs for the condominiums in Example 2–5, the estimate was segmented into the initial and monthly costs, and monthly revenues. The example illustrated the segmenting model (division of the overall estimate into various categories and activities, such as costs and benefits) together with the unit factor model to make the subestimates for each category. Example 2–6 provides another example of the segmenting approach.

EXAMPLE 2-6

Clean Lawn Corp., a manufacturer of yard equipment, is planning to introduce a new high-end industrial-use lawn mower called the Grass

Grabber. The Grass Grabber is designed as a walk-behind, self-propelled mower. Clean Lawn engineers have been asked by the accounting department to estimate the material costs for the new mower. The material cost estimate will be used, along with estimates for labor and overhead to evaluate the potential of this new model.

SOLUTION

The engineers decide to decompose the design specifications for the Grass Grabber into its components, estimate the material costs for each of the components, and then sum these costs up to obtain the overall estimate. The engineers are using a segmenting approach to build up their estimate. After careful consideration, the engineers have divided the mower into the following major subsystems: chassis, drive train, controls, and cutting/collection system. Each of these was further divided as appropriate, and unit material costs were estimated at this lowest of levels as follows:

Cost Item	Unit Material Cost Estimate
A. Chassis	
A.1 Deck	$ 7.40
A.2 Wheels	10.20
A.3 Axles	4.85
	$22.45
B. Drive train	
B.1 Engine	$38.50
B.2 Starter assembly	5.90
B.3 Transmission	5.45

B.4 Drive disc assembly	10.00
B.5 Clutch linkage	5.15
B.6 Belt assemblies	7.70
	$72.70
C. Controls	
C.1 Handle assembly	$ 3.85
C.2 Engine linkage	8.55
C.3 Blade linkage	4.70
C.4 Speed control linkage	21.50
C.5 Drive control assembly	6.70
C.6 Cutting height adjuster	7.40
D. Cutting/Collection system	
D.1 Blade assembly	$10.80
D.2 Side chute	7.05
D.3 Grass bag and adapter	7.75
	$25.60

The total material cost estimate of $173.45 was calculated by summing up the estimates for each of the four major subsystem levels (chassis, drive train, controls, and cutting/collection system). It should be noted that this cost represents only the material portion of the overall cost to produce the mowers. Other costs would include labor and overhead items.

In Example 2–6 the engineers at Clean Lawn Corp. decomposed the cost estimation problem into logical elements. The scheme they used of decomposing cost items and numbering the material components (A.1, A.1, A.2, etc.) is known as a **work breakdown structure.** This technique is commonly used in engineering cost estimating and project management of large products, processes, or projects. A work breakdown structure decomposes a large “work package” into its constituent parts, which can then be estimated or managed individually. In Example 2–6 the work breakdown structure of the Grass Grabber has three levels. At the top level is the product itself, at the second level are the four major subsystems, and at the third level are the individual cost items. Imagine what the product work breakdown structure for a modern commercial airliner looks like. Then imagine trying to manage an aircraft's design, engineering, construction, and costing without a tool like the work breakdown structure.

Cost and Price Indexes

Indexes are dimensionless numerical values that reflect historical change in engineering (and other) costs and prices. They can reflect relative cost/price change in either individual items (labor, materials, utilities) or groups of costs (consumer costs, producers' costs). Single item indexes are called *commodity specific indexes* and those that group costs or prices are called composite indexes. Equation 2–2 gives the formulation for how indexes can be used to update historical values—where the ratio of the cost or price index numbers at two points in time (*A* and *B*) is equivalent to the dollar cost ratio of the item at the same time.

$$\frac{\text{Cost or price at time } A}{\text{Cost or price at time } B} = \frac{\text{Index value at time } A}{\text{Index value at time } B} \quad (2\text{-}2)$$

EXAMPLE 2-7

Miriam is interested in estimating the annual labor and material costs for a new production facility. She was able to obtain the following cost data:

Labor Costs

• Labor cost index value was at 124 ten years ago and is 188 today.
• Annual labor costs for a similar facility were $575,500 ten years ago.

Material Costs

• Material cost index value was at 544 three years ago and is 715 today.
• Annual material costs for a similar facility were $2,455,000 three years ago.

SOLUTION

Miriam will use Equation 2–2 to develop her cost estimates for annual labor and material costs.

Labor

$$\frac{\text{Annual cost today}}{\text{Annual cost 10 years ago}} = \frac{\text{Index value today}}{\text{Index value 10 years ago}} = \frac{188}{124}$$

$$\text{Annual cost today} = \$2{,}455{,}000 \times \frac{188}{124} = \$3{,}227{,}000$$

Materials

$$\frac{\text{Annual cost today}}{\text{Annual cost 3 years ago}} = \frac{\text{Index value today}}{\text{Index value 3 years ago}} = \frac{715}{544}$$

$$\text{Annual cost today} = \$575{,}500 \times \frac{715}{544} = \$872{,}500$$

Cost and price index data are collected and published by several private and public sources in the U.S. (and world). The U.S. government publishes data through the Bureau of Labor Statistics of the Department of Commerce. The *Statistical Abstract of the United States* publishes cost indexes for labor, construction, and materials. Another useful source for engineering cost price index data is the *Engineering News Record*.

Power-Sizing Model

The **power-sizing model** is used to estimate the costs of industrial plants and equipment. The model “scales up” or “scales down” known costs, thereby accounting for economies of scale that are common in industrial plant and equipment costs. Consider the cost to build a refinery. Would it cost twice as much to build the same facility with double the capacity? It is unlikely. The *power-sizing model* uses the exponent (x), called the *power-sizing exponent*, to reflect economies of scale in the size or capacity:

$$\frac{\text{Cost of equipment } A}{\text{Cost of equipment } B} = \left(\frac{\text{Size (capacity) of equipment } A}{\text{Size (capacity) of } B}\right)^x \quad (2\text{-}3)$$

where x is the power-sizing exponent, costs of A and B are at the same point in time (same dollar basis), and size or capacity is in the same physical units for both A and B.

The power-sizing exponent (x) has an average value of 0.6, which is used if the exponent is not known. For this reason, the power-sizing model is also known as the six-tenths rule. Most values will be below 1.0, demonstrating economies of scale as sizes or capacities increase. If the value is 1.0, then there is a linear cost-versus-size relationship. Values above 1.0 demonstrate *dis*economies of scale, which often occur when sizes become very large or very small. The value of the exponent is rarely above 1.2.

Table 2-1 Example Power-Sizing Exponent Values

Equipment	Size Range	Power-Sizing Exponent
Agitators	1–200 kW	0.7
Blower, centrifugal	50–80,000 kW	0.9
Centrifugal pump	0.01–270 kW	0.3
Compressor, reciprocating	10–2000 kW	1.0
Crushers	10–1000 kg/s	0.8
Drum dryers	2–100 m^2	0.6
Electric motors	10–8000 kW	0.9
Electrostatic precipitators	5–1200 m^3/s	0.8
Fan, centrifugal	10^3–10^4 ft^3/min	0.4
Fan, centrifugal	$2 \times 10^4 - 7 \times 10^4$ ft^3/min	1.2
Filters, plate and frame	1–170 m^2	0.75
Jacketed vessels	1–800 m^3	0.6
Tanks, floating roof	200–70,000 m^3	0.6

Source: National Council of Examiners for Engineering and Surveying (NCEES), *FE Reference Handbook*, chemical engineering section; Ulrich, Gael D., and Palligarnai T. Vasudevan, "Capital Costs Quickly Calculated," *Chemical Engineering*, Vol. 116, No. 4, 2009, pp. 46–52.

Current values for plants and equipment can be difficult to find in the literature. The power-sizing model is usually used for initial estimates, and should not be used for detailed cost forecasts. Table 2–1 provides power-sizing exponent values for several types of equipment. The exponent given only applies to the size range specified.

In Equation 2–3 equipment costs for both *A* and *B* occur at the same point in time. This equation is useful for scaling equipment costs but *not* for updating those costs. When the time of the desired cost estimate is different from the time in which the scaling occurs (per Equation 2–3) cost indexes accomplish the time updating. Thus, in cases like Example 2–8 involving both scaling and updating, we use the power-sizing model together with cost indexes.

EXAMPLE 2-8

Based on her work in Example 2–7, Miriam has been asked to estimate the cost today of a 2500-ft^2 heat exchange system for the new plant being analyzed. She has the following data.

• Her company paid $50,000 for a 1000-$ft^2$ heat exchanger 5 years ago.
• Heat exchangers within this range of capacity have a power-sizing exponent (x) of 0.55.
• Five years ago the Heat Exchanger Cost Index (HECI) was 1306; it is 1487 today.

SOLUTION

Miriam will first use Equation 2–3 to scale up the cost of the 1000-ft^2 exchanger to one that is 2500 ft^2 using the 0.55 power-sizing exponent.

$$\frac{\text{Cost of 2500-ft}^2\text{ equipment}}{\text{Cost of 1000-ft}^2\text{ equipment}} = \left(\frac{\text{2500-ft}^2\text{ equipment}}{\text{1000-ft}^2\text{ equipment}}\right)^{0.55}$$

$$\text{Cost of 2500-ft}^2\text{ equipment} = \left(\frac{2500}{1000}\right)^{0.55} \times 50{,}000 = \$82{,}800$$

Miriam knows that the \$82,800 reflects only the scaling up of the cost of the 1000-ft^2 model to a 2500-ft^2 model. Now she will use Equation 2–2 and the HECI data to estimate the cost of a 2500-ft^2 exchanger today. Miriam's cost estimate would be

$$\frac{\text{Equipment cost today}}{\text{Equipment cost 5 years ago}} = \frac{\text{Index value today}}{\text{Index value 5 years ago}}$$

$$\text{Equipment cost today} = \frac{1487}{1306} \times \$82{,}800 = \$94{,}300$$

Triangulation

Triangulation is used in engineering surveying. A geographical area is divided into triangles from which the surveyor is able to map points within that region by using three fixed points and horizontal angular distances to locate fixed points of interest (e.g., property line reference points). Since any point can be located with two lines, the third line represents an extra perspective and check. We will not use trigonometry to arrive at our cost and benefit estimates, but we can use the concept of triangulation. We should approach our economic estimate from different perspectives because such varied perspectives add richness, confidence, and quality to the estimate. **Triangulation** in estimating costs and benefits might involve using multiple sources of data or multiple quantitative models. As decision makers, we should always seek out varied perspectives.

Improvement and the Learning Curve

One common phenomenon observed, regardless of the task being performed, is that as the number of repetitions increases, performance becomes faster and more accurate. This is the concept of learning and improvement in the activities that people perform. From our own experience we all know that our fiftieth repetition is done in much less time than we needed to do the task the first time.

The **learning curve** captures the relationship between task performance and task repetition. In general, as output *doubles*, the unit production time will be reduced to some fixed percentage, the **learning-curve percentage** or **learning-curve rate.** For example, it may take 300 minutes to produce the third unit in a production run involving a task with a 95% learning time curve. In this case the sixth unit doubles the output, so it will take 300(0.95) = 285 minutes to produce. Sometimes the learning curve is also known as the progress curve, improvement curve, experience curve, or manufacturing progress function.

Equation 2–4 gives an expression that can be used for time estimating for repetitive tasks.

$$T_N = T_{\text{initial}} \times N^b \quad \text{(2-4)}$$

where T_N = time required for the N^{th} unit of production

T_{initial} = time required for the first (initial) unit of production

N = number of completed units (cumulative production)

b = learning-curve exponent (slope of the learning curve on a log–log plot)

As just given, a learning curve is often referred to by its percentage learning slope. Thus, a curve with $b = -0.074$ is a 95% learning curve because $2^{-0.074} = 0.95$. This equation uses 2 because the learning-curve percentage applies for doubling cumulative production. The learning-curve exponent is calculated by using Equation 2–5.

$$b = \frac{\log(\text{learning curve expressed as a decimal})}{\log 2} \quad \text{(2-5)}$$

EXAMPLE 2-9

Calculate the time required to produce the hundredth unit of a production run if the first unit took 32.0 minutes to produce and the learning-curve rate for production is 80%.

SOLUTION

Using Equation 2–4, we write

$$T_{100} = T_1 \times 100^b$$
$$T_{100} = T_1 \times 100^{\log 0.80/\log 2.0}$$
$$T_{100} = 32.0 \times 100^{-0.3219}$$
$$T_{100} = 7.27 \text{ minutes}$$

It is particularly important to account for the learning-curve effect if the production run involves a small number of units instead of a large number. When thousands or even millions of units are being produced, early inefficiencies tend to be "averaged out" because of the larger batch sizes. However, in the short run, inefficiencies of the same magnitude can lead to rather poor estimates of production time requirements, and thus production cost estimates may be understated. Consider Example 2–10 and the results that might be observed if the learning-curve effect is ignored. Notice in this example that a "steady-state" time is given. Steady state is the time at which the physical constraints of performing the task prevent the achievement of any more learning or improvement.

EXAMPLE 2-10

Green Energy Inc., a clean energy equipment manufacturer, is responding to a *request for bids* to produce 20 2.0 MW wind turbines for a new wind farm planned in a coastal municipality. From previous projects, they assembled the following data. Estimate the manufacturing labor cost to include in the bid.

• The learning curve rate for labor is 85%.
• Steady-state manufacturing will be reached with the 16th unit.
• The steady-state production rate per unit is 100 hours, with 15 workers per unit.
• The average labor rate with benefits is $25 per hour.

SOLUTION

From the learning-curve rate we can calculate the value of the learning-curve exponent, *b*. Then from the time required to produce the 16th unit, we can calculate the time required to produce the first unit.

$$b = \log 0.85 / \log 2 = -0.2345$$

$$T_{16} = T_1 \times 16^{-0.2345}$$

$$100 = T_1 \times 16^{-0.2345}$$

$$T_1 = 100 \times 16^{0.2345} = 191.6 \text{ hours}$$

Now that we know the two parameters for Equation 2.4, it is easier to use a spreadsheet for the rest of our calculations and to plot the data.

	A	B	C	D	E
1	–0.2345	learning curve rate	Unit	Time to	Cum. Time
2	191.6	time for first unit	(N)	Produce Nth	1 to N
3			1	191.6	191.6
4	100	steady state time	2	162.8	354.4
5	16	steady state unit	3	148.1	502.5
6			4	138.4	640.9
7	20	# units	5	131.4	772.3
8	15	workers per unit	6	125.9	898.1
9	25	labor rate ($/hr)	7	121.4	1019.5
10			8	117.6	1137.2
11	=$A$2*C11^$A$1		9	114.4	1251.6
12			10	111.7	1363.3
13			11	109.2	1472.5
14	Cost if steady state assumed		12	107.0	1579.4
15	$ 750,000	=A4*A7*A8*A9	13	105.0	1684.4
16			14	103.2	1787.6
17			15	101.5	1889.1
18			16	100.0	1989.1
19			17	100	2089.1
20			18	100	2189.1
21	Cost of 20 units		19	100	2289.1
22	$ 895,927	=A9*A8*E22	20	100	2389.1
23					
24	19.5%	increase due to learning curve			

From the spreadsheet the total labor cost estimate would have been underestimated by 19.5% had Green Energy not included learning-curve effects in the estimate. The underestimate would have lowered their bid and increased the chance that they would win the project. Had they won the bid this would have affected the project's profitability and with a 20% error, they would probably have lost money on the project.

Figures 2–9 and 2–10 illustrate learning curves using the Green Energy data in columns C and D of the spreadsheet. When plotted on a linear scale the time per unit decreases as a declining rate. When plotted on a *log-log* scale, the relationship is a straight line through the 16th unit, when a steady state is reached. The straight line is because the 2nd, 4th, 8th, and 16th units have a production time that is 85% of the 1st, 2nd, 4th, and 8th units respectively.

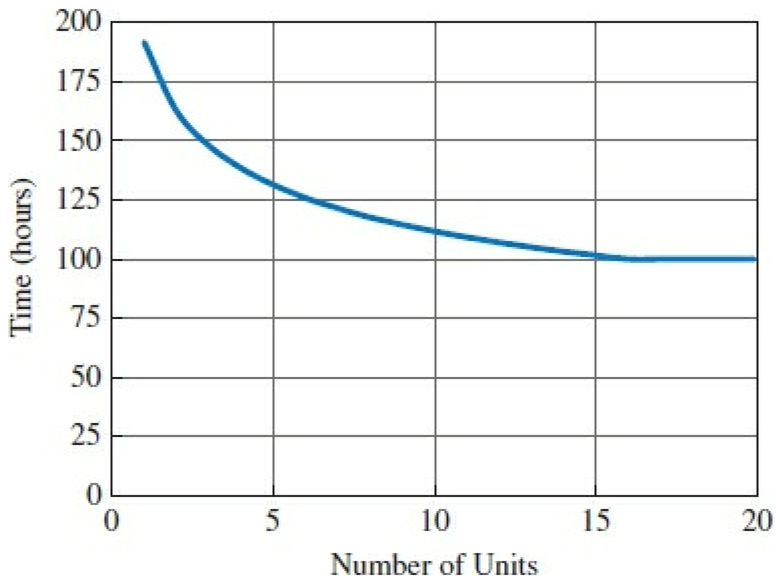


FIGURE 2-9 Learning curve of time vs. number of units.

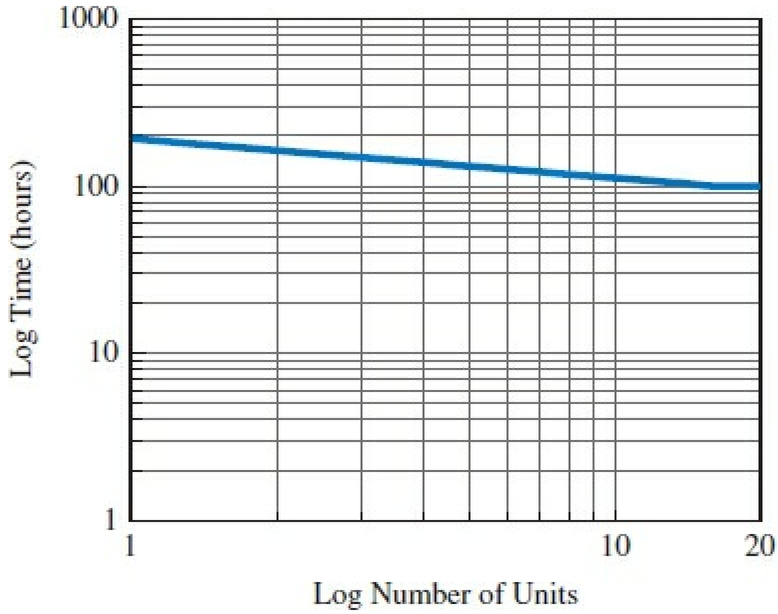


FIGURE 2-10 Learning curve on log–log scale.

CASH FLOW DIAGRAMS

The costs and benefits of engineering projects occur over time and are summarized on a cash flow diagram (CFD). Specifically, a CFD illustrates the size, sign, and timing of individual cash flows. In this way the CFD is the basis for engineering economic analysis.

A **cash flow diagram** is created by first drawing a segmented time-based horizontal line, divided into time units. The time units on the CFD can be years, months, quarters, or any other consistent time unit. Then at each time at which a cash flow will occur, a vertical arrow is added—pointing down for costs and up for revenues or benefits. These cash flows are drawn to scale.

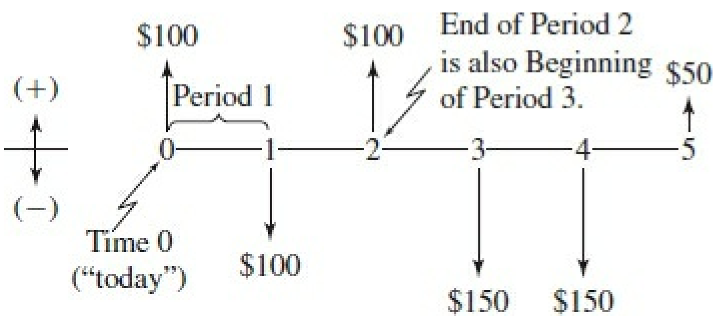


FIGURE 2-11 An example cash flow diagram (CFD).

Unless otherwise stated, cash flows are **assumed** to occur at time 0 or at the **end** of each period. Consider Figure 2–11, the CFD for a specific investment opportunity whose cash flows are described as follows:

Timing of Cash Flow	Size of Cash Flow
At time zero (now or today)	A positive cash flow of $100
1 time period from today	A negative cash flow of $100
2 time periods from today	A positive cash flow of $100
3 time periods from today	A negative cash flow of $150
4 time periods from today	A negative cash flow of $150
5 time periods from today	A positive cash flow of $50

Categories of Cash Flows

The expenses and receipts due to engineering projects usually fall into one of the following categories:

First cost ≡ expense to build or to buy and install
Operating and maintenance (O&M) ≡ annual expense, such as electricity, labor, and minor repairs
Salvage value ≡ receipt at project termination for sale or transfer of the equipment (can be a salvage cost)
Revenues ≡ annual receipts due to sale of products or services
Overhaul ≡ major capital expenditure that occurs during the asset's life

Individual projects will often have specific costs, revenues, or user benefits. For example, annual operating and maintenance (O&M) expenses on an assembly line might be divided into direct labor, power, and other. Similarly, a public-sector dam project might have its annual benefits divided into flood control, agricultural irrigation, and recreation.

Drawing a Cash Flow Diagram

The cash flow diagram shows when all cash flows occur. Look at Figure 2–11 and the $100 positive cash flow at the end of period 2. From the time line one can see that this cash flow can also be described as occurring at the *beginning* of period 3. Thus, in a CFD the end of *period t* is the same time as the beginning of *period t* + 1. Beginning-of-period cash flows (such as rent, lease, and insurance payments) are thus easy to handle: just draw your CFD and put them in where they occur. Thus O&M, salvages, revenues, and overhauls are assumed to be end-of-period cash flows.

The choice of time 0 is arbitrary. For example, it can be when a project is analyzed, when funding is approved, or when construction begins. When construction periods are assumed to be short, first costs are assumed to occur at time 0, and the first annual revenues and costs start at the end of the first period. When construction periods are long, time 0 is usually the date of commissioning—when the facility comes on stream.

Perspective is also important when one is drawing a CFD. Consider the simple transaction of paying $5000 for some equipment. To the firm buying the equipment, the cash flow is a cost and hence negative in sign. To the firm selling the equipment, the cash flow is a revenue and positive in sign. This simple example shows that a consistent perspective is required when one is using a CFD to model the cash flows of a problem. One person’s cash outflow is another person’s inflow.

Often two or more cash flows occur in the same year, such as an overhaul and an O&M expense or the salvage value and the last year’s O&M expense. Combining these into one total cash flow per year would simplify the cash flow diagram. However, it is better to show each individually, to ensure a clear connection from the problem statement to each cash flow in the diagram.

	A	B	C	D	E	F
1	Year	Capital Costs	O&M	Overhaul		
2	0	−80,000				
3	1		−12,000			
4	2		−12,000			
5	3		−12,000	−25,000		
6	4		−12,000			
7	5		−12,000			
8	6	10,000	−12,000			
9						

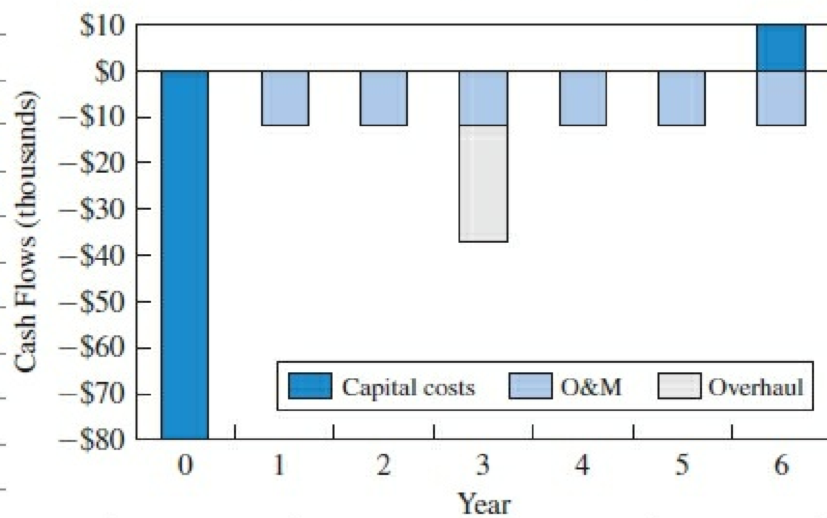


FIGURE 2-12 Example of cash flow diagram in spreadsheets.

Drawing Cash Flow Diagrams with a Spreadsheet

One simple way to draw cash flow diagrams with "arrows" proportional to the size of the cash flows is to use a spreadsheet to draw a stacked bar chart. The data for the cash flows are entered, as shown in the table part of Figure 2–12. To make a quick graph, select cells A1 to D8, which are the years and the three columns of the cash flows. Then select insert and chart, choose column chart, and select the stack option. Except for choosing the scale for the *y* axis, choosing the *y*-axis value for the *x*-axis, and adding titles, the cash flow diagram is done. Refer to the appendix for a review of basic spreadsheet use. (*Note*: A bar chart labels periods rather than using an *x* axis with arrows at times 0, 1, 2, …)

SUMMARY

This chapter has introduced several concepts and definitions for estimating the costs and benefits of proposed engineering projects. Covered were several cost concepts: fixed and variable, marginal and average, sunk, opportunity, recurring and nonrecurring, incremental, cash and book, and life-cycle. **Fixed costs** are constant and unchanging as volumes change, while **variable costs** change as output changes. Fixed and variable costs are used to find a breakeven value between costs and revenues, as well as the regions of net profit and loss. A **marginal cost** is for one more unit, while the **average cost** is the total cost divided by the number of units.

Sunk costs result from past decisions; they should not influence our attitude toward current and future opportunities. Remember, "sunk costs are sunk." **Opportunity costs** involve the benefit that is forgone when we choose to use a resource in one activity instead of another. **Recurring costs** can be planned and anticipated expenses; **nonrecurring costs** are one-of-a-kind costs that are often more difficult to anticipate.

Incremental costs are economic consequences associated with the differences between two choices of action. **Cash costs** are also known as **out-of-pocket costs** that represent actual cash flows. **Book costs** do not result in the exchange of money, but rather are costs listed in a firm's accounting books. **Life-cycle costs** are all costs that are incurred over the life of a product, process, or service. Thus engineering designers must consider life-cycle cash flows when choosing materials and components, tolerances, processes, testing, safety, service and warranty, and disposal. Historically, firms often focused on internal costs, but green engineering and its consideration of external costs have become increasingly influential.

Cost estimating is the process of "developing the numbers" for engineering economic analysis. Unlike a textbook, the real world does not present its challenges with neat problem statements that provide all the data. **Rough estimates** give us order-of-magnitude numbers and are useful for high-level and initial planning as well as judging the feasibility of alternatives. **Semidetailed estimates** are more accurate than rough-order estimates, thus requiring more resources (people, time, and money) to develop. These estimates are used in preliminary design and budgeting. **Detailed estimates** generally have an accuracy of −3% to 5%. They are used during the detailed design and contract bidding phases of a project.

Difficulties are common in developing estimates. **One-of-a-kind estimates** will have no basis in earlier work, but this disadvantage can be addressed through **estimation by analogy.** Lack of time is best addressed by planning and by matching the estimate's detail to the purpose—one should not spend money developing a detailed estimate when only a rough estimate is needed. **Estimator expertise** must be developed through work experiences and mentors.

Several general models and techniques for developing cost and benefit estimates were discussed. The **per-unit** and **segmenting models** use different levels of detail and costs and benefits per square foot or other unit. **Cost index data** are useful for updating historical costs to formulate current estimates. The **power-sizing model** is useful for scaling up or down a known cost quantity to account for economies of scale, with different power-sizing exponents for industrial plants and equipment of different types. **Triangulation** suggests that one should seek varying perspectives when developing estimates of a project's costs and benefits. Different information sources, databases, and analytical models can all be used to create unique perspectives. As the number of task repetitions increases, efficiency improves because of learning or improvement. This is summarized in the **learning-curve percentage,** where doubling the cumulative production reduces the time to complete the task, which equals the learning-curve percentage times the current production time.

Cash flow estimation must include project benefits. These include labor cost savings, avoided quality costs, direct revenue from sales, reduced catastrophic risks, improved traffic flow, and cheaper power supplies. Benefits are frequently overestimated resulting in an “optimist’s bias.” **Cash flow diagrams** are used to model the positive and negative cash flows of potential investment opportunities. These diagrams provide a consistent view of the problem (and the alternatives) to support economic analysis.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

2-1	DC Brick has fixed cost of production of $2,562,500 per year and a unit production cost of $0.75. Marvel Brick has a fixed cost of production of $1,000,000 a unit production cost of $2.00. At what number of units produced will the two companies have equal annual production costs?

SOLUTION

2-2	A household is considering installing solar panels, but they need to understand what they are paying for electricity. They use an average of 15 kilowatt hours of electricity per day, and an average month is 30 days. They pay $0.22 per kilowatt hour for the first 350 kilowatt hours used in a month, and $0.19 per kilowatt hour after that.

a. What is their average cost of electricity?
b. What is their marginal cost of electricity?

SOLUTION

2-3 Your Econ. Professor bought airline tickets to leave a conference early on Saturday morning. He later found out that he needed to present a paper later that Saturday, so he changed his tickets at a cost of $200. When he got to the conference, he discovered that he was scheduled twice: on Friday and on Saturday. He was given his choice of when to give the presentation. He said he would present on Saturday, because he paid an extra $200 in order to stay. Explain why you agree or disagree with his logic.

SOLUTION

2-4 LED lights are replacing incandescent lighting in many applications, including in industrial application. LED lights cost more (but prices continue to decrease), last longer, and consume less electricity. Identify fixed and variable costs that need to be considered when doing an economic analysis.

SOLUTION

2-5 You need to repair some vertical siding on your house. Each piece of siding is milled to overlap with others, and is 12 inches wide by 8 feet long. You need to replace a section that is 6 feet wide. Each milled board costs $15.50. You also need a gallon of paint ($35.00), sandpaper ($5.50), and a new paint brush ($9.95). Your labor is free. What will this project cost? If you need to replace an additional foot, how much more will it cost?

SOLUTION

2-6 Construction on your custom home has just been completed and the yard must be landscaped. The landscape contractor has estimated that 8 cubic yards of dirt and 3 cubic yards of sand will be required to “level” the yard and provide proper drainage. Dirt is priced at $35 per cubic yard and sand is priced at $27 per cubic yard. She has measured the yard and calculated the area requiring sod to be 3600 ft^2. Sod is sold in rolls that measure 3' × 30' at a cost of $75 per roll. Shrubbery installation will cost an additional $2500. Estimate the cost of the landscaping.

SOLUTION

2-7 The following data concern one of the most popular products of XYZ Manufacturing. Estimate the selling price per unit.

Labor	= 12.8 hours at $18.75/hour
Factory overhead	= 92% of labor
Material costs	= $65.10
Packing cost	= 10% of materials
Sales commission	= 10% of selling price
Profit	= 22% of selling price

SOLUTION

2-8 A tablet computer is being designed with the following estimated costs

Material costs	$16.50
Labor costs	6.00
Overheads	15.00

What wholesale price needs to be charged if they want to make a gross profit of 40%?

SOLUTION

2-9 A new 21-kW power substation was built in 2016 in Gibson County for $1.4 million. Weakley County, which is nearby, is planning to build a similar though smaller (18-kW) substation in 2019. The inflation rate has averaged 1.5% per year. If the power sizing exponent is .85 for this type of facility what is the estimated cost of construction of the Weakley County substation?

SOLUTION

American Petroleum (AP) recently completed construction on a large

2-10 refinery in Texas. The final construction cost was $27,500,000. The refinery covers a total of 340 acres. AP is currently working on plans for a new refinery for Oklahoma. The anticipated size is approximately 260 acres. If the power-sizing exponent for this type of facility is .67, what is the estimated cost of construction?

SOLUTION

2-11 The time required to produce the first gizmo is 1500 blips. Determine the time required to produce the 450th gizmo if the learning-curve coefficient is .85.

SOLUTION

2-12 A new training program at Arid Industries is intended to lower the learning-curve coefficient of a certain molding operation that currently costs $95.50/hour. The current coefficient is .87, and the program hopes to lower the coefficient by 10%. Assume that the time to mold the first product is 8 hours. If the program is successful, what cost savings will be realized when the 2000th piece is produced?

SOLUTION

2-13 Four operations are required to produce a certain product produced by ABC Manufacturing. Use the following information to determine the labor cost of producing the 1000th piece.

	Time Required for 1st Piece	Learning-Curve Coefficient	Labor Cost per Hour
Operation 1	1 hour, 15 minutes	.90	$ 8.50
Operation 2	2 hours	.82	12.00
Operation 3	2 hours, 45 minutes	.98	7.75

Operation 4	4 hours, 10 minutes	.74	10.50

SOLUTION

2-14 Draw a cash flow diagram for the following end-of-period cash flows.

EOP	Cash Flow
0	–$1000
1	200
2	–100
3	300
4	400
5	–400
6	500

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Fixed, Variable, Average, and Marginal Costs

2-1 G A New York renewable energy company pays \$0.18 per kilowatt-hour (kWh) for the first 10,000 units of electricity each month and \$0.15/kWh for all remaining units. If a firm uses 25,000 kWh/month, what is its average and marginal cost?

2-2 One of your firm's suppliers discounts prices for larger quantities. The first 1000 parts are \$15 each. The next 1500 are \$13 each. All parts in excess of 2500 cost \$11 each. What are the average cost and marginal cost per part for the following quantities?

A (*a*) 650
(*b*) 1250
(*c*) 2500
(*d*) 3500

2-3 A new machine comes with 200 free service hours over the first year. Additional time costs $180 per hour. What are the average and marginal costs per hour for the following quantities?
(*a*) 125
(*b*) 225
(*c*) 325

2-4 A Venus Robotics can produce 25,000 robots a year on its daytime shift. The fixed manufacturing costs per year are $2.3 million and the total labor cost is $9.5 million. To increase its production to 50,000 robots per year, Venus is considering adding a second shift. The unit labor cost for the second shift would be 15% higher than the day shift, but the total fixed manufacturing costs would increase only to $2.8 million from $2.3 million.
(*a*) Compute the unit manufacturing cost for the daytime shift.
(*b*) Would adding a second shift increase or decrease the unit manufacturing cost at the plant?

2-5 A labor-intensive process has a fixed cost of $338,000 and a variable cost of $143 per unit. A capital-intensive (automated) process for the same product has a fixed cost of $1,244,000 and a variable cost of $92.50 per unit. How many units must be produced and sold at $197 each for the automated process to be preferred to the labor-intensive process?
Contributed by Paul R. McCright, University of South Florida

2-6 A G CleanTech manufactures equipment to mitigate the environmental effects of waste.
(*a*) If Product *A* has fixed expenses of $15,000 per year and each unit of product has a $0.20 variable cost, and Product *B* has fixed expenses of $5000 per year and a $0.50 variable cost, at what number of units of annual production will *A* have the same overall cost as *B*?
(*b*) As a manager at CleanTech what other data would you need to evaluate these two products?

Heinrich is a manufacturing engineer with the Miller Company. He

has determined the costs of producing a new product to be as follows:
Equipment cost: $288,000/year
Equipment salvage value at EOY5 = $41,000
2-7 Variable cost per unit of production: $14.55
Overhead cost per year: $48,300
If the Miller Company uses a 5-year planning horizon and the product can be sold for a unit price of $39.75, how many units must be produced and sold each year to break even? *Contributed by Paul R. McCright, University of South Florida*

2-8 A

An assembly line can produce 80 units per hour. The line's hourly cost is $4250 on straight time (the first 8 hours). Workers are guaranteed a minimum of 5 hours. There is a 35% premium for overtime, however, productivity for overtime drops by 10%. What are the average and marginal costs per unit for the following daily quantities?
(*a*) 350
(*b*) 500
(*c*) 640
(*d*) 850

2-9

Christine Lynn travels from her home to a remote island. Her trip involves: car travel of 250 miles, air travel of 400 miles, and a boat ride of 75 miles. She is interested in calculating the average fuel cost per mile (per person) of her trip. Assume the fuel efficiency of car, air, and boat travel is 20, 0.20, and 2 miles per gallon, respectively, and that fuel cost per gallon is equal for all and is $3.00/gallon. She was alone in the car and among 180 people on the plane and 15 on the boat. What is the average fuel cost per mile, per person?

2-10 A

This month your vendor invoiced $31,500 in testing charges for your production run. The unit cost for testing is twice as much for each of the first 400 units per month as compared to each unit over 400. If we shipped 500 units to the vendor this month, find:
(*a*) Average cost per unit;
(*b*) Cost per unit below the price break point;
(*c*) Marginal cost for the 650th unit.

The Country Fields Retirement Community charges $6000/month for a single senior citizen to reside in an efficiency apartment with assisted living care. The facility has operating expenses of $600,000 per month. Staffing levels are dependent on the number of residents. Each senior who enters the community requires additional food, personal care, and support staff time. The estimated cost for each person is $4000 per person per month.

2-11

(*a*) How many senior citizen residents does the facility need to have in order to reach the breakeven point?

(*b*) What is the company's annual profit or loss if they maintain an average residency level of 350 senior citizens?

Contributed by Gillian Nicholls, Southeast Missouri State University

2-12 A

The Ozzie Chocolate Company is preparing to offer a new product in its candy offerings, the Minty Dark Chocolate Bite bar. Material costs per new candy bar are $0.25 for chocolate, $0.02 for sugar, and $0.03 for mint flavoring. Labor costs of the new product are approximately $0.15 per bar. Adding a production line devoted to the new candy will cost $250,000 per year.

(*a*) If the sales price is $1.40 per candy bar, how many must the company make per year in order to break even? Assume that each bar made is sold at full price.

(*b*) What is the company's profit or loss if they make and sell 270,000 candy bars at the $1.40 price in the first year?

(*c*) About 20% of the food consumed in the U.S. is imported. Production in many industries has been offshored. What ethical issues do companies face when presented with the decision to move operations?

Contributed by Gillian Nicholls, Southeast Missouri State University

2-13

A small machine shop, with 30 hp of connected load, purchases electricity at the following monthly rates (assume any demand charge is included in this schedule) per hp of connected load:

First 50 kWh at 12.6¢ per kWh
Next 50 kWh at 10.6¢ per kWh
Next 150 kWh at 9.0¢ per kWh
Over 250 kWh at 7.7¢ per kWh

The shop uses 2800 kWh per month.

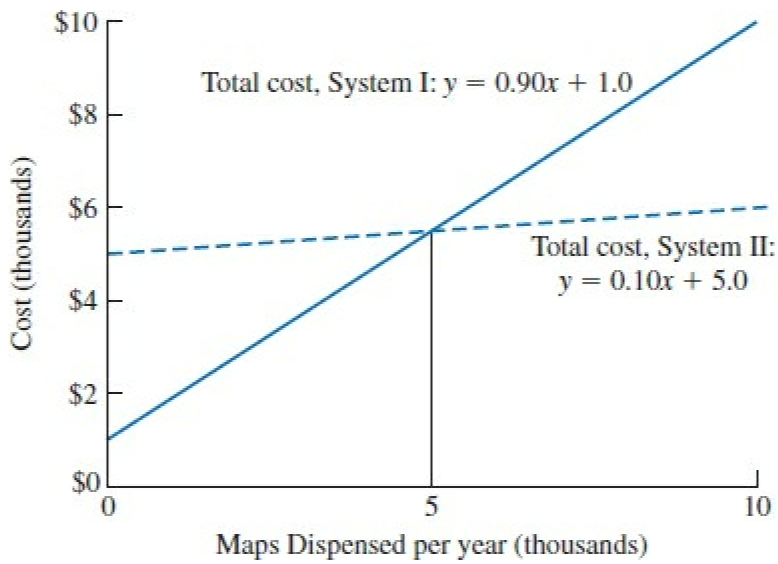
(*a*) Calculate the monthly bill for this shop. What are the marginal and average costs per kilowatt-hour?

(*b*) A contract for additional business would require more operating

hours per day. This will use an extra 1200 kWh per month. What is the “cost” of this additional energy? What is this per kilowatt-hour?
(*c*) New machines would reduce the labor time required on certain operations. These will increase the connected load by 10 hp, but since they will operate only on certain special jobs, will add only 100 kWh per month. In a study to determine the economy of installing these new machines, what should be considered as the “cost” of this energy? What is this per kilowatt-hour?

Two automatic systems for dispensing maps are being compared by the state highway department. The accompanying breakeven chart of the comparison of these systems (System I vs. System II) shows total yearly costs for the number of maps dispensed per year for both alternatives. Answer the following questions.
(*a*) What is the fixed cost for System I?
(*b*) What is the fixed cost for System II?
(*c*) What is the variable cost per map dispensed for System I?
(*d*) What is the variable cost per map dispensed for System II?
(*e*) What is the breakeven point in terms of maps dispensed at which the two systems have equal annual costs?
(*f*) For what range of annual number of maps dispensed is System I recommended?
(*g*) For what range of annual number of maps dispensed is System II recommended?
(*h*) At 3000 maps per year, what are the marginal and average map costs for each system?

2-14



A privately owned summer camp for youngsters has the following data for a 12-week session:

Charge per camper	$480 per week
Fixed costs	$192,000 per session
Variable cost per camper	$320 per week
Capacity	200 campers

2-15 (*a*) Develop the mathematical relationships for total cost and total revenue.
(*b*) What is the total number of campers that will allow the camp to *just break even*?
(*c*) What is the profit or loss for the 12-week session if the camp operates at 80% capacity?
(*d*) What are marginal and average costs per camper at 80% capacity?
(*e*) Would it be ethical to charge campers different rates depending on their family's socioeconomic status? Identify and describe two points pro and two points con for such a policy.

Two new rides are being compared by a local amusement park in terms of their annual operating costs. The two rides would generate the same level of revenue (thus the focus on costs). The Tummy Tugger has fixed costs of $10,000 per year and variable costs of $2.50 per visitor. The Head Buzzer has fixed costs of $4000 per year and variable costs of $4 per visitor.
(*a*) Mathematically find the number of visitors per year for the two

2-16 rides to have equal annual costs.

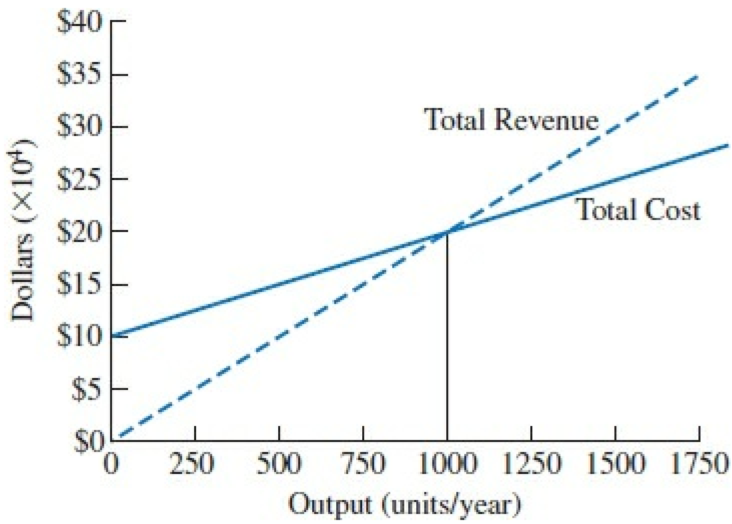
(*b*) Develop a breakeven graph to show:

• Accurate total cost lines for the two alternatives (show line, slopes, and equations).

• The breakeven point for the two rides in terms of number of visitors.

• The ranges of visitors per year where each alternative is preferred.

2-17 Consider the accompanying breakeven graph for an investment, and answer the following questions.



(*a*) Give the equation for total revenue for *x* units per year.

(*b*) Give the equation for total costs for *x* units per year.

(*c*) What is the "breakeven" level of *x*?

(*d*) If you sell 1500 units this year, will you have a profit or loss? How much?

(*e*) At 1500 units, what are your marginal and average costs?

2-18 A

Quatro Hermanas, Inc. is investigating implementing some new production machinery as part of its operations. Three alternatives have been identified, and they have the following fixed and variable costs:

Alternative	Annual Fixed Costs	Annual Variable Costs per Unit
A	$70,000	$14.00
B	115,000	7.00

C	150,000	4.00

Determine the ranges of production (units produced per year) over which each alternative would be recommended up to 30,000 units per year.

2-19 G

Three alternative designs have been created by engineers for a new machine that inspects solar power photovoltaic cells for home use. The costs for the three designs (where x is the annual production rate) follow:

Design	**Fixed Cost**	**Variable Cost ($/*x*)**
A	$100,000	20.5x
B	350,000	10.5x
C	600,000	8.0x

(*a*) Management is interested in the production interval of 0–150,000 cells per year. Mathematically determine the production volume over which each design (*A* or *B* or *C*) would be chosen.

(*b*) Depict your solution from part (*a*) graphically, clearly labeling your axes and including a *title* for the graph, so that management can easily see the following:

i. Accurate total cost lines for each alternative (show line, slopes, and line equations).

ii. Any relevant breakeven or crossover points.

iii. Ranges of annual production where each alternative is preferred.

(*c*) Our decision rule is to minimize total cost across the range of output. How would your analysis approach change if some alternative produced an ethical dilemma? Describe your thinking.

2-20 A

A painting operation is performed by a production worker at a labor cost of $1.15 per unit. A robot spray-painting machine, costing $16,000, would reduce the labor cost to $0.18 per unit. If the device would be valueless at the end of 4 years, what is the minimum number of units that would have to be painted each year to justify the purchase of the robot machine?

Mr. Sam Spade, the president of Ajax, recently read in a report that a competitor named Bendix has the following relationship between cost and production quantity:

$$C = \$3{,}000{,}000 - \$18{,}000Q + \$75Q^2$$

where C = total manufacturing cost per year and Q = number of units
2-21 produced per year.

A newly hired employee, who previously worked for Bendix, tells Mr. Spade that Bendix is now producing 110 units per year. If the selling price remains unchanged, Sam wonders if Bendix is likely to increase the number of units produced per year in the near future. He asks you to look at the information and tell him what you are able to deduce from it.

A small company manufactures a certain product. Variable costs are \$20 per unit and fixed costs are \$10,875. The price–demand relationship for this product is $P = -0.25D + 250$, where P is the unit sales price of the product and D is the annual demand.

- Total cost = Fixed cost + Variable cost
- Revenue = Demand × Price
- Profit = Revenue − Total cost

2-22 Set up your graph with dollars on the y axis (between 0 and \$70,000) and, on the x axis, demand D: (units produced or sold), between 0 and 1000 units.

(*a*) Develop the equations for total cost and total revenue.
(*b*) Find the breakeven quantity.
(*c*) What profit is earned if total revenue is maximized?
(*d*) What is the company's maximum possible profit?
(*e*) Graph the solutions to each part.

A firm believes a product's sales volume (S) depends on its unit selling price (P) as $S = \$100 - P$. The production cost ($C$) is $\$1000 + 10S$.

(*a*) Graph the sales volume (S) from 0 to 100 on the x axis, total cost and total income from \$0 to \$2500 on the y axis, $C = \$1000 + 10S$,
2-23 and plot the curve of total income. Mark the breakeven points on the graph.
(*b*) Determine the breakeven point (lowest sales volume at which total sales income just equals total production cost).
(*c*) Determine the sales volume (S) at which the firm's profit is a

maximum.

Sunk and Other Costs

2-24 Define each of the costs below as either sunk, opportunity, cash, or book:
(*a*) Amount you could have sold a piece of equipment for last month.
(*b*) Value claimed for tax purposes on a depreciated soda filling machine.
(*c*) Price paid to a consultant for a feasibility study on a prospective project.
(*d*) Monthly utility expense.
(*e*) Salary you could have earned while you were on non-paid leave.
(*f*) Value of floor space in a warehouse facility left empty the past 10 quarters.
(*g*) Low price of a stock that you chose not to invest in.

2-25 Define the difference between a "cash cost" and a "book cost." Is engineering economic analysis concerned with both types of cost? Give an example of each, and provide the context in which it is important.

2-26 In your own words, develop a statement of what the authors mean by "life-cycle costs." It is important for a firm to be aware of life-cycle costs. Can you explain why?

2-27 Most engineering students own a computer. What costs have you incurred at each stage of your computer's life cycle? Estimate the total cost of ownership. Estimate the benefits of ownership. Has it been worth it?

2-28 In looking at Figures 2–4 and 2–5, restate in your own words what the authors are trying to get across with these graphs. Do you agree that this is an important effect for companies? Explain.

2-29 In the context of green engineering, what is the difference between internal and external costs? Which type is easier to estimate and quantify, and why?

2-30 List and classify your costs in this academic year as recurring or nonrecurring.

2-31 Last year to help with your New Year's resolutions you purchased a $1450 piece of fitness equipment. However, you use it only twice a week on average. It is December, and you can sell the equipment for $400 (to someone with a New Year's resolution) and rely on the university gym until you graduate in May. If you don't sell until May, you will get only $200. If you keep the heavy piece, you'll have to pay $35 to move it to the city of your new job (where you interned last summer). There is no convenient gym at the new location. What costs and intangible consequences are relevant to your decision? What should you do?

2-32 G You are reevaluating the industrial heat pump choice that was made last year by your boss. The expected energy savings have not occurred because the pump is too small. Choice *A*, at $90,000, is to replace the pump with one that is the right size and sell the old one. Choice *B*, at $100,000, is to buy a small pump to use in tandem with the existing pump. The two-pump solution has slightly higher maintenance costs, but it is more flexible and reliable. What criteria should you use? Which choice would you recommend, and why?

2-33 Consider the situation of owning rental properties that local university students rent from you for the academic year.
(*a*) Develop a set of costs that you could classify as recurring and others that could be classified as nonrecurring.
(*b*) Research and list the ethical issues that student housing landlords face. What recourse do renters have when they encounter unethical landlords?

2-34 A A pump has failed in a facility that will be completely replaced in 3 years. A brass pump costing $6000 installed will last 3 years. However, a used stainless steel pump that should last 3 more years has been sitting in the maintenance shop for a year. The pump cost $13,000 new. The accountants say the pump is worth $7000 now. The maintenance supervisor says that it will cost an extra $500 to reconfigure the pump for the new use and that he could sell it used (as is) for $4000.
(*a*) What is the book cost of the stainless steel pump?
(*b*) What is the opportunity cost of the stainless steel pump?
(*c*) How much cheaper or more expensive would it be to use the

stainless steel pump rather than a new brass pump?

2-35 Owning, operating, and maintaining an automobile carries with it private internal costs to the driver, as well as public external societal costs. List five internal and external costs.

2-36 Bob Johnson decided to buy a new home. After looking at tracts of new homes, he decided that a custom-built home was preferable. He hired an architect to prepare the plans for a fee of $7000. While a building contractor was working on a bid to construct the home on a lot Bob already owned, Bob found a standard house plans on the Internet for $200 that he and his wife liked better. Bob then asked the contractor to provide a bid to construct this “stock plan” home. The building contractor submitted the following bids:

Custom-designed home	$258,000
Stock plan home	261,000

Bob was willing to pay the extra $3000 for the stock-plan home. Bob’s wife, however, felt they should go ahead with the custom-designed home, for, as she put it, “We can’t afford to throw away a set of plans that cost $7000.” Bob agreed, but he disliked the thought of building a home that is less desirable than the stock plan home. Which house would you advise him to build? Explain.

2-37 Identify the following costs as either internal to the firm or external to society:

Surveying	Lost animal habitat	Materials
Land	Public health	Reduced fisheries
Reduced water	Recreational loss	Administrative
quality	Roadway congestion	Air quality
Legal	Community	Noise pollution
Overhead	Construction	Virus tolerances
Roadway	Species extinction	Health benefits
Design	Waste storage	
Equipment	Labor	

A paper manufacturer creates industrial waste that is treated before being released into a local waterway. The EPA strictly monitors the

2-38 A level of this harmful waste. To maintain low levels of parts-per-million (PPM) of harmful waste requires expense on the part of the firm, and as a result they try to stay just under the allowable level. There is currently a 15% chance at each quarterly inspection that the firm will receive a $350,000 fine.

(*a*) How much should the firm be willing to pay on an annual basis to lease a new process that will decrease the risk of non-compliance to 6% per inspection?

(*b*) If the EPA fine represents the external cost, what is the average cost to society for each scenario?

2-39 LED light bulbs are more energy efficient, last longer, cost more, and have a smaller environmental impact from manufacture to recycling or disposal. The question is which costs less: incandescent, compact fluorescent (CFL), or LED lights?

	Incandescent	CFL	LED
Cost/bulb	$0.50	$2	$7
Life in hours	1250	8333	25,000
Watts used	60	14	7

(*a*) Ignore the time value of money. What is the average cost per hour for 25,000 hours of lighting in a community with $0.15/kWh electricity for each choice?

(*b*) If the light is on 5%, 30%, or 60% of the time, how long until does it take for the 25,000 hours?

(*c*) Which type of light should a homeowner buy?

Estimating Process

In the text we describe three effects that complicate the process of

2-40 making estimates to be used in engineering economy analyses. List these three effects and comment on which might be the most influential.

2-41 Develop an estimate for each of the following situations.
(*a*) The cost of a 500-mile automobile trip, if gasoline is $4 per gallon, vehicle wear and tear is $0.65 per mile, and our vehicle gets 25 miles per gallon.
(*b*) The total number of hours in the average human life, if the average life is 75 years.
(*c*) The number of days it takes to travel around the equator using a hot air balloon, if the balloon averages 100 miles per day, the diameter of the earth is ~4000 miles.
(*d*) The total area in square miles of the United States of America, if Kansas is an average-sized state. Kansas has an area of 390 miles × 200 miles.

2-42 A local minor league baseball team plays home games in a stadium that has four sections that each hold 2500 fans. On an average night, attendance in each higher section is filled to 80% the capacity of one below it. Estimate attendance on an average night if the bottom section is completely filled: What percent of the stadium is full?

2-43 Your dining room (11′× 13′) has a hardwood floor discolored by uneven fading from sunlight exposure, and your living room (12′× 12′) has wall-to-wall carpet ruined by your aging dog. You have obtained some cost figures (all per square foot) from a local flooring store for refinished and new hardwood floors. The cost to purchase and install red oak flooring is $3.55 (includes tax), and labor is $1.50. Refinishing costs the same $1.50. For new or old floors the labor/material/tax cost to apply a new light stain is $1.00 and two coats of polyurethane cost another $1.00. The store's sales manager estimates that you should include a 5% wastage factor on the wood purchase. Assume you will be removing and disposing of the carpet yourself without cost. Estimate the cash cost of this project.
Contributed by Gillian Nicholls, Southeast Missouri State University

Segmenting Models

2-44 Northern Tundra Telephone (NTT) has received a contract to install emergency phones along a new 160-mile section of the Snow-Moose Turnpike. Eighty emergency phone systems will be installed about 2 miles apart. The material cost of a unit is \$155. NTT will need to run underground communication lines that cost NTT \$7800 per mile (including labor) to install. There will also be a one-time cost of \$12,500 to network these phones into NTT's current communication system.

A

(*a*) Develop a cost estimate of the project from NTT's perspective.
(*b*) If NTT adds a profit margin of 40% to its costs, how much will it cost the state to fund the project?

2-45 You and your spouse are planning a second honeymoon to the Cayman Islands this summer and would like to have your house painted while you are away. Estimate the total cost of the paint job from the information given below, where:

$$\text{Cost}_{\text{total}} = \text{Cost}_{\text{paint}} + \text{Cost}_{\text{labor}} + \text{Cost}_{\text{fixed}}$$

Paint information: Your house has a surface area of 6000 ft^2. One can of paint can cover 300 ft^2. You are estimating the cost to put on *two coats* of paint for the entire house.

Number of Cans Purchased	Cost per Can
First 10 cans purchased	\$15.00
Second 15 cans purchased	\$12.50
Up to next 50 cans purchased	\$12

Labor information: You plan to hire five painters who will paint for 8 hours per day each. You estimate that the job will require 4.5 days of their painting time. The painter's rate is \$18 per hour.
Fixed cost information: The painting company charges \$1200 per job to cover travel expenses, clothing, cloths, thinner, administration, and so on.

You want a mountain cabin built for weekend trips, vacations, to host family, and perhaps eventually to retire in. After discussing the project with a local contractor, you receive an estimate that the total construction cost of your 2500-ft^2 lodge will be \$325,000. The percentage of costs is broken down as follows:

Cost Items	Percentage of Total Costs
Construction permits, legal and title fees	7
Roadway, site clearing, preparation	17
Foundation, concrete, masonry	19
Wallboard, flooring, carpentry	8
Heating, ventilation, air conditioning (HVAC)	14
Electric, plumbing, communications	10
Roofing, flooring	9
Painting, finishing	16
	100

2-46 A

(*a*) What is the cost per square foot of the 2500-ft^2 lodge?

(*b*) If you are also considering a 4500-ft^2 layout option, estimate your construction costs if:

i. All cost items (in the table) change proportionately to the size increase.

ii. The first two cost items do not change at all; HVAC changes by 50%; and all others are proportionate.

SungSam, Inc. is designing a new digital camcorder that is projected to have the following per-unit costs to manufacture:

Cost Categories	Unit Costs
Materials costs	$ 63
Labor costs	24
Overhead costs	110
Total unit cost	$97

2-47 SungSam adds 30% to its manufacturing cost for corporate profit.

(*a*) What unit profit would SungSam realize on each camcorder?

(*b*) What is the overall cost to produce a batch of 10,000 camcorders?

(*c*) What would SungSam's profit be on the batch of 10,000 if historical data show that 1% of product will be scrapped in manufacturing, 3% of finished product will go unsold, and 2% of *sold*

product will be returned for refund?
(*d*) How much can SungSam afford to pay for a contract that would lock in a 50% reduction in the unit material cost previously given? If SungSam does sign the contract, the sales price will not change.

Indexes and Sizing Models

2-48 A Estimate the cost of expanding a planned new clinic by 25,000 ft^2. The appropriate capacity exponent is 0.62, and the budget estimate for 185,000 ft^2 was $17 million.

2-49 G A new electrostatic precipitator has been requested to clean the air of a new work area. A complete package unit with a capacity of 5.7 cubic meters per second was recently purchased for $85,000. The new unit needs to be twice that size.
(*a*) Use the data in Table 2–1 to estimate the cost of the new electrostatic precipitator.
(*b*) Electrostatic precipitators do a good job at cleaning the air, but they use a lot of electricity. Are they thought to be environmentally friendly?

2-50 A Fifty years ago, Grandma Bell purchased a set of gold-plated dinnerware for $48, and last year you inherited it. Unfortunately a fire at your home destroyed the set. Your insurance company is at a loss to define the replacement cost and has asked your help. You do some research and find that the Aurum Flatware Cost Index (AFCI) for gold-plated dinnerware, which was 127 when Grandma Bell bought her set, is at 1989 today. Use the AFCI to update the cost of Bell's set to today's cost to show to the insurance company.

2-51 Your boss is the director of reporting for the Athens County Construction Agency (ACCA). It has been his job to track the cost of construction in Athens County. Twenty-five years ago he created the ACCA Cost Index to track these costs. Costs during the first year of the index were $12 per square foot of constructed space (the index value was set at 100 for that first year). This past year a survey of contractors revealed that costs were $90 per square foot. What index number will your boss publish in his report for this year? If the index value was 600 last year, what was the cost per square foot last year?

2-52 A G

A refinisher of antiques named Constance has been so successful with her small business that she is planning to expand her shop. She is going to start enlarging her shop by purchasing the following equipment.

Equipment	Original Size	Cost of Original Equipment	Power-Sizing Exponent	New Equipmen Size
Varnish bath	50 gal	$3500	0.80	75 gal
Power scraper	3/4 hp	$250	0.22	1.5 hp
Paint booth	3 ft^3	$3000	0.6	12 ft^3

(*a*) What would be the *net* cost to Constance to obtain this equipment? Assume that she can trade the old equipment in for 15% of its original cost. Assume there has been no inflation in equipment prices.
(*b*) Suggest a green engineering approach to constance for disposing of the solvents and lacquers used in her business.

2-53 Refer to Problem 2-52 and now assume the prices for the equipment that Constance wants to replace have not been constant. Use the cost index data for each piece of equipment to update the costs to the price that would be paid today. Develop the overall cost for Constance, again assuming the 15% trade-in allowance for the old equipment. Trade-in value is based on original cost.

Original Equipment	**Cost Index When Originally Purchased**	**Cost Index Today**
Varnish bath	154	171

Power scraper	780	900
Paint booth	49	76

2-54 A Five years ago, when the relevant cost index was 135, a nuclear centrifuge cost $32,000. The centrifuge had a capacity of separating 1250 gallons of ionized solution per hour. Today, it is desired to build a centrifuge with capacity of 3500 gallons per hour, but the cost index now is 270. Assuming a power-sizing exponent to reflect economies of scale, *x*, of 0.72, use the power-sizing model to determine the approximate cost (expressed in today's dollars) of the new reactor.

2-55 Padre works for a trade magazine that publishes lists of power-sizing exponents (PSE) that reflect economies of scale for developing engineering estimates of various types of equipment. Padre has been unable to find any published data on the VMIC machine and wants to list its *PSE* value in his next issue. Given the following data calculate the *PSE* value that Padre should publish. (*Note:* The VMIC-100 can handle twice the volume of a VMIC-50.)

Cost of VMIC-100 today $250,000
Cost of VMIC-50 5 years ago $95,000
VMIC equipment index today = 350
VMIC equipment index 5 years ago = 235

2-56 A Remerowski Corporation Inc. asks you to estimate the cost to purchase a new piece of production equipment. The company purchased this same type of equipment in the past for $8500. The original equipment had a capacity of 1500 units, while the new equipment has a capacity of 900 units. The power-sizing exponent for this type of equipment is 0.3. In addition, the cost index for this type of equipment was 111 when the original unit was purchased and is now 195. If Remerowski uses an 8% annual interest rate to evaluate equipment purchases, estimate the cost to purchase the new piece. *Contributed by Gillian Nicholls, Southeast Missouri State University*

Sage Lorimer owns a boutique that sells jewelry made by her husband and other local artists. She is considering expanding the business by opening a second store. The current store has 1200 square feet of retail space and 400 square feet of space for administrative

2-57 offices. For the new store she would like to double the retail space and triple the administrative space of the current store. At the time the current shop was purchased, retail space sold for $75 per square foot and general office space cost $30 per square foot. The cost indexes at the time were 153 for retail space and 120 for office space. The costs indexes are now 140 for retail space and 132 for office space. Estimate the cost for Sage to purchase a second store at this time. *Contributed by Gillian Nicholls, Southeast Missouri State University*

Learning-Curve Models

2-58 A If 200 labor hours were required to produce the 1^{st} unit in a production run and 60 labor hours were required to produce the 7^{th} unit, what was the *learning-curve rate* during production?

2-59 Ima New is a recent employee who initially requires 20 minutes to complete a job task. If she experiences an 80% learning rate, how much time will it take her to complete the sixth task?

2-60 A Rose is a project manager at the civil engineering consulting firm of Sands, Gravel, Concrete, and Waters, Inc. She has been collecting data on a project in which concrete pillars were being constructed; however not all the data are available. She has been able to find out that the 8^{th} pillar required 225 person-hours to construct and that a 68% learning curve applied. She is interested in calculating the time required to construct the 1^{st} and 25^{th} pillars. Compute the values for her.

2-61 Home Building Inc. (HBI) seeks to schedule manual labor for 18 new homes being constructed. Historical data leads HBI to apply a 92% learning curve rate to the manual labor portions of the project. If the first home requires 3500 manual labor hours to build, estimate the time required to build (*a*) the fifth house, (*b*) the tenth house, (*c*) all 18 houses. What would the manual labor estimate be for all 18 of the HBI houses in the problem above if the learning curve rate is (*d*) 70%, (*e*) 75%, (*f*) 80%?

2-62 Ima Neworker requires 43 minutes to produce her first unit of output. If her learning curve rate is 76%, how many units will be produced

A before the output rate exceeds 5 units per hour?

2-63 An industrial engineering consulting firm usually observes a 90% learning curve rate in the installation of enterprise level software with its clients. If the first installation required 75 hours, estimate the time required for (*a*) the fifth, (*b*) the tenth, and (*c*) the twentieth installations. (*d*) Research the AMCF Code of Ethics. How are these similar to and different from engineering society ethics statements from your discipline?

2-64 A Sally Statistics is implementing a system of statistical process control (SPC) charts in her factory in an effort to reduce the overall cost of scrapped product. The current cost of scrap is $*X* per month. If a 75% learning curve is expected in the use of the SPC charts to reduce the cost of scrap, what would the *percentage reduction* in monthly scrap cost be after the charts have been used for 6 months? (*Hint:* Model each month as a unit of production.)

2-65 G A new product made from recycled bio-plastics needs 20 labor hours to complete the build for the first unit. If production operates at a 85% learning curve rate, calculate the average labor hours required per unit to produce five units.

2-66 New technicians in an oncology department process patients at rates shown below. Steady state occurs at the eighth unit.

Patient number:	1	2	3	4	5	6	7	8
Process time (min):	84	76	61	54	50	48	45	43

(*a*) Calculate the learning curve rate for units 1–8.
(*b*) What is the total time needed to process 11 patients?

Benefit Estimation

2-67 Estimating benefits is often more difficult than cost estimation. Use the example of car ownership to describe the complicating factors in estimating the costs and benefits.

2-68 The authors mention in a few places in the chapter that benefits are overestimated while costs tend to be underestimated—they called this the "optimist's bias." What do they mean by this, and what effect can it have on the fair evaluation of projects?

2-69 Create a 2 × 2 table labeling the columns "costs" and "benefits," and the rows "recurring" and "non-recurring costs." Think about your life and list two to three items/examples in each cell.

2-70 Projects A and B have the same cost to implement, maintain, and dispose of over their life cycles. What impact will any incremental benefits play in the comparison of the alternatives?

2-71 Many types of employees are on performance-based contracts—which combine fixed and variable salary amounts depending on the performance (productivity) of the worker. Give examples of employees who are on these types of contracts.

2-72 Develop a statement that expresses the extent to which cost-estimating topics also apply to estimating benefits. Provide examples to illustrate.

2-73 Large projects, such as a new tunnel under the Hudson, the Big Dig in Boston, the Denver airport, a new military jet, and a natural gas pipeline from Alaska to the Midwest, often take 5 to 15 years from concept to completion.
(*a*) Should benefit and cost estimates be adjusted for the greater influences and impacts of inflation, government regulatory changes, and changing local economic environments? Why or why not?
(*b*) How does the public budget-making process interact with the goal of accurate benefit and cost estimating for these large projects?
Contributed by Morgan Henrie, University of Alaska Anchorage

2-74 A A home run king's team pays him a base salary of $10 million per season plus $65,000 per home run over 40. If the king hits 68 home runs next season, what will his salary be for the year?

Cash Flow Diagrams

On December 1, Al Smith purchased a car for $25,000. He paid $7500 immediately and agreed to pay three additional payments of $8000 each (which includes principal and interest) at the end of 1, 2, and 3 years.

2-75 Maintenance for the car is projected at $1000 at the end of the first year and $2000 at the end of each subsequent year. Al expects to sell the car at the end of the fourth year (after paying for the maintenance work) for $12,000. Using these facts, prepare a table of cash flows.

2-76 Bonka Toys is considering a robot that will cost $85,000 to buy. After 6 years its salvage value will be $22,000. An overhaul costing $13,000 will be needed in Year 4. O&M costs will be $2500 per year. Draw the cash flow diagram.

2-77 Pine Village needs some additional recreation fields. Construction will cost $300,000, and annual O&M expenses are $85,000. The city council estimates that the value of added youth leagues is about $200,000 annually. In year 6 another $75,000 will be needed to refurbish the fields. The salvage value is estimated to be $50,000 after 10 years. Draw the cash flow diagram.

2-78 Identify your major cash flows for the current school term as first costs, O&M expenses, salvage values, revenues, overhauls, and so on. Using a week as the time period, draw the cash flow diagram.

2-79 HiTech Inc. is investing in a new production line to manufacture their newest high volume consumer product. Design costs for the past three years, leading up to production, have been $0.75 million per year. Today production machinery totaling $3.65 million is being purchased to equip the new line. Operating and maintenance expenses will total $50,000 per year, and materials and overhead costs will be $1.75 million annually. The new line will operate for 7 years, at which time equipment will be sold at 10% of purchase price. Revenue from the new product is expected to be $2.0 million the first year, increasing by $0.50 million per year for the next 4 years, then decreasing by $0.75 million in years 6 and 7. HiTech has asked you to create a cash flow table and diagram.

Multiple Objectives A

A firm is bidding on a contract for a government building, which will be awarded using an augmented lowest bidder model. Most points will be awarded based on the firm’s bid, but 3 extra points will be awarded if (1) the firm is minority, female, or veteran owned, or (2)

2-80 G

the design will satisfy the platinum level LEED (Leadership in Energy and Environmental Design) criteria. The government estimate is $38M. The firm does not satisfy the ownership category, but it is deciding whether or not to develop its bid to meet the LEED standard. The firm's rough estimate (including profit) is $28M without meeting LEED. The estimated cost for meeting the LEED criteria is $875,000. The government evaluation function is

$$\text{Score} = (\text{Gov't. Estimate / Bid}) \times 100$$
$$+ \{0 \text{ or } 3\} \text{ bonus points.}$$

Should the firm's bid be developed with or without a platinum LEED design?

2-81 G

A large international firm has decided to include external costs in its decision making about its products. The firm's leaders believe this approach is better than focusing on meeting a variety of changing national regulations. While the firm is developing its knowledge base on estimating social and environmental costs, it plans to add 5% of the external costs to its internal costs. In the long run, its goal is use a 20% value for external costs. One process now costs $25M per year in internal costs, with an estimated $8M in external costs. Rough estimates indicate that spending an extra $1.2M internally would reduce the external costs to $3M. What total *cost* values should be used to evaluate this change?

(*a*) Now

(*b*) Later with a 20% weight

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 3	**Wildcat Oil in Kasakstan**
	Rough order-of-magnitude estimation of total facility cost and annual revenue. Option for NPV and different size facility.

CASE 25	**Raster Blaster**
	Questions to guide students. Includes breakeven analysis.

TRUST ME: YOU’LL USE THIS

Contributed by Kate Abel, Stevens Institute of Technology

■ Engineering in the real world is about money.
• Projects cost money now. The benefits may come in for decades.
■ Engineering economy also applies to your personal life.
• This “trust me” introduces some of the ways.

Most of us borrow money using credit cards or loans for cars, college, houses, etc.

■ How much do you trust the entity offering the loan? Are there hidden fees? Do payments match the verbal terms?
■ With these tools you can calculate the payments. Know the rules of the game so you cannot be taken advantage of.

Your credit score is between 350 (very bad) and 850 (outstanding).

■ What does it depend on?
• Do you have a credit history? It helps if you do.
• What is your debt/income ratio? Not too high is best.
• Do you make payments on time? Very important.
■ Why does it matter?
• Follows you for the rest of your life.
• Lower credit scores → loan not available or at a higher interest rate.
• Some employers use credit scores to screen resumes.

Why save now, when you can borrow when needed? See also Chapters 4–7.

■ If young parents invested $200 a month at 7% for 10 years, then the $24,000 in savings would earn $11,000 in interest. A college fund of $35,000 results.

■ Suppose instead a $35,000 loan is used. If paid back over the same 10 years and at the same 7% interest rate, then an additional $13,000 in interest or a total of $48,000 would be paid.

Which debt to pay off first? Goal is to minimize interest paid. See also Chapters 6 and 7.

■ The best approach is to pay off the highest interest debt as fast as possible, and just pay the minimum on the other debts.

■ Payday loans have the highest rate.

■ Credit card debts are usually next.

■ If you've missed a payment or paid late, some credit cards may charge you 25–40% interest. So making at least the minimum payment on time each month is very important.

■ Pay credit card balances in full each month if you can, then most cards charge *no* interest for the next month.

■ Sometimes the best strategy is to pay off a credit card with a lower interest rate and a lower balance, and then use only that card for new charges which are paid in full each month, while you pay off any other credit card balances.

When should one save money vs. pay off debt? See Appendix 12A.

■ Having an emergency fund is important! Cars, computers, phones, . . . may be stolen or need repair—now. A job can be lost or sickness/injury may mean you can't work for a while. An emergency fund comes first.

■ Saving is paying yourself and it should become a habit as early in life as possible.

■ Often investment returns are higher than low cost debt such as a mortgage.

Why save and invest when I have so much college debt? See Chapters 4 and 7 and Appendix 9A.

■ The best investment tool is the *power of time*.

■ If you want $20,000 in 15 years for a down payment on a house, how much must you save assuming a 6% return?

• Start today and $69/month is enough.

• Start after 5 years and $123/month is needed.

• Start after 10 years and $286/month is needed.
• That $69 a month may be a challenge right out of college, but it will feel like nothing later on.
■ You may feel you can't afford to save. It seems easier to save later when your income is higher, but your expenses will also be higher. See Appendix 12A.

Other important questions you will learn how to answer in Chapters 4–7 and Appendices 9A, 10A, and 12A.

■ What's the real interest rate on the car loan at 0% the dealer is advertising?
■ How long will it take to pay off this debt if I pay the minimum, make an extra payment, or pay $100 extra each month?
■ What are reasonable expectations for different types of investment?
■ How is insurance included in financial planning?

CHAPTER 3

INTEREST AND EQUIVALENCE

A Prescription for Success

A pharmaceutical company manufactured a prescription drug that contained a tablet inside another tablet. This inner tablet, called a "core," needed to be in the exact center of the larger tablet, and tolerances were measured in tenths of millimeters. The process was not robust, and the placement of the inner tablet sometimes drifted, requiring the scrapping of off-quality product and the adjustment of the tablet press. This resulted in significant scrap and tablet press downtime.

A process change was invented in Japan to correct the problem, using a new process to place the inner tablet in the die of the press that made the outer tablet. There were three of these tablet presses in use in the U.S., but modifications were made to one tablet press as a test. The modification to the first press cost $27,000. During the first batch the modified press ran the entire batch without a quality problem and without quality losses. The batch finished compressing in 16 hours, which was considerably faster than the typical time of 24 hours (however, core centering problems could cause a delay of several days).

Additional test batches were run, all with excellent results. A detailed quality examination proved that the modification performed as desired, reducing variation and nearly eliminating product quality scrap. The other two tablet presses were later modified. The total cost for all modifications, including spare parts, was $90,000.

Over the next year, the results of the change were analyzed. Product yield increased from 92.4% to 96.5%. Because less of the expensive active drug was scrapped and instead became good product, each 1% increase in yield was valued at $2.4 million per year. Operating efficiency improved, resulting in higher output because of less scrap and less downtime due to quality problems. Production plans called for 240 batches to be processed over the year after the tablet press modification was made. This product was produced daily, but production was reduced from three shifts to two because of the improved efficiency. Production planning could now plan effectively; they knew that a batch could be processed in two shifts, not one to five days.

Year-end accounting showed $10 million saved in the first year. Because the product's patent was about to expire, production was expected to be greatly reduced beyond this time.

QUESTIONS TO CONSIDER

1. One year of production had a value of $240 million. What is the value of one batch of product?
2. How many batches needed to be produced to break even on the initial $27,000 investment? (Assume all batches improved the yield by 4.2%. Do not consider the time value of money.)

3. If the first-year savings is considered to be a single end-of-year cash flow, and the entire $90,000 investment is considered to occur at time 0, what is the present value of the project? Assume an interest rate of 15%.
4. If one batch is produced per day, how often are the savings actually compounded?
5. Does a company face any ethical considerations when it improves process efficiency resulting in lost labor hours for employees? Discuss and explain.

After Completing This Chapter…

The student should be able to:

• Define and provide examples of the *time value of money*.
• Distinguish between *simple* and *compound interest*, and use compound interest in engineering economic analysis.
• Explain *equivalence* of cash flows.
• Solve problems using the single-payment compound interest formulas.
• Distinguish and apply *nominal* and *effective interest rates*.

Key Words

annual percentage rate (APR)

cash flows

compound interest

continuous compounding

disbursement

effective interest rate

equivalence

equivalent value

interest

interest formula

nominal interest rate

receipt

simple interest

technique of equivalence

time value of money

In the first chapter, we discussed situations where the economic consequences of an alternative were immediate or took place in a very short period of time, as in Example 1–2 (design of a concrete aggregate mix) or Example 1–3 (change of manufacturing method). We totaled the various positive and negative aspects, compared our results, and could quickly reach a decision. But can we do the same if the economic consequences occur over a considerable period of time?

No we cannot, because *money has value over time*. Would you rather (1) receive $1000 today or (2) receive $1000 ten years from today? Obviously, the $1000 today has more value. Money’s value over time is expressed by an interest rate. This chapter describes two introductory concepts involving the *time value of money:* interest and cash flow equivalence. Later in the chapter, nominal and effective interest are discussed. Finally, equations are derived for situations where interest is compounded continuously.

Most examples in this chapter are loans and savings accounts for individuals. This keeps the examples simple and emphasizes that engineering economy is part of everyday life, and not just used for evaluation of large engineering projects. Values in the hundreds or thousands are also easier to read. All of the extra zeros in millions or hundreds of millions can get in the way of understanding.

COMPUTING CASH FLOWS

Installing expensive machinery in a plant obviously has economic consequences that occur over an extended period of time. If the machinery was bought on credit, the simple process of paying for it could take several years. What about the usefulness of the machinery? Certainly it was purchased because it would be a beneficial addition to the plant. These favorable consequences may last as long as the equipment performs its useful function. In these circumstances, we describe each alternative as cash **receipts** or **disbursements** at different points in **time.** Since earlier cash flows are more valuable than later cash flows, we cannot just add them together. Instead, each alternative is resolved into a set of **cash flows,** as in Examples 3–1 and 3–2.

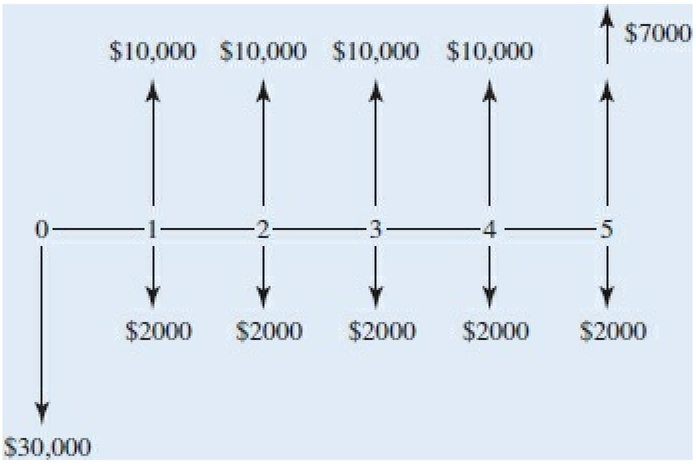
Example 3–1 is a classic engineering economy problem for both teaching and practice. There is a cash flow at the beginning, the annual cash flows are estimated to be uniform, and there is a cash flow at the end. Until the cash flow diagram is instantly visualized as you read the problem, it should be sketched. (Just like a free-body diagram in statics.) Figure 2–11 shows how a spreadsheet can be used to create a cash flow diagram.

EXAMPLE 3-1

A machine will cost $30,000 to purchase. Annual operating and maintenance costs (O&M) will be $2000. The machine will save $10,000 per year in labor costs. The salvage value of the machine after 5 years will be $7000. Draw the cash flow diagram.

SOLUTION

Even though some cash flows occur at the same time, they should not be combined. Not only does showing each cash flow clearly link the text and the diagram, we will find that it also makes it easier to calculate equivalent values.



EXAMPLE 3-2

A man borrowed $1000 from a bank at 8% interest. He agreed to repay the loan in two end-of-year payments. At the end of each year, he will repay half of the $1000 principal amount plus the interest that is due. Compute the borrower's cash flow.

SOLUTION

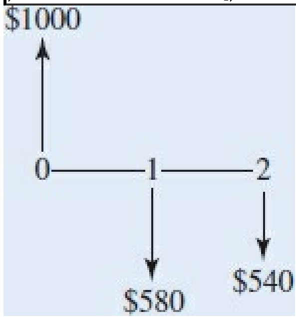
In engineering economic analysis, we normally refer to the beginning of the first year as "time 0." At this point the man receives $1000 from the bank. (A positive sign represents a receipt of money and a negative sign, a disbursement.) The time 0 cash flow is + $1000.

At the end of the first year, the man pays 8% interest for the use of $1000 for that year. The interest is 0.08 × $1000 = $80. In addition, he repays half the $1000 loan, or $500. Therefore, the end-of-year-1 cash flow is −$580.

At the end of the second year, the payment is 8% for the use of the balance of the principal ($500) for the one-year period, or 0.08 × 500 = $40. The $500 principal is also repaid for a total end-of-year-2 cash flow of − $540. The cash flow is:

End of Year	Cash Flow
0 (now)	+$1000

1	− 580
2	− 540



Techniques for comparing the value of money at different dates are the foundation of engineering economic analysis. We must be able to compare, for example, a low-cost pump with a higher-cost pump. If there were no other consequences, we would obviously prefer the low-cost one. But what if the higher-cost pump is more durable? Then we must consider whether to spend more money now to postpone the future cost of a replacement pump. This chapter will provide the methods for comparing the alternatives to determine which pump is preferred.

TIME VALUE OF MONEY

We often find that the monetary consequences of any alternative occur over a substantial period of time—say, a year or more. When monetary consequences occur in a short period of time, we simply add up the various sums of money and obtain a net result. But can we treat money this way when the time span is greater?

Which would you prefer, $1000 cash today or the assurance of receiving $1000 a year from now? You might decide you would prefer the $1000 now because that is one way to be certain of receiving it. But suppose you were convinced that you would receive the $1000 in one year. Now what would be your answer? A little thought should convince you that it *still* would be more desirable to receive the $1000 now. If you had the money now, rather than in a year, you would have the use of it for an extra year. And if you had no current use for $1000, you could let someone else pay you to use it.

Money is quite a valuable asset—so valuable that people are willing to pay to have money available for their use. Money can be rented in roughly the same way one rents an apartment; only with money, the charge is called **interest** instead of rent. The importance of interest is demonstrated by banks and savings institutions continuously offering to pay for the use of people's money, to pay interest.

If the current interest rate is 4% per year and you put $1000 into the bank for one year, how much will you receive back at the end of the year? You will receive your original $1000 together with $40 interest, for a total of $1040. This example demonstrates that there is a **time value of money** in the form of the willingness of banks, businesses, and people to pay interest for the use of various sums.

Simple Interest

While simple interest is rarely used, it is a good starting point to understand how interest is calculated. **Simple interest** is interest that is computed only on the original sum, not on accrued interest. Thus if you were to loan a present sum of money P to someone at a simple interest rate i (stated as a decimal) for n periods, the amount of interest you would receive from the loan would be

$\text{Total interest earned} = P \times i \times n = Pin$(3-1)

At the end of n periods the amount of money due you, F, would equal the amount of the loan P plus the total interest earned. That is, the amount of money due at the end of the loan would be

$F = P + Pin$(3-2)

or $F = P(1 + in)$.

EXAMPLE 3-3

You have agreed to loan a friend $5000 for 5 years at a simple interest rate of

8% per year. How much interest will you receive from the loan? How much will your friend pay you at the end of 5 years?

SOLUTION

$$\text{Total interest earned} = Pin = \$5000 \times 0.08 \times 5 \text{ yr} = \$2000$$

$$\text{Amount due at end of loan} = P + Pin = 5000 + 2000 = \$7000$$

In Example 3–3 the interest earned at the end of the first year is (5000)(0.08)(1) = $400, but this money is not paid to the lender until the end of the fifth year. As a result, the borrower has the use of the $400 for 4 years without paying any interest on it. This is how simple interest works, and it is easy to see why lenders seldom agree to make simple interest loans.

Compound Interest

With simple interest, the amount earned (for invested money) or due (for borrowed money) in one period does not affect the principal for interest calculations in later periods. However, this is not how interest is normally calculated. In practice, interest is computed by the **compound interest** method. For a loan, any interest owed but not paid at the end of the year is added to the balance due. Then the next year's interest is calculated on the unpaid balance due, which includes the unpaid interest from the preceding period. In this way, compound interest can be thought of as *interest on top of interest.* This distinguishes compound interest from simple interest. In this section, the remainder of the book, and in practice you should **assume that the rate is a compound interest rate**. The very few exceptions, such as some end-of-chapter problems, will clearly state "simple interest."

EXAMPLE 3-4

To highlight the difference between simple and compound interest, rework Example 3–3 using an interest rate of 8% per year compound interest. How will this change affect the amount that your friend pays you at the end of 5 years?

Original loan amount (original principal) = \$5000

Loan term = 5 years

Interest rate charged = 8% per year compound interest

SOLUTION

In the following table we calculate on a year-to-year basis the total dollar amount due at the end of each year. Notice that this amount becomes the principal upon which interest is calculated in the next year (this is the compounding effect).

Year	Total Principal (*P*) on Which Interest Is Calculated in Year *n*	Interest (*I*) Owed at the End of Year *n* from Year *n*'s Unpaid Total Principal	Total Amount Due at the End of Year *n*, New Total Principal for Year *n + 1*
1	\$5000	\$5000 × 0.08 = 400	\$5000 + 400 = 5400
2	5400	5400 × 0.08 = 432	5400 + 432 = 5832
3	5832	5832 × 0.08 = 467	5832 + 467 = 6299
4	6299	6299 × 0.08 = 504	6299 + 504 = 6803
5	6803	6803 × 0.08 = 544	6803 + 544 = 7347

The total amount due at the end of the fifth year is \$7347. This is \$347 more than you received for loaning the same amount, for the same period, at simple interest. This is because of the effect of interest being earned on top of interest.

Repaying a Debt

To better understand the mechanics of interest, let us say that \$5000 is owed and is to be repaid in 5 years, together with 8% annual interest. There are a great many ways in which debts are repaid; for simplicity, we have selected four specific ways for our example. Table 3–1 tabulates the four plans.

Plan 1 (Constant Principal), like Example 3–2, repays $1/n^{\text{th}}$ of the principal each year. So in Plan 1, \$1000 will be paid at the end of each year plus the interest due at the end of the year for the use of money to that point. Thus, at the end of Year 1, we will have had the use of \$5000. The interest owed is 8% × \$5000 = \$400. The end-of-year payment is \$1000 principal *plus* \$400 interest, for a total payment of \$1400. At the end of Year 2, another \$1000 principal plus interest will be repaid on the money owed during the year. This time the amount owed has declined from \$5000 to \$4000 because of the Year 1 \$1000 principal payment. The interest payment is 8% × \$4000 = \$320, making the end-of-year payment a total of \$1320. As indicated in Table 3–1, the series of payments continues each year until the loan is fully repaid at the end of the Year 5.

In *Plan 2 (Interest Only)* only the interest due is paid each year, with no principal payment. Instead, the \$5000 owed is repaid in a lump sum at the end of the fifth year. The end-of-year payment in each of the first 4 years of Plan 2 is 8% × \$5000 = \$400. The fifth year, the payment is \$400 interest *plus* the \$5000 principal, for a total of \$5400.

Plan 3 (Constant Payment) calls for five equal end-of-year payments of \$1252 each. In Example 4–3, we will show how the amount of \$1252 is computed. By following the computations in Table 3–1, we see that a series of five payments of \$1252 repays a \$5000 debt in 5 years with interest at 8%.

Plan 4 (All at Maturity) repays the \$5000 debt like Example 3–4. In this plan, no payment is made until the end of Year 5, when the loan is completely repaid. Note what happens at the end of Year 1: the interest due for the first year—8% × \$5000 = \$400—is not paid; instead, it is added to the debt. At the second year then, the debt has increased to \$5400. The Year 2 interest is thus 8% × \$5400 = \$432. This amount, again unpaid, is added to the debt, increasing it further to $5832. At the end of Year 5, the total sum due has grown to $7347 and is paid at that time.

Table 3-1 Four Plans for Repayment of $5000 in 5 Years with Interest at 8%

(a) Year	(b) Amount Owed at Beginning of Year	(c) Interest Owed for That Year, 8% × (b)	(d) Total Owed at End of Year, (b) + (c)	(e) Principal Payment	(f) Total End-of-Year Payment
Plan 1: Constant principal payment *plus* interest due.					
1	$5000	$ 400	$5400	$1000	$1400
2	4000	320	4320	1000	1320
3	3000	240	3240	1000	1240
4	2000	160	2160	1000	1160
5	1000	80	1080	1000	1080
		$1200		$5000	$6200
Plan 2: Annual interest payment and principal payment at end of 5 years.					
1	$5000	$ 400	$5400	$ 0	$ 400
2	5000	400	5400	0	400
3	5000	400	5400	0	400
4	5000	400	5400	0	400
5	5000	400	5400	5000	5400
		$2000		$5000	$7000
Plan 3: Constant annual payments.					
1	$5000	$ 400	$5400	$ 852	$1252*
2	4148	331	4479	921	1252
3	3227	258	3485	994	1252
4	2233	178	2411	1074	1252
5	1159	93	1252	1159	1252
		$1260		$5000	$6260
Plan 4: All payment at end of 5 years.					
1	$5000	$ 400	$5400	$ 0	$ 0
2	5400	432	5832	0	0

3	5832	467	6299	0	0
4	6299	504	6803	0	0
5	6803	544	7347	5000	7347
		$2347		$5000	$7347

* The exact value is $1252.28, which has been rounded to an even dollar amount.

Note that in Plan 4 when the $400 interest was not paid at the end of Year 1, it was added to the debt and, in Year 2 there was interest charged on this unpaid interest. That is, the $400 of unpaid interest resulted in 8% × $400 = $32 of additional interest charge in Year 2. That $32, together with 8% × $5000 = $400 interest on the $5000 original debt, brought the total interest charge at the end of Year 2 to $432. Charging interest on unpaid interest is called compound interest.

With Table 3–1 we have illustrated four different ways of accomplishing the same task, that is, to repay a debt of $5000 in 5 years with interest at 8%. Having described the alternatives, we will now use them to present the important concept of *equivalence*.

EQUIVALENCE

When we are indifferent as to whether we have a quantity of money now or the assurance of some other sum of money in the future, or series of future sums of money, we say that the present sum of money is **equivalent** to the future sum or series of future sums at a specified interest rate.

If an industrial firm believed 8% was a reasonable interest rate, it would have no particular preference about whether it received $5000 now or the payments from any plan in Table 3–1. In fact, *all four repayment plans are equivalent to each other and to $5000 now at 8% interest*.

Equivalence is an essential concept in engineering economic analysis. In Chapter 2, we saw how an alternative could be represented by a cash flow table. For example, consider the cash flows from Table 3–1:

Year	Plan 1	Plan 2	Plan 3	Plan 4
1	−$1400	−$400	−$1252	$0
2	−1320	− 400	−1252	0
3	−1240	− 400	−1252	0
4	−1160	− 400	−1252	0
5	−1080	− 5400	−1252	−7347
Total	− $6200	− $7000	−$6260	−$7347
Interest Paid	$1200	$2000	$1260	$2347

If you were given your choice between the alternatives, which one would you choose? Obviously the plans have cash flows that are different, and you cannot compare the totals, the cash flows, or the interest paid. To make a decision, we must use the **technique of equivalence**.

We can determine an **equivalent value** at some point in time for each plan, based on a selected interest rate. Then we can judge the relative attractiveness of the alternatives, not from their cash flows, but from comparable equivalent values. Since each plan repays a *present* sum of $5000 with interest at 8%, all plans are equivalent to $5000 *now* at an interest rate of 8%. This cannot be deduced from the given cash flows alone. It is necessary to learn this by determining the equivalent values for each alternative at some point in time, which in this case is “the present.”

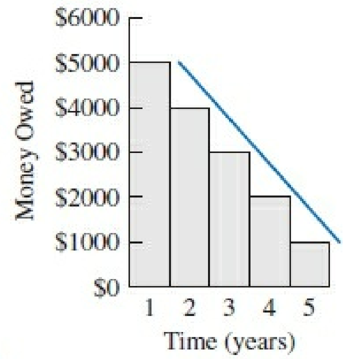
Difference in Repayment Plans

The four plans computed in Table 3–1 are equivalent in nature but different in structure. Table 3–2 repeats the end-of-year payment schedules from Table 3–1 and also graphs each plan to show the debt still owed at any point in time. Since $5000 was borrowed at the beginning of the first year, all the graphs begin at that point. We see, however, that the four plans result in quite different amounts of money owed at any other point in time. In Plans 1 and 3, the money owed declines as time passes. With Plan 2 the debt remains constant, while Plan 4 increases the debt until the end of the fifth year. These graphs show an important difference among the repayment plans—the areas under the curves differ greatly. Since the axes are *Money Owed* and *Time*, the area is their product: Money owed × Time, in dollar-years.

Table 3-2 End-of-Year Payment Schedules and Their Graphs

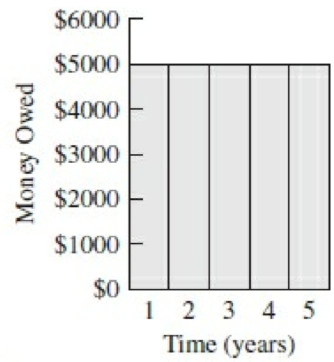
Plan 1 (Constant Principal): At end of each year pay \$1000 principal *plus* interest due.

Year	End-of-Year Payment
1	\$1400
2	1320
3	1240
4	1160
5	1080
	\$6200



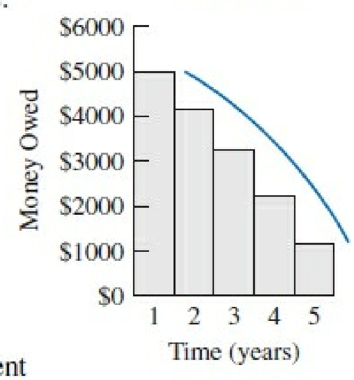
Plan 2 (Interest Only): Pay interest due at end of each year and principal at end of 5 years.

Year	End-of-Year Payment
1	\$ 400
2	400
3	400
4	400
5	5400
	\$7000



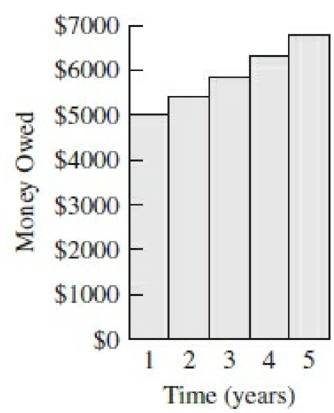
Plan 3 (Constant Payment): Pay in five equal end-of-year payments.

Year	End-of-Year Payment
1	\$1252
2	1252
3	1252
4	1252
5	1252
	\$6260



Plan 4 (All at Maturity): Pay principal and interest in one payment at end of 5 years.

Year	End-of-Year Payment
1	\$ 0
2	0
3	0
4	0
5	7347
	\$7347



The dollar-years for the four plans would be as follows:

Dollar-Years

	Plan 1	Plan 2	Plan 3	Plan 4
Money owed in Year 1 × 1 year	$5,000	$5,000	$5,000	$5,000
Money owed in Year 2 × 1 year	4,000	5,000	4,148*	5,400
Money owed in Year 3 × 1 year	3,000	5,000	3,227*	5,832
Money owed in Year 4 × 1 year	2,000	5,000	2,233*	6,299
Money owed in Year 5 × 1 year	1,000	5,000	1,159*	6,803
Total dollar-years	$15,000	$25,000	$15,767	$29,334

* Chapter 4 details the calculations of the amount owed each year with a constant total $ payment.

With the area under each curve computed in dollar-years, the ratio of total interest paid to area under the curve may be obtained:

Plan	Total Interest Paid	Area Under Curve (dollar-years)	Ratio of Total Interest Paid to Area Under Curve

1	$1200	15,000	0.08
2	2000	25,000	0.08
3	1260	15,767	0.08
4	2347	29,334	0.08

We see that the ratio of total interest paid to the area under the curve is constant and equal to 8%. Stated another way, the total interest paid equals the interest rate *times* the area under the curve.

From our calculations, we more easily see why the repayment plans require the payment of different total sums of money, yet are actually equivalent. The key factor is that the four repayment plans provide the borrower with different quantities of dollar-years. Since dollar-years times interest rate equals the interest charge, the four plans result in different total interest charges.

Equivalence Is Dependent on Interest Rate

In the example of Plans 1 to 4, all calculations were made at an 8% interest rate. At this interest rate, it has been shown that all four plans are equivalent to a present sum of $5000. But what would happen if we were to change the interest rate?

If the interest rate were increased to 9%, we know that the interest payment for each plan would increase, and the current calculated repayment schedules (Table 3–1, column **f**) would repay a sum *less* than the principal of $5000. By some calculations (to be explained later in this chapter and in Chapter 4), the equivalent present sum that each plan will repay at 9% interest is:

Plan	Repay a Present Sum of

1	$4877
2	4806
3	4870
4	4775

At the higher 9% interest, each of the repayment plans of Table 3–1 repays a present sum less than $5000. And, they do not repay the *same* present sum. Plan 1 would repay $4877 with 9% interest, while Plan 4 would repay only $4775. Thus, with interest at 9%, none of the plans is economically equivalent, for each will repay a different present sum. The series of payments were equivalent at 8%, but not at 9%. This leads to the conclusion **that equivalence is dependent on the interest rate.** Changing the interest rate destroys the equivalence between the series of payments.

Could we create revised repayment schemes that would be equivalent to $5000 now with interest at 9%? Yes, of course we could—but the revised plans would not be equivalent at 8%.

Thus far we have discussed computing equivalent present sums for a cash flow. But the technique of equivalence is not limited to a present computation. Instead, we could compute the equivalent sum for a cash flow at any point in time. We could compare alternatives in "Equivalent Year 10" dollars rather than "now" (Year 0) dollars. Furthermore, the equivalence need not be a single sum; it could be a series of payments or receipts. In Plan 3 of Table 3–1, the series of equal payments was equivalent to $5000 now. But the equivalency works both ways. Suppose we ask, What is the equivalent equal annual payment continuing for 5 years, given a present sum of $5000 and interest at 8%? The answer is $1252.

Differences in Economically Equivalent Plans

While Plans 1 to 4 are economically equivalent at 8% interest, they represent different approaches to paying back $5000. For example, most consumers when buying a car or a home are offered a repayment schedule similar to Plan 3 with a uniform payment. With a uniform repayment schedule the borrower is paying *all* of the interest due from that period along with at least a small payment toward reducing the balance due. Lenders prefer to reduce the risk of repayment problems by offering either Plan 1 or Plan 3. Borrowers prefer Plan 3 to Plan 1 because it has a lower initial payment for the same principal.

Plan 2 has only an interest payment every period, and the principal is repaid at the end. As will be described in Chapter 5, most long-term borrowing by firms and governments is done by issuing bonds that fit this pattern. Short-term borrowing by firms and governments often follows Plan 4.

Thus economically equivalent cash flows may have differences, such as the risk of nonpayment, that are important for decision making.

SINGLE PAYMENT COMPOUND INTEREST FORMULAS

To facilitate equivalence computations, a series of **interest formulas** will be derived. We use the following notation:

i = *interest rate per interest period;* in the equations the interest rate is stated as a decimal (that is, 9% interest is 0.09)
n = *number of interest periods*
P = *a present sum of money*
F = *a future sum of money* at the end of the n^{th} interest period, which is equivalent to P with interest rate i

Suppose a present sum of money P is invested for one year[1] at interest rate i. At the end of the year, we should receive back our initial investment P, together with interest equal to iP, or a total amount $P + iP$. Factoring P, the sum at the end of one year is $P(1 + i)$.

Let us assume that, instead of removing our investment at the end of one year, we agree to let it remain for another year. How much would our investment be worth at the end of Year 2? The end-of-first-year sum $P(1 + i)$ will draw interest in the second year of $iP(1 + i)$. This means that at the end of Year 2 the total investment will be

$P(1+i)+i[P(1+i)]$

This may be rearranged by factoring out $P(1 + i)$, which gives

$P(1+i)(1+i)$ or $P(1+i)^2$

If the process is continued for Year 3, the end-of-the-third-year total amount will be $P(1+i)^3$; at the end of n years, we will have $P(1+i)^n$. The progression looks like:

Year	Amount at Beginning of Interest Period	+ Interest for Period	= Amount at End of Interest Period
1	P	$+ iP$	$= P(1 + i)$
2	$P(1 + i)$	$+ iP(1 + i)$	$= P(1+i)^2$
3	$P(1+i)^2$	$+ iP(1+i)^2$	$= P(1+i)^3$
n	$P(1+i)^{n-1}$	$+ iP(1+i)^{n-1}$	$= P(1+i)^n$

In other words, a present sum P increases in n periods to $P(1+i)^n$. We therefore have a relationship between a present sum P and its equivalent future sum, F.

$$\text{Future sum} = (\text{Present sum})\ (1+i)^n$$

$$F = P(1+i)^n \qquad (3\text{-}3)$$

This is the **single payment compound amount formula** and is written in factor or functional notation as

$$F = P(F/P, i, n) \qquad (3\text{-}4)$$

The notation in parentheses (F/P, i, n) can be read as follows:

To find a future sum F, given a present sum, P, at an interest rate i per interest period, and n interest periods hence.

or

Find F, given P, at i, over n.

Functional notation is designed so that the compound interest factors may be written in an equation in an algebraically correct form. In Equation 3–4, for example, the functional notation is interpreted as

$$F = \cancel{P}\left(\frac{F}{\cancel{P}}\right)$$

which is dimensionally correct. Without proceeding further, we can see that when we derive a compound interest factor to find a present sum P, given a future sum F, the factor will be (P/F, i, n); so, the resulting equation would be

$$P = F(P/F, i, n)$$

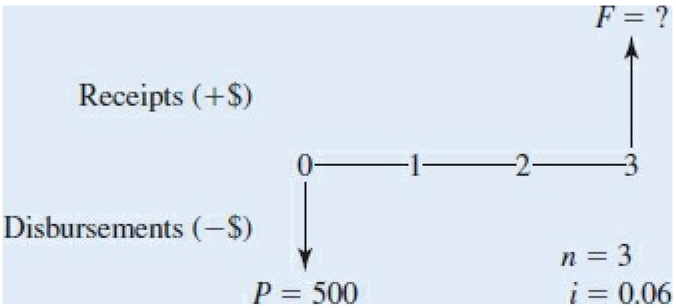
which is dimensionally correct.

EXAMPLE 3-5

If \$500 were deposited in a bank savings account, how much would be in the account 3 years from now if the bank paid 6% interest compounded annually?

FORMULA SOLUTION

From the viewpoint of the person depositing the \$500, the cash flows are:



The present sum P is \$500. The interest rate per interest period is 6%, and in 3 years there are three interest periods. The future sum F is found by using Equation 3–3, where P = \$500, i = 0.06, n = 3, and F is unknown:

$$F = P(1+i)^n = 500(1+0.06)^3 = \$595.50$$

Thus if we deposit \$500 now at 6% interest, there will be \$595.50 in the account in three years.

TABLE SOLUTION

The equation $F = P(1+i)^n$ need not be solved. Instead, *the single payment compound amount factor*, (F/P, i, n), is readily found in the tables given in Appendix C.[2] In this case the factor is

$$(F/P, 6\%, 3)$$

Knowing n = 3, locate the proper row in the 6% table of Appendix C. To find F given P, look in the first column, which is headed "Single Payment, Compound Amount Factor": or F/P for n = 3, we find 1.191.

Thus,

$$F = 500(F/P, 6\%, 3) = 500(1.191) = \$595.50$$

5-BUTTON SOLUTION

Appendix B describes spreadsheet annuity functions, financial calculators, and programmable scientific calculators that can be used instead of or in addition to the tables. Programmable scientific calculators can be used on the FE exam.

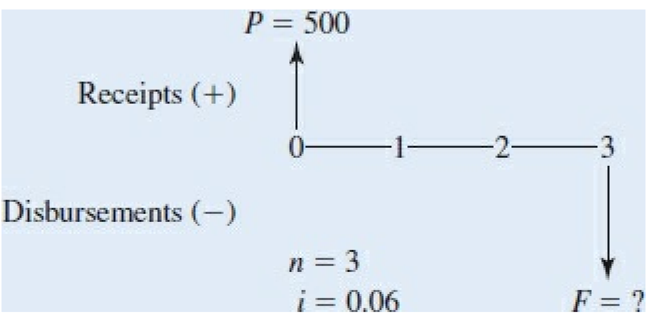
These spreadsheet functions and calculators make the same assumptions as the tabulated factors. But fewer numbers have to be entered, and many calculations can be completed much more quickly. Many problems can be solved by entering four values chosen from i, n, P, A, and F, and solving for the fifth value. Thus, these solutions are shown as 5-BUTTON SOLUTIONS.

As detailed in Appendix B, this solution corresponds to the more powerful spreadsheet and financial calculator functions that can also be programmed into HP calculators that can be used on the FE exam. Note that the A or PMT variable is the focus of Chapter 4, and it is 0 here.

	A	B	C	D	E	F	G	H	I	
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula	
2	Exp. 3-5	6%	3	0	-500		FV	$595.51	=FV(B2,C2,D2,E2)	
3									FV(rate, nper, pmt, [pv],[type])	
4										

BANK'S POINT OF VIEW

Before leaving this problem, let's draw another diagram of it, this time from the bank's point of view.



This indicates that the bank receives $500 now and must make a disbursement of F at the end of 3 years. The computation, from the bank's point of view, is

$$F = 500(F/P, 6\%, 3) = 500(1.191) = \$595.50$$

This is exactly the same as what was computed from the depositor's viewpoint, since this is just the other side of the same transaction. The bank's future disbursement equals the depositor's future receipt.

If we take $F = P(1+i)^n$ and solve for P, then

$$P = F\frac{1}{(1+i)^n} = F(1+i)^{-n}$$

This is the **single payment present worth formula.** The equation

$$P = F(1+i)^{-n} \quad (3\text{-}5)$$

in our notation becomes

$$P = F(P/F, i, n) \quad (3\text{-}6)$$

EXAMPLE 3-6

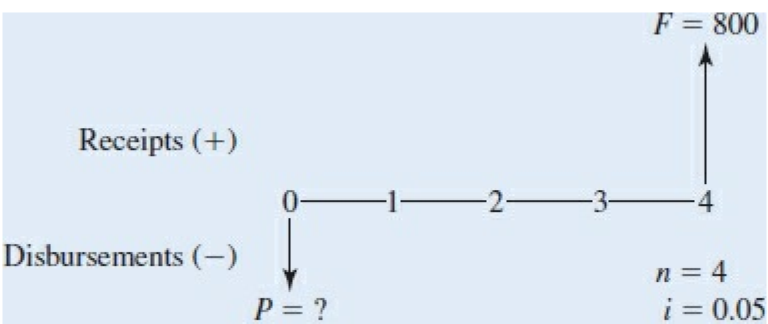
If you wish to have $800 in a savings account at the end of 4 years, and 5% interest will be paid annually, how much should you put into the savings account now?

FORMULA SOLUTION

$$F = \$800, \qquad i = 0.05, \qquad n = 4, \qquad P = \text{unknown}$$
$$P = F(1+i)^{-n} = 800(1+0.05)^{-4} = 800(0.8227) = \$658.16$$

Thus to have $800 in the savings account at the end of 4 years, we must deposit $658.16 now.

TABLE SOLUTION



$$P = F(P/F, i, n) = \$800(P/F, 5\%, 4)$$

From the compound interest tables,

$$(P/F, 5\%, 4) = 0.8227$$

$$P = \$800(0.8227) = \$658.16$$

5-BUTTON SOLUTION

As detailed in Appendix B, this solution corresponds to the spreadsheet and financial calculator functions that can also be programmed into HP calculators that can be used on the FE exam. Note that the *A* or *PMT* variable is the focus of Chapter 4, and it is 0 here.

Note that a positive future value results in a negative present value.

	A	B	C	D	E	F	G	H	I	
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula	
2	Exp. 3-6	5%	4	0		*800*	PV	-$658.16	=PV(B2,C2,D2,F2)	
3									PV(rate, nper, pmt, [fv],[type])	
4										

Here the problem has an exact answer. In many situations, however, the answer is rounded off, since it can be only as accurate as the input information on which it is based.

It is useful to examine how compound interest *adds up*, by examining the future values from Eq. 3-3 over time for different interest rates. Example 3–7 builds a spreadsheet to tabulate and graph a set of (*F*/*P*, *i*, *n*) values.

EXAMPLE 3-7

Tabulate the future value factor for interest rates of 5%, 10%, and 15% for *n*’s from 0 to 20 (in 5’s).

SOLUTION

This example has been built in Excel and some of the described shortcuts may not work in other spreadsheet packages—and they may work differently for you even in Excel as menus and capabilities are continually evolving.

It is probably most efficient to simply enter the three interest rates in row 1. However, it is probably more efficient to enter the first two years of 0 and 5, select them, and then extend the selection to autofill the values of 10, 15, and 20.

Then the formula $(1+i)^n$ is entered into cell B2 as =(1+B$1)^$A2. Note that the interest rates are all in row 1, so that the B$1 cell reference fixes the row as an absolute address. Since the years are all in column A, the $A2 cell reference fixes the column as an absolute address (see Appendix A for a more complete explanation of relative and absolute references). The formula for cell D6 is included in the table to show how the relative and absolute addresses change as the formula is copied. In the example the formatting of cell B2 was changed to 3 decimals before doing the copying.

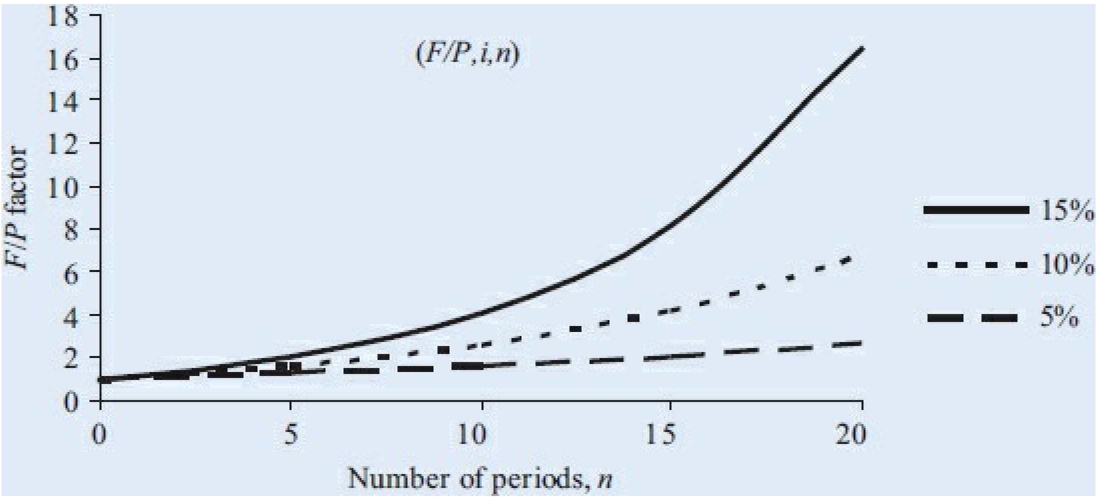
This formula can be copied into the rest of column B by simply double-clicking on the cell's lower right corner. Then select cells B2:B6, and drag the lower right corner to D6. The table is complete.

	A	B	C	D	E	F
1	*n* | *i*	5%	10%	15%		
2	0	1.000	1.000	1.000		
3	5	1.276	1.611	2.011		
4	10	1.629	2.594	4.046		
5	15	2.079	4.177	8.137		
6	20	2.653	6.727	16.367	=(1+D$1)^$A6	

To graph the values, it is easiest to select cells A2:D6 and INSERT a scatter or *xy* plot. In some versions of Excel, the figure may look strange until the data are selected and the row and column are switched (A2:A6 needed to be the *x* variable). The 3 series are then edited one-by-one so that the interest rates cells were entered as NAMES for the series. The data series was reordered (up/down arrows under SELECT DATA) so that the 15% legend label would be on top and the 5% label on the bottom (to match the graph).

So that the curves would "fill" the figure, each axis was selected so it could be formatted for maximum values and for the *y*-axis the number of decimals.

Finally under CHART LAYOUT titles were added to the chart and each axis. Since this was designed for print, the data series also had to be edited to change the line-type rather than relying on color to distinguish the series.



[NOMINAL AND EFFECTIVE INTEREST]()

EXAMPLE 3-8

Suppose the bank changed the interest policy in [Example 3–5]() to “6% interest, compounded quarterly.” For this situation, how much money would be in the account at the end of 1 year, assuming a $100 deposit now?

TABLE SOLUTION

First, we must be certain to understand the meaning of *6% interest, compounded quarterly.* There are two elements:

6% interest: Unless otherwise described, it is customary to assume that the stated interest is for a one-year period. *If the stated interest is for any other period, the time frame must be clearly stated.*
Compounded quarterly: This indicates there are four interest periods per year; that is, an interest period is 3 months long.

We know that the 6% interest is an annual rate because if it were for a different period, it would have been stated. Since we are dealing with four interest periods per year, it follows that the interest rate per interest period is 1½% (=6%/4). For the total 1-year duration, there are 4 interest periods. Thus

$$P = \$100, \qquad i = 0.06/4 = 0.015, \qquad n = 1 \times 4 = 4, \qquad F = \text{unknown}$$

$$\begin{aligned} F &= P(1+i)^n = P(F/P, i, n) \\ &= \$100(1+0.015)^4 = \$100(F/P, 1^1/_2\%, 4) \\ &= \$100(1.061) = \$106.1 \end{aligned}$$

A $100 deposit now would yield $106.1 in 1 year.

5-BUTTON SOLUTION

As detailed in Appendix B, this solution corresponds to the spreadsheet and financial calculator functions that can also be programmed into HP calculators that can be used on the FE exam. The *A* or *PMT* variable is the focus of Chapter 4, and it is 0 here.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 3-8	1.50%	4	0	-100		FV	-$106.14	=FV(B2,C2,D2,E2)

Nominal interest rate per year, *r*, is the annual interest rate without considering the effect of any compounding.

In Example 3–8, the bank pays 1½% interest every 3 months. The nominal interest rate per year, *r*, therefore, is 4 × 1½% = 6%. The federal government mandates that lenders provide the **annual percentage rate (APR)** for any loan. For credit cards and many consumer loans this is the nominal interest rate. Calculating the APR for a mortgage with origination fees and points is more complex.

Effective interest rate per year, i_a, is the *annual* interest rate taking into account the effect of any compounding during the year.

In Example 3–8, we saw that $100 left in the savings account for one year increased to $106.14, so the interest paid was $6.14. The effective interest rate per year, i_a, is \$6.14/\$100.00 = 0.0614 = 6.14%. The effective annual interest rate earned on savings accounts, certificates of deposit (CD), bonds, and so on is often called the annual percentage yield **(APY)**. To calculate the effective annual interest rate i_a, we use the following variables:

r = nominal interest rate per interest period (usually one year)

i = effective interest rate per interest period

m = number of compounding subperiods per time period

Using the method presented in Example 3–8, we can derive the equation for the effective interest rate. If a \$1 deposit were made to an account that compounded interest m times per year and paid a nominal interest rate per year, r, the *interest rate per compounding subperiod* would be r/m, and the total in the account at the end of one year would be

$$\$1\left(1+\frac{r}{m}\right)^m \qquad \text{or simply} \qquad \left(1+\frac{r}{m}\right)^m$$

If we deduct the \$1 principal sum, the expression would be

$$\left(1+\frac{r}{m}\right)^m - 1$$

Therefore,

Effective annual interest rate $$i_a = \left(1+\frac{r}{m}\right)^m - 1 \qquad (3\text{-}7)$$

where

r = nominal interest rate per year

m = number of compounding subperiods per year

Or, substituting the effective interest rate per compounding subperiod, $i = (r/m)$,

Effective annual interest rate $$i_a = (1+i)^m - 1 \qquad (3\text{-}8)$$

where

i = effective interest rate per compounding subperiod

m = number of compounding subperiods per year

Either Equation 3–7 or 3–8 may be used to compute an effective interest rate per year. This and many other texts use i_a for the effective annual interest rate. The Fundamentals of Engineering (FE) exam uses i_e for the effective annual interest rate in its supplied material.

EXAMPLE 3-9

If a credit card charges 1½% interest every month, what are the nominal and effective interest rates per year?

SOLUTION

Nominal interest rate per year $\quad r = 12 \times 1\frac{1}{2}\% = 18\%$

Effective annual interest rate
$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$
$$= \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 0.1956$$
$$= 19.56\%$$

Alternately,

Effective annual interest rate
$$i_a = (1 + i)^m - 1$$
$$= (1 + 0.015)^{12} - 1 = 0.1956 = 19.56\%$$

5-BUTTON SOLUTION

This solution corresponds to the spreadsheet and financial calculator functions. See Appendix B for details.

If $PV = -1$, then $FV = 1$ + effective rate (for the given number of compounding periods and interest rate per compounding period). The complete formula for the interest rate is *effective rate* = (*FV* − *PV*)/*PV* since the total period is 1 year.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 3-9	1.50%	12	0	-1		FV	1.1956	=FV(B2,C2,D2,E2)

So subtracting 1 from the FV value and restating as an interest rate, the effective rate is 19.56%.

SPREADSHEET SOLUTION

This problem can also be solved with the function =EFFECT(nominal_rate, npery).

	A	B	C	D	E
1	Nominal annual rate, r	Periods per year, m	Effective rate, i_a	Answer	Spreadsheet Function
2	18.00%	12		19.56%	=EFFECT(A2,B2)

There is a corresponding =NOMINAL(effect_rate, npery) function to convert effective rates to nominal.

EXAMPLE 3-10

A payday lender lends money on the following terms: “If I give you $100 today, you will write me a check for $120, which you will redeem or I will cash on your next payday.” Noting that calculated rates would be even higher for closer paydays, assume that the payday is two weeks away.

(a) What nominal interest rate per year (*r*) is the lender charging?
(b) What effective interest rate per year (i_a) is the lender charging?
(c) If the lender started with $100 and was able to keep it, as well as all the money received, loaned out at all times, how much money does the lender

have at the end of one year?

SOLUTION TO PART a

$$F = P(F/P, i, n)$$

$$120 = 100(F/P, i, 1)$$

$$(F/P, i, 1) = 1.2$$

Therefore, i = 20% per 2-week period

Nominal interest rate per year $= 26 \times 0.20 = 5.20 = 520\%$

SOLUTION TO PART b

$$\text{Effective annual interest rate} \quad i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{5.20}{26}\right)^{26} - 1 = 114.48 - 1 = 113.48 = 11{,}348\%$$

Or

$$\text{Effective annual interest rate} \quad i_a = (1 + i)^m - 1$$

$$= (1 + 0.20)^{26} - 1 = 113.48 = 11{,}348\%$$

SOLUTION TO PART c

$$F = P(1 + i)^n = 100(1 + 0.20)^{26} = \$11{,}448$$

$$= \$100 \text{ principal} + \$11{,}348 \text{ interest}$$

5-BUTTON/ SPREADSHEET SOLUTION

This solution corresponds to the spreadsheet and financial calculator functions.

If $PV = -1$, then $FV = 1$ + effective rate (for the given number of compounding periods and interest rate per compounding period). The complete formula for the interest rate is *effective rate* = $(FV - PV)/PV$ since the total period is 1 year.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 3-10		1	0	-100	120	i	20%	=RATE(C2,D2,E2,F2)
3									
4	Nominal	20%	26				r	520%	=B3*C3
5									
6	Effective	520%	26				i_a	11348%	=EFFECT(B6,C6)
7	or	20%	26	0	-1		FV	114.48	=FV(B9,C9,D9,E9)
8							i_a	113.48	=FV-1
9									
10	FV	20%	26	0	-100		FV	$11,448	=FV(B8,C8,D8,E8)

One should note that *i* was described earlier simply as the interest rate per interest period. We were describing the effective interest rate without making any fuss about it. A more precise definition, we now know, is that *i* is the *effective* interest rate per interest period. Although it seems more complicated, we are describing the same exact situation, but with more care.

The nominal interest rate *r* is often given for a one-year period (but it could be given for either a shorter or a longer time period). In the special case of a nominal interest rate that is given per compounding subperiod, the effective interest rate per compounding subperiod, i_m, equals the nominal interest rate per subperiod, *r*.

In the typical effective interest computation, there are multiple compounding subperiods ($m > 1$). The resulting effective interest rate is either the solution to the problem or an intermediate solution, which allows us to use standard compound interest factors to proceed to solve the problem.

For **continuous compounding** the effective annual interest rate is,

$i_a = e^r - 1$(3-9)

Table 3–3 tabulates the effective interest rate for a range of compounding frequencies and nominal interest rates. It should be noted that when a nominal interest rate is compounded annually, the nominal interest rate equals the effective interest rate. Also, it should be noted that increasing the frequency of compounding to semiannually or monthly matters more than increasing the frequency to daily or continuously.

Continuous compounding and cash flows that occur throughout a year would be a *better theoretical* model for most engineering economy problems. However, such models are extremely rare and are becoming more so because they are not supported by Excel functions. Thus, continuous compounding is most interesting as the limit for an effective interest rate, as shown in Example 3−11.

Table 3-3 Nominal and Effective Interest

Nominal Interest Rate per Year	Effective Annual Interest Rate, i_a When Nominal Rate Is Compounded				
r (%)	Yearly	Semiannually	Monthly	Daily	Continuously
1	1%	1.0025%	1.0046%	1.0050%	1.0050%
2	2	2.0100	2.0184	2.0201	2.0201
3	3	3.0225	3.0416	3.0453	3.0455
4	4	4.0400	4.0742	4.0809	4.0811

5	5	5.0625	5.1162	5.1268	5.1271
6	6	6.0900	6.1678	6.1831	6.1837
8	8	8.1600	8.3000	8.3278	8.3287
10	10	10.2500	10.4713	10.5156	10.5171
15	15	15.5625	16.0755	16.1798	16.1834
25	25	26.5625	28.0732	28.3916	28.4025

EXAMPLE 3-11 (Example 3–10 Revisited)

If the savings bank in Example 3–10 changes its interest policy to 6% interest, compounded continuously, what are the nominal and the effective interest rates?

SOLUTION

The nominal interest rate remains at 6% per year.

$$\begin{aligned}\text{Effective interest rate} &= e^r - 1\\ &= e^{0.06} - 1 = 0.0618\\ &= 6.18\%\end{aligned}$$

SUMMARY

This chapter describes cash flow tables, the time value of money, and equivalence. The single payment compound interest formulas were derived. These concepts and these interest formulas are the foundation of the rest of this book and the practice of engineering economy.

Time value of money: The continuing offer of banks to pay interest for the temporary use of other people's money is ample proof that there is a time value of money. Thus, we would always choose to receive $1000 today rather than the promise of $1000 to be paid at a future date.
Equivalence: What sum would a person be willing to accept in one year instead of $1000 today? At an interest rate of 3%, $1030 would be required. If $1000 today and $1030 a year from today are considered equally desirable, we say the two sums of money are equivalent. But, if we decided that a 6% interest rate is applicable, then $1030 a year from today would no longer be equivalent to $1000 today. Equivalence depends on the interest rate.

This chapter also defined simple interest, where interest does not carry over and become part of the principal in subsequent periods. Unless otherwise specified, all interest rates in this text are compound rates.

Single Payment Formulas

These formulas are for compound interest, which is used in engineering economy.

Compound amount $F = P(1+i)^n = P(F/P, i, n)$

Present worth $P = F(1+i)^{-n} = F(P/F, i, n)$

where

i = interest rate per interest period (stated as a decimal)

n = number of interest periods

P = a present sum of money

F = a future sum of money at the end of the n^{th} interest period that is equivalent to *P* with interest rate *i*

Nominal Annual Interest Rate, *r*

The annual interest rate without considering the effect of any compounding. Also called the annual percentage rate (APR).

Effective Annual Interest Rate, i_a

The annual interest rate taking into account the effect of any compounding during the year.

Effective annual interest rate (periodic compounding):

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

or

$$i_a = (1 + i)^m - 1$$

Effective annual interest rate (continuous compounding):

$$i_a = e^r - 1$$

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

3-1	John Buck opens a savings account by depositing $1000. The account pays 6% simple interest. After 3 years John makes another deposit, this time for $2000. Determine the amount in the account when John withdraws the money 8 years after the first deposit.

SOLUTION

3-2 If you had $1000 now and invested it at 4%, how much would it be worth 12 years from now?

SOLUTION

3-3 Mr. Ahmed deposited $200,000 in the Old Third National Bank. If the bank pays 3% interest, how much will he have in the account at the end of 10 years?

SOLUTION

3-4 If you can earn 2% interest on your money, how much would $1000 paid to you 12 years in the future be worth to you now?

SOLUTION

3-5 One thousand dollars, deposited into an account that pays interest monthly, is allowed to remain in the account for 3 years. Calculate the balance at the end of the 3 years if the annual interest rate is 6%.

SOLUTION

3-6 On July 1 and September 1, Abby placed $2000 into an account paying 3% compounded monthly. How much was in the account on October 1?

SOLUTION

3-7 Determine the amount of money accumulated in 5 years with an initial deposit of $10,000, if the account earned 12% compounded monthly the first 3 years and 15% compounded semiannually the last 2 years.

SOLUTION

3- An investment of $10,000 six years ago has now grown to $20,000. Determine the annual interest rate on this investment, assuming annual

8 compounding.

SOLUTION

3-9 Four hundred dollars is deposited into an account that compounds interest quarterly. After 10 quarters the account balance is $441.85. Determine the nominal interest paid on the account.

SOLUTION

3-10 Margaret M. withdrew $630,315 from an account into which she had invested $350,000. If the account paid interest at 4% per year, she kept the money in the account for how many years?

SOLUTION

3-11 Downtown has been experiencing an explosive population growth of 10% per year. At the end of 2017 the population was 16,000. If the growth rate continues unabated, how many years will it take the population to triple?

SOLUTION

3-12 If the interest rate is 6% compounded quarterly, how long (number of quarters) will it take to earn $100 interest on an initial deposit of $300?

SOLUTION

3-13 Two years ago, Luckett Land Developers Inc. borrowed $350,000 at a nominal interest rate of 4% compounded quarterly. Due to an economic slowdown, Luckett will be unable to pay off the loan, which is due today. Johnson City Bank has agreed to refinance the loan amount due, plus another $100,000 at a nominal interest rate of 3% compounded monthly. The new loan must be paid off 2 years from now. How much will Luckett owe when the new loan must be paid off?

SOLUTION

3-14 One thousand dollars, deposited into an account that pays interest monthly, is allowed to remain in the account for 3 years. The balance at the end of the 3 years is $1309.00. Calculate the nominal interest paid on this account.

SOLUTION

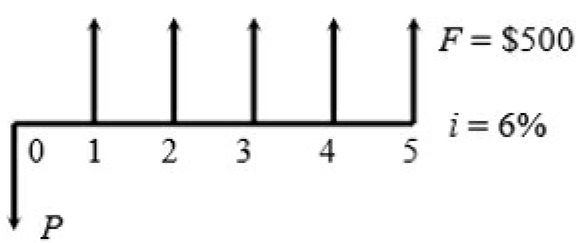
3-15 Isabella started saving for her retirement 15 years ago. If she invested $30,000 in a stock fund that averaged a 15% rate of return over the 15-year period, and expects to make no more investments and average a 9% return in the future, how long will it be before she has $1,000,000 in her retirement account?

SOLUTION

3-16 A man decides to put $1000 per month beginning 1 month from today into an account paying 3% compounded monthly. Determine how much (to the nearest penny) will be in the account immediately after the fourth deposit; use only basic concepts.

SOLUTION

3-17 Determine the value of *P* using the appropriate factor.



SOLUTION

3- The Block Concrete Company borrowed $20,000 at 8% interest, compounded semiannually, to be paid off in one payment at the end of 4 years. At the end of the 4 years, Block made a payment of $8000 and

18 refinanced the remaining balance at 6% interest, compounded monthly, to be paid at the end of 2 years. Calculate the amount Block owes at the end of the 2 years.

SOLUTION

3-19 The multistate Powerball Lottery, worth $182 million, was won by a single individual who had purchased five tickets at $1 each. The winner was given two choices: receive 26 payments of $7 million each, with the first payment to be made now and the rest to be made at the end of each of the next 25 years, or receive a single lump-sum payment now that would be equivalent to the 26 payments of $7 million each. If the state uses an interest rate of 4% per year, find the amount of the lump sum payment.

SOLUTION

3-20 A woman deposited $10,000 into an account at her credit union. The money was left on deposit for 10 years. During the first 5 years, the woman earned 9% interest, compounded monthly. The credit union then changed its interest policy; as a result, in the second 5 years the woman earned 6% interest, compounded quarterly.

a. How much money was in the account at the end of the 10 years?
b. Calculate the rate of return that the woman received.

SOLUTION

3-21 Lexie C. deposits $4000 into an account paying 3% simple interest. After 3.5 years, she moves the money into an account that will earn 4% compounded quarterly. After another 4 years, she deposits another $2000 into the account. How much will be in the account 1 year after the final deposit?

SOLUTION

3- Justin P. deposited $2000 into an account 5 years ago. Simple interest

22 was paid on the account. He has just withdrawn $2876. What interest rate did he earn on the account?

SOLUTION

3-23 A sum of $5000 is invested for 5 years with annual interest rates of 9% the first, 8% the second, 12% the third, 6% the fourth, and 15% the fifth years, respectively. Determine the future amount after 5 years.

SOLUTION

3-24 Callis Construction LLC is planning to expand its site planning division 4 years from now. In order to expand, the CFO of the company has determined that $400,000 will be required. Eighth National Bank has agreed to pay 8% simple interest for the first 2 years the money is on deposit with the terms changing to a 6% nominal rate compounded monthly for the last 2 years. How much should Callis deposit today in order to have the needed $400,000?

SOLUTION

3-25 Five years ago Mary Skinner inherited a sum of $50,000 and bought 1000 shares of IBEM at $50 per share. She sold 400 shares of the stock 3 years ago for $57 per share and invested the money in a CD that paid 2% interest per year compounded quarterly. Mary cashed the CD today and sold the remaining 600 shares of stock for $65 per share. What nominal interest rate did she earn on her inheritance?

SOLUTION

3-26 A local bank is advertising to savers a rate of 6% compounded monthly, yielding an effective annual rate of 6.168%. If $2000 is placed in savings now and no withdrawals are made, how much interest (to the penny) will be earned in 1 year?

SOLUTION

3-27 A small company borrowed $10,000 to expand the business. The entire principal of $10,000 will be repaid in 2 years, but quarterly interest of $330 must be paid every 3 months. What nominal annual interest rate is the company paying?

SOLUTION

3-28 Cole's Home Solutions policy is to charge 2¼% interest each month on unpaid credit balances. What nominal interest is Cole's charging? What is the effective interest?

SOLUTION

3-29 EZ Pay Loans will lend you $100 today with repayment of $117.50 due in 1 month. Determine the nominal and effective rate of this loan.

SOLUTION

3-30 For a nominal interest rate of 6%, what is the effective interest rate if interest is

a. compounded quarterly?
b. compounded monthly?
c. compounded continuously?

SOLUTION

3-31 Which is the better investment, a fund that pays 5% compounded annually or one that pays 4.8% compounded continuously?

SOLUTION

3-32 The effective interest rate is 9.38%. If interest is compounded monthly, what is the nominal interest rate?

SOLUTION

3-33	The effective interest rate on a mortgage with monthly payments is 9.38%. What is the monthly interest rate on the mortgage? What is the nominal interest rate?

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Simple and Compound Interest

3-1	How would you describe the difference between simple and compound interest to a class of 3rd graders? If you are not sure if the interest that you are paying is simple or compound, how will you find out?
3-2 A E	A student borrowed $2000 from her parents and agreed to repay it at the end of 3 years, together with 5% simple interest. (*a*) How much is repaid? (*b*) Do you ever borrow or lend money with family and friends? Given the time value of money, is it ethical not to pay interest? How does the amount or duration of the loan matter?
3-3	A $5000 loan was to be repaid with 5% simple annual interest. A total of $6000 was paid. How long had the loan been outstanding? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*
3-4 A	At an interest rate of 8% per year, $50,000 today is equivalent to how much a year from now? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*

Calculate the interest and total amount due at the end of the loan for both simple and compound interest.

	Loan	Years	Rate
(*a*)	$ 1000	2	5%
(*b*)	$ 1500	5	6%
(*c*)	$10,000	10	10%
(*d*)	$25,000	15	15%
(*e*)	$47,750	20	20%

3-5

3-6 A How long will it take for an investment to double at a 3% per year ______?
(*a*) simple interest rate
(*b*) compound interest rate
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

3-7 G Carolina Land Trust purchases private land for clean waterway conservation. A benefactor invested $1,000,000 for 15 years and gave the principal and accrued interest of $1,200,000 to the trust. (*a*) What simple interest rate did the investment earn? (*b*) Had the interest been compounded annually at that rate, what would have been the value of the gift?

Equivalence

Which is more valuable, $20,000 received now or $5000 per year for

3-8 4 years? Why? Explain the term “time value of money” in your own words.

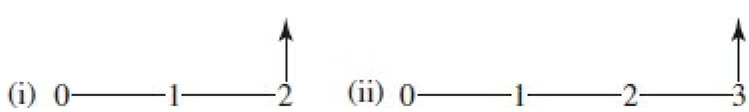
3-9 Magdalen, Miriam, and Mary June were asked to consider two different cash flows: $500 that they could receive today and $1000 that would be received 3 years from today. Magdalen wanted the $500 dollars today, Miriam chose to collect $1000 in 3 years, and Mary June was indifferent between these two options. Can you offer an explanation of the choice made by each woman?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

3-10 (*a*) If $160 at Time “0” will be worth $170 a year later and was $150 a year ago, compute the interest rate for the past year and the interest rate next year.
(*b*) Assume that $150 invested a year ago will return $170 a year from now. What is the annual interest rate in this situation?

3-11 A firm has borrowed $5,000,000 for 5 years at 10% per year compound interest. The firm will make no payments until the loan is due, when it will pay off the interest and principal in one lump sum. What is the total payment?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

3-12 What sum of money now is equivalent to $9500 three years later, if
A interest is 3% per 6-month period?

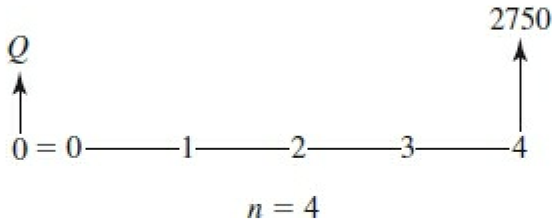
3-13 The following cash flows are equivalent in value if the interest rate is i. Which one is more valuable if the interest rate is $2i$?



Single Payment Factors

Solve the diagram for the unknown Q assuming a 6% interest rate.

3-14
A



3-15 A student has inherited \$50,000. If it is placed in a savings account that earns 3% interest, how much is in the account in 30 years?

3-16
A

We know that a certain piece of equipment will cost \$275,000 in 6 years. How much must be deposited today using 8% interest to pay for it?

3-17 You are planning to withdraw \$100 in Year 1, \$150 in Year 3, and \$200 in Year 5. At a 5% interest rate, what is the present worth of these withdrawals?

Contributed by Gana Natarajan, Oregon State University

3-18
A

Suppose that \$2700 is deposited in an account that earns 4% interest. How much is in the account

(*a*) after 5 years?
(*b*) after 10 years?
(*c*) after 20 years?
(*d*) after 50 years?
(*e*) after 100 years?

3-19 An inheritance will be \$25,000. The interest rate for the time value of money is 6%. How much is the inheritance worth now, if it will be received

(*a*) in 10 years?
(*b*) in 20 years?
(*c*) in 35 years?
(*d*) in 50 years?

3-20
A

Rita borrows \$2000 from her parents. She repays them \$2600. What is the interest rate if she pays the \$2600 at the end of

(*a*) Year 2?
(*b*) Year 3?
(*c*) Year 6?
(*d*) Year 10?

A savings account earns 2.5% interest. If \$3000 is invested, how many years is it until each of the following amounts is on deposit?

3-21

(*a*) $3394

(*b*) $3655

(*c*) $4035

(*d*) $5165

3-22 to 3-24 *Contributed by Paul R. McCright, University of South Florida*

3-22 A

Alvin's Uncle Arnold gave him $16,000 from selling the old family farm. Alvin wants to start college and have $12,000 available to buy a used car when he graduates in 4 years. Alvin earns 2% in his savings account. How much can he spend on a motorcycle now and still have enough to grow to the $12,000 he needs when he graduates?

3-23 G

Ace Manufacturing is building a new Platinum Level LEED certified facility that will cost $44M. Ace will borrow $40M from First National Bank and pay the remainder immediately as a down payment. Ace will pay 7% interest but will make no payments for 4 years, at which time the entire amount will be due.

(*a*) How large will Ace's payment be?

(*b*) What is LEED, by whom was it devised, and why?

3-24 A

Maheera can get a certificate of deposit (CD) at his bank that will pay 3.5% annually for 10 years. If he places $2442 in this CD, how much will it be worth when it matures?

(*a*) Use the formula.

(*b*) Use the interest tables and interpolation.

(*c*) Use a calculator or spreadsheet for a 5-button solution.

3-25

How much must you invest now at 4.2% interest to accumulate $175,000 in 46 years?

3-26 A

In 1995 an anonymous private collector purchased a painting by Picasso entitled *Angel Fernandez de Soto* for $29,152,000. The picture depicts Picasso's friend de Soto seated in a Barcelona cafe drinking absinthe. The painting was done in 1903 and was valued then at $600. If the painting was owned by the same family until its sale in 1995, what rate of return did they receive on the $600 investment?

In 1990 Mrs. John Hay Whitney sold her painting by Renoir, *Au*

3-27 *Moulin de la Galette*, depicting an open-air Parisian dance hall, for $71 million. The buyer also had to pay the auction house commission of 10%, or a total of $78.1 million. The Whitney family had purchased the painting in 1929 for $165,000.
(*a*) What rate of return did Mrs. Whitney receive on the investment?
(*b*) Was the rate of return really as high as you computed in **(*a*)**? Explain.

3-28 A The local bank pays 5% interest on savings deposits. In a nearby town, the bank pays 1.25% per quarter. A man who has $4000 to deposit wonders whether the higher interest paid in the nearby town justifies driving there. If all money is left in the account for 3 years, how much interest would he obtain from the out-of-town bank?

3-29 One thousand dollars is borrowed for one year at an interest rate of ½ % per month. If the same sum of money could be borrowed for the same period at an interest rate of 6% per year, how much could be saved in interest charges?

3-30 A A sum of money invested at 2% per 6-month period (semiannually) will double in amount in approximately how many years?

3-31 The tabulated factors for i = 2.5% to 12% stop at n = 100. How can they be used to calculate (P/F, i, 150)? (P/F, i, 200)?

3-32 A If lottery winnings of Q are invested now at an interest rate of 9%, how much is available to help fund an early retirement in Year 25?

3-33 A firm paid $160,000 for a building site two years ago. It is now worth $200,000, and the firm's plans have changed so that no building is planned. The firm estimates that the land will be worth $240,000 in four years. If the firm's interest rate is 5%, what should it do?

3-34 A What is the present worth of a two-part legal settlement if the interest rate is 4%? $100,000 is received at the end of Year 1 and $400,000 at the end of Year 5.

3-35 An R&D lab will receive $250,000 when a proposed contract is signed, a $200,000 progress payment at the end of Year 1, and $400,000 when the work is completed at the end of Year 2. What is the present worth of the contract at 15%?

3-36 An engineer invested $5000 in the stock market. For the first 6 years

A the average return was 9% annually, and then it averaged 3% for 4 years. How much is in the account after 10 years?

3-37 Camila Vega made an investment of $10,000 in a savings account 10 years ago. This account paid interest of 4% for the first 4 years and 6% interest for the remaining 6 years. How much is this investment worth now?

Nominal and Effective Interest Rates

3-38 A thousand dollars is invested in Green Bonds for 7 months at an interest rate of 0.75% per month. What is the nominal interest rate?

A (*a*) What is the effective interest rate?

G (*b*) What are Green Bonds? What type of projects do the bonds encourage?

3-39 A firm charges its credit customers 12/3% interest per month. What is the effective interest rate?

3-40 If the nominal annual interest rate is 9% compounded quarterly, what

A is the effective annual interest rate?

3-41 A local store charges 1½% each month on the unpaid balance for its charge account. What nominal annual interest rate is being charged? What is the effective interest rate?

3-42 What interest rate, compounded monthly, is equivalent to a 10.04%

A effective interest rate?

3-43 A bank advertises it pays 4% annual interest, compounded daily, on savings accounts, provided the money is left in the account for 5 years. What is the effective annual interest rate?

3-44 At the Central Furniture Company, customers who buy on credit pay an effective annual interest rate of 16.1%, based on monthly compounding.

A (*a*) What is the nominal annual interest rate that they pay?

E (*b*) Research the effective annual interest rates charged on a credit card that you or a friend has. How does the rate change if there is a late or skipped payment? What drives these rates? Are the rates ethical? Why or why not?

3-45 A student bought a $75 used guitar and agreed to pay for it with a single $80 payment at the end of 3 months. What is the nominal annual interest rate? What is the effective interest rate?

3-46 A bank is offering to sell 6-month certificates of deposit for $12,000. At the end of 6 months, the bank will pay $13,000 to the certificate owner. Compute the nominal annual interest rate and the effective annual interest rate.
A

3-47 A firm spent $2 million for new equipment that reduces greenhouse gases in their operations. The process improvement saved them $14,000 in the first month.
G
(*a*) What is the first month's rate of return on this investment? Express that value on an annual effective basis.
(*b*) If this investment was judged not to make economic sense, should some entity pay to reduce such emissions? Who and why?

3-48 Steelgrave Financing offers payday loans. The firm charges a $10 interest fee for a one-week period on a $250 loan. What are the nominal and effective annual interest rates on this loan? *Contributed by Gillian Nicholls, Southeast Missouri State University*
A

3-49 The treasurer of a firm noted that many invoices were received with the following terms of payment: "2%—10 days, net 30 days." Thus, if the bill is paid within 10 days of its date, he could deduct 2%. Or the full amount would be due 30 days from the invoice date. Assuming a 20-day compounding period, the 2% deduction for prompt payment is equivalent to what effective annual interest rate?

3-50 First Bank is sending university alumni an invitation to obtain a credit card, with the name of their university written on it, for a nominal 11.9% interest per year after 6 months of 0% interest. Interest is compounded monthly. If you fail to make the minimum payment in any month, your interest rate could increase (without notice) to a nominal 23.9% per year. Calculate the effective annual interest rates the credit company is charging in both cases.
Contributed by D. P. Loucks, Cornell University

Mona Persian is considering a new investment fund with a semiannual interest rate of 2.5%. Any money she invests would have to be left in it for at least five years if she wanted to withdraw it

3-51 without a penalty.

(*a*) What is the nominal interest rate?

(*b*) What is the annual effective interest rate?

(*c*) If Mona deposits $10,000 in the fund now, how much should it be worth in five years?

Contributed by Gillian Nicholls, Southeast Missouri State University

3-52 A A department store charges 1.5% interest per month, compounded continuously, on its customers' charge accounts. What is the nominal annual interest rate? What is the effective interest rate?

3-53 A bank pays 4% nominal annual interest on special three-year certificates. What is the effective annual interest rate if interest is compounded

(*a*) Every three months?

(*b*) Daily?

(*c*) Continuously?

3-54 A friend was left $50,000 by his uncle. He has decided to put it into a savings account for the next year or so. He finds there are varying interest rates at savings institutions: 2.25% compounded every two months, 2.30% compounded quarterly, and 2.20% compounded continuously. He wishes to select the savings institution that will give him the highest return on his money. What interest rate should he select?

3-55 Jill deposited $8000 into a bank for 6 months. At the end of that time, she withdrew the money and received $8250. If the bank paid interest based on continuous compounding:

(*a*) What was the effective annual interest rate?

(*b*) What was the nominal annual interest rate?

Minicases

The local garbage company charges $6 a month for garbage collection. It had been their practice to send out bills to their 100,000 customers at the end of each 2-month period. Thus, at the end of February it would send a bill to each customer for $12 for garbage collection during January and February.

Recently the firm changed its billing date: it now sends out the 2-month

3.56 A bills after one month's service has been performed. Bills for January and February, for example, are sent out at the end of January. The local newspaper points out that the firm is receiving half its money before the garbage collection. This unearned money, the newspaper says, could be temporarily invested for one month at 1% per month interest by the garbage company to earn extra income.

Compute how much extra income the garbage company could earn each year if it invests the money as described by the newspaper.

3-57 The Apex Company sold a water softener to Marty Smith. The price of the unit was $350. Marty asked for a deferred payment plan, and a contract was written. Under the contract, the buyer could delay paying for the water softener if he purchased the coarse salt for recharging the softener from Apex. At the end of 2 years, the buyer was to pay for the unit in a lump sum, with interest at a quarterly rate of 1.5%. According to the contract, if the customer ceased buying salt from Apex at any time prior to 2 years, the full payment due at the end of 2 years would automatically become due.

Six months later, Marty decided to buy salt elsewhere and stopped buying from Apex, whereupon Apex asked for the full payment that was to have been due 18 months hence. Marty was unhappy about this, so Apex offered as an alternative to accept the $350 with interest at 10% per semiannual period for the 6 months. Which alternative should Marty accept? Explain.

3-58 The U.S. recently purchased $1 billion of 30-year zero-coupon bonds from a struggling foreign nation. The bonds yield 4½% per year interest. Zero coupon means the bonds pay no annual interest payments. Instead, all interest is at the end of 30 years.

A U.S. senator objected, claiming that the correct interest rate for bonds like this is 7¼%. The result, he said, was a multimillion dollar gift to the foreign country without the approval of Congress. Assuming the senator's rate is correct, how much will the foreign country have saved in interest?

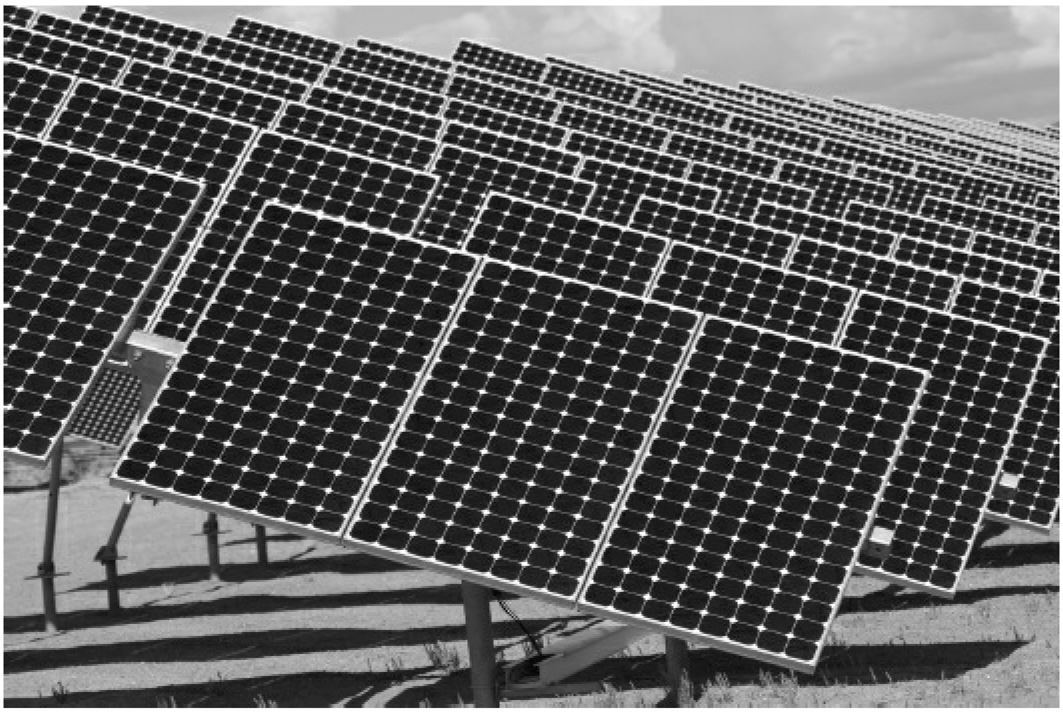
[1] A more general statement is to specify "one interest period" rather than "one year." Since it is easier to visualize one year, the derivation uses one year as the interest period.

[2] Appendix C contains compound interest tables for rates between 1/4 and

60%.

CHAPTER 4

EQUIVALENCE FOR REPEATED CASH FLOWS



Student Solar Power

Indiana State University (ISU) mechanical and manufacturing engineering technology students undertook a project to design a photovoltaic (PV) system to make use of solar energy in the College of Technology Building. The students considered different types of PV solar tracking systems and selected a two-axis tracking system (north/south for summer/winter and east/west for morning/evening). This PV system was designed to provide emergency lighting for the building as well as a learning opportunity for students concerning renewable energy and its functioning.

The system included four PV panels of 123 watts each with a life of 25 years as per the manufacturer's specifications. Panel tilt was fixed at 45 degrees. A two-axis system was able to track altitude and azimuth for Terre Haute, where ISU is located. Most of the electrical parts, such as the converter, PLC controller, and wiring, were provided for free by the college CIM Lab. More recently, engineering economy students have analyzed the system's economic viability, based on the city's available solar potential, as a case study.

Contributed by M. Affan Badar, Indiana State University

QUESTIONS TO CONSIDER

1. The panels were purchased by ISU about five years ago. Should the purchase cost be considered as sunk cost?
2. How much difference does the longitude of a city make if the same panel is installed in a different city (east or west coast of the U.S.)? Can latitude make a difference in the value of the summer/winter shift?
3. How important are latitude and yearly days of sunshine in system economics?
4. What costs must be considered, and how can they be estimated over time? In particular, battery performance and costs are getting better every year. Given this trend, how do you determine the system's equivalent present cost?
5. Electricity generated from the system is a saving, as this amount of electricity is not going to be purchased from the utility company (or, if it's a grid-connected system, electricity can be sold back to the utility company). How do you compute the annual dollar amount of the saving? Do the panels decline in efficiency each year?

After Completing This Chapter...

The student should be able to:

- Solve problems with the uniform series compound interest factors and annuity functions.
- Use arithmetic and geometric gradients to solve appropriately modeled problems.

• Understand when and why the assumed uniformity of *A*, *G*, and *g* is a good engineering economic model.
• Use spreadsheets and financial functions to model and solve engineering economic analysis problems.

Key Words

annuity

arithmetic gradient series

data block

feasibility analysis

geometric gradient series

ordinary annuity

scenario

spreadsheet annuity functions

spreadsheet block functions

uniform payment series

uniform rate

what-if analysis

Chapter 3 presented the fundamental components of engineering economic analysis, including formulas to compute equivalent single sums of money at different points in time. Most problems we will encounter are much more complex. This chapter develops formulas for cash flows that are a uniform series or are increasing on an arithmetic or geometric gradient.

UNIFORM SERIES COMPOUND

INTEREST FORMULAS

Many times we will find uniform series of receipts or disbursements. Automobile loans, house payments, and many other loans are based on a **uniform payment series.** Future costs and benefits are often estimated to be the *same* or *uniform* every year. This is the simplest assumption; it is often sufficiently accurate; and often there is no data to support what might be a more accurate model.

Since most engineering economy problems define "a period" as one year, the uniform cash flow is an *annual* cash flow denoted by *A* for all period lengths. More formally, *A* is defined as

A = an end-of-period cash receipt or disbursement in a uniform series, continuing for n periods

Engineering economy practice and textbooks (including this one) generally assume that cash flows after Time 0 are end-of-period cash flows. Thus the equations and tabulated values of *A* and *F* assume end-of-period timing. Tables for beginning- or middle-of-period assumptions have been built, but they are rarely used. When the uniform cash flow is assumed to be an end-of-period cash flow, it is an **annuity** or **ordinary annuity**.

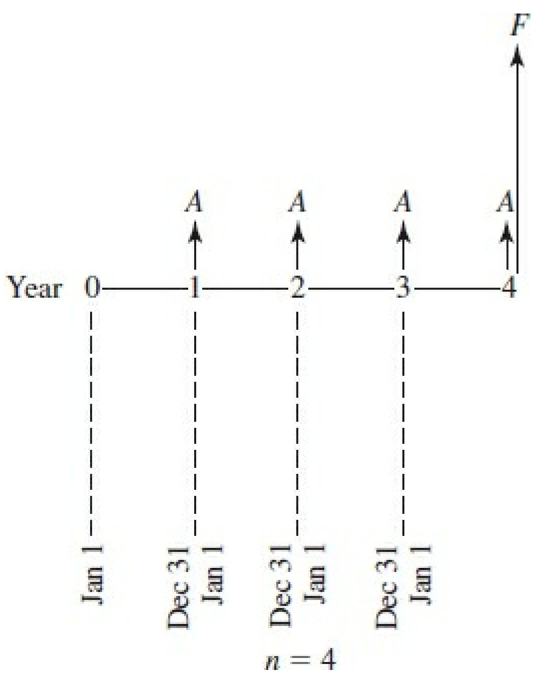


FIGURE 4-1 The general relationship between *A* and *F*.

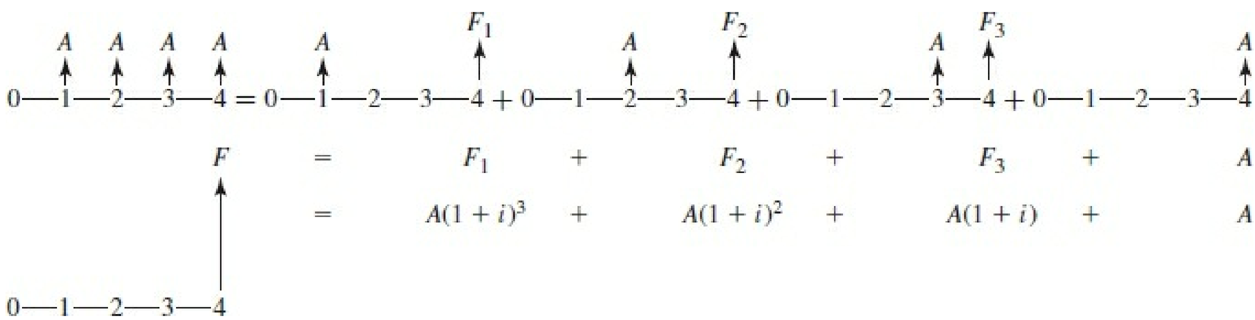
The horizontal line in Figure 4–1 is a representation of time with four interest periods illustrated. Uniform payments *A* have been placed at the end of each interest period, and there are as many *A*’s as there are interest periods *n*. (Both these conditions are specified in the definition of *A*.) Figure 4–1 uses January 1 and December 31, but other 1-year or other length periods could be used.

In Chapter 3’s section on single payment formulas, we saw that a sum *P* at one point in time would increase to a sum *F* in *n* periods, according to the equation

$$F = P(1 + i)^n$$

We will use this relationship in our uniform series derivation.

Looking at Figure 4–1, we see that if an amount *A* is invested at the end of each year for 4 years, the total amount *F* at the end of 4 years will be the sum of the compound amounts of the individual investments.



In the general case for *n* years,

$$F = A(1+i)^{n-1} + \cdots + A(1+i)^3 + A(1+i)^2 + A(1+i) + A \quad (4\text{-}1)$$

Multiplying Equation 4-1 by (1 + *i*), we have

$$(1+i)F = A(1+i)^n + \cdots + A(1+i)^4 + A(1+i)^3 + A(1+i)^2 + A(1+i) \quad (4\text{-}2)$$

Factoring out *A* and subtracting Equation 4-1 gives

$$\begin{aligned}(1+i)F &= A[(1+i)^n + \cancel{\cdots} + \cancel{(1+i)^4} + \cancel{(1+i)^3} + \cancel{(1+i)^2} + \cancel{(1+i)}] \\ -\quad F &= A[\cancel{(1+i)^{n-1}} + \cancel{\cdots} + \cancel{(1+i)^3} + \cancel{(1+i)^2} + \cancel{(1+i)} + 1] \\ \hline iF &= A[(1+i)^n - 1]\end{aligned}$$

(4-3)

TABLE 4-1 Excel Functions for Use with Annuities

To find the equivalent *P*	−PV (*i*, *n*, *A*, *F*, Type)
To find the equivalent *A*	−PMT (*i*, *n*, *P*, *F*, Type)
To find the equivalent *F*	−FV (*i*, *n*, *A*, *P*, Type)
To find *n*	NPER (*i*, *A*, *P*, *F*, Type)
To find *i*	RATE (*n*, *A*, *P*, *F*, Type, guess)

Solving Equation 4-3 for *F* gives the **uniform series compound amount factor**

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(\boldsymbol{F/A,i\%,n}) \quad (4\text{-}4)$$

These annuity functions, which are listed in Table 4-1 and detailed in Appendix B, are the spreadsheet version of the 5-BUTTON SOLUTION calculations. The spreadsheet annuity functions list four variables chosen from *n*, *A*, *P*, *F*, and *i*, and solve for the fifth variable.

The *Type* variable is optional. If it is omitted or 0, then the *A* value is assumed to be an end-of-period cash flow. If the *A* value represents a beginning-of-period cash flow, then a value of 1 is entered for the Type variable. RATE also has an optional term, *guess*, which has a default of 10%. This is the starting interest rate in calculating an answer. If the function does not converge on an answer, change the guess. An input is usually not needed.

When solving for PV, PMT, or FV, entering positive values produces a negative result. This sign convention, which seems odd to some students, is explained in Appendix B. The sign convention affects spreadsheets and calculators in the same way, and must be managed. The difficulty becomes minor with a little practice.

We suggest here and in Appendix B that you build a spreadsheet calculator with one row for each of the functions listed in Table 4−1. We suggest that it look like a 5-BUTTON SOLUTION with all 5 functions. Then when you’re solving homework problems you can simply create new rows in your spreadsheet by copying the rows that you need. The cash flow diagram and your understanding of the factors and functions determine what the values are. The spreadsheet just does the arithmetic.

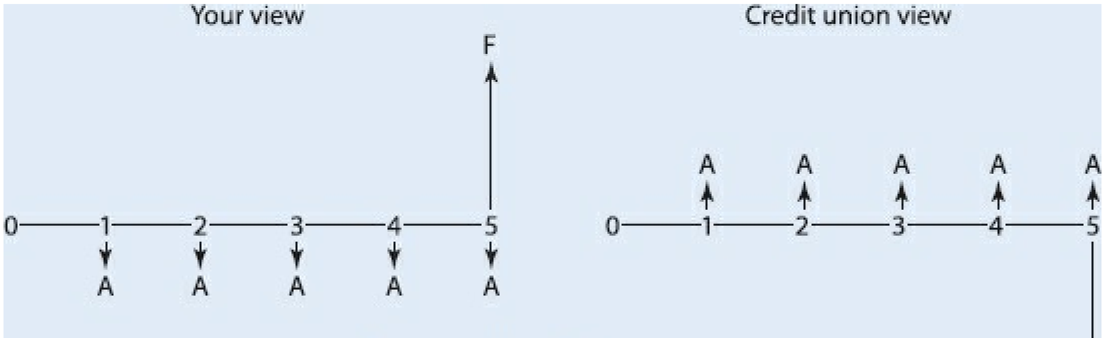
Appendix B also describes financial and programmable scientific calculators that can be used instead of or in addition to the tables. Two programmable scientific calculators can be used on the FE exam. These spreadsheet functions and calculators make the same assumptions as the tabulated factors.

EXAMPLE 4-1

You deposit $500 in a credit union at the end of each year for 5 years. The credit union pays 5% interest, compounded annually. Immediately after the fifth deposit, how much can you withdraw from your account?

TABLE SOLUTION

Both diagrams of the five deposits and the desired computation of the future sum F duplicates the situation for the uniform series compound amount formula



$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i\%, n)$$

where A = \$500, n = 5, i = 0.05, F = unknown. Filling in the known variables gives

$$F = \$500(F/A, 5\%, 5) = \$500(5.526) = \$2763$$

There will be $2763 in the account following the fifth deposit.

5-BUTTON SOLUTION

As detailed in Appendix B, this solution corresponds to the more powerful spreadsheet and financial calculator functions that can also be programmed into HP calculators that can be used on the FE exam.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-1	5%	5	-500	0		FV	$2,762.82	=FV(B2,C2,D2,E2)

Because the spreadsheet function is not rounded off like the interest factor, the spreadsheet answer is slightly more precise.

EXAMPLE 4-2

A new engineer wants to save money for a down payment on a house. The initial deposit is $685, and $375 is deposited at the end of each month. The savings account earns interest at an annual nominal rate of 6% with monthly compounding. How much is on deposit after 48 months?

5-BUTTON SOLUTION

Because deposits are made monthly, the nominal annual interest rate of 6% must be converted to ½% per month for the 48 months. Note both the initial and periodic deposits are negative cash flows from the engineer to the savings account.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-2	0.5%	48	-375	-685		FV	$21,156.97	=FV(B2,C2,D2,E2)

TABLE SOLUTION

Because there are multiple types of cash flows, the uniform series compound amount factor (Eq. 4-4) must be used for the monthly deposit and the compound amount factor (Eq. 3-4) for the initial deposit.

$$\begin{aligned} F &= 375(F/A, 0.5\%, 48) + 685(F/P, 0.5\%, 48) \\ &= 375(54.098) + 685(1.270) \\ &= \$21{,}156.7 \end{aligned}$$

If Equation 4-4 is solved for *A*, we have the **uniform series sinking fund**[1] factor, which is written as (*A*/*F*, *i*, *n*).

$$\begin{aligned} A &= F\left[\frac{i}{(1+i)^n - 1}\right] \\ &= F(A/F, i\%, n) \end{aligned} \qquad (4\text{-}5)$$

EXAMPLE 4-3

Jim Hayes wants to buy some electronic equipment for $1000. Jim has decided to save a uniform amount at the end of each month so that he will have the required $1000 at the end of one year. The local credit union pays 6% interest, compounded monthly. How much does Jim have to deposit each month?

TABLE SOLUTION

In this example,

$$F = \$1000, \qquad n = 12, \qquad i = 1/2\%, \qquad A = \text{unknown}$$

$$A = 1000(A/F, 1/2\%, 12) = 1000(0.0811) = \$81.10$$

Jim would have to deposit $81.10 each month.

5-BUTTON SOLUTION

As detailed in Appendix B, this solution corresponds to the more powerful spreadsheet and financial calculator functions that can also be programmed into HP calculators that can be used on the FE exam.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-3	0.5%	12		0	1000	PMT	-$81.07	=PMT(B2,C2,E2,F2)

If we use the sinking fund formula (Equation 4-5) and substitute for *F* the single payment compound amount formula (Equation 3-3), we obtain the **uniform series capital recovery factor**, which has the notation (*A*/*P*, *i*, *n*).

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right]$$

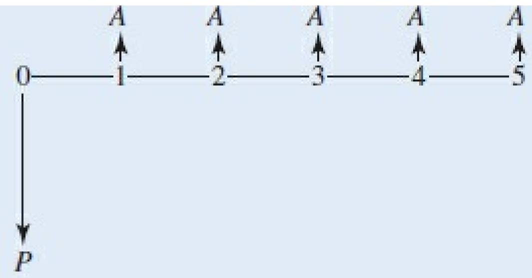
$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P(A/P, i\%, n) \qquad (4\text{-}6)$$

The name *capital recovery factor* comes from asking the question, How large does the annual return, *A*, have to be to “recover” the capital, *P*, that is invested at Time 0? In other words, find *A* given *P*. This is illustrated in Example 4–4.

EXAMPLE 4-4

An energy-efficient machine costs $5000 and has a life of 5 years. If the interest rate is 8%, how much must be saved every year to recover the cost of the capital invested in it?

TABLE SOLUTION



$P = \$5000, \quad n = 5, \quad i = 8\%, \quad A = \text{unknown}$

$A = P(A/P, 8\%, 5) = 5000(0.2505) = \$1252$

The required annual savings to recover the capital investment is $1252.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-4	8.0%	5		-5000	0	PMT	$1,252.28	=PMT(B2,C2,E2,F2)

In Example 4–4, with interest at 8%, a present sum of $5000 is equivalent to five equal end-of-period disbursements of $1252. This is another way of stating Plan 3 of Table 3–1. The method for determining the annual payment that would repay $5000 in 5 years with 8% interest has now been explained. The calculation is simply

$$A = 5000(A/P, 8\%, 5) = 5000(0.2505) = \$1252$$

If the capital recovery formula (Equation 4-6) is solved for the present sum *P*, we obtain the uniform series present worth formula or **uniform series present worth factor.**

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P/A,i\%,n) \quad (4\text{-}7)$$

EXAMPLE 4-5 Example 3–1 Revisited

A machine will cost $30,000 to purchase. Annual operating and maintenance costs (O&M) will be $2000. The machine will save $10,000 per year in labor costs. The salvage value of the machine after 5 years will be $7000. Calculate the machine's present worth for an interest rate of 10%.

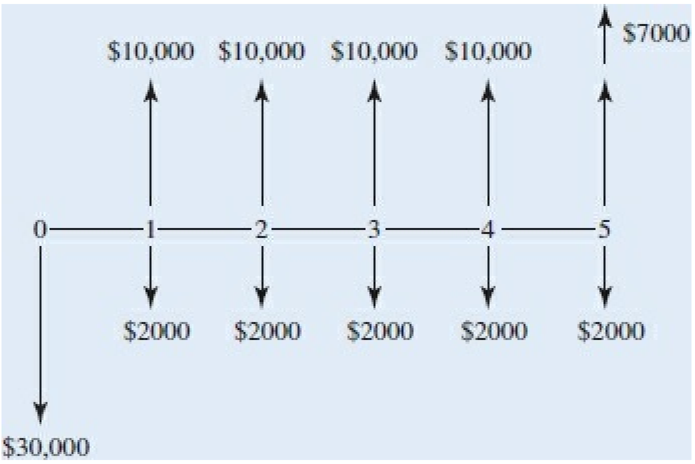


TABLE SOLUTION

Because there are multiple types of cash flows, the uniform series present worth factor (Eq. 4–7) must be used for the annual savings and O&M costs in conjunction with the present worth factor (Eq. 3–5) for the salvage value. No factor is need for the first cost since it is already a present worth.

$$\begin{aligned} P &= -30{,}000 + (10{,}000 - 2000)(P/A, 10\%, 5) + 7000(P/F, 10\%, 5) \\ &= -30{,}000 + 8000(3.791) + 7000(0.6209) \\ &= \$4672 \end{aligned}$$

5-BUTTON SOLUTION

The *n* is 5 for both the annual series of $8000 (= $10,000 – $2000) and the salvage value of $7000, so the present worth of both can be found in one step.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-5	10%	5	8000		7000	PV	-$34,672.74	=PV(B2,C2,D2,F2)
3							change sign	$34,672.74	=-H2
4							first cost	-$30,000.00	-$30,000.00
5							PW	$4,672.74	=H3+H4

EXAMPLE 4-6 Example 3–1 and 4–5 Revisited

What is the rate of return on the machine in Example 4–5?

5-BUTTON SOLUTION

This is one type of problem where the annuity functions of spreadsheets and calculators save a lot of arithmetic. This is a one-step solution with the rate function.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-6		5	8000	-3000	7000	RATE	15.38%	=RATE(C2,D2,E2,F2)

TABLE SOLUTION

The starting point for finding the rate of return on the machine is the same equation that was developed in Example 4−5. However, instead of specifying an interest rate of 10%, that is now our unknown. We are solving for the interest rate that makes the total present worth equal to 0. We are finding the interest rate that makes the $30,000 first cost equivalent to the other cash flows.

$$0 = -30{,}000 + (10{,}000 - 2000)(P/A, i, 5) + 7000(P/F, i, 5)$$

Adding up the cash flows gives a total of $17,000 (= −30, 000 + 8000 × 5 + 7000) at an interest rate of 0%. Example 4−4 tells us that the present worth was $4672 at 10%. So the rate of return is higher than 10%. There is a bit of trial and error in solving this problem with the tabulated factors, so this solution saves some time by using the answer from the 5-BUTTON SOLUTION to choose a 15% interest rate for our next calculation. It is common to use a subscript on the *P* to indicate the interest rate.

$$\begin{aligned} P_{15} &= -30{,}000 + (10{,}000 - 2000)(P/A, 15\%, 5) + 7000(P/F, 15\%, 5) \\ &= -\$30{,}000 + 8000(3.352) + 7000(0.4972) \\ &= \$296.4 \end{aligned}$$

A higher interest rate is needed to discount the future receipts to below $30,000. The next higher tabulated rate is 18%.

$$\begin{aligned} P_{18} &= -30{,}000 + (10{,}000 - 2000)(P/A, 18\%, 5) + 7000(P/F, 18\%, 5) \\ &= -30{,}000 + 8000(3.127) + 7000(0.4371) \\ &= -\$1924.3 \end{aligned}$$

We know that the interest rate is between 15% and 18%—and it is much closer to 15%. Doing an approximation before we start the arithmetic of interpolation helps catch arithmetic and data entry errors. The interpolation equation can be built using the simple tabular graphic that follows and knowing that $a/b = c/d$ so $a = b(c/d)$.

Interest Rate	Present Worth
15%	$296.4
i	0
18%	−$1924.3

Using ratios, $\frac{a}{b} = \frac{c}{d}$

$$\frac{i - 15\%}{18\% - 15\%} = \frac{0 - 296.4}{-1924.3 - 296.4}$$

$$\frac{i - 15\%}{3\%} = \frac{-296.4}{-2220.7} = \frac{296.4}{2220.7}$$

$$i - 15\% = \frac{3\% \times 296.4}{2220.7}$$

$$i = 15\% + \frac{3\% \times 296.4}{2220.7} = 15.4\%$$

EXAMPLE 4-7

A new engineer buys a car with 0% down financing from the dealer. The cost with all taxes, registration, and license fees is $15,732. If each of the 48 monthly payments is $398, what is the monthly interest rate? What is the effective annual interest rate?

5-BUTTON SOLUTION

The engineer is receiving the car valued at $15,732 at time 0. So that is the PV. The payment is a negative cash flow. The unknown is the monthly interest rate.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-7		48	$398	$15,732	0	i	0.822%	=RATE(C2,D2,E2,F2
3		monthly						annual	

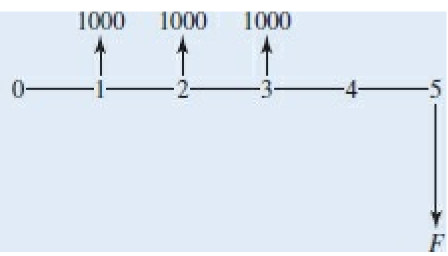
4	Effective	0.822%	12	0	-1	FV	1.1033	=FV(B4,C4,D4,E4)
5	or					i_a	10.33%	=FV-1
6	Normal	0.822%	12			r	9.87%	=B6*C6
7	Effective	9.87%	12			i_a	10.33%	=EFFECT(B7,C7)

CASH FLOWS THAT DO NOT MATCH BASIC PATTERNS

EXAMPLE 4-8

A student is borrowing $1000 per year for 3 years. The loan will be repaid 2 years later at a 15% interest rate. Compute how much will be repaid. This is *F* in the following cash flow table and diagram.

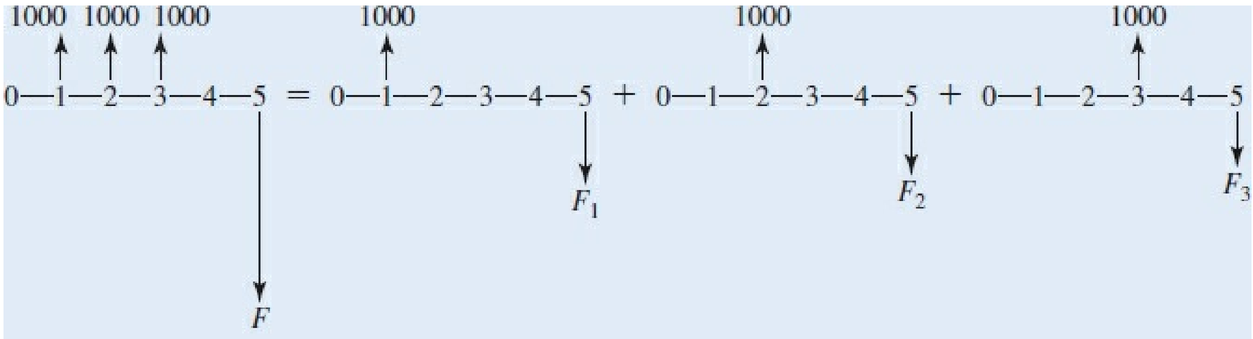
Year	Cash Flow
1	+1000
2	+1000
3	+1000
4	0
5	$-F$



SOLUTION

We see that the cash flow diagram does not match the sinking fund factor diagram: *F* occurs two periods later, rather than at the same time as the last *A*. Since the diagrams do not match, the problem is more difficult than those we've discussed so far. The approach to use in this situation is to convert the cash flow from its present form into standard forms, for which we have compound interest factors and compound interest tables.

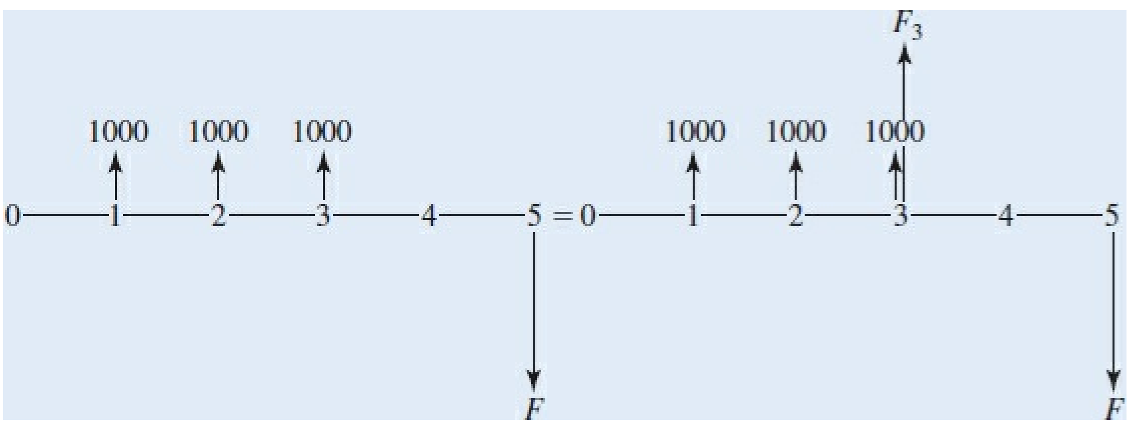
One way to solve this problem is to consider the cash flow as a series of single payments *P* and then to compute their sum *F*. In other words, the cash flow is broken into three parts, each one of which we can solve.



$$F = F_1 + F_2 + F_3 = 1000(F/P, 15\%, 4) + 1000(F/P, 15\%, 3) + 1000(F/P, 15\%, 2)$$
$$= 1000(1.749) + 1000(1.521) + 1000(1.322)$$
$$= \$4592$$

ALTERNATE SOLUTION

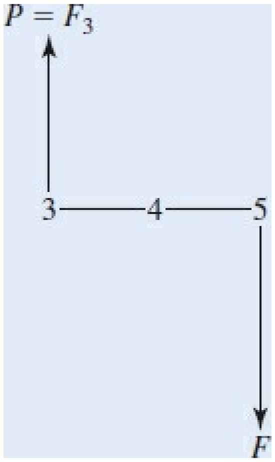
A second approach is to calculate an equivalent F_3 at the end of Period 3.



Looked at this way, we first solve for F_3.

$$F_3 = 1000(F/A, 15\%, 3) = 1000(3.472) = \$3472$$

Now F_3 can be considered a present sum P in the diagram



and so

$$
\begin{aligned}
F &= F_3(F/P, 15\%, 2)\\
&= 3472(1.322)\\
&= \$4590
\end{aligned}
$$

The $2 difference in calculated values is due to rounding in the interest tables.

This two-step solution can be combined into a single equation:

$$
\begin{aligned}
F &= 1000(F/A, 15\%, 3)(F/P, 15\%, 2)\\
&= 1000(3.472)(1.322)\\
&= \$4590
\end{aligned}
$$

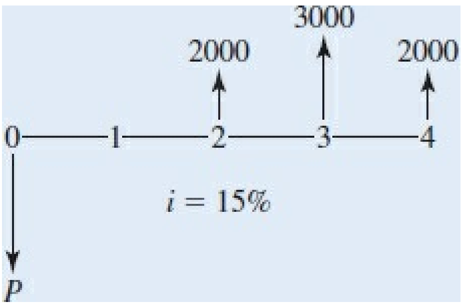
5-BUTTON SOLUTION

First find F_3 as in the previous solution; then that becomes the P for finding F.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 4-8	15%	3	1000	0		F_3	-$3,472.50
3		15%	2	0	3472.50		F	-$4,592.38
4					=-H2			

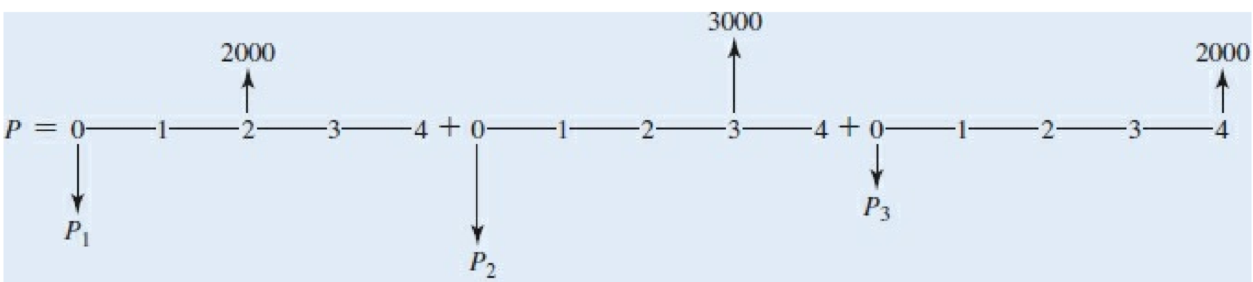
EXAMPLE 4-9

Consider the following situation, where *P* is deposited into a savings account and three withdrawals are made. Find *P*.



The diagram is not in a standard form, indicating that there will be a multiple-step solution. The solutions that follow are the most common approaches that are used. Understanding why all of them are correct requires understanding *economic equivalence*—which is the foundation of engineering economy. Not only does visualizing and understanding the different ways help develop overall understanding. Each way is sometimes the easiest way to solve a particular problem.

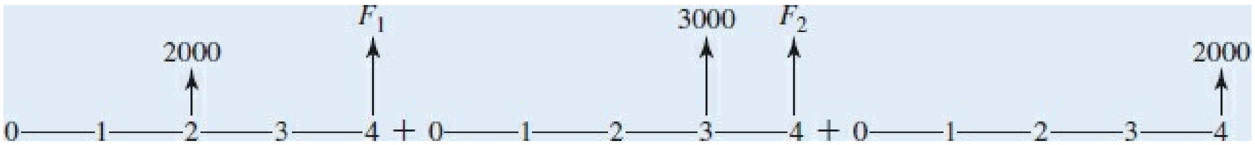
SOLUTION 1



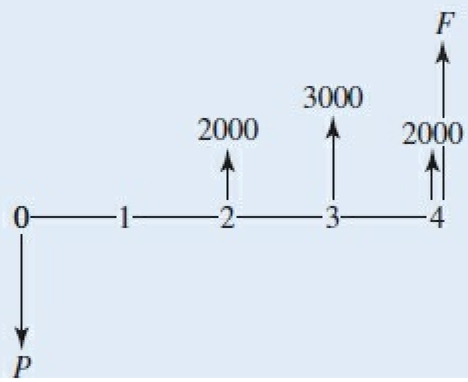
$$\begin{aligned} P &= P_1 + P_2 + P_3 \\ &= 2000(P/F, 15\%, 2) + 3000(P/F, 15\%, 3) + 2000(P/F, 15\%, 4) \\ &= 2000(0.7561) + 3000(0.6575) + 2000(0.5718) \\ &= \$4628 \end{aligned}$$

SOLUTION 2

The second approach converts each withdrawal into an equivalent value at the end of Period 4.



$$
\begin{aligned}
F &= F_1 + F_2 + 2000 \\
&= 2000(F/P, 15\%, 2) + 3000(F/P, 15\%, 1) + 2000
\end{aligned}
$$



The relationship between *P* and *F* in the diagram is

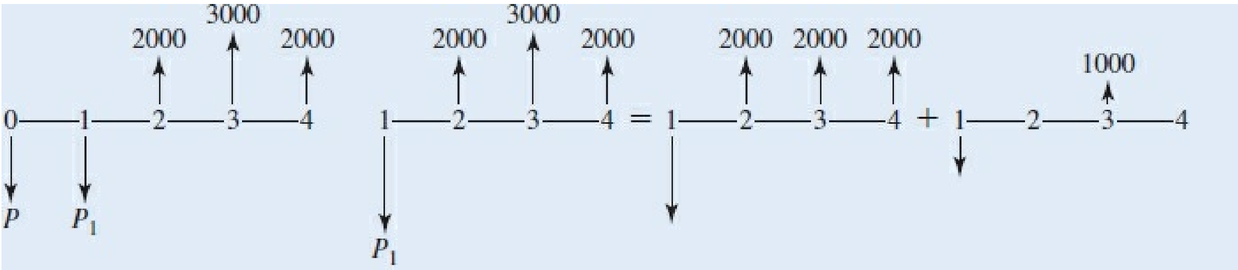
$$P = F(P/F, 15\%, 4)$$

Combining the two equations, we have

$$
\begin{aligned}
P &= [F_1 + F_2 + 2000](P/F, 15\%, 4) \\
&= [2000(F/P, 15\%, 2) + 3000(F/P, 15\%, 1) + 2000](P/F, 15\%, 4) \\
&= [2000(1.322) + 3000(1.150) + 2000](0.5718) \\
&= \$4628
\end{aligned}
$$

SOLUTION 3

The third approach finds how much would have to be deposited (P_1) at $t = 1$ and then converts that into an equivalent value (P) at $t = 0$.



$P = P_1(P/F, 15\%, 1)$ $\qquad$ $P_1 = 2000(P/A, 15\%, 3) + 1000(P/F, 15\%, 2)$

Combining, we have

$$
\begin{aligned}
P &= [2000(P/A, 15\%, 3) + 1000(P/F, 15\%, 2)] \times (P/F, 15\%, 1) \\
&= [2000(2.283) + 1000(0.7561)](0.8696) \\
&= \$4628
\end{aligned}
$$

5-BUTTON SOLUTION

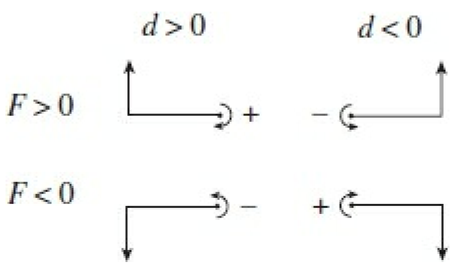
Solution 1 is probably the easiest approach, with changes in *n* and *F* as needed.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 4-9	15%	2	0		2000	P_1	-$1,512.29
3		15%	3	0		3000	P_2	-$1,972.55
4		15%	4	0		2000	P_3	-$1,143.51
5							P	-$4,628.34

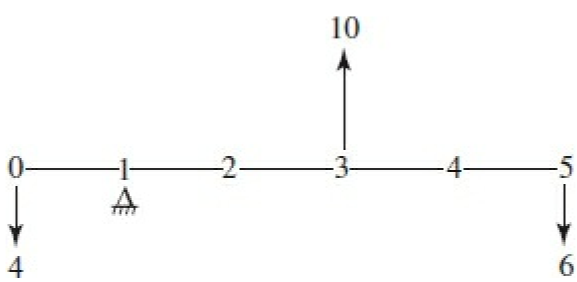
ECONOMIC EQUIVALENCE VIEWED AS A MOMENT DIAGRAM

The similarity between cash flow and free body diagrams allows an analogy that helps some students better understand economic equivalence.[2] Think of the cash flows as forces that are always perpendicular to the axis. Then the time periods become the distances along the axis.

When we are solving for unknown forces in a free body diagram, we know that a moment equation about any point will be in equilibrium. Typically moments are calculated by using the right-hand rule so that counterclockwise moments are positive, but it is also possible to define moments so that a clockwise rotation is positive. We are assuming that **clockwise rotations are positive**. This allows us to make the normal assumptions that positive forces point up and positive distances from the force to the pivot point are measured from left to right. Thus, negative forces point down, and negative distances are measured from the force on the right to the pivot point on the left. These assumptions are summarized in the following diagram.



With these assumptions we can write force moment equations at equilibrium for the following diagram.



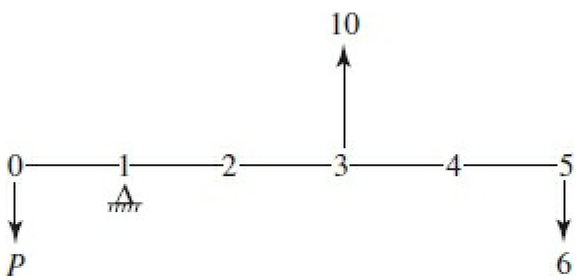
For example, in the force moment equation about Point 1, the force at 0 is –4, and the distance from the force to the pivot point is 1. Similarly the force at 5 is –6, and the distance from the force on the right to the pivot point on the left is –4. The equilibrium equation for force moments about Point 1 is

$$0 = -4 \times 1 + 10 \times (-2) + -6 \times (-4)$$

To write the cash flow moment equation for the cash flow diagram we need:

1. A sign convention for cash flows, such that positive values point up.
2. A way to measure the moment arm for each cash flow. This moment arm must be measured as $(1 + i)^T$, where T is the number of periods measured *from* the cash flow *to* the pivot point or axis of rotation.
a. Thus the sign of the distance is moved to the exponent.
b. For cash flows at the pivot point, $T = 0$, and $(1+i)^0 = 1$.

We can redraw the simple example with an unknown present cash flow, *P*.



Since *P* is drawn as a negative cash flow, we put a minus sign in front of it when we write the cash flow moment equation. To rewrite the cash flow moment equation at Year 1, we use the distances from the diagram as the exponents for $(1+i)^t$:

$$0 = -P \times (1+i)^1 + 10 \times (1+i)^{-2} + -6 \times (1+i)^{-4}$$

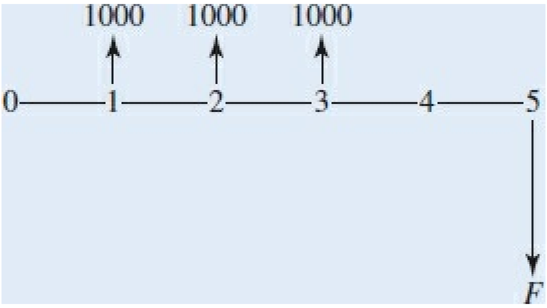
If $i = 5\%$, the value of *P* can be calculated.

$$0 = -P \times 1.05^1 + 10 \times 1.05^{-2} + -6 \times 1.05^{-4}$$

$$P = 10 \times 1.05^{-3} + -6 \times 1.05^{-5} = 8.64 - 4.70 = 3.94$$

EXAMPLE 4-10

For the cash flow diagram in Example 4–8 (repeated here), write the cash flow moment equations at Years 0, 3, and 5. Solve for *F* when $i = 15\%$.



SOLUTION

With Year 0 as the pivot point, the cash flow moment equation is

$$0 = 1000 \times (1 + i)^{-1} + 1000 \times (1 + i)^{-2} + 1000 \times (1 + i)^{-3} - F \times (1 + i)^{-5}$$

With Year 3 as the pivot point, the cash flow moment equation is

$$0 = 1000 \times (1 + i)^{2} + 1000 \times (1 + i)^{1} + 1000 \times (1 + i)^{0} - F \times (1 + i)^{-2}$$

With Year 5 as the pivot point, the cash flow moment equation is

$$0 = 1000 \times (1 + i)^{4} + 1000 \times (1 + i)^{3} + 1000 \times (1 + i)^{2} - F \times (1 + i)^{0}$$

In each case, the cash flow moment equation simplifies to

$$F = 1000 \times (1 + i)^{4} + 1000 \times (1 + i)^{3} + 1000 \times (1 + i)^{2}$$

If i = 15%, then

$$F = 1000 \times (1.749 + 1.521 + 1.323) = \$4592.3$$

RELATIONSHIPS BETWEEN COMPOUND INTEREST FACTORS

From the derivations, we see there are several simple relationships between the compound interest factors. They are summarized here.

Single Payment

$$\text{Compound amount factor} = \frac{1}{\text{Present worth factor}}$$

$$(F/P, i, n) = \frac{1}{(P/F, i, n)} \qquad (4\text{-}8)$$

Uniform Series

$$\text{Capital recovery factor} = \frac{1}{\text{Present worth factor}}$$

$$(A/P, i, n) = \frac{1}{(P/A, i, n)} \qquad (4\text{-}9)$$

$$\text{Compound amount factor} = \frac{1}{\text{Sinking fund factor}}$$

$$(F/A, i, n) = \frac{1}{(A/F, i, n)} \qquad (4\text{-}10)$$

The uniform series present worth factor is simply the sum of the *n* terms of the single payment present worth factor

$$(P/A, i, n) = \sum_{t=1}^{n} (P/F, i, t) \qquad (4\text{-}11)$$

For example:

$$(P/A, 5\%, 4) = (P/F, 5\%, 1) + (P/F, 5\%, 2) + (P/F, 5\%, 3) + (P/F, 5\%, 4)$$

$$3.546 = 0.9524 + 0.9070 + 0.8638 + 0.8227$$

The uniform series compound amount factor equals 1 *plus* the sum of (n − 1) terms of the single payment compound amount factor

$$(F/A, i, n) = 1 + \sum_{t=1}^{n-1} (F/P, i, t) \qquad (4\text{-}12)$$

For example,

$$(F/A, 5\%, 4) = 1 + (F/P, 5\%, 1) + (F/P, 5\%, 2) + (F/P, 5\%, 3)$$
$$4.310 = 1 + 1.050 + 1.102 + 1.158$$

The uniform series capital recovery factor equals the uniform series sinking fund factor *plus i*:

$$(A/P, i, n) = (A/F, i, n) + i \quad (4\text{-}13)$$

For example,

$$(A/P, 5\%, 4) = (A/F, 5\%, 4) + 0.05$$
$$0.2820 = 0.2320 + 0.05$$

This may be proved as follows:

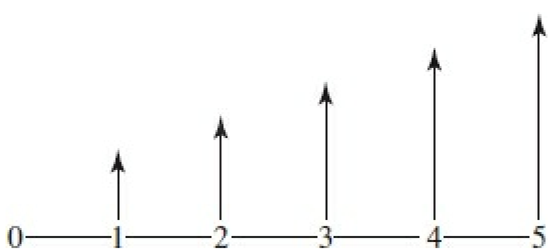
$$(A/P, i, n) = (A/F, i, n) + i$$
$$\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = \left[\frac{1}{(1+i)^n - 1}\right] + i$$

Multiply by $(1+i)^n - 1$ to get

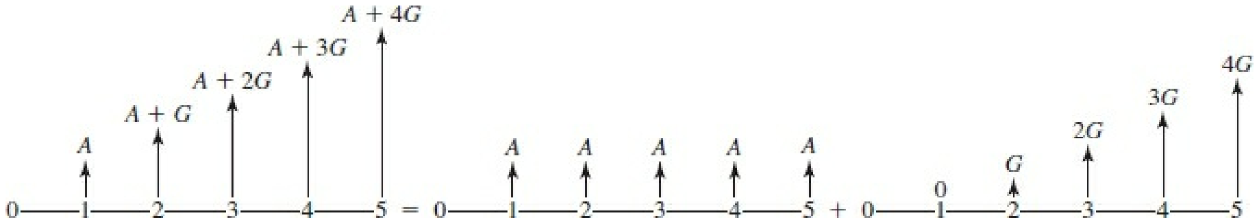
$$i(1+i)^n = i + i(1+i)^n - i = i(1+i)^n$$

ARITHMETIC GRADIENT

It frequently happens that the cash flow series is not of constant amount *A*. Instead, there is a uniformly increasing series as shown:



Cash flows of this form may be resolved into two components:



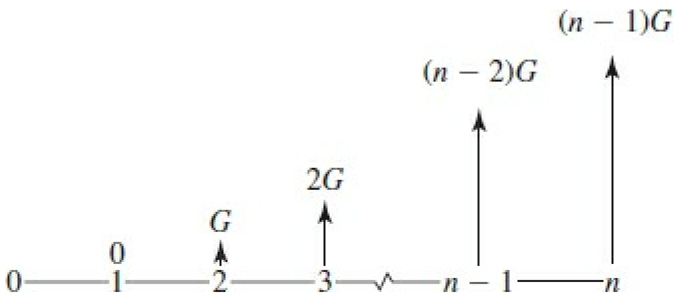
The *first* cash flow in the **arithmetic gradient series** is zero. Thus, G is the *change* from period to period. The gradient (G) series normally is used along with a uniform series (A).

$$P = A(P/A, i, n) + G(P/G, i, n)$$

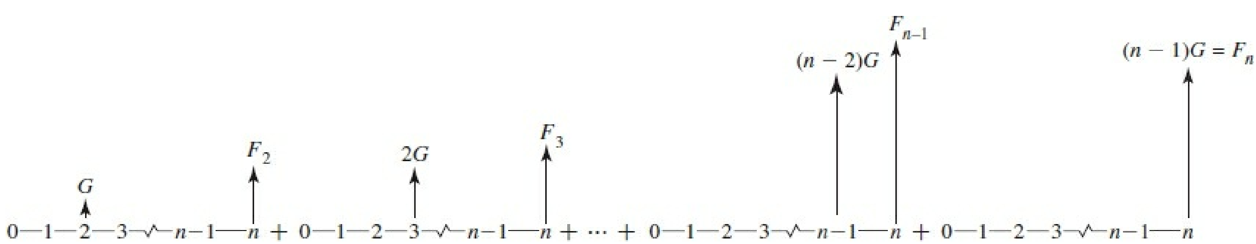
Pause here and look at the tables for $n = 1$. What is the value of $(P/G, i, 1)$ for any i?

Derivation of Arithmetic Gradient Factors

The arithmetic gradient is a series of increasing cash flows as follows:



The arithmetic gradient series may be thought of as a series of individual cash flows that can individually be converted to equivalent final cash flows at the end of period n.



The value of F for the sum of the cash flows = $F_2 + F_3 + \cdots + F_{n-1} + F_n$, or

$$F = G(1+i)^{n-2} + 2G(1+i)^{n-3} + \cdots + (n-2)(G)(1+i)^1 + (n-1)G \quad (4\text{-}14)$$

Multiply Equation 4–14 by (1 + *i*) and factor out *G*, or

$$(1+i)F = G[(1+i)^{n-1} + 2(1+i)^{n-2} + \cdots + (n-2)(1+i)^2 + (n-1)(1+i)^1] \quad (4\text{-}15)$$

Rewrite Equation 4–14 to show other terms in the series,

$$F = G[(1+i)^{n-2} + \cdots + (n-3)(1+i)^2 + (n-2)(1+i)^1 + n - 1] \quad (4\text{-}16)$$

Subtracting Equation 4—6 from Equation 4–15, we obtain

$$F + iF - F = G[(1+i)^{n-1} + (1+i)^{n-2} + \cdots + (1+i)^2 + (1+i)^1 + 1] - nG \quad (4\text{-}17)$$

In the derivation of Equation 4–4, the terms inside the brackets of Equation 4–17 were shown to equal the series compound amount factor:

$$[(1+i)^{n-1} + (1+i)^{n-2} + \cdots + (1+i)^2 + (1+i)^1 + 1] = \frac{(1+i)^n - 1}{i}$$

Thus, Equation 4–17 becomes

$$iF = G\left[\frac{(1+i)^n - 1}{i}\right] - nG$$

Rearranging and solving for *F*, we write

$$F = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i} - n\right] \quad (4\text{-}18)$$

Multiplying Equation 4–18 by the single payment present worth factor gives

$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i} - n\right]\left[\frac{1}{(1+i)^n}\right]$$

$$= G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right]$$

$$(P/G, i, n) = \left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right] \qquad (4\text{-}19)$$

Equation 4–19 is the **arithmetic gradient present worth factor.** Multiplying Equation 4-18 by the sinking fund factor, we have

$$A = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i} - n\right]\left[\frac{i}{(1+i)^n - 1}\right] = G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$(A/G, i, n) = \left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right] = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right] \qquad (4\text{-}20)$$

Equation 4–20 is the **arithmetic gradient uniform series factor.**

EXAMPLE 4-11

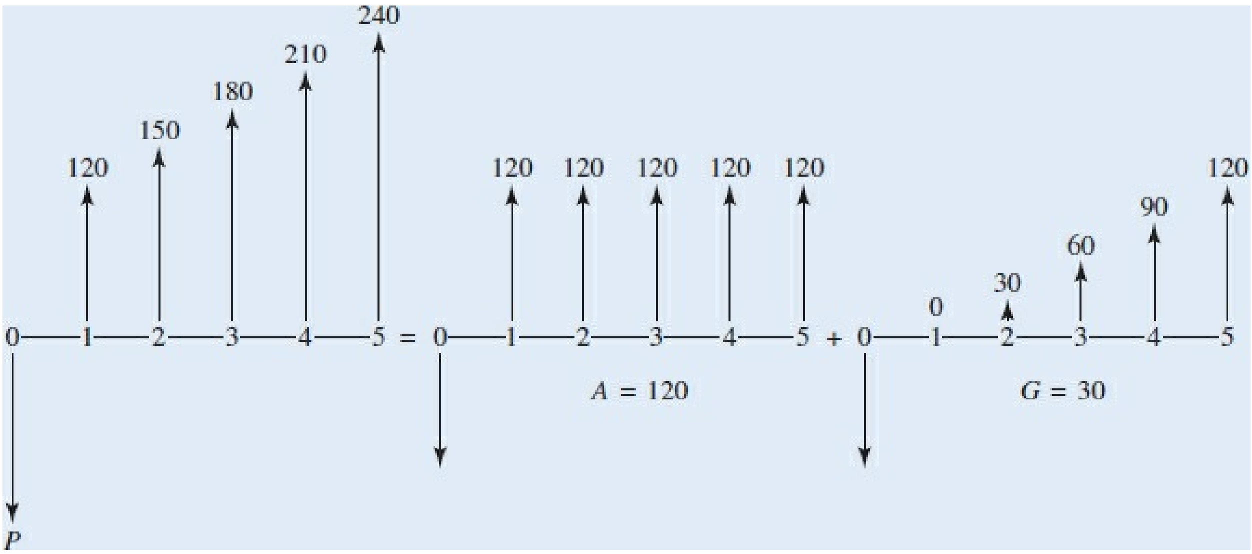
Andrew has purchased a new car. He wishes to set aside enough money in a bank account to pay the maintenance for the first 5 years. It has been estimated that the maintenance cost of a car is as follows:

Year	Maintenance Cost
1	$120
2	150
3	180
4	210
5	240

Assume the maintenance costs occur at the end of each year and that the bank pays 5% interest. How much should Andrew deposit in the bank now?

SOLUTION

The cash flow diagram may be broken into its two components:



Both components represent cash flows for which compound interest factors have been derived. The first is a uniform series present worth, and the second is an arithmetic gradient series present worth:

$$P = A(P/A, 5\%, 5) + G(P/G, 5\%, 5)$$

Note that the value of *n* in the gradient factor is 5, not 4. In deriving the gradient factor, the cash flow in the first period is zero followed by ($n - 1$) terms containing *G*. Here there are four terms containing *G*, and it is a 5-period gradient.

$$\begin{aligned} P &= 120(P/A, 5\%, 5) + 30(P/G, 5\%, 5) \\ &= 120(4.329) + 30(8.237) \\ &= 519 + 247 \\ &= \$766 \end{aligned}$$

Andrew should deposit \$766 in the bank now.

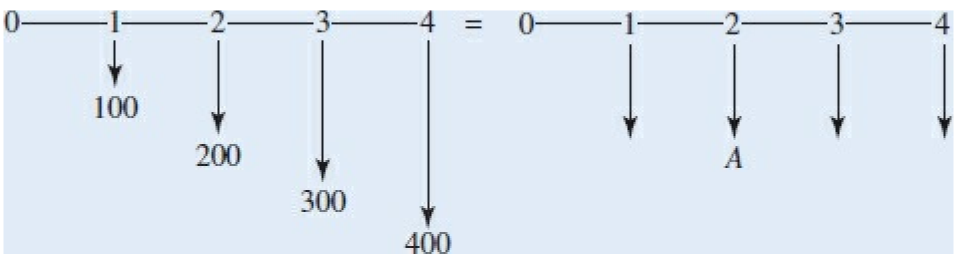
EXAMPLE 4-12

On a certain piece of machinery, it is estimated that the maintenance expense will be as follows:

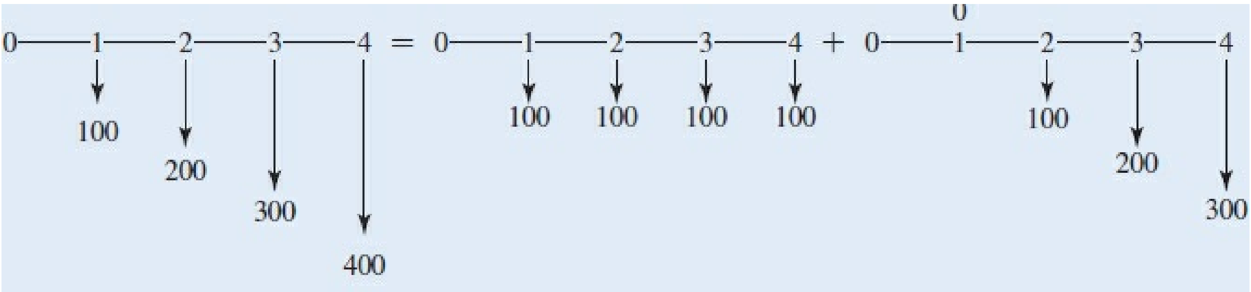
Year	Maintenance
1	$100
2	200
3	300
4	400

What is the equivalent uniform annual maintenance cost for the machinery if 6% interest is used?

SOLUTION



The most common mistake with arithmetic gradients is to solve problems like this as only a $100 gradient. The first cash flow in the arithmetic gradient series is zero, hence the diagram is *not* in proper form for the arithmetic gradient equation. As in Example 4–9, the cash flow must be resolved into two components:



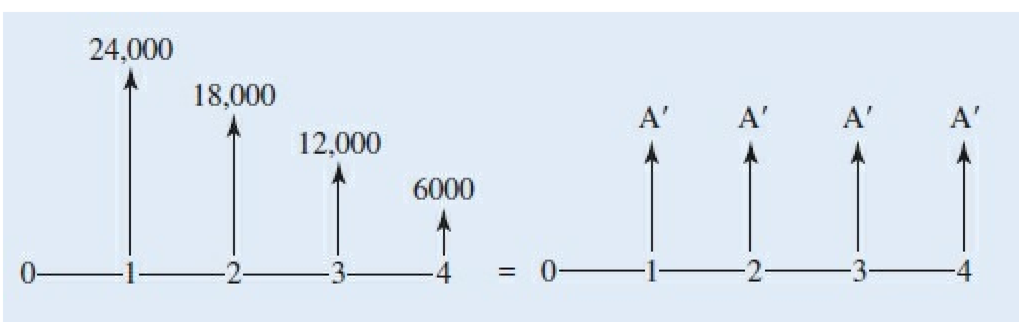
$$A = 100 + 100(A/G, 6\%, 4) = 100 + 100(1.427) = \$242.70$$

The equivalent uniform annual maintenance cost is $242.70.

EXAMPLE 4-13

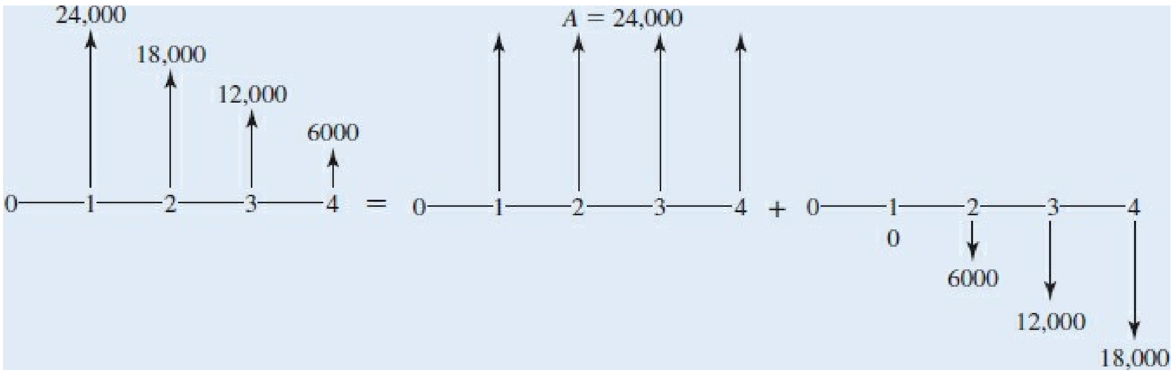
Demand for a new product will decline as competitors enter the market. If interest is 10%, what is an equivalent uniform value?

Year	Revenue
1	$24,000
2	18,000
3	12,000
4	6,000



SOLUTION

The projected cash flow is still a cash flow in Period 1 ($24,000) that defines the uniform series. However, now the gradient or change each year is −$6000.

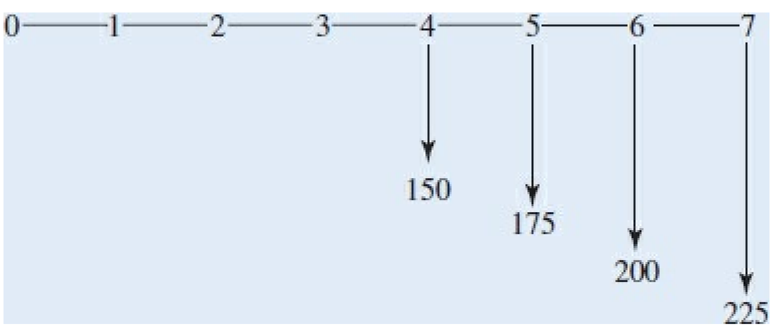


$$
\begin{aligned}
A' &= 24{,}000 - 6000(A/G, 10\%, 4) \\
&= 24{,}000 - 6000(1.381) \\
&= \$15{,}714
\end{aligned}
$$

The projected equivalent uniform value is $15,714 per year.

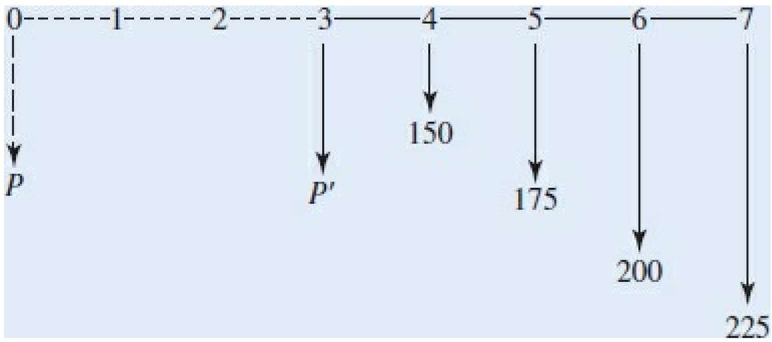
EXAMPLE 4-14

A car's warranty is 3 years. Upon expiration, annual maintenance starts at $150 and then climbs $25 per year until the car is sold at the end of Year 7. Use a 10% interest rate and find the present worth of these expenses.



SOLUTION

With the uniform series and arithmetic gradient series present worth factors, we can compute a present sum P'.



It is important that you closely examine the location of P'. Based on the way the factor was derived, there will be one zero value in the gradient series to the right of P'. (If this seems strange or incorrect, review the beginning of this section on arithmetic gradients.)

$$\begin{aligned} P' &= A(P/A, i, n) + G(P/G, i, n) \\ &= 150(P/A, 10\%, 4) + 25(P/G, 10\%, 4) \\ &= 150(3.170) + 25(4.378) = 475.50 + 109.45 = 584.95 \end{aligned}$$

Then

$$P = P'(P/F, 10\%, 3) = 584.95(0.7513) = \$439.47$$

Reality and the Assumed Uniformity of *A*, *G*, and *g*

The reality of engineering projects is that the annual revenues from selling a new product or annual benefits from using a new highway change each year as demand and traffic levels change. Most annual cash flows are not really uniform.

Thus, why do we define and start with an *A* that is a uniform annual cost, a *G* that is a uniform annual gradient, and a *g* (next section) that is a uniform annual rate of increase?

1. It is easier to start with simpler models. We use cash flow tables and spreadsheets when needed for more complex models.
2. These model cash flows are the basis of the formulas and tabulated factors that are often used in engineering economic analysis.

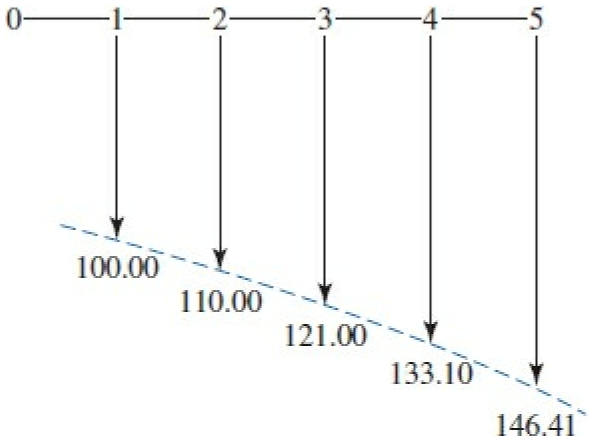
3. Often in the real world, engineering economy is applied in a **feasibility** or preliminary **analysis**. At this stage, annual cash flows for costs and revenues are typically estimated using *A*, *G*, and/or *g*. Not enough is known about the problem for more detailed estimates.

GEOMETRIC GRADIENT

Earlier, we saw that the arithmetic gradient is applicable where the period-by-period change in a cash receipt or payment is a *constant* amount. There are other situations where the period-by-period change is a **uniform rate,** *g*. Often **geometric gradient series** can be traced to population levels or other levels of activity where changes over time are best modeled as a percentage of the previous year. The percentage or *rate* is constant over time, rather than the *amount* of the change as in the arithmetic gradient.

For example, if the maintenance costs for a car are $100 the first year and they increase at a uniform rate, *g*, of 10% per year, the cash flow for the first 5 years would be as follows:

Year		Cash Flow
1	100.00	= $100.00
2	100.00 + 10%(100.00) = $100(1 + 0.10)^1$	= 110.00
3	110.00 + 10%(110.00) = $100(1 + 0.10)^2$	= 121.00
4	121.00 + 10%(121.00) = $100(1 + 0.10)^3$	= 133.10
5	133.10 + 10%(133.10) = $100(1 + 0.10)^4$	= 146.41



From the table, we can see that the maintenance cost in any year is

$$\$100(1+g)^{t-1}$$

Stated in a more general form,

$$A_t = A_1(1+g)^{t-1} \quad (4\text{-}21)$$

where

1. g = uniform rate of cash flow increase/decrease from period to period, that is, the geometric gradient
2. A_1 =value of cash flow at Year 1 ($100 in the example)
3. A_t =value of cash flow at any year t

Since the present worth P_t of any cash flow A_t at interest rate i is

$$P_t = A_t(1+i)^{-t} \quad (4\text{-}22)$$

we can substitute Equation 4-21 into Equation 4-22 to get

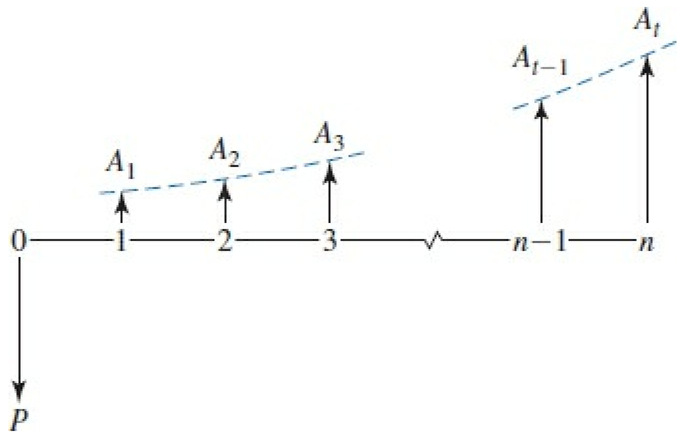
$$P_t = A_1(1+g)^{t-1}(1+i)^{-t}$$

This may be rewritten as

$$P_t = A_1(1+i)^{-1}\left(\frac{1+g}{1+i}\right)^{t-1} \quad (4\text{-}23)$$

The present worth of the entire gradient series of cash flows may be obtained by expanding Equation 4-23:

$$P = A_1(1+i)^{-1}\sum_{t=1}^{n}\left(\frac{1+g}{1+i}\right)^{t-1} \qquad (4\text{-}24)$$



In the general case, where $i \neq g$, Equation 4-23 may be written out as follows:

$$P = A_1(1+i)^{-1} + A_1(1+i)^{-1}\left(\frac{1+g}{1+i}\right) + A_1(1+i)^{-1}\left(\frac{1+g}{1+i}\right)^2 + \cdots + A_1(1+i)^{-1}\left(\frac{1+g}{1+i}\right)^{n-1} \qquad (4\text{-}25)$$

Let $a = A_1(1+i)^{-1}$ and $b = (1 + g)/(1 + i)$. Equation 4-25 becomes

$$P = a + ab + ab^2 + \cdots + ab^{n-1} \qquad (4\text{-}26)$$

Multiply Equation 4-26 by b:

$$bP = ab + ab^2 + ab^3 + \cdots + ab^{n-1} + ab^n \qquad (4\text{-}27)$$

Subtract Equation 4-27 from Equation 4-26:

$$P - bP = a - ab^n$$
$$P(1 - b) = a(1 - b^n)$$
$$P = \frac{a(1 - b^n)}{1 - b}$$

Replacing the original values for *a* and *b*, we obtain

$$P = A_1(1+i)^{-1}\left[\frac{1 - \left(\frac{1+g}{1+i}\right)^n}{1 - \left(\frac{1+g}{1+i}\right)}\right] = A_1\left[\frac{1 - \left(\frac{1+g}{1+i}\right)^n}{(1+i) - \left(\frac{1+g}{1+i}\right)(1+i)}\right]$$

$$= A_1\left[\frac{1 - (1+g)^n(1+i)^{-n}}{1 + i - 1 - g}\right]$$

$$P = A_1\left[\frac{1 - (1+g)^n(1+i)^{-n}}{i - g}\right] \qquad (4\text{-}28)$$

where $i \neq g$.

The expression in the brackets of Equation 4-28 is the **geometric series present worth factor**

$$(P/A,g,i,n) = \left[\frac{1 - (1+g)^n(1+i)^{-n}}{i - g}\right], \qquad \text{where } i \neq g \qquad (4\text{-}29)$$

In the special case of $i = g$, Equation 4-28 becomes

$$P = A_1 n(1+i)^{-1}$$
$$(P/A, g, i, n) = [n(1+i)^{-1}], \qquad \text{where } i = g \qquad (4\text{-}30)$$

EXAMPLE 4-15

The first-year maintenance cost for a new car is estimated to be \$100, and it increases at a uniform rate of 10% per year. Using an 8% interest rate, calculate the present worth (PW) of the cost of the first 5 years of maintenance.

STEP-BY-STEP SOLUTION

Year n			Maintenance Cost		$(P/F, 8\%, n)$		PW of Maintenance
1	100.00	=	100.00	×	0.9259	=	\$ 92.59
2	100.00 + 10%(100.00)	=	110.00	×	0.8573	=	94.30
3	110.00 + 10%(110.00)	=	121.00	×	0.7938	=	96.05
4	121.00 + 10%(121.00)	=	133.10	×	0.7350	=	97.83
5	133.10 + 10%(133.10)	=	146.41	×	0.6806	=	99.65
							\$480.42

SOLUTION USING GEOMETRIC SERIES PRESENT WORTH FACTOR

$$P = A_1 \left[\frac{1-(1+g)^n(1+i)^{-n}}{i-g}\right], \qquad \text{where } i \neq g$$

$$= 100.00 \left[\frac{1-(1.10)^5(1.08)^{-5}}{-0.02}\right] = \$480.42$$

The present worth of cost of maintenance for the first 5 years is \$480.42.

SPREADSHEETS FOR ECONOMIC ANALYSIS

Spreadsheets: *The* Tool for Engineering Practice

Spreadsheets are used to model annual revenues from selling a new product or annual benefits from using a new highway that change each year. Often this follows a geometric gradient, which spreadsheets can easily include; but more complex patterns are common. For example, cumulative sales and market penetration of a new product often follow a technology growth curve or *S-curve*, where the volume in each year must be individually estimated.

Similarly, if a monthly model of energy use is built, then fluctuations in air-conditioning and heating costs over the year can be accurately modeled in a spreadsheet. As a final example, consider the construction cost for a large project such as new tunnels under the Hudson River, the new airport that Denver built, a new power plant, or a new professional sports stadium. The construction period may be 3 to 10 years long, and the costs will be spread in a project-unique way over that time span. Spreadsheets are a powerful tool to model and analyze complex projects.

Spreadsheets are used in most real-world applications of engineering economy. Common tasks include the following:

1. Constructing tables of cash flows.
2. Using annuity functions to calculate a *P*, *F*, *A*, *n*, or *i*, as in the 5-BUTTON SOLUTION.
3. Using a block function to find the present worth or internal rate of return for a table of cash flows.
4. Making graphs for analysis and convincing presentations (virtually all graphs in text).
5. Conducting what-if analysis for different assumed values of problem variables (see scenarios in Example 4–16 and Chapter 9).

Constructing tables of cash flows relies mainly on spreadsheet basics that are covered in Appendix A. These basics include using and naming spreadsheet variables, using a **data block** to enter the variable values, understanding the difference between absolute and relative addresses when copying a formula, and formatting a cell.

Spreadsheet Block Functions for Cash Flow Tables

Cash flows can be specified period by period as a block of values. These cash flows are analyzed by **block functions** that identify the row or column entries for which a present worth or an internal rate of return should be calculated. In Excel the two functions are NPV(*i*,values) and IRR(values,guess).

Economic Criterion	Excel Function	Values for Periods

Net present value	NPV(*i*,values); guess argument is optional	1 to *n*
Internal rate of return	IRR(values,guess);	0 to *n*

Excel's IRR function can be used to find the interest rate for a loan with irregular payments (other applications are covered in Chapter 7). As in the RATE function, the IRR function's optional guess has a default value of 10%. If the IRR function cannot converge on the answer, change the guess.

These block functions make different assumptions about the range of years included. The NPV(*i*,values) function assumes that Year 0 is **not** included, while IRR(values,guess) assumes that Year 0 is included. These functions require that a cash flow be identified for each period. You cannot leave cells blank even if the cash flow is $0. The cash flows for 1 to *n* are assumed to be end-of-period flows. All periods are assumed to be the same length of time.

Also, the NPV function returns the present worth equivalent to the cash flows, unlike the PV annuity function, which returns the negative of the equivalent value.

For cash flows involving only constant values of *P*, *F*, and *A* this block approach seems to be inferior to the annuity functions. However, this is a conceptually easy approach for more complicated cash flows, such as arithmetic or geometric gradients. Suppose the years (row 1) and the cash flows (row 2) are specified in columns B through E.

	A	B	C	D	E	F
1	Year	0	1	2	3	4
2	Cash flow	−25,000	6000	8000	10,000	12,000

NPV(.08,C2:F2)

IRR(B2:F2)

If an interest rate of 8% is assumed, then the present worth of the cash flows can be calculated as =B2+NPV(.08,C2:F2), which equals $4172.95. This is the present worth equivalent to the five cash flows, rather than the negative of the present worth equivalent returned by the PV annuity function. The internal rate of return calculated using IRR(B2:F2) is 14.5%. Notice how the NPV function does not include the Year 0 cash flow in B2 but the IRR function does.

• PW = B2+NPV(.08,C2:F2)   NPV range without Year 0
• IRR = IRR(B2:F2)   IRR range with Year 0

The following example illustrates the ease with which cash flow tables can model and find the present worth and rate of return for arithmetic and geometric gradients. It also illustrates how spreadsheets can be used to model other patterns of cash flows and to answer “what-if ” questions by examining multiple **scenarios**.

EXAMPLE 4-16

A new product will cost $750,000 to design, test prototypes, and set up for production. Net revenue the first year is projected to be $225,000. Marketing is unsure whether future year revenues will (a) increase by $25,000 per year as the product’s advantages become more widely known or (b) decrease by 10% per year due to competition. A third pattern of increasing by $25,000 for one year and then decreasing by 10% per year has been suggested as being more realistic. The firm evaluates projects with a 12% interest rate, and it believes that this product will have a 5-year life. Calculate the present worth and rate of return for each scenario.

SOLUTION

The first cost, interest rate, and year 1 revenues are the same for all three scenarios so they are entered into the **data block** in A1:B3. Since the gradients are different for each scenario, the type of gradient is described in rows 5 and 6 and the value of each gradient is stated in row 7. The present worth formulas in row 16 must each include the year 0 cash flows in row 9.

	A	B	C	D	E
1	$750,000	First cost			
2	12%	Interest rate			
3	$225,000	Year 1 net revenue			
4	Scenario	a	b	c	
5	Gradient	Arithmetic	Geometric		
6		*G*	*g*	both	
7	Value	$ 25,000	-10%		
8	Year				
9	0	-$750,000	-$750,000	-$750,000	
10	1	225,000	225,000	225,000	=$A$3
11	2	250,000	202,500	250,000	=B10+B7
12	3	275,000	182,250	225,000	=C10*(1+$C$7)
13	4	300,000	164,025	202,500	
14	5	325,000	147,623	182,250	=D13*(1+$C$7)
15					
16	PW	$221,000	-$69,948	$42,448	=D9+NPV($A$2,D10:D14)
17	Rate of return	22.6%	7.9%	14.4%	=IRR(D9:D14)

Under scenarios (a) and (c), the new product is profitable. However, under scenario (b) the new product has a negative present worth and a rate of return below the 12% interest rate the firm uses. If scenario (c) truly is more realistic, then the project should go ahead.

COMPOUNDING PERIOD AND PAYMENT PERIOD DIFFER

When the various time periods in a problem match, we generally can solve the problem by simple calculations. Thus in Example 4–4, where we had $5000 in an account paying 8% interest, compounded annually, the five equal end-of-year withdrawals are simply computed as follows:

$$A = P(A/P, 8\%, 5) = 5000(0.2505) = \$1252$$

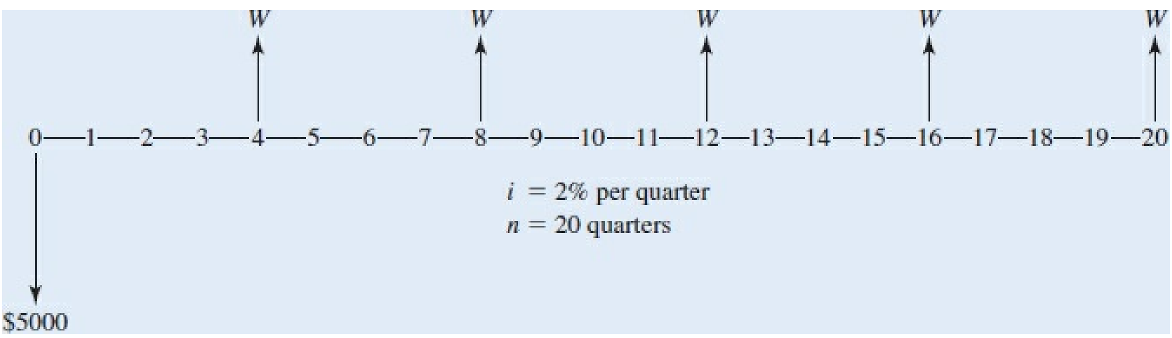
Consider how this simple problem becomes more difficult if the compounding period is changed so that it no longer matches the annual withdrawals.

EXAMPLE 4-17

On January 1, a woman deposits $5000 in a credit union that pays 8% nominal annual interest, compounded quarterly. She wishes to withdraw all the money in five equal yearly sums, beginning December 31 of the first year. How much should she withdraw each year?

SOLUTION

Since the 8% nominal annual interest rate *r* is compounded quarterly, we know that the effective interest rate per interest period, *i*, is 2%; and there are a total of 4 × 5 = 20 interest periods in 5 years. For the equation $A = P(A/P, i, n)$ to be used, there must be as many periodic withdrawals as there are interest periods, *n*. In this example we have 5 withdrawals and 20 interest periods.

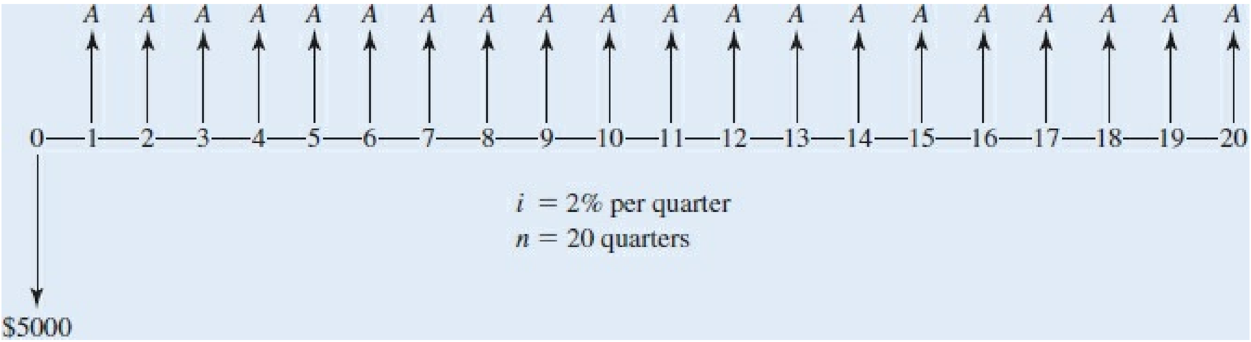


To solve the problem, we must adjust it so that it is in one of the standard forms for which we have compound interest factors. This means we must first compute either an equivalent *A* for each 3-month interest period or an effective *i* for each time period between withdrawals. Let's solve the problem both ways.

SOLUTION 1

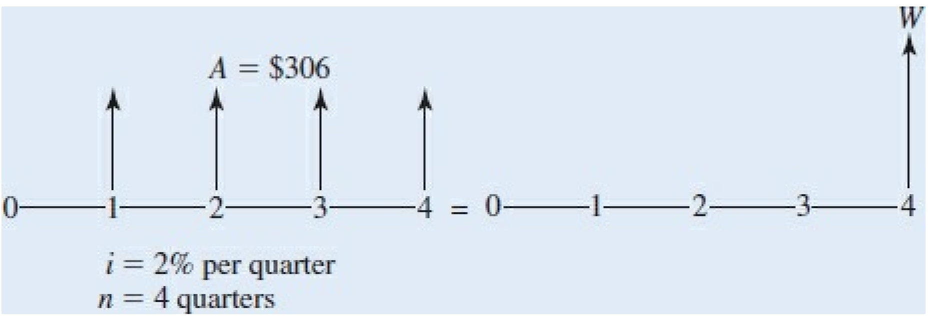
Compute an equivalent *A* for each 3-month time period.

If we had been required to compute the amount that could be withdrawn quarterly, the diagram would have been as follows:



$$A = P(A/P, i, n) = 5000(A/P, 2\%, 20) = 5000(0.0612) = \$306$$

Looking at each one-year period,



$$W = A(F/A, i, n) = 306(F/A, 2\%, 4) = 306(4.122)$$
$$= \$1260$$

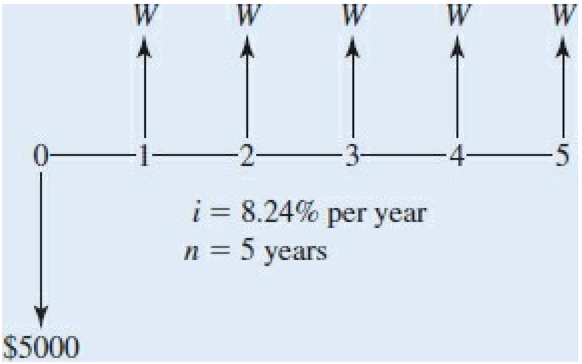
SOLUTION 2

Compute an effective *i* for the time period between withdrawals.

Between withdrawals, *W*, there are four interest periods, hence $m = 4$ compounding subperiods per year. The nominal interest rate per year, *r*, is 8%, so the effective annual interest rate is:

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.08}{4}\right)^4 - 1$$
$$= 0.0824 = 8.24\% \text{ per year}$$

Now the problem may be redrawn as follows:



The annual withdrawal W is found using the capital recovery factor:

$$
\begin{aligned}
W &= P(A/P, i, n) = 5000(A/P, 8.24\%, 5) \\
&= P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = 5000\left[\frac{0.0824(1+0.0824)^5}{(1+0.0824)^5 - 1}\right] \\
&= 5000(0.2520) = \$1260
\end{aligned}
$$

The depositor should withdraw $1260 per year.

SUMMARY

The compound interest formulas described in this chapter, along with those in Chapter 3, will be referred to throughout the rest of the book. It is very important that the reader understand the concepts presented and how these formulas are used. The following notation is used consistently:

i	=	effective interest rate per interest period[3] (stated as a decimal)
n	=	number of interest periods
P	=	a present sum of money
F	=	a future sum of money at the end of the n^{th} interest period, which is equivalent to P with interest rate i
A	=	an end-of-period cash receipt or disbursement in a uniform series continuing for n periods; the entire series equivalent to P or F at some interest rate i
		uniform period-by-period increase or decrease in cash receipts or

G	=	disbursements; the arithmetic gradient
g	=	uniform rate of cash flow increase or decrease from period to period; the geometric gradient
r	=	nominal interest rate per interest period (see footnote 3)
i_a	=	effective annual interest rate
m	=	number of compounding subperiods per period (see footnote 3)

Single Payment Formulas (Derived in Chapter 3)

Compound amount:

$$F = P(1+i)^n = P(F/P, i, n)$$

Present worth:

$$P = F(1+i)^{-n} = F(P/F, i, n)$$

Uniform Series Formulas

Compound amount:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i, n)$$

Sinking fund:

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = F(A/F, i, n)$$

Capital recovery:

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P(A/P, i, n)$$

Present worth:

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P/A, i, n)$$

Arithmetic Gradient Formulas

Arithmetic gradient present worth:

$$P = G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right] = G(P/G, i, n)$$

Arithmetic gradient uniform series:

$$A = G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right] = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right] = G(A/G, i, n)$$

Geometric Gradient Formulas

Geometric series present worth, where $i \neq g$:

$$P = A_1\left[\frac{1 - (1+g)^n(1+i)^{-n}}{i - g}\right] = A_1(P/A, g, i, n)$$

Geometric series present worth, where $i = g$:

$$P = A_1[n(1+i)^{-1}] = A_1(P/A, g, i, n) = A_1(P/A, i, i, n)$$

Spreadsheet Functions

Annuity functions
=PV(*i*, *n*, pmt, [fv], [type])
=FV(*i*, *n*, pmt, [pv], [type])
=PMT(*i*, *n*, pv, [fv], [type])
=NPER(*i*, pmt, pv, [fv], [type])
=RATE(*n*, pmt, pv, [fv], [type], [guess])
Block functions for cash flow tables
=NPV(rate, CF_1:CF_n)

=IRR(CF_0:CF_n, [guess])
Nominal and effective interest rate functions
=EFFECT(nominal_rate, npery)
=NOMINAL(effective_rate, npery)

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

4-1 Dylan deposits $10,000 now and makes an additional deposit of $5,000 at end of Year 3 in the same account. Determine the balance in the account at end of 10 years if the interest rate is a nominal 8%, compounded quarterly.

SOLUTION

4-2 The Smiths are planning ahead for their daughter's education. She's 8 now will start college in 10 years. How much will they need to set aside each ye to have $65,000 in 10 years if the annual interest rate is 7%?

SOLUTION

4-3 A young engineer wishes to buy a house but can afford monthly payments only $500. Thirty-year loans are available at 6% interest compounded mon If she can make a $5000 down payment, what is the price of the most expensive house that she can afford to purchase?

SOLUTION

4-4 A person borrows $15,000 at an interest rate of 6%, compounded monthly, be paid with payments of $456.33.

a. What is the length of the loan in years?
b. What is the total amount that would be required at the end of the twelfth month to pay off the entire loan balance?

SOLUTION

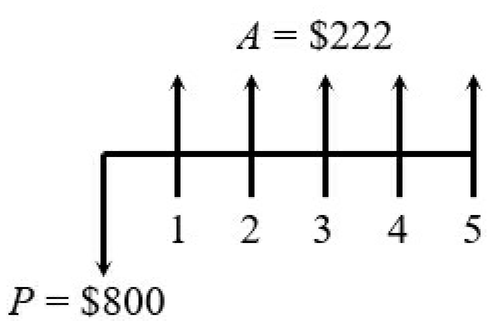
4-5 A $50,000 loan with a nominal interest rate of 6% is to be repaid over 30 y with payments of $299.77. The borrower wants to know how many payme *N**, he will have to make until he owes only half of the amount he borrowe initially.

SOLUTION

4-6 While in college, Ellen received $40,000 in student loans at 8% interest. Sl will graduate in June and will repay the loans in either 5 or 10 equal annua payments. Compute her yearly payments for both repayment plans.

SOLUTION

4-7 Given:



Find: i

SOLUTION

4-8 J. D. Homeowner has just bought a house with a 20-year, 9%, $70,000 mortgage on which he is paying $629.81 per month.

a. If J. D. sells the house after 10 years, how much must he pay the bank to completely pay off the mortgage at the time of the 120th payment?
b. How much of the first $629.81 payment on the loan is interest?

SOLUTION

4-9 Suppose you wanted to buy a $180,000 house. You have $20,000 cash to u as the down payment. The bank offers to lend you the remainder at 6% nominal interest. The term of the loan is 20 years. Compute your monthly l payment.

SOLUTION

4-10 To offset the cost of buying a $120,000 house, Jose and Sophia borrowed $25,000 from their parents at 6% nominal interest, compounded monthly. T loan from their parents is to be paid off in 5 years in equal monthly paymen The couple has saved $12,500. Their total down payment is therefore $25,(+ 12,500 = $37,500. The balance will be mortgaged at 9% nominal interest compounded monthly for 30 years. Find the combined monthly payment th the couple will be making for the first 5 years.

SOLUTION

4-11 Martinez and Martinez Inc. makes monthly payments of $152.11 and pays interest on a current loan. The initial loan was scheduled to be paid off in 3 years. Determine the loan amount.

SOLUTION

4-12 Abby's cat, Si, has convinced her to set up an account that will assure him his Meow Mix for the next four years. Abby will deposit an amount today will allow Felix to make end-of-month withdrawals of $10 for the next 48 months. Assume an interest rate of 6% and that the account will have a zer balance when the last withdrawal is made. What is the amount that Abby should deposit? What is the account balance immediately after the 24th withdrawal is made?

SOLUTION

4-13 Ben Spendlove just purchased a new stereo system for $975 and will be making payments of $45 per month. How long will it take to completely pa off the stereo at 18% nominal interest?

SOLUTION

4-14	Henry Fuller has agreed to purchase a used automobile for $13,500. He wi to limit his monthly payment to $350 for a period of 2 years. What down payment must he make to complete the purchase if the interest rate on the l will be 6%?

SOLUTION

4-15	Beginning 1 month from today David B. will deposit each month $200 into account paying 6% nominal interest. He will make a total of 240 deposits (years). After the last deposit the money in the account will begin to earn 4% interest compounded annually. After another 10 years David will begin to withdraw annual amounts for a total of 10 years. How much can be withdra each year if the account is to be depleted (zero balance) after another 10 ye

SOLUTION

4-16	Ray Witmer, an engineering professor at UTM, is preparing to retire to his farm and care for his cats and dogs. During his many years at UTM he inve well and has a balance of $1,098,000 in his retirement fund. How long will be able to withdraw $100,000 per year, beginning today, if his account earn interest at a rate of 4% per year?

SOLUTION

4-17	Benita deposits $125 per month into an account paying 6% interest for 2 ye to be used to purchase a car. The car she selects costs more than the amoun the account. She agrees to pay $175 per month for 2 more years at 9% inte and also uses a gift from her uncle of $500 as part of the down payment. W is the cost of the car to the nearest dollar?

SOLUTION

	Lenagar Lumber Inc. is making monthly payments of $572.39 and paying nominal 9% interest on a current loan. The initial loan was scheduled to be

4-18 paid off in 3 years. Immediately after the 15th payment business conditions
allow Lenagar to pay off the loan. Determine the amount of the loan balan
due after the 15th payment.

SOLUTION

4-19 Jason W. bought a Mercedes when he came to UTM as an engineering stu
The Mercedes was purchased by taking a loan that was to be paid off in 20
equal, quarterly payments. The interest rate on the loan was 12%. Four yea
later, after Jason made his 16th payment, he got married (no more dating!)
sold the Mercedes to his buddy Houston S. Houston made arrangements w
Jason's bank to refinance the loan and to pay Jason's unpaid balance by ma
16 equal, quarterly payments at the same interest rate that Jason was payin
Houston flunked out of UTM (too many dates!) 3¼ years later, after havin
made his 13th payment; he then sold the car to Jeff M. Jeff paid the bank $
cash (he had a good summer job!) to pay the loan balance. How much had
Jason borrowed to buy the new Mercedes?

SOLUTION

4-20 You have just taken out a mortgage of $50,000 for 30 years, with monthly
payments at 6% interest. The same day you close on the mortgage you rece
a $25,000 gift from your parents to be applied to the mortgage principal. W
amount of time will now be required to pay off the mortgage if you contin
make the original monthly payments? What is the amount of the last paym
(Assume any residual partial payment amount is added to the last payment

SOLUTION

4-21 Mr. Deere just purchased a new riding lawn mower. The first year mainten
is free. His maintenance costs are estimated to be $15 the second year and
increase by $15 each year thereafter. At 8% interest how much money sho
Mr. Deere set aside when he buys the mower to pay for maintenance for th
next 10 years?

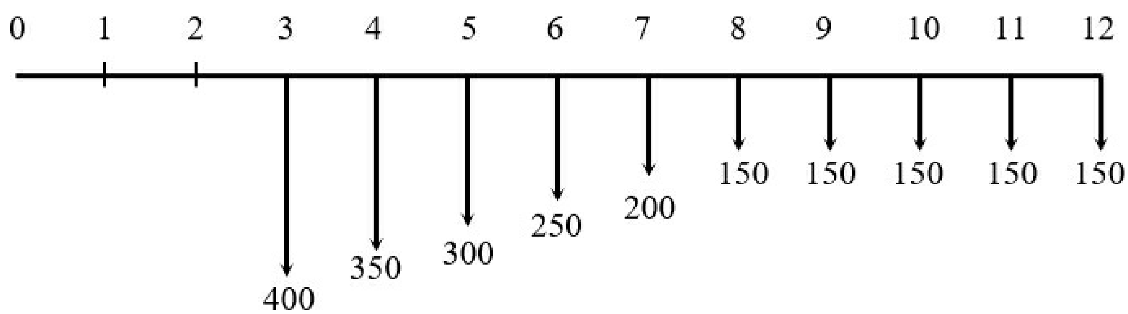
SOLUTION

4-22 Today Sam Keel deposits $5,000 in an account that earns 4% compounded quarterly. Additional deposits are made at the end of each quarter for the n 10 years. The deposits start at $100 and increase by $50 each quarter there Determine the amount that has accumulated in the account at the end of 10 years.

SOLUTION

Find the uniform annual equivalent for the following cash flow diagram if 10%. Use the appropriate gradient and uniform series factors.

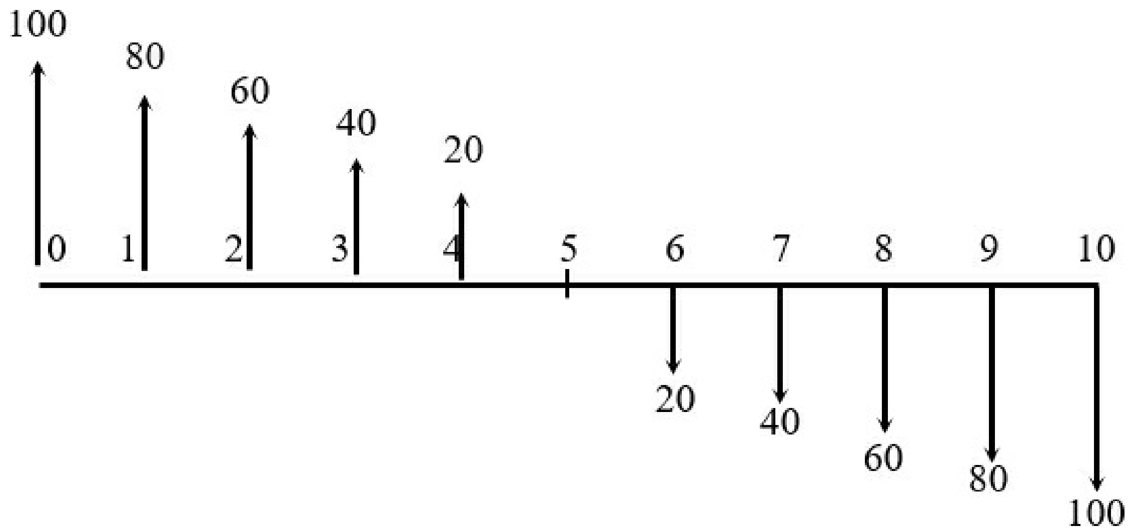
4-23



SOLUTION

Find the present equivalent of the following cash flow diagram if i = 8%.

4-24



SOLUTION

4-25 Holloman Hops has budgeted $300,000 per year to pay for labor over the r
5 years. If the company expects the cost of labor to increase by $10,000 ea
year, and the interest rate is 10%, what is the expected cost of the labor in t
first year?

SOLUTION

4-26 For the cash flow shown, determine the value of G that will make the futur
worth at the end of Year 6 equal to $8000 at an interest rate of 6% per year

Year	0	1	2	3	4	5	6
Cash Flow	0	600	600 + G	600 + 2G	600 + 3G	600 + 4G	600 + 5G

SOLUTION

4-27 Deposits are made at the end of Years 1 through 7 into an account paying 3
The first deposit equals $5,000 and each deposit will increase by $1,000 ea
year thereafter. After the last deposit assume no deposits or withdrawals ar
made. Determine the amount in the account after 10 years.

SOLUTION

4-28 Determine the uniform annual equivalent of the following cash flows at an interest rate of 6%.

t	0	1	2	3	4	5	6	7	8	9	10
CF_t	$100	$90	$80	$70	$60	$50	$50	$60	$70	$80	$90

SOLUTION

4-29 A set of cash flows begins at $25,000 the first year, increasing 10% each y until n = 10 years. If the interest rate is 5%, what is the present value?

SOLUTION

4-30 A set of cash flows begins at $25,000 the first year, decreasing 10% each y until n = 10 years. If the interest rate is 4%, what is the present value?

SOLUTION

4-31 Maintenance costs on some mining equipment is expected to be $20,000 th first year, increasing 10% each year until the equipment is retired after 4 ye If the interest rate is 12%, what the net present value of the maintenance cc

SOLUTION

4-32 How much will accumulate in an Individual Retirement Account (IRA) in years if $5000 is deposited in the account at the end of each quarter during time? The account earns 4% interest, compounded quarterly. What is the effective interest rate?

SOLUTION

4-33 Patrick J. just purchased a Zündapp Janus (a model and brand of car) for $4,000. He agreed to pay for the car with monthly payments of $138.80 ov 36-month period. What nominal interest rate is he paying on the loan? Wha the effective rate?

SOLUTION

4-34 A bank is offering a loan of $20,000 with an interest rate of 9%, payable w
monthly payments over a 4-year period.
a. Calculate the monthly payment required to repay the loan.
b. This bank also charges a loan fee of 4% of the amount of the loan, payal
the time of the closing of the loan (that is, at the time the borrower receive
money). What effective interest rate is the bank charging?

SOLUTION

4-35 The effective interest rate on a mortgage with monthly payments is 9.38%.
What is the monthly payment if the original mortgage amount is $200,000
the mortgage is to be paid over 30 years?

SOLUTION

4-36 Big John Sipes, owner of Sipes's Sipping Shine, has decided to replace the
distillation system his company now uses. After some research, he finds ar
acceptable distiller that costs $62,500. The current machine has approxima
1200 lbs. of copper tubing that can be salvaged and sold for $4.75/lb. to us
a down payment on the new machine. The remaining components of the
distillation machine can be sold as scrap for $3000. This amount will also l
used to pay for the replacement equipment. The remaining money will be
obtained through a 10-year mortgage with quarterly payments at an interes
rate of 8%. Determine the quarterly payment required to pay off the mortga
Also determine the effective interest rate on the loan.

SOLUTION

4-37 Twelve monthly payments are needed to pay off a $2000 loan. The annual
interest rate is 12.5%. Build a table that shows the balance due, principal
payment, and the interest payment for each month. How much interest was
paid?

SOLUTION

4-38 Determine the present value of cash flows that start at $25,000 and increas
per year, ending in year 10. The interest rate is 5%.

SOLUTION

4-39 Find the *annual* worth of a quarterly lease payment of $500 at 8% interest.

SOLUTION

4-40 A person would like to retire 15 years from now. He currently has $132,00 savings, and he plans to deposit $800 per month, starting next month, in a special retirement plan. The $132,000 is earning 8% interest, while the monthly deposits will pay him 6% annual interest. Once he retires, he will deposit the total of the two sums of money into an account that he expects earn a 4% annual interest rate. Assuming that he will spend only the intere earns, how much will he collect in annual interest, starting in Year 16?

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Uniform Annual Cash Flows

4-1 E

Rose recently graduated in engineering. Her employer will give her a raise of $6500 per year if she passes the FE exam (Fundamentals of Engineering).

(*a*) Over a career of 45 years, what is the present worth of the raise if the interest rate is 4%?

(*b*) What is the future worth at Year 45?

(*c*) Incentive pay systems can create ethical dilemmas in the workplace. Describe one each from the perspective of the employer and the employee.

4-2 Jose graduated in engineering 5 years ago. His employer will give him a raise of $8,000 per year if he passes the PE exam (Professional Engineer).

A (*a*) Over a career of 30 years, what is the present worth of the raise if the interest rate is 7%?

(*b*) What is the future worth at year 30?

4-3 Brad will graduate next year. When he begins working, he plans to deposit $6000 at the end of each year into a retirement account that pays 6% interest. How much will be in his account after 40 deposits? *Contributed by Paul R. McCright, University of South Florida*

4-4 If the university's College of Engineering can earn 5% on its investments, how much should be in its savings account to fund one

A $10,000 scholarship each year for 4 years? *Contributed by Paul R. McCright, University of South Florida*

4-5 How much must be invested now at 8% interest to produce $5000 at the end of every year for 10 years?

4-6 A man buys a car for $33,000 with no money down. He pays for the

A car in 48 equal monthly payments with interest at 9% per annum, compounded monthly. What is his monthly loan payment?

4-7 A car may be purchased with a $3500 down payment now and 72 monthly payments of $480. If the interest rate is 9% compounded monthly, what is the price of the car?

4-8 A manufacturing firm spends $350,000 annually for a required safety inspection program. A new monitoring technology would eliminate the need for such inspection. If the interest rate is 8% per year, how much can the firm afford to spend on this new

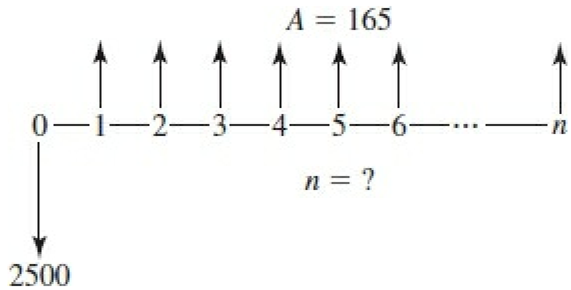
A technology? The firm wants to recover its investment in 20 years. *Contributed by Hamed Kashani, Saeid Sadri, Baabak Ashuri, Georgia Institute of Technology*

4-9 A city engineer knows that she will need $25 million in 3 years to implement new automated toll booths on a toll road in the city. Traffic on the road is estimated to be 3 million vehicles per year.

How much per vehicle should the toll be to cover the cost of the toll booth replacement project in 3 years? Interest is 8%. (Simplify your analysis by assuming that the toll receipts are received at year end as a lump sum.)

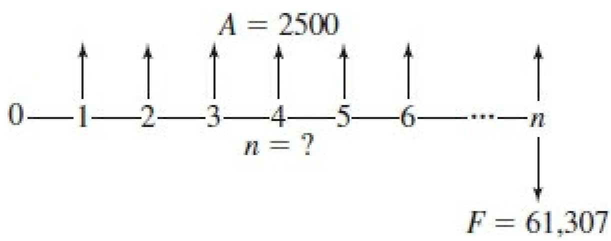
4-10
A

For what value of *n*, based on a 5% interest rate, do these cash flows have a present value of 0?



4-11

The cash flows have a present value of 0. Compute the value of *n*, assuming a 10% interest rate.



4-12
A

How many months will it take to pay off a $705 debt, with monthly payments of $25 at the end of each month, if the interest rate is 15%, compounded monthly?

4-13

Assume that you save $1 a day for 50 years, that you deposit it in the bank at the end of each month, and that there are 30.5 days per month (you save $30.50 each month). How much do you have after 50 years, if:

(*a*) The bank does not pay any interest.

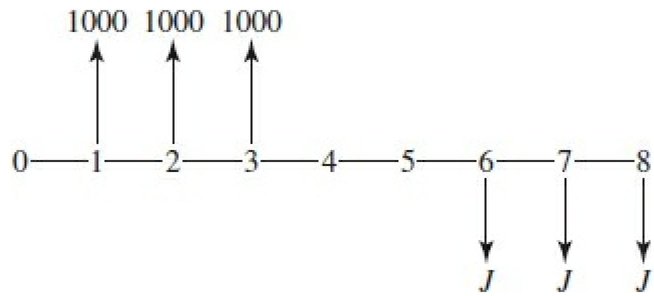
(*b*) The bank pays $1/2\%$ per month interest.

Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

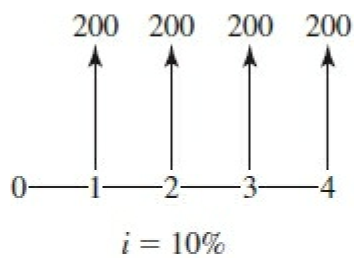
A local finance company will loan $15,000 to a homeowner. It is to

4-14 A be repaid in 36 monthly payments of $588 each. The first payment is due 30 days after the $15,000 is received. What interest rate per month are they charging?

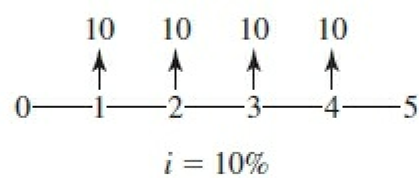
4-15 The cash flows have a present value of 0. Compute the value of *J*, assuming a 6% interest rate.



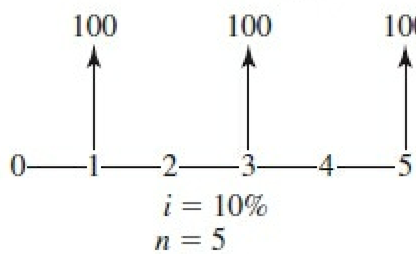
4-16 A For diagrams (*a*) to (*c*), compute the unknown values *B*, *C*, *V*, using the minimum number of compound interest factors.



(*a*) B = value at $t = 0$



(*b*) C = value at $t = 5$



(*c*) V = value at $t = 0$

4-17 G The automated toll booth system from Problem 4-9 improves traffic flow and thus reduces greenhouse emissions in the city. Rather than wait three years, the city can do it now with a 2% loan. If the toll is set at $0.75 per vehicle, and the state subsidizes the project at a rate of $0.25 per vehicle, how many years will it take for the city to pay off the loan?

4-18 A company deposits $3500 in a bank at the end of every year for 12 years. The company makes no deposits during the subsequent 8 years. If the bank pays 6% interest, how much would be in the account at the end of 20 years?
A

4-19 Kelsey Construction has purchased a crane that comes with a 5-year warranty. Repair costs are expected to average $5000 per year beginning in Year 6 when the warranty expires. Determine the present worth of the crane's repair costs over its 15-year life. The interest rate is 10%.

4-20 A company buys a machine for $17,000, which it agrees to pay for in six equal annual payments, beginning one year after the date of purchase, at an annual interest rate of 5%. Immediately after the second payment, the terms of the agreement are changed to allow the balance due to be paid off in a single payment the next year. What is the final single payment?
A

4-21 Using linear interpolation, determine the value of (*a*) (*F*/*A*, 11%, 15) and (*b*) (*F*/*P*, 16%, 25) from the compound interest tables. Compute this same value using the equation or a 5-BUTTON SOLUTION. Why do the values differ?

4-22 A student is buying a new car. The car's price is $37,500, the sales tax is 6%, and the title, license, and registration fee is $1250 to be paid in cash. The dealer offers to finance 95% of the car's price for 48 months at a nominal interest rate of 9% per year, compounded monthly.
A
(*a*) How much cash is paid when the car is purchased?
(*b*) How much is the monthly payment?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

4-23 Jennifer Creek is saving up for a new car. She wants to finance no more than $10,000 of the $26,000 estimated price in two years. She deposits $5000 into a savings account now and will make monthly deposits for the next two years. If the savings account pays a nominal interest rate of 3% per year with monthly compounding, how much must she deposit each month? *Contributed by Gillian Nicholls, Southeast Missouri State University*

4-24 A Tori is planning to buy a car. The maximum payment she can make is $3400 per year, and she can get a car loan at her credit union for 7.3% interest. Assume her payments will be made at the end of each year 1–4. If Tori's old car can be traded in for $3325, which is her down payment, what is the most expensive car she can purchase?
Contributed by Paul R. McCright, University of South Florida

4-25 Determine the breakeven resale price 15 years from now of an apartment house that can be bought today for $250,000. Its annual net income is $22,000. The owner wants a 10% annual return on her investment.
Contributed by D. P. Loucks, Cornell University

4-26 A Martin pays rent of $650 per month for the 9-month academic year. He is going to travel the world this summer and won't be working. How much must he set aside in his savings account for the 3-month summer to cover his rent for next year? The savings account earns 6% with monthly compounding.

4-27 Tameshia deposits $5500 in her retirement account every year. If her account pays an average of 6% interest and she makes 38 deposits before she retires, how much money can she withdraw in 20 equal payments beginning one year after her last deposit?
Contributed by Paul R. McCright, University of South Florida

4-28 A A young engineer wishes to become a millionaire by the time she is 60 years old. She believes that by careful investment she can obtain a 15% rate of return. She plans to add a uniform sum of money to her investment program each year, beginning on her 20th birthday and continuing through her 59th birthday. How much money must the engineer set aside in this project each year?

4-29 What amount will be required to purchase, on an engineers's 40th birthday, an annuity to provide him with 25 equal semiannual payments of $10,000 each, the first to be received on his 60th birthday, if nominal interest is 4% compounded semiannually?

4-30 A A man wants to help provide a college education for his young daughter. He can afford to invest $1500/yr for the next 5 years, beginning on the girl's 5th birthday. He wishes to give his daughter $10,000 on her 18th, 19th, 20th, and 21st birthdays, for a total of $40,000. Assuming 6% interest, what uniform annual investment will he have to make on the girl's 9th through 17th birthdays?

4-31 To provide for a college education for her son, a woman opened an escrow account in which equal deposits were made. The first deposit was made on January 1, 1998, and the last deposit was made on January 1, 2015. The yearly college expenses including tuition were estimated to be $9000, for each of the 4 years. Assuming the interest rate to be 4.5%, how much did the mother have to deposit each year in the escrow account for the son to draw $9000 per year for 4 years beginning January 1, 2013?

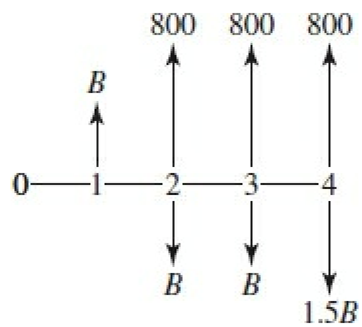
4-32 A Jennifer is saving up for the closing costs ($4250) and down payment on a home. For a better interest rate and savings on mortgage insurance, she must have a down payment of 10%. She can afford a monthly payment of $900 based on her current earnings and expenses. The amount available for the mortgage is reduced by an estimated $175 per month to cover home insurance and real estate taxes. The current nominal annual interest rate is 3% for a 30-year fixed-rate mortgage loan. How much of a loan can she afford? What is the corresponding house price? How much must she save? *Contributed by Gillian Nicholls, Southeast Missouri State University*

4-33 Abby and Jason are building a new house. They obtained a construction loan of $100,000, which will be rolled over into a conventional 20-year mortgage when the house is completed in 14 months. Simple interest of 1/2% per month will be charged on the construction loan. The 20-year mortgage will carry a 6% interest rate with monthly payments. What is the monthly payment that Abby and Jason will make? If they make each payment as scheduled for the life of the 20-year mortgage, how much total interest will they pay on the house? *Contributed by Ed Wheeler, University of Tennessee at Martin*

4-34 Jerry bought a house for $400,000 and made an $80,000 down payment. He obtained a 30-year loan for the remaining amount. Payments were made monthly. The nominal annual interest rate was 6%. After 10 years (120 payments) he sold the house and paid off the loan's remaining balance.
(*a*) What was his monthly loan payment?

(*b*) What must he have paid (in addition to his regular 120th monthly payment) to pay off the loan?

4-35 If $i = 5\%$, for what value of B is the present value = 0.



4-36 An engineer graduates at age 22, and she gets a job that pays \$65,000 per year. She wants to invest enough to fund her own retirement without relying on an employer pension program or Social Security. Her goal is to have \$1.5 million saved for retirement at age 67. She is relatively confident that her investments will earn an average interest rate of at least 4% per year.

(*a*) Assume that she makes equal annual deposits starting on her 23rd birthday and continuing through her 67th birthday. How much must she invest each year to meet her goal?

(*b*) Suppose she invests the same amount from part (a) every year starting on her 33rd birthday. How much money will she have in the account on her 67th birthday under this scenario?

Contributed by Gillian Nicholls, Southeast Missouri State University

4-37 On January 1, Frank bought a used car for \$7200 and agreed to pay for it as follows: 1/4 down payment; the balance to be paid in 36 equal monthly payments; the first payment due February 1; an annual interest rate of 9%, compounded monthly.

(*a*) What is the amount of Frank's monthly payment?

(*b*) During the summer, Frank made enough money to pay off the entire balance due on the car as of October 1. How much did Frank owe on October 1?

An engineering student bought a car at a local used car lot. Including tax and insurance, the total price was \$19,999. He is to pay for the car in 16 equal monthly payments, beginning with the first payment

4-38 immediately (the first payment is the down payment). Nominal interest on the loan is 9%, compounded monthly. Immediately after making the down payment and six more payments he decides to sell the car. A buyer agrees to pay off the loan in full and to pay the engineering student $1500. If there are no penalty charges for this early payment of the loan, how much will the car cost the new buyer?

4-39 Liam dreams of starting his own business to import consumer electronic products to his home country. He estimates he can earn 5% on his investments and that he will need to have $300,000 at the end of year 10 if he wants to give his business a good solid foundation. He now has $28,850 in his account, and he believes he can save $12,000 each year from his income, beginning now. He plans to marry at about the end of Year 6 and will skip the investment contribution that year. How far below or above his $300,000 goal will he be? *Contributed by Paul R. McCright, University of South Florida*

4-40 Table 3–1 presented four plans for the repayment of $5000 in 5 years with interest at 8%. Still another way to repay the $5000 would be to make four annual end-of-year payments of $750 each, followed by a final payment at the end of the fifth year. How much would the final payment be?

4-41 An engineer borrowed $3000 from the bank, payable in six equal end-of-year payments at 8%. The bank agreed to reduce the interest on the loan if interest rates declined before the loan was fully repaid. After the third payment, the bank agreed to reduce the interest rate on the remaining debt to $6\frac{1}{4}\%$. What was the amount of the equal annual end-of-year payments for each of the first 3 years? What was the amount of the equal annual end-of-year payments for each of the last 3 years?

4-42 A woman made 7 annual end-of-year purchases of $3500 worth of common stock. The stock paid no dividends. Then after holding the stock for 8 years, she sold all the stock for $38,000. What interest rate did her investment earn?

A bank recently announced an “instant cash” plan for holders of its

4-43 bank credit cards. A cardholder may receive cash from the bank up to a preset limit (about \$500). There is a special charge of the minimum of \$15 or 4% made at the time the "instant cash" is sent to the cardholders. Each month the bank charges $1\frac{1}{2}\%$ on the unpaid balance. The monthly payment, including interest, may be as little as \$10. Assume the cardholder makes the minimum monthly payment of \$10. How many months are required to repay \$150 of instant cash? How much interest is paid?

Moment Diagram Equations

4-44 Write the cash flow equivalence equation as a moment equation for Problem 4-15.
(*a*) About Year 4
(*b*) About Year 5

4-45 Write the cash flow equivalence equation as a moment equation about Year 4 for Problem 4-16.
(*a*) Include *B*
(*b*) Include *C*
(*c*) Include *V*

4-46 Write the cash flow equivalence equation as a moment equation about Year 3 for Problem 4-35.

Relationships Between Factors

4-47 How can the tables be used to compute (*P*/*A*, 5%, 150)? (*P*/*A*, 7%, 200)?

4-48 A For some interest rate *i* and some number of interest periods *n*, the uniform series capital recovery factor is 0.1408 and the sinking fund factor is 0.0408. What is the interest rate? What is *n*?

4-49 For some interest rate *i* and some number of interest periods *n*, the uniform series capital recovery factor is 0.1468 and the sinking fund factor is 0.0268. What is the interest rate? What is *n*?

4-50 A

Derive an equation to find the end-of-year future sum F that is equivalent to a series of n beginning-of-year payments B at interest rate i. Then use the equation to determine the future sum F equivalent to six B payments of \$100 at 8% interest.

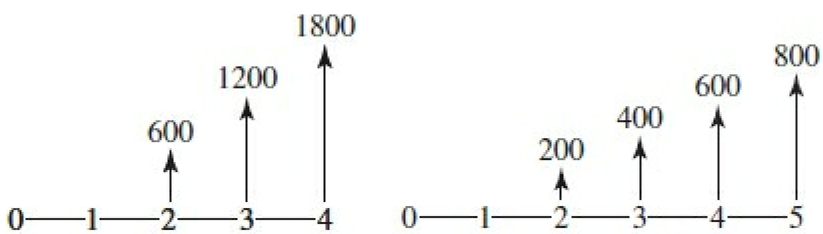
4-51

Prove the following relationships algebraically
(a) $(A/F, i, n) = (A/P, i, n) - i$
(b) $(P/F, i, n) = (P/A, i, n) - (P/A, i, n - 1)$
(c) $(P/A, i, n) = (P/F, i, 1) + (P/F, i, 2) + \cdots + (P/F, i, n)$
(d) $(F/A, i, n) = [(F/P, i, n) - 1]/i$

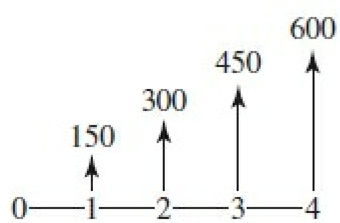
Arithmetic Gradients

4-52 A

Assume an 8% interest rate and find S, T, and X.



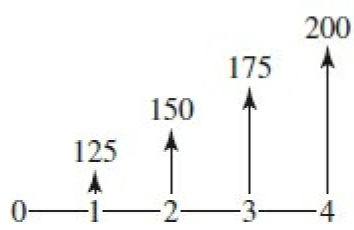
(a) S = value at $t = 0$ (b) T = uniform value for 5 periods



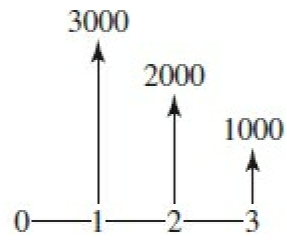
(c) X = value at $t = 0$

For diagrams (a) to (d), compute the present values of the cash flows.

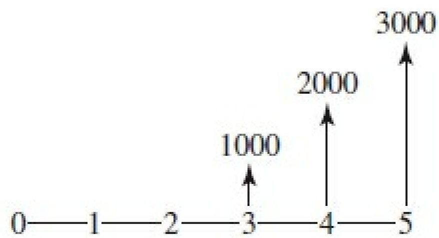
4-53



(*a*) $i = 10\%$



(*b*) $i = 10\%$



(*c*) $i = 10\%$

4-54
A

It is estimated that the maintenance cost on a new car will be $350 the first year. Each subsequent year, this cost is expected to increase by $250. How much would you need to set aside when you bought a new car to pay all future maintenance costs if you planned to keep the vehicle for 15 years? Assume interest is 6% per year.

4-55

A firm expects to install smog control equipment on the exhaust of a gasoline engine. The local smog control district has agreed to pay to the firm a lump sum of money to provide for the first cost of the equipment and maintenance during its 10-year useful life. At the end of 10 years the equipment, which initially cost $10,000, is valueless. The firm and the smog control district have agreed that the following are reasonable estimates of the end-of-year maintenance costs:

Year 1	2	3	4	5
$75	100	125	150	175
Year 6	7	8	9	10
$200	225	250	275	300

Assuming interest at 6% per year, how much should the smog control district pay to the firm now to provide for the first cost of the equipment and its maintenance for 10 years?

4-56 A The council members of a small town have decided that the earth levee that protects the town from flooding should be rebuilt and strengthened. The town engineer estimates that the cost of the work at the end of the first year will be $68,000. He estimates that in subsequent years the annual repair costs will decline by $7000, making the second-year cost $61,000; the third-year $54,000, and so forth. The council members want to know what the equivalent present cost is for the first 5 years of repair work if interest is 6%.

4-57 G A construction firm can achieve a $15,000 cost savings in Year 1 and increasing by $2000 each year for the next 5 years by converting their diesel engines for biodiesel fuel.
(*a*) At an interest rate of 15%, what is the equivalent annual worth of the savings?
(*b*) What is biodiesel fuel and why is it considered green energy?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

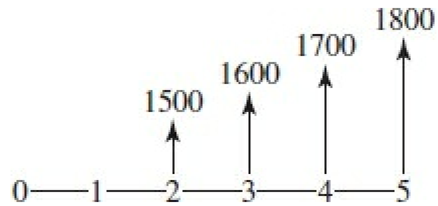
4-58 A Helen can earn 3% interest in her savings account. Her daughter Roberta is 11 years old today. Suppose Helen deposits $4000 today, and one year from today she deposits $1000. Each year she increases her deposit by $500 until she makes her last deposit on Roberta's 18th birthday. How much is on deposit after the 18th birthday, and what is the annual equivalent of her deposits? *Contributed by Paul R. McCright, University of South Florida*

4-59 Perry is a freshman. He estimates that the cost of tuition, books, room and board, transportation, and other incidentals will be $21,000 this year. He expects these costs to rise about $1500 each year while he is in college. If it will take him 5 years to earn his BS, what is the present cost of his degree at an interest rate of 4%? If he earns an extra $15,000 annually for 40 years, what is the present worth of his degree? *Contributed by Paul R. McCright, University of South Florida*

4-60 A sports star can sign a 6-year contract that starts at $12M with increases of $3M each year for his expected playing career of 6 years. It is also possible to sign a contract that starts at $8M for the first year and then increases at $2M each year for 10 years (note some income is deferred until after he retires). If his interest rate for the time value of money is 8%, what is the value of each choice? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*

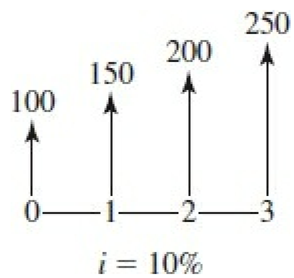
Use a 7% interest rate to compute the present value of the cash flows.

4-61



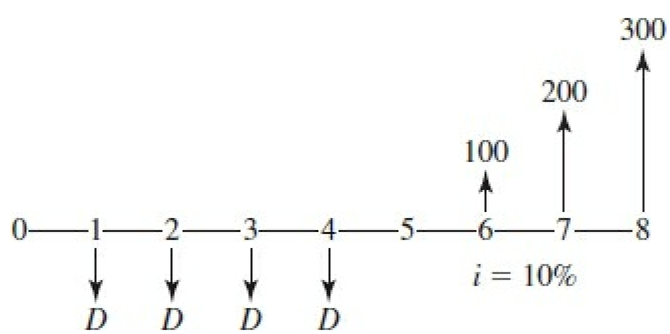
Compute the present value of the cash flows.

4-62
A



The cash flows have a present value equal 0. Compute the value of *D* in the diagram.

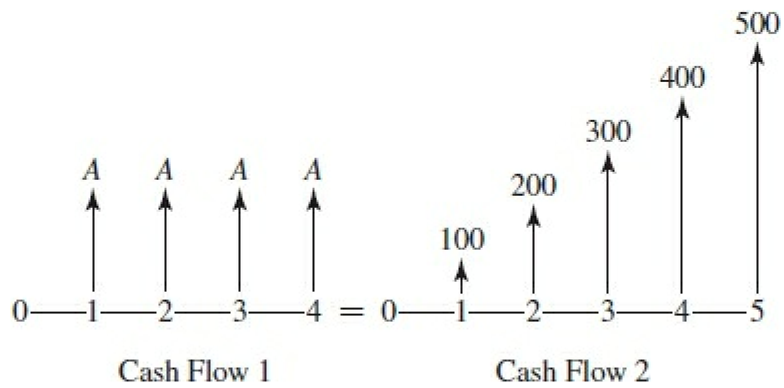
4-63



These cash flow transactions are said to be equivalent in terms of economic desirability at an interest rate of 9% compounded annually. Determine the unknown value *A*.

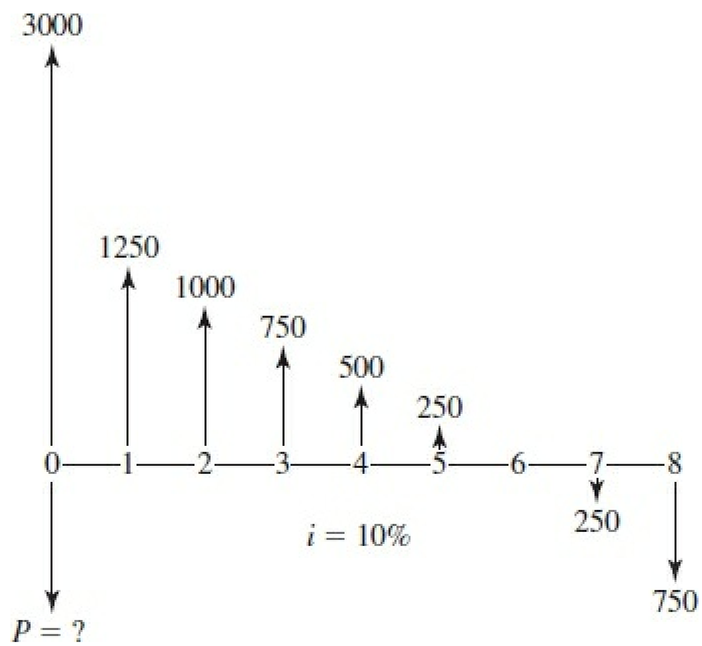
4-64

A



4-65

For what value of P in the cash flow diagram does the present value equal 0?



4-66

A

A debt of \$5000 can be repaid, with interest at 8%, by the following payments. How much is X?

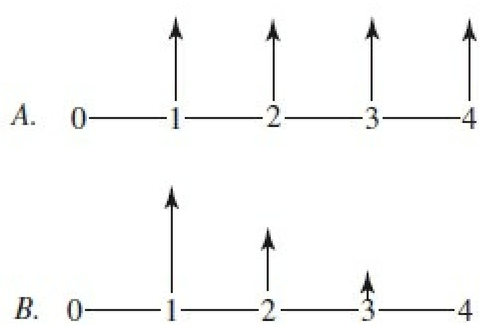
Year	Payment
1	\$ 500
2	1000
3	1500
4	2000

5 *X*

4-67 G A man is buying a new eco-friendly electric riding mower. There will be no maintenance cost during the first 2 years because the mower is sold with 2 years free maintenance. For the third year, the maintenance is estimated at $100. In subsequent years the maintenance cost will increase by $50 per year.
(*a*) How much would need to be set aside now at 8% interest to pay the maintenance costs on the tractor for the first 6 years of ownership?
(*b*) What are the advantages and disadvantages of electric powered riding mowers?

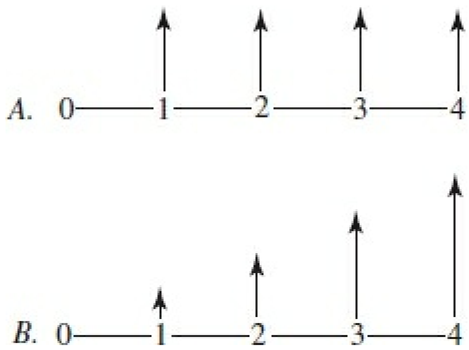
4-68 A college student is buying a new subcompact car, which costs $25,000 plus 7% sales tax. The title, license, and registration fees are $1100. The dealer offers her a financing program that starts with a monthly payment of $500, and each successive payment will increase by a constant dollar amount *x*. The dealer offers to finance 90% of the car's price for 48 months at a nominal interest rate of 6% per year, compounded monthly.
(*a*) How much is the constant amount *x*?
(*b*) How much is the 48th payment?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

4-69 The following cash flows are equivalent in value if the interest rate is *i*. Which one is more valuable if the interest rate is 2*i*?



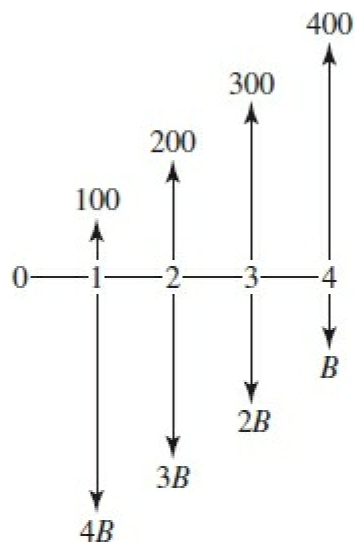
The following cash flows are equivalent in value if the interest rate is *i*. Which one is more valuable if the interest rate is 2*i*?

4-70



Using a 10% interest rate, for what value of *B* does the present value equal 0?

4-71



Consider the following cash flow:

Year	Cash Flow
0	–$100
1	50
2	60
3	70
4	80
5	140

4-72

If the present value is zero, which one of the following is correct?

A. $100 = 50 + 10(A/G, i, 5) + 50(P/F, i, 5)$

B. $$1 = \frac{50(P/A, i, 5) + 10(P/G, i, 5) + 50(P/F, i, 5)}{100}$$

C. $100(A/P, i, 5) = 50 + 10(A/G, i, 5)$

D. None of the equations are correct.

4-73 Consider the following cash flow:

Year	Cash Flow
1	\$1000
2	\$850
3	\$700
4	\$550
5	\$400
6	\$400
7	\$400
8	\$400

Which of the equations below is correct to compute the present value of the cash flows at 8% interest?

A. $P = 1000(P/A, i, 8) - 150(P/G, i, 8) + 150(P/G, i, 4)(P/F, i, 4)$

B. $P = 400(P/A, i, 8) + 600(P/A, i, 5) - 150(P/G, i, 4)$

C. $P = 150(P/G, i, 4) + 850(P/A, i, 4) + 400(P/A, i, 4)(P/F, i, 4)$

Geometric Gradients

4-74 A The market for a product is expected to increase at an annual rate of 6%. First-year sales are estimated at \$45,000, the horizon is 12 years, and the interest rate is 8%. What is the present value?

4-75 G Fred is evaluating whether a more efficient motor with a life of 5 years should be installed on an assembly line. If the interest rate is 10%, what is the present value of the energy savings?

(*a*) Energy savings are estimated at \$4000 for the first year, then increasing by 7% annually.

(*b*) What if the energy savings are increasing by 12% annually?

4-76 A
A set of cash flows begins at $20,000 the first year, with an increase each year until n = 10 years. If the interest rate is 8%, what is the present value when
(*a*) the annual increase is $2000?
(*b*) the annual increase is 10%?

4-77
A set of cash flows begins at $80,000 the first year, with an increase each year until n = 10 years. If the interest rate is 5%, what is the present value when
(*a*) the annual increase is $10,000?
(*b*) the annual increase is 15%?

4-78 A
A set of cash flows begins at $15,000 the first year, with an increase each year until n = 12 years. If the interest rate is 8%, what is the present value when
(*a*) the annual increase is $1000?
(*b*) the annual increase is 8%?

4-79
A set of cash flows begins at $20,000 the first year, with a decrease each year until n = 10 years. If the interest rate is 7%, what is the present value when
(*a*) the annual decrease is $2000?
(*b*) the annual decrease is 10%?

4-80 A
Suzanne is a recent chemical engineering graduate who has been offered a 5-year contract at a remote location. She has been offered two choices. The first is a fixed salary of $80,000 per year. The second has a starting salary of $70,000 with annual raises of 8% starting in Year 2. (For simplicity, assume that her salary is paid at the end of the year, just before her annual vacation.) If her interest rate is 10%, which should she take?

4-81
The football coach at a midwestern university was given a 5-year employment contract that paid $800,000 the first year, and increased at an 8% uniform rate in each subsequent year. At the end of the first year's football season, the alumni demanded that the coach be fired. The alumni agreed to buy his remaining years on the contract by paying him the equivalent present sum, computed using a 6% interest rate. How much will the coach receive?

4-82 A contractor estimates maintenance costs for a new backhoe to be $500 for the first month with a monthly increase of 0.75%. The contractor can buy a 4-year maintenance contract for $20,000 at any point. If the contract is purchased at the same time as the backhoe is purchased, the dealer has offered a 10% discount. Use i = 1% per month. What should the contractor do? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*

4-83 Eddie is a production engineer for a major supplier of component parts for cars. He has determined that a robot can be installed on the production line to replace one employee. The employee earns $20 per hour and benefits worth $8 per hour for a total annual cost of $58,240 this year. Eddie estimates this cost will increase 6% each year. The robot will cost $16,500 to operate for the first year with costs increasing by $1500 each year. The firm uses an interest rate of 15% and a 10-year planning horizon. The robot costs $75,000 installed and will have a salvage value of $5000 after 10 years. Should Eddie recommend that purchase of the robot?
Contributed by Paul R. McCright, University of South Florida

4-84 An engineer will deposit 10% of her salary each year into a retirement fund. If her current annual salary is $95,000 and she expects that it will increase by 6% each year, what will be the present worth of the fund after 30 years if it earns 6% per year? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*

4-85 Mark Johnson invests a fixed percentage of his salary at the end of each year. This year he invested $1500. For the next 5 years, he expects his salary to increase 8% annually, and he plans to increase his savings at the same rate. How much will the investments be worth at the end of 6 years if the average increase in the stock market is
(*a*) 8%?
(*b*) 5%?
(*c*) 3%?

Zachary has opened a retirement account that will pay 4% interest

each year. He plans to deposit 8% of his annual salary into the
account for 35 years before he retires. His first year's salary is
4-86 \$75,000, and he expects the salary to grow 6% each year. How much
will be in his account after he makes the last deposit? What uniform
amount can he withdraw from the account for 30 years beginning
one year after his last deposit?
Contributed by Paul R. McCright, University of South Florida

A 25-year-old engineer is opening an individual retirement account
(IRA) at a bank. Her goal is to accumulate \$1 million in the account
by the time she retires from work in 40 years. The bank manager
4-87 estimates she may expect to receive 6% nominal annual interest,
compounded quarterly, throughout the 40 years. The engineer
believes her income will increase at a 5% annual rate during her
career. She wishes to start her IRA with as low a deposit as possible
and increase it at a 5% rate each year. Assuming end-of-year
deposits, how much should she deposit the first year?

Nominal and Effective Interest Rates

Pete borrows \$15,000 to purchase a used car. He must repay the loan
in 36 equal end-of-period monthly payments. Interest is calculated at
4-88 $1\frac{1}{4}\%$ per month. Determine the following:
A (*a*) The amount of the monthly payment
(*b*) The nominal annual interest rate
(*c*) The effective annual interest rate

Picabo borrows \$1000. To repay the amount, she makes 12 equal
monthly payments of \$93.12. Determine the following:
4-89 (*a*) The monthly interest rate
(*b*) The nominal annual interest rate
(*c*) The effective annual interest rate

In the 1500s King Henry VIII borrowed money from his bankers on
the condition that he pay 5% of the loan at each fair (there were four
4-90 fairs per year) until he had made 40 payments. At that time the loan
A would be considered repaid. What effective annual interest did King
Henry pay?

4-91 Quentin has been using his credit card too much. His plan is to use only cash until the balance of \$8574 is paid off. The credit card company charges 18% interest, compounded monthly. What is the effective interest rate? How much interest will he owe in the first month's payment? If he makes monthly payments of \$225, how long until it is paid off?
Contributed by Paul R. McCright, University of South Florida

4-92 A E One of the largest car dealers in the city advertises a 3-year-old car for sale as follows:

Cash price \$24,000, or a down payment of \$2400 with 48 monthly payments of \$634.50.

Susan DeVaux bought the car and made a down payment of \$4000. The dealer charged her the same interest rate used in his advertised offer.

(*a*) What is the monthly interest rate? How much will Susan pay each month for 48 months? What effective interest rate is being charged?
(*b*) Find the Ethics Guide of the National Automobile Autodealers Association (NADA). If you were selling an auto on-line would you practice these guidelines?

4-93 You are taking a \$5000 loan. You will pay it back in four equal amounts, paid every 6 months starting 5 years from now. The interest rate is 12% compounded semiannually. Calculate:
(*a*) The effective interest rate
(*b*) The amount of each semiannual payment
(*c*) The total interest paid

4-94 The *Bourbon Street Journal* costs \$625, payable now, for a 3-year subscription. The newspaper is published 252 days per year (5 days per week, except holidays). If an 8.5% nominal annual interest rate, compounded every two months, is used:
(*a*) What is the effective annual interest rate in this problem?
(*b*) Compute the equivalent interest rate per $1/252$ of a year.

(*c*) What is a subscriber's cost per copy of the newspaper, taking interest into account?

4-95 A bank is offering a loan of $10,000 with a nominal interest rate of 6% compounded monthly, payable in 72 months. There is a loan origination fee of 3% that is taken out from the loan amount.

(*a*) What is the monthly payment?

(*b*) What is the effective interest rate?

4-96 A local car dealer offers a customer a 3-year car loan of $15,000 using "add-on" interest.

Money to pay for car	$15,000
Two years' interest at 8%: $3 \times 0.08 \times 15{,}000$	3,600
	$18,600

$$36 \text{ monthly payments} = \frac{18{,}600}{36} = \$516.67$$

The first payment must be made in 30 days. What are the nominal and effective annual interest rates?

4-97 A local lending institution advertises the "51–50 Club." A person may borrow $2000 and repay $51 for the next 50 months, beginning 30 days after receiving the money. Compute the nominal annual interest rate for this loan. What is the effective interest rate?

The **Rule of 78s** is a commonly used method of computing the amount of interest when the balance of a 1-year loan is repaid in advance. Adding the numbers from month 1 to month 12 equals 78.

Now the first month's interest is 12/78 of the year's interest. The second month's interest is 11/78 of the year's interest. Thus after 11 months the total interest charged would be 77/78 of the total year's interest.

4-98 Helen borrowed $10,000 at 15% annual interest, compounded monthly. The loan was to be repaid in 12 equal end-of-period payments. After making the first two payments she decided to pay off the balance along with the third payment. Calculate the amount of this additional sum

(*a*) based on the rule of 78s.
(*b*) based on exact economic analysis methods.
(*c*) How close is the approximation?

Spreadsheets for Loans

4-99 Develop a complete amortization table for a loan of $20,000, to be paid back in 18 uniform monthly installments, based on an interest rate of 6%. The amortization table should include the following column headings:

Payment Number, Principal Owed, Interest Owed, Principal Paid, and Balance Due Monthly Payment

You must also show the equations used to calculate each column of the table.

4-100 A Ten annual payments at an interest rate of 7% are made to repay a loan of $9000. Build the table that shows the balance due, principal payment, and interest payment for each payment. What is the annual payment? What interest is paid in the last year?

4-101 Using the loan and payment plan developed in Problem 4-99, determine the month that the final payment is due, and the amount of the final payment, if $1500 is paid for Payment 5 and $2500 is paid for Payment 10.

4-102 A A newly graduated engineer bought furniture for $4500 from a local store. Monthly payments for 1 year will be made. Interest is computed at a nominal rate of 9%. Build the table that shows the balance due, principal payment, and interest payment for each payment. What is the monthly payment? What interest is paid in the last month?

4-103 Calculate and print out the balance due, principal payment, and interest payment for each period of a used-car loan. The nominal interest is 6% per year, compounded monthly. Payments are made monthly for 3 years.

The original loan is for $18,000.

4-104 Calculate and print out the balance due, principal payment, and interest payment for each period of a new car loan. The nominal interest is 8% per year, compounded monthly. Payments are made monthly for 5 years. The original loan is for $27,000.

4-105 For the used car loan of Problem 4-103, graph the monthly payment.
(*a*) As a function of the interest rate (5–15%).
(*b*) As a function of the number of payments (24–48).

4-106 For the new car loan of Problem 4-104, graph the monthly payment.
(*a*) As a function of the interest rate (4–14%).
(*b*) As a function of the number of payments (36–84).

4-107 Develop a general-purpose spreadsheet to calculate the balance due, principal payment, and interest payment for each period of a loan. The user inputs to the spreadsheet will be the loan amount, the number of payments per year, the number of years payments are made, and the nominal interest rate. Submit printouts of your analysis of a loan in the amount of $15,000 at 8.9% nominal rate for 36 months and for 60 months of payments.

4-108 Use the spreadsheet developed for Problem 4-107 to analyze 180-month and 360-month house loan payments for a $100,000 mortgage loan at a nominal interest rate of 4.5%. Submit a graph of the interest and principal paid over time. What is the total interest paid for each number of payments?

Spreadsheets for Gradients

4-109 What is the present worth of cash flows that begin at $10,000 and increase at 5% per year for 10 years? The interest rate is 12%.

4-110 A What is the present worth of cash flows that begin at $15,000 and increase at 8% per year for 10 years? The interest rate is 6%.

4-111 What is the present worth of cash flows that begin at $30,000 and decrease at 10% per year for 5 years? The interest rate is 15%.

4-112 A What is the present worth of cash flows that begin at $25,000 and decrease at 11% per year for 10 years? The interest rate is 7%.

4-113 Net revenues at an older manufacturing plant will be $2 million for this year. The net revenue will decrease 15% per year for 5 years, when the assembly plant will be closed (at the end of Year 6). If the firm's interest rate is 10%, calculate the PW of the revenue stream.

4-114 Your beginning salary is $70,000. You deposit 12% at the end of each year in a savings account that earns 3% interest. Your salary increases by 2% per year. What value does your savings book show after 40 years?

4-115 The market volume for widgets is increasing by 15% per year from current profits of $300,000. Investing in a design change will allow the profit per widget to stay steady; otherwise profits will drop 3% per year. What is the present worth of the design change over the next 5 years? Ten years? The interest rate is 9%.

4-116 G In an effort to be more environmentally conscious, a homeowner may upgrade a furnace that runs on fuel oil to a natural gas unit. The investment will be $2500 installed. The cost of the natural gas will average $60 per month over the year, instead of the $145 per month that the fuel oil costs. Assume energy costs increase 3% per year. If the interest rate is 9% per year, how long will it take to recover the initial investment?

Compounding and Payment Periods Differ

4-117 Upon the birth of his first child, Dick Jones decided to establish a savings account to partly pay for his son's education. He plans to deposit $200 per month in the account, beginning when the boy is 13 months old. The savings and loan association has a current interest policy of 3% per annum, compounded monthly, paid quarterly. Assuming no change in the interest rate, how much will be in the savings account when Dick's son becomes 16 years old?

Ann deposits $500 at the end of each month into her bank savings

4-118 account. The bank paid 8% nominal interest, compounded and paid quarterly. No interest was paid on money not in the account for the full 3-month period. How much was in Ann's account at the end of 4 years?

A

4-119 What is the present worth of a series of equal quarterly payments of $5000 that extends over a period of 8 years if the interest rate is 12% compounded monthly?

4-120 What single amount on April 1, 2019, is equivalent to a series of equal, semiannual cash flows of $1500 that starts with a cash flow on January 1, 2017, and ends with a cash flow on January 1, 2027? The interest rate is 14% and compounding is quarterly.

4-121 A contractor wishes to set up a special fund by making uniform semiannual end-of-period deposits for 20 years. The fund is to provide $10,000 at the end of each of the last 5 years of the 20-year period. If interest is 8%, compounded semiannually, what is the required semiannual deposit?

4-122 The State University is considering funding options for a new engineering building on campus. The money has been raised for the construction costs and now the focus is raising funds for the annual upkeep and maintenance (U&M) expenses. For this building, contractors will be hired with a series of 3-year agreements over the 30 years. Under each contract the university will pay $125,000 at the beginning of each 3-year agreement to cover all U&M building expenses over that 3-year period. The first 3-year agreement begins when the building opens.

A wealthy alumnus has agreed to donate enough at the building's opening to cover the U&M expenses over the 30-year term. If money invested by the school's engineering foundation earns 6% interest compounded quarterly, how much must be donated?

4-123 A series of monthly cash flows is deposited into an account that earns 12% nominal interest compounded monthly. Each monthly deposit is equal to $2100. The first monthly deposit occurred on June 1, 2012 and the last monthly deposit will be on January 1, 2019. The account (the series of monthly deposits, 12% nominal interest, and monthly compounding) also has equivalent quarterly withdrawals from it. The first quarterly withdrawal is equal to $5000 and occurred on October 1, 2012. The last $5000 withdrawal will occur on January 1, 2019. How much remains in the account after the last withdrawal?

4-124 Jing, a recent engineering graduate, never took engineering economics. When she graduated, she was hired by a prominent engineering firm. The earnings from this job allowed her to deposit $1000 each quarter into a savings account. There were two banks that offered savings accounts in her town (a small town!). The first bank was offering 5.0% interest compounded continuously. The second bank offered 5.125% compounded monthly. Jing decided to deposit in the first bank because it offered continuous compounding. Did she make the right decision?

Minicases

4-125 Assume that you plan to retire 40 years from now and that you expect to need $2M to support the lifestyle that you want.
(*a*) If the interest rate is 10%, is the following statement approximately true? "Waiting 5 years to start saving doubles what you must deposit each year."
(*b*) If the interest rate is 12%, is the required multiplier higher or lower than for the 10% rate in (*a*)?
(*c*) At what interest rate is the following statement exactly true? "Waiting 5 years to start saving doubles what you must deposit each year."

4-126 For winners of the California SuperLotto Plus, the choice is between a lump sum and annual payments that increase from 2.5% for the first year to 2.7% for the second year and then increase by 0.1% per year to 5.1% for the 26^{th} payment. The lump sum is equal to the net proceeds of bonds purchased to fund the 26 payments. This is estimated at 45% to 55% of the lump sum amount. At what interest rate is the present worth of the two payment plans equivalent if the lump sum is 45%? If it is 55%?

CASES

The following case from *Cases in Engineering Economy* (www.oup.com/us/newnan) is suggested as matched with this chapter.

CASE 27 **Can Crusher**
Basic time value of money, costs, and breakeven analysis.

[1] A *sinking fund* is a separate fund into which one makes a uniform series of money deposits (A) to accumulate a desired future sum (F) by the end of period n.

[2] We thank David Elizandro and Jessica Matson of Tennessee Tech for developing, testing, and describing this approach (*The Engineering Economist*, 2007, Vol. 52, No. 2, "Taking a Moment to Teach Engineering Economy," pp. 97–116).

[3] Normally the interest period is one year, but it could be some other period (e.g., quarter, month, half-year).

CHAPTER 5

PRESENT WORTH ANALYSIS

The Present Value of 30 Years of Benefits

The Columbia River, from its headwaters west of Banff in the Canadian Rockies (Columbia Lake, elevation 820 m) flows 2000 km through British Columbia and the state of Washington before entering the Pacific Ocean at Astoria, Oregon. Measured by the volume of its flow, the Columbia is the largest river flowing into the Pacific from North America. Measured by elevation drop (0.41 m/km compared to the Mississippi's 0.12 m/km), the Columbia alone possesses one-third of the hydroelectric potential of the U.S. Because of the steep mountain trenches and high snowfall in its catchment area, the Columbia's water levels used to fluctuate wildly, and vulnerable areas along its banks were subject to seasonal flooding.

Anxious to exploit that hydroelectric power and control flooding, the U.S. and Canadian governments negotiated the Columbia River Treaty. It was signed in 1961 and ratified by the U.S. in 1961 and by Canada in 1964.

The Treaty requires Canada to store 15.5 Million Acre Feet [the volume of water that would cover an acre of land one foot deep], for flood control in perpetuity. This storage was accomplished with the construction of the Duncan, Hugh Keenleyside, and Mica Dams in Canada. In return for constructing the dams and regulating the water levels, the Province of British Columbia is entitled to half of the electrical downstream power benefits that the water generates on the dams located in the U.S. Canada was also entitled to half of the estimated value of the future flood control benefits in the U.S. in a one-time payment of $69.9 million. (*Virtual Museum of Canada*)

It took Canada several years to ratify the treaty because British Columbia in the 1950s was sparsely populated and not very industrialized, and the prospects of turning a large part of the interior of the province into a reservoir to provide benefits that the residents could not use was not appealing to the premier, W. A. C. Bennett. The impasse was resolved when the U.S. agreed to pay in cash to British Columbia the discounted present value of the first 30 years of the province's benefit entitlement. The treaty said:

The purchase price of the entitlement shall be $254,400,000, in United States funds as of October 1, 1964, subject to adjustment, in the event of an earlier payment of all or part thereof, to the then present worth, at a discount rate of 4.5% per annum.

The 30 years was up in 1994, and British Columbia then started receiving annual benefits. Today, Canada's 50% share in the downstream benefits is worth annually about $250 to 350 million, which is paid to the government of British Columbia.

Contributed by John Whittaker, University of Alberta

QUESTIONS TO CONSIDER

1. Canadian Premier Bennett used the $254 million to finance the Portage Mountain Dam (later renamed the W. A. C. Bennett Dam) and an associated 2730-megawatt power station in northern British Columbia. Was this a good use for the money from the Canadian perspective?

2. Negotiating downstream rights can be a curious tangle. The people at the bottom of the hill can argue that the water is going to flow there anyway and they are free to use it as they want. The people upstream can argue that while the water is on their land, they should be free to use or divert it as they please. Who is right- and wrong-minded in this situation? What are the ethical considerations from both perspectives?
3. Was 4.5%, the discount rate used in 1964, a reasonable one for the governments to choose? Explain and justify your conclusion.

After Completing This Chapter…

The student should be able to:

• Define and apply the *present worth criteria*.
• Compare two competing alternative choices using present worth (PW).
• Apply the PW model in cases with equal, unequal, and infinite project lives.
• Compare multiple alternatives using the PW criteria.
• Develop and use spreadsheets to make *present worth* calculations.
• Compute bond prices and yields.

Key Words

analysis period

capitalized cost

coupon interest rate

economic efficiency

financing

investment

least common multiple

minimum attractive rate of return (MARR)

net present worth criterion

par (face) value

planning horizon

present worth analysis

project life

XNPV

In Chapters 3 and 4 we accomplished two important tasks. First, we presented the concept of equivalence. We can compare cash flows only if we can resolve them into equivalent values. Second, we derived a series of compound interest factors to find those equivalent values.

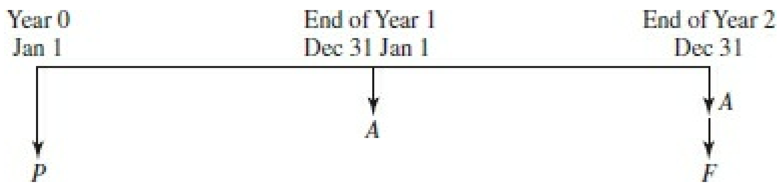
ASSUMPTIONS IN SOLVING ECONOMIC ANALYSIS PROBLEMS

One of the difficulties of problem solving is that most problems tend to be very complicated. It becomes apparent that *some* simplifying assumptions are needed to make complex problems manageable. The trick, of course, is to solve the simplified problem and still be satisfied that the solution is applicable to the *real* problem! In the subsections that follow, we will consider six different items and explain the customary assumptions that are made. These assumptions apply to all problems and examples, unless other assumptions are given.

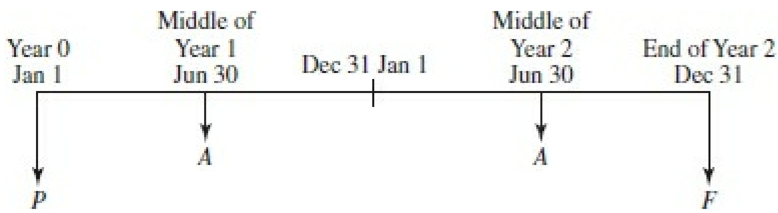
End-of-Year Convention

As we indicated in Chapter 4, economic analysis textbooks and practice follow the end-of-period convention. This makes "*A*" a series of end-of-period receipts or disbursements.

A cash flow diagram of *P*, *A*, and *F* for the end-of-period convention is as follows:



If one were to adopt a middle-of-period convention it would only apply to the annual cash flows, and the diagram would be:



As the diagrams illustrate, only *A* shifts; *P* remains at the beginning of the period and *F* at the end of the period, regardless of the convention. The compound interest tables in Appendix C are based on the end-of-period convention.

Viewpoint of Economic Analysis Studies

When we make economic analysis calculations, we must proceed from a point of reference. Generally, we will want to take the point of view of a total firm when doing industrial economic analyses. Example 1–1 vividly illustrated the problem: a firm's shipping department decided it could save money by outsourcing its printing work rather than by using the in-house printing department. An analysis from the viewpoint of the shipping department supported this, as it could get for $688 the same printing it was paying $793 for in-house. Further analysis showed, however, that its printing department costs would decline *less* than using the commercial printer would save. From the viewpoint of the firm, the net result would be an increase in total cost.

From Example 1–1 we see it *is* important that the **viewpoint of the study** be carefully considered. Selecting a narrow viewpoint, like that of the shipping department, may result in a suboptimal decision from the firm's viewpoint. It is the total firm's viewpoint that is used in industrial economic analyses. For public-sector problems, the combined viewpoint of the government and the citizenry is chosen, since for many public projects the benefits of faster commuting, newer schools, and so on are received by individuals and the costs are paid by the government.

Sunk Costs

We know that it is the *differences between alternatives* that are relevant in economic analysis. Events that have occurred in the past really have no bearing on what we should do in the future. When the judge says, “$200 fine or 3 days in jail,” the events that led to these unhappy alternatives really are unimportant. It is the current and future differences between the alternatives that *are* important. Past costs, like past events, have no bearing on deciding between alternatives unless the past costs somehow affect the present or future costs. In general, past costs do not affect the present or the future, so we refer to them as *sunk costs* and disregard them.

Borrowed Money Viewpoint

In most economic analyses, the proposed alternatives inevitably require money to be spent, and so it is natural to ask the source of that money. Thus, each problem has two monetary aspects: one is the **financing**—the obtaining of money; the other is the **investment**—the spending of money. Experience has shown that these two concerns should be distinguished. When separated, the problems of obtaining money and of spending it are both logical and straightforward. Failure to separate financing and investment sometimes produces confusing results and poor decision making.

The conventional assumption in economic analysis is that the money required to finance alternatives is considered to be *obtained from the bank or the firm at interest rate i*.

Effect of Inflation and Deflation

For the present we will assume that prices are stable or stated in constant-value dollars. This means that a machine that costs $5000 today can be expected to cost the same amount several years hence. Inflation and deflation may need to be considered for after-tax analysis and for cost and revenues whose inflation rates differ from the economy's inflation rates (see Chapter 14), but we assume stable or constant dollar prices for now.

Income Taxes

Income taxes, like inflation and deflation, must be considered to find the real payoff of a project. However, taxes will often affect alternatives similarly, allowing us to compare choices without considering income taxes. We will introduce income taxes into economic analyses in Chapter 12.

ECONOMIC CRITERIA

We have shown how to manipulate cash flows in a variety of ways, and we can now solve many kinds of compound interest problems. But engineering economic analysis is more than simply solving interest problems. The decision process (see Figure 1–1) requires that the outcomes of feasible alternatives be arranged so that they may be judged for **economic efficiency** in terms of the selection criterion. The economic criteria were previously stated in general terms, and they are restated in Table 5–1.

We will now examine ways to resolve engineering problems, so that criteria for economic efficiency can be applied.

Equivalence provides the logic by which we may adjust the cash flow for a given alternative into some equivalent sum or series. We must still choose which comparable units to use. In this chapter we will learn how analysis can resolve alternatives into *equivalent present consequences*, referred to simply as **present worth analysis.** Chapter 6 will show how alternatives are converted into an *equivalent uniform annual cash flow*, and Chapter 7 solves for the interest rate at which favorable consequences—that is, *benefits*—are equivalent to unfavorable consequences—or *costs*.

As a general rule, any economic analysis problem may be solved by any of these three methods. This is true because *present worth*, *annual cash flow*, and *rate of return* are exact methods that will always yield the same recommendation for selecting the best alternative from among a set of mutually exclusive alternatives. Remember that “mutually exclusive” means that selecting one alternative precludes selecting any other alternative. For example, constructing a gas station and constructing a drive-in restaurant on a particular piece of vacant land are mutually exclusive alternatives.

Table 5-1 Present Worth Analysis

Input/Output	Situation	Criterion
Neither input nor output is fixed	Typical, general case	Maximize (present worth of benefits *minus* present worth of costs), that is, maximize net present worth
Fixed input	Amount of money or other input resources are fixed	Maximize present worth of benefits or other outputs
Fixed output	There is a fixed task, benefit, or other output to be accomplished	Minimize present worth of costs or other inputs

Some problems, however, may be more easily solved by one method. Present worth analysis is most frequently used to determine the present value of future money receipts and disbursements. It would help us, for example, to determine the present worth of an income-producing property, like an oil well or an apartment house. If the future income and costs are known, then we can use a suitable interest rate to calculate the property’s present worth. This should provide a good estimate of the price at which the property could be bought or sold. Another application is valuing stocks or bonds based on the anticipated future benefits of ownership.

TIME PERIOD FOR ANALYSIS

In present worth analysis, careful consideration must be given to the time period covered by the analysis. Usually the task to be accomplished has a time period associated with it. The consequences of each alternative must be considered for this period of time, which is usually called the **analysis period, planning horizon, or project life.**

The analysis period for an economy study should be determined from the situation. In some industries with rapidly changing technologies, a rather short analysis period or planning horizon might be in order. Industries with more stable technologies (like steel making) might use a longer period (say, 10–20 years), while government agencies frequently use analysis periods extending to 50 years or more.

Three different analysis-period situations are encountered in economic analysis problems with multiple alternatives:

1. The useful life of each alternative equals the analysis period.
2. The alternatives have useful lives different from the analysis period.
3. There is an infinite analysis period, $n = \infty$.

Useful Lives Equal the Analysis Period

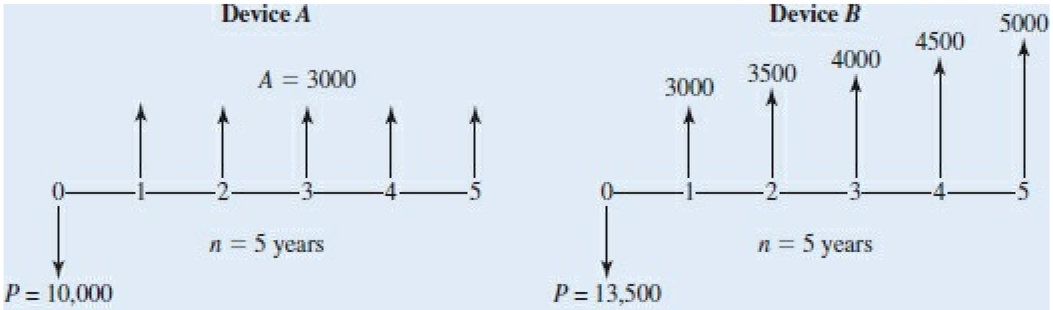
Since different lives and an infinite analysis period present some complications, we will begin with four examples in which the useful life of each alternative equals the analysis period.

EXAMPLE 5-1

A firm will install one of two mechanical devices to reduce costs. Both devices have useful lives of 5 years and no salvage value. Device *A* costs $10,000 and can be expected to result in $3000 savings annually. Device *B* costs $13,500 and will provide cost savings of $3000 the first year, but savings will increase $500 annually, making the second-year savings $3500, the third-year savings $4000, and so forth. With interest at 7%, which device should the firm purchase?

SOLUTION

The analysis period can conveniently be selected as the useful life of the devices, or 5 years. The appropriate decision criterion is to choose the alternative that maximizes the net present worth of benefits minus costs.



$$PW_A = -10{,}000 + 3000(P/A, 7\%, 5) = -10{,}000 + 3000(4.100) = \$2300$$

$$\begin{aligned} PW_B &= -13{,}500 + 3000(P/A, 7\%, 5) + 500(P/G, 7\%, 5) \\ &= -13{,}500 + 3000(4.100) + 500(7.647) = \$2624 \end{aligned}$$

Device *B* has the larger present worth and is the preferred alternative.

EXAMPLE 5-2

Wayne County will build an aqueduct to bring water in from the upper part of the state. It can be built at a reduced size now for \$300 million and be enlarged in 25 years for an additional \$350 million. An alternative is to construct the full-sized aqueduct now for \$400 million.

Both alternatives would provide the needed capacity for the 50-year analysis period. Maintenance costs are small and may be ignored. At 6% interest, which alternative should be selected?

TABLE SOLUTION

This problem illustrates staged construction. The aqueduct may be built in a single stage, or in a smaller first stage followed many years later by a second stage to provide the additional capacity when needed.

For the Two-Stage Construction

$$
\begin{aligned}
\text{PW of cost} &= \$300 \text{ million} + 350 \text{ million}(P/F, 6\%, 25) \\
&= \$300 \text{ million} + 81.6 \text{ million} \\
&= \$381.6 \text{ million}
\end{aligned}
$$

For the Single-Stage Construction

$$\text{PW of cost} = \$400 \text{ million}$$

The two-stage construction has a smaller present worth of cost and is the preferred construction plan.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Two-Stage	6%	25	0		-350	PV	$81.55
3							year 0	$300.00
4							total	$381.55

This is less than the $400M for the one-stage construction, so the two-stage is preferred.

EXAMPLE 5-3

A firm is trying to decide which of two weighing scales it should install to check a package-filling operation in the plant. The ideal scale would allow better control of the filling operation, hence less overfilling. If both scales have lives equal to the 6-year analysis period, which one should be selected? Assume an 8% interest rate.

Alternatives	Cost	Uniform Annual Benefit	End-of-Useful-Life Salvage Value
Atlas scale	$2000	$450	$200
Tom Thumb scale	3000	600	700

TABLE SOLUTION

Atlas Scale

$$\begin{aligned}\text{PW of benefits} - \text{PW of cost} &= 450(P/A, 8\%, 6) + 200(P/F, 8\%, 6) - 2000\\ &= 450(4.623) + 200(0.6302) - 2000\\ &= 2080 + 126 - 2000 = \$206\end{aligned}$$

Tom Thumb Scale

$$\begin{aligned}\text{PW of benefits} - \text{PW of cost} &= 600(P/A, 8\%, 6) + 700(P/F, 8\%, 6) - 3000\\ &= 600(4.623) + 700(0.6302) - 3000\\ &= 2774 + 441 - 3000 = \$215\end{aligned}$$

The salvage value of each scale, it should be noted, is simply treated as another positive cash flow. Because the PWs for the two alternatives are nearly identical, it is likely that there are other tangible or intangible differences that should determine the decision. If there are no such differences, buy the Tom Thumb equipment.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Atlas	8%	6	450		200	PV	-$2,206
3					-2000	total = -H2+E3		$206
4								
5	Tom Thumb	8%	6	600		700	PV	-$3,215
6					-3000	total = -H8+E9		$215

In Examples 5–1 and 5–3, we compared two alternatives and selected the one in which present worth of benefits *minus* present worth of cost was a maximum. The criterion is called the **net present worth criterion** and written simply as **NPW:**

$$\text{Net present worth} = \text{Present worth of benefits} - \text{Present worth of cost}$$

$$\text{NPW} = \text{PW of benefits} - \text{PW of cost} \qquad (5\text{-}1)$$

The field of engineering economy and this text often use present worth (PW), present value (PV), net present worth (NPW), and net present value (NPV) as synonyms. Sometimes, as in the foregoing definition, *net* is included to emphasize that both costs and benefits have been considered.

Useful Lives Different from the Analysis Period

In present worth analysis, there always must be an identified analysis period. It follows, then, that each alternative must be considered for the entire period. In Examples 5–1 to 5–3, the useful life of each alternative was equal to the analysis period. While often this is true, in many situations at least one alternative will have a useful life different from the analysis period. This section describes one way to evaluate alternatives with lives different from the study period.

For present worth calculations, it is important that we select an analysis period and judge the consequences of each of the alternatives over that period. As such, in Example 5–4 it is not a fair comparison to compare the NPW of Pump *A* over its 12-year life against the NPW of Pump *B* over its 6-year life.

The firm, its economic environment, and the specific situation are important in selecting an analysis period. If Pump *A* (Example 5–4) has a useful life of 12 years, and Pump *B* will last 6 years, one method is to select an analysis period that is the **least common multiple** of their useful lives. Thus we would compare the 12-year life of Pump *A* against an initial purchase of Pump *B* *plus* its replacement with new Pump *B* in 6 years. The result is to judge the alternatives on the basis of a 12-year requirement.

EXAMPLE 5-4

Two pumps are being considered for purchase. If interest is 7%, which pump should be bought? Their maintenance costs are the same.

	Pump *A*	Pump *B*

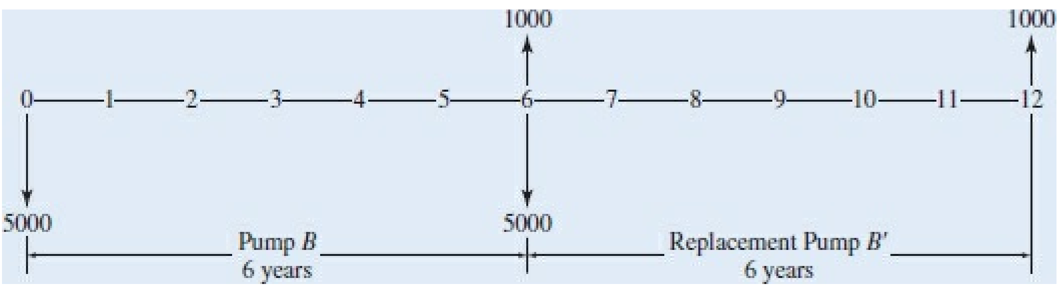
Initial cost	$7000	$5000
End-of-useful-life salvage value	1200	1000
Useful life, in years	12	6

TABLE SOLUTION

Since the maintenance costs are the same, they can be omitted from the comparison. The present worth of Pump *A* over 12 years is

$$\begin{aligned} PW_A &= -7000 + 1200(P/F, 7\%, 12) \\ &= -7000 + 1200(0.4440) \\ &= -\$6467 \end{aligned}$$

For a common analysis period of 12 years, we need to replace Pump *B* at the end of its 6-year useful life. If we assume that another pump B' can be obtained, having the same $5000 initial cost, $1000 salvage value, and 6-year life, the cash flow will be as follows:



For the 12-year analysis period, the present worth for Pump *B* is

$$\begin{aligned} PW_B &= -5000 + 1000(P/F, 7\%, 6) - 5000(P/F, 7\%, 6) \\ &\quad + 1000(P/F, 7\%, 12) \\ &= -5000 + (1000 - 5000)(0.6663) + 1000(0.4440) \\ &= -\$7221 \end{aligned}$$

By assuming that the shorter-life equipment is replaced by equipment with identical economic consequences, we have avoided a lot of calculations. Select Pump *A*.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H	I
1	Exp. 5-4	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Pump A	7%	12	0		1200	PV	-$533	=PV(B2,C2,D2,F2)
3								$533	change sign
4								-$7,000	Initial cost
5								-$6,467	PW
6	Pump B yr 6	7%	6	0		-4000	PV	$2,665	=PV(B6,C6,D6,F6)
7	yr 12	7%	12	0		1000	PV	-$444	=PV(B7,C7,D7,F7)
8					-7000			-$2,221	sum & change sign
9	yr 0							-$5,000	Initial cost
10								-$7,221	PW

We have seen that setting the analysis period equal to the least common multiple of the lives of the two alternatives seems reasonable in Example 5–4. However, what if the alternatives had useful lives of 7 and 13 years? Here the least common multiple of lives is 91 years. An analysis period of 91 years hardly seems realistic. Instead, a suitable analysis period should be based on how long the equipment is likely to be needed. This may require that terminal values be estimated for the alternatives at some point prior to the end of their useful lives.

As Figure 5–1 shows, it is not necessary for the analysis period to equal the useful life of an alternative or some multiple of the useful life. To properly reflect the situation at the end of the analysis period, an estimate is required of the market value of the equipment at that time. The calculations might be easier if everything came out even, but this is not essential.

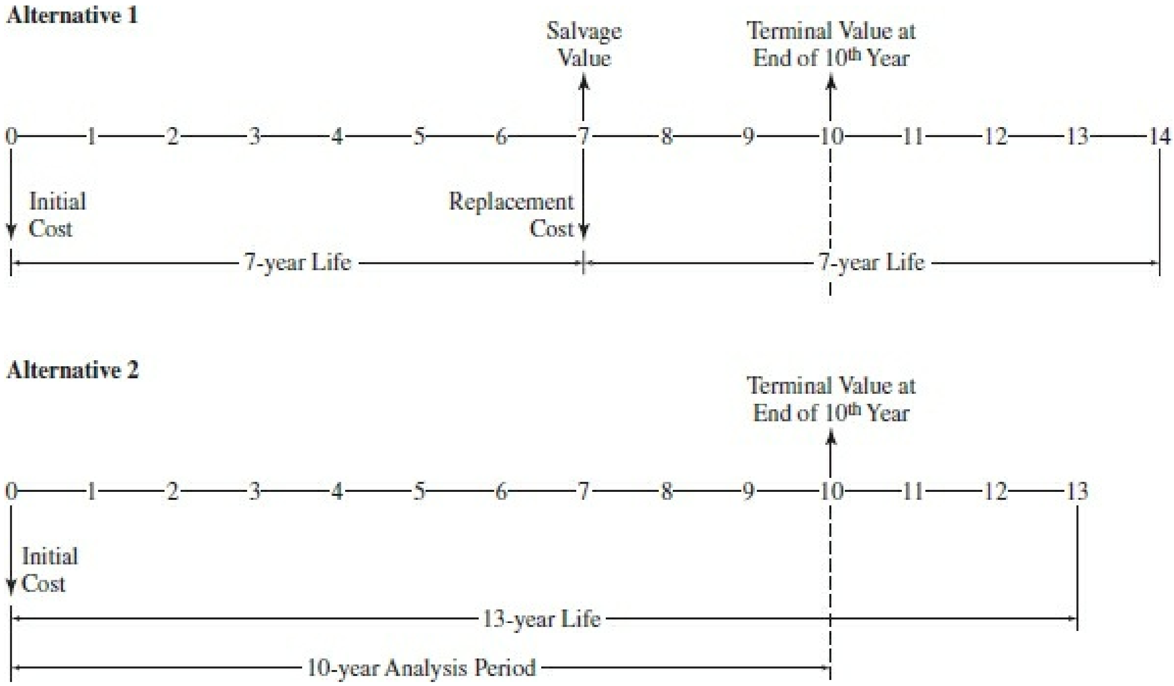


FIGURE 5-1 Superimposing a 10-year analysis period on 7- and 13-year alternatives.

EXAMPLE 5-5

A diesel manufacturer is considering the two alternative production machines graphically depicted in Figure 5–1. Specific data are as follows:

	Alt. 1	**Alt. 2**
Initial cost	$50,000	$75,000
Estimated salvage value at end of useful life	$10,000	$12,000
Useful life of equipment, in years	7	13

The manufacturer uses an interest rate of 8% and wants to use the PW method to compare these alternatives over an analysis period of 10 years. To do so, the market values at year 10 must be estimated. Alt. 1 will be 3 years into its “second” life and Alt. 2 will be nearing the end of its “first” life.

	Alt. 1	**Alt. 2**

Estimated market value, end of 10-year analysis period	$20,000	$15,000

SOLUTION

In this case, the decision maker is setting the analysis period at 10 years rather than accepting a common multiple of the lives of the alternatives, or assuming that the period of needed service is infinite (to be discussed in the next section). This is a legitimate approach—perhaps the diesel manufacturer will be phasing out this model at the end of the 10-year period. In any event, we need to compare the alternatives over 10 years.

As illustrated in Figure 5–1, we may assume that Alternative 1 will be replaced by an identical machine after its 7-year useful life. Alternative 2 has a 13-year useful life. The diesel manufacturer has provided an estimated market value of the equipment at the time of the analysis period. We can compare the two choices over 10 years as follows:

$$\begin{aligned}\text{PW (Alt. 1)} &= -50{,}000 + (10{,}000 - 50{,}000)(P/F, 8\%, 7) + 20{,}000(P/F, 8\%, 10)\\ &= -50{,}000 - 40{,}000(0.5835) + 20{,}000(0.4632)\\ &= -\$64{,}076\end{aligned}$$

$$\begin{aligned}\text{PW (Alt. 2)} &= -75{,}000 + 15{,}000(P/F, 8\%, 10)\\ &= -75{,}000 + 15{,}000(0.4632)\\ &= -\$68{,}052\end{aligned}$$

To minimize PW of costs the diesel manufacturer should select Alt. 1.

Infinite Analysis Period: Capitalized Cost

Some present worth analyses use an infinite analysis period $(n = \infty)$. In governmental analyses, a service or condition sometimes must be maintained for an infinite period. The need for roads, dams, pipelines, and so on, is sometimes considered to be permanent. In these situations a present worth of cost analysis would have an infinite analysis period. We call this particular analysis **capitalized cost.**

Infinite lives are rare in the private sector, but a similar assumption of “indefinitely long” horizons is sometimes made. This assumes that the facility will need electric motors, mechanical HVAC equipment, and forklifts as long as it operates and that the facility will last far longer than any individual unit of equipment. So the equipment can be analyzed as though the problem horizon is *infinite* or indefinitely long.

Capitalized cost is the present sum of money that would need to be set aside now, at some interest rate, to yield a uniform cash flow indefinitely. To accomplish this, the money set aside for future expenditures must not decline. The interest received on the money set aside can be spent, but not the principal. When one stops to think about an infinite analysis period, we see that an unchanged principal sum is essential.

In Chapter 3 we saw that

Principal sum + Interest for the period = Amount at end of period, or

$$P \quad + \quad iP \quad = \quad P + iP$$

If we spend iP, then in the next interest period the principal sum P will again increase to $P + iP$. Thus, we can again spend iP.

This concept may be illustrated by a numerical example. Suppose you deposited \$200 in a bank that paid 4% interest annually. How much money could be withdrawn each year without reducing the balance in the account below the initial \$200? At the end of the first year, the \$200 would have earned 4%(\$200) = \$8 interest. If this interest were withdrawn, the \$200 would remain in the account. At the end of the second year, the \$200 balance would again earn 4%(\$200) = \$8. This \$8 could also be withdrawn and the account would still have \$200. This procedure could be continued indefinitely and the bank account would always contain \$200. If more or less than \$8 is withdrawn each year, over time the account will either increase to ∞ or decrease to 0.

The year-by-year situation would be depicted like this:

$$\textit{Year 1:}\ \$200 \text{ initial } P \rightarrow 200 + 8 = 208$$

$$\text{Withdrawal } iP = -\ 8$$

$$\textit{Year 2:}\ \$200 \rightarrow 200 + 8 = 208$$

$$\text{Withdrawal } iP = \underline{-\ 8}$$

$$\$200$$

and so on

Thus, for any initial present sum *P*, there can be an end-of-period withdrawal of *A* equal to *iP* each period, and these withdrawals can continue forever without diminishing the initial sum *P*. This gives us the basic relationship:

$$\text{For} \quad n = \infty, \quad A = iP$$

This relationship is the key to capitalized cost calculations. Earlier we defined capitalized cost as the present sum of money that would need to be set aside at some interest rate to yield the funds to provide the desired task or service forever. Capitalized cost is therefore the *P* in the equation *A* = *iP*. It follows that:

$$\textbf{Capitalized cost} \qquad P = \frac{A}{i} \quad (5\text{-}2)$$

If we can resolve the desired task or service into an equivalent *A*, the capitalized cost can be computed. The following examples illustrate such computations.

EXAMPLE 5-6

How much should one set aside to pay $5000 per year for maintenance on a small park if interest is assumed to be 4%? For perpetual maintenance, the principal sum must remain undiminished after the annual disbursement is made.

SOLUTION

$$\text{Capitalized cost } P = \frac{\text{Annual disbursement } A}{\text{Interest rate } i}$$

$$P = \frac{5000}{0.04} = \$125{,}000$$

One should set aside $125,000.

EXAMPLE 5-7

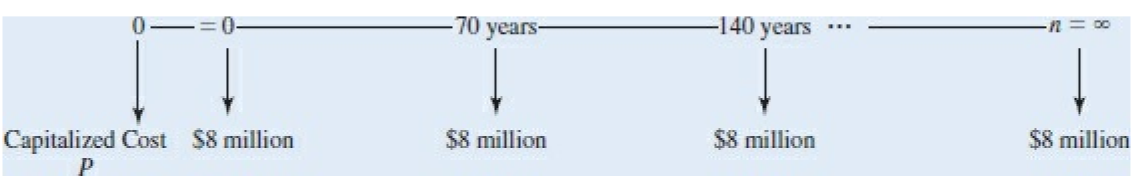
A city plans a pipeline to transport water from a distant watershed area to the city. The pipeline will cost $8 million and will have an expected life of 70 years. The city expects to keep the water line in service indefinitely. Compute the capitalized cost, assuming 7% interest.

SOLUTION

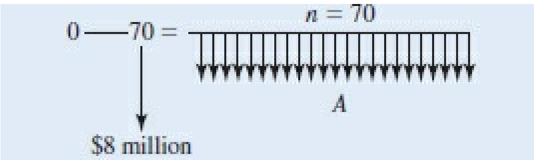
The capitalized cost equation

$$P = \frac{A}{i}$$

is simple to apply when there are end-of-period disbursements *A*. Here we have renewals of the pipeline every 70 years. To compute the capitalized cost, it is necessary to first compute an end-of-period disbursement *A* that is equivalent to $8 million every 70 years.



The $8 million disbursement at the end of each 70-year period may be resolved into an equivalent *A*.



$$A = F(A/F, i, n) = \$8 \text{ million}(A/F, 7\%, 70)$$
$$= \$8 \text{ million}(0.00062)$$
$$= \$4960$$

Each 70-year period is identical to this one, and the infinite series is shown in Figure 5–2.

$$\text{Capitalized cost } P = 8 \text{ million} + \frac{A}{i} = 8 \text{ million} + \frac{4960}{0.07}$$
$$= \$8{,}071{,}000$$

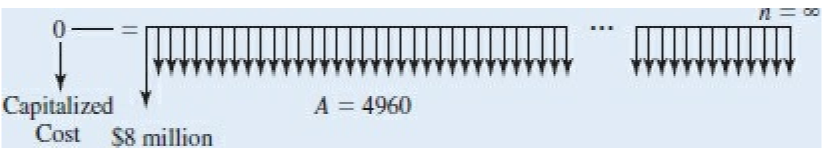


FIGURE 5-2 Using the sinking fund factor to compute an infinite series.

ALTERNATE SOLUTION 1

Instead of solving for an equivalent end-of-period payment *A* based on a *future* $8 million disbursement, we could find *A*, given a *present* $8 million disbursement.

$$A = P(A/P, i, n) = 8 \text{ million}(A/P, 7\%, 70)$$
$$= 8 \text{ million}(0.0706) = \$565{,}000$$

On this basis, the infinite series is shown in Figure 5–3. Carefully note the difference between this and Figure 5–2. Now:

$$\text{Capitalized cost } P = \frac{A}{i} = \frac{565{,}000}{0.07} = \$8{,}071{,}000$$

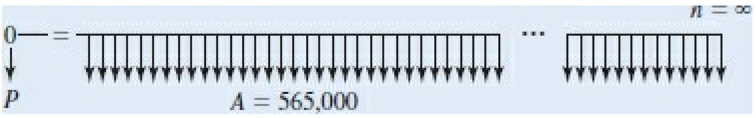


FIGURE 5-3 Using the capital recovery factor to compute an infinite series.

ALTERNATE SOLUTION 2

Another way of solving the problem is to assume the interest period is 70 years long and compute an equivalent interest rate for the 70-year period. Then the capitalized cost may be computed by using Equation 3–8 for *m* = 70

$$i_{70\,\text{yr}} = (1 + i_{1\,\text{yr}})^{70} - 1 = (1 + 0.07)^{70} - 1 = 112.989$$

$$\text{Capitalized cost} = 8\text{ million} + \frac{8\text{ million}}{112.989} = \$8{,}070{,}803$$

MULTIPLE ALTERNATIVES

So far the discussion has been based on examples with only two alternatives. But multiple-alternative problems may be solved by exactly the same methods. (The only reason for avoiding multiple alternatives was to simplify the examples.) Examples 5–8 and 5–9 have multiple alternatives.

EXAMPLE 5-8

A contractor has been awarded the contract to construct a 6-mile-long tunnel in the mountains. During the 5-year construction period, the contractor will need water from a nearby stream. She will construct a pipeline to carry the water to the main construction yard. An analysis of costs for various pipe sizes is as follows:

	Pipe Sizes (in.)			
	2	3	4	6
Installed cost of pipeline and pump	$22,000	$23,000	$25,000	$30,000

Cost per hour for pumping	\$1.20	\$0.65	\$0.50	\$0.40

At the end of 5 years, the pipe and pump will have a salvage value equal to the cost of removing them. The pump will operate 2000 hours per year. The lowest interest rate at which the contractor is willing to invest money is 7%. (The minimum required interest rate for invested money is called the **minimum attractive rate of return,** or MARR.) Select the alternative with the least present worth of cost.

SOLUTION

We can compute the present worth of cost for each alternative. For each pipe size, the present worth of cost is equal to the installed cost of the pipeline and pump plus the present worth of 5 years of pumping costs.

	Pipe Size (in.)			
	2	3	4	6
Installed cost of pipeline and pump	\$22,000	\$23,000	\$25,000	\$30,000
1.20 × 2000 hr × (*P*/*A*, 7%, 5)	9,840			
0.65 × 2000 hr × 4.100		5,330		
0.50 × 2000 hr × 4.100			4,100	
0.40 × 2000 hr × 4.100				3,280
Present worth of cost	\$31,840	\$28,330	\$29,100	\$33,280

Select the 3-in. pipe, since the lowest present worth of cost.

EXAMPLE 5-9

An investor paid \$8000 to a consulting firm to analyze possible uses for a small parcel of land on the edge of town that can be bought for \$30,000. In their report, the consultants suggested four alternatives:

Alternatives	Total Investment Including Land*	Uniform Net Annual Benefit	Terminal Value at End of 20 yr

A	Do nothing	$ 0	$ 0	$ 0
B	Vegetable market	50,000	5,100	30,000
C	Gas station	95,000	10,500	30,000
D	Small motel	350,000	36,000	150,000

* Includes the land and structures but does not include the $8000 fee to the consulting firm.

Assuming 10% is the minimum attractive rate of return, what should the investor do?

SOLUTION

Alternative *A* is the "do-nothing" alternative. Generally, one feasible alternative is to remain in the present status and do nothing. The investor could decide that the most attractive alternative is not to purchase the property at all.

Note that if the investor does nothing, then the total venture would appear to be an unsatisfactory one because of the $8000 spent on professional advice. However, because the $8000 is a past cost, it is a **sunk cost.** The only relevant costs in an economic analysis are *present* and *future* costs. Sunk costs are gone and should not be allowed to affect future planning. (Past costs may be relevant in computing depreciation charges and income taxes, but nowhere else.) The past should not deter the investor from making the best decision now.

This problem is one of neither fixed input nor fixed output, so our criterion will be to maximize the present worth of benefits *minus* the present worth of cost; that is, to maximize net present worth.

Alternative *A*, Do Nothing

$$\text{NPW} = 0$$

Alternative *B*, Vegetable Market

$$\begin{aligned}
\text{NPW} &= -50{,}000 + 5100(P/A, 10\%, 20) + 30{,}000(P/F, 10\%, 20) \\
&= -50{,}000 + 5100(8.514) + 30{,}000(0.1486) \\
&= -50{,}000 + 43{,}420 + 4460 \\
&= -\$2120
\end{aligned}$$

Alternative *C*, Gas Station

$$\begin{aligned}
\text{NPW} &= -95{,}000 + 10{,}500(P/A, 10\%, 20) + 30{,}000(P/F, 10\%, 20) \\
&= -95{,}000 + 89{,}400 + 4460 = -\$1140
\end{aligned}$$

Alternative *D*, Small Motel

$$\begin{aligned}
\text{NPW} &= -350{,}000 + 36{,}000(P/A, 10\%, 20) + 150{,}000(P/F, 10\%, 20) \\
&= -350{,}000 + 306{,}500 + 22{,}290 = -\$21{,}210
\end{aligned}$$

The criterion is to maximize net present worth. In this situation, one alternative has NPW equal to zero, and three alternatives have negative values for NPW. We will select the best of the four alternatives, namely, the do-nothing Alt. *A*, with NPW equal to zero.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Vegetable market	10%	20	5100		30,000	PV	-$47,878
3					50,000	total = -H2+E3		-$2,122
4								
5	Gas station	10%	20	10,500		30,000	PV	-$93,852
6					-95,000	total = -H5+E6		-$1,148
7								
8	Small Motel	10%	20	36,000		150,000	PV	-$328,785
9					-350,000	total = -H8+E9		-$21,215

Since the total present worth of each action alternative is negative, the best choice is to do nothing. Note that in this case the PV of the positive annual revenue and positive salvage value is negative. To compute the present worth of each alternative, we must subtract the PV values in H2, H5, and H8.

APPLICATIONS AND COMPLICATIONS

EXAMPLE 5-10

Two pieces of construction equipment are being analyzed.

Year	Alt. *A*	Alt. *B*
0	− \$15, 000	− \$12, 000
1	5, 000	3, 500
2	4, 500	3, 500
3	4, 000	3, 500
4	4, 000	3, 600
5	4, 000	3, 700

At an 8% interest rate, which alternative should be selected?

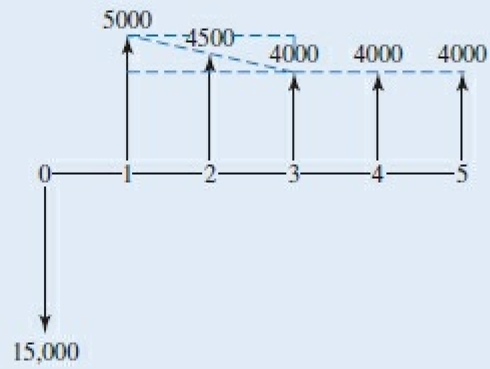
SPREADSHEET SOLUTION

In this case the cash flows are so irregular that it is easier and clearer to treat them as individual data block entries. Time can be saved by dragging or double-clicking on the cell corners to copy the cash flows and gradients.

	A	B	C
1	8%	Interest rate	
2	Year	Alt. *A*	Alt. *B*
3	0	-\$15,000	-\$12,000
4	1	5,000	3,500
5	2	4,500	3,500
6	3	4,000	3,500
7	4	4,000	3,600
8	5	4,000	3,700
9		\$2,325	\$2,184
10			
11	=B3+NPV(\$A\$1,B4:B8)		
12		=C3+NPV(\$A\$1,C4:C8)	

TABLE SOLUTION

Alternative *A*



$$\begin{aligned} \text{PW of benefits} &= 4000(P/A, 8\%, 5) + (5000 - 4000)(P/A, 8\%, 3) - 500(P/G, 8\%, 3) \\ &= 4000(3.993) + 1000(2.577) - 500(2.445) = 17,326 \\ \text{PW of cost} &= 15,000 \\ \text{Net present worth} &= 17,326 - 15,000 = \$2326 \end{aligned}$$

Alternative *B*

$$\begin{aligned} \text{PW of benefits} &= 3500(P/A, 8\%, 5) + 100(P/G, 8\%, 3)(P/F, 8\%, 2) \\ &= 3500(3.993) + 100(2.445)(0.8573) = 14,185 \\ \text{PW of cost} &= 12,000 \\ \text{Net present worth} &= 14,185 - 12,000 = \$2185 \end{aligned}$$

To maximize NPW, choose Alt. *A*.

EXAMPLE 5-11

A piece of land may be purchased for $610,000 to be strip-mined for the underlying coal. Annual net income will be $200,000 for 10 years. At the end of the 10 years, the surface of the land will be restored as required by a federal law on strip-mining. The reclamation will cost $1.5 million more than the resale value of the land after it is restored. Using a 10% interest rate, determine whether the project is desirable.

SOLUTION

The investment opportunity may be described by the following cash flow:

Year	Cash Flow (thousands)
0	− $610
1–10	+ 200 (per year)
10	− 1500

$$
\begin{aligned}
\text{NPW} &= -610 + 200(P/A, 10\%, 10) - 1500(P/F, 10\%, 10) \\
&= -610 + 200(6.145) - 1500(0.3855) \\
&= -610 + 1229 - 578 \\
&= +\$41
\end{aligned}
$$

Since NPW is positive, the project seems desirable. However, at interest rates below 4.07% or above 18.29%, the NPW is negative, indicating an undesirable project. This indicates that the project is undesirable at 4% and desirable at 10%. This does not make sense. The results warn us that NPW may not always be a proper criterion for judging whether or not an investment should be undertaken. In this example the disbursements ($610,000 + $1,500,000) exceed the benefits (10 × $200,000), which certainly does not portray a desirable investment. Thus Example 5–11 shows that NPW calculations in certain infrequent conditions can lead to unreliable results. Very large environmental cleanup costs can cause this, as in this example. Appendix 7A addresses this in more detail.

Spreadsheets make it easy to build more accurate models with shorter time periods. When one is using factors, it is common to assume that costs and revenues are uniform for *n* years. With spreadsheets it is easy to use 120 months instead of 10 years, and the cash flows can be estimated for each month. For example, energy costs for air conditioning peak in the summer, and in many areas there is little construction during the winter.

Example 4–14 illustrated how spreadsheets can be used for arithmetic and geometric gradients. The latter are important, because cash flows that depend on population often increase at *x*% per year, such as for electric power and transportation costs. Example 5–12 is another illustration of how spreadsheets support models that more closely match reality.

EXAMPLE 5-12

Regina Industries has a new product whose sales are expected to be 1.2, 3.5, 7, 5, and 3 million units per year over the next 5 years. Production, distribution, and overhead costs decline 10% per year from $140 per unit in the first year. The price will be $200 per unit for the first 2 years and then $180, $160, and $140 for the next 3 years. The remaining R&D and production costs are $300 million. If *i* is 15%, what is the present worth of the new product?

SOLUTION

All of the variable values are entered in the spreadsheet's data block, except for the yearly volume. Since each year is different, these values are simply entered into column B. Note that the gradient for revenue per unit first makes a difference in Year 3.

	A	B	C	D	E	F	G
1	$300	First cost ($M)					
2	$140	Initial unit cost					
3	-10%	Geometric gradient for cost					
4	$200	Initial unit revenue					
5	-$20	Arithmetic change in unit revenue for years 3 to 5					
6	15%	Interest rate					
7	Year	Volume (M)	Revenue unit	Cost unit	Net revenue unit	Cash flow	
8	0					-$300.0	=-$A$1
9	1	1.2	$200	$140	$60	72.0	
10	2	3.5	200	126	74	259.0	=B10*E10
11	3	7.0	180	113	67	466.2	
12	4	5.0	160	102	58	289.7	
13	5	3.0	140	92	48	144.4	
14							
15			=D12*(1+$A$3)		PW ($M)	$502	
16		=C12+$A$5				=F8+NPV($A$6,F9:F13)	

With a present worth of over $500M, this is a desirable project.

Example 5–13 uses the **XNPV** spreadsheet function, which finds the present worth of a series of cash flows that occur on specific dates. The NPV function assumes that time 0 is one period before the first cash flow, so time 0 must *not* be included in the function's range. XNPV is different. Time 0 must be specified just like every other date—even if there is no cash flow at time 0. The time 0 date specifies the *when* for the net present value or present worth. The interest rate for the XNPV function is an effective annual rate, and XNPV is using calendar dates, so leap year has an extra day and the daily interest rate is slightly smaller in leap years.

EXAMPLE 5-13

A construction firm has just won a contract for $2.25 million. If work is completed on schedule, it will receive a series of progress payments. The first $100,000 will be received 15 days after the contract is signed. Further payments of $450,000 each will be received at 45 and 180 days. The remaining $1.25 million will be received 390 days after the contract is signed. If the firm's interest rate is 9.5%, what is the present worth of the progress payments on the contract signing date of September 23, 2019?

SOLUTION

Notice that the XNPV function specifies the range of cash flows and then the range of dates.

	A	B	C
1	9.50%	Interest rate	
2			
3	Days	Dates	Cash flows
4	0	9/23/2019	0
5	15	10/08/2019	$100,000
6	45	11/07/2019	450,000
7	180	3/21/2020	450,000
8	390	10/17/2020	1,250,000
9			
10		=$B$4+A8	
11	$2,109,403	=XNPV(A1,C4:C8,	B4:B8)

The two key differences between the NPV and XNPV functions can be seen in Example 5–13. XNPV includes:

• The time 0 cash flow, as in cell C4.
• A final argument of the range of dates for the cash flows, as in cells B4 to B8.

Bond Pricing

The calculation in Example 5–14 is done routinely when bonds are bought and sold during their life. Bonds are issued at a **face or par value** (usually $1000), which is received when the bonds mature. There is a **coupon interest rate**, which is set when the bond is originally issued or sold. The term *coupon interest* dates from the time when bonds were paper rather than electronic, and a paper coupon was detached from the bond to be redeemed in cash. This rate is a nominal interest rate. Most bonds pay interest semi-annually which is calculated as (face value) × (coupon rate)/2. Some bonds have other compounding periods. A cash flow diagram for the remaining interest payments and the final face value is used with a current interest rate to calculate a present worth. This is the bond's price.

Bond pricing calculations are important because selling bonds is the principal source of new external funds for existing firms. Bonds are also how governments fund many projects for bridges, schools, airports, dams, and so on. Bonds are also considered a safer way for individuals, pension funds, and other entities to invest.

Appendix 10A is about the return and risks of portfolios including bonds and stocks. A bond is a contract to deliver interest payments when due and the face value of the bond at maturity. The cash flows are on known dates and of known amounts. When interest rates rise above the coupon rate, those future cash flows are discounted more and the bond sells at a discount. When interest rates fall below the coupon rate, future cash flows are discounted less and the bond sells at a premium. Bonds of the federal government are safer than bonds of a firm or many cities and nations. The U.S. government is less likely to *default* or not pay interest or face value when due. See Appendix 9A for past interest rates on government bonds.

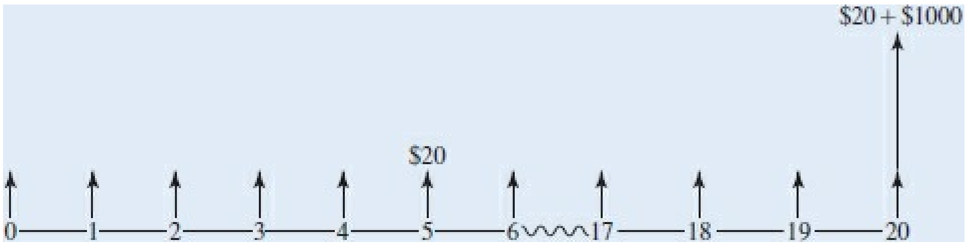
EXAMPLE 5-14

A 15-year municipal bond was issued 5 years ago. Its coupon interest rate is 4%, interest payments are made semiannually, and its face value is $1000. If the current market interest rate is 6.09%, what should be the bond's price? *Note*: The issuer of the bond (city, state, company) makes interest payments to the bondholder (at the coupon rate), as well as a final value payment.

TABLE SOLUTION

The first 5 years are past, and there are 20 more semiannual payments. The coupon interest rate is the nominal annual rate or APR. Half that, or 2%, of $1000 or $20 is paid at the end of each 6-month period.

The bond's price is the PW of the cash flows that will be received if the bond is purchased. The cash flows are $20 at the end of each of the 20 semiannual periods and the face value of $1000 at the end of period 20.



Since the $20 in interest is received semiannually, the market or effective annual interest rate (i_a) must be converted to a semiannual rate. Using Equation 3-8, we obtain

$$
\begin{aligned}
(1+i)^2 &= 1 + i_a = 1.0609 \\
(1+i) &= 1.03 \\
i &= 3\% \text{ semiannual interest rate} \\
PW &= 20(P/A, 3\%, 20) + 1000(P/F, 3\%, 20) \\
&= 20(14.877) + 1000(0.5537) = \$851.24
\end{aligned}
$$

The $851.24 is the discounted price; that is, the PW at 6.09% of the cash flows from the $1000 bond. The $148.77 discount raises the investment's rate of return from a nominal 4% for the face value to 6.09% on an investment of $851.24.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 5-14	3%	20	20		1000	PV	-$851.24

This example also illustrates why it is better to separately state cash flows. At the end of period 20, there are two cash flows, $20 and $1000. The $20 is part of the 20-period uniform series, and the $1000 is a single cash flow. All of these numbers come directly from the problem statement. If the two final cash flows are combined into $1020, then the $20 uniform series has only 19 periods—and it is easy to err and forget that change.

SUMMARY

Present worth analysis is suitable for almost any economic analysis problem. But it is particularly desirable when we wish to know the present worth of future costs and benefits. We frequently want to know the value today of such things as income-producing assets, stocks, and bonds.

For present worth analysis, the proper economic criteria are:

Neither input nor	Maximize (PW of benefits − PW of costs)
output is fixed	or, more simply stated:
	Maximize NPW
Fixed input	Maximize the PW of benefits
Fixed output	Minimize the PW of costs

To make valid comparisons, we need to analyze each alternative in a problem over the same **analysis period** or **planning horizon.** If the alternatives do not have equal lives, some technique must be used to achieve a common analysis period. One method is to select an analysis period equal to the least common multiple of the alternative lives. Another method is to select an analysis period and estimate end-of-analysis-period salvage values for the alternatives.

Capitalized cost is the present worth of cost for an infinite analysis period $(n = \infty)$. When $n = \infty$, the fundamental relationship is $A = iP$.

The numerous assumptions routinely made in solving economic analysis problems include the following.

1. Present sums (P) are beginning-of-period, and all series receipts or disbursements (A) and future sums (F) occur at the end of the interest period. The compound interest tables were derived on this basis.
2. In industrial economic analyses, the point of view for computing the consequences of alternatives is that of the total firm. Narrower views can result in suboptimal solutions.
3. Only the differences between the alternatives are relevant. Past costs are sunk costs and generally do not affect present or future costs. For this reason they are ignored.
4. The investment problem is isolated from the financing problem. We

generally assume that all required money is borrowed at interest rate *i*.
5. For now, stable prices are assumed. The inflation–deflation problem is deferred to Chapter 14. Similarly, our discussion of income taxes is deferred to Chapter 12.
6. Often uniform cash flows or arithmetic gradients are reasonable assumptions. However, spreadsheets simplify the finding of PW in more complicated problems.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

5-1 A tax refund expected 1 year from now has a present worth of $3000 if i = What is its present worth if i = 4%?

SOLUTION

5-2 Investment in a crane is expected to produce profit from its rental of $15,0 during the first year of service. The profit is expected to decrease by $250 each year thereafter. At 12% interest, find the present worth of the profits.

SOLUTION

5-3 Strickland Storage Inc. leases storage units for $200/month. Calculate the present worth of 12 lease payments at 6%.

SOLUTION

5-4 The winner of a sweepstakes prize is given the choice of a one-time payme of $1,000,000 or a guaranteed $80,000 per year for 20 years. If the value o money is 5%, which option should the winner choose?

SOLUTION

5-5 A local car wash charges $3 per wash, or one can pay $12.98 for 5 washes, payable in advance with the first wash. If you normally washed your car or month, and your cost of money is 1% compounded monthly, would the opt be worthwhile?

SOLUTION

5-6 A project being considered by the XYZ Company will have $100,000 in construction costs in each of the first 3 years of the project. Income of $100,000 will begin flowing in Year 4 and will continue through Year 10. the net present worth at 4% of the project

SOLUTION

5-7 The community theater spends $10,000 annually to produce a musical extravaganza. Immediately *before* this year's extravaganza, the theater had balance of $60,000 in an account paying 4% interest. *After* this year, how r extravaganzas can be sponsored without raising more money?

SOLUTION

5-8 The annual income from an apartment house is $20,000. The annual expen estimated to be $2000. If the apartment house can be sold for $100,000 at t end of 10 years, how much should you be willing to pay for it now, with a required return of 10%?

SOLUTION

Your company has been presented with an opportunity to invest in the following project. The facts on the project are:

5-

Investment required	$90,000,000
Salvage value after 10 years	0
Gross income	20,000,000
Annual operating costs:	
Labor	2,500,000

9

Materials, licenses, insurance, etc.*	1,000,000
Fuel and other costs	1,500,000
Maintenance costs	500,000

*Beginning-of-period cash flow

The project will operate for 10 years. If management expects 8% on its investments before taxes, would you recommend this project?

SOLUTION

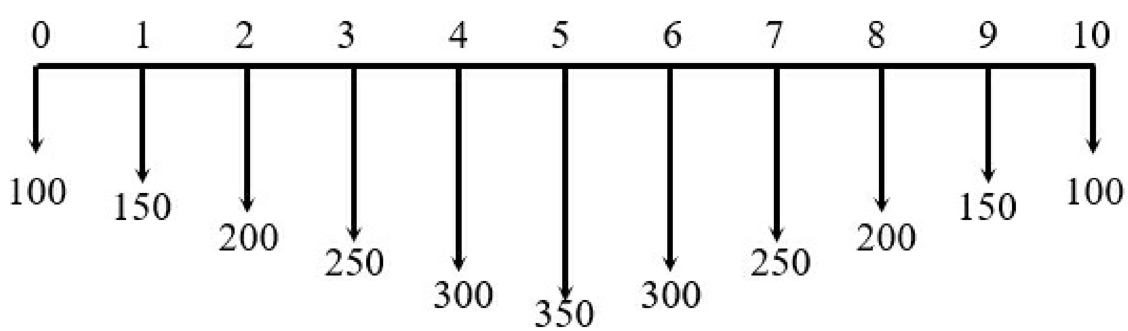
5-10 Sarah Bishop, having become a very successful engineer, wishes to start ar endowment at UTM that will provide scholarships of $10,000 per year to f engineering students beginning in Year 6 and continuing indefinitely. Determine the amount Sarah must donate now if the university earns 10% year on endowment funds.

SOLUTION

5-11 What is the maximum amount you should be willing to pay for the followi piece of equipment? First-year income is anticipated to be $18,000 and decrease by $1500 each year thereafter. First-year costs are anticipated to b $5000 and increase by $500 each year thereafter. The salvage value is estimated to be 8.5% of the first cost. Use an interest rate of 4% and useful of 5 years.

SOLUTION

5-12 Find the present worth of the following cash flow diagram if i = 8 %.



SOLUTION

5-13 A couple wants to begin saving money for their daughter's education. $16,(will be needed on the child's 18th birthday, $18,000 on the 19th birthday, $20,000 on the 20th birthday, and $22,000 on the 21st birthday. Assume 5% interest with annual compounding. The couple is considering two methods accumulating the money.

a. How much money would have to be deposited into the account on the ch first birthday to accumulate enough money to cover the education expense: (*Note*: A child's "first birthday" is celebrated 1 year after the child is born.) b. What uniform annual amount would the couple have to deposit each yea the child's first through seventeenth birthdays to accumulate enough mone cover the education expenses?

SOLUTION

5-14 Assume that you borrowed $50,000 at an interest rate of 1% per month, to repaid in uniform monthly payments for 30 years. How much of the 163rd payment would be interest, and how much would be principal?

SOLUTION

5-15 A town is seeking a new tourist attraction, and the town council has voted budget $500,000 for the project. A survey shows that an interesting cave ca enlarged and developed for a contract price of $400,000. The proposed attraction is expected to have an infinite life. The estimated annual expense operation total $50,000. The price per ticket is to be based on an average o 12,000 visitors per year. If money is worth 8%, what should be the price of each ticket?

SOLUTION

Two alternatives are being considered for recovering aluminum from garba Alternative 1 has a capital cost of $100,000 and a first-year maintenance co of $12,000, with maintenance increasing by $500 per year for each year aft the first.

5-16 Alternative 2 has a capital cost of $140,000 and a first-year maintenance c of $4000, with maintenance increasing by $1000 per year after the first.

Revenues from the sale of aluminum are $20,000 in the first year, increasi $2000 per year for each year after the first. The life of both alternatives is years. There is no salvage value. The before-tax MARR is 4%. Use presen worth analysis to determine which alternative is preferred.

SOLUTION

5-17 As a temporary measure, a brewing company is deciding between two use filling machines: the Kram and the Zanni.

a. The Kram filler has an initial cost of $85,000; the estimated annual maintenance is $8000.
b. The Zanni filler has a purchase price of $42,000, with annual maintenan costs of $8000.

The Kram filler has a higher efficiency than the Zanni, and it is expected tl savings would amount to $4000 per year if the Kram filler were installed. filling machine will not be needed after 5 years, and at that time, the salva value for the Kram filler would be $25,000, while the Zanni would have lit or no value. Assuming a MARR of 10%, which filling machine should be purchased?

SOLUTION

McClain, Edwards, Shiver, and Smith (MESS) LLC is considering the purchase of new automated cleaning equipment. The industrial engineer fo company has been asked to calculate the present worth of the two best alternatives based on the following data.

	Mess Away	Quick Clean
First cost	$65,000	$78,000
Annual savings	0,000	24,000

5-18	Annual operating costs	4,000	2,750
	Scheduled maintenance	$1500 at the end of 3rd year	$3000 at the end of 3rd year
	Annual insurance*	2,000	2,200
	Salvage value	10% of first cost	12.5% of first cost
	Useful life	5 years	5 years

* Assume beginning-of-period payments.

Determine which equipment should be purchased, given an interest rate of

SOLUTION

5-19 Helbing Construction has just been awarded a contract that will require the company to either purchase or lease a new track hoe. The equipment will b needed for the entire length of the 4-year contract. The equipment can be leased for $85,000 per year. All operating costs must be paid by Helbing. These are estimated to be $56,000 per year. All other expenses will be paic the leasing company. If the track hoe is purchased, the first cost will be $485,000 and the anticipated salvage value is $365,000 at the end of the 4 years. The annual operating costs are expected to be $75,000 with an addit maintenance costing $5000 occurring at the end of Year 2. If Helbing's M. is 5%, should the company lease or purchase the track hoe?

SOLUTION

5-20 Gullett Glue must replace a machine used to fill glue tubes. The relevant d concerning the machines under consideration are presented in the table bel If the MARR for GG is 5%, determine which machine should be purchase

	Fill-Rite	Best-Fill
First cost	$72,000	$68,000

Annual costs	$6,000 the first year increasing by $750 each year thereafter	$7,800 per year
Overhaul	$7,525 at the end of Year 4	$8,000 at the end of Year 4
Salvage value	7.5% of first cost	$5,500
Useful life	8 years	8 years

SOLUTION

5-21 Two alternatives are under consideration by XYZ Inc. Alternative A has a NPV of $1,243 and a life of 4 years. Alternative B has a NPV of 2,196 and life of 8 years. Which alternative should XYZ select if the MARR is 5%?

SOLUTION

5-22 A project has a first cost of $10,000, net annual benefits of $2000, and a salvage value of $3000 at the end of its 10-year useful life. The project wil replaced identically at the end of 10 years, and again at the end of 20 years What is the present worth of the entire 30 years of service, given an interes rate of 10%?

SOLUTION

5-23 Using a 10% interest rate, determine which alternative, if any, should be selected.

Alternative	*A*	B
First cost	$5300	$10,700
Uniform annual benefit	1800	2,100
Useful life	4 years	8 years

SOLUTION

5- The lining of a chemical tank in a certain manufacturing operation is replac every 5 years at a cost of $5000. A new type of lining is now available that would last 10 years, but it costs $9000. The manufacturer's tank needs a ne

24 lining now, and the company intends to use the tank for 40 years, replacing linings when necessary. Compute the present worth of costs of 40 years of service for the 5-year and 10-year linings if *i* = 10%.

[SOLUTION]

5-25 Be-low Mining, Inc., is trying to decide whether it should purchase or leas new earthmoving equipment. If purchased, the equipment will cost $175,0 and will be used 6 years, at which time it can be sold for $72,000. At Year an overhaul costing $20,000 must be performed. The equipment can be lea for $30,000 per year. Be-low will not be responsible for the midlife overha the equipment is leased. If the equipment is purchased, it will be leased to mining companies when possible; this is expected to yield revenues of $15 per year. The annual operating cost regardless of the decision will be approximately equal. What would you recommend if the MARR is 6%?

[SOLUTION]

Two technologies are currently available for the manufacture of an importa and expensive food and drug additive.

Laboratory *A* is willing to release the exclusive right to manufacture the additive in this country for $50,000 payable immediately, and a $40,000 payment each year for the next 10 years. The production costs are $1.23 pe unit of product.

Laboratory *B* is also willing to release similar manufacturing rights, with th following schedule of payments:

5-26 on the closing of the contract, $10,000

from Years 1 to 5, at the end of each year, a payment of $25,000 each

from Years 6 to 10, also at the end of each year, a payment of $20,000

The production costs are $1.37 per unit of product.

Neither lab is to receive any money after 10 years for this contract. It is anticipated there will be an annual production of 100,000 items for the nex years. On the basis of analyses and trials, the products of *A* and *B* are practically identical in quality. Assuming a MARR of 12%, which lab shou be chosen?

SOLUTION

5-27 An engineering analysis by net present worth (NPW) is to be made for the purchase of two devices, *A* and *B*. If an 8% interest rate is used, recommen the device to be purchased.

	Cost	Uniform Annual Benefit	Salvage	Useful Life
Device *A*	$600	$100	$250	5 years
Device *B*	700	100	180	10 years

SOLUTION

5-28 An engineer is considering buying a life insurance policy for his family. H currently owes $50,500, and he would like his family to have an annual available income of $55,000 indefinitely upon receipt of the policy procee the engineer believes that money from the policy can be invested in an acc paying 4% annual interest, how much life insurance should he buy?

SOLUTION

5-29 A successful engineer wishes to establish a scholarship at her alma mater tl will pay two industrial engineering students $4000 per year. The university earns 5% on endowment accounts. Determine the amount that must be deposited today if the scholarships will be awarded 1 year from today.

SOLUTION

5-30 A bridge is being considered at a cost of $220M. The annual maintenance c are estimated to be $150K. A major renovation costing $50M is required e 25 years. What is the capitalized cost of the bridge at 5% interest?

SOLUTION

5-31 A resident will give money to his town to purchase a statue honoring the tc founders and will pay to maintain the work at a cost of $500 per year forev If an interest rate of 5% is used, and the resident gives a total of $25,000; h much can be paid for the statue?

SOLUTION

5-32 A minor league baseball stadium is built at a cost of $12,000,000. The annu maintenance costs are expected to cycle every 6 years with a cost of $18,0(the first year and $2000 each year thereafter until the cost is $28,000 at the of Year 6. The maintenance costs cycle will repeat starting with $18,000 ir Year 7. Every 6 years an expenditure of $1,500,000 must be made to renov and update the stadium. Determine the capitalized cost at 4%.

SOLUTION

5-33 A was transporting tunnel through a mountain range initially cost $1,000,0 and has expected maintenance costs that will occur in a 6-year cycle as shc

End of Year:	1	2	3	4	5	6
Maintenance:	$35,000	$35,000	$35,000	$45,000	$45,000	$60,000

Determine the capitalized cost at 8% interest.

SOLUTION

5- A new runway at Chester International Airport was recently built at a cost $16,000,000. Maintenance and upkeep costs will be $450,000 per year. Th runway lighting will require replacement at a projected cost of $2,000,000

34 every 4 years. Every 8 years the runway will require resurfacing at a projec cost of $6,000,000. Determine the capitalized cost of the runway at an inte rate of 5%.

SOLUTION

5-35 Cheap Motors Manufacturing must replace one of its tow motors. The NPV Alternative *A* is –$5,876; for Alternative *B* it is –$7547, and for Alternative –$3409. Alternatives *A* and *B* are expected to last for 12 years, and Alterna *C* is expected to last for 6 years. If Cheap's MARR is 4%, which alternativ should be chosen?

SOLUTION

5-36 A company decides that it must provide repair service for the equipment it sells. Based on the following, which alternative for providing repair servic should be selected?

Alternative	NPW
A	–$9241
B	–6657
C	–8945

SOLUTION

5-37 A farmer must borrow $20,000 to purchase a used tractor. The bank has offered the following choice of payment plans, each determined by using a interest rate of 8%. If the farmer's MARR is 15%, which plan should he choose?

Plan *A*:	$5010 per year for 5 years
Plan *B*:	$2956 per year for 4 years plus $15,000 at end of 5 years
Plan *C*:	Nothing for 2 years, then $9048 per year for 3 years

SOLUTION

5-38

A firm is considering the purchase of a new machine to increase the output an existing production process. Of all the machines considered, manageme has narrowed the field to the machines represented by the following cash flows.

Machine	Initial Investment	Annual Operating Income
1	$ 50,000	$22,815
2	60,000	25,995
3	75,000	32,116
4	80,000	34,371
3	100,000	42,485

If each of these machines provides the same service for 3 years and the minimum attractive rate of return is 6%, which machine should be selected

SOLUTION

5-39

The city council wants the municipal engineer to evaluate three alternative supplementing the city water supply. The first alternative is to continue dee well pumping at an annual cost of $10,500. The second alternative is to ins an 18-inch pipeline from a surface reservoir. First cost is $25,000 and annu pumping cost is $7000.

The third alternative is to install a 24-inch pipeline from the reservoir at a f cost of $34,000 and annual pumping cost of $5000. The life of each alterna is 20 years. For the second and third alternatives, salvage value is 10% of f cost. With interest at 8%, which alternative should the engineer recommen Use present worth analysis.

SOLUTION

The following data are associated with three grape-crushing machines unde consideration by Rabbit Ridge Wineries LLC.

	Smart Crush	Super Crush	Savage Crush

5-40	First cost	$52,000	$63,000	$105,000
	O&M costs	15,000	9,000	12,000
	Annual benefits	38,000	31,000	37,000
	Salvage value	13,000	19,000	22,000
	Useful life	4 years	6 years	12 years

If Rabbit Ridge uses a MARR of 12%, which alternative, if any, should be chosen?

SOLUTION

5-41 A used car dealer states that if you put $2000 down on a particular car, you monthly payments will be $199.08 for 4 years at an interest rate of 9%. Wh the cost of the car to you?

SOLUTION

5-42 Mary Ann requires approximately 30 pounds of bananas each month, Janu through June, and 35 pounds of bananas each month, July through Decemb to make banana cream pies for her friends. Bananas can be bought at a loca market for 40 cents/lb. If Mary Ann's cost of money is 3%, approximately much should she set aside at the beginning of each year to pay for the bana

SOLUTION

5-43 Three purchase plans are available for a new car.

Plan *A*:	$5000 cash immediately
Plan *B*:	$1500 down and 36 monthly payments of $97.75
Plan *C*:	$1000 down and 48 monthly payments of $96.50

If a customer expects to keep the car for 5 years, and her cost of money is (which payment plan should she choose?

SOLUTION

5-83 Mod

5-44 A firm has the following monthly costs. Find the present worth of the costs the annual interest rate is 15%.

SOLUTION

Parker Designs has a profitable project, and the customer has agreed to payments as follows. Use the XNPV function to find the present worth of t cash flows as of January 1, 2019 if the interest rate is 8.0%.

5-45

Date	Income
1/1/2019	$75,000
5/6/2019	75,000
7/8/2019	75,000
9/2/2019	75,000
12/2/2019	100,000

SOLUTION

A retail sunscreen's manufacturer has income for one season as shown. Us XNPV to find the present worth as of March 1, 2019 if the MARR is 15%.

5-46

Date	Income
3/1/2019	−$15.0 million
4/1/2019	6.1 million
5/6/2019	1.3 million
6/3/2019	3.5 million
7/1/2019	4.6 million
8/5/2019	2.5 million
9/2/2019	0.9 million

SOLUTION

5-47 If the current market interest rate on bonds of a certain type is 3%, compounded semiannually, what should be the market price of a 5% bond having a $1000 face value? The bond will mature (pay its face value) 6½ y from today, and the next interest payment to the bondholder will be due in months.

[SOLUTION]

5-48 A bond issued by Golden Key casinos has a face value of $100,000 and a f rate of 3.75% payable semiannually. If the bond matures 6 years from the (of purchase and an investor requires a 5% return, what is the maximum the investor should pay for the bond?

[SOLUTION]

[PROBLEMS]

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Present Value of One Alternative

5-1 A project has a net present worth of −$20,000 as of January 1, 2022. If a 4% interest rate is used, what is the project NPW as of December 31, 2017?

5-2 [A] The annual income from a rented house is $26,400. The annual expenses are $7200. If the house can be sold for $255,000 at the end of 12 years, how much could you afford to pay for it now, if you considered 8% to be a suitable interest rate?

5-3 A machine costs $250,000 to purchase and will provide $60,000 a year in benefits. The company plans to use the machine for 12 years and then will sell the machine for scrap, receiving $15,000. The company interest rate is 10%. Should the machine be purchased?

5-4 A

IBP Inc. is considering establishing a new machine to automate a meatpacking process. The machine will save $55,000 in labor annually. The machine can be purchased for $225,000 today and will be used for 10 years. It has a salvage value of $12,500 at the end of its useful life. The new machine will require an annual maintenance cost of $11,000. The corporation has a minimum rate of return of 9%. Do you recommend automating the process?

5-5

A student has a job that leaves her with $300 per month in disposable income. She decides that she will use the money to buy a car. Before looking for a car, she arranges a 100% loan whose terms are $300 per month for 48 months at 9% nominal annual interest. What is the maximum car purchase price that she can afford with her loan?

5-6 A

The student in Problem 5-5 finds a car she likes and the dealer offers to arrange financing. His terms are 6% interest for 72 months and no down payment. The car's sticker price is $24,000. How expensive a car can she afford on the dealer's terms?

5-7 G

A road building contractor has received a major highway construction contract that will require 50,000 m^3 of crushed stone each year for 5 years. The stone can be obtained from a quarry for $7.80/$m^3$. As an alternative, the contractor has decided to try to buy the quarry. He believes that if he owned the quarry, the stone would cost him only $6.30/$m^3$. He thinks he could resell the quarry at the end of 5 years for $200,000.

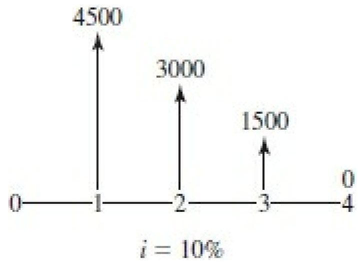
(*a*) If the contractor uses a 12% interest rate, how much would he be willing to pay for the quarry?

(*b*) The extraction of needed quarry stone and other aggregates via mining can pose serious social consequences to various stakeholders. Provide a list of concerns and mitigating potential actions that the road-building contractor may want to consider if he purchases the quarry.

5-8

Compute the present value, *P*, for the following cash flows.

A



5-9 You supervise an aging production line that constantly needs maintenance and new parts. Last month you spent $25,000 replacing a failed controller. Should the following plan be accepted if the interest rate is 15%? The net installed cost of the new line to replace the old line is $600,000 with a useful life of six years. In the first year its operating cost will be $100,000, and it will generate annual revenues of $300,000. Each year the operating cost will increase by $5000 and the revenues will fall by $15,000. After six years the equipment will have a value of $100,000 in the next re-building of the line.

Contributed by Yasser Alhenawi, University of Evansville

5-10 A E

(*a*) How much would the owner of a building be justified in paying for a sprinkler system that will save $1300 a year in insurance premiums if the system has to be replaced every 20 years and has a salvage value equal to 8% of its initial cost? Assume money is worth 6%.

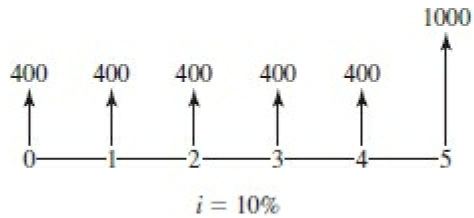
(*b*) Research the International Code Council (ICC). Find and read their code of ethics. Why are organizations such as ICC important from a building safety perspective?

5-11 A wholesale company has signed a contract with a supplier to purchase goods for $2,000,000 annually. The first purchase will be made now to be followed by 10 more. Determine the contract's present worth at a 7% interest rate.

Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

Compute the present value, *P*, for the following cash flows.

5-12
A



5-13

By installing some elaborate inspection equipment on its assembly line, the Robot Corp. can avoid hiring an extra worker who would have earned \$36,000 a year in wages and an additional \$9500 a year in employee benefits. The inspection equipment has a 6-year useful life and no salvage value. Use a nominal 18% interest rate in your calculations. How much can Robot afford to pay for the equipment if the wages and worker benefits were to have been paid

(*a*) At the end of each year

(*b*) Monthly

Explain why the answer in (*b*) is larger.

5-14
A

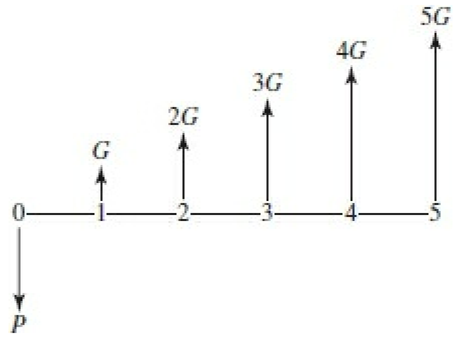
On February 1, the Miro Company needs to purchase some office equipment. The company is short of cash and expects to be short for several months. The treasurer has said that he could pay for the equipment as follows:

Date	Payment
April 1	$100
June 1	220
Aug. 1	340
Oct. 1	460
Dec. 1	580

A local office supply firm will agree to sell the equipment to Miro now and accept payment according to the treasurer's schedule. If interest will be charged at 2.5% every 2 months, with compounding once every 2 months, how much office equipment can the Miro Company buy now? What is the effective interest rate?

A stonecutter, assigned to carve the headstone for a well-known engineering economist, began with the following design.

5-15



He started the equation

$$P = G(P/G, i, 6)$$

He realized he had made a mistake, but he does not want to discard the stone and start over. What one compound interest factor can be added to make the equation correct?

$$P = G(P/G, i, 6)(\quad, i, \quad)$$

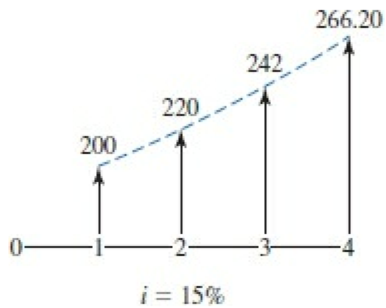
A young industrial engineer analyzed some equipment to replace one production worker. The present worth of employing one less production worker just equaled the present worth of the equipment costs, assuming a 10-year useful life for the equipment. It was decided not to buy the equipment.
A short time later, the production workers won a new 3-year union
5-16 contract that granted them an immediate 40¢-per-hour wage increase,
A plus an additional 25¢-per-hour wage increase in years 2 and 3.
E Assume that in future years, a 25¢-per-hour wage increase will be granted.

(*a*) By how much does the present worth of replacing one production employee increase? Assume an interest rate of 8%, a single 8-hour shift, and 250 days per year.

(*b*) What are the ethical issues of replacing workers with advanced technologies from the firm's, the workers', and society's perspective?

Use a geometric gradient formula to compute the present value, *P*, for the following cash flows.

5-17



5-18 A firm has installed a manufacturing line for packaging materials. The firm plans to produce 45 tons of packing peanuts at $4000 per ton annually for 3 years, and then 60 tons of packing peanuts per year at $5000 per ton for the next 5 years. What is the present worth of the expected income? The firm's interest rate is 15% per year.
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

5-19 Annual maintenance costs for a particular section of highway pavement are $2500. The placement of a new surface would reduce the annual maintenance cost to $500 per year for the first 3 years and to $1000 per year for the next 7 years. After 10 years the annual maintenance would again be $2500. If maintenance costs are the only saving, what investment can be justified for the new surface? Assume interest at 5%.

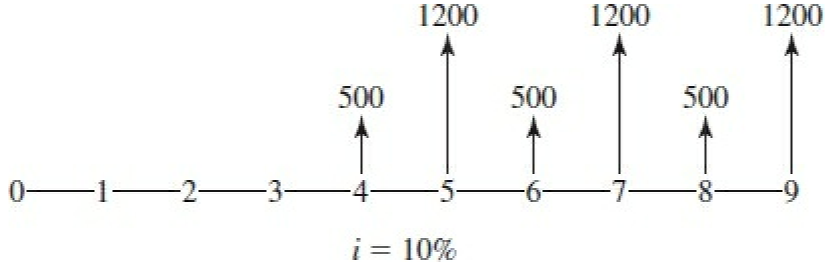
5-20 Luis is responsible for buying some specialized manufacturing equipment that has a purchase price of $10,000 and annual operating costs of $1000. The vendor is offering a special buyer incentive that provides free maintenance for the first four years. After that time, the maintenance is $500 per year over the 10-year life, and there is an overhaul expense in year 5 of $2000. The equipment has a salvage value of $1000. If the interest rate is 8%, what is the present value?
Contributed by Gillian Nicholls, Southeast Missouri State University

A new office building was constructed 5 years ago by a consulting engineering firm. At that time the firm obtained a bank loan for $600,000 with a 12% annual interest rate, compounded quarterly. The loan terms call for equal quarterly payments for 10 years. The loan also allows for its prepayment at any time without penalty.
The firm proposes to refinance the loan through an insurance

5-21 company. The new loan would be for a 20-year term with an interest rate of 8% per year, compounded quarterly. The insurance company requires the payment of a 5% loan initiation charge (often described as a “5-point loan fee”), which will be added to the starting balance.
(*a*) What is the balance due on the original mortgage if 16 payments have been made?
(*b*) What is the difference between the old and new quarterly payments?

5-22 Argentina is considering constructing a bridge across the Rio de la Plata to connect its northern coast to the southern coast of Uruguay. If this bridge is constructed, it will reduce the travel time from Buenos Aires, Argentina, to São Paulo, Brazil, by over 10 hours, and there is the potential to significantly improve the flow of manufactured goods between the two countries. The cost of the new bridge, which will be the longest bridge in the world, spanning over 50 miles, will be $700 million. The bridge will require an annual maintenance of $10 million for repairs and upgrades and is estimated to last 80 years. It is estimated that 550,000 vehicles will use the bridge during the first year of operation and an additional 50,000 vehicles per year until the tenth year. The annual traffic for the remainder of the life of the bridge will be 1,000,000 vehicles per year. These data are based on a toll charge of $90 per vehicle. The Argentine government requires a minimum rate of return of 9% to proceed with the project.
(*a*) Does this project provide sufficient revenues to offset its costs?
(*b*) What considerations are there besides economic factors in deciding whether to construct the bridge?

5-23 Compute the present value, *P*, for the following cash flows.



Lives Match

5-24 [A]

Walt Wallace Construction Enterprises may buy a new dump truck with a 10-year life. Interest is 6%. The cash flows for two likely models are as follows:

Model	First Cost	Annual Operating Cost	Annual Income	Salvage Value
A	$65,000	$8,000	$16,000	$25,000
B	59,000	10,000	17,500	19,000

(*a*) Using present worth analysis, which truck should the firm buy, and why?
(*b*) Before the construction company can close the deal, the dealer sells out of Model *B* and cannot get any more. What should the firm do now, and why?

5-25

A new tennis court complex is planned. Each of two alternatives will last 18 years, and the interest rate is 7%. Use present worth analysis to determine which should be selected.
Contributed by D. P. Loucks, Cornell University

	Construction Cost	Annual O&M
A	$500,000	$25,000
B	640,000	10,000

5-26 [A]

Two alternative courses of action have the following schedules of disbursements:

Year	*A*	*B*
0	− $21, 000	
1	0	− $2, 000
2	0	−3, 500
3	0	−5, 000
4	0	−6, 500
5	0	−8, 000
	− $21,000	− $25,000

Based on a 4% interest rate, which alternative should be selected?

5-27 If produced by Method *A*, a product's initial capital cost will be $100,000, its annual operating cost will be $20,000, and its salvage value after 3 years will be $20,000. With Method *B* there is a first cost of $150,000, an annual operating cost of $10,000, and a $50,000 salvage value after its 3-year life. Based on a present worth analysis at a 15% interest rate, which method should be used? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*

5-28 A For a new punch press. Company *A* charges $250,000 to deliver and install it. Company *A* has estimated that the machine will have operating and maintenance (O & M) costs of $4000 a year. You estimate an annual benefit of $89,000. Company *B* charges $205,000 to deliver and install the device. Company *B* has estimated O & M of the press at $4300 a year. You estimate an annual benefit of $86,000. Both machines will last 5 years and can be sold for $15,000. Use an interest rate of 12%. Which machine should your company buy?

5-29 G Quinton's refrigerator has just died. He can get a basic refrigerator or a more efficient and environmentally friendly refrigerator with an Energy Star designation. Quinton earns 4% compounded annually on his investments, he wants to consider a 10-year planning horizon, and he will use present worth analysis to determine the best alternative. What is your recommendation? *Contributed by Paul R. McCright, University of South Florida*

	Basic Unit	Energy Star Unit
Initial cost	700	800
Delivery and installation	60	60
Professional servicing (year 5)	100	100
Annual energy costs	120	55
Salvage value (year 10)	150	175

A battery manufacturing plant has been ordered to cease discharging acidic waste liquids containing mercury into the city sewer system. As a result, the firm must now adjust the pH and remove the mercury from its waste liquids. Quotations from three firms are included in the following table of costs.

5-30
A
G

Bidder	Installed Cost	Annual Operating Cost	Annual Income from Mercury Recovery	Salvage Value
Foxhill	$ 70,000	$5000	$6500	$10,000
Instrument				
Quicksilver	50,000	5000	2500	0
Almaden	90,000	5000	7800	10,000

If the installation will last 15 years and money is worth 12%, which equipment should be purchased?

5-31 Teléfono Mexico is expanding its facilities to serve a new manufacturing plant. The new plant will require 2000 telephone lines this year, and another 2000 lines after expansion in 10 years. The plant will operate for 30 years.
Option 1 Provide one cable now with capacity to serve 4000 lines. The cable will cost $20,000 and annual maintenance costs will be $1500.
Option 2 Provide a cable with capacity to serve 2000 lines now and a second cable to serve the other 2000 lines in 10 years. Each cable will cost $15,000 and will have an annual maintenance of $1000.

The telephone cables will last at least 30 years, and the cost of removing the cables is offset by their salvage value.

(*a*) Which alternative should be selected, assuming a 10% interest rate?
(*b*) Will your answer change if the demand for additional lines occurs in 5 years?

A consulting engineer has been hired to advise a town how best to proceed with the construction of a 200,000-m^3 water supply reservoir. Since only 120,000 m^3 of storage will be required for the next 25 years, an alternative to building the full capacity now is to

5-32 build the reservoir in two stages. Initially, the reservoir could be built with 120,000 m^3 of capacity and then, 25 years hence, the additional 80,000 m^3 of capacity could be added by increasing the height of the reservoir. If interest is computed at 4%, which construction plan is preferred?

A

	Construction Cost	Annual Maintenance Cost
Build in two stages		
First stage	$14,200,000	$75,000
Second stage	12,600,000	add $25,000
Build full capacity now	22,400,000	100,000

5-33 In order to improve evacuation routes out of New Orleans in the event of another major disaster such as Hurricane Katrina, the Louisiana Department of Transportation (L-DoT) is planning to construct an additional bridge across the Mississippi River. The department uses an interest rate of 8% and plans a 50-year life for either bridge. Which design has the better present worth? *Contributed by Paul R. McCright, University of South Florida*

All Costs in $M	Suspension Bridge	Cantilever Bridge
Initial construction	$585	$470
Land acquisition	120	95
Annual O&M	2	3
Annual increase	4%	0.3
Major maintenance	185	210
(Year 25)		
Salvage cost	30	27

Javier is an IE at Lobos Manufacturing. He has been studying process line G to determine if an automated system would be preferred to the existing labor-intensive system. If Lobos wants to earn at least 20% and uses a 15-year planning horizon, which alternative is preferred? *Contributed by Paul R. McCright, University of South Florida*

5-34

	Labor Intensive	Automated
Initial cost	$0	$110,000
Installation cost	0	18,500
First-year O&M	2,000	4,800
Annual increase	450	950
First-year labor costs	72,000	47,500
Annual increase	5%	5%
Salvage value (EOY15)	2,500	20,000

5-35 In an analysis one alternative has a net present worth of $4200, based on a 6-year analysis period that matches its useful life. A 9% interest rate was used. The replacement will be an identical item with the same cost, benefits, and useful life. Using a 9% interest rate, compute the net present worth for a 12-year analysis period.

Lives Differ

5-36 A

Use a 10-year analysis period and an 8% interest rate to determine which alternative should be selected:

	A	B
First cost	$6500	$12,000
Uniform annual benefit	$1800	$2000
Useful life, in years	5	10

5-37 The Larkspur Furniture Company needs a new grinder. Compute the present worth for these mutually exclusive alternatives and identify which you would recommend given *i* = 6% per year. Larkspur uses a 10-year planning horizon.

Alternative	A	B
Initial cost	$4500	$5500
Annual costs	$300	$400
Salvage value	$500	$0
Life	5 years	10 years

Contributed by Gillian Nicholls, Southeast Missouri State University

5-38 Which process line should be built for a new chemical? The expected market for the chemical is 16 years. An 18% rate is used to evaluate new process facilities, which are compared with present worth. How much does the better choice save?

A

	First Cost	O & M Cost/year	Salvage	Life
A	$14M	$3.5M	$2M	8 years
B	22M	3M	7.5M	16 years

5-39 Which equipment is preferred if the firm's interest rate is 15%? In PW terms how great is the difference?

Alternative	*A*	*B*
First cost	$45,000	$35,000
Annual O&M	3,900	4,200
Salvage value	0	5,000
Overhaul (Year 6)	10,000	Not required
Life, in years	10	5

5-40 A weekly business magazine offers a 1-year subscription for $24.99 and a 4-year subscription for $85. If you thought you would read the magazine for at least the next 4 years, and consider 18% as a minimum rate of return, which way would you purchase the magazine: With four 1-year subscriptions or a single 4-year subscription?

A

5-41 A man had to have the muffler replaced on his 2-year-old car. The repairman offered two alternatives. For $250 he would install a muffler guaranteed for 2 years. But for $400 he would install a muffler guaranteed "for as long as you own the car." Assuming the present owner expects to keep the car for about 3 more years, which muffler would you advise him to have installed if you thought 10% was a suitable interest rate and the less expensive muffler would only last 2 years?

North City must choose between two new snow-removal machines. The SuperBlower has a $70,000 first cost, a 20-year life, and an

5-42 A

$8000 salvage value. At the end of 9 years, it will need a major overhaul costing $19,000. Annual maintenance and operating costs are $9000. The Sno-Mover will cost $50,000, has an expected life of 10 years, and has no salvage value. The annual maintenance and operating costs are expected to be $12,000. Using a 12% interest rate, which machine should be chosen?

5-43

A new alloy can be produced by Process *A*, which costs $200,000 to implement. The operating cost will be $10,000 per quarter with a salvage value of $25,000 after its 2-year life. Process *B* will have a first cost of $250,000, an operating cost of $15,000 per quarter, and a $40,000 salvage value after its 4-year life. The interest rate is 8% per year compounded quarterly. What is the present value difference between *A* and *B*?

Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

5-44 G

An elevator company has redesigned their product to be 50% more energy efficient than hydraulic designs. Two designs are being considered for implementation in a new building.

(*a*) Given an interest rate of 20% which bid should be accepted?

Alternatives	*A*	*B*
Installed cost	$45,000	$54,000
Annual cost	2700	2850
Salvage value	3000	4500
Life, in years	10	15

(*b*) Research and list a few attributes that the company might be using in the elevators' major systems: drive, cab, hoist, and control mechanisms that make them more energy efficient.

5-45

The Crockett Land Winery must replace its present grape-pressing equipment. The two alternatives are the Quik-Skwish and the Stomp-Master. The annual operating costs increase by 12% each year as the machines age. If the interest rate is 9%, which press should be chosen?

	Quik-Skwish	Stomp-Master
First cost	$350,000	$500,000

Annual operating costs	28,000	22,500
Salvage value	35,000	50,000
Useful life, in years	5	10

A railroad branch line to a landfill site is to be constructed. It is expected that the railroad line will be used for 15 years, after which the landfill site will be closed and the land turned back to agricultural use. The railroad track and ties will be removed at that time.

5-46 G

In building the railroad line, either treated or untreated wood ties may be used. Treated ties have an installed cost of $6 and a 10-year life; untreated ties are $4.50 with a 6-year life. If at the end of 15 years the ties then in place have a remaining useful life of 4 years or more, they will be used by the railroad elsewhere and have an estimated salvage value of $3 each. Any ties that are removed at the end of their service life, or too close to the end of their service life to be used elsewhere, can be sold for 50¢ each.

(*a*) Determine the most economical plan for the initial railroad ties and their replacement for the 15-year period. Make a present worth analysis assuming 8% interest.
(*b*) Plastics in landfills do not degrade quickly and the land may not be useful for agriculture. Research and write a summary of what is being done with landfills now to make them more earth friendly and reduce negative impacts.

Perpetual Life

5-47

Dr. Fog E. Professor is retiring and wants to endow a chair of engineering economics at his university. It is expected that he will need to cover an annual cost of $250,000 forever. What lump sum must he donate to the university today if the endowment will earn 5% interest?

An elderly lady decided to distribute most of her considerable wealth to charity and to keep for herself only enough money to provide for her living. She feels that $4500 a month will amply provide for her

5-48 needs. She will establish a trust fund at a bank that pays .6% interest,
A compounded monthly. Upon her death, the balance is to be paid to her niece, Susan. If she deposits enough money to last forever, how much will Susan receive when her aunt dies?

5-49 A local symphony association offers memberships as follows:

Continuing membership, per year	$ 300
Patron lifetime membership	5000

The patron membership has been based on the symphony association's belief that it can obtain a 3% rate of return. If you agree that 3% is appropriate, would you be willing to purchase the patron membership? Explain why or why not.

5-50 A depositor puts $30,000 in a savings account that pays 4% interest, compounded quarterly. Equal annual withdrawals are to be made
A from the account, beginning one year from now and continuing forever. What is the maximum annual withdrawal?

5-51 The local botanical society wants to ensure that the gardens in the town park are properly cared for. The group recently spent $100,000 to plant the gardens. The members want to set up a perpetual fund to provide $100,000 for future replantings every 10 years.

(*a*) If interest is 5%, how much money is needed for the fund?

(*b*) If the last replanting is in year 100, how much is needed for the fund?

5-52 What amount of money deposited 40 years ago at 5% interest would
A provide a perpetual payment of $50,000 a year beginning this year?

5-53 A subdivision developer must construct a sewage treatment plant and deposit sufficient money in a perpetual trust fund to pay the $5000 per year operating cost and to replace the treatment plant every 40 years. The plant and future replacement plants will cost $150,000 each. If the trust fund earns 8% interest, what is the developer's capitalized cost?

Use capitalized cost to determine which type of road surface is preferred on a particular section of highway. Use a 12% interest rate.

5-54 A

	A	*B*
Initial cost	$500,000	$700,000
Annual maintenance	35,000	25,000
Periodic resurfacing	350,000	450,000
Resurfacing interval	10 years	15 years

A new bridge project is being evaluated at i = 5%. Recommend an alternative based on the capitalized cost for each.

5-55

	Construction Cost	Annual O & M	Life (years)
Concrete	$50 million	$250,000	70
Steel	40 million	500,000	50

A new stadium is being evaluated at i = 8%. Recommend an alternative for the main structural material based on the capitalized cost for each.

5-56

	Construction Cost	Annual O & M	Life (years)
Concrete	$28 million	$1.4 million	55
Steel	24 million	1.5 million	45

5-57 G

An open-pit mine must fund an account now to pay for maintenance of a tailing pond in perpetuity (after the mine shuts down in 30 years). The costs until shutdown are part of the mine's operating costs. The maintenance costs begin in 31st year at $300,000 annually.
(*a*) How much must be deposited now if the fund will earn 5% interest? How much does this change if the interest rate is 4%?
(*b*) What is a tailing pond? Is the mine building this because they are a good corporate citizen or because they are required to do so? By whom? Could it be both?

5-58 A

A firm wants to sponsor a new engineering lab at a local university. This requires $4.0M to construct the lab, $1.5M to equip it, and $750,000 every 6 years for new equipment. What is the required endowment if the university will earn 8% interest on the funds?

A city has developed a plan to provide for future municipal water needs. The plan proposes an aqueduct that passes through 500 feet of tunnel in a nearby mountain. The first alternative is to build a full-capacity tunnel now for $556,000. The second alternative is to build a

5-59 half-capacity tunnel now (cost = $402,000), which should be adequate for 20 years, and then to build a second parallel half-capacity tunnel. The maintenance cost of the tunnel linings every 10 years for the full-capacity tunnel is $40,000 and for each half-capacity tunnel it is $32,000.

The friction losses in the half-capacity tunnels will be greater than in the full-capacity tunnel. The estimated additional pumping costs in each half-capacity tunnel will be $2000 per year. Based on capitalized cost and a 7% interest rate, which alternative should be selected?

5-60 A

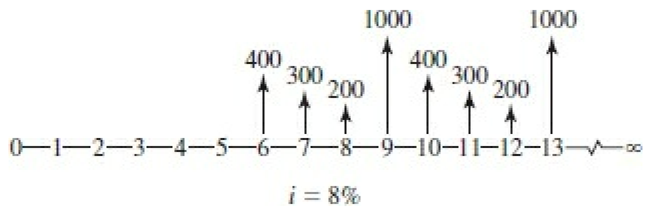
A rather wealthy man decides to arrange for his descendants to be well educated. He wants each child to have $65,000 for his or her education. He plans to set up a perpetual trust fund so that five children will receive this assistance in each generation. He estimates that generations will be spaced 25 years apart. He expects the trust to be able to obtain a 5% rate of return and the first recipients to receive the money 15 years hence. How much money should he now set aside in the trust?

5-61 Kansas Public Service Company wishes to determine the capitalized worth of a new windmill at an interest rate of 9% and following costs.

Purchase	$ 725,000	Installation	$ 143,000
Annual O & M	12,000	Overhaul (Year 25)	260,000
Expected life	40 years	Salvage value	32,000

Contributed by Paul R. McCright, University of South Florida

5-62 Compute the present value, *P*, for the following cash flows (assume series repeats forever).



Multiple Alternatives

5-63 G

A steam boiler is needed as part of the design of a new plant. The boiler can be fired by natural gas, fuel oil, or coal. A cost analysis shows that natural gas would be the cheapest at $30,000; for fuel oil it would be $55,000; and for coal it would be $180,000. If natural gas is used rather than fuel oil, the annual fuel cost will decrease by $7500. If coal is used rather than fuel oil, the annual fuel cost will be $15,000 per year less.

(*a*) Assuming 8% interest, a 20-year analysis period, and no salvage value, which is the most economical installation?

(*b*) Find resources and build a *green effects* table to compare the major advantages and disadvantages of using natural gas, fuel oil, or coal to heat the steam boiler. Use *Advantages* and *Disadvantages*as the column headers and then each choice as a row.

5-64 A

Austin General Hospital is evaluating new lab equipment. The interest rate is 12% and in each case the equipment's useful life is 5 years. Use NPW analysis to pick which company you should purchase from.

Company	A	B	C
First cost	$13,500	$20,000	$15,000
O & M costs	1,500	1,800	1,100
Annual benefit	9,500	11,000	9,000
Salvage value	3,500	6,000	4,000

A building contractor obtained bids for some asphalt paving, based on a specification. Three paving subcontractors quoted the following prices and terms of payment:

The building contractor uses a 12% nominal interest rate, compounded monthly. Which paving subcontractor should be awarded the paving job?

5-65

Paving Co.	Price	Payment Schedule
Quick	$85,000	50% payable immediately
		25% payable in 6 months
		25% payable at year end
Tartan	82,000	Payable immediately
Faultless	84,000	25% payable immediately
		75% payable in 6 months

Six mutually exclusive alternatives, *A–F*, are being examined. For an 8% interest rate, which alternative should be selected? What is the highest present worth? Each alternative has a 6-year useful life.

5-66
A

	Initial	Uniform Annual Benefit
A	$ 20	$ 6.00
B	35	9.25
C	55	13.38
D	60	13.78
E	80	24.32
F	100	24.32

The management of an electronics manufacturing firm believes it is desirable to automate its production facility. The automated equipment would have no salvage value at the end of a 10-year life. The plant engineering department has suggested eight mutually exclusive alternatives. If the firm expects a 12% rate of return, which plan, if any, should it adopt?

5-67

Plan	Initial Cost (thousands)	Net Annual Benefit (thousands)
1	$265	$51
2	220	39
3	180	26
4	100	15

5	300	57
6	130	23
7	245	47
8	165	33

5-68 A

Consider *A*–*E*, five mutually exclusive alternatives:

	A	*B*	*C*	*D*	*E*
Initial cost	$800	$800	$800	$800	$800
Uniform annual benefits					
For first 6 years	125	150	100	125	150
For last 6 years	40	80	120	50	50

The interest rate is 8%. If all the alternatives have a 12-year useful life and no salvage value, which alternative should be selected?

5-69

A firm is considering three mutually exclusive alternatives as part of a production improvement program. The alternatives are:

	A	*B*	*C*
Installed cost	$10,000	$15,000	$20,000
Annual benefit	1,625	1,530	1,890
Useful life (yrs)	10	20	20

The salvage value of each alternative is zero. At the end of 10 years, Alternative *A* could be replaced with another *A* with identical cost and benefits.

(*a*) Which alternative should be selected if interest is 6%?
(*b*) 3%,
(*c*) If there is a difference between parts (*a*) and (*b*) can you explain it?

The following costs are associated with three tomato-peeling machines being considered for use in a canning plant. If the canning company uses an interest rate of 12%, which is the best alternative? Use NPW to make your decision. (*Note:* Consider the least common multiple as the study period.)

5-70 A

Machine	A	B	C
First cost	$52,000	$63,000	$67,000
O & M costs	15,000	9,000	12,000
Annual benefit	38,000	31,000	37,000
Salvage value	13,000	19,000	22,000
Useful life (yrs)	4	6	12

5-71

Consider the following three alternatives. There is also a "do nothing" alternative.

	A	*B*	*C*
Cost	$50	$30	$40
Net annual benefit	12	4.5	6
Useful life, in years	5	10	10

At the end of the 5-year useful life of *A*, a replacement is made. If a 10-year analysis period and a 8% interest rate are selected, which is the preferred alternative?

5-72

A cost analysis is to be made to determine what, if anything, should be done in a situation offering three "do-something" and one "do-nothing" alternatives. Estimates of the cost and benefits are as follows.

Alternatives	Cost	Uniform Annual Benefit	End-of-Useful-Life Salvage Value	Useful Life (years)
1	$450	$150	$ 0	4
2	550	125	300	4
3	800	135	250	8
4	0	0	0	0

Use an 8-year analysis period. At the end of 4 years, Alternatives 1 and 2 may be replaced with identical alternatives. Which alternative should be selected?

(*a*) If a 7% interest rate is used?
(*b*) If a 14% interest rate is used?

5-73 Given the following data, use present worth analysis to find the best alternative, *A*, *B*, or *C*.

	A	*B*	*C*
Initial cost	$10,000	15,000	$12,000
Annual benefit	6,000	10,000	5,000
Salvage value	1,000	−2,000	3,000
Useful life	2 years	3 years	4 years

Use an analysis period of 12 years and 15% interest.

5-74 A

An investor has carefully studied a number of companies and their common stock. From his analysis, he has decided that the stocks of six firms are the best of the many he has examined. They represent about the same amount of risk, and so he would like to determine one single stock in which to invest. He plans to keep the stock for 4 years and requires a 10% minimum attractive rate of return.
Which stock if any, should the investor consider buying?

Common Stock	Price per Share	Annual End-of-Year Dividend per Share	Estimated Price at End of 4 Years
Western House	$23¾	$1.25	$32
Fine Foods	45	4.50	45
Mobile Motors	30⅝	0	42
Spartan Products	12	0	20
U.S. Tire	33⅜	2.00	40
Wine Products	52½	3.00	60

Nominal and Effective Interest

5-75 Assume monthly car payments of $500 per month for 4 years and an interest rate of 0.75% per month. What initial principal or PW will this repay?

5-76 A Assume annual car payments of $6000 for 4 years and an interest rate of 9% per year. What initial principal or PW will this repay?

5-77 Assume annual car payments of $6000 for 4 years and an interest rate of 9.381% per year. What initial principal or PW will this repay?

5-78 Why do the values in Problems 5-75, 5-76, and 5-77 differ?

5-79 Assume mortgage payments of $1000 per month for 30 years and an interest rate of 0.5% per month. What initial principal or PW will this repay?

5-80 A Assume annual mortgage payments of $12,000 for 30 years and an interest rate of 6% per year. What initial principal or PW will this repay?

5-81 Assume annual mortgage payments of $12,000 for 30 years and an interest rate of 6.168% per year. What initial principal or PW will this repay?

5-82 Why do the values in Problems 5-79, 5-80, and 5-81 differ?

Applications and Complications

5-83 A construction project has the following end-of-month costs. Calculate the PW at a nominal interest rate of 18%.

January	$ 30,000	May	$620,000
February	50,000	June	460,000
March	210,000	July	275,000
April	530,000	August	95,000

A factory has averaged the following monthly heating and cooling costs over the last 5 years. Calculate the PW at a nominal interest rate of 9%.

January	$23,000	July	$15,000

5-84 A

February	18,000	August	25,000
March	16,000	September	19,000
April	9,000	October	10,000
May	28,000	November	15,000
June	12,000	December	21,000

5-85 Ding Bell Imports requires a return of 15% on all projects. If Ding is planning an overseas development project with these cash flows, what is the project's net present value?

Year	1	2	3	4	5	6	7
Net Cash ($)	− 60,000	− 110,000	20,000	40,000	80,000	100,000	60,000

5-86 A Maverick Enterprises is planning a new product. Annual sales, unit costs, and unit revenues are as tabulated; the first cost of R&D and setting up the assembly line is $42,000. If *i* is 10%, what is the PW?

Year	Annual Sales	Cost/unit	Price/unit
1	$4,000	$3.50	$6
2	6,000	2.75	5.75
3	8,000	3.00	5.45
4	10,000	2.15	5.25
5	9,000	2.5	4.6
6	4,000	3.25	3.9

5-87 Northern Engineering is analyzing a mining project. Annual production, unit costs, and unit revenues are in the table. The first cost of the mine setup is $6 million. If *i* is 15%, what is the PW?

Year	Annual Production (tons)	Cost per Ton	Price per Ton
1	90,000	$25	$35
2	120,000	20	36
3	120,000	22	37
4	100,000	24	38
5	80,000	26	39
6	60,000	28	40

7	40,000	30	41

XNPV

5-88 [A]

An investment has the following cash flows. Use the XNPV function to find the present worth as of December 1, 2019. The interest rate is 6%.

Date	Cash Flow
12/1/2019	−$11,000
1/12/2020	100
7/15/2020	200
1/15/2021	375
7/15/2021	425
1/16/2022	12,500

5-89

An engineering firm is doing design work on a client’s project. It has $40,000 in expenses at the beginning of each month starting in February 2018 through December 2018. The client has agreed to a payment schedule, if the firm meets milestone delivery dates. Use the XNPV function to find the present worth as of December 11, 2017. The interest rate is 12%.

Date	Income
1/1/2018	$150,000
5/14/2018	150,000
8/6/2018	150,000
10/22/2018	150,000
1/1/2019	200,000

5-90 [A]

Use the XNPV function to find the present worth on January 1, 2018. The interest rate is 10%.

Date	Cash Flow
4/16/2018	−$5000
12/10/2018	1500
4/1/2019	1800

7/22/2019	2000
11/11/2019	1900
2/4/2020	1500
6/2/2020	1200

5-91 A cold remedy's cash flows for one season's cycle are shown below. Use XNPV to find the present worth as of June 1, 2018 if the MARR is 15%.

Date	Cash Flow (in $M)
6/1/2018	−$30
9/1/2018	12
11/1/2018	2
12/17/2018	6
1/7/2019	8
2/25/2019	5
4/1/2019	2

Bonds

5-92 A A corporate bond has a face value of $1000 with maturity date 15 years from today. The bond pays interest semiannually at a rate of 6% nominal per year based on the face value. The interest rate paid on similar corporate bonds has decreased to a current rate of 3%. Determine the market value of the bond.

5-93 You bought a $1000 corporate bond for $900 three years ago. It is paying $30 in interest at the end of every 6 months, and it matures in 5 more years.

(*a*) Compute its coupon rate.

(*b*) Compute its current value, assuming the market interest rate for such investments is 4% per year, compounded semiannually.

Contributed by D. P. Loucks, Cornell University

5-94 An investor is considering buying a 20-year corporate bond. The bond has a face value of $1000 and pays 4% interest per year in two semiannual payments. To receive 6% interest, compounded

A semiannually, how much should be paid for the bond?

Problems 5-95 to 5-98 contributed by Meenakshi Sundaram, Tennessee Tech University

5-95 A 4% coupon rate bond has a face value of $1000, pays interest semiannually, and will mature in 10 years. If the current market rate is 2% interest compounded semiannually, what is the bond's price?

5-96 A A Treasury bond with a face value of $5000 and a coupon rate of 6% payable semiannually was bought when the market's nominal rate was 8%. The bond matures 20 years from now. What was paid for the bond?

5-97 A zero-coupon bond (coupon rate = 0%) has a face value of $10,000 and a maturity date in 5 years. The current market interest rate is a nominal 6%, compounded quarterly. How much should be paid for the bond?

5-98 A city government wants to raise $3 million by issuing bonds. By ballot proposition, the bond's coupon interest rate was set at 8% per year with semiannual payments. However, market interest rates have risen to a nominal 9% interest rate. If the bonds mature in 20 years, how much will the city raise from issuing $3M in bonds.

Minicases

Bayview's growth is constrained by mountains on one side and the bay on the other. A bridge across the bay is planned, but which plan is best? It can be built with a single deck to meet the needs of the next 20 years, or it can be built with two decks to meet the needs of the next 50 years. The piers can also be built to support two decks, but with only one deck being built now.

5-99 Building it all now will cost $160M, and leaving the top deck for later will save $40M. Building that top deck later will cost $70M including the cost of traffic disruption. A single-deck bridge will cost $105M now and $120M in 20 years. Deck maintenance is $1.4M per year per deck. Pier maintenance is $1.2M per year per bridge.

If the interest rate is 5%, which design should be built?If the two-deck bridge is built immediately, then dedicated lanes for buses, carpools, and bicycles can be added. To economically evaluate this use, estimate the cost of the underutilized capacity for the bridge.

5-100 Florida Power and Light has committed to building a solar power plant. JoAnne, an IE working for FPL, has been tasked with evaluating the three current designs. FPL uses an interest rate of 10% and a 20-year horizon.

Design 1: Flat Solar Panels

A field of “flat” solar panels angled to best catch the sun will yield 2.6 MW of power and will cost $87 million initially with first-year operating costs at $2 million, growing $250,000 annually. It will produce electricity worth $6.9 million the first year and will increase by 8% each year thereafter.

Design 2: Mechanized Solar Panels

A field of mechanized solar panels rotates from side to side so that they are always positioned parallel to the sun’s rays, maximizing the production of electricity. This design will yield 3.1 MW of power and will cost $101 million initially with first-year operating costs at $2.3 million, growing $300,000 annually. It will produce electricity worth $8.8 million the first year and will increase 8% each year thereafter.

Design 3: Solar Collector Field

This design uses a field of mirrors to focus the sun’s rays onto a boiler mounted in a tower. The boiler then produces steam and generates electricity the same way a coal-fired plant operates. This system will yield 3.3 MW of power and will cost $91 million initially with first-year operating costs at $3 million, growing $350,000 annually. It will produce electricity worth $9.7 million the first year and will increase 8% each year thereafter.

5-101 Your grandparents are asking you for advice on when they should start collecting social security payments. If they wait until age 66, they will collect $2000 per month; but if they start collecting at age 62, they will collect $1500 per month. Assume they live to be 85, and simplify by assuming annual payments.

(*a*) When do the higher payments catch up in total dollars received with

the lower payment that starts earlier?
(*b*) If their interest rate is 6%, which plan has a higher PW?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 7	**The Board Looks to You** Bond valuation with realistic business details including early-call premium. Medium difficulty. Good example of employer taking a small fact and making big assumptions about individual talents which implies why engineering students must keep learning.
CASE 8	**Picking a Price** Simplified real estate analysis. Demonstrates analysis to screen before acquiring more data and further decision making.
CASE 9	**Recycling?** Financial analysis of recycling cardboard and selling unusable pallets to recycler.

CHAPTER 6

ANNUAL CASH FLOW ANALYSIS

Typical FTC required EnergyGuide label for a clothes washer. According to the California Energy Commission, the typical American household does 400 loads of laundry per year using 40 gallons of water per load with a typical, non–ENERGY STAR clothes washer. An ENERGY STAR appliance would reduce water and energy consumption by 40%. For more information see: http://www.energystar.gov/.

Are More Efficient Appliances Cost Effective?

A typical residence is used for 75 years, and its major appliances last from 10 to 30 years. The long-term trend in energy prices (1–3% per year) and reduced operating costs are not usually enough to induce home owners to replace existing appliances—except when replacing failed appliances.

One driver in the slow adoption of energy efficient appliances is that many investment decisions are made by homebuilders, landlords, and property managers, rather than those who pay the monthly energy bills. When consumers do decide to replace appliances, EnergyGuide labels show the typical energy cost so that different models can be compared by all consumers, not just by those who have studied engineering economy.

From the EnergyGuide label for a clothes washer, the range of energy usage for comparable ENERGY STAR clothes washers goes from 113 kWh/year to 680 kWh/year. The energy consumption estimate is based on ratings of electricity and water usage factors. If electricity costs $0.08/kWh, the estimated electricity costs for comparable models range from $9.04/year to $54.40/year.

When selecting a clothes washer, the consumer should consider initial cost, the annual energy worth (benefit or cost?), and select a model that suits the need. Choices are based on needs; making choices requires tools that are part of the skill set provided by engineering economics.

Contributed by Gene Dixon, East Carolina University

QUESTIONS TO CONSIDER

1. Should average use be based on family size or lifestyle? How can lifestyle be properly analyzed? For example, a family of two in which one or more work outdoors in construction may have larger laundry demands than a family of four in which the parents work in offices. What about the impact of geography and climate; for example, does wearing shorts year-round (versus jeans) create a significant difference in laundry demands?
2. How would a homebuilder, landlord, or property manager justify the use of ENERGY STAR appliances in pricing properties and competing with other home-builders, landlords, or property managers who may use less efficient but still ENERGY STAR–rated appliances?

3. Based on type, major appliances have life spans of 7 to 15 years, with refrigerators having the longest life. If a family stays in a residence on average for 8 years, what considerations should a family use in deciding whether or not to keep their major appliances when moving?
4. Which appliance replacement decisions would be more sensitive to changes in the cost of electricity? How would you explain your answer to someone not skilled in engineering economics?
5. EnergyGuide labels are not required for TVs, ranges, ovens, and clothes dryers. Why do you think this is true?

After Completing This Chapter…

The student should be able to:

• Define *equivalent uniform annual cost (EUAC)*, *equivalent uniform annual benefits (EUAB)*, and *equivalent uniform annual worth (EUAW)*.
• Resolve an engineering economic analysis problem into its annual cash flow equivalent.
• Calculate equivalent annual values for irregular cash flows such as overhaul costs.
• Use EUAW, EUAC, and EUAB to compare alternatives with equal, common multiple, or continuous lives, or over some fixed study period.
• Develop and use spreadsheets to analyze loans for purposes of building an amortization table, calculating interest versus principal, finding the balance due, and determining whether to pay off a loan early.
• Use annuity due for beginning of period cash flows such as leases, insurance, and tuition payments.

Key Words

amortization schedule

annuity due

capital recovery cost

equivalent uniform annual benefit (EUAB), cost (EUAC), worth (EUAW) infinite analysis period

salvage value

This chapter is devoted to annual cash flow analysis—the second of the three major analysis techniques. With present worth analysis, we resolved an alternative into an equivalent net present worth, a present worth of cost, or a present worth of benefit. Here we compare alternatives based on their equivalent annual cash flows: the **equivalent uniform annual cost (EUAC)**, the **equivalent uniform annual benefit (EUAB)**, or their difference, the **equivalent uniform annual worth: (EUAW)** = (EUAB − EUAC).

To prepare for a discussion of annual cash flow analysis, we will review some annual cash flow calculations, then examine annual cash flow criteria.

ANNUAL CASH FLOW CALCULATIONS

Resolving Cash Flows to an Annual Equivalent

In annual cash flow analysis, the goal is to convert money to an equivalent uniform annual cost or benefit. The simplest case is to convert a present sum P to a series of equivalent uniform end-of-period cash flows. This is illustrated in Example 6–1.

EXAMPLE 6-1 (Example 3–1 and 4–5 Revisited)

A machine will cost $30,000 to purchase. Annual operating and maintenance costs (O&M) will be $2000. The machine will save $10,000 per year in labor costs. The salvage value of the machine after 5 years will be $7000. Calculate the machine's equivalent uniform annual worth (EUAW) for an interest rate of 10%.

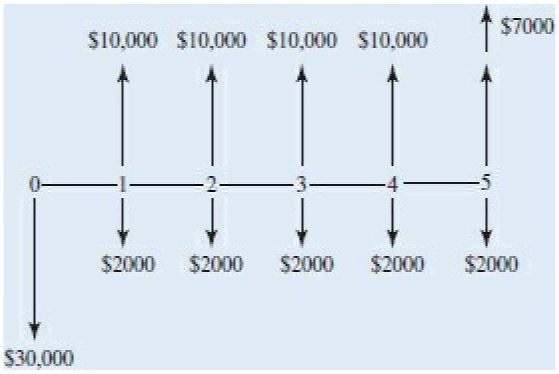


TABLE SOLUTION

Because there are multiple types of cash flows, the capital recovery factor (Eq. 4–6) must be used for the initial cost and the sinking fund factor (Eq. 4–5) for the salvage value. No factor is needed for the annual savings and O&M costs since they are already annual cash flows.

$$\begin{aligned} EUAW &= -30{,}000(A/P, 10\%, 5) + (10{,}000 - 2000) + 7000(A/F, 10\%, 5) \\ &= -30{,}000(0.2638) + 8000 + 7000(0.1638) \\ &= \$1232 \end{aligned}$$

5-BUTTON SOLUTION

The *n* is 5 for both the initial cash flow of −$30,000 and the salvage value of $7000, so the annual equivalent of both can be found in one step.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*PV*	Solve for	Answer	Formula
2	Exp. 6-1	10%	5		-30,000	7000	PMT	$6,767	=PMT(B2,C2,E2,I
3							change sign	-6,767	=-H2
4							annual revenue	10,000	
5							annual O&M	-2,000	
6							EUAW	$1,233	=SUM(H3:H5)

Figure 6–1 generalizes Example 6–1, and Equation 6–1 converts the cash flows to annual equivalents one by one from left to right.

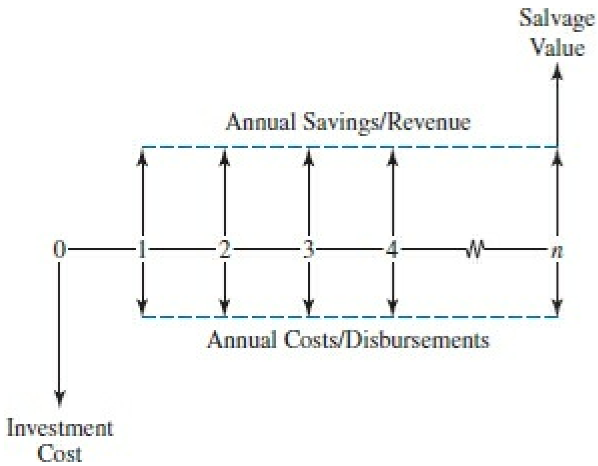


FIGURE 6-1 Common cash flows in EUAW problems.

$$\text{EUAW} = -P(A/P, i, n) + (R - E) + S(A/F, i, n)$$

Where P = investment cost

R = annual revenues (or savings)

E = annual expenses (or disbursements)

S = salvage value (6-1)

Irregular Cash Flows

Individual cash flows do not always occur at time 0 or time *n*. An overhaul, a refurbishing, and a capacity expansion are common examples of midlife cash flows. Call such a mid-life cash flow F_t. To convert F_t to an EUAW, we could either first convert it to a present value (P) using the (P/F, i, t) factor (as in Equation 6–2), or convert it to a future value (F) using the (F/P, i, t) factor (as in Equation 6–3). Then the second factor in each equation calculates the EUAW over *n* periods. This process is depicted in Figure 6–2.

$$\text{EUAW} = F_t(P/F, i, t)(A/P, i, n) \quad (6\text{-}2)$$

$$\text{EUAW} = F_t(F/P, i, n - t)(A/F, i, n) \quad (6\text{-}3)$$

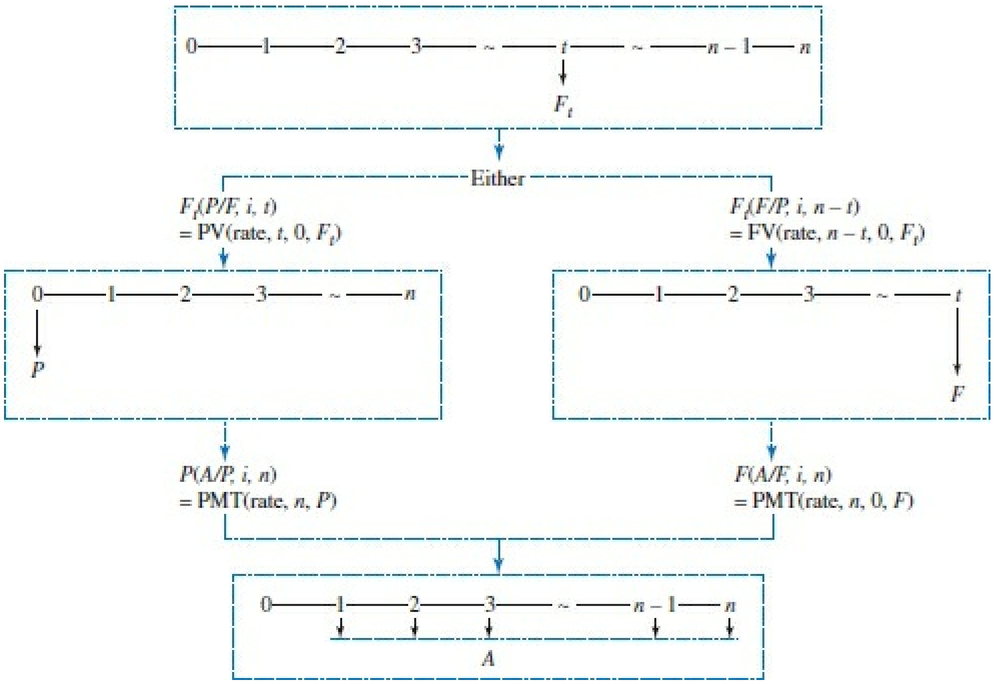


FIGURE 6-2 EUAC for single, irregular cash flow.

EXAMPLE 6-2 EUAC for an Overhaul Cost

Find the EUAC over a 5-year period for a $10,000 overhaul in year 3. Let i = 10%.

TABLE SOLUTION

Finding the present value first,

$$P = 10{,}000(P/F, 10\%, 3) = 10{,}000(0.7513) = 7513$$
$$\text{EUAC} = 7513(A/P, 10\%, 5) = 7513(0.2638) = \$1982$$

or, using Equation 6–2,

$$\text{EUAC} = 10{,}000(P/F, 10\%, 3)(A/P, 10\%, 5) = 10{,}000(0.7513)(0.2638) = \$1982$$

Finding the future value first,

$$F = 10{,}000(F/P, 10\%, 2) = 10{,}000(1.210) = 12{,}100$$
$$\text{EUAC} = 12{,}100(A/F, 10\%, 5) = 12{,}100(0.1638) = \$1982$$

or, using Equation 6-3,

$$\text{EUAC} = 10{,}000(F/P, 10\%, 2)(A/F, 10\%, 5) = 10{,}000(1.210)(0.1638) = \$1982$$

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 6-2	10%	3	0		10,000	PV	-\$7,513
3		10%	5		-7513	0	PMT	\$1,982
4	or							
5		10%	2	0	10,000		FV	-\$12,100
6		10%	5		0	-12,100	PMT	\$1,982

Capital Recovery Costs

Capital costs typically include the first cost (*P*) and a salvage value (*S*). The **capital recovery cost** is the equivalent uniform annual cost of the net capital expenditure. The EUAC can be calculated three ways. Equation 6–4 is a cash flow perspective, where the annualized salvage value is subtracted from the annualized first cost. It is the most commonly used.

$$\text{EUAC} = P(A/P, i, n) - S(A/F, i, n) \quad (6\text{-}4)$$

$$\text{EUAC} = (P - S)(A/F, i, n) + Pi \quad (6\text{-}5)$$

$$\text{EUAC} = (P - S)(A/P, i, n) + Si \quad (6\text{-}6)$$

Equation 6–5 considers the drop in value (*P* – *F*) at the end and includes the interest on the capital tied up from the beginning. Equation 6–6 considers the drop in value (*P* – *F*) at the beginning and includes the interest on the capital tied up until the end.

EXAMPLE 6-3 (Example 6–1 Revisited)

Consider only the capital costs from Example 6–1. A machine will cost \$30,000 to purchase. The salvage value of the machine after 5 years will be \$7000. Calculate the machine's capital recovery cost (EUAC) for an interest rate of 10%.

TABLE SOLUTION

This may be solved using Equation 6–4, 6–5, or 6–6. Equation 6–4 is used.

$$\begin{aligned} \text{EUAC} &= P(A/P, i, n) - S(A/F, i, n) \\ &= 30{,}000(A/P, 10\%, 5) - 7000(A/F, 10\%, 5) \\ &= 30{,}000(0.2638) - 7000(0.1638) \\ &= \$6767 \end{aligned} \tag{6-4}$$

Using equation 6–5 or 6–6 will result in an identical result.

5-BUTTON SOLUTION

The n is 5 for both the initial cash flow of −\$30,000 and the salvage value of \$7000, so the annual equivalent of both can be found in one step.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 6-2	10%	5		-30,000	7000	PMT	\$6,767	=PMT(B2,C2,E2,F2)

Annual Equivalent with Gradient and Overhaul

EXAMPLE 6-4

The maintenance costs for a generator have been recorded over its 5-year life.

Compute the generator's equivalent uniform annual cost (EUAC) assuming 7% interest and end-of-year cash flows.

	Maintenance	
Year	Cost	Overhaul Cost
1	$545	
2	590	
3	635	$750
4	680	
5	725	

TABLE SOLUTION

Maintenance costs are an increasing gradient series and are easily converted into an EUAC. The overhaul cost is first converted to a present worth (future worth also possible), and then the EUAC is calculated as follows:

$$\begin{aligned}\text{EUAC} &= 545 + 45(A/G, 7\%, 5) + 750(P/F, 7\%, 3)(A/P, 7\%, 5)\\ &= 545 + 45(1.865) + 750(0.8163)(0.2439) = \$778.24\end{aligned}$$

SPREADSHEET SOLUTION

When recording the repair and overhaul expenses in the spreadsheet, it is best to use separate columns. This ensures that the calculations for the gradient remain simple.

	A	B	C	D	E
1	7%	interest rate			
2	5	horizon			
3	545	first year maintenance			
4	45	maintenance gradient			
5	750	overhaul in yr 3			
6	Year	Maintenance	Overhaul	Total cost	

7	0	0		0	
8	1	545		545	
9	2	590		590	
10	3	635	750	1385	
11	4	680		680	
12	5	725		725	
13			Present cost	$3,190.93	=D7+NPV($A$1,D8:D12)
13			EUAC	$778.24	=PMT($A$1,A2,-D13)

The examples have shown four essential points concerning cash flow calculations:

1. There is a direct relationship between the present worth of cost and the equivalent uniform annual cost. It is

$$\text{EUAC} = (\text{PW of cost})(A/P, i, n)$$

2. In a problem, spending money increases the EUAC, while receiving money—for example, from an item's salvage value—decreases the EUAC.
3. When there are irregular cash disbursements over the analysis period, a convenient method of solution is to first determine the PW of cost; then use the equation in Item 1 to calculate the EUAC.
4. Where there is an arithmetic gradient, EUAC may be rapidly computed by using the arithmetic gradient uniform series factor, $(A/G, i, n)$.

ANNUAL CASH FLOW ANALYSIS

The criteria for economic efficiency are presented in Table 6–1. One notices immediately that the table is quite similar to Table 5–1. It is apparent that, if you are maximizing the present worth of benefits, simultaneously you must be maximizing the equivalent uniform annual worth. This is illustrated in Example 6–5.

TABLE 6-1 Annual Cash Flow Analysis

Input/Output	Situation	Criterion
Neither input nor output is fixed	Typical, general situation	Maximize equivalent uniform annual worth (EUAW = EUAB − EUAC)
Fixed input	Amount of money or other input resources is fixed	Maximize equivalent uniform annual benefit (maximize EUAB)
Fixed output	There is a fixed task, benefit, or other output to be accomplished	Minimize equivalent uniform annual cost (minimize EUAC)

EXAMPLE 6-5 Example 5–1 Revisited

A firm is considering which of two devices to install to reduce costs. Both devices have useful lives of 5 years with no salvage value. Device *A* costs \$10,000 and can be expected to result in \$3000 savings annually. Device *B* costs \$13,500 and will provide cost savings of \$3000 the first year; however, savings will increase \$500 annually, making the second-year savings \$3500, the third-year savings \$4000, and so forth. With interest at 7%, which device should the firm purchase?

SOLUTION

Device *A*

$$\begin{aligned}\text{EUAW} &= -10{,}000(A/P, 7\%, 5) + 3000\\ &= -10{,}000(0.2439) + 3000 = \$561\end{aligned}$$

Device *B*

$$\begin{aligned}\text{EUAW} &= -13{,}500(A/P, 7\%, 5) + 3000 + 500(A/G, 7\%, 5)\\ &= -13{,}500(0.2439) + 3000 + 500(1.865) = \$640\end{aligned}$$

To maximize EUAW, select Device *B*.

Example 6–5 was presented earlier, as Example 5–1, where we found:

$$\begin{aligned}\mathrm{PW}_A &= -10{,}000 + 3000(P/A, 7\%, 5)\\ &= -10{,}000 + 3000(4.100) = \$2300\end{aligned}$$

This is converted to EUAW by multiplying by the capital recovery factor:

$$\mathrm{EUAW}_A = 2300(A/P, 7\%, 5) = 2300(0.2439) = \$561$$

Similarly, for machine *B*

$$\begin{aligned}\mathrm{PW}_B &= -13{,}500 + 3000(P/A, 7\%, 5) + 500(P/G, 7\%, 5)\\ &= -13{,}500 + 4000(4.100) + 500(7.647) = \$2624\end{aligned}$$

and, hence,

$$\begin{aligned}\mathrm{EUAW}_B &= 2624(A/P, 7\%, 5) = 2624(0.2439)\\ &= \$640\end{aligned}$$

We see, therefore, that it is easy to convert the present worth analysis results into the annual cash flow analysis results. We could go from annual cash flow to present worth just as easily, by using the series present worth factor. And, of course, both methods show that Device *B* is the preferred alternative.

EXAMPLE 6-6

Three alternatives are being considered to improve an assembly line along with the "do-nothing" alternative. Each of Plans *A*, *B*, and *C* has a 10-year life and a salvage value equal to 10% of its original cost.

	Plan *A*	Plan *B*	Plan *C*
Installed cost of equipment	$15,000	$25,000	$33,000
Material and labor savings per year	14,000	9,000	14,000

Annual operating expenses	8,000	6,000	6,000
End-of-useful life salvage value	1,500	2,500	3,300

If interest is 8%, which plan, if any, should be adopted? Consider the salvage value as a benefit, because it is a cash inflow.

TABLE SOLUTION

Since neither installed cost nor output benefits are fixed, the economic criterion is to maximize EUAW = EUAB − EUAC.

	Plan *A*	Plan *B*	Plan *C*
Equivalent uniform annual benefit (EUAB)			
Material and labor per year	\$14,000	\$9,000	\$14,000
Salvage value (*A*/*F*, 8%, 10)	104	172	228
EUAB =	\$14,104	\$9,172	\$14,228
Equivalent uniform annual cost (EUAC)			
Installed cost (*A*/*P*, 8%, 10)	\$2,235	\$3,725	\$4,917

Annual operating expenses	8,000	6,000	6,000
EUAC =	$10,235	$9,725	$10,917
EUAW = EUAB − EUAC =	$3,869	− $ 553	$3,311

Based on our criterion of maximizing EUAW, Plan *A* is the best of the four alternatives. Since the do-nothing alternative has EUAW = 0, it is a more desirable alternative than Plan *B*.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Plan A	8%	10		-15,000	1500	PMT	$2,132
3	Plan B	8%	10		-25,000	2500	PMT	3,553
4	Plan C	8%	10		-33,000	3300	PMT	4,690
5					PMT sign change	+NAME & L savings	- Operating	EUAW
6				Plan A	-$2,132	$14,000	-$8,000	$3,868
7				Plan B	-3,553	9,000	-6,000	-553
8				Plan C	-4,690	14,000	-6,000	3,310

ANALYSIS PERIOD

In Chapter 5, we saw that the analysis period is an important consideration in computing present worth comparisons. In such problems, a common analysis period must be used for all alternatives. In annual cash flow comparisons, we again have the analysis period question. Example 6–7 will help in examining the problem.

EXAMPLE 6-7 (Example 5–4 Revisited)

Two pumps are being considered for purchase. If interest is 7%, which pump should be bought? Consider salvage value as a cost, because it is part of the capital recovery cost.

	Pump *A*	**Pump *B***
Initial cost	$7000	$5000
End-of-useful-life salvage value	1200	1000
Useful life, in years	12	6

TABLE SOLUTION

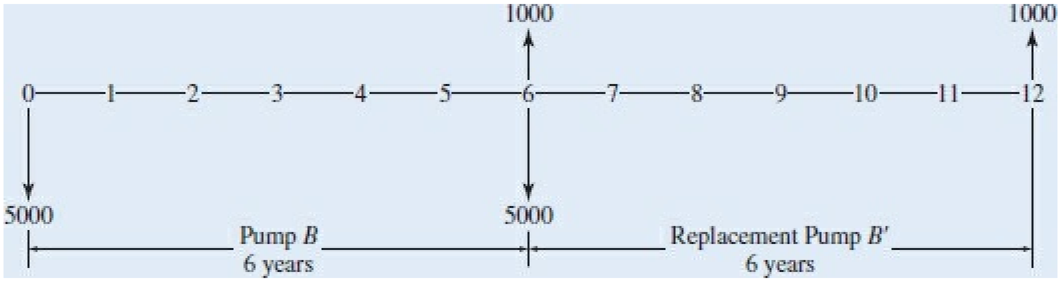
The annual cost for 12 years of Pump *A* can be found using Equation 6–4.

$$\begin{aligned} \text{EUAC} &= P(A/P, i, n) - S(A/F, i, n) \\ &= 7000(A/P, 7\%, 12) - 1200(A/F, 7\%, 12) \\ &= 7000(0.1259) - 1200(0.0559) = \$814 \end{aligned}$$

Now compute the annual cost for 6 years of Pump *B*:

$$\begin{aligned} \text{EUAC} &= P(A/P, i, n) - S(A/F, i, n) \\ &= 5000(A/P, 7\%, 6) - 1000(A/F, 7\%, 6) \\ &= 5000(0.2098) - 1000(0.1398) = \$909 \end{aligned}$$

For a common analysis period of 12 years, we need to replace Pump *B* at the end of its 6-year useful life. If we assume that another pump B' can be obtained, having the same $5000 initial cost, $1000 salvage value, and 6-year life, it will have the same $909 EUAC over its 6-year life. Analyzing the cash flows over 12 years will confirm this as follows:



For the 12-year analysis period, the annual cost for Pump *B* is

$$\begin{aligned}
\text{EUAC} &= [5000 - 1000(P/F, 7\%, 6) + 5000(P/F, 7\%, 6) \\
&\quad - 1000(P/F, 7\%, 12)] \times (A/P, 7\%, 12) \\
&= [5000 - 1000(0.6663) + 5000(0.6663) - 1000(0.4440)] \times (0.1259) \\
&= (5000 - 666 + 3331 - 444)(0.1259) \\
&= (7211)(0.1259) = \$909
\end{aligned}$$

The annual cost of *B* for the 6-year analysis period is the same as the annual cost for the 12-year analysis period. Thus the lengthy calculation of EUAC for 12 years of Pump *B* and *B′* was not needed. Assuming that the shorter-life equipment is replaced by equipment with identical economic consequences avoids a lot of calculations. Select Pump *A*.

5-BUTTON SOLUTION

The EUAC of Pump *A* over 12 years is a 1-step calculation. However, finding the EUAC of Pump *B* either requires breaking the problem into several steps or recognizing that both 6-year periods have the same EUAC.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Pump A	7%	12		-7000	1200	PMT	$814
3	Pump B	7%	6		-5000	1000	PMT	909

Note that the sign assumption of the PMT function produces results that match the previous EUAC calculation.

Analysis Period Equal to Alternative Lives

If the analysis period for an economy study coincides with the useful life for each alternative, then the economy study is based on this analysis period.

Analysis Period a Common Multiple of Alternative Lives

When the analysis period is a common multiple of the alternative lives a "replacement with an identical item with the same costs, performance, and so forth" is frequently assumed. This means that when an alternative has reached the end of its useful life, we assume that it will be replaced with an identical item. As shown in Example 6–7, the result is that the EUAC for an alternative with a 6-year useful life is equal to the EUAC for the entire analysis period based on that alternative being replaced with an *identical* unit.

Under these circumstances of identical replacement, we can compare the annual cash flows computed for alternatives based on their own service lives. In Example 6–7, the annual cost for Pump *A*, based on its 12-year service life, was compared with the annual cost for Pump *B*, based on its 6-year service life.

Analysis Period for a Continuing Requirement

Many times an economic analysis is undertaken to determine how to provide for a more or less continuing requirement. One might need to pump water from a well as a continuing requirement. There is no distinct analysis period. In this situation, the analysis period is assumed to be long but undefined.

If, for example, we had a continuing requirement to pump water and alternative Pumps *A* and *B* had useful lives of 7 and 11 years, respectively, what should we do? The customary assumption is that Pump *A*'s annual cash flow (based on a 7-year life) may be compared to Pump *B*'s annual cash flow (based on an 11-year life). This is done without much concern that the least common multiple of the 7- and 11-year lives is 77 years. This comparison of "different-life" alternatives assumes identical replacement (with identical costs, performance, etc.) when an alternative reaches the end of its useful life.

This continuing requirement, which can also be described as an *indefinitely long horizon*, is illustrated in Example 6–8. Since this is longer than the lives of the alternatives, we can make the best decision possible given current information by minimizing EUAC or maximizing EUAW or EUAB. At a later time, we will make another replacement and there will be more information on costs *at that time*.

EXAMPLE 6-8

Pump *B* in Example 6–7 is now believed to have a 9-year useful life. Assuming the same initial cost and salvage value, compare it with Pump *A* using the same 7% interest rate.

TABLE SOLUTION

If we assume that the need for *A* or *B* will exist for some continuing period, the comparison of costs per year for the unequal lives is an acceptable technique. For 12 years of Pump *A*, using equation 6–5:

$$\text{EUAC} = (7000 - 1200)(A/P, 7\%, 12) + 1200(0.07) = \$814$$

For 9 years of Pump *B*:

$$\text{EUAC} = (5000 - 1000)(A/P, 7\%, 9) + 1000(0.07) = \$684$$

For minimum EUAC, select Pump *B*.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Pump A	7%	12		-7000	1200	PMT	$814
3	Pump B	7%	6		-5000	1000	PMT	684

With the longer assumed life, Pump *B* now has the lowest EUAC.

Infinite Analysis Period

At times we have an alternative with a limited (finite) useful life in an **infinite analysis period** situation. The equivalent uniform annual cost may be computed for the limited life. The assumption of identical replacement (replacements have identical costs, performance, etc.) is often appropriate. Based on this assumption, the same EUAC occurs for each replacement of the limited-life alternative. The EUAC for the infinite analysis period is therefore equal to the EUAC computed for the limited life. With identical replacement,

$$\text{EUAC}_{\text{infinite analysis period}} = \text{EUAC}_{\text{for limited life } n}$$

A somewhat different situation occurs when there is an alternative with an infinite life in a problem with an infinite analysis period:

$$\text{EUAC}_{\text{infinite analysis period}} = P(A/P, i, \infty) + \text{Any other annual costs}$$

When $n = \infty$, we have $A = Pi$ and, hence, $(A/P, i, \infty)$ equals i.

$$\text{EUAC}_{\text{infinite analysis period}} = Pi + \text{Any other annual costs}$$

EXAMPLE 6-9

In the construction of an aqueduct to expand the water supply of a city, there are two alternatives for a particular portion of the aqueduct. Either a tunnel can be constructed through a mountain, or a pipeline can be laid to go around the mountain. If there is a permanent need for the aqueduct, should the tunnel or the pipeline be selected for this particular portion of the aqueduct? Assume a 6% interest rate.

SOLUTION

	Tunnel Through Mountain	Pipeline Around Mountain
Initial cost	$5.5 million	$5 million
Maintenance	0	0

Useful life	Permanent	50 years
Salvage value	0	0

Tunnel

For the tunnel, with its permanent life, we want $(A/P, 6\%, \infty)$. For an infinite life, the capital recovery is simply interest on the invested capital. So $(A/P, 6\%, \infty) = i$, and we write

$$\text{EUAC} = Pi = \$5.5 \text{ million}(0.06)$$
$$= \$330{,}000$$

Pipeline

$$\text{EUAC} = \$5 \text{ million}(A/P, 6\%, 50)$$
$$= \$5 \text{ million}(0.0634) = \$317{,}000$$

For fixed output, minimize EUAC. Select the pipeline.

The difference in annual cost between a long life and an infinite life is small unless an unusually low interest rate is used. In Example 6–9 the tunnel is assumed to be permanent. For comparison, compute the annual cost if an 85-year life is assumed for the tunnel.

$$\text{EUAC} = \$5.5 \text{ million}(A/P, 6\%, 85)$$
$$= \$5.5 \text{ million}(0.0604)$$
$$= \$332{,}000$$

The difference in time between 85 years and infinity is great indeed; yet the difference in annual costs equivalent is only \$2000.

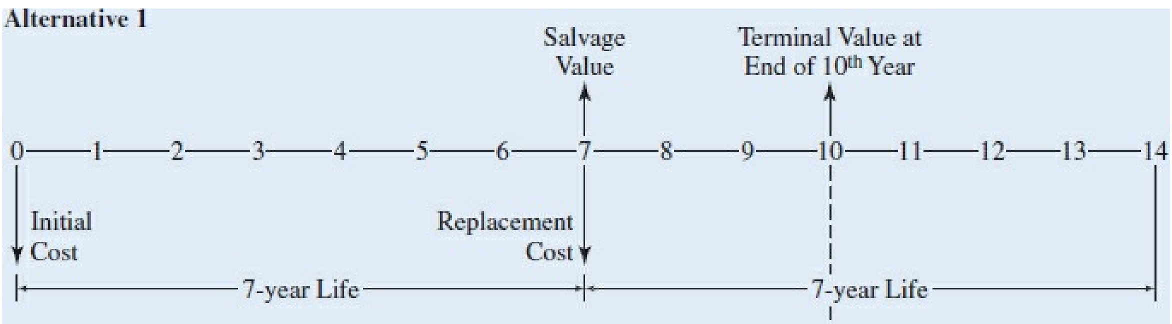
Some Other Analysis Period

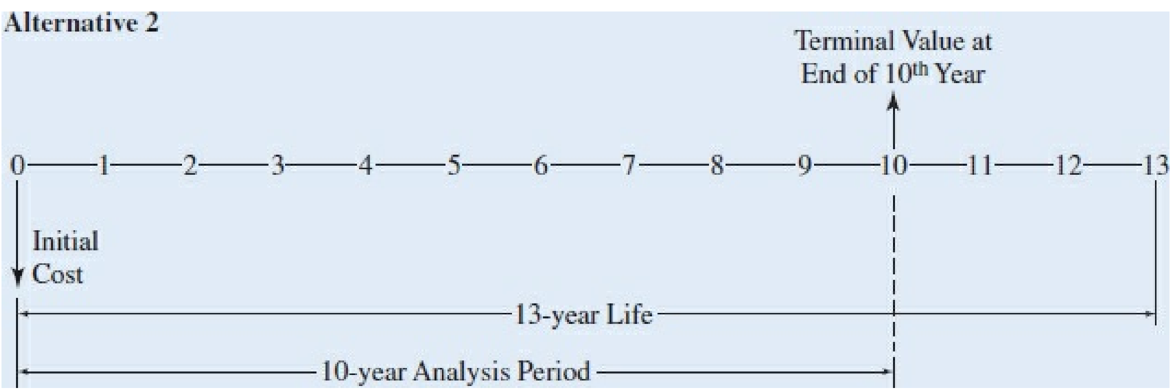
The analysis period in a particular problem may be something other than one of the four we have so far described. It may be equal to the life of the shorter-life alternative, the longer-life alternative, or something entirely different. One must carefully examine the consequences of each alternative throughout the analysis period and, in addition, see what differences there might be in salvage values, and so forth, at the end of the analysis period.

EXAMPLE 6-10 (Example 5–5 Revisited)

Suppose that Alternative 1 has a 7-year life and a salvage value at the end of that time. The replacement cost at the end of 7 years may be more or less than the original cost. If the replacement is retired prior to 7 years, it will have a terminal value that exceeds the end-of-life salvage value. Alternative 2 has a 13-year life and a terminal value whenever it is retired. If the situation indicates that 10 years is the proper analysis period, set up the equations to compute the EUAC for each alternative. Use results from Example 5–5 to compute the results.

SOLUTION





Alternative 1

$$\begin{aligned}
\text{EUAC}_1 &= [\text{Initial cost} + (\text{Replacement cost} - \text{Salvage value})(P/F,\ i, 7) \\
&\quad - (\text{Terminal value})(P/F,\ i, 10)](A/P,\ i, 10) \\
&= 64{,}076(A/P, 8\%, 10) \quad \text{using results from Example 5–5} \\
&= 64{,}076(0.1490) = \$9547
\end{aligned}$$

Alternative 2

$$\begin{aligned}
\text{EUAC}_2 &= [\text{Initial cost} - (\text{Terminal value})(P/F,\ i, 10)](A/P,\ i, 10) \\
&= 68{,}052(A/P, 8\%, 10) \quad \text{using results from Example 5–5} \\
&= 68{,}052(0.1490) = \$10{,}140
\end{aligned}$$

Select Alternative 1.

Example 6–10 requires estimating terminal values for both alternatives due to the project life being set at 10 years. For Alternative 1 the replacement asset is terminated after three years (7 + 3 = 10), and for Alternative 2 termination occurs at year 10. Where do these termination values come from? They are estimates—like most quantities in engineering economy analysis. Estimates are done with the best tools and data available.

ANALYZING LOANS

Loan and bond payments are made by firms, agencies, and individuals. Usually, the payments in each period are constant. Spreadsheets make it easy to:

- Calculate the loan’s amortization schedule
- Demonstrate how a payment is split between principal and interest
- Find the balance due on a loan
- Calculate the number of payments remaining on a loan.

If the interest rate is tabulated, some of these can also be solved with tabulated factors.

Building an Amortization Schedule

As illustrated in previous chapters and Appendix A, an **amortization schedule** lists for each payment period: the loan payment, interest paid, principal paid, and remaining balance. For each period, the interest paid equals the interest rate times the balance remaining from the period before. Then the principal payment equals the payment minus the interest paid. Finally, this principal payment is applied to the balance remaining from the preceding period to calculate the new remaining balance. As a basis for comparison with spreadsheet loan functions, Figure 6–1 shows this calculation for Example 6–11.

EXAMPLE 6-11

An engineer wanted to celebrate graduating and getting a job by spending $2400 on new furniture. Luckily the store was offering 6-month financing at the low interest rate of 6% per year nominal (really $^1/_2$% per month). Calculate the amortization schedule.

SOLUTION

The first step is to calculate the monthly payment:

$$A = 2400(A/P, {}^1\!/\!_2\%, 6) = 2400(0.1696) = \$407.04$$

or

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 6-11	0.5%	6		2400	0	PMT	-$407,03

With this information the engineer can use a spreadsheet like Figure 6–1 to obtain the amortization schedule.

	A	B	C	D	E
1	2400	Initial balance			
2	0.50%	i			
3	6	N			
4	$407.03	Payment	= −PMT(A2,A3,A1)		
5					
6			Principal	Ending	
7	Month	Interest	Payment	Balance	
8	0			2400.00	=A1
9	1	12.00	395.03	2004.97	=D8−C9
10	2	10.02	397.00	1607.97	
11	3	8.04	398.99	1208.98	
12	4	6.04	400.98	807.99	
13	5	4.04	402.99	405.00	
14	6	2.03	405.00	0.00	
15				=$A$4−B14	
16				=Payment−Interest	
17			=$A$2*D13		
18			=rate*previous balance		

FIGURE 6-1 Amortization schedule for furniture loan.

Finding the Balance Due on a Loan

An amortization schedule is one used to calculate the balance due on a loan. A second, easier way is to remember that the balance due equals the present worth of the remaining payments. Interest is paid in full after each payment, so later payments are simply based on the balance due.

EXAMPLE 6-12

A car is purchased with a 48-month, 9% nominal loan with an initial balance of $15,000. What is the balance due halfway through the 4 years?

TABLE SOLUTION

The first step is to calculate the monthly payment, at a monthly interest rate of 1/4%. This equals

$$\text{Payment} = 15{,}000(A/P, 0.75\%, 48) = (15{,}000)(0.0249) = \$373.5$$

Note that there are only three significant digits in the tabulated factor. After 24 payments and with 24 left, the remaining balance equals (P/A, i, $N_{\text{remaining}}$) payment

$$\text{Balance} = (P/A, 0.75\%, 24)\$373.5 = (21.889)(373.28) = \$8176$$

Thus halfway through the repayment schedule, 54.5% (= $8176/$15, 000) of the original balance is still owed.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Monthly payment	0.75%	48		15,000	0	PMT	-$373.28
3	Balance due 1/2 way	0.75%	24	-$373.28		0	PV	$8,170.68

How Much to Interest? How Much to Principal?

For a loan with constant payments, we can answer these questions for any period without the full amortization schedule. For a loan with constant payments, the functions IPMT and PPMT directly answer these questions. For simple problems, both functions have four arguments (i, t, n, $-P$), where t is the time period being calculated. Both functions have optional arguments that permit adding a balloon payment (an F) and changing from end-of-period payments to beginning-of-period payments.

For example, consider Period 4 of Example 6–11. The spreadsheet formulas give the same answer as shown in Figure 6–1.

$$\text{Interest period 4} = \text{IPMT}(0.5\%, 4, 6, -2400) = \$6.04$$
$$\text{Principal payment period 4} = \text{PPMT}(0.5\%, 4, 6, -2400) = \$400.98$$

This can also be solved with factors by first finding the balance due after 3 periods.

$$\text{Interest period 4} = 407.04(P/A, 0.5\%, 3) \times \text{interest rate}$$
$$= 407.04(2.970)(0.005) = \$6.04$$
$$\text{Principal payment} = 407.04 - 6.04 = \$401.00$$

Pay Off Debt Sooner by Increasing Payments

Paying off debt can be a good investment because the investment earns the rate of interest on the loan. For example, this could be 8% for a mortgage, 10% for a car loan, or 19% for a credit card. When one is making extra payments on a loan, the common question is: How much sooner will the debt be paid off? Until the debt is paid off, the savings from any early payments are essentially locked up, since the same payment amount is owed each month. An early payment reduces the principal owed—which reduces future interest which increases future principal payments. This compounding cycle further speeds paying off the loan.

The first reason that spreadsheets and TVM calculators are convenient relates to fractional interest rates. For example, an auto loan might be at a nominal rate of 13% with monthly compounding or 1.08333% per month. The second reason is that the function NPER or the *n* key calculates the number of periods remaining on a loan. Thus we can calculate how much difference is made by one extra payment or by increasing all payments by *x*%. Extra payments are applied entirely to principal, so the interest rate, remaining balance, and payment amounts are all known. $N_{\text{remaining}}$ equals NPER(i, payment, remaining balance) with optional arguments for beginning-of-period cash flows and balloon payments. The signs of the payment and the remaining balance must be different.

EXAMPLE 6-13

Maria has a 7.5% mortgage with monthly payments for 30 years. Her original balance was $100, 000, and she just made her twelfth payment. Each month she also pays into a reserve account, which the bank uses to pay her fire and liability insurance ($900 annually) and property taxes ($1500 annually). By how much does she shorten the loan if she makes an extra *loan* payment today? If she makes an extra *total* payment? If she increases each total payment to 110% of her current total payment?

5-BUTTON SOLUTION

At a nominal rate of 7.5%, the monthly interest rate is 0.625%. The reserve payment for insurance and taxes of $200 [= ($900 + $1500)/12] is added to the loan payment each month to arrive at the total payment. Thus, the remaining balance in E6 is $200 less than in E5.

The extra loan payment lowers the remaining balance by $699.21. The extra total payment lowers the remaining balance by $899.21. A 110% loan payment would shorten the loan by 101.5 (= 348 − 246.5) months, or 8.5 years.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Monthly payment	0.625%	360		1,00,000	0	PMT	-$699.21
3	After 12 payments	0.625%	348	-$699.21		0	PV	$99,078.17
4								
5	Extra loan payment now	0.625%		-$699.21	$98,378.95	0	*n*	339.5
6	Extra total payment now	0.625%		-$699.21	$98,178.95	0	*n*	337.1
7	110% payment to end	0.625%		-$789.14	$99,078.17	0	*n*	246.5

Formulas can be shown for all cells by clicking on icon within the Formula Auditing panel.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer

2	Monthly payment	=0.075/12	360		100000	0	PMT	=PMT(B
3	After 12 payments	=0.075/12	=$C$2-12	=$H$2		0	PV	=PV(B3,
4								
5	Extra loan payment now	=0.075/12		=$H$2	=$H$3+D5	0	*n*	=NPER(
6	Extra total payment now	=0.075/12		=$H$2	=$H$3+D6-200	0	*n*	=NPER(
7	110% payment to end	=0.075/12		=($H$2-200)*1.1+200	=$H$3	0	*n*	=NPER(

ANNUITY DUE

Equivalent uniform annual worths and costs have been defined as end-of-period values or ordinary annuities. However, there are some uniform cash flows that are beginning-of-period cash flows. These cash flows are referred to as an **annuity due**. Lease, rent, insurance, and tuition payments are normally beginning-of-the-period cash flows. Some things must be paid for in advance. Example 6–14 illustrates 3 different ways to convert these beginning-of-period cash flows to present worth or EUAC values.

EXAMPLE 6-14

Find the present worth and equivalent uniform monthly cost of lease payments of $1200 per month for a year. The monthly interest rate is 1%.

SOLUTION

The easiest way to find the present worth of the lease payments is to use the *Type* argument in the PV function. When the *Type* argument is non-zero (a value of 1 is normally used) then beginning-of-period cash flows are assumed for *only* the uniform cash flow. For comparison purposes, the present worth for end-of-period payments is also shown.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 6-14	1%	12	1200			PV	-$13,641	=PV(B2,C2,D2,F2,1)
3	ordinary								
4	annuity	1%	12	1200			PV	-$13,506	=PV(B2,C2,D2,F2)

The second method for calculating the present worth is to recognize that the first payment is already at time 0 and that there are payments at the end of the next 11 months. The beginnings of months 2 to 12 are the same as the endings of months 1 to 11.

$$\begin{aligned} PW &= -1200 - 1200(P/A, 1\%, 11) \\ &= -1200 - 1200(10.368) = -\$13{,}642 \end{aligned}$$

To calculate an equivalent uniform monthly cost or ordinary annuity, the easiest approach is to calculate an end-of-period equivalent for each beginning-of-period value by multiplying by (1 + *i*). This could also be calculated from the present worth value with an (*A*/*P*) factor or PMT function.

$$\begin{aligned} \text{EUmonthlyC} &= 1200(1 + 0.01) = \$1212 \\ \text{PW} &= -1212(P/A, 1\%, 12) = -\$13{,}642 \end{aligned}$$

SUMMARY

Annual cash flow analysis is the second of the three major methods of resolving alternatives into comparable values. One of the commonest applications of equivalent uniform annual worth (EUAW) analysis uses:

$$\text{EUAW} = -P(A/P, i, n) + (R - E) + S(A/F, i, n)$$

Where P = investment cost

R = annual revenues (or savings)

E = annual expenses (or disbursements)

S = salvage value (6-1)

To calculate the EUAW of an overhaul or other midlife cash flow, F_t, you must first find a present or future value, then convert into an EUAW over all *n* periods.

$$\text{EUAW} = F_t(P/F, i, t)(A/P, i, n) \quad (6\text{-}2)$$

$$\text{EUAW} = F_t(F/P, i, n - t)(A/F, i, n) \quad (6\text{-}3)$$

The capital recovery cost is the EUAC of the cost to buy an asset minus its final salvage value. It can be calculated three ways.

- $\text{EUAC} = P(A/P, i, n) - S(A/F, i, n)$ (6-4)
- $\text{EUAC} = (P - S)(A/F, i, n) + Pi$ (6-5)
- $\text{EUAC} = (P - S)(A/P, i, n) + Si$ (6-6)

All three equations give the same answer. This quantity is also known as the *capital recovery cost* of the project.

The relationship between the present worth of cost and the equivalent uniform annual cost is

• EUAC = (PW of cost)(A/P, i, n)

The three annual cash flow criteria are:

Neither input nor output fixed	Maximize EUAW = EUAB - EUAC
For fixed input	Maximize EUAB
For fixed output	Minimize EUAC

In present worth analysis there must be a common analysis period. Annual cash flow analysis, however, allows some flexibility provided the necessary assumptions are suitable in the situation being studied. The analysis period may be different from the lives of the alternatives, and provided the following criteria are met, a valid cash flow analysis may be made.

1. When an alternative has reached the end of its useful life, it is assumed to be replaced by an identical replacement (with the same costs, performance, etc.).
2. The analysis period is a common multiple of the useful lives of the alternatives, or there is a continuing or perpetual requirement for the selected alternative.

If neither condition applies, it is necessary to make a detailed study of the consequences of the various alternatives over the entire analysis period with particular attention to the difference between the alternatives at the end of the analysis period.

There is very little numerical difference between a long-life alternative and a perpetual alternative. As the value of *n* increases, the capital recovery factor approaches *i*. At the limit, $(A/P, i, \infty) = i$.

One of the most common uniform payment series is the repayment of loans. Spreadsheets and TVM calculators are useful in analyzing loans (balance due, interest paid, etc.) for several reasons: they have specialized functions, different loan lives are easy, and any interest rate can be used.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

6-1	Deere Construction just purchased a new track hoe attachment costing \$12,500. The CFO expects the implement will be used for 5 years, at whicl time its salvage value is estimated to be \$4000. Maintenance costs are estimated at \$0 the first year, increasing by \$100 each year thereafter. If a 12% interest rate is used, what is the equivalent uniform annual cost of the implement?

SOLUTION

6-2 The survey firm of Myers, Anderson, and Pope (MAP) LLP is considering the purchase of new GPS equipment. Data concerning the alternative unde consideration are as follows.

First cost	$28,000
Annual income	7,000
Annual costs	2,500
Recalibration at end of Year 4	4,000
Salvage value	2,800

If the equipment has a life of 8 years and MAP's minimum attractive rate o return (MARR) is 5%, what is the annual worth of the equipment?

SOLUTION

6-3 Ronald McDonald decides to install a fuel storage system for his farm that will save him an estimated 6.5 cents/gallon on his fuel cost. He uses an estimated 20,000 gallons/year on his farm. Initial cost of the system is $10,000, and the annual maintenance the first year is $25, increasing by $2 each year thereafter. After a period of 10 years the estimated salvage is $3000. If money is worth 12%, is the new system a wise investment?

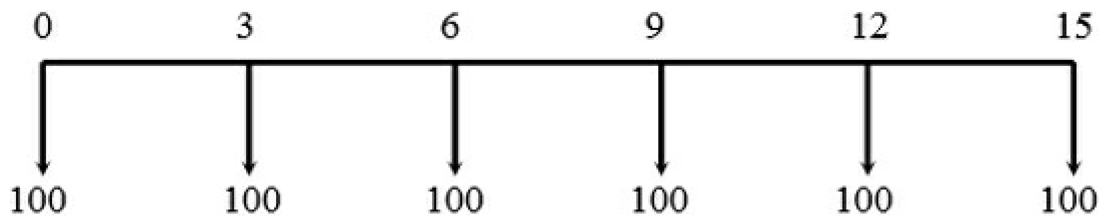
SOLUTION

6-4 The incomes for a business for 5 years are as follows: $8250, $12,600, $9,750, $11,400, and $14,500. If the value of money is 8%, what is the equivalent uniform annual benefit for the 5-year period?

SOLUTION

For the cash flow diagram shown, write the equation that properly calculat the uniform equivalent.

6-

5 

SOLUTION

6-6 A project has a first cost of $75,000, operating and maintenance costs of $10,000 during each year of its 8-year life, and a $15,000 salvage value. If the interest rate is 12%, what is its equivalent uniform annual cost (EUAC)

SOLUTION

6-7 A foundation supports an annual campus seminar by using the earnings of $50,000 gift. It is felt that 10% interest will be realized for 10 years but tha plans should be made to anticipate an interest rate of 6% after that time. W uniform annual payment may be established from the beginning, to fund th seminar at the same level into infinity?

SOLUTION

6-8 A project requires an initial investment of $10,000 and returns benefits of $3000 at the end of every fifth year thereafter. If the minimum attractive ra of return (MARR) is 5%, calculate the equivalent uniform annual worth.

SOLUTION

6-9 At an interest rate of 10% per year, determine the perpetual equivalent ann cost of: $70,000 now, $100,000 at the end of Year 6, and $10,000 per year from the end of Year 10 through infinity.

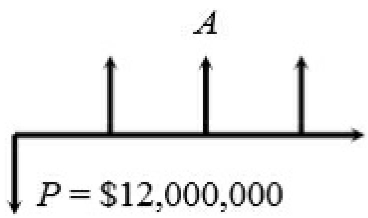
SOLUTION

6-10 A recent engineering graduate makes a donation of $20,000 now and will p $375 per month for 10 years to endow a scholarship. If interest is a nomina 9%, what annual amount can be awarded? Assume that the first scholarshi will be bestowed at the end of the first year.

SOLUTION

Given

6-11



Find: *A*

SOLUTION

6-12 The first cost of a fairly large flood control dam is expected to be $5 millio
The maintenance cost will be $60,000 per year, and a $100,000 outlay will
required every 5 years. At interest of 10%, find the EUAC of the dam proj

SOLUTION

6-13 Determine the equivalent annual worth of a 5-year lease with annual payments of $5000 at 5%.

SOLUTION

6-14 Twenty-five thousand dollars is deposited in a bank trust account that pays
3% interest, compounded semiannually. Equal annual withdrawals are to b
made from the account, beginning one year from now and continuing forev
Calculate the maximum amount of *W*, the annual withdrawal.

SOLUTION

6-15 A tractor costs $32,500 and will be used for 5 years, at which time its
estimated salvage value will be $14,000. Maintenance costs are estimated t
be $500 for the first year, increasing by $150 each year thereafter. If i = 6%

what is the equivalent uniform annual cost (EUAC) for the tractor?

SOLUTION

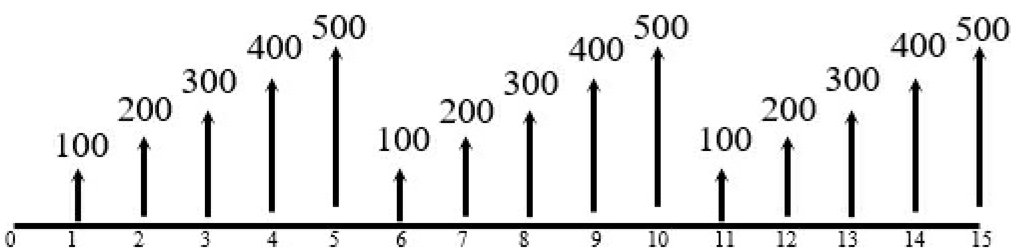
6-16 If in the last week of February 2016 Ellen won $250,000 and invested it by March 1, 2016, in a “sure thing” that paid 8% interest, compounded annual what uniform annual amount can she withdraw on the first of March for 15 years starting in 2022?

SOLUTION

6-17 A machine having a first cost of $20,000 is expected to save $1500 in the first year of operation, and the savings should increase by $200 every year until (and including) the 9th year; thereafter, the savings will decrease by $150 until (and including) the 16th year. Using equivalent uniform annual worth, is this machine economical? Assume a MARR of 10%.

SOLUTION

6-18 Calculate the equivalent uniform annual cost of the following schedule of payments.



SOLUTION

6-19 The initial cost of a van is $22,800; its salvage value after 5 years will be $8500. Maintenance is estimated to be a uniform gradient amount of $225 year (with no maintenance costs the first year), and the operation cost is estimated to be 36 cents/mile for 400 miles/month. If money is worth 6%, what is the approximate equivalent uniform annual cost (EUAC) for the va expressed as a monthly cost?

SOLUTION

6-20 Granny Gums has established a scholarship at the Martin College of Dentistry. She will make deposits into an endowment account that pays 12 per year based on the following schedule.

Year:	0	1	2	3	4	5	6
Deposit amount ($):	100	90	80	70	60	50	40

If the first scholarship is to be awarded 1 year after the first deposit is mad and thereafter the award will be given indefinitely, what is the scholarship amount?

SOLUTION

6-21 A proposed steel bridge has an indefinite life. The initial cost of the bridge $3,750,000, and annual maintenance costs are estimated to be $25,000. Th bridge deck will be resurfaced every 10 years for $900,000, and anticorros paint will be applied every 5 years for $250,000. If the interest rate is 8%, what is the EUAC?

If 650,000 axles will cross the bridge each year, what approximate toll per axle should be charged? Give your answer to the nearest nickel.

SOLUTION

6-22 A college has been willed $100,000 to establish a permanent scholarship. I funds are invested at 6% and all funds earned are disbursed yearly, what w be the value of the scholarship in the sixth year of operation?

SOLUTION

6-23 Calculate the annual worth of a quarterly lease payment of $500 at 8% interest.

SOLUTION

6-24 An airport expansion that is expected to be used indefinitely is under way a Jackson Hole Metro Airport. Land acquisition and major earthworks that w last as long as the airport is used are expected to cost $600 million. Termin construction that will last 20 years is budgeted at $200 million. (Assume th the terminal is to be identically replaced every 20 years.) Runway construction will cost $150 million. The runways will also be used indefinitely, with repaving required every 10 years at a cost of $10 million The operating and maintenance costs are estimated to be $15 million per year. What is the annual cost of the project if i = 5% and the airport will be used indefinitely?

SOLUTION

6-25 Green County is planning to construct a bridge across the south branch of Carey Creek to facilitate traffic flow though Clouser Canyon. The first cos for the bridge will be $9,500,000. Annual maintenance and repairs the first year of operation, estimated to be $10,000, are expected to increase by $10 each year thereafter. In addition to regular maintenance, every 5 years the roadway will be resurfaced at a cost of $750,000, and the structure must be painted every 3 years at a cost of $100,000. If Green County uses 5% as its cost of money and the bridge is expected to last for 20 years, what is the EUAC?

SOLUTION

6-26 Two alternative investments are being considered. What is the minimum uniform annual benefit that will make Investment *B* preferable to Investme *A*? Assume that interest is 10%.

Year	A	B
0	–$500	–$700
1–5	+150	?

SOLUTION

Consider two investments:

1. Invest $1000 and receive $110 at the end of each month for the next 10 months.

6-27 2. Invest $1200 and receive $130 at the end of each month for the next 10 months.

If this were your money, and you wanted to earn at least 12% interest on it which investment would you make, if any? Solve the problem by annual ca flow analysis.

SOLUTION

Morton and Moore LLC (M^2) is trying to decide between two machines tha are necessary in its manufacturing facility. If M^2 has a MARR of 10%, whi of the following machines should be chosen?

6-28

	Machine *A*	Machine *B*
First cost	$45,000	$24,000
Annual operating costs	31,000	35,000
Overhaul in Years 2 and 4	—	6,000
Overhaul in Year 5	12,000	—
Salvage value	10,000	8,000
Useful life	8 years	6 years

SOLUTION

A land surveyor just starting in private practice needs a van to carry crew a equipment. He can lease a used van for $8000 per year, paid at the beginni of each year, in which case maintenance is provided. Alternatively, he can
6-29 buy a used van for $16,000 and pay for maintenance himself. He expects to keep the van for 3 years, at which time he would sell it for an anticipated $3500. Given a MARR of 6%, what is the most the surveyor should pay fo uniform annual maintenance to make it worthwhile to buy the van instead o leasing it?

SOLUTION

Assuming monthly payments, which would be the better financing plan on the same $19,000 car?

6-30

Plan A:	6% interest on the full amount for 48 months
Plan B:	a $2500 rebate (discount) and 12% interest on the remaining amou for 48 months

SOLUTION

6-31 The town of Dry Hole needs an additional supply of water from Duck Cree The town engineer has selected two plans for comparison. The gravity plar would divert water at a point 10 miles up Duck Creek and carry it through pipeline by gravity to the town. A system using a pumping station would divert water at a point closer to town and pump it into the town. The pump plant would be built in two stages, with 75% of its capacity installed initial and the remaining 25% installed 10 years later. The engineer has assumed that each plan will last 40 years and be worthless at the end of its life. Use following data and an interest rate of 8% to find the maximum that should paid for the gravity plan.

	Gravity	Pumping
Initial investment	$??????	$1,800,000
Completion cost in 10th year		350,000
Annual operating and maintenance costs	$10,000	$25,000
Annual power costs:		
Average costs the first 10 years	0	$ 50,000
Average costs the next 30 years	0	$100,000

SOLUTION

Fitzgerald, Ivy, Garcia, Nichols, Eudy, Williams, Thomas, Owens, and Na (FIGNEWTON) Inc. must replace its fig-crushing equipment. The alternatives under consideration are presented below.

Alternative	First Cost	Net Annual Costs	Useful Life

6-32

A	$170,500	$14,675	5 years
B	205,000	17,000	7 years
C	242,500	16,350	8 years
D	290,000	14,825	10 years

FIGNEWTON's cost of capital is 8%. Which alternative should be chosen'

SOLUTION

6-33

The Tennessee Department of Highways is trying to decide whether it shou "hot-patch" a short stretch of an existing highway or resurface it. If the hot patch method is chosen, approximately 500 cubic meters of material woulc be required at a cost of $800/cubic meter (in place). If hot-patched, the shoulders will have to be improved at the same time at a cost of $24,000. T shoulders must be maintained at a cost of $3000 every 2 years. The annual cost of routine maintenance on the patched road is estimated to be $6000.

Alternatively, the state can resurface the road at a cost of $500,000. If maintained properly, at a cost of $2000 per year beginning in the second ye the surface will last for 10 years. The shoulders would require reworking a the end of the fifth year at a cost of $15,000. Regardless of the method selected, the road will be completely rebuilt in 10 years. At an interest rate 9%, which alternative should be chosen?

SOLUTION

A semiconductor manufacturer has been ordered by the city to stop discharging acidic waste liquids into the city sewer system. Your analysis shows that the company should select one of the following three systems.

6-34

System	Installed Cost	Annual Operating Cost	Salvage Value
CleanH_2O	$30,000	$6000	$ 2,000

Acid Free	35,000	5000	5,000
Evergreen	80,000	1000	40,000

If the system is expected to be used for 20 years and to last that long, as we
and money is worth 8%, which system should be purchased?

SOLUTION

Consider Projects *A* and *B*. Which project would you approve if a project must be selected? The expected period of service is 15 years, and the intere rate is 10%.

6-35

	Project *A*	Project *B*
Initial cost	$50,000	$75,000
Annual operating costs	15,000	10,000
Annual repair costs	5,000	3,000
Salvage value	5,000	10,000

SOLUTION

6-36

The construction costs and annual maintenance costs of two alternatives fo canal are given. a. Use equivalent uniform annual cost (EUAC) analysis to decide which alternative you would recommend. Assume 7% interest and infinite life. b. What is the capitalized cost of maintenance for the alternati you choose?

	Alternative *A*	Alternative *B*
Construction cost	$25,000,000	$50,000,000
Annual maintenance costs	3,500,000	2,000,000

SOLUTION

The manager of Cats-N-The-Pond Inc. is trying to decide between two alternative designs for an aquacultural facility. Both facilities produce the same number of fish for sale. The first alternative costs $250,000 to build a has a first-year operating cost of $110,000. Operating costs are estimated t increase by $10,000 per year for each year after the first.

6-37 The second alternative costs $450,000 to build and has a first-year operatin cost of $40,000 per year, escalating at $5000 per year for each year after th first. The estimated life of both plants is 10 years, and each has a salvage value that is 10% of construction cost.

Assume an 8% interest rate. Use equivalent uniform annual cost (EUAC) analysis to determine which alternative should be selected.

SOLUTION

The plant engineer of a major food processing corporation is evaluating alternatives to supply electricity to the plant. He will pay $3 million for electricity purchased from the local utility at the end of the first year and estimates that this cost will increase thereafter at $300,000 per year. He desires to know if he should build a 4000-kilowatt power plant. His operati costs (other than fuel) for such a power plant are estimated to be $130,000 per year. He is considering two alternative fuels:

6-38 a. Wood: Installed cost of the power plant is $1200/kW. Fuel consumption 30,000 tons per year. Fuel cost for the first year is $20/ton and is estimated increase at a rate of $2/ton for each year after the first. No salvage value.
b. Oil: Installed cost is $1000/kW. Fuel consumption is 46,000 barrels per year. Fuel cost is $34 per barrel for the first year and is estimated to increa at $1/barrel per year for each year after the first. No salvage value.

If interest is 12%, and the analysis period is 10 years, which alternative should the engineer choose? Solve the problem by equivalent uniform annu cost analysis (EUAC).

SOLUTION

Two alternatives are being considered by a food processor for the warehousing and distribution of its canned products in a sales region. Thes canned products come in standard cartons of 24 cans per carton. The two alternatives are as follows.

Alternative *A*: To have its own distribution system. The administrative cos are estimated at $43,000 per year, and other general operating expenses are calculated at $0.009 per carton. A warehouse will have to be purchased, at cost of $300,000.

6-39 Alternative *B*: To sign an agreement with an independent distribution company that is asking a payment of $0.10 per carton distributed.

Assume a study period of 10 years and that the warehouse can be sold at th end of this period for $200,000.

a. Which alternative should be chosen, if management expects that the number of cartons to be distributed will be 600,000 per year?
b. Find the minimum number of cartons per year that will make the alternative of having a distribution system (Alt. *A*) more profitable than to sign an agreement with the distribution company (Alt. *B*).

SOLUTION

6-40 Dorf Motors Manufacturing must replace one of its tow motors. The net present cost of Alternative *A* is $8956, Alternative *B* is $5531, and Alternative *C* is $4078. Alternative *A* is expected to last for 12 years; Alternative *B* has an expected life of 7 years; and Alternative *C* is expectec last for 5 years. If Dorf's MARR is 5%, which Alternative (if any) should l chosen using EUAC.

SOLUTION

6-41 According to the manufacturers' literature, the costs of running automatic grape peelers, if maintained according to the instruction manuals, are as follows.

	Slippery	Grater
First cost	$500	$300
Maintenance	$100 at end	Year 1 $ 0
	of Years 2,	2 50
	4, 6, and 8	3 75

		4 100
		5 125
Useful life	10 years	5 years

Which alternative is preferred if MARR = 8%?

SOLUTION

6-42 Two options are available for the reroofing of Pinkley Pickles Inc. The firs option, a traditional tar and gravel roof, costs $75,000 and has a life of 15 years. The second option, a neoprene membrane roof, costs $85,000 and ha a life of 20 years. The company expects to occupy the building for 50 year Using an interest rate of 5% and annual worth analysis, which roof should chosen?

SOLUTION

6-43 The following alternatives describe possible projects for the use of a vacan lot. In each case the project cost includes the purchase price of the land.

	Parking Lot	Gas Station
Investment cost	$50,000	$100,000
Annual income	35,000	85,000
Annual operating expenses	25,000	$70,000 in Year 1, then increasing by 1000/yr
Salvage value	10,000	10,000
Useful life	5 years	10 years

a. If the minimum attractive rate of return (MARR) equals 18%, what shou be done with the land?
b. Is it possible that the decision would be different if the MARR were higl

than 18%? Why or why not? (No calculations necessary.)

SOLUTION

6-44 Given the following information about possible investments being conside by the ABC Block Company, what is the best choice at a minimum attracti rate of return (MARR) of 10%?

	A	B
Investment cost	$5000	$8000
Annual benefits	1200	800
Useful life	5 years	15 years

SOLUTION

6-45 You are considering purchasing the Press-o-Matic or Steam-It-Out automa ironing system to allow you to handle more dry cleaning business. Both machines have the same cost, $5000. The Press-o-Matic will generate a positive cash flow of $1300 per year for 5 years and then be of no service (salvage value. The Steam-It-Out will generate a positive cash flow of $800 per year for 10 years and then be of no service or salvage value. You plan t be in the dry cleaning business for the next 10 years. How would you inves the $5000 you have in your hand if you feel the time value of money is wo the same as your high-interest bank account offers, which is

a. 8%?
b. 12%?

SOLUTION

6-46 Data for Machines *X* and *Y* are listed. With an interest rate of 8%, and base upon equivalent uniform annual cost (EUAC), which machine should be selected?

	X	Y
First cost	$5000	$10,000
Annual maintenance	500	200

Salvage value	600	1,000
Useful life	5 years	15 years

SOLUTION

Assuming a 10% interest rate, determine which alternative should be selected.

6-47

	A	B
First cost	$5300	$10,700
Uniform annual benefit	1800	2,100
Salvage value	0	200
Useful life	4 years	8 years

SOLUTION

A company must decide whether to buy Machine *A* or Machine *B*. After 5 years, Machine *A* will be replaced with another *A*.

6-48

	Machine *A*	Machine *B*
First cost	$10,000	$20,000
Annual maintenance	1,000	0
Salvage value	10,000	10,000
Useful life	5 years	10 years

With the minimum attractive rate of return (MARR) = 10%, which machin should be purchased?

SOLUTION

6-49 Your company bought equipment for $25,000 by paying 10% down and th rest on credit. The credit arrangement is for 5 years at 7% interest. After making 20 payments, you want to pay off the loan. How much do you owe

SOLUTION

6-50 A personal loan for $20,000 has an interest rate of 12% over 5 years with monthly payments. What is the monthly uniform payment? What is owed o the loan after 3 years?

SOLUTION

6-51 You are interested in leasing a car for $425 per month, due at the beginnin of each month. Using an interest rate of 4% annually, what is the present worth of a one-year lease for this car?

SOLUTION

6-52 Your uncle paid $300,000 for an annuity that will pay $2200 per month fo life. The insurance company that sold the annuity says that they are paying interest of 4%. How long (in months) do they plan on paying the annuity?

SOLUTION

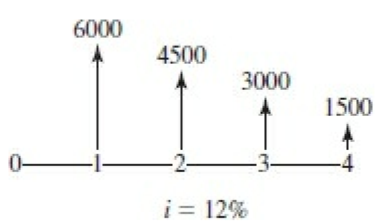
PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Annual Calculations

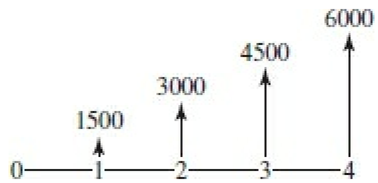
Compute the EUAB for these cash flows.

6-1



Compute the EUAB for these cash flows based on a 10% interest rate.

6-2 A



6-3 G

A production machine costs $40,000 and has a 7-year useful life. At the end of 7 years, it can be sold for $10,000.
(*a*) If interest is 10% compounded quarterly, what is the equivalent annual cost of ownership of this machine over its useful life?
(*b*) The company that buys the 7-year old machine uses it for an additional 5 years and then wants to responsibly dispose of it. What *green* options does the company have? Research and report to the firm on what happens to most industrial machinery after its useful life.

6-4 A

An electronics firm invested $50,000 in a precision inspection device. They spend $1500 each year to operate and maintain it. An additional $750 is spent in years 2 and 4 for software updates. At the end of 5 years, the firm changed their inspection procedure, eliminating the need for the device. The purchasing agent was very fortunate to sell the inspection device for $40,000, the original price. Compute the equivalent uniform annual cost during the 5 years the device was used. Assume interest at 10% per year.

6-5

A firm is buying an adjacent 10-acre parcel for a future plant expansion. The price has been set at $30,000 per acre. The payment plan is 25% down, and the balance 2 years from now. If the transaction interest rate is 9% per year, what are the two payments?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

6-6 A

When he started work on his twenty-first birthday, D. B. Cooper decided to invest money each month with the objective of becoming a millionaire by the time he reaches age 60. If he expects his investments to yield 9% per annum, compounded monthly, how much should he invest each month?

The average age of engineering students at graduation is a little over

6-7 23 years. This means that the working career of most engineers is almost exactly 500 months. How much would an engineer need to save each month to accrue $5 million by the end of her working career? Assume a 9% interest rate, compounded monthly.

6-8 An engineer wishes to have $3 million by the time he retires in 35 years. Assuming 8% nominal interest, compounded continuously, what annual sum must he set aside?

6-9 G To reduce her personal carbon footprint, Zooey is buying a new hybrid. She has negotiated a price of $21,900 and will trade in her old car for $2350. She will put another $850 with it and borrow the remainder at 6% interest compounded monthly for 4 years. How large will her monthly payments be?

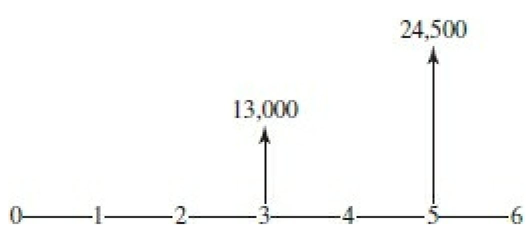
Contributed by Paul R. McCright, University of South Florida

6-10 A Zwango Plus Manufacturing expects that fixed costs of keeping its Zephyr Hills Plant operating will be $2.1M this year. If the fixed costs increase by $125,000 each year, what is the EUAC for a 15-year period? Assume the interest rate is 10%.*Contributed by Paul R. McCright, University of South Florida*

6-11 A firm purchased some equipment at a very favorable price of $30,000. The equipment reduced costs by $1000 per year during 8 years of use. After 8 years, the equipment was sold for $40,000. Assuming interest at 9%, did the equipment purchase prove to be desirable?

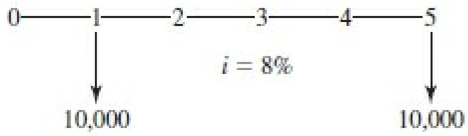
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

6-12 A If i = 6%, compute the EUAB over 6 years that is equivalent to the two receipts shown.



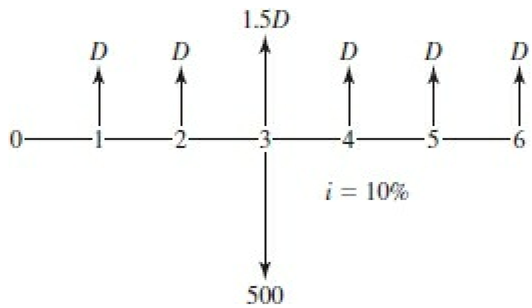
Compute the EUAC for these cash flows.

6-13



For the diagram, compute the value of *D* that results in a net equivalent uniform annual worth (EUAW) of 0.

6-14
A



6-15

Amanda and Blake have found a house, which owing to a depressed real estate market costs only $201,500. They will put $22,000 down and finance the remainder with a 30-year mortgage loan at 3% interest (compounded monthly).
(*a*) How much is their monthly loan payment?
(*b*) How much interest will they pay in the second payment?
(*c*) They will also have the following expenses: property taxes of $2100, homeowners' insurance of $1625, and $290 mortgage insurance (in case one of them dies before the loan is repaid, a requirement of the bank). These annual amounts are paid in 12 installments and added to the loan payment. What is their full monthly cost?
(*d*) If they can afford $1200 per month, can Amanda and Blake afford this house?

Contributed by Paul R. McCright, University of South Florida

6-16
A

How much should a new graduate pay in 15 equal annual payments, starting 3 years from now, in order to repay a $33,500 loan he has received today? The interest rate is 6% per year.
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

6-17 A firm manufactures and sells high quality business printers and ink toners. Each printer sells for $650 and each toner for $100. The average user keeps the printer for 5 years and consumes 4 toners every year. In response to a recent significant drop in printer sales (which will reduce future toner sales as well) the firm wants to lower the printer price to $500. Assume that income from toner sales occurs at year-end and the firm's cost of capital is 10%. How much of an increase is needed in the toner price to cover the loss in printer price?
Contributed by Yasser Alhenawi, University of Evansville

6-18 A

A motorcycle is for sale for $20,200. The dealer is willing to sell it on the following terms:

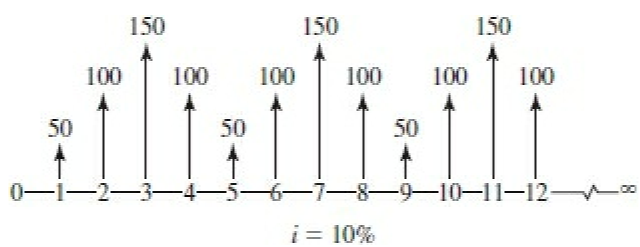
No down payment; pay $300 at the end of each of the first 6 months; pay $500 at the end of each month after that until the loan has been paid in full.

At a 12% annual interest rate compounded monthly, how many $500 payments will be required?

6-19 A couple is saving for their newborn daughter's college education. She will need $25,000 per year for a four-year college program, which she will start when she is 18. What uniform deposits starting 2 years from now and continuing through year 17 are needed, if the account earns interest at 4%?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

6-20 A

There is an annual receipt of money that varies from $50 to $150 in a fixed pattern that repeats forever. If interest is 8%, compute the EUAB, also continuing forever, that is equivalent to the fluctuating disbursements.



6-21 An engineer has a fluctuating future budget for the maintenance of a particular machine. During each of the first 5 years, $10, 000 per year will be budgeted. During the second 5 years, the annual budget will be $15, 000 per year. In addition, $5000 will be budgeted for an overhaul of the machine at the end of the fourth year, and again at the end of the eighth year. What uniform annual expenditure would be equivalent, if interest is 8% per year?

6-22 A

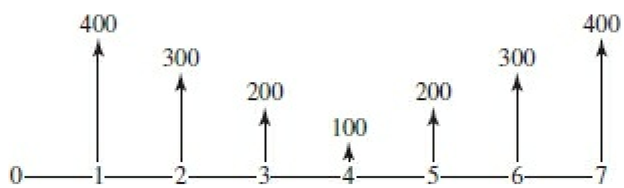
The maintenance foreman of a plant in reviewing his records found that maintenance costs on a large press had increased with sales of a product that will decline in the future.

4 years ago	$550
3 years ago	650
2 years ago	750
Last year	850

He believes that maintenance will be $950 this year and will decrease by $50 yearly for 4 years. What will be the equivalent uniform annual maintenance cost for the 9-year period? Assume interest at 7%.

6-23

If interest is 12%, what is the EUAB?



6-24 A

A machine has a first cost of $126,000, an annual operation and maintenance cost of $1850, a life of 8 years, and a salvage value of $27,000. At the end of Years 3 and 6, it requires a major service, which costs $14,000 and $16,500, respectively. What is the equivalent uniform annual cost of owning and operating this particular machine if interest is 6%?

6-25

LaQuesha Jackson has made a considerable fortune. She wishes to start a perpetual scholarship for engineering students at her school. The scholarship will provide a student with an annual stipend of $10,000 for each of 4 years (freshman through senior), plus an additional $5000 during the senior year to cover job search expenses. Assume that students graduate in 4 years, a new award is given every

4 years, and the money is paid at the beginning of each year with the first award at the beginning of Year 1. The interest rate is 10%.
(*a*) Determine the equivalent uniform annual cost (EUAC) of providing the scholarship.
(*b*) How much money must LaQuesha donate?

6-26 A Linda O'Shay deposited \$50,000 in a savings account as a perpetual trust. She believes the account will earn 8*%* annual interest during the first 10 years and 4*%* interest thereafter. The trust is to provide a uniform end-of-year scholarship at the university. What uniform amount could be used for the student scholarship each year, beginning at the end of the first year and continuing forever?

6-27 Curtis Lowe must pay his property taxes in two equal installments on December 1 and April 1. The two payments are for taxes for the fiscal year that begins on July l and ends the following June 30. Curtis purchased a home on September 1. Assuming the annual property taxes remain at \$3400 per year for the next several years, Curtis plans to open a savings account and to make uniform monthly deposits the first of each month. The account is to be used to pay the taxes when they are due.

To open the account, Curtis deposits a lump sum equivalent to the monthly payments that will not have been made for the first year's taxes. The savings account pays 4*%* interest, compounded quarterly (March 31, June 30, September 30, and December 31). How much money should Curtis put into the account when he opens it on September 1? What uniform monthly deposit should he make from that time on?

6-28 A Your company must make a \$500,000 balloon payment on a lease 2 years and 9 months from today. You have been directed to deposit an amount of money quarterly, beginning today, to provide for the \$500,000 payment. The account pays 4% per year, compounded quarterly. What is the required quarterly deposit? *Note:* Lease payments are due at the beginning of the quarter.

If the owner earns 5% interest on her investments, determine the equivalent annual cost of owning a car with the following costs (EOY = end of year).

6-29

Initial down payment = $2200
Annual payments = $5500, EOY1–EOY4
Prepaid insurance = $1500, growing 8% annually
Gas & oil & minor maintenance = $2000, growing 10% annually
Replacement tires = $650 at EOY4 & $800 at EOY8
Major maintenance = $2400 at EOY5
Salvage value = $3750 at EOY9
Contributed by Paul R. McCright, University of South Florida

Annual Comparisons

6-30 A G

The Johnson Company pays $1700 a month to a trucker to haul wastepaper and cardboard to the city dump. The material could be recycled if the company were to buy a $48,000 hydraulic press baler and spend $21,000 a year for labor to operate the baler. The baler has an estimated useful life of 15 years and no salvage value. Strapping material would cost $1500 per year for the estimated 600 bales a year that would be produced. A wastepaper company will pick up the bales at the plant and pay Johnson $27 per bale for them. Use an annual cash flow analysis and an interest rate of 8% to recommend whether it is economical to install and operate the baler.

6-31 G

A municipal power plant uses natural gas from an existing pipeline at an annual cost of $40,000 per year. A new pipeline would initially cost $100,000, but it would reduce the annual cost to $10,000 per year.
(*a*) Assume an analysis period of 25 years and no salvage value for either pipeline. The interest rate is 6%. Using the equivalent uniform annual cost (EUAC), should the new pipeline be built?
(*b*) The power plant uses natural gas. What are some of the non-economic benefits to the municipality of this energy source over others? Develop three primary advantages and disadvantages.

A construction firm needs a new small loader. It can be leased from the dealer for 3 years for $5500 per year including all maintenance, or it can be purchased for $27,000. The firm expects the loader to have a

6-32 A salvage value of \$8500 after 8 years. The maintenance will be \$650 the first year and then it will increase by \$250 each year. The firm's interest rate is 10% per year. Compare the EUACs for leasing and buying the loader.
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

6-33 Hinson's Homegrown Farms needs a new irrigation system. System one will cost \$145,000, have annual maintenance costs of \$10,000, and need an overhaul at the end of year six costing \$30,000. System two will have first-year maintenance costs of \$5000 with increases of \$500 each year thereafter. System two would not require an overhaul. Both systems will have no salvage value after 12 years. If Hinson's cost of capital is 4%, using annual worth analysis determine the maximum Hinson's should be willing to pay for system two.
Contributed by Ed Wheeler, University of Tennessee at Martin

6-34 A When he financed his firm's building, Al Silva borrowed \$280,000 at 9% interest to be repaid in 30 equal annual end-of-year payments. After making 10 payments, Al found he could refinance the loan at 8% interest for the remaining 20 years.

To refinance the loan, Al must pay the balance due plus a prepayment penalty charge of 4% of the balance due, and he must pay a \$2000 service charge on the new loan. All payments are financed by the new loan. Should Al refinance the loan, assuming that he will keep the firm's building for the next 20 years? Use an annual cash flow analysis.

6-35 Claude James, a salesman, needs a new car for business use. He expects to be promoted to a supervisory job after 3 years, and he will no longer be "on the road." The company reimburses salesmen each month at the rate of 55 per mile driven. Claude believes he should use a 12% interest rate. If the car could be sold for \$7500 at the end of 3 years, which method should he use to obtain it?

A. Pay cash: the price is \$26,000.

B. Lease the car: the monthly charge is \$700 on a 36-month lease, payable at the end of each month; at the end of the 3-year period, the car is returned to the leasing company.

C. Lease the car with an option to buy at the end of the lease: pay $720 a month for 36 months; at the end of that time, Claude could buy the car, if he chooses, for $7000.

An RV manufacturer estimates that annual profits will increase if a mobile model is built and taken to trade shows to market their product line. A finance and engineering team has looked at the issue and has developed two options:

1. A large model can be developed at a cost of $75,000, and it should increase annual profits by $25,000 per year.

6-36 A

2. A small model can be developed for $40,000, but it will only increase annual profits by $14,500 per year.

The salvage value for the large model is $6000 more than the small model after their common useful life of 6 years, and it costs $1500 more a year to transport to the trade shows. The manufacturer uses an interest rate of 18%. Use an annual worth comparison to make a recommendation on which, if either, option should be chosen.

6-37 E

The town of Dry Gulch needs more water from Pine Creek. The town engineer has selected two plans for comparison: a *gravity plan* (divert water at a point 10 miles up Pine Creek and pipe it by gravity to the town) and a *pumping plan* (divert water at a point closer to town). The pumping plant would be built in two stages, with half-capacity installed initially and the other half installed 10 years later.

(*a*) Use an annual cash flow analysis to find which plan is more economical. The analysis will assume a 40-year life, 10% interest, and no salvage value.

	Gravity	Pumping
Initial investment	$2,800,000	$1,400,000
Investment in 10th year	None	200,000
Operation and maintenance	10,000/yr	25,000/yr
Power cost		
Average first 10 years	None	50,000/yr

Average next 30 years	None	100,000/yr

(*b*) The situation described in this problem is a common one for cities that are upstream and downstream of a water supply. Search using the term "water ethics" and write a short paragraph that describes what you find from the perspectives of Dry Gulch and Pine Creek.

6-38 A — A manufacturer is considering replacing a production machine tool. The new machine, costing $40,000, would have a life of 5 years and no salvage value, but would save the firm $5000 per year in direct labor costs and $2000 per year in indirect labor costs. The existing machine tool was purchased 4 years ago at a cost of $40,000. It will last 5 more years and will have no salvage value. It could be sold now for $15,000 cash. Assume that money is worth 10% and that differences in taxes, insurance, and so forth are negligible. Use an annual cash flow analysis to determine whether the new machine should be purchased.

6-39 G — Two possible routes for a power line are under study. In both cases the power line will last 15 years, have no salvage value, have annual property taxes of 2% of first cost, and have a yearly power loss of $500/km.

	Around the Lake	Under the Lake
Length	15 km	5 km
First cost	$9000/km	$25,000/km
Maintenance	$200/km/yr	$300/km/yr

(*a*) If 6% interest is used, should the power line be routed around the lake or under the lake?
(*b*) Search using the term "environmental impact of power lines" and write a short paragraph on what you learn about this topic. What aspects of the environment seem to be most affected, and what policies and technologies are being used to eliminate or mitigate concerns?

6-40 — An oil refinery must now begin sending its waste liquids through a costly treatment process before discharging them. The engineering department estimates costs at $450,000 for the first year. It is estimated that if process and plant alterations are made, the waste treatment cost will decline $43,000 each year. As an alternate, a

A specialized firm, Hydro-Clean, has offered a contract to process the
G waste liquids for 15 years for $225,000 per year. Either way, there should be no need for waste treatment after 15 years. Use an 10% interest rate and annual cash flow analysis to determine whether the Hydro-Clean offer should be accepted.

Bill Anderson buys a car every 2 years as follows: initially he makes a down payment of $12,000 on a $30,000 car. The balance is paid in 24 equal monthly payments with annual interest at 12%. When he has made the last payment on the loan, he trades in the 2-year-old car for $12,000 on a new $30,000 car, and the cycle begins over again.

Doug Jones decided on a different purchase plan. He thought he would be better off if he paid $30,000 cash for a new car. Then he would make a monthly deposit in a savings account so that, at the end
6-41 of 2 years, he would have $18,000 in the account. The $18,000 plus the $12,000 trade-in value of the car will allow Doug to replace his 2-year-old car by paying $30,000 for a new one. The bank pays 3% interest, compounded monthly.

(*a*) What is Bill's monthly loan payment?
(*b*) What is Doug's monthly savings account deposit?
(*c*) Why is Doug's monthly savings account deposit smaller than Bill's payment?

One of two mutually exclusive alternatives must be selected. Alternative *A* costs $30,000 now for an annual benefit of $8450.
6-42 Alternative *B* costs $50,000 now for an annual benefit of $14,000.
A Using a 15% nominal interest rate, compounded continuously, which do you recommend? Solve by annual cash flow analysis with 5 year lives.

North Plains Biofuels (NPB) has negotiated a contract with an oil firm to sell 150,000 barrels of ethanol per year, for 10 years beginning in end of year (EOY) 4. The oil firm will pay NPB $10M annually beginning from EOY0 to EOY3 and then $110 per barrel.
(*a*) If NPB uses an interest rate of 15%, which method should be used to produce the biofuels? Annual O&M costs increase 2% per year and raw materials costs 3% per year.

6-43 G

	Corn	Algae
Purchase of land (EOY0)	$1,900,000	$ 3,800,000
Facility construction (EOY1)	$5,300,000	$7,100,000
Annual O&M (EOY4)	$2,450,000	$2,800,000
Raw materials (EOY4)	$1,500,000	$250,000
Salvage value (EOY13)	$3,000,000	$3,600,000

(*b*) What are biofuels? How are they used in the automotive industry, and what are the pros and cons of this use? What are some of the important non-auto industry uses of biofuels?

Contributed by Paul R. McCright, University of South Florida

6-44 A

Which car has a lower EUAC if the owner can earn 5% in his best investment?

Contributed by Paul R. McCright, University of South Florida

	Midsize	Hybrid
Initial cost	$19,200	$25,500
Annual maintenance	1,000	1,500
Annual gas & oil	2,500	1,200
(increasing 15% yearly)		
Salvage value (Year 8)	8,000	10,000

Different Lives

6-45

A pump is needed for 10 years at a remote location. The pump can be driven by an electric motor if a power line is extended to the site. Otherwise, a gasoline engine will be used. Use an annual cash flow analysis and a 6% interest rate. How should the pump be powered?

	Gasoline	Electric
First cost	$2400	$9000
Annual operating cost	1200	250

Annual maintenance	300	50
Salvage value	300	600
Life, in years	5	10

A firm is choosing between machines that perform the same task in the same time. Assume the minimum attractive return is 7%. Which machine would you choose?

6-46
A

	Machine *X*	**Machine *Y***
First cost	$7500	$9250
Estimated life, in years	7	12
Salvage value	0	$2000
Annual maintenance cost	0	165

Alternative *B* may be replaced with an identical item every 20 years at the same cost and annual benefit. Using a 9% interest rate and an annual cash flow analysis, which alternative should be selected?

6-47

	A	***B***
Cost	$100,000	$150,000
Uniform annual benefit	30,000	45,000
Useful life, in years	∞	20

Road Runner LLC (RRL) is considering three alternate routes in the desert. RRL uses a MARR of 5%. Using equivalent annual worth over the least common multiple horizon, which choice is best?

6-48
A

	Route 105	**Route 205**	**Route 305**
First cost	$520,000	$460,000	$395,000
Savings/year	135,000	100,000	95,000
Life	5	7	6

Contributed by Ean Ng and Gana Natarajan, Oregon State University

A suburban taxi company is considering buying taxis with diesel engines instead of gasoline engines. The cars average 80,000 km a year. Use an annual cash flow analysis to determine the more economical choice if interest is 6%.

6-49

	Diesel	Gasoline
Vehicle cost	$24,000	$19,000
Useful life, in years	5	4
Fuel cost per liter	88¢	92¢
Mileage, in km/liter	16	11
Annual repairs	$ 900	$ 700
Annual insurance premium	1,000	1,000
End-of-useful-life resale value	4,000	6,000

6-50 A

Assuming that Alternatives *B* and *C* are replaced with identical units at the end of their useful lives, and an 8% interest rate, which alternative should be selected? Use an annual cash flow analysis.

	A	*B*	*C*
cost	$12,500	$15,000	$17,500
Annual benefit	1,500	3,500	2,500
useful life (yrs)	∞	7	15

6-51

The manager in a canned food processing plant is trying to decide between two labeling machines.

	Machine *A*	**Machine *B***
First cost	$15,000	$25,000
Maintenance and operating costs	1,600	400
Annual benefit	8,000	13,000
Salvage value	3,000	6,000
Useful life, in years	6	10

Assume an interest rate of 6%. Use annual cash flow analysis to determine which machine should be chosen.

A college student has been looking for new tires. The student feels that the warranty period is a good estimate of the tire life and that a 10% interest rate is appropriate. Using an annual cash flow analysis, which tire should be purchased?

6-52 A

Tire Warranty (months)	**Price per Tire**
12	$39.95
24	59.95
36	69.95
48	90.00

6-53 Carp, Inc. wants to evaluate two methods of packaging their products. Use an interest rate of 15% and annual cash flow analysis to decide which is the most desirable alternative.

	A	***B***
First cost	$700,000	$1,700,000
O&M costs (yr 1)	18,000	29,000
+ Cost gradient	+ 900/yr	+ 750/yr
Annual benefit	154,000	303,000
Salvage value	142,000	210,000
Useful life, in years	10	20

6-54 A The analysis period is 10 years, but there will be no replacement for Alternative *B* after 5 years. Based on a 15% interest rate, which alternative should be selected? Use an annual cash flow analysis.

	A	*B*
Cost	$5000	$18, 000
Uniform annual benefit	1500	6,000
Useful life, in years	10	5

6-55 A small manufacturing company is evaluating trucks for delivering their products. Truck *A* has a first cost of $32,000, its operating cost will be $5500 per year, and its salvage after 3 years will be $7000. Truck *B* has a first cost of $37,000, an operating cost of $5200, and a resale value of $12,000 after 4 years. At an interest rate of 12% which model should be chosen? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*

6-56 Dick Dickerson Construction, Inc. has asked you to help them select a new backhoe. You have a choice between a wheel-mounted version, which costs $60,000 and has an expected life of 5 years and a salvage value of $2000, and a track-mounted one, which costs $80,000, with a 7-year life and an expected salvage value of $10,000. Both machines will achieve the same productivity. Interest is 8%. Which one will you recommend? Use an annual worth analysis.

6-57 A job can be done with Machine *A* that costs $12,500 and has annual end-of-year maintenance costs of $5000; its salvage value after 3 years is $2000. Or the job can be done with Machine *B*, which costs $15,000 and has end-of-year maintenance costs of $4000 and a salvage value of $1500 at the end of 4 years. These investments can be repeated in the future, and your work is expected to continue indefinitely. Use present worth, annual worth, and capitalized cost to compare the machines. The interest rate is 5%/year. *Contributed by D. P. Loucks, Cornell University*

6-58 Hospitality Enterprises is planning to build a new 112 room inn in Martin. The initial cost of land leases and construction is anticipated to be $3.4 million. The annual operating and maintenance costs are expected to average $25,000 for the 20-year life of the inn. Every 4 years the interior of the inn must be painted at a cost of $15,000. The exterior must be painted and refurbished every 5 years at a cost of $60,000. The carpet and furniture must be replaced every 6 years at a cost of $100,000. Every 8 years $80,000 will be spent on paving and striping the parking areas. The inn will have a net demolition cost of $100,000 at the end of its life. If the MARR for Hospitality is 5%, determine the EUAC for the inn. *Contributed by Ed Wheeler, University of Tennessee at Martin*

Spreadsheets and Loans

6-59 A new car is purchased for $22,000 with a 10% down, 6% loan. The loan is for 4 years. After making 30 monthly payments, the owner wants to pay off the loan's remaining balance. How much is owed?

A year after buying her car, Anita has been offered a job in Europe.

6-60 Her car loan is for $27,000 at a 6% nominal interest rate for 48
A months. If she can sell the car for $20,000, how much does she get to
keep after paying off the loan?

(*a*) You are paying off a debt at a nominal 8% per year by paying $400 at the end of each quarter for the next year. Find the interest paid in the last $400 payment.
(*b*) If this debt were to be paid off in two equal payments of $1650 at
6-61 the end of this year and next year, find the interest paid in the first
$1650 payment. Again the loan rate is a nominal 8% per year compounded quarterly.
Contributed by D. P. Loucks, Cornell University

A student loan totals $30,000 at graduation. The interest rate is 6%,
6-62 and there will be 60 payments beginning 1 month after graduation.
A What is the monthly payment? What is owed after the first 2 years of
payments?

The student in Problem 6-62 received $2500 as a graduation present.
6-63 If an extra $2500 is paid at Month 1, when is the final payment
made? How much is it?

Sam can afford to spend $500 per month on a car. He figures he needs half of it for gas, parking, and insurance. He has been to the bank, and they will loan him 100% of the car's purchase price. (*Note*: If he had a down payment saved, then he could borrow at a lower rate.)
6-64 (*a*) If his loan is at a nominal 12% annual rate over 36 months, what
A is the most expensive car he can purchase?
(*b*) The car he likes costs $14,000 and the dealer will finance it over 60 months at 12%. Can he afford it? If not, for how many months will he need to save his $500 per month?
(*c*) What is the highest interest rate he can pay over 60 months and stay within his budget if he buys the $14,000 car now?

EnergyMax Engineering constructed a small office building for their firm 5 years ago. They financed it with a bank loan for $450,000 over 15 years at 6% interest with quarterly payments and compounding. The loan can be repaid at any time without penalty. The loan can be refinanced through an insurance firm for 4% over 20 years—still with
6-65

quarterly compounding and payments. The new loan has a 5% loan initiation fee, which will be added to the new loan.
(*a*) What is the balance due on the original mortgage (20 payments have been made in the last 5 years)?
(*b*) How much will EnergyMax's payments drop with the new loan?
(*c*) How much longer will the proposed loan run?

6-66 A E
Suppose you graduate with a debt of $42,000 that you or someone must repay. One option is to pay off the debt in constant amounts at the beginning of each month over the next 10 years at a nominal annual interest rate of 10%.
(*a*) What is the constant beginning-of-month payment?
(*b*) Of the first payment, what is the interest and the principal paid?
(*c*) Of the last payment, what is the interest and the principal paid?
(*d*) How are student loans treated in bankruptcy? What are the practical and ethical reasons for and against treating student loans differently from other loans?
Contributed by D. P. Loucks, Cornell University

6-67
An $11,000 mortgage has a 30-year term and a 6% nominal interest rate.
(*a*) What is the monthly payment?
(*b*) After the first year of payments, what is the outstanding balance?
(*c*) How much interest is paid in Month 13? How much principal?

6-68 A
A 30-year mortgage for $175,000 is issued at a 6% nominal interest rate.
(*a*) What is the monthly payment?
(*b*) How long does it take to pay off the mortgage, if $1250 per month is paid?
(*c*) How long does it take to pay off the mortgage, if double payments are made?

6-69
Solve Problem 6-39(*a*) for the breakeven first cost per kilometer of going under the lake.

6-70
Redo Problem 6-49 to calculate the EUAW of the alternatives as a function of miles driven per year to see if there is a crossover point in the decision process. Graph your results.

6-71
Develop a spreadsheet to solve Problem 6-37(*a*). What is the

breakeven cost of the additional pumping investment in Year 10?

Annuity Due

6-72 A You are leasing some furniture for your apartment. The monthly lease is for $395/month, payable at the beginning of the month. If your personal cost of capital is a 3% nominal annual rate, what is the present value of a one-year lease?

6-73 It is the first of October, and you are developing cost estimates for creating an engineering consulting business with a small group of friends. Liability insurance beginning January 1 will cost $475 per month, payable at the beginning of each month. What is the PW of this insurance for the first year as of today's date? The firm's MARR is a 12% nominal annual rate.

6-74 You are interested in leasing a new car for 36 months.

- The value of the car is $22,555.
- You must pay $3025 at signing, which does not include the first month's lease payment.
- The monthly lease cost for the car is $154 for 36 months.
- At the end of the lease, you will need to pay a lease termination fee of $2000.
- The interest rate for this type of new car is 1.90% APR.

Calculate the present worth of leasing the car.

6-75 Your grandmother set up an annuity, in which she will receive a monthly payment from the bank of $1200 per month, payable at the beginning of each month. She paid $200,000 for this annuity. The bank says they are paying an interest rate of 4%. How many months does the bank plan to pay the annuity?

Minicases

An office building should last 60 years, but this owner will sell it at 20 years for 40% of its construction cost. For the first 20 years it can be leased as Class A space, which is all this owner operates. When the building is sold, the land's cost will be recovered in full.

6-76	$2.2M	Land
	$4.1M	Building
	$640,000	Annual operating and maintenance
	4%	Annual property taxes and insurance (% of initial investment)

(*a*) If the owner wants a 12% rate of return, what is the required monthly leasing cost?
(*b*) Assuming that the building is vacant 5% of the time, what is the required monthly lease?
(*c*) What is an example monthly cost per square foot for Class A space in your community?

A 30-unit apartment building should last 35 years, when it will need to be either replaced or undergo major renovation. Assume the building's value at 35 years will be 10% of its construction cost. Assume it will be sold and that the land's cost will be recovered in full.

6-77	$3.2M	Land
	$4.8M	Building
	$850,000	Annual operating and maintenance
	6%	Annual property taxes and insurance (% of initial investment)
	12%	Vacancy rate

(*a*) If the owner wants a 15% rate of return, what is the required monthly leasing cost for each unit?
(*b*) If turning 2 units into an exercise facility would decrease the vacancy rate by 5%, would that be a good decision?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 6	**Lease a Lot**
	Compares leasing and ownership. Both financing and investing

	decisions are needed.
CASE 10	**The Cutting Edge**
	Make versus buy and machine selection.
CASE 27	**Harbor Delivery Service**
	Focus is treatment of sunk costs. More complicated than most. Some discoveries in the data gathering process. Solution uses equation rather than cash flow table.

CHAPTER 7

RATE OF RETURN ANALYSIS

What a Marvel! Iron Man

Common byproducts of war are advances in technology. Past examples include wireless communications, operations management, and rocketry. Current examples are exoskeletons and biomechanical assistive devices that help humans carry more weight, move faster, and work longer. With these devises, troops could carry more equipment and still be in fighting shape on arrival at their destination. Research and development (R&D) is addressing how a human integrates into the equipment, how the equipment operates, and how the equipment is powered—moving us closer to having deployable products.

These technologies are also finding uses closer to home for civilian applications. For example, construction workers move large, unwieldy materials to changing locations all day long. Mechanically assisted workers may be better than robots at key tasks in manufacturing and warehousing. The devices would take more of the day-to-day wear, thus protecting the worker's body. These devices would make the workplace safer for employees by reducing the risks they take and by replacing other support equipment that is not as mobile.

Medical integration of these technologies focuses on how we use these devices for rehabilitation and prosthetics. One driver for R&D has been the serious injuries of veterans returning home, who need these devices for their daily lives. Another motivation for R&D has been to help individuals who have suffered since birth, from disease, or from an accident. Advances include how the injured can sense the world, how they can control devices with their minds, and how they can better integrate these technologies into their daily lives. Better interfaces make the equipment more intuitive and user friendly. These devices assist not only those who have lost limbs, but also those who have lost or never had control over their limbs. The benefits realized from these devices apply to the entire population. These technologies allow individuals to retain or regain their independence and improve their quality of life.

Contributed by Billy Gray, Tarleton University

QUESTIONS TO CONSIDER

1. Why does it seem to take technological advancements so long to become available to the general public?
2. Does the influence of insurance affect how medical companies invest in new technologies?
3. What are some of the specialized requirements that result in the technology becoming cost prohibitive to produce?
4. How would patients and workers be trained, and what services would be needed to support the integration of assistive devices?

After Completing This Chapter

The student should be able to:

• Evaluate project cash flows with the *internal rate of return* (IRR) measure.
• Plot a project's present worth (PW) against the interest rate.
• Use an *incremental* rate of return analysis to evaluate competing alternatives.
• Develop and use spreadsheets to make IRR and incremental rate of return calculations.
• Identify when multiple roots exist and select the correct alternative. (Appendix 7A)

Key Words

balloon payment

increment of borrowing

increment of investment

incremental rate of return

internal rate of return

MARR

NPW plot

rate of return

XIRR

In this chapter we will examine four aspects of rate of return, the third major analysis method. First, the meaning of “rate of return” is explained; second, calculating the rate of return is illustrated; third, rate of return analysis problems are presented; and fourth, incremental analysis is presented. In an appendix to the chapter, we describe difficulties sometimes encountered when computing an interest rate for cash flow series with multiple sign changes.

Rate of return is the most frequently used measure in industry. Problems in computing the rate of return sometimes occur, but its major advantage is that it is a single figure of merit that is readily understood.

Consider these statements:

- The net present worth on a project is $32,000.
- The equivalent uniform annual net benefit is $2800.
- The project will produce a 23% rate of return.

While none of these statements tells the complete story, the third one measures the project’s desirability in terms that are widely and easily understood. Thus, this measure is accepted by engineers and business leaders alike.

There is another advantage to rate of return analysis. In both present worth and annual cash flow calculations, one must select an interest rate before calculations start—and the exact value may be a difficult and controversial item. In rate of return analysis, no interest rate is introduced into the calculations (except as described in Appendix 7A). Instead, we compute the rate of return (more accurately called *internal rate of return*) from the cash flows. To decide how to proceed, the calculated rate of return is compared with a preselected **minimum attractive rate of return,** or simply **MARR**. This is the same value of i used for present worth and annual cash flow analysis, but the comparison is after calculations are complete. Often the project’s IRR is well above or below the MARR so an exact value for the MARR is not needed.

INTERNAL RATE OF RETURN

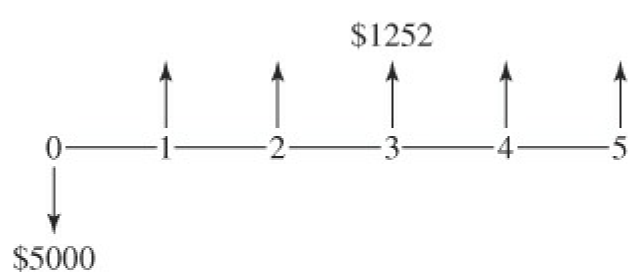
Internal rate of return is the interest rate at which the present worth and equivalent uniform annual worth are equal to 0.

This definition is easy to remember, and it also tells us how to solve for the rate of return. In earlier chapters we did this when we solved for the interest rate on a loan or investment.

Other definitions based on the unpaid balance of a loan or the unrecovered investment can help clarify why this rate of return is also called the *internal rate of return* or IRR. In Chapter 3 we examined four plans to repay $5000 in 5 years with interest at 8% (Table 3–1). In each case the amount loaned ($5000) and the loan duration (5 years) were the same. Yet the total interest paid to the lender varied from $1200 to $2347. In each case the lender received 8% interest each year on the amount of money actually owed. And, at the end of 5 years, the principal and interest payments exactly repaid the $5000 debt with interest at 8%. We say the lender received an “8% rate of return.”

Internal rate of return can also be defined as the interest rate paid on the unpaid balance of a *loan* such that the payment schedule makes the unpaid loan balance equal to zero when the final payment is made.

Instead of lending money, we might invest $5000 in a machine tool with a 5-year useful life and an equivalent uniform annual benefit of $1252. The question becomes, What rate of return would we receive on this investment?



We recognize these cash flows as reversed from Plan 3 of Table 3–1. We know that five payments of $1252 are equivalent to a present sum of $5000 when interest is 8%. Therefore, the rate of return on this investment is 8%.

Internal rate of return can also be defined as the interest rate earned on the unrecovered *investment* such that the payment schedule makes the unrecovered investment equal to zero at the end of the investment's life.

It must be understood that the 8% rate of return does not mean an annual return of 8% on the $5000 investment, or $400 in each of the 5 years with $5000 returned at the end of Year 5. Instead, each $1252 payment represents an 8% return on the unrecovered investment *plus* a partial return of the investment. This may be tabulated as follows:

TABLE 7-1 Amortization of an Investment

Year	Cash Flow	Unrecovered Investment at Beginning of Year	8% Return on Unrecovered Investment	Investment Repayment at End of Year	Unrecovered Investment at End of Year
0	–$5000				
1	1252	$5000	$ 400	$ 852	$4148
2	1252	4148	331	921	3227
3	1252	3227	258	994	2233
4	1252	2233	178	1074	1159
5	1252	1159	93	1159	0
			$1260	$5000	

This cash flow represents a $5000 investment with benefits that produce an 8% rate of return on the unrecovered investment. Except for the language shift to return on unrecovered balance from interest on balance due, the table above is like the amortization schedules covered in Chapter 6.

Although the definitions of internal rate of return are stated differently for a loan and for an investment, there is only one fundamental concept. It is that **the internal rate of return is the interest rate at which the benefits are equivalent to the costs**, or the present worth (PW) is 0. Since we are describing the funds that remain within the investment throughout its life, the resulting rate of return is described as the internal rate of return, *i*.

CALCULATING RATE OF RETURN

To calculate a rate of return on an investment, we must convert the various consequences of the investment into a cash flow series. Then we solve the cash flow series for the unknown value of the internal rate of return (IRR). Five forms of the cash flow equation are as follows:

$$\text{PW of benefits} - \text{PW of costs} = 0 \quad (7\text{-}1)$$

$$\frac{\text{PW of benefits}}{\text{PW of costs}} = 1 \quad (7\text{-}2)$$

$$\text{Present worth} = \text{Net present worth}^{1} = 0 \quad (7\text{-}3)$$

$$\text{EUAW} = \text{EUAB} - \text{EUAC} = 0 \quad (7\text{-}4)$$

$$\text{PW of costs} = \text{PW of benefits} \quad (7\text{-}5)$$

The five equations represent the same concept in different forms. They relate costs and benefits with the IRR as the only unknown. The calculation of rate of return is illustrated by the following examples.

Example 7-1

An engineer invests $5000 at the end of every year for a 40-year career. If the engineer wants $1 million in savings at retirement, what interest rate must the investment earn?

While this example has been defined in terms of an engineer's personal finances, we could just as easily have said, "a mining firm makes annual deposits of $50,000 into a reclamation fund for 40 years. If the firm must have $10 million when the mine is closed, what interest rate must the investment earn?" Since the annual deposit and required final amount are 10 times larger the answer is obviously the same.

TABLE SOLUTION

Using the net future worth version of Equation 7–3, we write

$$\text{Net FW} = 0 = -\$5000(F/A, i, 40) + \$1{,}000{,}000$$

Rewriting, we see that

$$(F/A, i, 40) = \$1{,}000{,}000/\$5000 = 200$$

We then look at the compound interest tables for the value of *i* where (*F*/*A*, *i*, 40) = 200. If no tabulated value of *i* gives this value, then we will interpolate, using the two closest values or solve exactly with a calculator or spreadsheet. In this case (*F*/*A*, 0.07, 40) = 199.636, which to three significant digits equals 200. Thus the required rate of return for the investment is 7%.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 7-1		40	-5000	0	1,000,000	*i*	7.007%

Example 7-2

An investment resulted in the following cash flow. Compute the internal rate of return.

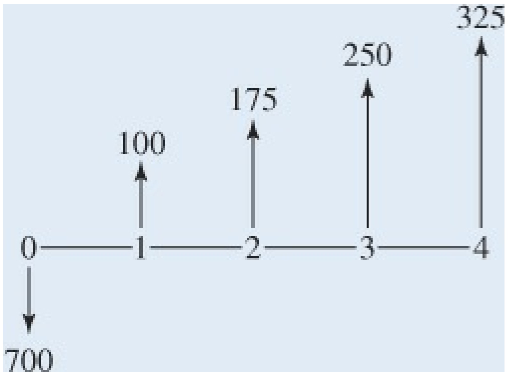


TABLE SOLUTION

$$\text{EUAW} = \text{EUAB} - \text{EUAC} = 0 = 100 + 75(A/G, i, 4) - 700(A/P, i, 4)$$

Here, we have two different interest factors in the equation, and we will solve the equation by trial and error. The EUAW value is a function of *i*. Try *i* = 5*%* first:

$$\begin{aligned} \text{EUAW}_{5\%} &= 100 + 75(A/G, 5\%, 4) - 700(A/P, 5\%, 4) \\ &= 100 + 75(1.439) - 700(0.2820) \\ &= \text{EUAB} - \text{EUAC} = 207.9 - 197.4 = +10.5 \end{aligned}$$

The EUAW is too high. If the interest rate is increased, EUAW will decrease. Try *i* = 8*%*:

$$\begin{aligned} \text{EUAW}_{8\%} &= 100 + 75(A/G, 8\%, 4) - 700(A/P, 8\%, 4) \\ &= 100 + 75(1.404) - 700(0.3019) \\ &= \text{EUAB} - \text{EUAC} = 205.3 - 211.3 = -6.0 \end{aligned}$$

This time the EUAW is too low. We see that the true rate of return is between 5% and 8%. Try *i* = 7*%*:

$$\begin{aligned} \text{EUAW}_{7\%} &= 100 + 75(A/G, 7\%, 4) - 700(A/P, 7\%, 4) \\ &= 100 + 75(1.416) - 700(0.2952) \\ &= \text{EUAB} - \text{EUAC} = 206.2 - 206.6 = -0.4 \end{aligned}$$

The IRR is about 7*%*. Interpolating between the $EUAW_{6\%}$ and $EUAW_{7\%}$ values would yield a more precise answer.

SPREADSHEET SOLUTION

Since there is a gradient a 5-BUTTON SOLUTION is not possible. Best practice is to build or re-use a spreadsheet with a data block. However, it is also possible to build a small spreadsheet with hard coded values for one-time use to find that the rate of return is 6.91%.

	A	B	C	D	E	F	G	H
1	Year	0	1	2	3	4	Solve for	Answer
2	Cash flow	-700	100	175	250	325	IRR	6.91%
3				=C2+75				=IRR(B2:F2)

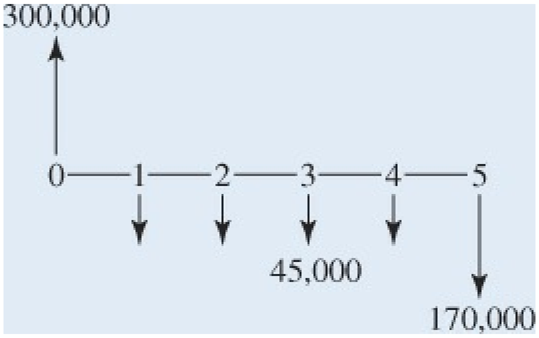
Example 7–3 is an example of a **balloon payment** loan. A lender may offer a lower interest rate, if the interest rate can be re-set before the principal is completely paid off. This may be done with a variable interest rate loan or by using a balloon payment loan, which is paid off by taking out a new loan. The borrower may prefer a lower rate now, even if there is a risk that the rate may be higher later.

Example 7-3

A firm borrows $300,000 to be repaid with 5 annual payments of $45,000 and a final balloon payment of $170,000. What interest rate is the firm paying on this loan?

5-BUTTON SOLUTION

The first step is to draw the cash flow diagram.



This can be solved in one step with a spreadsheet or calculator annuity function.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 7-3		5	-45,000	300,000	-170,000	RATE	7.546%	=RATE(C2,D2,E2,F

TABLE SOLUTION

This can also be solved with tabulated factors. A good starting point is the present worth equation.

$$\text{PW}_i = 300{,}000 - 45{,}000(P/A, i, 5) - 170{,}000(P/F, i, 5)$$

$\text{PW}_{7\%}$ is −\$5717 and $\text{PW}_{8\%}$ is \$4629. Thus the interpolated interest rate is:

$$i = 0.07 + 0.01(5717)/(5717 + 4629) = 7.553\%$$

The interest rates are shown with an extra decimal point to illustrate the difference between the exact and the interpolated values here.

Example 7-4

A local firm sponsors a student loan program for the children of employees.

No interest is charged until graduation, and then the interest rate is 5%. Maria borrows $9000 per year, and she graduates after 4 years. Since tuition must be paid ahead of time, assume that she borrows the money at the start of each year.

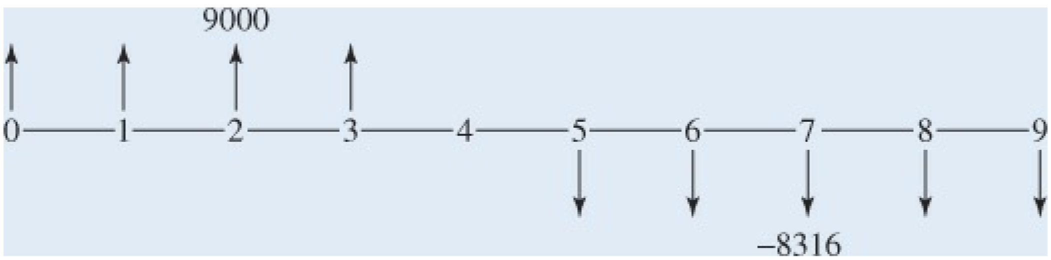
If Maria makes five equal annual payments, what is each payment? Use the cash flow from when she started borrowing the money to when it is all paid back, and then calculate the internal rate of return for Maria's loan. Is this arrangement attractive to Maria?

TABLE SOLUTION

Maria owes $36,000 at graduation. The first step is to calculate the five equal annual payments to repay this loan at 5%.

$$\text{Loan payment} = \$36{,}000(A/P, 5\%, 5) = 36{,}000(0.2310) = \$8316$$

Maria receives $9000 by borrowing at the start of each year. She graduates at the end of Year 4. At the end of Year 4, which is also the beginning of Year 5, interest starts to accrue. She makes her first payment at the end of Year 5, which is one year after graduation.



The next step is to write the present worth equation in factor form, so that we can apply Equation 7-3 and set it equal to 0. This equation has three factors, so we will have to solve the problem by picking interest rates and substituting values. The present worth value is a function of *i*.

$$\text{PW}_i = 9000[1 + (P/A, i, 3)] - 8316(P/A, i, 5)(P/F, i, 4)$$

The first two interest rates used are 0% (because it is easy) and 3% because the subsidized rate will be below the 5% that is charged after graduation.

At 0% any P/A factor equals n, and any P/F factor equals 1.

$$\text{PW}_{0\%} = 9000(4) - 8316(5) = -5180$$

$$\text{PW}_{3\%} = 9000(1 + 2.829) - 8316(4.580)(0.8885) = 620.5$$

Since PW_i has opposite signs for 0 and 3, there is a value of i between 0 and 3% which is the IRR. Because the value for 3% is closer to 0, the IRR will be closer to 3%. Try 2% next.

$$\text{PW}_{2\%} = 9000(1 + 2.884) - 8316(4.713)(0.9238) = -1251$$

As shown in Figure 7–1, interpolating between 2 and 3% leads to

$$\text{IRR} = 2\% + (3\% - 2\%)[1251/(1251 + 620.5)] = 2.67\%.$$

This rate is quite low, and it makes the loan look like a good choice.

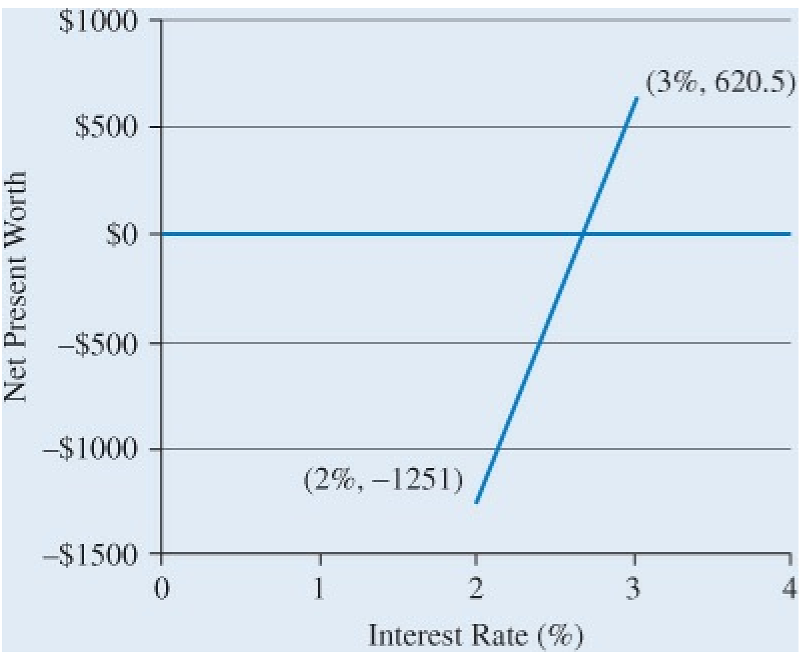


FIGURE 7-1 Plot of PW versus interest rate *i*.

If in Figure 7–1 net present worth (NPW) had been computed for a broader range of values of *i*, Figure 7–2 would have been obtained. From this figure it is apparent that the error resulting from linear interpolation increases as the interpolation width increases.

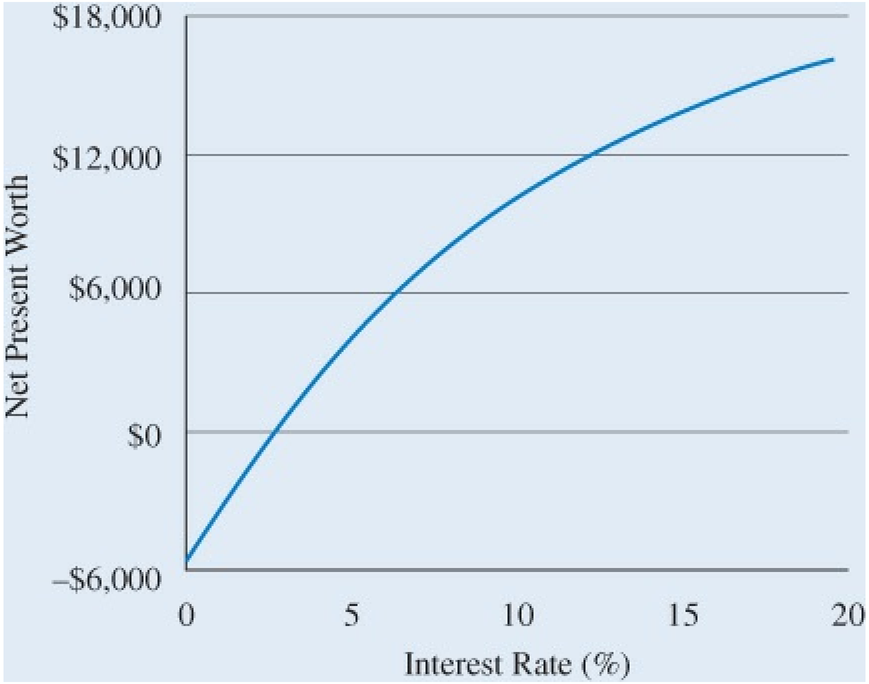


FIGURE 7-2 Replot of NPW versus interest rate *i* over a larger range of values.

SPREADSHEET SOLUTION

Note that cell F6 cannot be blank—a zero must be entered. If it is blank, then the cell is ignored by the IRR function and the IRR is calculated as though the cash flows occur through year 8.

	A	B	C	D	E	F	G
1	$9,000	Annual tuition			Year	Cash flow	
2	4	# years in college			0	$9,000	
3	5%	Interest rate after graduation			1	9,000	
4	5	# payments			2	9,000	
5					3	9,000	
6	$36,000	Amount owed at graduation			4	0	Cannot be blank!!
7	-$8,315.09	Annual payment			5	-8,315	
8					6	-8,315	
9		=PMT(A3,A4,A6,0)			7	-8,315	
10					8	-8,315	
11					9	-8,315	
12					IRR	2.66%	=IRR(F2:F11)

An amortization table like Table 7–1 can be used to confirm the rate of return.

Plot of NPW Versus Interest Rate i

The plot of NPW versus interest rate *i* is an important source of information. For a cash flow where borrowed money is repaid, the NPW plot would appear as in Figure 7–3. The borrowed money is received early in the time period with a later repayment of an equal sum, plus payment of interest on the borrowed money. In all cases in which interest is charged and the amount borrowed is fully repaid, the NPW at 0% will be negative.

For a cash flow representing an investment followed by benefits from the investment, the plot of NPW versus *i* (we will call it an **NPW plot** for convenience) would have the form of Figure 7–4. As the interest rate increases, future benefits are discounted more heavily and the NPW decreases.

Year	Cash Flow
0	$+P$
1	$-A$
2	$-A$
3	$-A$
4	$-A$
.	.
.	.
.	.

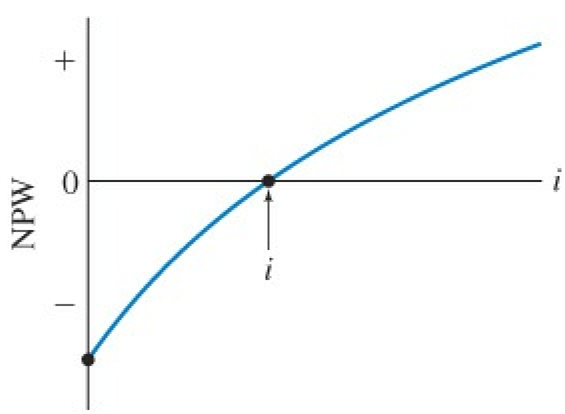


FIGURE 7-3 Typical NPW plot for borrowed money.

Year	Cash Flow
0	$-P$
1	$+$Benefit A
2	$+A$
3	$+A$
4	$+A$
.	.
.	.
.	.

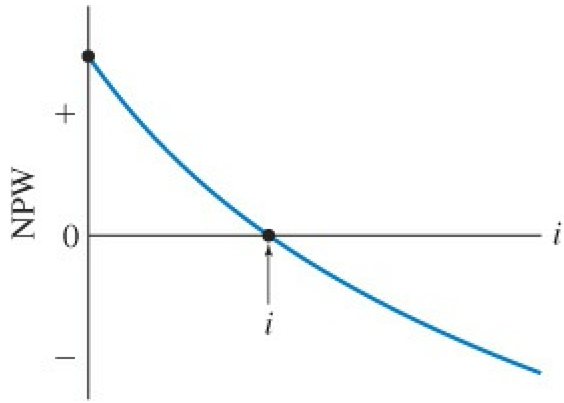


FIGURE 7-4 NPW plot for a typical investment.

Thus, interest is a charge for the use of someone else's money or a receipt for letting others use our money. The interest rate is almost always positive, but negative interest rates do occur. A loan with a forgiveness provision (not all principal is repaid) can have a negative rate. Some investments perform poorly and have negative rates.

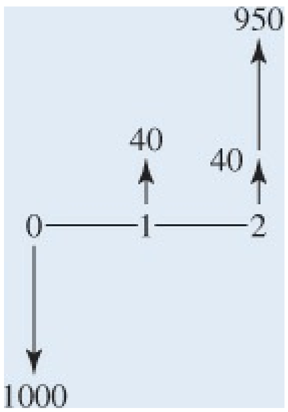
Example 7-5

A new corporate bond with a coupon rate of 8% was initially sold by a stockbroker to an investor for $1000. The issuing corporation promised to pay the bondholder $40 interest on the $1000 face value of the bond every 6 months, and to repay the $1000 at the end of 10 years. After one year the bond was sold by the original buyer for $950.

(a) What rate of return did the original buyer receive on his investment?
(b) What rate of return can the new buyer (paying $950) expect to receive if he keeps the bond for its remaining 9-year life?

TABLE SOLUTION TO PART a

The original bondholder sold the bond for less ($950) than the purchase price ($1000). So the semiannual rate of return is less than the 4% semiannual interest rate on the bond.



Since $40 is received each 6 months, we will solve the problem using a 6-month interest period.

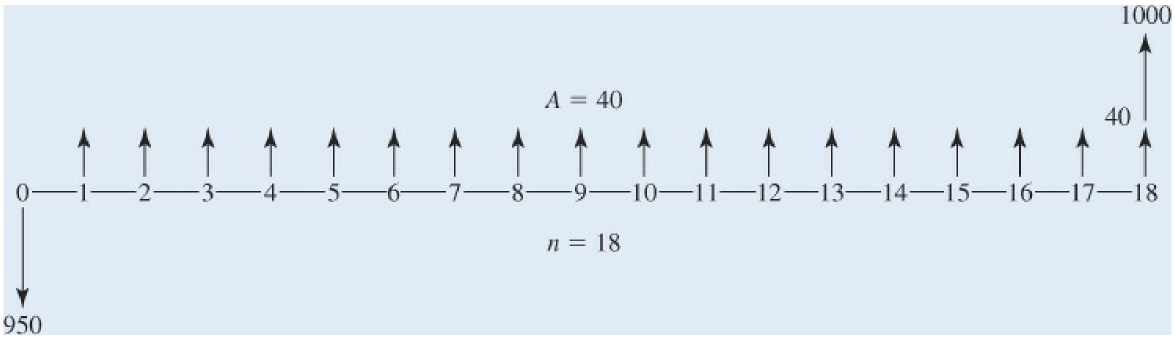
$$\mathrm{PW} = -1000 + 40(P/A, i, 2) + 950(P/F, i, 2)$$

$$\mathrm{PW}_{1^1/_2\%} = -1000 + 40(1.956) + 950(0.9707) = \$0.41$$

The interest rate per 6 months, $\mathrm{IRR}_{6\ \mathrm{mon}}$, is very close to $1^1/_2$ %. This means the nominal (annual) interest rate is 2 × 1.5% = 3%. The effective (annual) interest rate or IRR is $(1 + 0.015)^2 - 1 = 3.02\%$.

TABLE SOLUTION TO PART b

The new buyer will redeem the bond for more ($1000) than the purchase price ($950). So the semiannual rate of return is more than the 4% semiannual interest rate on the bond.



Given the same $40 semiannual interest payments, for 6-month interest periods we write

$$\text{PW} = -950 + 40(P/A, i, 18) + 1000(P/F, i, 18)$$

$$\text{PW}_{5\%} = -950 + 40(11.690) + 1000(0.4155) = -66.90$$

Try a lower interest rate, say, $i = 4\%$:

$$\text{PW}_{4\%} = -950 + 40(12.659) + 1000(0.4936) = 49.96$$

The value of the 6-month rate *i*, is between 4 and 5%. By interpolation,

$$i = 4\% + (1\%)\left(\frac{49.96 - 0}{49.96 - (-66.90)}\right) = 4.43\%$$

The nominal interest rate is 2 × 4.43% = 8.86%. The effective interest rate or IRR is $(1 + 0.0443)^2 - 1 = 9.05\%$.

SPREADSHEET SOLUTION

Entering the information from the cash flow diagrams into the spreadsheet produces more precise answers.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 7-5a		2	40	-1000	950	$i6_{mon}$	1.519%
3		3.04%	2				effective	3.061%
4	Exp. 7-5b		18	40	-950	1000	$i6_{mon}$	4.408%
5		8.82%	2				effective	9.011%
6		=C5*H4					=EFFECT(B5,C5)	

In Example 7–5a the $50 change lowers the interest rate by 1% for 1 year and in 7–5b it raises it by 0.4% for 9 years.

INTEREST RATES WHEN THERE ARE FEES OR DISCOUNTS

Example 7–6 shows how a cash discount is "unstated interest." For higher cost items, there is sometimes a choice between a "cash" rebate or discount and a low-interest loan. Fees and cash discounts are unstated interest that raise the interest rate on a loan, and the true cost of the loan is its internal rate of return.

When firms and individuals borrow money, there are often fees charged in addition to the interest. This can be as simple as the underwriting fee that a firm is charged when it sells a bond. In Example 7–7 we add that underwriting fee to the bond in Example 7–5, and look at the bond from the firm's perspective rather than the investor's.

Example 7-6

A regional furniture company is offering either a discount equal to twice the state sales tax or interest-free financing for 60 months. The state sales tax is 6%. What is the effective annual interest rate for financing versus the discount? Assume you are buying $1000 worth of furniture. How will you decide which option to take?

TABLE SOLUTION

We set each option's PW equal to each other. By paying cash, we get a discount of 12% and pay $880 now [= $1000(1 − 0.12)]. If we finance with no interest, the monthly payments are $1000/60 = $16.67. Thus,

$$\begin{aligned} 1000(1-0.12) &= (16.67)(P/A, i, 60) \\ 880 &= 16.67(P/A, i, 60) \\ (P/A, i, 60) &= 880/16.67 = 52.789 \end{aligned}$$

From Appendix C,

$$(P/A, 0.25\%, 60) = 55.653$$
$$(P/A, 0.50\%, 60) = 51.726$$

Interpolating,

$$\begin{aligned} \text{Monthly interest rate} &= 0.432\% \\ \text{Nominal annual interest rate} &= 0.432\% \times 12 = 5.184\% \\ \text{Effective annual interest rate} &= (1.00432)^{12} - 1 = 0.0531 = 5.31\% \end{aligned}$$

SPREADSHEET SOLUTION

Paying cash is equivalent to financing the purchase at some interest rate. If we pay cash, we will get a discount of 12%, so the present value is $880. If we finance with no interest, monthly payments are $1000/60 = $16.67, so

	A	B	C	D	E	F	G	H	I

1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	
2	Exp. 7-6		60	-16.67	880	0	RATE	0.430%	monthly
3							nominal	5.16%	=+H2*12
4							effective	5.28%	=EFFECT(H3,12)

The interest-free financing carries an effective annual interest rate of 5.3% relative to paying cash. This rate is much lower than most credit card rates and higher than most savings account rates. How does this rate compare with your MARR?

Example 7-7 (Example 7–5 Revisited)

The corporate bond in Example 7–5 was part of a much larger offering that the firm arranged with the underwriter. Each of the bonds had a face value of $1000 and a life of 10 years. Since $40 or 4% of the face value was paid in interest every 6 months, the bond had a nominal or coupon interest rate of 8% per year. If the firm paid the underwriter a 1% fee to sell the bond, what is the effective annual interest rate that the firm is paying on the bond?

TABLE SOLUTION

From the firm’s perspective, it receives $1000 minus the fee at time 0, then it pays interest every 6 months for 10 years, and then it pays $1000 to redeem the bond. The 1% fee reduces what the firm receives when the bond is sold to $990. The interest payments are $40 every 6 months. This is easiest to model using twenty 6-month periods.

$$\mathrm{PW}_i = 990 - 40(P/A, i, 20) - 1000(P/F, i, 20)$$

Since the nominal interest rate is 4% every 6 months, we know that the fee will raise this some. So let us use the next higher table of 4.5%.

$$\begin{aligned} PW_{4.5\%} &= 990 - 40(P/A, 4.5\%, 20) - 1000(P/F, 4.5\%, 20) \\ &= 990 - 40(13.008) - 1000(0.4146) = \$55.08 \end{aligned}$$

We know that the PW of the interest and final bond payoff is $1000 at 4%.

$$PW_{4\%} = 990 - 1000 = -10$$

Now we interpolate to find the interest rate for each 6-month period.

$$i = 4\% + (4.5\% - 4\%) \times 10/(10 + 55.08) = 4.077\%$$

The effective annual rate is

$$i_a = 1.04077^2 - 1 = 0.0832 = 8.32\%$$

SPREADSHEET SOLUTION

As discussed in the table solution, the firm receives 1% less than the bond's face value at time 0, and 4% of the face value is paid as interest every 6 months. Thus, the cash flows begin with a positive $990, and the interest payments and final payment of the face value are negative cash flows.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 7-6		20	-40	990	-1000	$i_{6\ \text{mon}}$	4.074%
3		8.15%	2				effective	8.314%
4		=C3*H2					=EFFECT(B3,C3)	

LOANS AND INVESTMENTS ARE EVERYWHERE

Examples 7–3 to 7–7 were about borrowing money through bonds and loans, but many applications for the rate of return are stated in other ways. Example 7–8 is a common problem on university campuses—buying parking permits for an academic year or a term at a time.

Buying a year's parking permit is investing more money now to avoid paying for another shorter permit later. Choosing to buy a shorter permit is a loan, where the money saved by not buying the annual permit is borrowed to be repaid with the cost of the second semester permit.

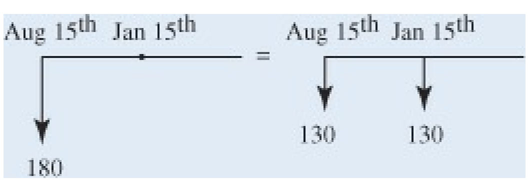
Example 7-8

An engineering student is deciding whether to buy two 1-semester parking permits or an annual permit. The annual parking permit costs $180 due August 15th; the semester permits are $130 due August 15th and January 15th. What is the rate of return for buying the annual permit?

TABLE SOLUTION

Before we solve this mathematically, let us describe it in words. We are equating the $50 cost difference now between the two permits with the $130 cost to buy another semester permit in 5 months. Since the $130 is 2.6 times the $50, it is clear that we will get a high interest rate.

This is most easily solved by using monthly periods, and the payment for the second semester is 5 months later. The cash flow table adds a column for the incremental difference to the information in the cash flow diagram.



Time	Annual Pass	Two Semester Passes	Incremental Difference
Aug. 15th	–$180	–$ 130	–$ 50
Jan. 15th		–130	130

Setting the two PWs equal to each other we have

$$-\$180 = -\$130[1 + (P/F, i_{\text{mon}}, 5)]$$
$$(P/F, i_{\text{mon}}, 5) = -\$50/-\$130 = 5/13 = 0.3846$$

Rather than interpolating, we can use the formula for the P/F factor.

$$1/(1 + i_{\text{mon}})^5 = 0.3846$$
$$(1 + i_{\text{mon}})^5 = 2.600$$
$$1 + i_{\text{mon}} = 1.2106$$
$$i_{\text{mon}} = 21.06\%, \text{ which is an extremely high rate per month}$$

On an annual basis, the effective interest rate is $(1.2106^{12} - 1) = 891\%$. Unless the student is planning to graduate in January, it is clearly better to buy the permit a year at a time.

5-BUTTON SOLUTION

The first step as detailed in the table solution is to identify the incremental cash flows between the two choices. Paying an extra $50 in August saves $130 in January. The resulting interest rate of 21.06% per month is extremely high!

Compounding the monthly interest rate for 12 periods with a PV = −1 leads to an FV that equals 1 + effective interest rate. Subtracting 1 from the FV value of 9.907 shows the annual rate is 890.7%. Unless the student is planning to graduate in January or sell the car, at this university permits should only be purchased by the year.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 7-8		5	0	-50	130	i_{mon}	21.06%
3		21.06%	12	0	-1		FV	9.907
4						effective annual rate		890.7%

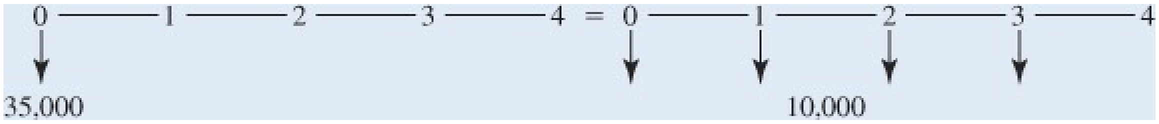
Examples 7–9 and 7–10 reference two common situations faced by firms and by individuals—buying insurance with a choice of payment plans and deciding whether to buy or lease a vehicle or equipment. Like the parking permit case, these situations can be described as investing more now to save money later, or borrowing money now to be paid later. Lease, insurance, and tuition payments are annuities due with cash flows that have beginning-of-period payments (see also Example 6–14).

Example 7-9

An engineering firm can pay for its liability insurance on an annual or quarterly basis. If paid quarterly, the insurance costs $10,000. If paid annually, the insurance costs $35,000. What is the rate of return for paying on annual basis? As shown in the cash flow diagrams, insurance payments are made in advance at the beginning of the period covered.

TABLE SOLUTION

This is most easily solved by using quarterly periods. As shown in the cash flow diagram and table, insurance must be paid at the start of each quarter. The cash flow table adds a column for the incremental difference to the information in the cash flow diagram.



Quarter	Annual Payment	Quarterly Payments	Incremental Difference
0	–$35,000	–$10,000	–$25,000
1		–10,000	–10,000
2		–10,000	–10,000
3		–10,000	–10,000

Setting the two PWs equal to each other we have

$$-\$35{,}000 = -\$10{,}000(1 + (P/A,\ i_{qtr}, 3))$$

$$(P/A,\ i_{qtr}, 3) = -\$25{,}000/-\$10{,}000 = 2.5$$

$$(P/A, 9\%, 3) = 2.531$$

$$(P/A, 10\%, 3) = 2.487$$

$$i_{qtr} = 0.09 + (0.10 - 0.09)(2.531 - 2.5)/(2.531 - 2.487)$$

$$= 0.09 + 0.01(0.7045) = 9.705\%$$

On an annual basis the effective interest rate is $(1.09705^4 - 1) = 44.8\%$.

Unless the firm is planning to go out of business, it is clearly better to buy the insurance a year at a time.

5-BUTTON SOLUTION

The incremental cash flows were identified as −$25,000 at time 0 and $10,000 at the end of the first three quarters. The resulting interest rate is shown in the first calculation of the 5-BUTTON SOLUTION. It is 9.70% for 3 months, which is a high rate.

The second calculation shows four quarterly payments and an initial investment of −$35,000. These annuity due payments are identified as beginning-of-period cash flows by using BEGIN on a financial calculator or Type = 1 in a spreadsheet.

Compounding the quarterly interest rate for 4 periods with a PV = −1 leads to an FV that equals 1 + effective interest rate. Subtracting 1 from the FV value of 1.448 shows the annual rate is 44.8%, which means the firm should buy its insurance a year at a time.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 7-9		3	10,000	-25,000	0	i_{qtr}	9.70%

3	using Type=1 or Begin		4	10,000	-35,000	0	i_{qtr}	9.70%
4		9.70%	4	0	-1		FV	1.448
5						effective annual rate		44.8%

Example 7-10

Mountain Environmental Consulting may buy some field equipment for $40,000 or lease it for $2500 per month. In either case, the equipment will be replaced in 2 years. The salvage value of the equipment after 2 years is $6000. What is the IRR or cost of the lease?

TABLE SOLUTION

The first step is to summarize the cash flows in a table. We must use monthly periods and remember that lease fees (an annuity due) are paid at the start of the period.

Month	Lease	Buy	Buy – Lease
0	−$2500	−$40,000	−$37,500
1–23	−2500		2,500
24	0	6,000	6,000

The easiest way to analyze this example is to simply set the present worths of buying and leasing equal to each other.

$$-\$2500 - \$2500(P/A, i_{\text{mon}}, 23) = -\$40{,}000 + \$6000(P/F, i_{\text{mon}}, 24)$$

$$0 = \$37{,}500 - \$2500(P/A, i_{\text{mon}}, 23) - \$6000(P/F, i_{\text{mon}}, 24)$$

The cash flow table was arranged so that the alternative with the lower cost now (leasing) was listed first. This ensures that the column of incremental cash flows (buy − lease) is an investment—the negative cash flows come first. An additional $37,500 must be paid or invested at time 0 to buy rather than lease.

The order of the two alternatives can be reversed. If Mountain Environmental leases the equipment, it avoids the expenditure of \$37,500 at time 0. It incurs a cost of \$2500 at the ends of Months 1 through 23, and it gives up the salvage value of \$6000 at the end of Month 24. This cash flow pattern (positive at time 0 and negative in later years) occurs because leasing is a way to borrow money.

The column of incremental difference describes the decision to buy rather than lease, which invests \$37,500 more now to avoid the later lease payments.

This equation is easier to solve if n = 24 for both factors. There is a total cash flow of −\$6000 at the end of Month 24. This cash flow can be split into −\$2500 and −\$3500. Then both the uniform and single payment periods continue to the end of Month 24.

$$0 = \$37{,}500 - \$2500(P/A, i_{\text{mon}}, 24) - \$3500(P/F, i_{\text{mon}}, 24)$$

$$PW_{0\%} = \$37{,}500 - \$2500(24) - \$3500(1) = -\$26{,}000$$

$$PW_{5\%} = \$37{,}500 - \$2500(13.799) - \$3500(0.3101) = \$1917$$

The answer is clearly between 0% and 5%, and closer to 5%. Try 4% next.

$$PW_{4\%} = \$37{,}500 - \$2500(15.247) - \$3500(0.3901) = -\$1983$$

$$i_{\text{mon}} = 4\% + (5\% - 4\%)(1983)/(1983 + 1917) = 4.51\%$$

The effective annual rate is

$$i_{\text{a}} = 1.0451^{12} - 1 = 0.6975 = 69.8\%$$

5-BUTTON SOLUTION

As noted above, using n = 24 makes things easier. The resulting interest rate is shown in the first calculation of the 5-BUTTON SOLUTION. It is 4.49% per month, which is a very high rate.

The second calculation identifies the payments as beginning-of-period cash flows by using BEGIN on a financial calculator or Type=1 in a spreadsheet. Then the initial investment is −$40,000 and the final salvage value is $6000.

Compounding the monthly interest rate for 12 periods with a PV = −1 leads to an FV that equals 1 + effective interest rate. Subtracting 1 from the FV value of 1.694 shows the annual rate is 69.4%, which means the firm should buy (not lease) the equipment.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 7-10		24	2,500	-37,500	3500	i_{qtr}	4.49%
3	using Type=1 or Begin		24	2,500	-40,000	6000	i_{qtr}	4.49%
4		4.49%	12	0	-1		FV	1.694
5						effective annual rate		69.4%

In this case leasing is an extremely expensive way to obtain the equipment.

INCREMENTAL ANALYSIS

When there are two alternatives, rate of return analysis is performed by computing the **incremental rate of return**—ΔIRR—on the difference between the alternatives. In Examples 7–7 through 7–10 this was done by setting the present worths of the two alternatives equal to each other. Chapter 8 will address incremental analyses with more than 2 alternatives.

Now we will calculate the increment more formally. Since we want to look at increments of investment, the cash flow for the difference between the alternatives is computed by taking the higher initial-cost alternative *minus* the lower initial-cost alternative (as in Example 7–10). The incremental investment is justified if its rate of return is greater than or equal to the MARR.

Two-Alternative Situation	Decision	Want Higher Rate

ΔIRR ≥ MARR	Choose the higher-cost alternative	Invest increment at ΔIRR
ΔIRR < MARR	Choose the lower-cost alternative	Invest increment elsewhere at MARR

Example 7–11 illustrates a *very* important point that was not part of our earlier examples of parking permits, insurance, and equipment to be bought or leased. In those cases, the decision to purchase the insurance, permit, or equipment had already been made. We only needed to decide the best way to obtain it. In Example 7–11, there is a *do-nothing* alternative that must be considered.

Because Example 7–11 includes a do-nothing alternative, the rates of return of each alternative have meaning. When rates are high, a common mistake is to select the alternative with the highest rate. Example 7–11 shows why this is incorrect. If both mutually exclusive alternatives are acceptable, it is the incremental rate that determines which one should be chosen. This will be applied to multiple alternatives in Chapter 8.

Example 7-11

You may select one of two mutually exclusive alternatives. Doing nothing is allowed. (*Note*: Engineering economists often use the term “mutually exclusive alternatives” to emphasize that selecting one alternative precludes selecting any other.) The alternatives are as follows:

Year	Alt. 1	Alt. 2
0	− $1000	− $2000
1	+ 1500	+ 2800

Any money not invested here may be invested elsewhere at the MARR of 6%. If you can choose at most one alternative one time, using the internal rate of return (IRR) analysis method, which one would you select?

SOLUTION

For Alternative 1, if \$1000 increases to \$1500 in a year, then the rate of return must be 50% (= 500/1000). For Alternative 2, if \$2000 increases to \$2800 in a year, then the rate of return must be 40% (= 800/2000). Both rates are *very* attractive, so both alternatives are clearly better than doing nothing and investing elsewhere at 6%. To decide which is better—incremental analysis is needed.

Year	Alt. 1	Alt. 2	Alt. 2 − Alt. 1
0	−\$1000	−\$2000	−\$1000
1	1500	2800	1300

One can see that if \$1000 increases to \$1300 in one year, the interest rate must be 30%, which is far higher than the 6% MARR. The additional \$1000 investment to obtain Alt. 2 is superior to investing the \$1000 elsewhere at 6%. The interest rate can also be easily calculated using a future worth equation.

$$\begin{aligned}\text{FW} = 0 &= -1000(1+i) + 1300\\ 1+i &= 1300/1000\\ i &= 30\%\end{aligned}$$

To understand an intuitively appealing—but *incorrect*—approach to Example 7–11, consider the rate of return for each alternative. The interest rates for Alternatives 1 and 2 were 50% and 40%. The higher rate of return of Alt. 1 is attractive, but Alt. 1 is not the correct solution. This analysis only compares each alternative with doing nothing rather than with the other alternative.

Figure 7–5 shows why comparing the alternatives with nothing is not enough. Those rates of 50% and 40% are very attractive, but it is not correct to choose the alternative with the highest rate. It is also not correct to choose the largest project with a good rate of return. The alternatives must be directly compared with incremental analysis.

On the graph we can see that Alternative 1 has a rate of return of 50%, Alternative 2 has a rate of 40%, and the increment Alternative 2-1 has a rate of 30%. All of these rates are high compared with a MARR of 6%. The incremental rate determines that Alternative 2 is better.

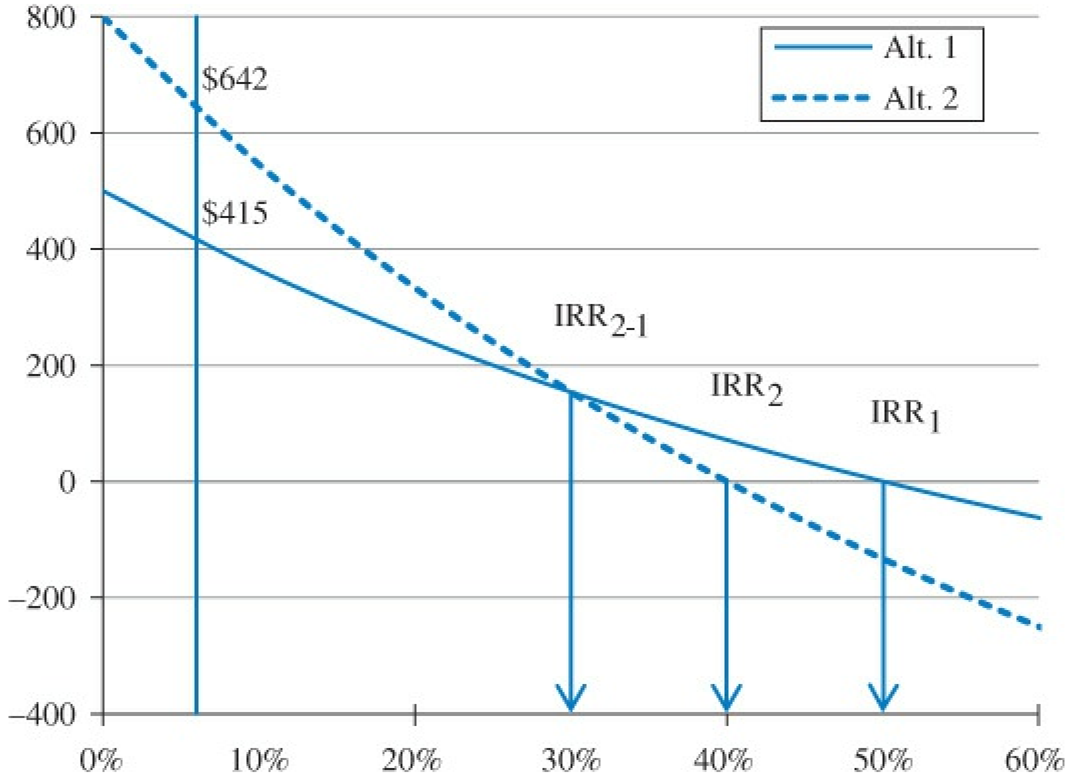


FIGURE 7-5 PW vs. *i* for two alternatives.

These results are consistent with a present worth analysis. At 6% Alternative 2 clearly has a higher PW. The exact values are $415 for Alternative 1 and $642 for Alternative 2.

Example 7-12

Consider the alternatives in Example 7–11 again, but this time compute the interest rate on the increment (Alt. 1 – Alt. 2) instead of (Alt. 2 – Alt. 1). How do you interpret the results?

SOLUTION

The signs of the incremental cash flows are reversed.

Year	Alt. 1 – Alt. 2
0	$1000
1	− 1300

The present worth equation is:

$$\text{PW} = 0 = +1000 - 1300/(1 + i)$$

$$1 + i = \frac{1300}{1000} \Longrightarrow i = 30\%$$

Once again the interest rate is found to be 30%. The critical question is, What does the 30% represent? The incremental cash flows do *not* represent an investment, but rather a loan. It is as if we borrowed $1000 in Year 0 ($1000 represents a receipt of money) and repaid it in Year 1 (− $1300 represents a disbursement). The 30% interest rate means this is the amount *we would pay* for the loan.

This is not a desirable borrowing scenario. The MARR on investments is 6%; it is reasonable to assume our maximum interest rate on borrowing would be less. Here the interest rate is 30%, which means the borrowing is undesirable. Alternative 2 is preferred to Alternative 1—the same conclusion reached in Example 7–11.

Example 7–12 illustrated that one can analyze either **increments of investment** or **increments of borrowing.** When looking at increments of investment, we accept the increment when the incremental rate of return equals or exceeds the minimum attractive rate of return (ΔIRR ≥ MARR). When looking at increments of borrowing, we accept the increment when the incremental interest rate is less than or equal to the *minimum* attractive rate of return (ΔIRR ≤ MARR). One way to avoid much of the possible confusion is to organize the solution to any problem so that one is examining increments of investment. This is illustrated in the next example.

Example 7-13 (Examples 5–1 and 6–5 Revisited)

A firm is considering which of twodevices to install to reduce costs. Both devices have useful lives of 5 years and no salvage value. Device *A* costs $10,000 and can be expected to result in $3000 savings annually. Device *B* costs $13,500 and will provide cost savings of $3000 the first year but will increase $500 annually, making the second-year savings $3500, the third-year savings $4000, and so forth. For a 7% MARR, which device should the firm purchase?

TABLE SOLUTION

This problem has been solved by present worth analysis (Example 5–1) and annual cost analysis (Example 6–5). This time we will use rate of return analysis, which must be done on the incremental investment.

Year	Device *A*	Device *B*	Difference Between Alternatives: Device *B* − Device *A*
0	−$10,000	−$13,500	−$3500
1	3000	3000	0
2	3000	3500	500
3	3000	4000	1000
4	3000	4500	1500
5	3000	5000	2000

For the difference between the alternatives, write a single equation with *i* as the only unknown.

$$\mathrm{PW}(i) = 0 = -3500 + 500(P/G, i, 5)$$

$$(P/G, i, 5) = 7;\ \text{so } i \text{ is between } 9\%, (P/G, 9\%, 5) = 7.111$$

$$\text{and } 10\%, (P/G, 10\%, 5) = 6.862$$

$$i = 9\% + (10\% - 9\%)(7.111 - 7)/(7.111 - 6.862) = 9.45\%$$

The 9.45% IRR is greater than the 7% MARR; therefore, the increment is desirable. Reject Device *A* and choose Device *B*.

SPREADSHEET SOLUTION

	A	B	C	D	E	F	G
1	$3,500	Incremental initial cost for B vs. A			Year	Cash flow	
2	0	Initial incremental savings for B vs. A			0	-$3,500	
3	$500	Gradient in incremental savings for B vs. A			1	0	=A2
4	5	Horizon			2	500	=F3+$A$3
5					3	1,000	
6					4	1,500	
7					5	2,000	
8					IRR	9.44%	=IRR(F2:F7)

Example 7–15 will show how GOAL SEEK can be used to solve for the interest rate that makes two values, such as NPV_A and NPV_B, equal.

ANALYSIS PERIOD

In discussing present worth analysis and annual cash flow analysis, an important consideration is the analysis period. This is also true in rate of return analysis. The solution method for two alternatives is to examine the differences between the alternatives. Clearly, the examination must cover the selected analysis period. For now, we can only suggest that the assumptions made should reflect one’s perception of the future as accurately as possible.

In Examples 7–14 and 7–15 the analysis period is a common multiple of the alternative service lives and identical replacement is assumed. Example 7–14 explicitly includes a second Machine X that is identical to the first. Example 7–15 implicitly assumes identical repetitions until the lives match. The two approaches are shown to demonstrate their consistency.

Example 7-14

Two machines are being considered for purchase. If the MARR is 10%, which machine should be bought? Use an IRR analysis comparison.

All $ values in 1000s	Machine *X*	Machine *Y*
Initial cost	$200	$700
Uniform annual benefit	95	120
End-of-useful-life salvage value	50	150
Useful life, in years	6	12

SOLUTION

The solution is based on a 12-year analysis period and a replacement machine *X* that is identical to the present Machine *X*. The cash flow for the differences between the alternatives is as follows:

			Difference Between Alternatives (Alts):
Year	Machine X	Machine Y	Machine Y − Machine X
0	−\$200	−\$700	−\$500
1	95	120	25
2	95	120	25
3	95	120	25
4	95	120	25
5	95	120	25
6	95	120	25
	50		150
	−200		
7	95	120	25
8	95	120	25
9	95	120	25
10	95	120	25
11	95	120	25
12	95	120	25
	50	150	100

$$\text{PW} = -500 + 25(P/A, i, 12) + 150(P/F, i, 6) + 100(P/F, i, 12)$$

The cash flow sum over years 1 to 12 is \$550, which is only a little greater than the \$500 additional cost. This indicates that the rate of return is quite low. Try i = 1%.

$$\text{PW}_{1\%} = -500 + 25(11.255) + 150(0.942) + 100(0.887) = 11$$

The interest rate is too low. Try $i = 1^1/_2\,\%$:

$$\text{PW}_{1.5\%} = -500 + 25(10.908) + 150(0.914) + 100(0.836) = -6$$

The internal rate of return on the $Y - X$ increment, IRR_{Y-X}, is about 1.3%, far below the 10% minimum attractive rate of return. The additional investment to obtain Machine Y yields an unsatisfactory rate of return, therefore X is the preferred alternative.

Many problems compare mutually exclusive alternatives for a continuing requirement where the horizon is longer than the lives of the current alternatives. In Example 6–8 pumps with lives of 9 and 12 years were compared, and the pump with the lower EUAC was recommended. The discussion before the example noted that this common approach assumes identical cost repetition for a horizon equal to the least common multiple of the lives.

To analyze that problem for an incremental rate of return, a horizon of 36 years could be used with repetitions like Machine X in Example 7–14. Example 7–15 in the next section illustrates another approach.

GOAL SEEK

Examples throughout this chapter have shown that the spreadsheet functions are particularly useful in calculating internal rates of returns (IRRs). If a cash flow diagram can be reduced to at most one P, one A, and/or one F, then the RATE *annuity function* can be used. Otherwise the IRR *block function* is used with a cash flow in each period.

Example 7–15 explains the spreadsheet feature, GOAL SEEK. In general, this is used to vary one cell in a spreadsheet in order to achieve a goal for another cell. The goal may be to maximize or minimize the cell's value or to make it equal to a specified value. In this case, the goal will be to make the difference between two EUACs equal zero.

Example 7–15 shows how to calculate incremental rates of return when alternatives have different lives, and it is appropriate to assume that alternatives repeat with the same costs within the indefinite continuing requirement.

Example 7-15 (Example 6–7 Revisited)

Compare two pumps that are being used for a continuing indefinite requirement. What is the incremental rate of return for buying Pump A, which is longer lasting and more expensive?

	Pump A	Pump B
Initial cost	$7000	$5000
End-of-useful-life salvage value	1200	1000
Useful life, in years	12	6

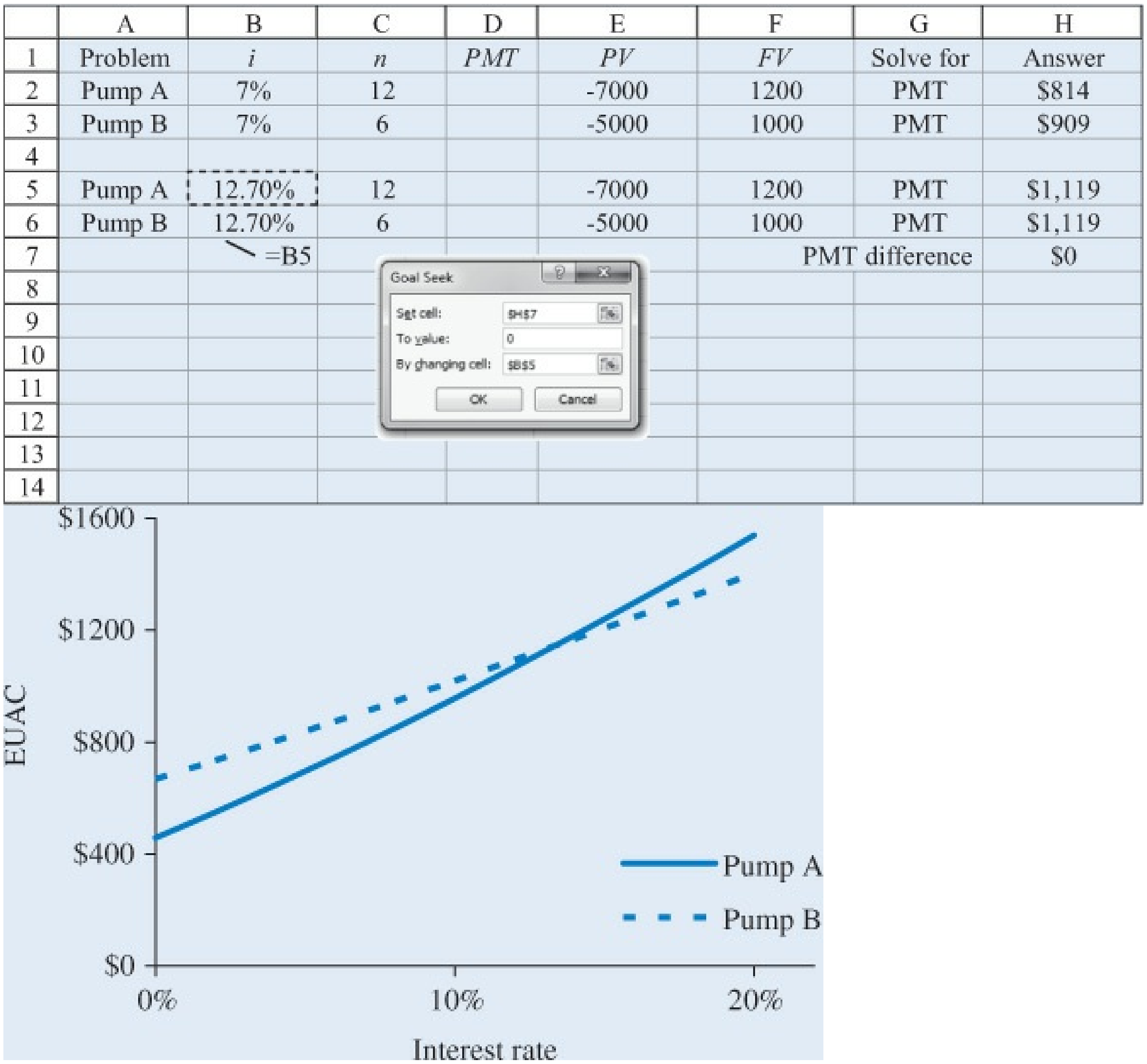
SPREADSHEET SOLUTION

The incremental rate of return is the interest rate that will make the EUACs for each pump equal. In Example 6–7 an interest rate of 7% was used, and Pump A had the lower EUAC at $814 per year (Pump B was $909). That incremental rate is what is earned on cash flows that start with an extra $2000 at time 0.

This rate of return cannot be solved for using the RATE function, since the alternatives have different lives. Instead an interest rate is specified for one alternative and then also used for the other alternative. Annual equivalent payments are calculated for each alternative, and then GOAL SEEK is used to vary the interest rate. This is much easier than building a cash flow table for the 12 years it takes for Pumps A and B to match when they end.

GOAL SEEK is found under the DATA tab in Excel by selecting WHAT-IF ANALYSIS. The 5 button calculations shown in rows 2 and 3 are the EUAC calculations for Example 6–7. Rows 5 and 6 show the result when GOAL SEEK is applied to make the two PMT values the same—the incremental rate of return is 12.70%. The difference of the EUACs is calculated in cell H7. Then GOAL SEEK changes the interest rate (B5) until the difference is 0.

Figure 7–6 graphs the EUAC for each pump for interest rates from 0% to +20%. It verifies this result. The graph in Figure 7–6 is a powerful tool for understanding the EUACs at different interest rates. The cross-over interest rate is the incremental rate of return. Graphs like this are a key tool in Chapter 8 to examine which alternatives and/or projects are preferred at different interest rates.



	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Pump A	7%	12		-7000	1200	PMT	$814
3	Pump B	7%	6		-5000	1000	PMT	$909
4								
5	Pump A	12.70%	12		-7000	1200	PMT	$1,119
6	Pump B	12.70%	6		-5000	1000	PMT	$1,119
7		=B5				PMT difference		$0
8								
9								
10								
11								
12								
13								
14								

FIGURE 7-6 Graphing present worth versus *i*.

If Pump B’s modified life of 9 years from Example 6–8 is used, then Pump B has a lower average cost and the EUAC at 7% falls to $684. The incremental rate of return for the difference between the two pumps falls to a very surprising −3.85%. This result happens because the average cost per year for Pump A (the more expensive and longer lived pump) is higher. This is true both for initial cost ($7000/12 for A is > $5000/9 for B) and for total costs ($7000 − $1200)/12 for A is > ($5000 − $1000)/9 for B).

XIRR

The tools of this chapter can solve for the rate of return—if the transaction dates are or can be approximated as uniformly spaced end-of-period cash flows. Using Excel's XIRR function, it is possible to solve for a rate of return with cash flows that occur on any set of dates. A daily interest rate is calculated, and then an effective annual interest rate is returned. This rate can be negative. There can be multiple sign changes in the cash flows, but there cannot be multiple roots (see Appendix 7A). This XIRR function is similar to the XNPV function that was used in Example 5–13.

The dates do not need to be in order, so it is possible to group cash flows that occur periodically together (for example, cash flows that occur every year on June 15, the first day of each month, or at the end of every other year). This is an easier way to build the cash flow table using copy and fill.

Example 7-16

An engineer's employer offers a 401(k) that is invested in the stock market. The engineer deposited $1500 on September 16, 2019, and another $2200 on June 8, 2020. If the account is worth $3840 on August 10, 2020, what annual rate of return has the engineer's account earned?

SOLUTION

	A	B	C
1	Dates	Cash flows	
2	9/16/2019	-1500	
3	6/8/2020	-2200	
4	8/10/2020	3840	
5			
6	8.17%	=XIRR(B2:B4, A2:A4)	
7	= XIRR(Dates, Cash flows, *guess*)		

SUMMARY

Rate of return is the interest rate *i* at which the equivalent worth of benefits and costs are equal, or the net present worth equals zero.

There are a variety of ways of writing the cash flow equation in which the rate of return *i* may be the single unknown. Five of them are as follows:

$$\text{PW of benefits} - \text{PW of costs} = 0$$

$$\frac{\text{PW of benefits}}{\text{PW of costs}} = 1$$

$$\text{NPW} = 0$$

$$\text{EUAB} - \text{EUAC} = 0$$

$$\text{PW of costs} = \text{PW of benefits}$$

Rate of return analysis: Rate of return is the most frequently used measure in industry, as the resulting rate of return is readily understood. Also, the difficulties in selecting a suitable interest rate to use in present worth and annual cash flow analysis are avoided.

Criteria

Two Alternatives

Compute the incremental rate of return—ΔIRR—on the increment of *investment* between the alternatives. Then,

- if ΔIRR ≥MARR, choose the higher-cost alternative, or,
- if ΔIRR < MARR, choose the lower-cost alternative

When an increment of *borrowing* is examined, where ΔIRR is the incremental interest rate,

- if ΔIRR ≤MARR, the increment is acceptable, or
- if ΔIRR > MARR, the increment is not acceptable

Three or More Alternatives

See Chapter 8.

Looking Ahead

Rate of return is further described in Appendix 7A. This material concentrates on the difficulties that occur with some cash flows series with multiple sign changes that may yield more than one root for the rate of return equation.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

7-1 Andrew T. invested $15,000 in a high-yield account. At the end of 30 years he closed the account and received $539,250. Compute the effective interest rate he received on the account.

SOLUTION

7-2 The heat loss through the exterior walls of a processing plant is expected to cost the owner $3000 next year. A salesman from Superfiber, Inc. claims he can reduce the heat loss by 80% with the installation of $15,000 worth of Superfiber now. If the cost of heat loss rises by $200 per year, after next year (gradient), and the owner plans to keep the building 10 years, what is his rate of return, neglecting depreciation and taxes?

SOLUTION

7-3 Does the following project have a positive or negative rate of return? Show how this is known to be true.

Investment cost	$2500
Net benefits	300 in Year 1, increasing by $200 per year
Salvage value	50
Useful life	4 years

SOLUTION

7- At what interest rate would $1000 at the end of 2020 be equivalent to

4 $2000 at the end of 2027?

SOLUTION

7-5 A piece of art, purchased three months ago for $12,000, has just been sold for $15,000. What nominal annual rate of return did the seller receive on her investment?

SOLUTION

7-6 Some time ago a young engineer obtained a mortgage at a 12% interest rate, for a total of $102,000. She has to pay 240 more monthly payments of $1049.19. As interest rates are going down, she inquires about the conditions under which she could refinance the mortgage. If the bank charges an origination fee of 2% of the amount to be financed, and if the bank and the engineer agree that the fee will be paid by combining the fee with the refinanced mortgage, what percentage rate would make refinancing her mortgage attractive, if the new mortgage terms require 120 payments?

SOLUTION

7-7 A 9.25% coupon bond issued by Gurley Gears LLC is purchased January 1, 2020, and matures December 31, 2028. The purchase price is $1079 and interest is paid semiannually. If the face value of the bond is $1000, determine the effective internal rate of return.

SOLUTION

7-8 Sain and Lewis Investment Management (SLIM), Inc. is considering the purchase of a number of bonds to be issued by Southeast Airlines. The bonds have a face value of $10,000 with an interest rate of 7.5% payable annually. The bonds will mature 10 years after they are issued. The issue price is expected to be $8750. Determine the yield to maturity (IRR) for the bonds. If SLIM Inc. requires at least a 10% return on all investments, should the firm invest in the bonds?

SOLUTION

7-9 A bond with a face value of $1000 can be purchased for $800. The bond will mature 5 years from now, and the bond dividend rate is 6%. Dividends are paid every 6 months. What effective interest rate would an investor receive if she purchased the bond?

SOLUTION

7-10 You find a car you like that costs \$18,000. The dealer is offering 0% financing for 4 years, but you must pay a financing fee of \$800.

a. What is the effective annual interest of this ‘free financing’ deal?

b. If you get a cash discount and can pay \$17,250, what is the effective annual interest rate of the financing?

a. Monthly payment = (18,000 + 900) / 48 = 393.75

18,000 = 393.75(P/A, i, 48)

(P/A, i, 48) = 18,000 / 393.75 = 45.714

(P/A, 0.25%, 48) = 45.179

(P/A, 0%, 48) = 48

Interpolating, i = 0.20% monthly

Effective annual = $(1 + 0.0020)^{12} - 1 = 0.0243$ = 2.43%

b. 17,250 = 393.75(P/A, i, 48), or (P/A, i, 48) = 17,250 / 393.75 = 43.810

(P/A, 0.25%, 48) = 45.179

(P/A, 0.5%, 48) = 42.580

Interpolating, i = 0.382% monthly

Effective annual = $(1 + 0.00382)^{12} - 1 = 0.0468$ = 4.68%

SOLUTION

7-11 Processing equipment costs \$170,000. The manufacturer offers financing at 5% interest for 5 years. If your company pays cash, the manufacturer will offer a 6% decrease in price. What effective interest rate would you be paying for the financing?

SOLUTION

7-12 Find the rate of return for a \$10,000 investment that will pay \$1000 per year for 20 years.

SOLUTION

7-13 Your company has been presented with an opportunity to invest in a project that is summarized as follows.

Investment required	\$60,000,000
Annual operating income	14,000,000
Annual operating costs	5,500,000
Salvage value after 10 years	0

The project is expected to operate as shown for 10 years. If your management requires a return of 6% on its investments before taxes, would you recommend this project based on rate of return analysis?

SOLUTION

7-14 An investment that cost $15,000 is sold after 5 years for $18,917. What is the nominal rate of return on the investment, assuming annual compounding?

SOLUTION

7-15 Isabella made an initial investment of $5000 in a trading account with a stock brokerage house. After a period of 17 months, the value of the account had increased to $6400. Assuming that there were no additions or withdrawals from the account, what was the nominal annual interest rate earned on the initial investment?

SOLUTION

7-16 Whiplash Airbags has been presented the investment opportunity summarized as follows.

Year	0	1	2	3	4	5	6	7	8
Cash flow (1000s)	$(440)	20	40	60	80	100	120	140	160

Determine the IRR for the proposed investment.

SOLUTION

7-17 You have a choice of $2000 now or $250 now with $80 a month for 2 years. What interest rate will make these choices comparable?

SOLUTION

7-18 Tri-State Tire is considering the purchase of new inflation equipment for its Martin operation. From the following cash flows associated with the new equipment, determine the IRR.

Year	Cash Flow
0	$ (2000)
1	1000
2	750
3	500
4	250
5	0

6	–250

SOLUTION

7-19 One share of Harris, Andrews, and Tatum (HAT) Enterprises was purchased 5 years ago for $10.89. Dividends of 5¢ were paid each quarter over the 5-year period of ownership. The share is sold today for $18.31. Determine the effective rate of return on the stock.

SOLUTION

7-20 If a MARR of 12% is required, which alternative should be chosen?

Year	X	Y
0	−10,000	−10,000
1	−6,000	4,000
2	8,000	4,000
3	8,000	4,000
4	8,000	4,000

SOLUTION

7-21 Water purification facilities are in the planning stage, with expected lives of 10 years. Two final plans are being compared using a MARR of 15%. Which alternative is preferred using incremental IRR?

	Alt. A	Alt. B
First cost	$1,200,000	$800,000
O&M cost	30,000	20,000
Annual benefits	350,000	250,000
Salvage value	50,000	−20,000

SOLUTION

7-22 If the firm's MARR is 10%, which alternative should be chosen assuming identical replacement?

	A	B
First cost	$10,000	$17,500
Uniform Annual Benefit	3,500	4000
Useful life, in years	5	10

SOLUTION

Lena wants to buy a small microwave oven, but wants the best deal.

7-23 Two models are available at the discount store. A higher investment is expected to return at least 20% per year. Which should she buy?

	Nuke	Zap
First cost	$45	$80
Expected life	1 year	2 years

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Rate of Return

7-1 G (*a*) A mining firm makes annual deposits of $400,000 into a reclamation fund for 25 years. If the firm must have $17 million when the mine is closed, what interest rate must the investment earn?
(*b*) The $17 million above is to be used to reclaim the negative impacts of the mine. List 6 to 10 potential environmental or community impacts that the fund might be used for.

7-2 A An engineer invests $7350 at the end of every year for a 30-year career. If the engineer wants $1 million in savings at retirement, what interest rate must the investment earn?

7-3 An investment of $5000 in Biotech common stock proved to be very profitable. At the end of 5 years the stock was sold for $25,000. What was the rate of return on the investment?

7-4 A The Diagonal Stamp Company, which sells used postage stamps to collectors, advertises that its average price has increased from $1 to $9 in the last 10 years. Thus, management states, investors who had purchased stamps from Diagonal 10 years ago would have received a 100% rate of return each year. What is the annual rate of return?

7-5 A woman went to the Beneficial Loan Company and borrowed \$10,000. She must pay \$323.53 at the end of each month for the next 60 months. What is the monthly interest rate she is paying? What effective annual interest rate is she paying?

7-6 A Helen is buying a \$12,375 car with a \$3000 down payment, followed by 36 monthly payments of \$325 each. The down payment is paid immediately, and the monthly payments are due at the end of each month. What nominal annual interest rate is Helen paying? What effective interest rate?

7-7 Your cousin Jeremy has asked you to bankroll his proposed business painting houses in the summer. He plans to operate the business for 5 years to pay his way through college. He needs \$5000 to purchase an old pickup, some ladders, a paint sprayer, and some other equipment. He is promising to pay you \$1500 at the end of each summer (for 5 years) in return. Calculate your annual rate of return.
Contributed by Paul R. McCright, University of South Florida

7-8 A Compute the rate of return for the following cash flow.

Year	Cash Flow
0	− \$7400
1	0
2	1500
3	3000
4	4500
5	6000

7-9 E Peter Minuit bought an island from the Manhattoes Indians in 1626 for \$24 worth of glass beads and trinkets. The 1991 estimate of the value of land on this island was \$12 billion.
(*a*) What rate of return would the Indians have received if they had retained title to the island rather than selling it for \$24?
(*b*) What is your view of the ethics of this transaction? Do you believe deception was involved? In what circumstances do you view deception as ethical?

The student in Problem 5-6 wanted to buy a car costing \$24,000 from a dealer offering 0% down and financing at 6% interest over 60 months. Her disposable income is $500 per month.

7-10 A (*a*) What monthly interest rate can she afford? What effective annual rate is this?

(*b*) Insurance on this car will be $50 per month more than she had planned, which will leave her with only $450 per month for her car payment. Now what monthly interest rate can she afford? What effective annual rate is this?

7-11 You invest $2500 and in return receive two payments of $1800—one at the end of 2 years and the other at the end of 5 years. Calculate the resulting rate of return.

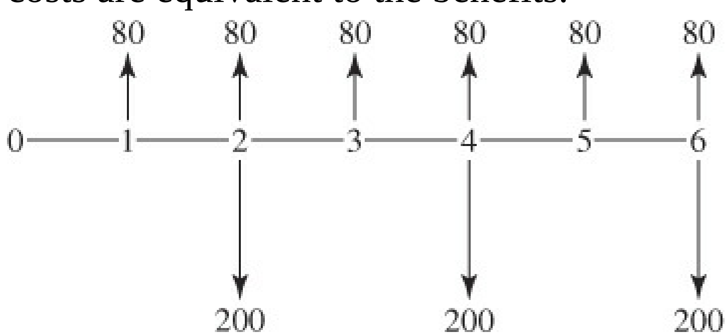
7-12 A Compute the rate of return on the following investment.

Year	Cash Flow
0	− $8000
1	0
2	2600
3	2600
4	2600
5	2600

7-13 Compute the rate of return for the following cash flow.

Year	Cash Flow
1–5	− $6, 209
6–10	10, 000

7-14 A For the following diagram, compute the interest rate at which the costs are equivalent to the benefits.



An investor has invested $250,000 in a new rental property. Her estimated annual costs are $6000 and annual revenues are $20,000.

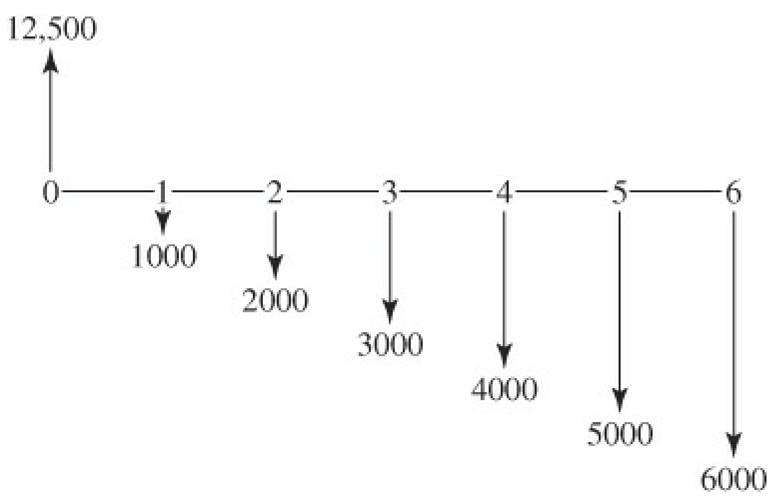
7-15 What rate of return per year will the investor make over a 30-year period ignoring the salvage value? If the property can be sold for $200,000 what is the rate of return?
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

7-16 [A](#) Installing an automated production system costing $300,000 is initially expected to save Zia Corporation $52,000 in expenses annually. If the system needs $7500 in operating and maintenance costs each year and has a salvage value of $30,000 at Year 10, what is the IRR of this system? If the company wants to earn at least 12% on all investments, should this system be purchased? *Contributed by Paul R. McCright, University of South Florida*

7-17 Compute the rate of return represented by the cash flow.

Year	Cash Flow
0	$-\$20,000$
1	6,000
2	6,500
3	7,000
4	7,500

7-18 [A](#) For the following diagram, compute the IRR.



Switching to powder coating technology will reduce the emission of volatile organic carbons (VOCs) for a firm's production process. The initial cost is $200,000 with annual costs of $50,000 and savings of $90,000 in the first year. Savings are projected to increase by

7-19 $3000 annually after Year 1. The salvage value 10 years from now is projected to be $30,000. What rate of return will the firm make on this investment?

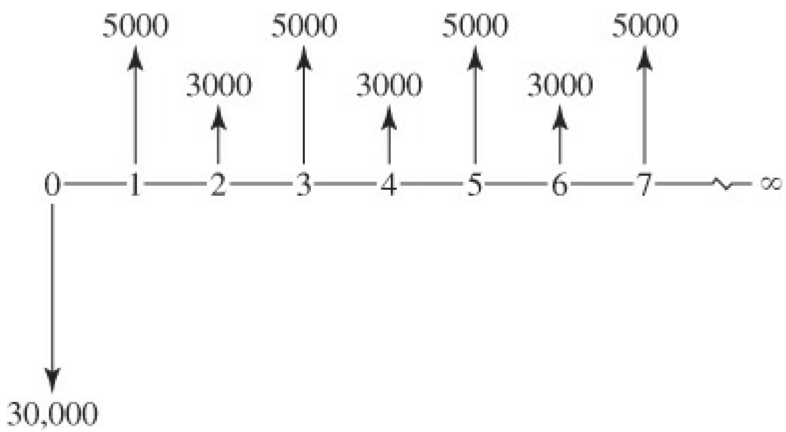
Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

7-20 A G To secure funding to convert their service vehicle fleet to hybrid technologies, MGL Industries is issuing *green bonds* to investors. You plan to buy $1 million of these bonds. This is how much you will raise by selling a vacation home that you acquired 25 years ago for $140,000.

(*a*) At what rate has the value of the vacation home increased?

(*b*) Write a short description of *green bonds*. How are they different from normal corporate, revenue, and debenture bonds? As an investor, would you buy *green bonds*? Explain.

7-21 For the following diagram, compute the rate of return on the $30,000 investment.



7-22 A Consider the following cash flow:

Year	Cash Flow
0	− $6000
1	0
2	3500
3	3000
4	2500
5	2000

Write one equation, with i as the only unknown, for the cash flow.

Use no more than two single payment compound interest factors. Solve the equation for *i*.

7-23 You have just been elected into the "Society of Honorable Engineers." First-year dues are waived in honor of your election. Thus, your first-payment of $200 is due at the end of the year, and annual dues are expected to increase 3% annually. After 40 years of paying dues you become a life member and pay no more dues. Instead of paying annual dues, however, you can pay a one-time $2000 life membership fee.
(*a*) Show the equation for determining the rate of return for buying a life membership.
(*b*) What is the rate of return?

Contributed by D. P. Loucks, Cornell University

7-24 A A bank proudly announces that it has changed its interest computation method to continuous compounding. Now $2000 left in the bank for 9 years will double to $4000. What nominal continuous interest rate is the bank paying? What effective interest rate is it paying?

Net Present Worth vs. *i* and Bonds

7-25 For Problem 7-7, graph the PW versus the interest rate for values from 0% to 50%. Is this the typical PW graph for an investment?

7-26 For Problem 7-8, graph the PW versus the interest rate for values from 0% to 50%. Is this the typical PW graph for an investment?

7-27 For Problem 7-18, graph the PW versus the interest rate for values from 0% to 50%. Is this the typical PW graph for an investment?

7-28 A A well-known industrial firm has issued $1000 bonds with a 3% coupon interest rate paid semiannually. The bonds mature 10 years from now. From the financial pages of your newspaper you learn that the bonds may be purchased for $800 each ($795 for the bond plus a $5 sales commission). What nominal and effective annual rate of return would you receive if you purchased the bond now and held it to maturity 10 years from now?

7-29 A man buys a corporate bond from a bond brokerage house for $875. The bond has a face value of $1000 and a 4% coupon rate paid semiannually. If the bond will be paid off at the end of 12 years, what rate of return will the man receive?

7-30 A
An engineer bought a $1000 bond of an American airline for $875 just after an interest payment had been made. The bond paid a 6% coupon interest rate semiannually. What nominal rate of return did the engineer receive from the bond if he held it 13.5 years until its maturity?

7-31 Mildred can purchase a municipal bond with a par (face) value of $1000 that will mature in 10 years. The bond pays 6% interest compounded quarterly. If she can buy this bond for $1050, what rate of return will she earn? *Contributed by Paul R. McCright, University of South Florida*

7-32 A
Mike buys a corporate bond with a face value of $1000 for $800. The bond matures in 10 years and pays a coupon interest rate of 5%. Interest is paid every quarter.

(*a*) Determine the effective rate of return if Mike holds the bond to maturity.

(*b*) What effective interest rate will Mike get if he keeps the bond for only 5 years and sells it for $900?

Contributed by Meenakshi Sundaram, Tennessee Tech University

7-33 An investor purchased a 5%, $1000 30-year bond for $850 with 22 years to maturity. The interest was payable quarterly. The bond was kept for only 9 years and sold for $950 immediately after the 36^{th} interest payment was received. What nominal and effective rates of return per year were made on this investment? *Contributed by Meenakshi Sundaram, Tennessee Tech University*

7-34 A
A 9%, $10,000 bond that has interest payable semiannually sells for $8500. Determine what the maturity date should be so that the purchaser may enjoy a 12% nominal rate of return on this investment. *Contributed by Meenakshi Sundaram, Tennessee Tech University*

7-35 ABC Corporation's recently issued bonds paying interest semiannually and maturing in 10 years. The face value of each bond is $1000, and 8% is the nominal interest rate.

(*a*) What is the effective interest rate an investor receives if $1000 is

paid for the bond?
(*b*) If a 1.75% fee is deducted by the brokerage firm from the initial $1000, what is the effective annual interest rate paid by ABC Corporation?

7-36 A ABC Corporation is issuing some *zero coupon bonds*, which pay no interest. At maturity in 15 years they pay a face value of $1000. The bonds are expected to sell for $275 when issued.
(*a*) What is the effective interest rate an investor receives?
(*b*) A 1.5% fee (based on the face value) is deducted by the brokerage firm from the initial revenue. What is the effective annual interest rate paid by ABC Corporation?

7-37 A zero coupon bond pays no interest—only its face value of $1000 at maturity. One such bond has a maturity of 18 years and an initial price of $130. What annual interest rate is earned if the bond is bought when issued and held to maturity?

Discounts and Fees

7-38 A The cash price of a machine tool is $12,500. The dealer is willing to accept a $3500 down payment and 24 end-of-month monthly payments of $425 each. At what effective interest rate are these terms equivalent?

7-39 E A local used car dealer calculates its "4%" financing as follows. If $3600 is borrowed to be repaid over a 3-year period, the interest charge is ($3600)(0.04)(3 years) = $432. The $432 of interest is deducted from the $3600 loan and the customer has $3168 toward the cost of a car. The customer must repay the loan with 1/36 of $3600, or $100, monthly payments for 36 months.
(*a*) What effective annual interest rate is being charged?
(*b*) What is your view of the ethics of this loan calculation? Do you believe deception is involved? In what circumstances do you view deception as ethical?

7-40 Jan purchased 100 shares of Peach Computer stock for $15 per share, plus a $14 brokerage commission. Every 6 months she received a dividend from Peach of 35 cents per share. At the end of 2 years, just

A after receiving the fourth dividend, she sold the stock for $21 per share and paid a $58 brokerage commission from the proceeds. What annual rate of return did she receive on her investment?

7-41 A used car dealer advertises financing at 0% interest over 3 years with monthly payments. You must pay a processing fee of $500 at signing. The car you like costs $9000.
(*a*) What is your effective annual interest rate?
(*b*) You believe that the dealer would accept $8200 if you paid cash. What effective annual interest rate would you be paying, if you financed with the dealer?

7-42 A A new car dealer advertises financing at 0% interest over 4 years with monthly payments or a $3000 rebate if you pay cash.
(*a*) The car you like costs $12,000. What effective annual interest rate would you be paying if you financed with the dealer?
(*b*) The car you like costs $18,000. What effective annual interest rate would you be paying if you financed with the dealer?
(*c*) The car you like costs $24,000. What effective annual interest rate would you be paying if you financed with the dealer?

7-43 A used car dealer advertises financing at 4% interest over 3 years with monthly payments. You must pay a processing fee of $500 at signing. The car you like costs $9000.
(*a*) What is your effective annual interest rate?
(*b*) You believe that the dealer would accept $8200 if you paid cash. What effective annual interest rate would be paying if you financed with the dealer?
(*c*) Compare these answers with those for Problem 7-41. What can you say about what matters the most for determining the effective interest rate?

7-44 A Some laboratory equipment sells for $85,000. The manufacturer offers financing at 6% with annual payments for 5 years for up to $65,000 of the cost. The salesman is willing to cut the price by 10% if you pay cash. What is the interest rate you would pay by financing?

A home mortgage with monthly payments for 30 years is available at 6% interest. The home you are buying costs $120,000, and you have saved $12,000 to meet the requirement for a 10% down payment. The lender charges “points” of 2% of the loan value as a loan origination

7-45 and processing fee. This fee is added to the initial balance of the loan.
(*a*) What is your monthly payment?
(*b*) If you keep the mortgage until it is paid off in 30 years, what is your effective annual interest rate?
(*c*) If you move to a larger house in 10 years and pay off the loan, what is your effective annual interest rate?
(*d*) If you are transferred in 3 years and pay off the loan, what is your effective annual interest rate?

Investments and Loans

7-46 A An investor bought a one-acre lot on the outskirts of a city for $12,700 cash. Each year she paid $175 of property taxes. At the end of 7 years, she sold the lot for a net value of $25,000. What rate of return did she receive on her investment?

7-47 A mine is for sale for $800,000. It is believed the mine will produce a profit of $250,000 the first year, but the profit will decline $25,000 a year after that, eventually reaching zero, whereupon the mine will be worthless. What rate of return would be earned on the mine?

7-48 A An apartment building in your neighborhood is for sale for $175,000. The building has four units, which are rented at $550 per month each. The tenants have long-term leases that expire in 7 years. Maintenance and other expenses for care and upkeep are $9500 annually. A new university is being built in the vicinity and it is expected that the building could be sold for $198,000 after 7 years. What is the internal rate of return for this investment?

7-49 An engineering student is deciding whether to buy multiple term length parking permits or an annual permit. The annual parking permit costs $250 due August 15th, and the semester permits are $160 due August 15th and January 15th. What is the rate of return for buying the annual permit?

7-50 G An engineering student is deciding whether to buy multiple term length parking permits or an annual permit. Using the dates and costs for your university, find the rate of return for the incremental cost of the annual permit.

7-51 Fifteen families live in Willow Canyon. Although several water wells have been drilled, none has produced water. The residents take turns driving a water truck to a fill station in a nearby town. The water is hauled to a storage tank in Willow Canyon. Last year truck and water expenses totaled $6200. What rate of return would the Willow Canyon residents receive on a new water supply pipeline costing $100,000 that would replace the truck? The pipeline is considered to last

(*a*) Forever.

(*b*) 100 years.

(*c*) 50 years.

(*d*) Would you recommend that the pipeline be installed? Explain.

7-52 A A new machine can be purchased today for $450,000. The annual revenue from the machine is calculated to be $72,000, and the equipment will last 10 years. Expect the maintenance and operating costs to be $4500 a year and to increase $750 per year. The salvage value of the machine will be $35,000. What is the rate of return for this machine?

7-53 An insurance company is offering to sell an annuity for $20,000 cash. In return the firm will guarantee to pay the purchaser 20 annual end-of-year payments, with the first payment amounting to $1100. Subsequent payments will increase at a uniform 10% rate each year (second payment is $1210; third payment is $1331, etc.). What rate of return would someone who buys the annuity receive?

7-54 A A popular magazine offers a lifetime subscription for $1000. Such a subscription may be given as a gift to an infant at birth (the parents can read it in those early years), or taken out by an individual for himself. Normally, the magazine costs $64.50 per year. Knowledgeable people say it probably will continue indefinitely at this $64.50 rate. What is the rate of return on a life subscription purchased for an infant?

7-55 A luxury car can be leased for $679 per month for 36 months. Terms are first month’s lease payment, a $625 refundable security deposit, a consumer down payment of $3500, and an acquisition fee of $725 due at lease signing. Tax, license, title fees, and insurance extra. Option to purchase at lease end for $37,775 plus a fee of $350.

Mileage charge of $0.20 per mile over 30,000 miles. Determine the interest rate (nominal and effective) for the lease. The MSRP is $64,025. You could buy the car for $58,000, if you arranged other financing.

7-56 A An engineering firm can pay for its liability insurance on an annual or monthly basis. If paid monthly, the insurance costs $4500. If paid annually, the insurance costs $50,000. What are the monthly rate of return and the nominal and effective interest rates for paying on an annual basis?

7-57 An engineering student must decide whether to pay for auto insurance on a monthly or an annual basis. If paid annually, the upfront annual cost is $2750. If paid monthly, the cost is $250 at the start of each month. What is the rate of return for buying the insurance on an annual basis expressed as an annual effective rate?

7-58 For your auto or home insurance, find out the cost of paying annually or on a shorter term. What is the rate of return for buying the insurance on an annual basis?

7-59 An investor bought 100 shares of Omega common stock for $14,000. He held the stock for 9 years. For the first 4 years he received annual end-of-year dividends of $800. For the next 4 years he received annual dividends of $400. He received no dividend for the ninth year. At the end of the ninth year he sold his stock for $6000. What rate of return did he receive on his investment?

7-60 One aspect of obtaining an engineering education is the prospect of improved future earnings in comparison to non-engineering graduates. Sharon Shay estimates that her engineering education has a $85,000 equivalent cost at graduation. She believes the benefits of her education will occur throughout 25 years of employment. She thinks that during the first 8 years out of college, her income will be higher than that of a non-engineering graduate by $25,000 per year. During the subsequent 10 years, she projects an annual income that is $35,000 per year higher. During the last 17 years of employment, she estimates an annual salary that is $52,000 above the level of the nonengineering graduate. If her estimates are correct, what rate of return will she receive as a result of her investment in an engineering education?

7-61 Upon graduation, every engineer must decide whether to go on to graduate school. Estimate the costs of going to the university full time to obtain a master of science degree. Then estimate the resulting costs and benefits. Combine the various consequences into a cash flow table and compute the rate of return. Nonfinancial benefits are probably relevant here too.

Incremental Analysis

7-62 A If 7% is considered the minimum attractive rate of return, which alternative should be selected?

Year	*A*	*B*
0	$-\$14,500$	$-\$23,000$
1	5,900	9,200
2	5,900	9,200
3	5,900	9,200

7-63 If the MARR is 8%, which alternative should be selected?

Year	*X*	*Y*
0	$-\$5000$	$-\$5000$
1	-3000	2000
2	4000	2000
3	4000	2000
4	4000	2000

7-64 A Alternatives A and B require investments of $10,310 and $13,400, respectively. For 5 years their respective net annual cash inflows are $3300 and $4000. What is the rate of return for each alternative and for the incremental difference? If the interest rate is 10%, which alternative should be selected? *Contributed by Yasser Alhenawi, University of Evansville*

Two mutually exclusive alternatives are being considered. Both have lives of 10 years. Alternative *A* has a first cost of $10,000 and annual benefits of \$4500. Alternative *B* costs \$25,000 and has annual benefits of \$8800.

7-65

If the minimum attractive rate of return is 6%, which alternative should be selected? Solve the problem by

(*a*) Present worth analysis
(*b*) Annual cash flow analysis
(*c*) Rate of return analysis

7-66 A

Two hazardous environment facilities are being evaluated, with the projected life of each facility being 10 years. The company uses a MARR of 15%. Using rate of return analysis, which alternative should be selected?

	Alt. *A*	Alt. *B*
First cost	\$615, 000	\$300, 000
O&M cost	10,000	25,000
Annual benefits	158,000	92,000
Salvage value	65,000	− 5, 000

7-67

The owner of a corner lot wants to find a use that will yield a desirable return on his investment. If the owner wants a minimum attractive rate of return on his investment of 15%, which of the two alternatives would you recommend? Both alternatives have a 20-year life and no salvage value.

	Build Fast Food Restaurant	Build Gas Station
First cost	\$800,000	\$1,200,000
Annual property taxes	30,000	50,000
Annual net income	150,000	200,000

7-68

A grocery distribution center is considering whether to invest in RFID or bar code technology to track its inventory within the warehouse and truck loading operations. The useful life of the RFID and bar code devices is projected to be 5 years with minimal or zero salvage value. The bar code investment cost is \$105,000 and can be expected to save at least \$33,000 in product theft and lost items annually. The RFID system is estimated to cost \$230,000 and will

A save $30,000 the first year, with an increase of $15,000 annually after the first year. For a 6% MARR, should the manager invest in the RFID system or the bar code system? Analyze incrementally using rate of return. *Contributed by Oliver Hedgepeth, American Public University*

7-69 A state's department of transportation (DOT) is considering whether to buy or lease an RFID tracking system for asphalt, concrete, and gravel trucks to be used in road paving. Purchasing the RFID system will cost $5000 per truck, with a salvage value of $1500 after the RFID system's useful life of 5 years. However, the DOT considering this purchase is also looking at leasing this same RFID system for an annual payment of $3500, which includes a full replacement warranty. Assuming that the MARR is 11% and on the basis of an internal rate of return analysis, which alternative would you advise the DOT to consider? Analyze incrementally using rate of return. The number of trucks used in a season varies from 5000 to 7500. Does this matter?

Contributed by Oliver Hedgepeth, American Public University

7-70 A A contractor is considering whether to buy or lease a new machine for her layout site work. Buying a new machine will cost $12, 000 with a salvage value of $1200 after the machine's useful life of 8 years. On the other hand, leasing requires an annual lease payment of $3000, which occurs at the start of each year. The MARR is 15%. On the basis of an internal rate of return analysis, which alternative should the contractor be advised to accept?

7-71 A bulldozer can be purchased for $380,000 and used for 6 years, when its salvage value is 15% of the first cost. Alternatively, it can be leased for $60,000 a year. (Remember that lease payments occur at the start of the year.) The firm's interest rate is 12%.

(*a*) What is the interest rate for buying versus leasing? Which is the better choice?

(*b*) If the firm will receive $65,000 more each year than it spends on operating and maintenance costs, should the firm obtain the bulldozer? What is the rate of return for the bulldozer using the best financing plan?

7-72 A

A diesel generator for electrical power can be purchased by a remote community for $350,000 and used for 10 years, when its salvage value is $50,000. Alternatively, it can be leased for $42,500 a year. (Remember that lease payments occur at the start of the year.) The community's interest rate is 7%.

(*a*) What is the interest rate for buying versus leasing? Which is the better choice?

(*b*) The community will spend $80,000 less each year for fuel and maintenance, than it currently spends on buying power. Should it obtain the generator? What is the rate of return for the generator using the best financing plan?

7-73

After 5 years of working for one employer, you accept a new job. Your retirement account total is $60,000. Half is "yours" and half is your employer's matching contributions. You now have two alternatives. (1) You may leave both contributions in the fund until retirement in 35 years, when you will receive its future value at 4% interest. (2) You may take out the total value of "your" contributions. You can do as you wish with the money you take out, but the other half will be lost as far as you are concerned. Which alternative is more attractive? Why?

7-74 A

Three mutually exclusive alternatives are being considered. All have a 10-year useful life. If the MARR is 12%, which alternative is preferred?

	A	B	C
Initial cost	$50,000	$54,000	$60,000
Uniform annual benefit	8,000	9,500	10,750

7-75

If the minimum attractive rate of return is 14%, which alternative should be selected?

Year	*w*	*x*	*Y*	*Z*
0	–$1000	–$500	–$1200	–$1500
1	350	165	420	500
2	350	165	420	500
3	350	165	420	500
4	350	165	420	500

Analysis Period

7-76 A

If the minimum attractive rate of return is 7%, which alternative should be selected assuming identical replacement?

	A	*B*
First cost	$5000	$9200
Uniform annual benefit	1750	1850
Useful life, in years	4	8

7-77

Jean has decided it is time to buy a new battery for her car. Her choices are:

	Zappo	Kicko
First cost	$48	$78
Guarantee period, in months	12	24

Jean believes the batteries can be expected to last only for the guarantee period. She does not want to invest extra money in a battery unless she can expect a 40% rate of return. If she plans to keep her present car another 2 years, which battery should she buy?

7-78 A

Two investment opportunities are as follows:

	A	*B*
First cost	$200	$100
Uniform annual benefit	25	20
End-of-useful-life salvage value	45	0
Useful life, in years	15	10

At the end of 10 years, Alt. *B* is not replaced. Thus, the comparison is 15 years of *A* versus 10 years of *B*. If the MARR is 8%, which alternative should be selected?

Spreadsheets

The Southern Guru Copper Company operates a large mine in a South American country. A legislator in the National Assembly said in a speech that most of the capital for the mining operation was provided by loans from the World Bank; in fact, Southern Guru has only $1.5 million of its own money actually invested in the property. The cash flow for the mine is:

7-79

Year	Cash Flow
0	$−0.5M
1	0.9M
2	3.5M
3	3.9M
4	8.6M
5	4.3M
6	3.1M
7	1.2M

The legislator divided the $25.5 million total profit by the $1.5 million investment. This produced, he said, a 1700% rate of return on the investment. Southern Guru, claiming the actual rate of return is much lower, asks you to compute it.

7-80 A A young engineer's starting salary is $52,000. The engineer expects annual raises of 3%. The engineer will deposit 10% of the annual salary at the end of each year in a savings account that earns 4%. How much will the engineer have saved for starting a business after 15 years? We suggest that the spreadsheet include at least columns for the year, the year's salary, the year's deposit, and the year's cumulative savings.

7-81 A young engineer's starting salary is $75,000. The engineer expects annual raises of 2%. The engineer will deposit 15% of annual salary at the end of each year in an investment account that averages 6% interest. How much will the engineer have saved for retirement after 40 years?

7-82 A A young engineer's starting salary is $65,000. The engineer expects annual raises of 2.5%. The engineer will deposit a constant percentage of annual salary at the end of each year in a savings account that earns 5%. What percentage must be saved so that there will be $1 million in savings for retirement after 35 years? (*Hint*: Use GOAL SEEK: see last section of Chapter 7.)

7-83 Find the average starting engineer’s salary for your discipline. Find and reference a source for the average annual raise you can expect. If you deposit 10% of your annual salary at the end of each year in a savings account that earns 4%, how much will you have saved for retirement after 40 years?

7-84 A An engineer believes a firm is well managed with an exciting new technology. He purchased \$4500 worth of its stock on March 14 and another \$3500 on June 17. It is now November 15, and the stock is worth \$8000. Use the XIRR function to find the IRR. If the stock is worth \$7000, what is the IRR?

7-85 An investment has the following cash flows. Use the XIRR function to find the IRR.

Date	Cash Flow
12/11/2017	−\$10,000
1/8/2018	50
7/2/2018	300
1/7/2019	400
7/1/2019	500
1/6/2020	10,700

7-86 A A cold remedy’s cash flows for one season’s cycle are shown below. Use XIRR to find the IRR.

Cash Flow	Date (in \$M)
6/1/2018	−\$40
9/1/2018	10
11/1/2018	8
12/17/2018	6
1/7/2019	10
2/25/2019	6
4/1/2019	3

Use the XIRR function to find the IRR.

Date	Cash Flow
4/16/2018	−\$5000

	12/10/2018	1500
7-87	4/1/2019	1800
	7/22/2019	2000
	11/11/2019	1900
	2/4/2020	1500
	6/2/2020	1200

Minicases

7-88 Some lenders charge an up-front fee on a loan, which is subtracted from what the borrower receives. This is typically described as "points" (where one point equals 1% of the loan amount). The federal government requires that this be accounted for in the APR that discloses the loan's cost.

(*a*) A 5-year auto loan for $18,000 has monthly payments at a 9% nominal annual rate. If the borrower must pay a loan origination fee of 2 points, what is the true effective cost of the loan? What would the APR be?

(*b*) If the car is sold after 2 years and the loan is paid off, what is the effective interest rate and the APR?

(*c*) Graph the effective interest rate as the time to sell the car and pay off the loan varies from 1 to 5 years.

7-89 Some lenders charge an up-front fee on a loan, which is added to what the borrower owes. This is typically described as "points" (where one point equals 1% of the loan amount). The federal government requires that this be accounted for in the APR that discloses the loan's cost.

(*a*) A 30-year mortgage for $220,000 has monthly payments at a 6% nominal annual rate. If a borrower's loan origination fee is 3% (3 points) and it is added to the initial balance, what is the true effective cost of the loan? What would the APR be?

(*b*) If the house is sold after 6 years and the loan is paid off, what is the effective interest rate and the APR?

(*c*) Graph the effective interest rate as the time to sell the house and pay off the loan varies from 1 to 15 years.

APPENDIX 7A
Difficulties in Solving for an Interest Rate

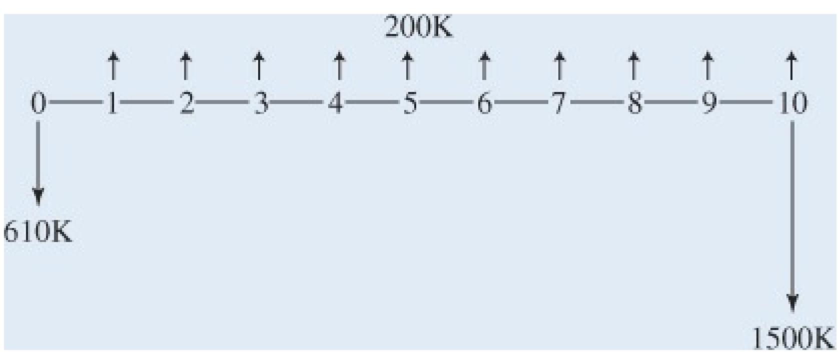
After completing this chapter appendix, students should be able to:

• Describe why some projects' cash flows cannot be solved for a single positive interest rate.
• Identify the multiple roots if they exist.
• Evaluate whether the multiple roots are a problem for decision making.
• Use the *modified internal rate of return (MIRR)* methodology in multiple-root cases.

Example 7A–1 illustrates the situation.

Example 7A-1 (Example 5–11 Revisited)

A piece of land may be purchased for $610,000 to be strip-mined for the underlying coal. Annual net income will be $200,000 for 10 years. At the end of 10 years, the surface of the land will be restored as required by a federal law on strip-mining. The reclamation will cost $1.5 million more than the land's resale value after it is restored. Is this a desirable project, if the minimum attractive rate of return is 10%?



SOLUTION

In Example 5–11 the NPW was calculated to be $41,000. Since the NPW is positive, the project seems desirable. However, at interest rates below 4.07% or above 18.29% (see Figure 7A–1), the NPW is negative, indicating an undesirable project. This indicates that the project is undesirable at 4% and desirable at 10% and not at 20%. This does not make sense.

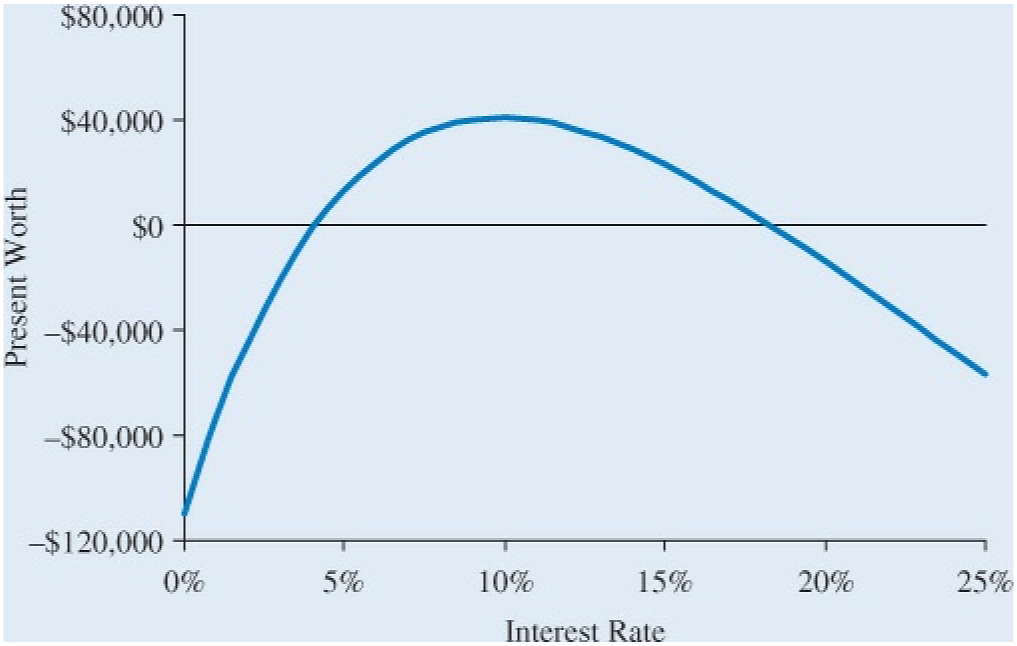


FIGURE 7A-1 Two positive roots.

The results warn us that NPW may not always be a proper criterion for judging whether or not an investment should be undertaken. In this example the disbursements ($610,000 + $1,500,000) exceed the benefits (10 × $200,000), which certainly does not portray a desirable investment. Thus Example 5–11 shows that NPW calculations in certain infrequent conditions can lead to unreliable results.

As outlined in this appendix, the problem is in the cash flows. It is *not* solved by relying on present worth as an economic measure over a confusing double root for the rate of return.

Example 7A–1 demonstrates that cash flows can have multiple solutions to the IRR equation (PW = 0). The example also demonstrates a common feature of such a problem—the cash flows fit neither the pattern of an investment nor the pattern of a loan. Both investments and loans have a single change in the sign of the cash flows over time. In contrast, Example 7A–1 has two sign changes in its series of cash flows.

In Example 7A–1 the open pit mine has a negative initial cash flow, which is followed by a series of positive cash flows—the pattern of an investment. However, the cash flows at the project's end follow the pattern of a loan—initial cash flows are positive and then the final cash flow for remediation is negative. Thus, Example 7A–1 is neither a loan nor an investment, but rather a combination. Thus it is impossible to know whether it is desirable to have a high interest rate for an investment or a low interest rate for a loan.

Two or more sign changes in the series of cash flows are the distinguishing feature of instances where multiple roots can but *may not* arise when solving for the rate of return for a set of cash flows.

Examples like the cash flow diagram with a first cost, uniform return, and final remediation expense have been thoroughly analyzed (Eschenbach, Baker, and Whittaker). Only problems with a very large final cost have double positive roots. More important, due to the uncertain remediation expenses being much larger than the initial costs, the sensitivity of the results means that the PW values seem also to be unreliable guides for decision making when there are two positive roots for the PW = 0 equation.

WHAT TO DO IF CASH FLOW DIAGRAM HAS TWO OR MORE SIGN CHANGES

Traditional coverage of multiple roots starts with Descartes' rule of signs for roots of a polynomial and often includes references to the tools of decades of research. This tends to focus attention on hypothetical possibilities. A simpler approach is to use the power of spreadsheets and graph present worth values over an appropriate range of negative and positive interest rates.

This approach identifies

• Interest rates where PW = 0.
• Present worth values for positive interest rates.
• Situations where a single positive interest rate is a useful internal rate of return (IRR).
• Situations where the pattern of present worth values suggest that neither PW nor IRR is reliable for decision making.

In Example 7A–1, graphing the present worth over a range of interest rates from 0% to 25% was sufficient to show that there were two positive roots of about 4% and 18%. The values of present worth did not behave as investments or loans that this text has analyzed. As explained after Example 7A–1, this happened because the cash flows are neither an investment nor a loan, but a combination. The solution to Example 7A–1 noted that the total of the cash flows is −$110,000, which is not a sign of a good investment.[1]

In general, the range (−100%, 100%] includes the interest rates and present worth values that are useful for decision making. Note that present value calculations can only approach the lower limit of $i = -1$, since the denominator $1 + i$ equals 0 at that limit. For the suggested upper limit there do not seem to be occasions involving multiple roots where interest rates of over 100% are earned over an engineering project's life. Unrealistic cash flow examples with very high rates have been created and published.

As will be detailed in several examples, when multiple roots occur there is usually a negative root that can be ignored and a positive root that can be used. When this positive root is used as the IRR it implies the exact same decision as a PW measure that appears to be reliable. If there is only one positive root, then it is a valid and useful IRR.

If there is only one root and it is negative, then it is a valid IRR. One example of a valid negative IRR comes from a bad outcome, such as the IRR for a failed research and development project. Another example would be a privately sponsored student loan set up so that if a student volunteers for the Peace Corps, only half of the principal need be repaid.

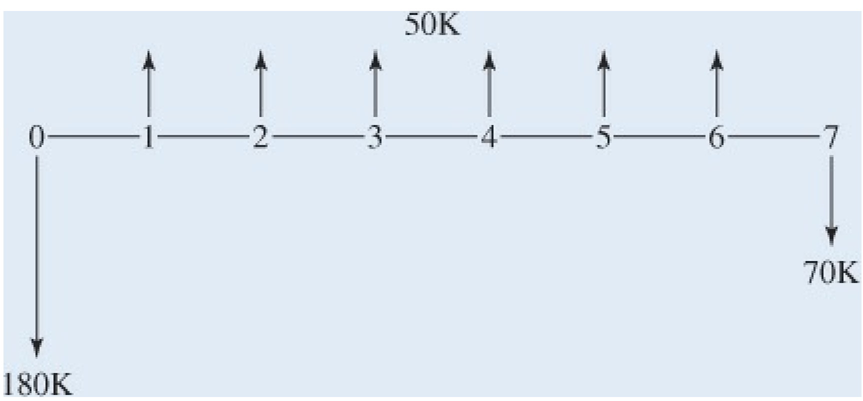
If a project has a negative and a positive root for i, then, as stated for most projects only the positive root of *i* is used.

PROJECTS WITH MULTIPLE SIGN CHANGES

The most common case of two or more sign changes in cash flows is projects with a salvage cost which typically have two sign changes. This salvage cost can be large for environmental restoration at termination. Examples include pipelines, open-pit mines, and nuclear power plants. Example 7A–2 is representative.

Example 7A-2

This project is representative of ones with a salvage cost. How many roots for the PW equation exist?



SOLUTION

Figure 7A–2 shows the spreadsheet calculations and the graph of PW versus *i*. In this case, there is one positive root of 10.45%. The value can be used as an IRR. There is also a negative root of $i = -38.29\%$. This root is not useful. Few firms would accept a project with only a 10% rate of return.

	A	B	C	D	E	F	G
1	Year	Cash Flow		*i*	PW		
2	0	−180		−40%	−126.39	=$B$2+NPV(D2,$B$3:$B$9)	
3	1	50		−30%	219.99		
4	2	50		−20%	189.89		
5	3	50		−10%	114.49		
6	4	50		0%	50.00		
7	5	50		10%	1.84		
8	6	50		20%	−33.26		
9	7	−70		30%	−59.02		
10				40%	−78.24		
11	IRR	10.45%		50%	−92.88		
12	root	−38.29%					

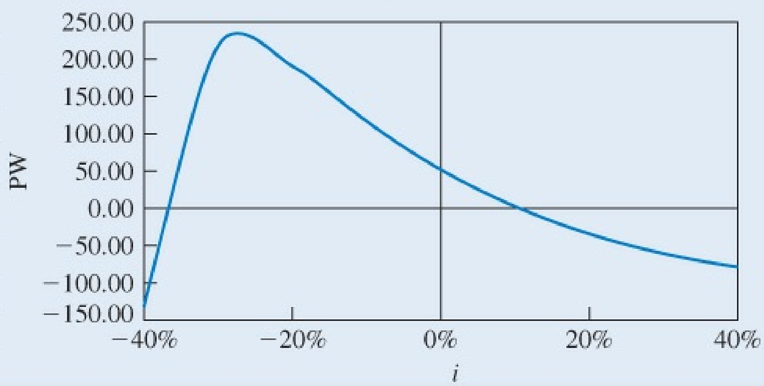


FIGURE 7A-2 PW versus *i* for project with salvage cost.

Examples like the cash flow diagram with a first cost, uniform return, and final remediation expense have been thoroughly analyzed (see footnote 1). Only problems with a very large final cost have double positive roots. More importantly, because the uncertain remediation expenses are much larger than the initial costs, the sensitivity of the results means that the PW values seem also to be unreliable guides for decision making when there are two positive roots for the PW = 0 equation.

Many enhancement projects for existing mines and deposits have a pattern of two sign changes. Example 7A–3 describes an oil well in an existing field. The initial investment recovers more of the resource and speeds recovery of resources that would have been recovered eventually. The resources shifted for earlier recovery can lead to two sign changes.

In Example 7A–4, we consider staged construction, where three sign changes are common but a single root is also common.

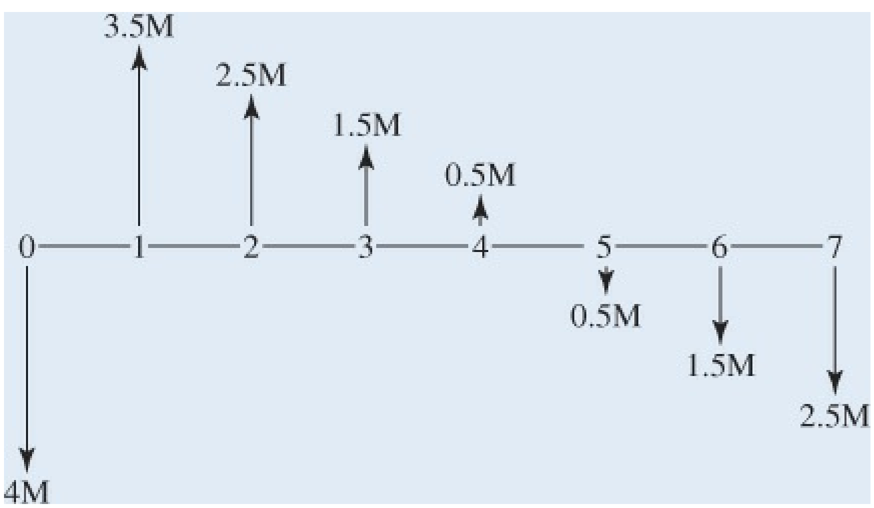
Other examples with multiple sign changes and a single root can be found in comparisons of alternatives with unequal lives. [2]

Example 7A-3

Adding an oil well to an existing field costs $4 million (4M). It will increase recovered oil by $3.5M, and it shifts $4.5M worth of production from Years 5, 6, and 7 to earlier years. Thus, the cash flows for Years 1 through 4 total $8M and Years 5 through 7 total −$4.5M. If the well is justified, one reason is that the oil is recovered sooner. How many roots for the PW equation exist? Is one useful as an IRR, and should the project be funded?

SOLUTION

The first step is to draw the cash flow diagram and count the number of sign changes. The following pattern is representative, although most wells have a longer life.



There are two sign changes, thus there may be multiple positive roots for the PW = 0 equation. The additional recovery corresponds to an investment, and the shifting of recovery to earlier years corresponds to a loan (positive cash flow now and negative later). Thus, the oil wells are neither an investment nor a loan; they are a combination of both.

Figure 7A–3 shows the spreadsheet calculations and the graph of PW versus *i*. In this case, there are positive roots at 4.73 and 37.20%. These roots are not useful. This project is a combination of an investment and a loan, so we don't even know whether we want a high rate or a low rate. If our interest rate is about 20%, then the project has a positive PW. However, small changes in the data can make for large changes in these results. At this stage no certain recommendation can be made.

It is useful to apply the modified internal rate of return described in the last section.

	A	B	C	D	E	F	G
1	Year	Cash Flow		*i*	PW		
2	0	−4.00		0%	−0.50	=$B$2+NPV(D2,$B$3:$B$9)	
3	1	3.50		5%	0.02		
4	2	2.50		10%	0.28		
5	3	1.50		15%	0.37		
6	4	0.50		20%	0.36		
7	5	−0.50		25%	0.29		
8	6	−1.50		30%	0.19		
9	7	−2.50		35%	0.06		
10				40%	−0.08		
11	root	4.73%		45%	−0.22		
12	root	37.20%		50%	−0.36		
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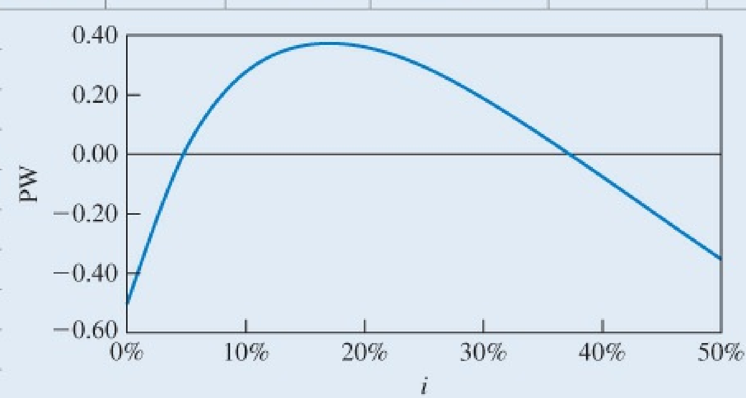


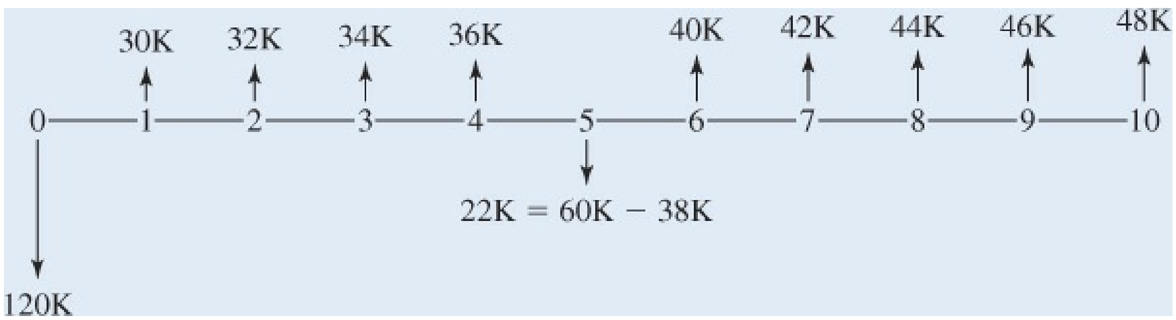
FIGURE 7A-3 PW versus *i* for oil well.

Example 7A-4

A project has a first cost of $120,000. Net revenues begin at $30,000 in Year 1 and then increase by $2000 per year. In Year 5 the facility is expanded at a cost of $60,000 so that demand can continue to expand at $2000 per year. How many sign changes in the cash flows are there? How many roots for the PW equation are there? Is one a useful IRR, and what decision is recommended?

SOLUTION

The first step is to draw the cash flow diagram. Then counting the three sign changes is easy.



With three sign changes, there may be multiple roots for the PW = 0 equation.

Figure 7A–4 shows the spreadsheet calculations and the graph of PW versus *i*. In this case, there is one positive root of 21.69%. The value can be used as an IRR, and the project is attractive.

	A	B	C	D	E	F	G
1	Year	Cash Flow		*i*	PW		
2	0	−120		−80%	579,646,330	=$B$2+NPV(D2,$B$3:$B$12)	
3	1	30		−60%	735,723		
4	2	32		−40%	17,651		
5	3	34		−30%	1460		
6	4	36		0%	210		
7	5	−22		10%	73		
8	6	40		20%	7		
9	7	42		30%	−28		
10	8	44		40%	−48		
11	9	46		50%	−62		
12	10	48		60%	−71		
13				70%	−78		
14	IRR	21.69%		80%	−83		
15				90%	−87		
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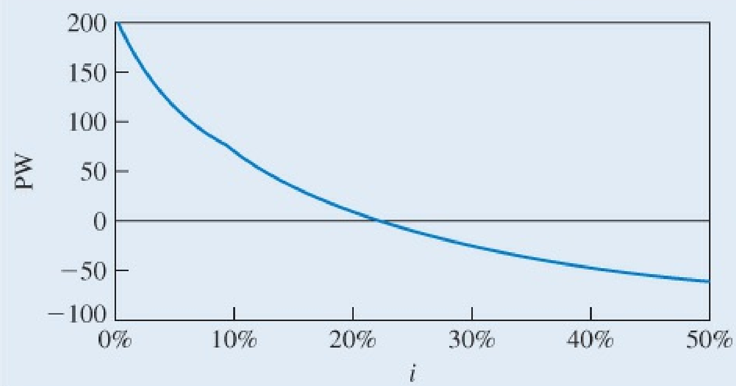


FIGURE 7A-4 PW versus *i* for project with staged construction.

MODIFIED INTERNAL RATE OF RETURN (MIRR)

Two external rates of return can be used to ensure that the resulting equation is solvable for a unique rate of return—the MIRR. The MIRR is a measure of the attractiveness of the cash flows, but it is also a function of the two external rates of return.

The rates that are *external* to the project's cash flows are (1) the rate at which the organization normally invests and (2) the rate at which it normally borrows. These are external rates for investing, e_{inv}, and for financing, e_{fin}. Because profitable firms invest at higher rates than they borrow at, the rate for investing is generally higher than the rate for financing. Sometimes a single external rate is used for both, but this requires the questionable assumption that investing and financing happen at the same rate.

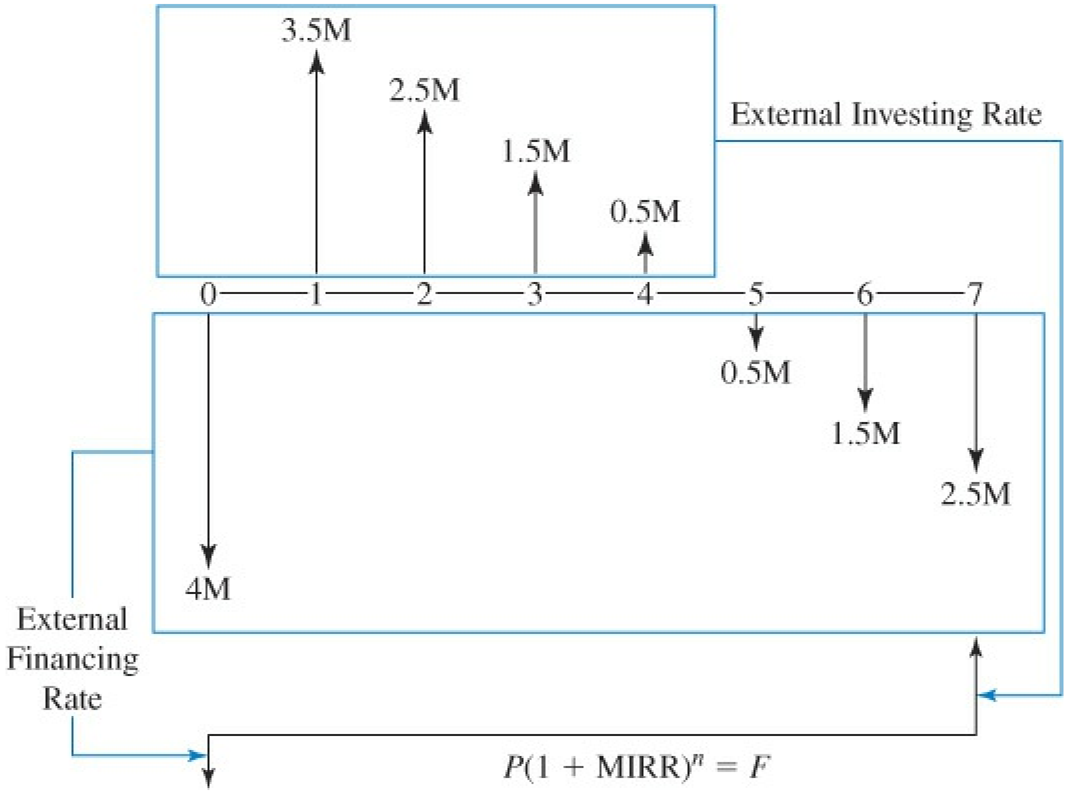


FIGURE 7A-5 MIRR for the oil well.

The approach is:

1. Combine cash flows in each period (t) into a single net receipt, R_t, or net expense, E_t.
2. Find the present worth of the expenses with the financing rate.
3. Find the future worth of the receipts with the investing rate.
4. Find the MIRR which makes the present and future worths equivalent.

The result is Equation 7A-1. This equation will have a unique root, since it has a single negative present worth and a single positive future worth. There is only one sign change in the resulting series.

$$(F/P, \text{MIRR}, n) \sum_t E_t(P/F, e_{\text{fin}}, t) = \sum_t R_t(F/P, e_{\text{inv}}, n-t) \quad (7A.1)$$

There are other external rates of return, but the MIRR has historically been the most clearly defined. All of the external rates of return are affected by the assumed values for the investing and financing rates, so none are a *true* rate of return on the project's cash flow. The MIRR also has an Excel function, so it now can easily be used. Example 7A–5 illustrates the calculation, which is also summarized in Figure 7A–5.

Example 7A-5 (Example 7A–3 Revisited)

Adding an oil well to an existing field had the cash flows summarized in Figure 7A–5. If the firm normally borrows money at 8% and invests at 15%, find the modified internal rate of return (MIRR).

SOLUTION

Figure 7A–6 shows the spreadsheet calculations.

	A	B	C	D	E	F	G	H	I
1	8%	external financing rate							
2	15%	external investing rate							
3	Year	0	1	2	3	4	5	6	7
4	Cash Flow	−4.00	3.50	2.50	1.50	0.50	−0.50	−1.50	−2.50
5	13.64%		Cell A5 contains = MIRR(B4:I4,A1,A2)						

FIGURE 7A-6 MIRR for oil well.

It is also possible to calculate the MIRR by hand. While more work, the process does clarify what the MIRR function is doing.

1. Each period's cash flow is already a single net receipt or expenditure.
2. Find the present worth of the expenses with the financing rate.

$$\begin{aligned} \text{PW} &= -4\text{M} - 0.5\text{M}(P/F, 8\%, 5) - 1.5\text{M}(P/F, 8\%, 6) - 2.5\text{M}(P/F, 8\%, 7) \\ &= -4\text{M} - 0.5\text{M}(0.6806) - 1.5\text{M}(0.6302) - 2.5\text{M}(0.5835) = -6.744\text{M} \end{aligned}$$

3. Find the future worth of the receipts with the investing rate.

$$\begin{aligned} \text{FW} &= 3.5\text{M}(F/P, 15\%, 6) + 2.5\text{M}(F/P, 15\%, 5) + 1.5\text{M}(F/P, 15\%, 4) + 0.5\text{M}(F/P, 15\%, 3) \\ &= 3.5\text{M}(2.313) + 2.5\text{M}(2.011) + 1.5\text{M}(1.749) + 0.5\text{M}(1.521) = 16.507\text{M} \end{aligned}$$

4. Find the MIRR that makes the present and future worths equivalent.

$$\begin{aligned} 0 &= (1 + \text{MIRR})^n(\text{PW}) + \text{FW} \\ 0 &= (1 + \text{MIRR})^7(-6.744\text{M}) + 16.507\text{M} \\ (1 + \text{MIRR})^7 &= 16.507\text{M}/6.744\text{M} = 2.448 \\ (1 + \text{MIRR}) &= 2.448^{1/7} = 1.1364 \\ \text{MIRR} &= 13.64\% \end{aligned}$$

The MIRR does allow calculation of a rate of return for *any* set of cash flows. However, the result is only as realistic as the external rates that are used. The MIRR value can depend as much on the external rates that are used, as it does on the cash flows that it is describing.

SUMMARY

In cash flows with more than one sign change, we find that solving the cash flow equation can result in more than one root for the rate of return. Typical situations include a new oil well in an existing field, a project with a significant salvage cost, and staged construction.

In a sign change, successive nonzero values in the cash flow have different signs (that is, they change from + to −, or vice versa). Zero sign changes indicates there is no rate of return, as the cash flow is either all disbursements or all receipts.

One sign change is the usual situation, and a single positive rate of return generally results. There will be a negative rate of return whenever loan repayments are less than the loan or an investment fails to return benefits at least equal to the investment.

Multiple sign changes may result in multiple positive roots for *i*. When they occur, none of the positive multiple roots is a suitable measure of the project's economic desirability. The PW may also not be a reliable guide for decision making. If multiple positive roots are identified by graphing the present worth versus the interest rate, then the modified internal rate of return can be used to evaluate the project.

Graphing the present worth versus the interest rate ensures that the analyst recognizes that the cash flow has multiple sign changes. Otherwise a rate could be found and used that is not in fact a meaningful descriptor of the project. Graphing the present worth also identifies the much more common situation with multiple roots where one root is negative and the other positive. The negative root can be ignored for decision making. The positive root can be used for the IRR—with results identical to PW for decision making.

The modified internal rate of return (MIRR) relies on rates for investing and borrowing that are external to the project. The number of sign changes are reduced to one, ensuring that the MIRR can be found.

Table 7A–1 summarizes the number of roots and the appropriate action for the IRR and MIRR methods.

Table 7A–1 Multiple Roots and the IRR

Roots	Action
1 positive root	Accept the answer as the IRR
1 negative root	Accept the answer as the IRR
1 positive and 1 or more negative roots	Accept the positive root as the IRR Ignore the negative roots
2 or more positive roots	Use MIRR

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

Do these cashflows have a unique IRR?

7A-1

Year	Cash Flow
0	−$20,000
1	10,000
2	−8,000
3	12,000
4	20,000

SOLUTION

Do these cashflows have a unique IRR? Should the project be built if a firm’s MARR is 25%?

7A-2

Year	Cash Flow
0	−$1250
1	2000
2	−400

SOLUTION

Do these cashflows have a unique IRR? Should the project be built if a firm’s MARR is 25%? If a MIRR is needed use 4% as the borrowing rate and 10% as the investing rate.

7A-3

Year	Cash Flow
0	−$9000
1	8000

2	5000
3	−6000

SOLUTION

Do these cashflows have a unique IRR? Should the project be built if a firm’s MARR is 25%? If a MIRR is needed use 4% as the borrowing rate and 10% as the investing rate.

7A-4

Year	Cash Flow
0	−$2000
1	7200
2	−8500
3	3300

SOLUTION

Find modified internal rate of return (MIRR) if a firm finances money at 2% and invests money at 8%

7A-5

Year	Cash Flow
0	–$1000
1	250
2	–250
3–5	425

SOLUTION

Find the modified internal rate of return (MIRR) if Fly-By-Night Shipping finances money at 4% and invests money at 10%.

7A-6

Year	Cash Flow
0	$–800,000
1	–150,000

2	250,000
3	–100,000
4	400,000
5–10	250,000

SOLUTION

Find the modified internal rate of return (MIRR) if a firm finances money at 5% and invests money at 10%

7A-7

Year	Cash Flow
0	–$10,000
1	–25,000
2–7	12,000
8	–14,250

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Unless the problem asks a different question or provides different data: (1) determine how many roots are possible and (2) graph the PW versus the interest rate to see whether multiple roots occur. If the root is a unique IRR, it is the project's rate of return. If there are multiple roots, then use an *external investing rate* of 12% and an *external borrowing rate* of 6%. Compute and use the MIRR as the project's rate of return.

Given the following cash flow, determine the rate of return on the investment.

7A-1

Year	Cash Flow
0	− $1000
1	3400
2	− 5700
3	3800

7A-2 A

Find the rate of return for the following cash flow:

Year	Cash Flow
0	−$25,000
1	15,000
2	− 4, 000
3	8,000
4	15,000

7A-3

Given the following cash flow, determine the rate of return on the investment.

Year	Cash Flow
0	− $5, 000
1	3,500
2	− 5, 000
3	11,000

7A-4 A

Given the following cash flow, determine the rate of return on the project.

Year	Cash Flow
0	− $1000
1	2750
2	− 1200
3	− 600

7A-

(*a*) Determine the rate of return on the investment for the following cash flow.

Year	Cash Flow
0	− $1000

5 E	1	3600
	2	− 4280
	3	1680

(*b*) If the firm's interest rate is 25%, is it ethical to recommend this investment?

7A-6 A

A firm invested $15,000 in a project that appeared to have excellent potential. Unfortunately, there was a lengthy labor dispute in Year 3. Compute the project's rate of return.

Year	Cash Flow
0	− $20, 000
1	12, 500
2	6, 000
3	− 9, 500
4	5, 000
5	5, 000
6	5, 000

7A-7

Compute the rate of return for this project.

Year	Cash Flow
0	−$16,000
1	−8,000
2	11,000
3	13,000
4	−7,000
5	14,000

7A-8 A

Determine the rate of return on the investment on the following cash flow.

Year	Cash Flow
0	− $4500
1–3	1500
4	− 4250
5–8	1850

7A-9

What is the rate of return associated with this project?

Year	Cash Flow
0	−$225,000
1	85,000
2	65,000
3	59,000
4	− 35, 000
5	−10,000
6	38,000
7	50,000

7A-10 A

A project has been in operation for 5 years. Calculate the rate of return and state whether it has been an acceptable rate of return.

Year	Cash Flow
0	−$103,000
1	102,700
2	−87,000
3	94,500
4	−8,300
5	38,500

7A-11

Bill bought a vacation lot he saw advertised on television for $9000 down and monthly payments of $500. When he visited the lot, he found it was not something he wanted to own. After 40 months he was finally able to sell the lot. The new owner assumed the balance of the loan on the lot and paid Bill $19,000. What rate of return did Bill receive on his investment?

7A-12 A

The project, which had a projected life of 5 years, was terminated early. Compute the interest rate.

Year	Cash Flow
0	− $7500
1	2000
2	2000

3	2000

Compute the rate of return on the investment.

7A-13 A E

Year	Cash Flow
0	− *$*5000
1	7500
2	1500
3	− 6000

Consider the following cash flow.

7A-14

Year	Cash Flow
0	− *$*100
1	240
2	− 143

(*a*) If the minimum attractive rate of return is 12%, should the project be undertaken?
(*b*) If the firm's interest rate is 25%, is it ethical to recommend this investment?

Compute the rate of return on an investment having the following cash flow.

7A-15

Year	Cash Flow
0	− *$* 1000
1	750
2–9	300
10	− 2500

Consider the following situation. What is the rate of return?

7A-16 A

Year	Cash Flow
0	− *$*200
1	350
2	− 100

An investor is considering two mutually exclusive projects. She can obtain a 6% before-tax rate of return on external investments, but she requires a minimum attractive rate of return of 7% for these projects. Use a 10-year analysis period to compute the incremental rate of return from investing in Project *A* rather than Project *B*.

		Project *A* : Build Drive-Up Photo Shop	Project *B*: Buy Land in Hawaii
7A-17	Initial capital investment	$58,500	$ 48,500
	Net uniform annual income	6648	0
	Salvage value 10 years hence	30,000	138,000
	Computed rate of return	8%	11%

7A-18 A

Compute the rate of return on the investment on the following cash flow.

Year	Cash Flow
0	-$1200
1–5	358
6	-200

7A-19

A problem often discussed in the engineering economy literature is the “oil-well pump problem.” Pump 1 is a small pump; Pump 2 is a larger pump that costs more, will produce slightly more oil, and will produce it more rapidly. If the MARR is 20%, which pump should be selected? Assume that any temporary external investment of money earns 10% per year and that any temporary financing is done at 6%.

Year	Pump 1 ($000s)	Pump 2 ($000s)
0	−$100	−$110
1	70	115
3	70	30

In January 2013, an investor bought a convertible debenture bond issued

7A-20 [A] by the XLA Corporation. The bond cost $1000 and paid $60 per year interest in annual payments on December 31. Under the convertible feature of the bond, it could be converted into 20 shares of common stock by tendering the bond, together with $400 cash. The next business day after the investor received the December 31, 2015, interest payment, he submitted the bond together with $400 to the XLA Corporation. In return, he received the 20 shares of common stock. The common stock paid no dividends. On December 31, 2017, the investor sold the stock for $1740, terminating his 5-year investment in XLA Corporation. What rate of return did he receive?

7A-21 An engineering firm is doing design work on a client's project. It has $40,000 in expenses at the beginning of each month from January 2018 through December 2018. The client has agreed to the following payment schedule, if the firm meets milestone delivery dates. Use the XIRR function to find the IRR.

Date	Income
5/14/2018	150,000
8/6/2018	140,000
10/22/2018	110,000
1/1/2019	140,000

CASES

The following case from *Cases in Engineering Economy* (www.oup.com/us/newnan) is suggested as matched with this chapter.

CASE 14 **Northern Gushers**
Incremental oil production investment with possible double root.

[1] Remember that present value (PV), present worth (PW), net present value (NPV), and net present worth (NPW) are synonyms in practice and in this text.

[1] All possible roots for combinations of *P*, *A*, and *F* are detailed in

Eschenbach, Ted G., Elisha Baker, and John Whittaker, “Characterizing the Real Roots for *P*, *A*, and *F* with Applications to Environmental Remediation and Home Buying Problems,” *The Engineering Economist,* Volume 52, Number 1, 2007, pp. 41–65.

[2] Eschenbach, “Multiple Roots and the Subscription/Membership Problem,” *The Engineering Economist* Volume 29, Number 3, Spring 1984, pp. 216–223.

CHAPTER 8

CHOOSING THE BEST ALTERNATIVE

Coauthored with John Whittaker

Selecting the Best Pavement

The U.S. highway system is critical to meeting our mobility and economic needs. The goal is ensuring that users travel on pavements that are safe, smooth, quiet, durable, economical, and constructed of suitable materials—typically asphalt or concrete. Poor road surfaces are estimated to cost the average driver $324 annually in vehicle repairs and increase fuel consumption by about 2%.

In choosing between rigid concrete and flexible asphalt pavements, the three key questions are: (1) initial cost, (2) time to rehabilitation, and (3) cost of rehabilitation. A complete analysis would include the costs of the greenhouse gases (higher for asphalt), the ability to recycle (higher for asphalt), and the costs of traffic disruptions during rehabilitation (more frequent for asphalt). While government agencies must often focus on direct costs, the proper measure is life cycle and not initial costs.

Looking at the roads you drive on will indicate which pavement is generally preferred in your community. Often this is asphalt, since a thick asphalt structure can be a perpetual pavement with the only rehabilitation being milling of the surface followed by an asphalt overlay over a life of 6o years or more. Asphalt pavement often provides the smoothest, quietest ride with the greatest satisfaction for the motoring public.

On the other hand, asphalt pavement is also subject to rutting and shoving. Most of this damage is due to truck traffic, since the damage increases as a function of axle load to the 4^{th} power. Deflection of flexible pavements under heavy truck loads can also increase fuel consumption. In northern climates, wear from studded tires also contributes to rutting. Thus in some cases rigid concrete pavement is a better choice than asphalt, which must be rehabilitated more frequently.

Historically, concrete pavements had design lives of 20 to 40 years and asphalt pavements had lives of 10 years. Today both can be designed for 50 to 60 years—if needed maintenance and rehabilitation occurs on schedule. In addition, combinations of pavements and overlays have increased the choices available to design engineers.

Contributed by Benedict N. Nwokolo, Grambling State University

QUESTIONS TO CONSIDER

1. Transportation agencies must choose the type of pavement to consider. Should the transportation agency be indifferent as to whether a rigid pavement or a flexible pavement be chosen? Why?
2. What engineering economic principles would you apply in the above problem if you are to choose from different types of pavement? What would

you consider as the most effective measures for the public forum?
3. Considering the life cycle cited in the narrative, how can that be used in selecting a pavement type? How does a focus on initial vs. life-cycle costs affect the choice between asphalt and concrete pavements? Which pavement choice is made for the different types of road in your community?
4. What uncertainties are likely in estimating the (1) initial cost, (2) time to rehabilitation, and (3) cost of rehabilitation? Which are greater or smaller for asphalt or concrete choices?
5. If federal money is a significant share of a highway's initial cost, how might this distort the decision making of a state agency responsible for rehabilitation costs?

After Completing This Chapter…

The student should be able to:

• Use a *graphical technique* to visualize and solve problems involving mutually exclusive choices.
• Define *incremental analysis* and differentiate it from a standard present worth, annual worth, and internal rate of return analyses.
• Use spreadsheets to solve incremental analysis problems.

Key Words

choice table

incremental analysis

mutually exclusive

graphical approach

INCREMENTAL ANALYSIS

The **incremental analysis** presented in Chapter 7 can be extended to multiple alternatives by using a series of numerical comparisons of challengers and defenders. That approach is presented in the last section of this chapter.

This chapter introduces a more powerful and easier-to-understand approach of graphing each alternative's present worth (PW), equivalent uniform annual cost (EUAC), or equivalent uniform annual worth (EUAW). These graphs and spreadsheets support the calculation of incremental rates of return.

The graphical approach has the added benefit of focusing on the difference between alternatives. Often the difference is much smaller than the uncertainty in our estimated data. For ease of grading and instruction, we assume in this text that answers are exact, but in the real world uncertainty in the data must always be considered.

The engineering design process involves selection from competing alternatives. In engineering economy the words **mutually exclusive** alternatives are often used to emphasize that only one alternative may be implemented. Thus the objective is selecting the best of these mutually exclusive alternatives.

In earlier chapters we did this by maximizing PW, minimizing EUAC, or maximizing EUAW. We do the same here. In Chapter 7 we compared two alternatives incrementally to decide whether the IRR on the increment was acceptable. Any two alternatives can be compared by recognizing that:

[Higher-cost alternative] = [Lower-cost alternative] + [Increment between them]

or

[Increment between them] = [Higher-cost alternative] − [Lower-cost alternative]

When there are two alternatives, only a single incremental analysis is required. With more alternatives, a series of comparisons is required. Also, only by doing the analysis step by step can we determine which pairs must be compared. For example, if there are 4 alternatives, then 3 of 6 possible comparisons must be made. For 5 alternatives then 4 of 10 possible comparisons must be made. For N alternatives, $N-1$ comparisons must be made from $N(N-1)/2$ possibilities.

The **graphical approach** provides more information, and it is easier to understand and to present to others. It is best implemented with spreadsheets.

GRAPHICAL SOLUTIONS

Examples 8–1 and 8–2 illustrate why incremental or graphical analyses for the IRR criterion are required. They show that graphing makes it easy to choose the best alternative. Chapter 4 presented *xy* plots done with spreadsheets, and in Chapter 7 spreadsheets were used to graph the present worth versus the rate of return. In this chapter the present worth of each alternative is one *y* variable for graphs with multiple alternatives (or variables). In Chapter 9, one of the spreadsheet sections will present some of the ways that graphs can be customized for a better appearance.

EXAMPLE 8-1

The student engineering society is building a snack cart to raise money. Members must decide what capacity the cart should be able to serve. To serve 100 customers per hour costs \$10,310, and to serve 150 customers per hour costs \$13,400. The 50% increase in capacity is less than 50% of \$10,310 because of economies of scale; but the increase in net revenue will be less than 50%, since the cart will not always be serving 150 customers per hour. The estimated net annual income for the lower capacity is \$3300, and for the higher capacity it is \$4000. After 5 years the cart is expected to have no salvage value. The engineering society is unsure of what interest rate to use to decide on the capacity. Make a recommendation.

SOLUTION

Since we do not know the interest rate, the easiest way to analyze the problem is to graph the PW of each alternative versus the interest rate, as in Figure 8–1. The Excel function[1]

$$= -\text{cost} + \text{PV(interest rate, life, }-\text{annual benefit)}$$

is used to graph these two equations:

$$\begin{aligned} PW_{\text{low}} &= -\$10{,}310 + \$3300(P/A, i, 5) \\ PW_{\text{high}} &= -\$13{,}400 + \$4000(P/A, i, 5) \end{aligned}$$

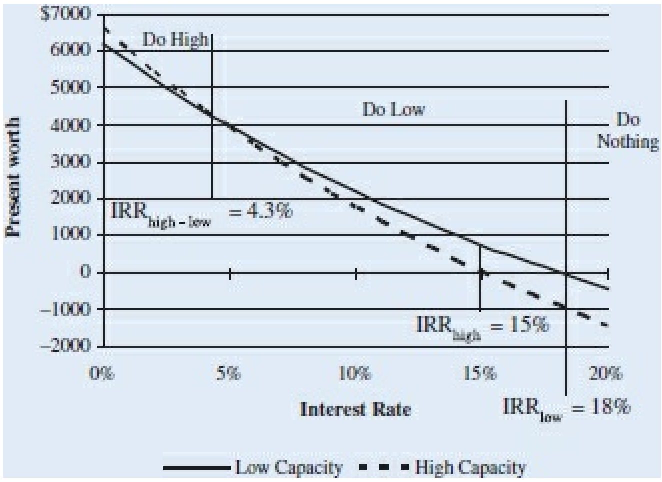


FIGURE 8-1 Maximizing PW to choose best alternative.

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Low		5	3300	10,310	0	i	18.04%
3	High		5	4000	13,400	0	i	15.03%
4	High - low		5	700	3,090	0	i	4.30%

We want to maximize the PW, so the choice between these alternatives is defined by where their curves intersect (the incremental rate of return of 4.3%), not by where those curves intersect the PW = 0 axis. Those internal rates of return of 15 and 18% are irrelevant to the choice between the low- and high-capacity carts.

Figure 8–1 also shows the main advantage of the graphical approach for real-world decision making. We can easily see that for interest rates between 0% and about 8%, there is little difference between the choices. In this region we are better off focusing our decision making on risk, benefits, or costs that we could not quantify, as well as on uncertainties in the data—since the PWs of our two alternatives are nearly the same. These topics are discussed in later chapters. Until then it is clearer if we analyze problems as though all numbers are known precisely.

From Figure 8–1, we see that if the interest rate is below 4.3%, the high-capacity cart has a higher PW. From 4.3 to 18% the lower-capacity cart has a higher PW. Above 18%, the third alternative of doing nothing is a better choice than building a low-capacity cart because the PW < 0. This can be summarized in a **choice table**:

Interest Rate	Best Choice
$0\% \le i \le 4.3\%$	High capacity
$4.3\% \le i \le 18.0\%$	Low capacity
$18.0\% \le i$	Do nothing

This example has been defined in terms of a student engineering society. But very similar problems are faced by a civil engineer sizing the weighing station for trucks carrying fill to a new earth dam, by an industrial engineer designing a package-handling station, and by a mechanical engineer sizing energy conservation equipment.

EXAMPLE 8-2 (Example 7–14 Revisited)

Solve Example 7–14 by means of an NPW graph. Two machines are being considered for purchase. If the minimum attractive rate of return (MARR) is 10%, which machine should be bought?

All $ values in 1000s	Machine *X*	Machine *Y*
Initial cost	$200	$700
Uniform annual benefit	95	120
End-of-useful-life salvage value	50	150
Useful life, in years	6	12

SOLUTION

Since the useful lives of the two alternatives are different, for an NPW analysis we must adjust them to the same analysis period. If the need seems continuous, then the “replace with an identical machine” assumption is reasonable and a 12-year analysis period can be used. The annual cash flows and the incremental cash flows are as follows:

	Cash Flows		
End of Year	Machine X	Machine Y	Y−X
0	-$200	-$700	-$500
1	95	120	25
2	95	120	25
3	95	120	25
4	95	120	25
5	95	120	25
6	-55	120	175
7	95	120	25
8	95	120	25
9	95	120	25
10	95	120	25
11	95	120	25
12	145	270	125

By means of a spreadsheet program, the NPWs are calculated for a range of interest rates and then plotted on an NPW graph (Figure 8–2).

For a MARR of 10%, Machine *X* is clearly the superior choice. In fact, as the graph clearly illustrates, Machine *X* is the correct choice for most values of MARR. The intersection point of the two graphs can be found by calculating ΔIRR, the rate of return on the incremental investment. From the spreadsheet IRR function applied to the incremental cash flows for $Y - X$:

$\Delta\text{IRR} = 1.32\%$

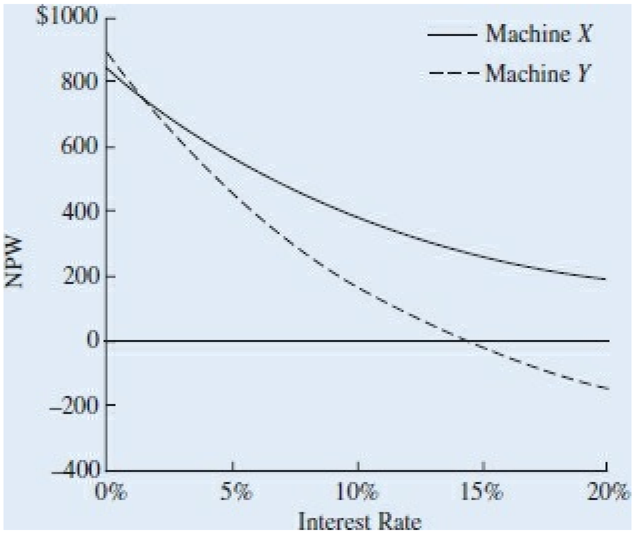


FIGURE 8-2 NPW graph.

We can see from the graph that for MARR greater than 1.32%, Machine *X* is the right choice, and for MARR values less than 1.32%, Machine *Y* is the right choice.

	NPW	
Rate	Machine *X*	Machine *Y*
0%	$840.00	$890.00
1.32	752.24	752.24
2	710.89	687.31
4	604.26	519.90
6	515.57	380.61
8	441.26	263.90
10	378.56	165.44
12	325.30	81.83
14	279.77	10.37
16	240.58	-51.08
18	206.65	-104.23
20	177.10	-150.47

Examples 8–3 through 8–6 increase the number of alternatives being considered, but the same approach is used: graph the PW, EUAW, or EUAC, and for each possible interest rate choose the best alternative. Generally this means maximising the PW or EUAW, but for Example 8–4 the EUAC is minimized. Once the best choices have been identified from the graph, calculate the incremental rates of return.

Example 8–4 is a typical design problem where the most cost-effective solution must be chosen, but the dollar value of the benefit is not defined. For example, every building must have a roof, but deciding whether it should be metal, shingles, or a built-up membrane is a cost decision. No value is placed on a dry building; it is simply a requirement. In Example 8–4 having the pressure vessel is a requirement.

EXAMPLE 8-3 (Examples 7–14 and 8–2 Revisited)

In Example 8–2 Machine *X* was assumed to be "replaced with an identical machine" for a 12-year analysis period. In Chapter 6 this was shown to be equivalent to using EUAW as the comparison measure. Construct a choice table using EUAW.

SOLUTION

The first step is to construct the EUAW graph for Machines *X* and *Y*. If Figures 8–2 and 8–3 are compared, it is easy to see that there are only two differences. The first is the scale of the *y*-axis, where one is for NPW and the other is for EUAW. The second difference is the shape of the curves, where the EUAW curves are straighter than the NPW curves. The important similarity is that in both cases Machine Y is preferred for very low interest rates, and Machine X is preferred for other interest rates.

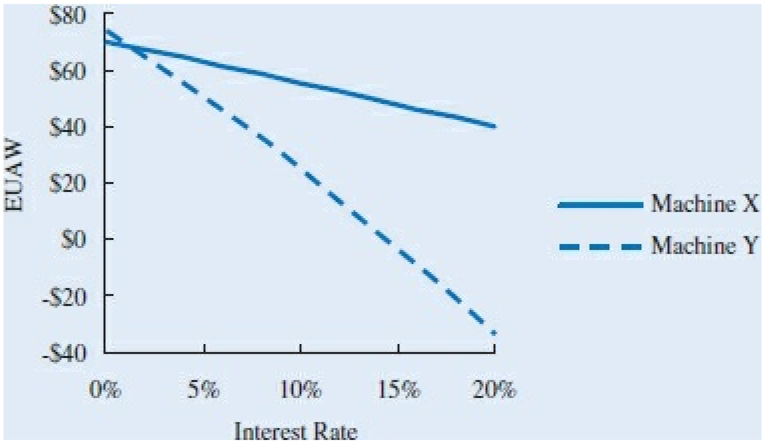


FIGURE 8-3 EUAW graph.

Finding the rate of return where Machines *X* and *Y* have the same EUAW is equivalent to finding the incremental IRR on *Y* − *X*. This was done for Example 8–2 by using the IRR spreadsheet function on the cash flows. Here we will use the GOAL SEEK function that was introduced in Example 7–15.

The 5-BUTTON SOLUTION starts by finding the EUAW for each machine at the 10% minimum attractive rate of return (MARR) that was used in Example 8–2. Note that this solution must add the annual benefit to the negative equivalent annual value of the initial cost and salvage value. To show this with the GOAL SEEK results, the 5-button calculations are duplicated in rows 9 to 16.

The GOAL SEEK function is found under DATA/WHAT-IF ANALYSIS. To use the GOAL SEEK function, the goal must be expressed in a single cell. Cell H16 simply calculates the difference in the EUAW values for each machine—the goal is a value of 0. GOAL SEEK varies a single cell to try to achieve the goal—in this case cell B10, the interest rate for Machine *X*. Cell B13 must contain =B10, so that both interest rates change at the same time.

When the lives of alternatives differ, this is the easiest way to find the breakeven interest rate. For the EUAWs of Machines *X* and *Y* the incremental IRR_{Y-X} is 1.32%.

	A	B	C	D	E	F	G	H
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer

2	Machine *X*	10%	6	-200	50	PMT	$39
3					Change sign		-39
4		annual benefit	95			EUAW	56
5	Machine *Y*	10%	12	-700	150	PMT	96
6					Change sign		-96
7		annual benefit	120			EUAW	24
8					Difference in EUAW		31
9							
10	Machine *X*	1.32%	6	-200	50	PMT	27
11					Change sign		-27
12		annual benefit	95			EUAW	68
13	Machine *Y*	1.32%	12	-700	150	PMT	52
14					Change sign		-52
15		annual benefit	120			EUAW	68
16					Difference in EUAW		0

EXAMPLE 8-4

A pressure vessel can be made out of brass, stainless steel, or titanium. The first cost and expected life for each material are:

	Brass	Stainless Steel	Titanium
Cost	$100,000	$175,000	$300,000
Life, in years	4	10	25

The pressure vessel will be in the nonradioactive portion of a nuclear power plant that is expected to have a life of 50 to 75 years. The public utility commission and the power company have not yet agreed on the interest rate to be used for decision making and rate setting. Build a choice table for the interest rates to show where each material is the best.

SOLUTION

The pressure vessel will be replaced repeatedly during the life of the facility, and each material has a different life. Thus, the best way to compare the materials is using EUAC (see Chapter 6). This assumes identical replacements.

Figure 8–4 graphs the EUAC for each alternative. In this case the best alternative at each interest rate is the material with the *lowest* EUAC. (We maximize worths, but minimize costs.) The Excel function is

$$= \text{PMT(interest rate, life, }-\text{first cost)}$$

The factor equation is

$$\text{EUAC} = \text{first cost}(A/P, \text{interest rate, life})$$

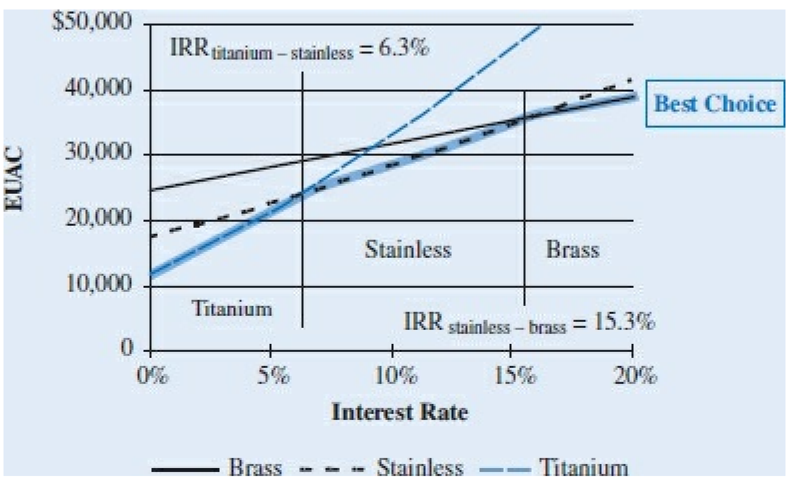


FIGURE 8-4 EUAC comparison of alternatives.

	A	B	C	D	E	F	G	H
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Brass	15.3%	4		-100,000	0	PMT	$35,206
	Stainless							

3	Steel	15.3%	10		-175,000	0	PMT	35,206
4	Titanium	15.3%	25		-300,000	0	PMT	47,114
5		15.3%		Difference in EUAW for brass & stainless				0
6		15.3%		Difference in EUAW for stainless & titanium				11,908
7								
8	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
9	Brass	6.3%	4		-100,000	0	PMT	29,043
10	Stainless Steel	6.3%	10		-175,000	0	PMT	24,091
11	Titanium	6.3%	25		-300,000	0	PMT	24,091
12		6.3%		Difference in EUAW for brass & stainless				-4,952
13		6.3%		Difference in EUAW for stainless & titanium				0

Because these alternatives have different-length lives, calculating the incremental IRRs is best done using the spreadsheet function GOAL SEEK. This spreadsheet is structured somewhat differently. In the top block of rows, each interest rate is set equal to the highlighted interest rate in cell B5. Copying the formula =$B$5 is easiest. In the lower block of rows, each interest rate is set equal to the highlighted interest rate in cell B13. Then two GOAL SEEKS are used to find the highlighted interest rates that make their respective differences in EUAW equal to zero.The choice table for each material is:

Interest Rate	Best Choice
0% ≤ *i* ≤ 6.3%	Titanium
6.3% ≤ *i* ≤ 15.3%	Stainless steel
15.3%≤ *i*	Brass

EXAMPLE 8-5

The following information refers to three mutually exclusive alternatives. The decision maker wishes to choose the right machine but is uncertain what MARR to use. Create a choice table that will help the decision maker to make the correct economic decision.

	Machine X	Machine *Y*	Machine *Z*
Initial cost	$2000	$7000	$4250
Uniform annual benefit	650	1100	1000
Useful life, in years	6	12	8

SOLUTION

In this example, the lives of the three alternatives are different. As we saw in Chapter 6 and Examples 8–3 and 8–4, when the service period is expected to be continuous and the assumption of identical replacement reasonable, we can assume a series of replacements and compare annual worth values just as we did with present worth values.

We will make these assumptions in this instance and plot EUAW. This was done in a spreadsheet using the Excel function = benefit + PMT (interest rate, life, initial cost), and the result is Figure 8–5.

The EUAW graph shows that for low values of MARR, *Y* is the correct choice. Then, as MARR increases, *Z* and *X* become the preferred machines. If the “do nothing” alternative is available, then it becomes the best choice for higher MARR values.

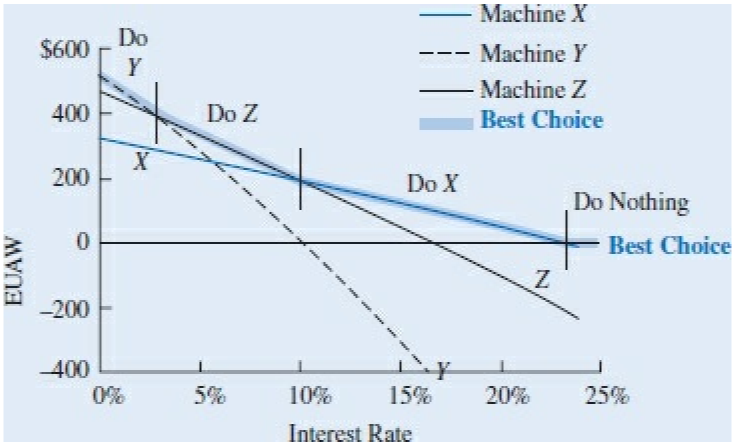


FIGURE 8-5 EUAW graph.

In this case, it is first necessary to solve for the EUAC of the initial cost. This is subtracted from the annual benefit of each machine to calculate the EUAWs. In the top block of rows, each interest rate is set equal to the highlighted interest rate in cell B5. Copying the formula =$B$5 is easiest. In the lower block of rows, each interest rate is set equal to the highlighted interest rate in cell B13. Then two GOAL SEEKS are used to find the highlighted interest rates that make their respective differences in EUAWs equal to zero. Notice that Figure 8–5 has been used to determine that finding the incremental or breakeven rates between *Y* & *Z*, *Z* & *X*, and *X* & doing nothing is necessary. The breakeven rate between *X* and doing nothing equals IRR_X, and it is found using the RATE function.

	A	B	C	D	E	F	G	H	I	J
1	Machine	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	EUAC	Ann. Benefit	Benefit - cost
2	*X*	3.1%	6		-2000	0	PMT	371	650	279
3	*Y*	3.1%	12		-7000	0	PMT	709	1100	391
4	*Z*	3.1%	8		-4250	0	PMT	609	1000	391
5		3.1%					Difference in EUAW for *Y* & *Z*			0
6	3.1%						Difference in EUAW for *Z* & *X*			112
7										

8	Machine	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	EUAC	Ann. Benefit	Benefit - cost
9	*X*	10.8%	6		-2000	0	PMT	470	650	180
10	*Y*	10.8%	12		-7000	0	PMT	1068	1100	32
11	*Z*	10.8%	8		-4250	0	PMT	820	1000	180
12		10.8%					Difference in EUAW for *Y* & *Z*			148
13	10.8%						Difference in EUAW for *Z* & *X*			0
14										
15	Machine	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	IRR		
16	*X*		6	650	-2000	0	*i*	23.2%		

If MARR ≥ 23.2% do nothing

If 23.2% ≥ MARR ≥ 10.8% choose *X*

If 10.8% ≥ MARR ≥ 3.1% choose *Z*

If 3.1% ≥ MARR ≥ 0% choose *Y*

If the “do nothing” alternative is not available, then the table reads

If MARR ≥ 10.8% choose *X*

If 10.8% ≥ MARR ≥ 3.1% choose *Z*

If 3.1% ≥ MARR ≥ 0% choose *Y*

The choice now is back in the decision maker's hands. There is still the need to determine MARR, but if the uncertainty was, for example, that MARR was some value in the range 12 to 18%, it can be seen that for this problem it doesn't matter. The answer is Machine *X* in any event. If, however, the uncertainty of MARR were in the 7 to 13% range, two things are clear. First, the decision maker would have to determine MARR with greater accuracy to maximize the EUAW. Second, there is little practical difference between the EUAWS for machines *X* and *Y*.

EXAMPLE 8-6

The following information is for five mutually exclusive alternatives that have 20-year useful lives. The decision maker may choose any one of the options or reject them all. Prepare a choice table.

	Alternatives				
	A	*B*	*C*	*D*	*E*
Cost	$4000	$2000	$6000	$1000	$9000
Uniform annual benefit	639	410	761	117	785

SOLUTION

Figure 8–6 is an NPW graph of the alternatives constructed by means of a spreadsheet.

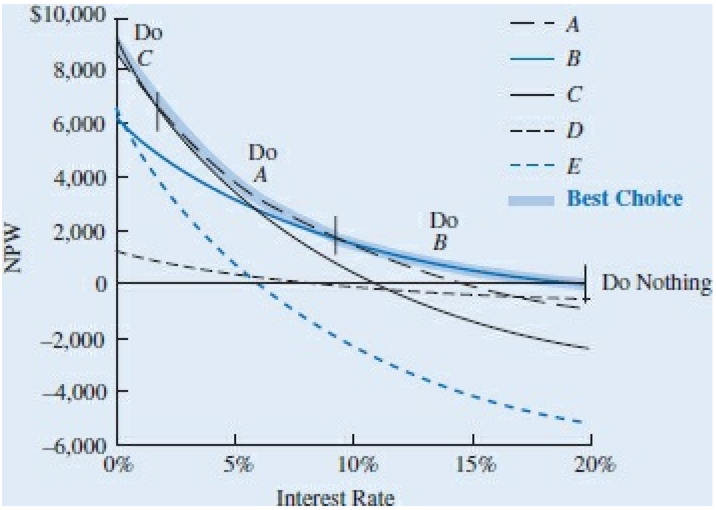


FIGURE 8-6 NPW graph.

The graph clearly shows that Alternatives *D* and *E* are never part of the solution. They are dominated by the other three. The crossover points can either be read from the graph (if you have plotted it at a large enough scale) or found by calculating the ΔIRR of the intersecting curves.

Entering the data for the five alternatives also makes it easy to solve for the IRR of each. However, only IRR_B is useful. As the highest rate, it is the rate above which doing nothing is the best choice. This can be seen from the right-hand side of Figure 8–6. Fortunately, the graph shows us that the rate is about 20%. Some spreadsheet packages fail to find rates that high, unless an *initial guess* is supplied. It turns out that the guess is almost exactly the IRR_B of 19.96%.

The figure also identifies the two incremental calculations that are needed—for *C* − *A* and for *A* − *B*. The first value appears to be about 2% and the second a bit less than 10%. The rate on the incremental investment is also the rate at which the two alternatives have the same NPW. The three “useful” IRRs are left-indented for emphasis.

	A	B	C	D	E	F	G	H
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	*A*		20	639	-4000	0	*i*	15.00%
3	*B*		20	410	-2000	0	*i*	20.0%

4 *C*	20 761	-6000 0	*i*	11.2%
5 *D*	20 117	-1000 0	*i*	9.9%
6 *E*	20 785	-9000 0	*i*	6.0%
7 *C* - *A*	20 122	-2000 0	*i*	2.0%
8 *A* - *B*	20 229	-2000 0	*i*	9.6%

Placing these numbers in a choice table:

If	MARR ≥ 20%	do nothing
If 20%	≥ MARR ≥ 9.6%	select *B*
If 9.6%	≥ MARR ≥ 2%	select *A*
If 2%	≥ MARR ≥ 0%	select *C*

Examining Figure 8–6 also makes it clear that *C* is only slightly better than *A* for MARRs below 2%.

ELEMENTS IN COMPARING MUTUALLY EXCLUSIVE ALTERNATIVES

1. **Be sure all the alternatives are identified.** In textbook problems the alternatives will be well-defined, but real-life problems may be less clear. Before proceeding, one must have all the mutually exclusive alternatives tabulated, including the “do-nothing” or “keep doing the same thing” alternative, if appropriate.
2. **Construct an NPW or EUAW or EAC graph showing all alternatives**

plotted on the same axes. This would be a difficult task were it not for spreadsheets.

3. **Examine the line of best values and determine which alternatives create it, and over what range.**

4. **Determine the changeover points** where the line of best values changes from one alternative to another. These can either be read directly off the graph or calculated, since they are the intersection points of the two curves and, what is more important and meaningful for engineering economy, they are **the ΔIRR of the incremental investment** between the two alternatives.

5. **Create a choice table** to present the information in compact and easily understandable form.

DOING A SET OF INCREMENTAL CHALLENGER–DEFENDER COMPARISONS

This chapter has focused on doing a graphical comparison of the PW of each alternative over a range of interest rates. We've calculated incremental interest rates, but we've looked at the curves to see which pair of intersecting PW curves we were analyzing. Before spreadsheets, this type of problem was solved by a series of challenger–defender comparisons, where defender ≡ best alternative identified at this stage of the analysis, and challenger ≡ next alternative being evaluated. Example 8–7 illustrates this approach.

The numerically based pairwise incremental comparisons is done at a single interest rate, and it does not show that "near" their intersection, two PW curves have "essentially" the same value. It also does not produce a choice table, since it is at a single interest rate. A choice table can be constructed, but it requires even more incremental comparisons.

EXAMPLE 8-7 (Example 8–6 Revisited)

For the alternatives in Example 8–6, conduct a pairwise incremental comparison. Which alternative is the best at an interest rate of 10%? Each of the five alternatives has no salvage value at the end of a 20-year useful life.

	Alternatives				
	A	*B*	*C*	*D*	*E*
Cost	\$4000	\$2000	\$6000	\$1000	\$9000
Annual benefit	639	410	761	117	785

SOLUTION

1. The first step is to reorder the alternatives in order of increasing cost. This ensures that for each pairwise comparison the increment is an investment. In this example the order becomes *D*, *B*, *A*, *C*, *E*.

	Alternatives				
	D	*B*	*A*	*C*	*E*
Cost	\$1000	\$2000	\$4000	\$6000	\$9000
Annual benefit	117	410	639	761	785

2. Calculate the IRR of the least expensive alternative to see if it is better than doing nothing at a MARR of 10%.

$$\mathrm{PW}_D = 0 = -\$1000 + \$117(P/A, \mathrm{IRR}_D, 20)$$

Solving this without a spreadsheet may take several tries and interpolation, but using the Excel function = RATE(life, annual benefit, –first cost) makes it easy and the answer exact:

$$\text{IRR}_D = 9.94\%$$

Because investment *D* earns less than 10%, doing nothing is preferred to doing *D*.

3. Doing nothing is still our *defender*, and the next Alternative, *B*, becomes the *challenger*. Calculate the IRR of Alternative *B* to see if it is better than doing nothing at a MARR of 10%.

$$\text{PW}_B = 0 = -\$2000 + \$410(P/A, IRR_B, 20)$$

Using the Excel function = RATE(life, annual benefit, –first cost), we find

$$\text{IRR}_B = 19.96\%$$

Because investment *B* earns more than 10%, Alternative *B* is preferred to doing nothing.

4. Alternative *B* is now our *defender*, and the next Alternative, *A*, becomes the *challenger*. This comparison must be made incrementally.

$$\text{PW}_{A-B} = 0 = -(\$4000 - \$2000) + (\$639 - \$410)(P/A, \text{IRR}_{A-B}, 20)$$

Using the Excel function = RATE(life, annual benefit, −first cost), we have

$$\text{IRR}_{A-B} = 9.63\%$$

Because the incremental investment $A - B$ earns less than 10%, Alternative *B* is preferred to doing Alternative *A*.

5. Alternative *B* is still our *defender*, and the next Alternative, *C*, becomes the *challenger*. This comparison must be made incrementally.

$$\text{PW}_{C-B} = 0 = -(\$6000 - \$2000) + (\$761 - \$410)(P/A, \text{IRR}_{C-B}, 20)$$

Using the Excel function = RATE(life, annual benefit, −first cost), we have

$$\text{IRR}_{C-B} = 6.08\%$$

Because the incremental investment $C - B$ earns less than 10%, Alternative *B* is preferred to doing Alternative *C*.

6. Alternative *B* is still our *defender*, and the final Alternative, *E*, becomes the *challenger*. This comparison must be made incrementally.

$$\text{PW}_{E-B} = 0 = -(\$9000 - \$2000) + (\$785 - \$410)(P/A, \text{IRR}_{E-B}, 20)$$

Using the Excel function = RATE(life, annual benefit, –first cost)

$$\text{IRR}_{E-B} = 0.67\%$$

Because the incremental investment $E - B$ earns less than 10%, Alternative *B* is preferred to doing Alternative *E*.

At an interest rate of 10%, Alternative *B* is the best choice. But only by referring to Figure 8–6 can we see that Alternative *B* is the best choice for all interest rates between 9.63% (where Alternative *A* is equally attractive) and 19.96% (where doing nothing is equally attractive). With other data, the curve for Alternative *B* might intersect at an incremental comparison we hadn't done. Also this pairwise comparison has not even compared Alternatives *A* and *C* so we cannot complete the choice table.

CHOOSING AN ANALYSIS METHOD

At this point, we have examined in detail the three major economic analysis techniques: present worth analysis, annual cash flow analysis, and rate of return analysis. A practical question is, which method should be used for a particular problem?

While the obvious answer is to use the method requiring the least computations, a number of factors may affect the decision.

1. Unless the MARR—minimum attractive rate of return (or minimum required interest rate for invested money)—is known, neither present worth analysis nor annual cash flow analysis is possible.
2. Present worth analysis and annual cash flow analysis often require far less computation than rate-of-return analysis.
3. In some situations, a rate-of-return analysis is easier to explain to people unfamiliar with economic analysis. At other times, an annual cash flow analysis may be easier to explain.

4. Business enterprises generally adopt one, or at most two, analysis techniques for broad categories of problems. If you work for a corporation and policy specifies the rate of return analysis, you would appear to have no choice in the matter.

Since one may not always be able to choose the analysis technique computationally best suited to the problem, this book illustrates how to use each of the three methods in all feasible situations. Ironically, the most difficult method to apply when using tabulated factors—rate-of-return analysis—is the one used most frequently by engineers in industry, which is one reason why spreadsheets and financial calculators are used so much in industry.

SUMMARY

For choosing from a set of mutually exclusive alternatives, the rate-of-return technique is more complex than the present worth or annual cash flow techniques. This results because in the latter two techniques the numbers can be compared directly, whereas with the rate of return it is necessary to consider the *increment of investment.* This is fairly straightforward if there are only two alternatives, but it becomes more and more complex as the number of alternatives increases.

A visual display of the problem can be created by using a spreadsheet to graph the economic value of the alternatives. The steps are as follows:

1. Be sure all the alternatives are identified.
2. Construct an NPW or EUAW (or EUAC) graph showing all alternatives plotted on the same axes.
3. Examine the line of maximum values (or minimum for the EUAC) and determine which alternatives create it, and over what range.
4. Determine the changeover points (ΔIRRs).
5. Create a choice table.

The graphical approach, where more values are calculated, is a more powerful one. By allowing the decision maker to see the range over which the choices are valid, it provides a form of sensitivity analysis. It also makes it clear that “close” to the changeover point the alternatives are very similar in value.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

8-1 Two mutually exclusive investment projects have been presented to your company. Project *A*’s rate of return is 6%, and Project *B*’s rate of return is 8%. The cost of Project *A* is less than the cost of Project *B*. Incremental analysis yields a rate of return of 4%. If both alternatives have the same useful life and the MARR is 5%, which project should be chosen?

SOLUTION

8-2 EA Construction must replace a piece of equipment. Cat and Volvo are the two best alternatives. Both alternatives are expected to last 6 years. If EA has a MARR of 11%, which alternative should be chosen? Use IRR analysis.

	Cat	Volvo
First cost	$160,000	$225,000
Annual operating cost	30,000	17,500
Salvage value	20,000	40,000

SOLUTION

8-3 Horizon Wireless must rebuild a cell tower. A tower made of normal steel (NS) will cost $30,000 to construct and should last 15 years. Maintenance will cost $1000 per year. If corrosion-resistant steel (CRS) is used, the tower will cost $36,000 to build, but the annual maintenance cost will be reduced to $250 per year. Determine the IRR of building the corrosion-resistant tower. If Horizon requires a return of 9% on its capital projects, which tower should be chosen?

SOLUTION

8-4 Gamma Inc. must replace its grinding machine. Relevant information about the three "best" alternatives is given below. Each alternative has a 10-year life. Gamma's MARR is 8%. Using ROR analysis, which alternative should be selected?

	A	B	C
First cost	$30,000	$35,000	$28,000
Annual costs	2,900	2,200	3,250

SOLUTION

8-5 Free-Flow Sanitation Inc. is considering the purchase of advanced software. Four software packages are under consideration. Relevant information for each package is given below. Each alternative has a projected 8-year life and the MARR for the company is 6%. Using ROR analysis, which alternative should be selected?

	A	B	C	D
First cost	$5,000.00	$7,000.00	$4,000.00	$6,500.00
Annual benefits	1,842.25	2,161.40	1,640.50	1,914.70
Annual costs	1,068.62	849.31	835.35	949.31

SOLUTION

8-6 Barber Brewing is considering investing in one of the following opportunities.

	A	B	C	D
First cost	$100.00	$130.00	$200.00	$330.00
Annual income	100.00	90.78	160.00	164.55
Annual cost	73.62	52.00	112.52	73.00

Each alternative has a 5-year useful life. Use net present worth to prepare a choice table.

SOLUTION

8-7 Abby W. is considering the following mutually exclusive investment projects.

	A	B	C	D	E
First cost	$100.00	$130.00	$200.00	$330.00	Do
Annual income	150.00	130.78	185.00	184.55	Nothing
Annual cost	123.62	92.00	137.52	93.00	

Each alternative has a 5-year useful life. If Abby requires at least a 10% return on her investments, which alternative should she select?

SOLUTION

8-8 An industrial laboratory is comparing options for getting new lab equipment. They can pay cash, finance the purchase, or lease the equipment.

a. Cash price is $35,000; no salvage value; life is 5 years.
b. If financed, price is $34,000; annual interest rate is 10%, payable monthly over 60 months. No salvage value.
c. Lease is $660 per month, payable at the beginning of the month for 60 months. Requires an extra payment of $500 at the beginning of the lease.

Develop a choice table based on EUAC for nominal interest rates from 0% to 20%.

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

These problems are organized such that the (*a*) parts are best done with graphical analysis and as such are much more easily done with spreadsheets, and the (*b*) parts require numerical incremental analysis. Some problems include only one approach. Usually only (*b*) answers are included in Appendix E.

Two Action Alternatives

8-1 Including a do-nothing alternative, construct a choice table for interest rates from 0% to 100%.

Year	*X*	*Y*
0	−$1200	−$2500
1	1750	3250

8-2 A Construct a choice table for interest rates from 0% to 100% for two mutually exclusive alternatives and the do-nothing alternative.

Year	Buy *Y*	Buy *X*
0	−$185	−$100.0
1-4	62	36

8- Consider three alternatives *A*, *B*, and “do-nothing.”
(*a*) Construct a choice table for interest rates from 0% to 100%.

Year	*A*	*B*
0	−$10,000	−$15,000

3

1-5	3,200	4,500

E

(*b*) Step #4 of the decision-making process described in Chapter 1 is *Identify Feasible Alternatives*. Would you view it as ethical to not consider an alternative *C*, if you knew it was competitive but your boss asked you to leave it off the list? What would you do and why?

8-4AG

A paper mill is considering two types of pollution control equipment.

	Neutralization	Precipitation
Initial cost	$850,000	$625,000
Annual chemical cost	55,000	125,000
Salvage value	200,000	150,000
Useful life, in years	5	5

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) The firm wants a 25% rate of return on any avoidable increments of investment. Which equipment should be purchased?

8-5E

A stockbroker has proposed two investments in low-rated corporate bonds paying high interest rates and selling at steep discounts (junk bonds). The bonds are rated as equally risky and both mature in 15 years.

Bond	Stated value	Annual Interest Payment	Current Market Price with Commission
Gen Dev	$1000	$ 67	$480
RJR	1000	98	630

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) Which, if any, of the bonds should you buy if your MARR is 20%?
(*c*) Are there professional ethics standards for stockbrokers in the U.S.? What are some common ethical pitfalls?

A firm is considering two alternatives that have no salvage value.

	A	*B*

Initial cost	$9000	$4700
Uniform annual benefits	1400	1650
Useful life, in years	10	5

8-6 A

At the end of 5 years, another *B* may be purchased with the same cost, benefits, and so forth.

(*a*) Graph the EUAC or EUAW for the alternatives. Construct a choice table for interest rates from 0% to 100%.

(*b*) If the MARR is 15%, which alternative should be selected?

8-7

Don Garlits is a landscaper. He is considering the purchase of a new commercial lawn mower, either the Atlas or the Zippy. Graph the EUAC or EUAW for the alternatives. Construct a choice table for interest rates from 0% to 100%.

	Atlas	Zippy
Initial cost	$6700	$16,900
Annual O&M	1500	1,800
Annual benefit	4000	5,500
Salvage value	1000	3,500
Useful life, in years	3	6

8-8 A

Your cat's summer kitty-cottage needs a new roof. You feel a 15-year analysis period is in line with your cat's remaining lives. (There is no salvage value for old roofs.)

	Thatch	Slate
First cost	$250	$425
Annual upkeep	65	30
Service life, in years	3	5

(*a*) Graph the EUAC or EUAW for the alternatives. Construct a choice table for interest rates from 0% to 100%.

(*b*) Which roof should you choose if your MARR is 12%? What is the actual value of the IRR on the incremental cost?

The South End bookstore has an annual profit of $285,000. The owner may open a new bookstore by leasing an existing building for 5 years with an option to continue the lease for a second 5-year period. If he

8-9 opens “The North End,” it will take $950,000 of store fixtures and inventory. He believes that the two stores will have a combined profit of $395,000 a year after all the expenses of both stores have been paid. The owner’s economic analysis is based on a 5-year period. He will be able to recover $750,000 at the end of 5 years by selling the store fixtures and moving the inventory to The South End.
(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If The North End is opened, what rate of return can he expect?

8-10 AE George is going to replace his car in 3 years when he graduates, but now he needs a radiator repair. The local shop has a used radiator, which will be guaranteed for 2 years, or they can install a new one, which is “guaranteed for as long as you own the car.” The used radiator is $250 and the new one is $425. If George assumes the used radiator will last 3 years, but will need to be replaced so he can sell the car, which should he buy?
(*a*) Graph the EUAC or EUAW for the alternatives. Develop a choice table for interest rates from 0% to 50%.
(*b*) George’s interest rate on his credit card is 20%. What should he do?
(*c*) Find the ASA auto repair ethics standards. How are the profession’s ethical standards similar and different from those of your engineering discipline?

8-11 G Using the current specifications, resurfacing a road will cost $1.5M initially, need $120K in annual maintenance, and need to be resurfaced every 10 years. A proposed new specification is expected to be more resistant to wear. The resurfacing cost will be $2.1M with $90K in annual maintenance and resurfacing every 15 years.
(*a*) Develop a choice table for interest rates from 0% to 25%.
(*b*) If the highway department’s interest rate is 6%, which specification is preferred?
(*c*) How significant is the economic difference between the two specifications?
(*d*) Research the relationship between road surfacing and environmental impact. You may be surprised!

Multiple Alternatives

Each alternative has a 10-year useful life and no salvage value.

		A	*B*	*C*
8-12 A	Initial cost	\$2000	\$7500	\$3900
	Uniform annual benefits	395	1150	650

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the MARR is 8%, which alternative should be selected?

The following three mutually exclusive alternatives have no salvage value after 5 years. Construct a choice table for interest rates from 0% to 100%.

		A	*B*	*C*
8-13	First cost	\$2000	\$3000	\$6000
	Uniform annual benefit	597	771	1652
	Computed rate of return	15%	9%	11.7%

The following four mutually exclusive alternatives have no salvage value after 10 years.

		A	*B*	*C*	*D*
8-14 A	First cost	\$8000	\$6000	\$6000	\$9500
	Uniform annual benefit	1750	1300	1425	1900

Computed rate of return 17.5% 17.3% 19.9% 15.1%

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) Using 12% for the MARR, which alternative should be selected?

8-15 Consider four mutually exclusive and a do-nothing alternatives, each having an 10-year useful life:

	A	*B*	*C*	*D*
First cost	$1000	$800	$600	$500
Uniform annual benefit	125	120	100	125
Salvage value	750	500	250	0

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the minimum attractive rate of return is 8%, which alternative should be selected?

8-16 A Three mutually exclusive alternatives are being considered.

	A	*B*	*C*
Initial investment	$43,000	$24,000	$17,000
Annual net income	4,150	2,500	1,700
Rate of return	7.3%	8.3%	6.0%

Each alternative has a 20-year useful life with no salvage value.
(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the minimum attractive rate of return is 7%, which alternative should be selected?

8-17 Each alternative has a 10-year useful life and no salvage value. Construct a choice table for interest rates from 0% to 100%, if doing nothing is allowed.

	A	*B*	*C*
Initial cost	$1500	$1000	$2035
Annual benefit for first 5 years	250	250	650
Annual benefit for subsequent 5 years	450	250	145

8- QZY, Inc. is evaluating new widget machines offered by three companies. The chosen machine will be used for 3 years.

	Company *A*	Company *B*	Company *C*
First cost	$15,000	$25,000	$20,000

18 A	Maintenance and operating	1,600	400	900
	Annual benefit	8,000	13,000	9,000
	Salvage value	3,000	6,000	4,500

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) MARR = 15%. From which company, if any, should you buy the widget machine? Use rate of return analysis.

8-19 Andrews Manufacturing offers three models for one of its products to its customers. You have been asked to analyze the choices from the customer's perspective. Which model should a customer choose if each model has a life of 12 years? Doing nothing is an alternative.

	Alternative		
	Deluxe	Regular	Economy
First cost	$220,000	$125,000	$75,000
Annual benefit	79,000	43,000	28,000
Maintenance and operating costs	38,000	13,000	8,000
Salvage value	16,000	6,900	3,000

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) MARR = 15%. Using incremental rate of return analysis, which alternative, if any, should the customer choose?

Wayward Airfreight, Inc. is considering a new automatic parcel sorter. Each choice has a 7-year life.

8-20 A

	SHIP-R	SORT-Of	U-SORT-M
First cost	$184,000	$235,000	$170,000
Salvage value	38,300	44,000	14,400
Annual benefit	75,300	89,000	68,000
Yearly O&M cost	19,000	19,000	12,000

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) Using a MARR of 15% and a rate of return analysis, which alternative, if any, should be selected?

A firm is considering the following alternatives, as well as a fifth choice: do nothing. Each alternative has a 5-year useful life.

8-21

	1	*2*	*3*	*4*
Initial cost	$100,000	$130,000	$200,000	$330,000
Uniform annual	26,380	38,780	47,480	91,550
Rate of return	10%	15%	6%	12%

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) The firm's minimum attractive rate of return is 8%. Which alternative should be selected?

Consider three mutually exclusive alternatives that have a uniform annual benefit of $450. The analysis period is 8 years. Assume identical replacements and construct a choice table for interest rates from 0% to 100%.
(*a*) Assume doing nothing is allowed.
(*b*) Assume *A*, *B*, or *C* must be chosen.

8-22 A

	A	*B*	*C*
Initial cost	$800	$1500	$2750
Useful life (years)	2	4	8
Rate of return	8.2%	7.7%	6.4%

8-23

Three mutually exclusive projects are being considered:

	A	*B*	*C*
First cost	$1000	$2000	$3000
Uniform annual benefit	250	350	525
Salvage value	200	300	400
Useful life (years)	5	6	7

Assume identical replacements.
(*a*) Construct a choice table for interest rates from 0% to 100%.

(*b*) If 8% is the desired rate of return, which project should be selected?

8-24 A business magazine is available for $58 for 1 year, $108 for 2 years, $153 for 3 years, or $230 for 5 years. Assume you will read the magazine for at least the next 5 years.

A (*a*) For what interest rates do you prefer each payment plan?

G (*b*) What is the environmental impact of glossy magazines? How are they recycled?

8-25 Three office furniture firms that offer different payment plans have responded to a request for bids from a state agency.

	Price	Payment Schedule
OfficeLess	$130,000	34% now,
		33% in 6 months,
		33% in 1 year
OfficeMore	128,500	50% now,
		50% in 6 months
OfficeStation	125,000	100% now

(*a*) Develop a choice table for nominal interest rates from 0% to 50%.

(*b*) If the agency's MARR is 10%, which vendor's plan is preferred?

Consider the following alternatives:

A	*B*	*C*

	Initial cost	$10,000	$15,000	$20,000
8-26 A	Uniform annual benefit	1000	1762	5548
	Useful life (years)	Infinite	20	5

Alternatives *B* and *C* are replaced at the end of their useful lives with identical replacements. Use an infinite analysis period.

(*a*) Construct a choice table for interest rates from 0% to 100%.

(*b*) At an 8% interest rate, which alternative is better?

8-27 Three mutually exclusive alternatives may replace the current equipment.

Year	*A*	*B*	*C*
0	−$20,000	−$24,000	−$25,000
1	10,000	10,000	5,000
2	5,000	10,000	5,000
3	10,000	8,000	5,000
4	5,000	5,000	25,000

(*a*) Construct a choice table for interest rates from 0% to 100%.

(*b*) If the MARR is 12%, which alternative should be selected?

A firm is considering three mutually exclusive alternatives as part of a production improvement program. The alternatives are as follows:

		A	B	C
	Installed cost	$8,000	$12,000	$16,000
8-28 A	Uniform annual benefit	1,600	1,750	2,050
	Useful life, in years	10	20	20

For each alternative, the salvage value at the end of useful life is zero. At the end of 10 years, Alt. *A* could be replaced by another *A* with identical cost and benefits.
(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) The MARR is 12%. If the analysis period is 20 years, which alternative should be selected?

8-29 A new 10,000-square-meter warehouse next door to the Tyre Corporation is for sale for $450,000. The terms offered are $100,000 down with the balance being paid in 60 equal monthly payments based on 15% interest. It is estimated that the warehouse would have a resale value of $600,000 at the end of 5 years.
Tyre has the cash and could buy the warehouse but does not need all the warehouse space at this time. The Johnson Company has offered to lease half the new warehouse for $2500 a month. Modifying the space for two tenants will cost $12,000.
Tyre presently rents and uses 7000 square meters of warehouse space for $2700 a month. It has the option of reducing the rented space to 2000 square meters, in which case the monthly rent would be $1000 a month. Furthermore, Tyre could cease renting warehouse space entirely. Tom Clay, the Tyre Corp. plant engineer, is considering three alternatives:
1. Buy the new warehouse and lease half the space to the Johnson Company. In turn, the Tyre-rented space would be reduced to 2000 square meters.
2. Buy the new warehouse and cease renting any warehouse space.
3. Continue as is, with 7000 square meters of rented warehouse space.

Construct a choice table for interest rates from 0% to 100%.

Construct a choice table for interest rates from 0% to 100%. Similar alternatives will repeat indefinitely.

		Alternatives			
		A	*B*	*C*	*D*
8-30 A	Initial cost	$2500	$4800	$4200	$3600
	Annual benefit	850	700	850	1300
	Salvage value	2500	1750	1250	3000
	Life, in years	5	6	8	4

One of these four mutually exclusive alternatives must be chosen. Each costs $13,000 and has no salvage value. Similar alternatives will repeat indefinitely.

	Alternative	Annual Cost	Life (yrs)
	A	$1000 first year; then increasing $575 per year	7
8-31	*B*	$100 first year; then increasing $500 per year	6
	C	$2500	7

	D	$4250 first year; then declining $500 per year	8

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the MARR is 8%, which alternative should be selected?

8-32 A more detailed examination of the situation in Problem 8-31 reveals that there are two additional mutually exclusive alternatives to be considered. Both have no salvage value after a useful life of 10 years.

Alternative	Initial Cost	Annual Cost
E	$4880	$4880
F	8000	$2800 for first year; then increasing $200 per year

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the MARR remains at 8%, which one of the six alternatives should be selected?

The owner of a downtown parking lot has employed a civil engineering consulting firm to advise him on the economic feasibility of constructing an office building on the site. Betty Samuels, a newly hired civil engineer, has been assigned to make the analysis. She has assembled the following data:

Alternative	Total Investment*	Total Net Annual Revenue
Sell parking lot	$ 0	$ 0
Keep parking lot	200,000	22,000
Build 1-story building	400,000	60,000

Build 2-story building	555,000	72,000
Build 3-story building	750,000	100,000
Build 4-story building	875,000	105,000
Build 5-story building	1,000,000	120,000

* Includes the value of the land.

The analysis period is to be 15 years. For all alternatives, the property has an estimated resale (salvage) value at the end of 15 years equal to the present total investment.
(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the MARR is 10%, what recommendation should Betty make?

8-33 A firm is considering moving its manufacturing plant from Chicago to a new location. The industrial engineering department was asked to identify the various alternatives together with the costs to relocate the plant and the benefits. The engineers examined six likely sites, together with the do-nothing alternative of keeping the plant at its present location. Their findings are summarized as follows:

Plant Location	First Cost ($000s)	Uniform Annual Benefit ($000s)
Denver	$300	$ 52
Dallas	550	137

8-34 A	San Antonio	450	117
	Los Angeles	750	167
	Cleveland	150	18
	Atlanta	200	49
	Chicago	0	0

The annual benefits are expected to be constant over the 8-year analysis period.
(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the firm uses 10% annual interest in its economic analysis, where should the manufacturing plant be located?

8-35 An oil company plans to purchase a piece of vacant land on the corner of two busy streets for $50,000. On properties of this type, the company installs businesses of three different types. Each has an estimated useful life of 15 years. The salvage land for each is estimated to be the $50,000 land cost.

Plan	Cost*	Type of Business	Net Annual Income
A	$ 83,000	Conventional gas station	$26,500
B	195,000	Add automatic carwash	39,750
C	115,000	Add quick carwash	31,200

* Improvements cost does not include $50,000 for the land.

(*a*) Construct a choice table for interest rates from 0% to 100%.
(*b*) If the oil company expects a 10% rate of return on its investments, which plan (if any) should be selected?

8-36 *The Financial Advisor* is a weekly column in the local newspaper. Assume you must answer the following question. “I recently retired at age 65, and I have a tax-free retirement annuity coming due soon. I have three options. I can receive (*A*) $30,976 now, (*B*) $359.60 per month for the rest of my life, or (*C*) $513.80 per month for the next 10 years. For option C if I die within 10 years, payments continue to my heirs. My interest rate is 9%. What should I do?” Ignore the timing of the monthly cash flows and assume that the payments are received at the end of year.
Contributed by D. P. Loucks, Cornell University
(a) Develop a choice table for life spans from age 66 to 100.
(b) If remaining life is 20 years and i = 9%, use an incremental rate of return analysis to recommend which option should be chosen.

8-37 A firm must decide which of three alternatives to adopt to expand its capacity. The firm wishes a minimum annual profit of 20% of the initial cost of each separable increment of investment. Any money not invested in capacity expansion can be invested elsewhere for an annual yield of 20% of initial cost.

Alt.	Initial Cost	Annual Profit	Profit Rate
A	$100,000	$30,000	30%
B	300,000	66,000	22%
C	500,000	80,000	16%

Which alternative should be selected? Use a challenger–defender rate of return analysis.

The New England Soap Company is considering adding some processing equipment to the plant to aid in the removal of impurities from some raw materials. By adding the processing equipment, the firm can purchase lower-grade raw material at reduced cost and upgrade it for use in its products.

Four different pieces of processing equipment with 15-year lives are being considered:

8-38 A

	A	*B*	*C*	*D*
Initial investment	$8,500	$18,500	26,500	$30,000
Annual saving in materials costs	3,500	6,000	7,400	9,000
Annual operating cost	2,100	3,000	3,200	4,100

The company can obtain a 13% annual return on its investment in other projects and is willing to invest money on the processing equipment only as long as it can obtain 14% annual return on each increment of money invested. Which one, if any, of the alternatives should be selected? Use a challenger–defender rate of return analysis.

Cash vs. Loan vs. Lease

Frequently we read in the newspaper that one should lease a car rather than buying it. For a typical 24-month lease on a car costing $9400, the monthly lease charge is about $267. At the end of the 24 months, the car is returned to the lease company (which owns the car). As an alternative, the same car could be bought with no down payment and 24 equal

8- monthly payments, with interest at a 12% nominal annual percentage rate.

39 At the end of 24 months the car is fully paid for. The car would then be worth about half its original cost.

(*a*) Over what range of nominal before-tax interest rates is leasing the preferred alternative?

(*b*) What are some of the reasons that would make leasing more desirable than is indicated in (*a*)?

The Financial Advisor is a weekly column in the local newspaper. Assume you must answer the following question. “I need a new car that I will keep for 4 years. I have three options. I can (*A*) pay \$32,999 now, (*B*) make monthly payments for a 7% 4-year loan with 0% down, or (*C*) make lease payments of \$425 per month for the next 4 years. The lease
8- option also requires an up-front payment of \$3500. What should I do?”
40 Assume that the number of miles driven matches the assumptions for the
A lease, and the vehicle’s value after 4 years is \$14,500. Remember that lease payments are made at the beginning of the month, and the salvage value is received only if you own the vehicle.

(*a*) Develop a choice table for nominal interest rates from 0% to 50%. (You do not know what the reader’s interest rate is.)

(*b*) If i = 8%, which option should be chosen?

The Financial Advisor is a weekly column in the local newspaper. Assume you must answer the following question. “I need a new car that I will keep for 5 years. I have three options. I can (*A*) pay \$25,999 now, (*B*) make monthly payments for a 9% 5-year loan with 0% down, or (*C*) make lease payments of \$470 per month for the next 5 years. The lease option also requires a security deposit of \$1500. What should I do?”
8- Assume that the number of miles driven matches the assumptions for the
41 lease, and the vehicle’s value after 5 years is \$7000. Remember that lease payments are made at the beginning of the month, and the salvage value is received only if you own the vehicle.

(*a*) Develop a choice table for nominal interest rates from 0% to 50%. (You do not know what the reader’s interest rate is.)

(*b*) If i = 9%, use an incremental rate of return analysis to recommend which option should be chosen.

Minicases

Contact a car dealer and choose a car to evaluate a buy-versus-lease decision (keep it reasonable—no Lamborghinis). Tell the people at the dealership that you are a student working on an assignment. Be truthful and don't argue; if they don't want to help you, leave and find a friendlier dealer. For both buying and leasing, show all assumptions, costs, and calculations. Do not include the cost of maintenance, gasoline, oil, water, fluids, and other routine expenses in your calculations.

Determine the car's sales price (no need to negotiate) and the costs for sales tax, license, and fees. Estimate the "Blue Book value" in 5 years. Determine the monthly payment based on a 5-year loan at 9% interest. Assume that your down payment is large enough to cover only the sales
8- tax, license, and fees. Calculate the equivalent uniform monthly cost of
42 owning the car.

Identify the costs to lease the car (if available assume a 5-year lease). This includes the monthly lease payment, required down payments, and any return fees that are required. Calculate the equivalent uniform monthly cost of leasing the car.

The salesperson probably does not have the answers to many of these questions. Write a one-to two-page memo detailing the costs. Make a recommendation: Should you own or lease your car? Include nonfinancial items and potential financial items in your conclusions, such as driving habits and whether you are likely to drive more than the allowed number of miles dictated in the lease.

Develop the costs and benefits to compare owning a car versus depending on public transit, friends, and/or a bicycle. Place a monetary value on each advantage or disadvantage.
8- (*a*) Develop a choice table for interest rates between 0% and 25%.
43 (*b*) Since costs are likely to be similar for most people, how sensitive are your results to environmental concerns? What external costs did you omit? How does including them and increasing their importance change your results?

Develop the costs and benefits to compare owning a new car with one
8- that is 2 years old. Place a monetary value on each advantage or

44 disadvantage. Develop a choice table for interest rates between 0% and 25%.

For a vehicle that you or a friend owns, determine the number of miles driven per year. Find three alternative sets of 4 tires that differ in their tread warranty. Assume that the life of the tires equals the tread warranty divided by the number of miles driven per year. Compare the EUACs of

8- the tires.

45 (*a*) For what interest rates is each choice the best?

(*b*) Develop a graph equivalent to Figure 8–4 to illustrate the results.

(*c*) For the interest rate that is in the "middle" of your range, how low and how high can the number of miles each year be without changing the best choice?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 6	**Lease a Lot** Compares leasing and ownership. Results show importance of separating financing and investing decisions.
CASE 10	**The Cutting Edge** Make vs. buy and machine selection.
CASE 11	**Harbor Delivery Service** Annual comparison of diesel versus gasoline engines.

[1] The cost term in the equation has a minus sign because it is a cost and we are calculating a present worth. The annual benefit term has a minus sign because of the sign convention for the PV function.

CHAPTER 9

OTHER ANALYSIS TECHNIQUES

Clean, Lower Cost Space Travel

Since 1960 there have been over 5000 orbital launches. Each launch has the potential to leave pieces of debris behind. NASA currently tracks more than 500,000 pieces of space debris in orbit. Space debris is defined as any man-made object in Earth's orbit that does not serve a useful function. When companies launch satellites into space or deliver payloads to the International Space Station, the rockets used to get them into orbit are either left floating in space or drop back through Earth's atmosphere, landing in their final resting place at the bottom of the ocean. About 24% of space debris consist of satellites, while 18% are spent rockets and other mission-specific objects. All of this debris is orbiting at speeds up to 17,000 mph, posing catastrophic threats to other spacecraft and satellites.

In addition to polluting space, the “disposable rocket” model is expensive. In 1969, NASA spent approximately $350 million for each launch of the Saturn V rocket as part of the Apollo space program. That is over $6.5 billion, adjusted to today’s dollars. In fact, the high cost of producing and launching these rockets was the main reason for the cancellation of the last three Apollo flights. Similarly, the average cost to design, build, prepare, and launch a Space Shuttle was close to $920 million per mission (adjusted for inflation).

We are entering the era of the reusable rocket. Private companies such as SpaceX and Blue Origin believe the future of space travel will be powered by reusable rockets. On October 7, 2018, SpaceX launched the Falcon 9, which successfully delivered an earth-observation satellite into orbit. Less than 10 minutes after liftoff, as seen in a widely viewed video, Falcon 9’s first stage safely returned to Earth at Vandenberg Air Force Base in California to be refurbished and reused in another launch. Falcon 9’s first-stage engines provide liftoff power and boost the rocket to the edge of space. The second stage carries the payload into orbit while the first stage returns back to Earth for reuse.

According to SpaceX, the first stage engine can be reused up to 15 times. By refurbishing rockets, the cost of each launch can be reduced from $62 million to $40 million. SpaceX’s chief operating officer Gwynne Shotwell suggests the reusable first stage reduces production costs by half. The first stage is estimated to be 75% of the total direct cost of each rocket. Much of these savings will be passed on to SpaceX customers—private firms launching satellites for commercial use or for government entities such as NASA. SpaceX hopes to decrease the overall cost of commercializing and exploring the solar system while having a positive impact on the environment—on Earth and in space!

Contributed by Christy Bozic, University of Colorado Boulder

QUESTIONS TO CONSIDER

1. What has happened more recently with reusable launches?
2. What are the potential costs and benefits of reusable rockets to producers

such as SpaceX? To a satellite company?
3. Which measures of economic value would be the most useful in evaluating reusable rockets? Why?
4. What ethical questions arise from companies who allow single-use rockets to land in the ocean or become space debris?

After Completing This Chapter …

The student should be able to:

- Use future worth, benefit–cost ratio, payback period, and sensitivity analysis methods to solve engineering economy problems.
- Link the use of *future worth* analysis to the present worth and annual worth methods developed earlier.
- Mathematically develop the *benefit–cost ratio*, and use this model to select alternatives and make economic choices.
- Understand the concept of the *payback period* of an investment, and be able to calculate this quantity for prospective projects.
- Demonstrate a basic understanding of *sensitivity* and *breakeven analyses* and the use of these tools in an engineering economic analysis.
- Use a spreadsheet to perform *sensitivity* and *breakeven analyses*.
- Consider expected returns and risks when investing or saving. (Appendix 9A)

Key Words

benefit–cost ratio

breakeven chart

discounted payback period

future worth analysis

liquidity

payback period

profitability

sensitivity analysis

staged construction

what-if analysis

Chapter 9 examines four topics:

• Future worth analysis
• Benefit–cost ratio or present worth index analysis
• Payback period
• Sensitivity, breakeven, and what-if analysis

Future worth analysis is very much like present worth analysis, dealing with *then* (future worth) rather than with *now* (present worth) situations.

Previously, we have written economic analysis relationships based on either

$$\text{PW of cost} = \text{PW of benefit} \qquad \text{or} \qquad \text{EUAC} = \text{EUAB}$$

Instead of writing it in this form, we could define these relationships as

$$\frac{\text{PW of benefit}}{\text{PW of cost}} = 1 \qquad \text{or} \qquad \frac{\text{EUAB}}{\text{EUAC}} = 1$$

When economic analysis is based on these ratios, the calculations are called benefit–cost ratio analysis. The PW ratio is also known as a present worth index.

Payback period is an approximate analysis technique, generally defined as the time required for cumulative benefits to equal cumulative costs.

Sensitivity describes how much a problem element must change to reverse a particular decision. Closely related is breakeven analysis, which determines the conditions under which two alternatives are equivalent. What-if analysis changes one or all variables to see how the economic value and recommended decision change. Thus, breakeven and what-if analysis are forms of sensitivity analysis.

FUTURE WORTH ANALYSIS

In present worth analysis, alternatives are compared in terms of their present consequences. In annual cash flow analysis, the comparison was in terms of equivalent uniform annual costs (or benefits). But the concept of resolving alternatives into comparable units is not restricted to a present or annual comparison. The comparison may be made at any point in time. In many situations we would like to know what the *future* situation will be, if we take some particular course of action *now*. This is called **future worth analysis.**

Example 9–1 illustrates how much impact seemingly small decisions can have on long-term objectives. A common variation of Example 9–1 defines a future worth goal in *n* years for an expensive vacation, a down payment on a house, a child's college fund, or for retirement. In these cases the question is how much must be saved each month to meet the goal. This type of savings and investing is the subject of Appendix 9A, which focuses on retirement because of its long horizon and importance. Appendix 9A can be applied to any future worth savings goal. Example 9–2 illustrates how to calculate the future worth at start-up of projects with multi-year construction periods.

EXAMPLE 9-1

A college student spends $35 a week on lottery tickets. He wonders how much money he could accumulate before retiring at 65 if he stopped buying lottery tickets and instead put the money into a savings account. The savings account would earn 5% interest, compounded semiannually. Compute the future worth of the savings at age 65.

Assume that the weekly $35 is from a 5-day per week habit where this makes sense, and that there is a $2 per day alternative. Possible examples include lattes, breakfasts from a drive-through, and lunches not brought from home. What are the weekly savings and the future worth of the savings account?

TABLE SOLUTION

Stop and Save

Semiannual saving = ($35/week)(26 weeks) = $910
$\text{FW} = A(F/A, 2^1/2\%, 90) = 910(329.2) = \$299{,}572$

Daily Habit to Weekly Treat[1]

A weekly $35 from a 5-day per week habit is $7 per day. Substituting a $2 per day alternative on 4 days per week would save $20 weekly (= 4 × $5). This would be $520 every 6 months.

$\text{FW} = 520(F/A, 2^1/2\%, 90) = 520(329.2) = \$171{,}184$

5 BUTTON SOLUTION

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Stop & save	2.5%	90	-910	0		FV	$299,530	FV =FV(B2,C2,D2,E2)
3									
4	Weekly treat	2.5%	90	-520	0		FV	$171,160	=FV(B2,C2,D2,E2)

EXAMPLE 9-2

An East Coast firm has decided to establish a second plant in Kansas City. There is a factory for sale for $850,000 that could be remodeled and used. As an alternative, the firm could buy vacant land for $85,000 and have a new plant constructed there. Either way, it will be 3 years before the firm will be able to get a plant into production. The timing of costs for the factory are:

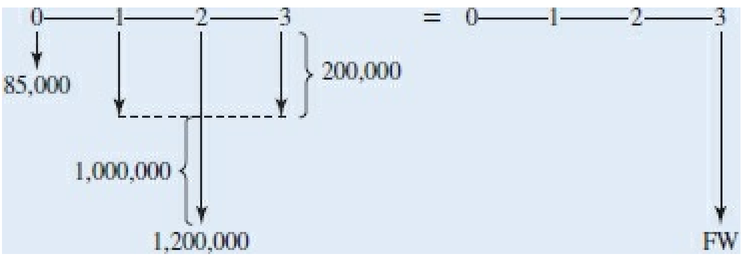
Year	Construct New Plant		Remodel Available Factory	
0	Buy land	$ 85,000	Purchase factory	$ 850,000
1	Design	200,000	Design	250,000

2	Construction	1,200,000	Remodeling	250,000
3	Production equipment	200,000	Production equipment	250,000

If interest is 8%, which alternative has the lower equivalent cost when the firm begins production at the end of Year 3?

SOLUTION

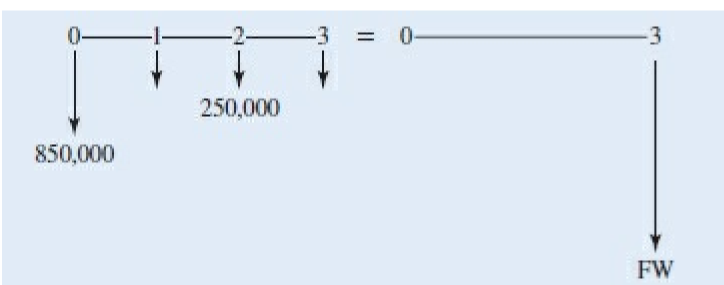
New Plant



Cash flow in year 2 can be viewed as \$1,000,000 plus \$200,000, creating a uniform cost of \$200,000 in years 1, 2, and 3 plus a one-time cost of \$1,000,000 in year 2.

$$\begin{aligned}\text{FW of cost} &= 85{,}000(F/P, 8\%, 3) + 200{,}000(F/A, 8\%, 3)\\ &\quad + 1{,}000{,}000(F/P, 8\%, 1) = \$1{,}836{,}000\end{aligned}$$

Remodel Available Factory



$$\begin{aligned}\text{FW of cost} &= 850{,}000(F/P, 8\%, 3) + 250{,}000(F/A, 8\%, 3)\\ &= \$1{,}882{,}000\end{aligned}$$

The total cost of remodeling the available factory (\$1,600,000) is smaller than the total cost of a new plant (\$1,685,000). However, the timing of the expenditures is better with the new plant. The new plant is projected to have the smaller future worth of cost and thus is the preferred alternative.

BENEFIT–COST RATIO ANALYSIS

At a given minimum attractive rate of return (MARR), we would consider an alternative acceptable, provided that

$$\text{PW of benefits} - \text{PW of costs} \geq 0 \qquad \text{or} \qquad \text{EUAB} - \text{EUAC} \geq 0$$

Table 9-1 Benefit–Cost Ratio Analysis

Input/Output	Situation	Criterion
Neither input nor output fixed	Neither amount of money or other inputs nor amount of benefits or other outputs are fixed	*One alternative:* B/C ≥ 1 Two or more alternatives: Solve by incremental analysis of benefit–cost ratios
Fixed input	Amount of money or other input resources are fixed	Maximize B/C
Fixed output	Fixed task, benefit, or other output to be accomplished	Maximize B/C

These could also be stated as a ratio of benefits to costs, or

$$\textbf{Benefit–cost ratio} = \frac{\text{PW of benefit}}{\text{PW of costs}} = \frac{\text{EUAB}}{\text{EUAC}} \geq 1$$

Rather than using present worth or annual cash flow analysis to solve problems, we can base the calculations on the benefit–cost ratio, B/C. The criteria are presented in Table 9–1. In Table 9–1 the two special cases where maximizing the B/C ratio is correct are listed below the more common situation where incremental analysis is required. In Chapter 16 we will detail how this measure is applied in the public sector. Its use there is so pervasive that the term *present worth index* is sometimes used to distinguish private-sector applications.

Present worth, annual cash flow, rate of return, and benefit−cost ratio approaches *all* require a common horizon when comparing alternatives. Example 9–3 shows that incremental analysis is required for mutually exclusive alternatives to correctly apply the benefit−cost ratio, just as it was required to use rate of return. The intuitively appealing approach of maximizing the B/C ratio does not work. Example 9–4 applies incremental analysis to a six-alternative problem.

EXAMPLE 9-3 Examples 5–1, 6–5, and 7–13 Revisited

A firm is trying to decide which of two devices to install to reduce costs. Both devices have useful lives of 5 years and no salvage value. Device *A* costs $10,000 and can be expected to result in $3000 savings annually. Device *B* costs $13,500 and will provide cost savings of $3000 the first year, but savings will increase by $500 annually, making the second-year savings $3500, the third-year savings $4000, and so forth. With interest at 7%, which device should the firm purchase?

SOLUTION

We have used three types of analysis thus far to solve this problem: present worth in Example 5–1, annual cash flow in Example 6–5, and rate of return in Example 7–13. First we correctly analyze this incrementally, then we look at each device's benefit–cost ratio.

Incremental *B*–*A*

$$\begin{aligned}\text{PW of incremental cost} &= \$3500\\ \text{PW of incremental benefits} &= 500(P/G, 7\%, 5)\\ &= 500(7.647) = \$3820\\ \frac{\text{B}}{\text{C}} = \frac{\text{PW of benefit}}{\text{PW of costs}} &= \frac{3820}{3500} = 1.09\end{aligned}$$

The increment is justified at the MARR of 7%. Device *B* should be purchased.

Device *A*

$$\begin{aligned}\text{PW of cost} &= \$10{,}000\\ \text{PW of benefits} &= 3000(P/A, 7\%, 5)\\ &= 3000(4.100) = \$12{,}300\\ \frac{\text{B}}{\text{C}} &= \frac{\text{PW of benefit}}{\text{PW of costs}} = \frac{12{,}300}{10{,}000} = 1.23\end{aligned}$$

Device *B*

$$\begin{aligned}\text{PW of cost} &= \$13{,}500\\ \text{PW of benefit} &= 3000(P/A, 7\%, 5) + 500(P/G, 7\%, 5)\\ &= 3000(4.100) + 500(7.647) = 12{,}300 + 3820 = 16{,}120\\ \frac{\text{B}}{\text{C}} &= \frac{\text{PW of benefit}}{\text{PW of costs}} = \frac{16{,}120}{13{,}500} = 1.19\end{aligned}$$

Maximizing the benefit–cost ratio indicates the wrong choice, Device *A*. Incremental analysis must be used.

EXAMPLE 9-4 Examples 8–6 and 8–7 Revisited

Consider the five mutually exclusive alternatives from Examples 8–6 and 8–7 plus an additional alternative, *F*. They have 20-year useful lives and no salvage value. If the minimum attractive rate of return is 6%, which alternative should be selected?

	A	*B*	*C*	*D*	*E*	*F*
Cost	\$4000	\$2000	\$6000	\$1000	\$9000	\$10,000
PW of benefit	7330	4700	8730	1340	9000	9,500
$\frac{\text{B}}{\text{C}} = \frac{\text{PW of benefits}}{\text{PW of cost}}$	1.83	2.35	1.46	1.34	1.00	0.95

SOLUTION

Incremental analysis is needed to solve the problem. The steps in the solution are the same as the ones presented in Example 8–7 for incremental rate of return, except here the criterion is ΔB/ΔC, and the cutoff is 1, rather than ΔIRR with a cutoff of MARR.

1. Be sure all the alternatives are identified.
2. (Optional) Compute the B/C ratio for each alternative. Since there are alternatives for which B/C ≥ 1, we will discard any with B/C < 1. Discard Alt. *F*.
3. Arrange the remaining alternatives in ascending order of investment.

	D	*B*	*A*	*C*	*E*
Cost (= PW of cost)	$1000	$2000	$4000	$6000	$9000
PW of benefits	1340	4700	7330	8730	9000
B/C	1.34	2.35	1.83	1.46	1.00

	B – *D* Increment	*A* – *B* Increment	*C* – *A* Increment
ΔCost	$1000	$2000	$2000
Δ Benefits	3360	2630	1400
ΔB/C	3.36	1.32	0.70

4. For each increment of investment, if ΔB/ΔC≥1 the increment is attractive. If ΔB/ΔC < 1 the increment of investment is not desirable. The increment *B* – *D* is desirable, so *B* is preferred to *D*. The increment *A* – *B* is desirable. Thus, Alt. *A* is preferred. Increment *C* – *A* is not attractive since ΔB/ΔC = 0.70. Now we compare *A* and *E*:

	E–*A* Increment
ΔCost	$5000
ΔBenefit	1670
ΔB/ΔC	0.33

The increment is undesirable. We choose Alt. *A* as the best of the six alternatives. [*Note:* The best alternative does not have the highest B/C ratio, nor is it the largest project with a B/C ratio ¿ 1. Alternative *A* does have the largest difference between the PW of its benefits and costs (= $3330).]

Benefit–cost ratio analysis may be graphically represented. Figure 9–1 is a graph of Example 9–4. We see that *F* has a B/C < 1 and can be discarded. Alternative *D* is the starting point for examining the separable increments of investment. The slope of line *B–D* indicates a ΔB/ΔC ratio of >1. This is also true for line *A–B*. Increment *C–A* has a slope much flatter than B/C = 1, indicating an undesirable increment of investment. Alternative *C* is therefore discarded and *A* retained. Increment *E–A* is similarly unattractive. Alternative *A* is therefore the best of the six alternatives.

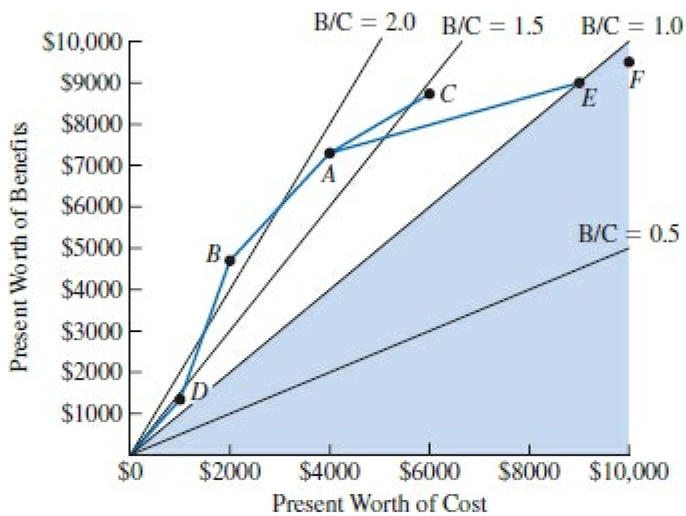


FIGURE 9-1 Benefit–cost ratio graph of Example 9–4.

Note three additional things about Figure 9–1: first, even if alternatives with B/C ratio < 1 had not been initially excluded, they would have been systematically eliminated in the incremental analysis. Second, Alt. *B* had the highest B/C ratio (B/C = 2.35), but it is not the best of the six alternatives. We saw the same situation in rate of return analysis of three or more alternatives. The reason is the same in both analysis situations. We seek to maximize the *total* profit, not the profit rate.

Third and most important, the total profit or B − C for each alternative is the vertical distance from the point to the B/C = 1.0 line. *A* maximizes that distance.

Variations on the Theme of the Benefit–Cost Ratio

The basic benefit–cost ratio has been defined as placing *all* benefits in the numerator of the ratio and *all* costs in the denominator. One variation of the ratio considers the salvage values as reducing the costs rather than as increasing the benefits.

In the public sector, it is common to define the benefit–cost ratio so that the numerator includes all consequences to the users or the public and the denominator includes all consequences to the sponsor or government. For example, the numerator might include the positive benefits of improved highway traffic flow and the disbenefits of congestion during construction, since both accrue to the public or users. The denominator in this case includes consequences to the government, such as the costs of construction and the reduced maintenance cost for the new highway. Example 9–5 illustrates this for a public project.

In Example 9–6 exactly the same numbers are put in a private context. Here the benefit–cost ratio is typically called a present worth index. The calculation is modified so that the denominator is the project's first cost, and all other consequences are placed in the numerator. This formulation of the benefit–cost ratio emphasizes the "bang for the buck" of how much return is gained for each dollar of investment.

We will examine the public-sector application of the benefit–cost ratio in more detail in Chapter 16, and the present worth index will be used in Chapter 15. Here and in those chapters, the same standard applies for all versions of the ratio. Is the ratio ≥1? More importantly, if one version of the ratio is ≥1, then all versions are ≥1. As shown in Examples 9–5 and 9–6, the values of the ratios may differ, but whether they are above or below 1 and the recommended decisions do not change.

EXAMPLE 9-5

Traffic congestion on Riverview Boulevard has reached a point where something must be done. Two suggested plans have a life of 15 years, because that is the scheduled time for completion of the new Skyway Highway. After that time traffic will fall well below current levels.

Adding right-turn lanes at key intersections will cost \$8.9M (million) with annual maintenance costs for signals and lane painting of \$150,000. Added congestion during construction is a disbenefit of \$900,000, but the reduced congestion after construction is an annual benefit of \$1.6M. This benefit actually starts lower and increases over time, but for a simple initial analysis we are assuming a uniform annual benefit.

Adding a second left-turn lane at a few key intersections will cost an additional \$3M with an added annual maintenance cost of \$75,000. This construction is more disruptive, and the total disbenefit for congestion during construction is \$2.1M. Upon completion, the total benefit for reduced congestion will be \$2.2M annually.

Which alternative is preferred if the interest rate is 10%? Analyze using a government B/C ratio (public in numerator and government in denominator).

SOLUTION

Since something *must* be done and we have only two identified alternatives, we could simply analyze the difference between the alternatives to see which is better. But we are going to start by analyzing the less expensive “right-turns” alternative to check that this is a reasonable choice for what *must* be done.

The user consequences include an annual benefit for reduced congestion and a first “cost” that is the disbenefit of increased congestion during construction.

$$\begin{aligned}\text{PW-benefits}_{\text{right turns}} &= -900{,}000 + 1{,}600{,}000(P/A, 10\%, 15)\\ &= -900{,}000 + 1{,}600{,}000(7.606) = \$11.27\text{M}\end{aligned}$$

The government costs include a first cost for construction and annual maintenance costs. Note that these are calculated as present *costs*.

$$\begin{aligned}\text{PW-costs}_{\text{right turns}} &= 8{,}900{,}000 + 150{,}000(P/A, 10\%, 15)\\ &= 8{,}900{,}000 + 150{,}000(7.606) = \$10.04\text{M}\end{aligned}$$

The benefit–cost ratio for public divided by government consequences is

$$\text{B/C ratio} = \$11.27\text{M}/\$10.04\text{M} = 1.122$$

Thus, the right-turns-only alternative is better than doing nothing.

Now we evaluate the incremental investment for also doing the left-turn improvements. Because we are using a benefit–cost ratio, this evaluation must be done incrementally. The user consequences include an incremental annual benefit for reduced congestion and an incremental first "cost" that is the disbenefit of increased congestion during construction.

$$\begin{aligned}\text{PW-benefits}_{\text{left turns} - \text{right turns}} &= -2{,}100{,}000 - (-900{,}000) + (2{,}200{,}000 - 1{,}600{,}000)(P/A, 10\%, 15)\\ &= -1{,}200{,}000 + 600{,}000(P/A, 10\%, 15)\\ &= -1{,}200{,}000 + 600{,}000(7.606) = \$3.364\text{M}\end{aligned}$$

The government costs include a first cost for construction and annual maintenance costs.

$$\begin{aligned}\text{PW-costs}_{\text{left turns} - \text{right turns}} &= 3{,}000{,}000 + 75{,}000(P/A, 10\%, 15)\\ &= 3{,}000{,}000 + 75{,}000(7.606) = \$3.570\text{M}\end{aligned}$$

The benefit–cost ratio for public divided by government consequences is

$$\text{B/C ratio} = \$3.364\text{M}/\$3.570\text{M} = 0.942$$

Thus, the right-turns-only alternative is better than adding the left-turn increment.

EXAMPLE 9-6

The industrial engineering department of Amalgamated Widgets is considering two alternatives for improving material flow in its factory. Both plans have a life of 15 years, because that is the estimated remaining life for the factory.

A minimal reconfiguration will cost $8.9M (million) with annual maintenance costs of $150,000. During construction there is a cost of $900,000 for extra material movements and overtime, but more efficient movement of materials will save $1.6M annually. The cost savings actually start lower and increase over time, but for a simple initial analysis we are assuming a uniform annual cost savings.

Reconfiguring a second part of the plant will cost an additional \$3M with an added annual maintenance cost of \$75,000. This construction is more disruptive, and the total cost for material movement and overtime congestion during construction is \$2.1M. Once complete, the total cost savings for more efficient movement of materials is \$2.2M annually.

Which alternative is preferred if the interest rate is 10%? Analyze using a present worth index (all consequences in Years 1 to *n* in numerator and all first costs in denominator).

SOLUTION

Since something *must* be done and we have only two identified alternatives, we could simply analyze the difference between the alternatives to see which is better. But we are going to start by analyzing the less expensive minimal reconfiguration alternative to check that this is a reasonable choice for what *must* be done.

The consequences in Years 1 to *n* include an annual cost savings for more efficient flow and annual maintenance costs.

$$\begin{aligned}\text{PW-benefits}_{\text{Years 1 to } n} &= (1{,}600{,}000 - 150{,}000)(P/A, 10\%, 15) \\ &= (1{,}600{,}000 - 150{,}000)(7.606) = \$11.03\text{M}\end{aligned}$$

The first costs include a first cost for construction and the cost for disruption during construction.

$$\text{PW-costs} = 8{,}900{,}000 + 900{,}000 = \$9.8\text{M}$$

The present worth index is

$$\text{PW-index} = \$11.03\text{M}/\$9.8\text{M} = 1.125$$

Thus, the minimal reconfiguration is better than doing nothing.

Now we evaluate the incremental investment for also reconfiguring the second part of the plant. Because we are using a present worth index, this evaluation must be done incrementally. The annual consequences include an incremental annual cost savings and incremental maintenance costs.

$$\begin{aligned}\text{PW-benefits}_{\text{Years 1 to } n} &= (600{,}000 - 75{,}000)(P/A, 10\%, 15)\\ &= 525{,}000(7.606) = \$3.993\text{M}\end{aligned}$$

There is a first cost for construction and for the associated disruption.

$$\text{PW-costs} = 3{,}000{,}000 + 1{,}200{,}000 = \$4.2\text{M}$$

The present worth index is

$$\text{PW-index} = \$3.993\text{M}/\$4.2\text{M} = 0.951$$

Thus, the minimal reconfiguration is better than reconfiguring the second part of the plant.

In Examples 9–5 and 9–6, the numbers that appeared in the numerator and denominator were changed, and the exact values of the B/C ratio and present worth index also changed. However, the conclusions did not. The ratios were above 1.0 for the minimal investment choice. The ratios were below 1.0 for the incremental investment. It was always best to make the minimal investment.

These examples demonstrate that present worth analysis and incremental benefit–cost ratio analysis lead to the same optimal decision. We saw in Chapter 8 that rate of return and present worth analysis led to identical decisions. Any of the exact analysis methods—present worth, annual cash flow, rate of return, or benefit–cost ratio—will lead to the same decision. Benefit–cost ratio analysis is extensively used in economic analysis at all levels of government.

PAYBACK PERIOD

Payback period measures the time required for the profit or other benefits from an investment to equal the cost of the investment. This is the general definition for payback period. Other definitions consider depreciation of the investment, interest, and income taxes; they, too, are simply called “payback period.” We will limit our discussion to the simplest form.

Payback period is the period of time required for the project’s profit or other benefits to equal the project’s cost.

The criterion in all situations is to minimize the payback period. The computation of payback period and its weaknesses relative to time value of money measures are illustrated in Examples 9–7 and 9-8.

EXAMPLE 9-7

The cash flows for two alternatives are as follows:

Year	*A*	*B*
0	− \$1000	− \$2783
1	200	1200
2	200	1200
3	1200	1200
4	1200	1200
5	1200	1200

You may assume the benefits occur throughout the year rather than just at the end of the year. Based on payback period, which alternative should be selected? For what interest rates is this the correct choice?

PAY BACK PERIOD SOLUTION

Because benefits occur throughout the year (like most engineering projects), fractional years have meaning.

Alternative *A*

Payback period is how long it takes for the profit or other benefits to equal the cost of the investment. In the first 2 years, only \$400 of the \$1000 cost is recovered. The remaining \$600 cost is recovered in the first half of Year 3.

Thus the payback period for Alt. *A* is 2.5 (= 2 + 600/1200) years.

Alternative *B*

Since the annual benefits are uniform, the payback period is simply

$2783/$1200 per year = 2.3 years

To minimize the payback period, choose Alt. *B*.

TIME VALUE OF MONEY SOLUTION

Previous chapters have shown that incremental analysis is needed to analyze mutually exclusive alternatives with rate of return. Since both alternatives have cash flows of $1200 in years 3 through 5, the incremental cash flows are zero, and the incremental analysis can focus on the cash flows for the first 2 years.

Year	*B*−*A*
0	−$1783
1	1000
2	1000

The incremental IRR is 8.0%, so for interest rates below 8% Alternative *B* will be preferred (matching the payback period solution). For interest rates above 8% Alternative *A* will be preferred. The easiest way to check this is to ask, which is preferred at 0% interest? By adding the cash flows we see that PW_{B-A} = $217 (= 2000 − 1783). So the PW of *B* is $217 higher than the PW of *A* at 0%. At 10% the PW of the increment is −$47, so *A* is preferred.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	IRR_{B-A}		2	1000	-1783	0	RATE	8.011%	=RATE(C2,D2,E2,F2)
3	$PW_{10\%}$	10%	2	1000			PV	1736	
4								-1783	

5		$PW_{B\text{-}A}$ 10%	-47

EXAMPLE 9-8

A product will be phased out in 5 years, but it currently has a quality problem that is costing $4000 per year. Machines *A* and *B* will both solve the quality problem. Machine *A* costs only $10,000 but it will have no salvage value in 5 years. Machine *B* costs $15,000 but in 5 years it will have a salvage value of $9000. Based on payback period, which alternative should be selected? For what interest rates is this the correct choice?

PAYBACK PERIOD SOLUTION

As long as payback does not depend on the salvage value (usually true), the payback period for uniform cash flows is simply the first cost divided by the annual benefit. In this case:

$$\text{Payback}_A = \$10{,}000/(\$4000/\text{year}) = 2.5 \text{ years}$$
$$\text{Payback}_B = \$15{,}000/(\$4000/\text{year}) = 3.75 \text{ years}$$

So choosing to minimize the payback period would result in choosing Machine *A*.

TIME VALUE OF MONEY SOLUTION

Previous chapters have shown that incremental analysis is needed to analyze mutually exclusive alternatives with rate of return. Since both alternatives have cash flows of $4000 in years 1 through 5, the incremental analysis can focus on the cash flows for the first cost and the salvage value.

Year	*B* −*A*
0	−$5000
5	9000

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	$IRR_{B\text{-}A}$		5		-5000	9000	RATE	12.5%	=RATE(C2,D2,E2,F2
3	$PW_{10\%}$	15%	5			1000	PV	4475	
4								-5000	
5						$PW_{B\text{-}A\ 15\%}$		-525	

The incremental IRR is 12.5%, so for interest rates below 12.5% Alternative *B* will be preferred (matching the payback period solution). For interest rates above 12.5% Alternative *A* will be preferred. The easiest way to check this is to ask, which is preferred at 0% interest? By adding the cash flows we see that at 0% PW_{B-A} = $4000 (= 9000 − 5000). So the PW of *B* is $4000 higher than the PW of *A* at 0%. At 15% the PW of the increment is −$525, so *A* is preferred.

There are four important points to be understood about payback period calculations:

1. This is an approximate, rather than an exact, economic analysis calculation.
2. All costs and all profits or savings of the investment before payback are included *without* considering differences in their timing.
3. All the economic consequences beyond the payback period are completely ignored.
4. Being an approximate calculation, payback period may or may not select the correct alternative.

This last point—that payback period may select the *wrong* alternative—was illustrated by Examples 9–7 and 9–8. But if payback period calculations are approximate and may lead to selecting the wrong alternative for some or all interest rates, why are they used? First, the calculations can be readily made by people unfamiliar with economic analysis. Second, good projects and alternatives are usually clearly superior by all measures. Third, payback period is easily understood. Earlier we pointed out that this is also an advantage to rate of return. The vignette in Chapter 3 described a high payout project. The NPV was about $8.6M and the IRR was over 11,000%. Both of these can be confusing to management, but the idea that the project paid out in one day was very clear.

Moreover, payback period *does* measure **liquidity**—how long it will take for the cost of the investment to be recovered from its benefits. Firms are often very interested in this time period: a rapid return of invested capital means that the funds can be reused sooner for other purposes. But one must not confuse the *speed* of the return of the investment, as measured by the payback period, with **profitability**. They are two distinctly separate concepts.

There is a refinement of the payback period that does include interest—the **discounted payback period**. The discounted payback period is longer than the payback period, because the benefits must also cover the interest on the capital invested in the project. If the annual benefits are uniform and the salvage value is $0, then NPER can be used to calculate the discounted payback period—as long as that period is less than the alternative's life. If the salvage value becomes involved in achieving payback, then the horizon is the payback period.

The discounted payback period includes some consideration of the time value of money, so it is a better measure than payback period, but discounted payback period is still not a valid time value of money measure that includes all cash flows. In Example 9–9, there are clearly salvage values that would make either alternative the preferred choice. Payback periods, whether discounted or not, ignore cash flows that occur after payback.

EXAMPLE 9-9

Two alternatives have been identified. Alternative *A* has a first cost of $10,000 and benefits of $3000 annually. Alternative *B* has a first cost of $12,000 and benefits of $3500 annually. The salvage values of each alternative are currently unknown. Using discounted payback period, which alternative is preferred at an interest rate of 10%?

SOLUTION

Even if the salvage values were known, they must be ignored for the NPER calculation. If they are included, then NPER assumes that the salvage value occurs at time of payback—which may dramatically shorten the calculated time period.

	A	B	C	D	E	F	G	H	I
1	Exp. 9-9	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	A	10%		3000	-10,100	0	NPER	4.25	=NPER(B2,D2,E2,F2)
3	B	10%		3500	-12,000	0	NPER	4.41	

Alternative *A* has a discounted payback period of 4.25 years versus 4.41 years for Alternative *B*. So *A* is preferred using the discounted payback period. Whether or not this is correct can only be determined by a valid time value of money computation that includes the unknown salvage values. We do not yet know which alternative has the better present worth at 10%.

From the discussion and the examples, we see that payback period can measure the speed of the return of the investment. This might be quite important, for example, for a company that is short of working capital or for a firm in an industry experiencing rapid changes in technology. Calculation of payback period alone, however, must not be confused with a careful economic analysis. Ignoring all cash flows after the payback period is seldom wise. We have shown that a short payback period does not always mean that the associated investment is desirable. Thus, payback period is not a suitable replacement for accurate economic analysis calculations.

SENSITIVITY AND BREAKEVEN ANALYSIS

Since many data gathered in solving a problem represent *projections* of future consequences, there may be considerable uncertainty regarding the data's accuracy. Since the goal is to make good decisions, an appropriate question is: To what extent do variations in the data affect my decision? When small variations in a particular estimate would change which alternative is selected, the decision is said to be **sensitive to the estimate.** To better evaluate the impact of any particular estimate, we compute "how much a particular estimate would need to change in order to change a particular decision." This is called **sensitivity analysis.** Chapter 8 compared economic values at different interest rates with graphs and choice tables. That was also a form of sensitivity analysis.

An analysis of the sensitivity of a problem's decision to its various parameters highlights the important aspects of that problem. For example, estimated annual maintenance and salvage values may vary substantially. Sensitivity analysis might indicate that a certain decision is insensitive to the salvage-value estimate over the full range of possible values. But, at the same time, we might find that the decision is sensitive to changes in the annual maintenance estimate. Under these circumstances, one should place greater emphasis on improving the annual maintenance estimate and less on the salvage-value estimate.

As indicated at the beginning of this chapter, breakeven analysis is a form of sensitivity analysis that is often presented as a **breakeven chart**. Another nomenclature that is sometimes used for the breakeven point is *point of indifference*. One application of these tools is **staged construction.** Should a facility be constructed now to meet its future full-scale requirement? Or should it be constructed in stages as the need for the increased capacity arises? What is the breakeven point on how soon the capacity is needed for this decision? Three examples are:

• Should we install a cable with 400 circuits now or a 200-circuit cable now and another 200-circuit cable later?
• A 10-cm water main is needed to serve a new area of homes. Should it be installed now, or should a 15-cm main be installed to ensure an adequate water supply to adjoining areas later, when other homes have been built?
• An industrial firm needs a new warehouse now and estimates that it will need to double its size in 4 years. The firm could have a warehouse built now and later enlarged, or the firm could have the larger warehouse built right away.

Examples 9–10 and 9–11 illustrate sensitivity and breakeven analysis. These examples have focused on the breakeven project life, because that value of *n* is often one of the most uncertain values in an economic analysis. How long will that bridge, machine, or product function and meet the need?

EXAMPLE 9-10

Consider a project that may be constructed to full capacity now or may be constructed in two stages.

Construction Alternative	Costs
Two-stage construction	
Construct first stage now	$100,000
Construct second stage *n* years from now	120,000
Full-capacity construction	
Construct full capacity now	140,000

After 40 years all facilities will have zero salvage value. The annual cost of operation and maintenance is the same for both alternatives. With an 8% interest rate, what is the breakeven *n*?

5-BUTTON SOLUTION

As emphasized previously, the choice between two mutually exclusive alternatives can be made by analyzing the incremental difference between them. For rate of return and benefit–cost methods, incremental analysis is required. In this case, building the full capacity now costs $40,000 more than the initial cost of the two-stage alternative. This will save the $120,000 required whenever the second stage would be built. NPER can be used to solve for the life that gives these cash flows the same worth at an interest rate of 10%.

	A	B	C	D	E	F	G	H	I
1	Exp. 9-9	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Full–staged	8%		0	-40,000	120,000	NPER	4.27	=NPER(B2,D2,E2,F

Example 9–12 will revisit this example to create and improve the graph of each alternative’s present worth.

TABLE SOLUTION

Since we are dealing with a common analysis period, the calculations may be either annual cost or present worth. Present worth calculations appear simpler and are used here. Incremental analysis with a simple factor equation matching the 5-BUTTON SOLUTION will be presented, but starting with a graphical approach (like Chapter 8) seems likely to support a better understanding.

Construct Full Capacity Now

$$\text{PW of cost} = \$140{,}000$$

Two-Stage Construction

If the first stage is to be constructed now and the second stage n years hence, compute the PW of cost for several values of n (years).

$$\text{PW of cost} = 100{,}000 + 120{,}000(P/F, 8\%, n)$$

$n = 5$ PW = 100, 000 + 120, 000(0.6806) = \$181, 700
$n = 10$ PW = 100, 000 + 120, 000(0.4632) = 155, 600
$n = 20$ PW = 100, 000 + 120, 000(0.2145) = 125, 700
$n = 30$ PW = 100, 000 + 120, 000(0.0994) = 111, 900

These data are plotted in the form of a breakeven chart in Figure 9–2.

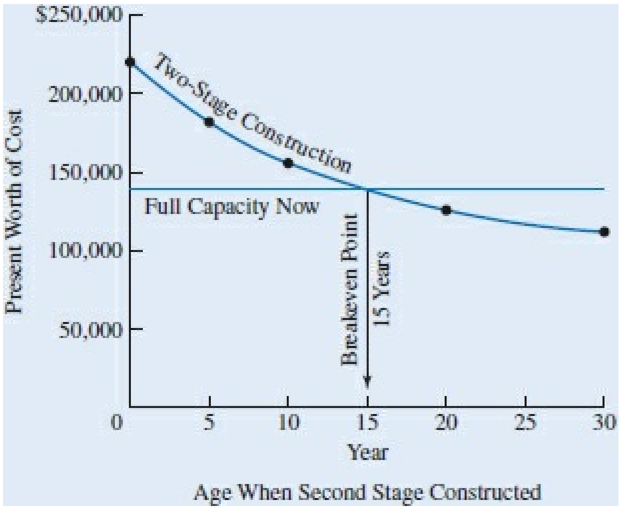


FIGURE 9-2 Breakeven chart for Example 9–10.

In Figure 9–2 we see that the PW of cost for two-stage construction naturally decreases as the second stage is deferred. The one-stage construction (full capacity now) is unaffected by the *x*-axis variable and, hence, is a horizontal line.

The breakeven point on the graph is where both alternatives have the same PW. This is about 15 years. If the second stage were to be needed prior to Year 15, then one-stage construction has the lower cost. On the other hand, if the second stage would not be required until after 15 years, two-stage construction has the lower cost.

This breakeven point can be more accurately calculated by setting the two alternatives equal to each other.

$$\text{PW} = 140{,}000 = 100{,}000 + 120{,}000(P/F, 8\%, n)$$
$$(P/F, 8\%, n) = \frac{40{,}000}{120{,}000} = 0.3333$$

From the tables

$$n = 14 + (15 - 14)(0.3405 - 0.3333)/(0.3405 - 0.3152)$$
$$n = 14.3 \text{ years}$$

SENSITIVITY DISCUSSION

The decision on how to construct the project is sensitive to the age at which the second stage is needed *only* if the range of estimates includes 15 years. For example, if one estimated that the second-stage capacity would be needed between 5 and 10 years hence, the decision is insensitive to that estimate. For any value within that range, the decision does not change. But, if the second-stage capacity were to be needed sometime between, say, 12 and 18 years, the decision would be sensitive to the estimate of when the full capacity would be needed.

EXAMPLE 9-11 Examples 9–5 and 9–6 Revisited

In both Examples 9–5 (traffic congestion on Riverview Boulevard) and 9–6 (reconfiguring the plant of Amalgamated Widgets), the life of 15 years is clearly subject to some uncertainty. While holding the other data constant, analyze the sensitivity of the recommended decisions to the project life. Use the present worth measure, since it is the same for both examples.

SOLUTION

The two alternatives have the following present worth values.

$$\begin{aligned}
PW_{\text{right turns or minimal}} &= -900{,}000 - 8{,}900{,}000 + (1{,}600{,}000 - 150{,}000)(P/A, 10\%, n) \\
&= -9{,}800{,}000 + 1{,}450{,}000(P/A, 10\%, n) \\
PW_{\text{left turns or 2}^{\text{nd}}\text{ part of plant}} &= -2{,}100{,}000 - 8{,}900{,}000 - 3{,}000{,}000 \\
&\quad + (2{,}200{,}000 - 225{,}000)(P/A, 10\%, n) \\
&= -14{,}000{,}000 + 1{,}975{,}000(P/A, 10\%, n)
\end{aligned}$$

These could be analyzed for breakeven values of *n*. However, it is easier to use the graphing technique for multiple alternatives that was presented in Chapter 8. Instead of using the interest rate for the *x* axis, use *n*.

As shown in Figure 9–3, the right-turn or minimal alternative is the best one for lives of 12 to 16 years. The left-turn increment or 2^{nd} part of the plant, is the best choice for lives of 17 or more years. If the life is 11 years or less, doing nothing is better. To keep the graph readable, Figure 9–3 includes only

Years 10 through 20.

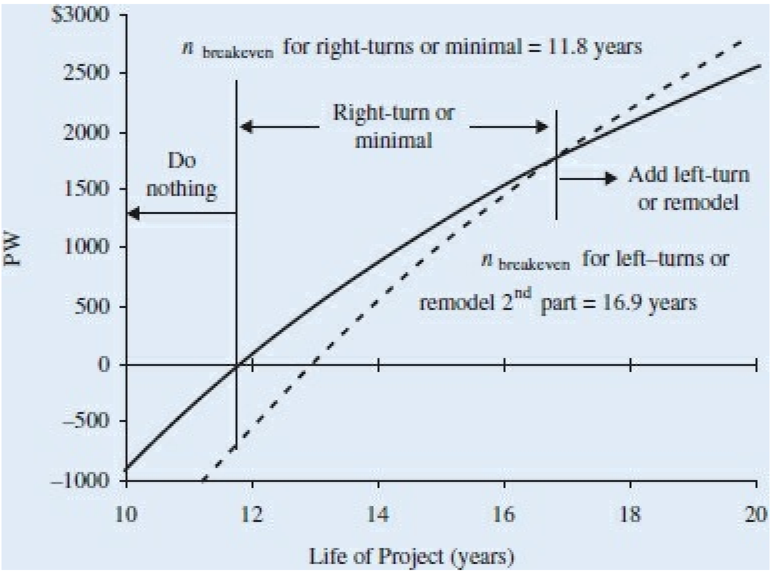


FIGURE 9-3 Breakeven chart for Example 9–11.

Breakeven points can be estimated from graphs or calculated with formulas or GOAL SEEK. Sensitivity analysis and breakeven point calculations can be very useful in identifying how different estimates affect the calculations. It must be recognized that these calculations assume that all parameters except one are held constant, and the sensitivity of the decision to that one variable is evaluated. The next section presents ways to modify your Excel chart to make it more effective.

GRAPHING WITH SPREADSHEETS FOR SENSITIVITY AND BREAKEVEN ANALYSIS

Chapter 4 introduced drawing *xy* plots with spreadsheets, and Chapter 8 relied on plots of PW, EUAW, and EUAC versus *i*. This section will present some of the spreadsheet tools and options that can make the *xy* plots more effective and attractive.

The spreadsheet tools and options can be used to:

• Modify the *x* or *y* axes
Specify the minimum or maximum value
Specify at what value the other axis intersects (default is 0)
• Match line types to data
Use line types to distinguish one curve from another
Use markers to show real data
Use lines without markers to plot curves (straight segments or smooth curves)
• Match chart colors to how displayed
Color defaults are fine for color computer screen
Color defaults are OK for color printers
Black-and-white printing is better with editing (use line types not colors)
• Annotate the graph
Add text, arrows, and lines to graphs
Add data labels

In most cases the menus of Excel are self-explanatory, so the main step is deciding what you want to achieve. Then you just look for the way to do it. Left clicks are used to select the item to modify, and right clicks are used to bring up the options for that item. Example 9–12 illustrates this process.

EXAMPLE 9-12 Example 9–10 Revisited

The staged construction choice described in Example 9–10 used a broad range of *x* values for the *x* axis. Create a graph that focuses on the 10- to 20-year period and is designed for printing in a report. The costs are:

Year	Full Capacity	Two Stages
0	$140,000	$100,000
n	0	120,000

SOLUTION

The first step is to create a table of values that shows the present worth of the costs for different values of $n \equiv$ the length of time until the second stage or full capacity is needed. Notice that the full capacity is calculated at $n = 0$. The only reason to calculate the corresponding value for staged construction is to see if the formula is properly entered, since building both stages at the same time will not really cost $220,000. The values for staged construction at 5, 10, 20, and 30 years check with the values in Example 9–10.

The next step is to select cells A8:C13, which includes the *x* values and two series of *y* values. Then the ChartWizard tool is selected. In the first step, the *xy* (scatter) plot is selected with the option of smoothed lines without markers. In Step 2 no action is required, since the cells A8:C13 were selected first. In Step 3 labels are added for the *x* and *y* axes. In Step 4 the chart is moved around on the worksheet page, so that it does not overlap with the data. The result is shown in Figure 9–4.

	A	B	C	D
1	Time	Full Capacity	Two Stage	
2	0	140,000	100,000	
3	*n*		120,000	
4				
5	Life	40		
6	*i*	8%		
7		Present Worth of		
8	*n*	Full Capacity	Two Stage	
9	0	140,000	220,000	=\$C\$2+PV(\$B\$6,A9,0,−\$C\$3)
10	5	140,000	181,670	
11	10	140,000	155,583	
12	20	140,000	125,746	
13	30	140,000	111,925	

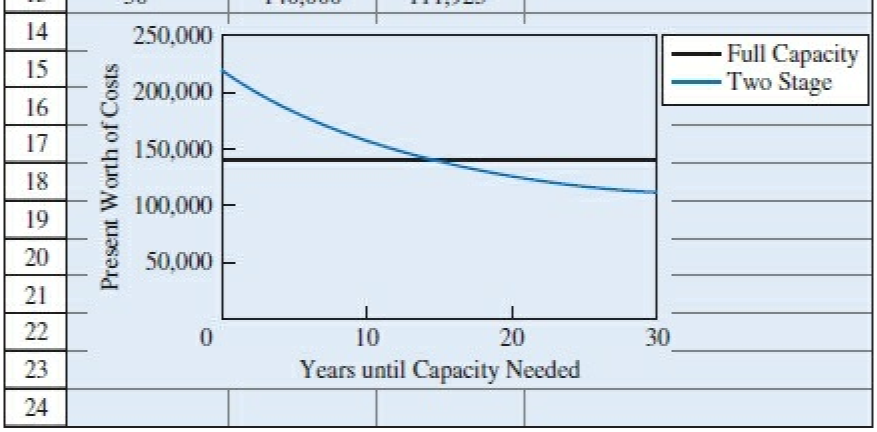


FIGURE 9-4 Automatic graph from spreadsheet.

Our first step in cleaning up the graph is to delete the formula in cell C9, since two-stage construction will not be done at Time 0. We also delete the label in the adjacent cell, which explains the formula. Then we create a new label for cell C10. As shown in Appendix A, the easy way to create that label is to insert an apostrophe or space, as the first entry in cell C10. This converts the formula to a label that we can copy to D10. Then we delete the apostrophe or space in cell C10.

The axis scales must be modified to focus on the area of concern. Select the *x* axis and change the minimum from automatic to 10 and the maximum to 20. Select the *y* axis and change the minimum to 125,000 and the maximum to 160,000.

Left-click on the plot area to select it. Then right-click to bring up the options. Select Format Plot Area and change the area pattern to “none.” This will eliminate the gray fill that made Figure 9–4 difficult to read.

Left-click on the two-stage curve to select it. Then right-click for the options. Format the data series using the Patterns tab. Change the line style from solid to dashed, the line color from automatic to black, and increase the line weight. Similarly, increase the line weight for the full-capacity line. Finally, select a grid line and change the line style to dotted. The result is far easier to read in black and white.

	A	B	C	D
1	Time	Full Capacity	Two Stage	
2	0	140,000	100,000	
3	*n*		120,000	
4				
5	life	40		
6	*i*	8%		
7		Present worth of		
8	*n*	Full Capacity	Two Stage	
9	0	140,000		
10	5	140,000	181,670	=$C$2+PV($B$6,A10,0,−$C$3)
11	10	140,000	155,583	
12	20	140,000	125,746	
13	30	140,000	111,925	
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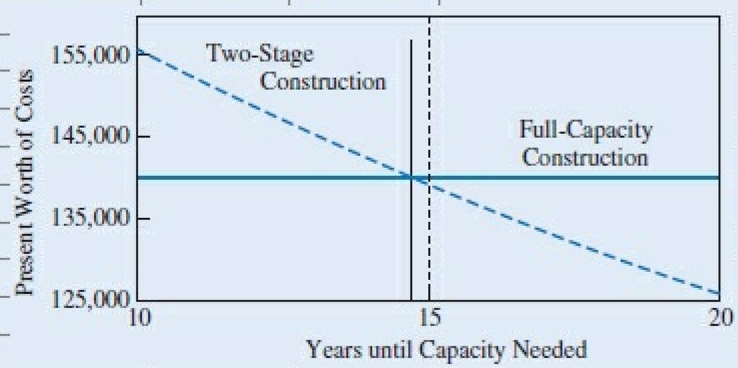


FIGURE 9-5 Spreadsheet of Figure 9–4 with improved graph.

To further improve the graph, we can replace the legend with annotations on the graph. Left-click somewhere in the white area around the graph to select "chart area." Right-click and then choose the chart options on the menu. The legends tab will let us delete the legend by turning "show legend" off. Similarly, we can turn the *x*-axis gridlines on. The line style for these gridlines should be changed to match the *y*-axis gridlines. This allows us to see that the breakeven time is between 14 and 15 years.

To make the graph less busy, change the scale on the *x* axis so that the interval is 5 years rather than automatic. Also eliminate the gridlines for the *y* axis (by selecting the Chart Area, Chart Options, and Gridlines tabs). The graph size can be increased for easier reading as well. This may require specifying an interval of 10,000 for the scale of the *y* axis.

Finally, to add the labels for the full-capacity curve and the two-stage curve, find the toolbar for graphics, which is open when the chart is selected (probably along the bottom of the spreadsheet). Select the text box icon, and click on a location close to the two-stage chart. Type in the label for two-stage construction. Notice how including a return and a few spaces can shape the label to fit the slanted line. Add the label for full construction. Figure 9–5 is the result.

DOING WHAT-IF ANALYSIS WITH SPREADSHEETS

Breakeven charts change one variable at a time, while **what-if analysis** may change many of the variables in a problem. However, spreadsheets remain a very powerful tool for this form of sensitivity analysis. In Example 9–13 a project appears to be very promising. However, what-if analysis indicates that a believable scenario raises some questions about whether the project should be done.

EXAMPLE 9-13

You are an assistant to the vice president for manufacturing. The staff at one of the plants has recommended approval for a new product with a new assembly line to produce it. The VP believes that the numbers presented are too optimistic, and she has added a set of adjustments to the original estimates. Analyze the project's benefit–cost ratio or present worth index as originally submitted. Reanalyze the project, asking "What if the VP's adjustments are correct?"

	Initial Estimate	Adjustment
First cost	$70,000	+10%
Units/year	1,200	−20%
Net unit revenue	$ 25	−15%
Life, in years	8	−3

Interest rate	12%	None

SOLUTION

Figure 9–6 shows that the project has a 2.13 benefit–cost ratio with the initial estimates, but only a value of 0.96 with the what-if adjustments. Thus we need to determine which set of numbers is more realistic. Real-world experience suggests that in many organizations the initial estimates are too optimistic. Auditing of past projects is the best way to develop adjustments for future projects.

	A	B	C	D
1		Initial Estimate	Adjust-ment	Adjusted Values
2	First cost	$70, 000	10%	$77, 000
3	Units/year	1,200	–20%	960
4	Net unit revenue	$25	–15%	$21
5	Life (years)	8	–3	5
6	Interest rate	12%	none	12%
7				
8	Benefits	149,029		73,537
9	Cost	70,000		77,000
10	B/C Ratio	2.13		0.96
11				
12		=PV(B6,B5,-B3*B4)		

FIGURE 9-6 Spreadsheet for what-if analysis.

SUMMARY

In this chapter, we have looked at four new analysis techniques.

Future worth: When the comparison between alternatives will be made in the future, the calculation is called future worth. This is very similar to present worth, which is based on the present, rather than a future point in time. *Benefit–cost ratio analysis:* This technique is based on the ratio of benefits to costs using either present worth or annual cash flow calculations. The method is graphically similar to present worth analysis. When neither input nor output is fixed, incremental benefit–cost ratios (ΔB/ΔC) are required. The method is similar in this respect to rate of return analysis. Benefit–cost ratio analysis is often used at the various levels of government.

Payback period: Here we define payback as the period of time required for the profit or other benefits of an investment to equal the cost of the investment. Although simple to use and understand, payback is a poor analysis technique for ranking alternatives. While it provides a measure of the liquidity or speed of the return of the investment, it is not an accurate measure of the profitability of an investment.

Sensitivity, breakeven, and what-if analysis: These techniques are used to see how sensitive a decision is to estimates for the various parameters. Breakeven analysis is done to locate conditions under which the alternatives are equivalent. This is often presented in the form of breakeven charts. Sensitivity analysis examines a range of values of some parameters to determine the effect on a particular decision. What-if analysis changes one or many estimates to see what results.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

FUTURE WORTH

9-1 A new automobile offers free maintenance during the first year of ownership. The maintenance costs the second year will be $100, increasing by $100 each year thereafter. Assume that you will own the automobile for 5 years and that your cost of money is 8%. Find the future worth of the maintenance costs.

SOLUTION

9-2 Determine the future worth of 20 quarterly lease payments of $500 at an interest rate of 8%.

SOLUTION

9-3 Macoupin Mining Inc. must purchase a new coring machine that costs $60,000 and will last 15 years with a salvage value of $12,000. The annual operating expenses will be $9000 the first year, increasing by $200 each year thereafter. The annual income is $15,000 per year. If Macoupin's MARR is 8%, determine the net future worth of the machine purchase.

SOLUTION

9-4 Zap Bug Killers, Inc. recently purchased new electrical shock equipment. The equipment cost $16,250 and has a useful life of 4 years. Each year the equipment will produce income of $5500. The costs to operate the equipment are $500 the first year, increasing by $250 year thereafter. The equipment should have a salvage value of $800. If ZAP's MARR is 8%, what is the net future worth of the equipment? Was the purchase a wise investment?

SOLUTION

9-5 Salty Nuts, Inc. must buy a new nut-shelling machine. The industrial engineer has collected the following information concerning the apparent best alternative. Calculate the net future worth of the alternative if the MARR is 6%.

First cost	$250,000
Annual benefits	73,000 the first year, decreasing by $1200 each year thereafter
Annual O & M costs	28,000 the first year, increasing by $1600 each year thereafter
Salvage value	42,000
Useful life	6 years

SOLUTION

9-6 An engineer is considering the purchase of a new set of batteries for an electric pallet jack. Given the cost, annual benefit, useful life, and *i* = 5%, conduct a net future worth analysis to decide which alternative to purchase.

	A	B
Cost	$19,000	$11,000
Annual benefit	4,000	4,250
Useful life	6 years	3 years

SOLUTION

9-7 Lucky Luis has just won $20,000 and wants to invest it for 12 years. There are three plans available to him.

a. A savings account that pays 3¾% per year, compounded daily.
b. A money market certificate that pays 6¾% per year, compounded semiannually.
c. An investment account that, based on past experience, is likely to pay 8½% per year, compounded annually.

If Luis did not withdraw any interest, how much would be in each of the three investment plans at the end of 12 years?

SOLUTION

9-8 The following investment opportunities are available. Use future worth analysis and a MARR of 6% to determine which, if either, alternative should be selected.

	A	B
First cost	$22,000	$30,000
Annual benefits	6,000	10,000

Annual cost	1,000	3,500
Midlife overhaul	4,000	7,500
Salvage value	3,000	8,000
Useful life	6 years	6 years

SOLUTION

BENEFIT/COST RATIO

Rash, Riley, Reed, and Rogers Consulting has a contract to design a major highway project that will provide service from Memphis to Tunica, Mississippi. R^4 has been requested to provide an estimated B/C ratio for the project, summarized as follows.

9-9

Initial cost	25,750,000
Right-of-way maintenance	550,000
Resurfacing (every 8 years)	10% of first cost
Shoulder grading and rework (every 6 years)	1,000,000
Average number of road users per year	1,950,000
Average time savings value per road user	$2

Determine the B/C ratio if *i* = 8%.

SOLUTION

A proposed bridge on the interstate highway system is being considered at the cost of $12 million. It is expected that the bridge will last 20 years. Construction costs will be paid by the federal and state governments.
9-10 Operation and maintenance costs will be $180,000 per year. Benefits to the public will be $1,500,000 per year. The building of the bridge will result in a cost of $200,000 per year to the general public. The project requires a 6% return. Determine the B/C ratio for the project. State any assumptions made about benefits or costs.

SOLUTION

9-11 The town of Podunk is considering building a new downtown parking lot. The land will cost $25,000, and the construction cost of the lot will be $150,000. Each year, costs associated with the lot will be $17,500. The income from the lot is $18,000 the first year, increasing by $3500 each year for the 12-year expected life of the lot. Determine the B/C ratio if Podunk uses a cost of money of 4%.

SOLUTION

9-12 Tires-R-Us is considering the purchase of new tire-balancing equipment. The machine, which will cost $12,699, will result in annual savings of $1500 with a salvage value at the end of 12 years of $250. For a MARR of 6%, use B/C analysis to determine whether the equipment should be purchased.

SOLUTION

9-13 Dunkin City wants to build a new bypass between two major roads that will cut travel time for commuters. The road will cost $16,000,000 and save 17,500 people $100/year on gas. The road will need to be resurfaced every year at a cost of $10,000. The road is to be used for 20 years. Use B/C analysis to determine whether Dunkin City should build the road. The cost of money is 8%.

SOLUTION

PAYBACK PERIOD

9-14 For calculating payback period, when is the following formula valid?

$$\text{Payback period} = \frac{\text{First Cost}}{\text{Annual Benefits}}$$

SOLUTION

Is the following statement true or false?

9-15 If two investors are considering the same project, the payback period will b longer for the investor with the higher minimum attractive rate of return (MARR).

SOLUTION

What is the payback period for a project with the following characteristics, given a MARR of 10%?

9-16

First cost	$20,000
Annual benefits	8,000
Annual maintenance	2,000 in Year 1, then increasing by $500 per year
Salvage value	2,000
Useful life	10 years

SOLUTION

Determine the payback period (to the nearest year) for the following projec the MARR is 10%.

9-17

First cost	$10,000
Annual maintenance	500 in Year 1, increasing by $200 per year
Annual income	3,000
Salvage value	4,000
Useful life	10 years

SOLUTION

Determine the payback period (to the nearest year) for the following projec

9-18

Investment cost	$22,000
Annual maintenance costs	1,000
Annual benefits	6,000

Overhaul costs	7,000 every 4 years
Salvage value	2,500
Useful life	12 years
MARR	10%

SOLUTION

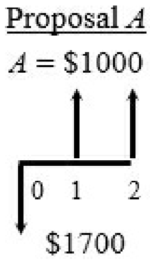
9-19 A cannery is considering different modifications to some of their can fillers two plants that have substantially different types of equipment. These modifications will allow better control and efficiency of the lines. The requ investments amount to $135,000 in Plant *A* and $212,000 for Plant *B*. The expected benefits (which depend on the number and types of cans to be fill each year) are as follows.

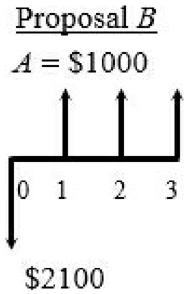
Year	Plant *A* Benefits	Plant *B* Benefits
1	$ 73,000	$ 52,000
2	73,000	85,000
3	80,000	135,000
4	80,000	135,000
5	80,000	135,000

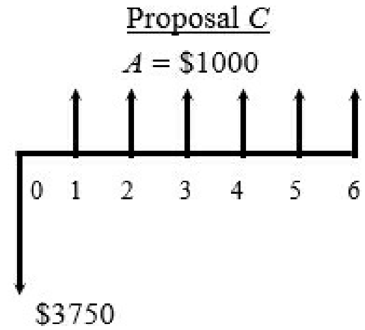
a. Assuming that MARR = 10%, which alternative is should be chosen?
b. Which alternative should be chosen based on payback period?

SOLUTION

9-20 In this problem the minimum attractive rate of return is 10%. Three propos are being considered.







a. Which proposal would you choose using future value analysis?
b. How many years for payback for each alternative?

SOLUTION

BREAKEVEN

9-21 A road can be paved with either asphalt or concrete. Concrete costs $20,000/mile and lasts for 20 years. What is the maximum that should be spent on asphalt, which lasts only 10 years? The annual maintenance costs are $500/mile for both pavements. The cost of money = 8%.

SOLUTION

9-22 What is the minimum acceptable annual income from a project that has a $70,000 investment cost and a $14,000 salvage value if the life is 15 years and the minimum attractive rate of return (MARR) is 9%?

SOLUTION

9-23 Junker Rental Car has a contract with a garage for major repair service for $450 per car, every 6 months. Management estimates that for $350,000, the company could have its own facility, financed at 8% interest for 20 years, and a salvage value of $20,000. They will do their own car repairs at a cost of $200 per car, every 6 months. Ignoring taxes and other economic factors, what is the minimum number of cars needed to make the change feasible?

SOLUTION

9-24 The annual income from an apartment house is $33,600. The annual expense is estimated to be $8000. If the apartment house can be bought today for $349,000, what is the breakeven resale price in 10 years with a 6% interest rate?

SOLUTION

9-25 A machine, costing $16,000 to buy and $1200 per year to operate, will produce savings of $2500 per year for 8 years. If the interest rate is 8%, what is the minimum salvage value that would make the machine an attractive investment?

SOLUTION

9-26 ABC Manufacturing has a MARR of 12% on new investments. What uniform annual benefit would Investment *B* have to generate to make it preferable to Investment *A*?

Year	Investment *A*	Investment *B*
0	–$60,000	–$45,000
1–6	+15,000	?

SOLUTION

9-27 Over the next 6 years, investment in a crane is expected to produce profit from its rental as shown. Assume that the salvage value is zero. Assuming 12% interest, what is the breakeven cost of the crane?

Year	Profit
1	$15,000
2	12,500
3	10,000

4	7,500
5	5,000
6	2,500

SOLUTION

9-28 A proposed building may be roofed in either composition roofing (C) or galvanized steel sheet (S). The composition roof costs $56,000 and is replaced every 5 years (assume at the same cost). The steel roof costs $68,000 but the useful life is very long. Neither roof has any salvage value, nor is maintenance needed. If the MARR is 10%, what minimum life must the steel roof have to make it the better alternative? (Report to the nearest whole year; don't bother interpolating.)

SOLUTION

9-29 What is the breakeven cost for Project *B* if interest equals 10%?

Year	*A*	*B*
0	–10,000	?
1–5	+3,500	+2,800

SOLUTION

9-30 The PARC Company can purchase gizmos to be used in building whatsits for $90 each. PARC can manufacture their own gizmos for $7000 per year overhead cost plus $25 direct cost for each gizmo, provided they purchase a gizmo maker for $100,000. PARC expects to use gizmos for 10 years. The gizmo maker should have a salvage value of $20,000 after 10 years. PARC uses 12% as its minimum attractive rate of return. At what annual production rate *N* should PARC make its own gizmos?

SOLUTION

Oliver Douglas decides to install a fuel storage system for his farm that

9-31 will save him an estimated 6.5 cents/gallon on his fuel cost. Initial cost of the system is $10,000, and the annual maintenance is $25 the first year, increasing by $25 each year thereafter. After a period of 10 years the estimated salvage is $3000. If money is worth 12%, what is the breakeven quantity of fuel?

SOLUTION

9-32 Given the following:

$\text{AW}_A = -23{,}000(A/P, 10\%, 10) + 4000(A/F, 10\%, 10) - 3000 - 3X$

$\text{AW}_B = -8000(A/P, 10\%, 4) - 2000 - 6X$

For these two AW relations, find the breakeven point X in miles per year.

SOLUTION

9-33 To produce an item in-house, equipment costing $250,000 must be purchased. It will have a life of 4 years and an annual cost of $80,000; each unit will cost $40 to manufacture. Buying the item externally will cost $100 per unit. At i = 12%, determine the breakeven production number.

SOLUTION

9-34 Data for two drill presses under consideration are listed. Assuming an interest rate of 12%, what salvage value of Press *B* will make the two alternatives equal?

	A	*B*
First cost	$30,000	$36,000
Annual maintenance	1,500	2,000
Salvage value	5,000	?
Useful life	6 years	6 years

SOLUTION

A fruit processing company is considering the purchase of new equipment. The data are as follows.

9-35

First cost	$78,750
Annual income	$25/ton of processed fruit
Annual operating costs	$5500 the first year, increasing $800 each year thereafter
Annual property taxes	8% of first cost
Annual insurance	4% of first cost, payable at the beginning of each year
Salvage value	15% of first cost + $1000
Useful life	10 years

The MARR is 4%. Determine the number of tons of fruit that must be processed annually to justify purchasing the machine.

SOLUTION

9-36 The state Department of Highways is trying to decide whether it should "hot-patch" a short stretch of an existing highway or resurface it. If the hot-patch method is chosen, approximately 300 cubic meters of material would be required at a cost of $600/cubic meter (in place). The shoulders would have to be improved at the same time, at a cost of $24,000. These shoulder improvements must be redone every 2 years (assume the same cost). The annual cost of routine maintenance on the patched road is estimated to be $5000. Alternatively, the state can resurface the road. This surface will last 10 years if maintained properly at a cost of $2000 per year. The shoulders would require reworking at the end of the fifth year at a cost of $5000. Regardless of the method selected, the road will be completely rebuilt in 10 years. At an interest rate of 8%, what is the maximum amount that should be paid for resurfacing the road?

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Future Worth

9-1 Pick a discretionary expense that you incur on a regular basis, such as buying premium coffees daily, buying fashion items monthly, buying sports tickets monthly, or going to movies weekly. Assume that you instead place the money in an investment account that earns 6% annually. After 40 years, how much is in the account?

9-2 A Sally deposited $250 a month in her savings account for 36 months. For the next 6 years she made no deposits. What is the future worth in Sally's savings account at the end of the 9 years, if the account earned 2% annual interest, compounded monthly?

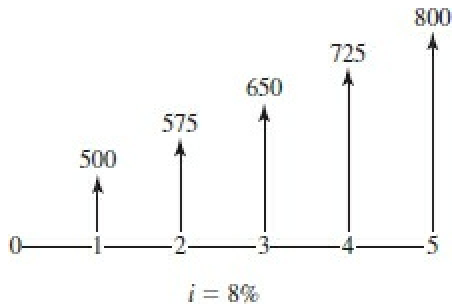
9-3 A new engineer is considering investing in an individual retirement account (IRA) with a mutual fund that has an average annual return of 8%. What is the future worth of her IRA at age 70 if she makes annual investments of $2000 into the fund beginning on her 22nd birthday? Assume that the fund continues to earn an annual return of 8%.

9-4 A You can buy a piece of vacant land for $40,000 cash. You plan to hold it for 20 years and then sell it at a profit. During this period, you would pay annual property taxes of $815. You would have no income from the property.
(*a*) Assuming that you want an 8% rate of return, at what net price

G would you have to sell the land 20 years hence?
(*b*) What is open space conservation and why is it important? What options would you have in selling your property with this in mind?

Compute the future worth for the following cash flows.

9-5

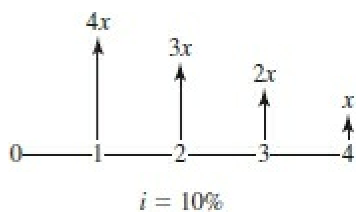


9-6 An individual who makes $32,000 per year anticipates retiring in 30 years. If his salary is increased by $600 each year and he deposits 10% of his yearly salary into a fund that earns 7% interest, what is the future worth at retirement?

A

For the following cash flows, compute the future worth.

9-7



9-8 The interest rate is 6% per year and there are 24 compounding periods per year. The principal is $30,000. What is the future worth in 4 years?

A

9-9 A company deposits $10,000 in a bank at the beginning of each year for 20 years. The account earns 4% interest, compounded every 6 months. What is in the account at the end of 20 years?

In the early 1980s, planners were examining alternate sites for a new London airport. At one potential site, the twelfth-century Norman church of St. Michaels, in the village of Stewkley, would have had to be demolished. The planners used the value of the fire insurance policy on the church—a few thousand pounds sterling—as the

9-10 church's value.

A

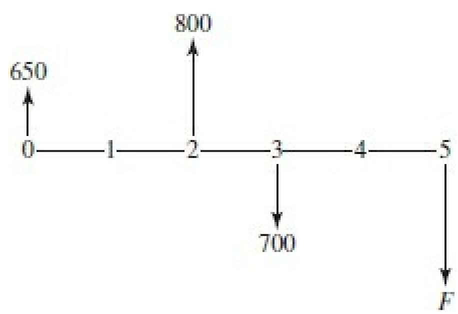
An outraged antiquarian wrote to the London *Times* that an equally plausible computation would be to assume that the original cost of the church (estimated at 150 pounds sterling) be increased at the rate of 8% per year for 750 years. Based on his proposal, what would be the future worth of St. Michaels? (*Note:* There was great public objection to tearing down the church, and it was spared.)

9-11 If you invested $5000 in a 24-month bank certificate of deposit (CD) paying 3%, compounded monthly, what is the future worth of the CD when it matures?

For a 12% interest rate, compute the value of *F* so the following cash flows have a future worth of 0.

9-12

A



9-13 Calculate the present worth and the future worth of a series of 15 annual cash flows with the first cash flow equal to $15,000 and each successive cash flow increasing by $750. The interest rate is 6%.

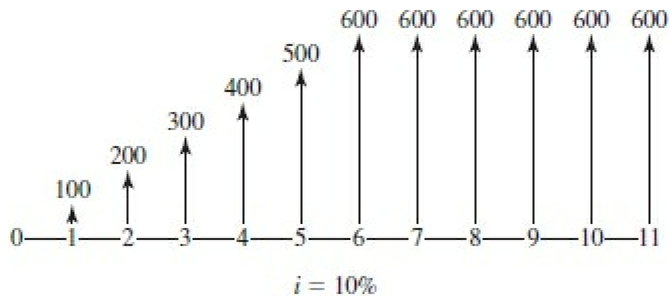
A 20-year-old student decided to set aside $100 on his 21st birthday for investment. Each subsequent year through his 55th birthday, he

9-14 plans to increase the investment on a $100 arithmetic gradient. He

A will not set aside additional money after his 55th birthday. If the student can achieve an 8% rate of return, what is the future worth of the investments on his 65th birthday?

For the following cash flows, compute the future worth.

9-15



9-16
A

Stamp collecting has become an increasingly and popular hobby, and stamps have been a good place to invest money over the last 10 years, as demand has caused resale prices to increase 18% each year. Suppose a collector purchased $100 worth of stamps 10 years ago and increased his purchases by $50 per year in each subsequent year. After 10 years of stamp collecting, what is the current worth of the stamp collection?

9-17

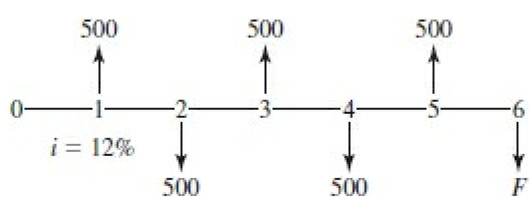
After receiving an inheritance of $50,000 on her 21^{st} birthday, Katlyn deposited the inheritance in a savings account with an effective annual interest rate of 3%. She decided to make regular deposits, beginning with $1000 on her 22^{nd} birthday and increasing by $200 each year (i.e., $1200 on her 23^{rd} birthday, $1400 on her 24^{th} birthday, etc.). What was the future worth of Katlyn's deposits after her deposit on her 66^{th} birthday?

9-18
A

Bill made a budget and planned to deposit $350 a month in a savings account, beginning September 1. He did this, but on the following January 1, he reduced the monthly deposits to $100. He made 20 deposits, 5 at $350 and 15 at $150. If the savings account paid 3% interest, compounded monthly, what was the future worth of his savings immediately after the last deposit?

9-19

Compute *F* so the following cash flows have a future worth of 0.



A family starts an education fund for their son Patrick when he is 8

9-20 years old, investing $500 on his eighth birthday, and increasing the
A yearly investment by $500 per year until Patrick is 21 years old. The fund pays 6% annual interest. What is the fund's future worth after the deposit when Patrick is 21?

9-21 IPS Corp. will upgrade its package-labeling machinery. It costs $850,000 to buy the machinery and have it installed. Operation and maintenance costs, which are $11,000 per year for the first 3 years, increase by $1000 per year for the machine's 10-year life. The machinery has a salvage value of 12% of its initial cost. Interest is 25%. What is the future worth of cost of the machinery?

9-22
A
A bank account pays 3% interest with monthly compounding. A series of deposits started with a deposit of $5000 on January 1, 2007. Deposits in the series were to occur each 6 months. Each deposit in the series is for $150 less than the one before it. The last deposit in the series will be due on July 1, 2021. What is the future worth of the account on July 1, 2024, if the balance was zero before the first deposit and no withdrawals are made?

9-23 Jamal Brown is a 55-year-old engineer. According to mortality tables, a male at age 55 has an average life expectancy of 21 more years. Jamael has accumulated $200,000 toward his retirement. He is now adding $10,000 per year to his retirement fund. The fund earns 5% interest. Jamal will retire when he can obtain an annual income from his retirement fund of $100,000, assuming he lives to age 76. He will make no provision for a retirement income after age 76. What is the youngest age at which Jamal can retire?

9-24
A
An engineering graduate starts a new job at $68,000 per year. Her investments are deposited at the end of the year into a mutual fund that earns a nominal interest rate of 5% per year with quarterly compounding. How much money will be in the account immediately after she makes the last deposit?

(*a*) She makes $4000 annual deposits for the next 40 years.

(*b*) She makes the $4000 deposits for 10 years, then stops all investments for the next 10 years, and then resumes deposits of $6000 per year for the next 20 years.

Contributed by Gillian Nicholls, Southeast Missouri State University

9-25 G

A company is considering buying a new bottle-capping machine. The initial cost of the machine is \$1.2M and it has a 10-year life. Monthly maintenance costs are expected to be \$2000 per month for the first 7 years and \$2500 per month for the remaining years. The machine requires a major overhaul costing \$100,000 at the end of the fifth year of service. Assume that all these costs occur at the end of the appropriate period.

(*a*) What is the future value of all the costs of owning and operating this machine if the nominal interest rate is 9% compounded monthly?

(*b*) Bottle caps are in the top 10 items found in beach cleanups. What are other items on such lists, and what are beach cities doing to reduce this debris?

9-26 A

The Association of General Contractors (AGC) is endowing a fund of \$1.5 million for the Construction Engineering Technology Program at Grambling State University. The AGC established an escrow account in which 15 equal end-of-year deposits that earn 8% compound interest were to be made. After seven deposits, the Louisiana legislature revised laws relating to the licensing fees AGC can charge its members, and there was no deposit at the end of Year 9. What must the amount of the remaining equal end-of-year deposits be, to ensure that the \$1.5 million is available on schedule for the Construction Engineering Technology Program?

9-27

A recent college graduate got a good job and began a savings account. He authorized the bank to automatically transfer \$500 each month from his checking account to the savings account. The bank made the first withdrawal on July 1, 2015, and is instructed to make the last withdrawal on January 1, 2040. The bank pays a nominal interest rate of 6% and compounds twice a month. What is the future worth of the account on January 1, 2040?

9-28

A business executive is offered a management job at Generous Electric Company, which offers him a 5-year contract that calls for a salary of \$62,000 per year, plus 600 shares of GE stock at the end of the 5 years. This executive is currently employed by Fearless Bus Company, which also has offered him a 5-year contract. It calls for a salary of \$65,000, plus 100 shares of Fearless stock each year. The Fearless stock is currently worth \$60 per share and pays an annual

A dividend of $2 per share. Assume end-of-year payments of salary and stock. Stock dividends begin one year after the stock is received. The executive believes that the value of the stock and the dividend will remain constant. If the executive considers 9% a suitable rate of return in this situation, what must the Generous Electric stock be worth per share to make the two offers equally attractive? Use the future worth analysis method in your comparison.

9-29 E Jean invests $1000 in Year 1 in a socially responsible fund, and doubles the amount each year after that (so the investment is $1000, 2000, …).

(*a*) If she does this for 10 years, and the investment pays 4% annual interest, what is the future worth of her investment?

(*b*) What are socially/ethically responsible investment funds? How do they differ from other types of investments? Why do people invest in them?

Benefit–Cost Ratio

9-30 A Each of the three mutually exclusive alternatives shown has a 5-year useful life. If the MARR is 10%, which alternative should be selected? Solve the problem by benefit–cost ratio analysis.

	A	*B*	*C*
Cost	$600.0	$500.0	$200.0
Uniform annual benefit	158.3	138.7	58.3

9-31 Consider three mutually exclusive alternatives, each with a 15-year useful life. If the MARR is 12%, which alternative should be selected? Solve the problem by benefit–cost ratio analysis.

	A	*B*	*C*
Cost	$800.0	$300.0	$150.0
Uniform annual benefit	130	60	35

An investor is considering buying some land for $100,000 and constructing an office building on it. Three different buildings are being analyzed.

9-32 A

	Building Height		
	2 Stories	5 Stories	10 Stories
Cost of building (excluding cost of land)	$500,000	$900,000	$2,200,000
Resale value* of land + building after 20-year horizon	200,000	300,000	350,000
Annual net rental income	70,000	110,000	215,000

* Resale value considered a reduction in cost—not a benefit.

Using benefit–cost ratio analysis and a 7% MARR, determine which alternative, if any, should be selected.

Using benefit–cost ratio analysis, determine which one of the three mutually exclusive alternatives should be selected. Each alternative has a 10-year useful life. Assume a 20% MARR.

9-33

	A	*B*	*C*
First cost	$560	$340	$120
Uniform annual benefit	140	100	40
Salvage value	40	0	0

A government agency is planning a new office building close to its current headquarters. Four proposed sites are to be evaluated. Any of these sites will save the agency $650,000 per year, since two of its current satellite offices will no longer need to be rented. The agency uses a 6% interest rate and assumes that the building and its benefits will last for 25 years. Based on a benefit–cost analysis what should the agency do?

9-34

A

	Site			
	A	*B*	*C*	*D*
Initial cost	$7.1M	$6.5M	$7.5M	$7.7M
Annual O&M	0.09M	0.125M	0.12M	0.025M

Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

Using benefit–cost ratio analysis, a 10-year useful life, and a 25% MARR, determine which of the following mutually exclusive alternatives should be selected.

9-35

	A	*B*	*C*	*D*	*E*
Cost	$100	$200	$300	$400	$500
Annual benefit	37	60	83	137	150

Five mutually exclusive investment alternatives have been proposed. Based on benefit–cost ratio analysis, and a MARR of 15%, which alternative should be selected?

9-36

A

Year	*A*	*B*	*C*	*D*	*E*
0	− $200	− $100	− $125	− $150	− $225
1–5	68	25	42	52	68

Three mutually exclusive projects are being considered by Sesame Street Productions (SSP). SSP uses a MARR of 8%. SSP has heard about your excellent analysis skills and wants you to help them make a decision. Using a B/C analysis, which project do you recommend to SSP? Assume all benefits and costs repeat for Project A.

9-37

	Project A	Project B	Project C
Initial cost	$300	$450	$765

Annual benefits	$200	$190	$300
Project life (years)	2	4	4
B/C ratio	1.19	1.40	1.30

Contributed by Gana Natarajan, Oregon State University

9-38 A G Burns City may build a garbage incinerator on the outskirts of town. Environmental impact statements and safety planning/inspection will cost $21,000 (payable at start of construction). The annual upkeep and operating costs are expected to be $34,000. The new incinerator will save $14 each annually for 24,000 billed customers. Consultants have estimated an annual disbenefit to the surrounding area of $36,500. At the end of a 10-year useful life the incinerator will be dismantled at a cost of $50,000. Using benefit-cost ratio analysis, and assuming a cost of money of 5% what is the maximum that Burns City can pay to build the incinerator? *Contributed by Ed Wheeler, University of Tennessee, Martin*

9-39 Looking at Figure 9-1 another way to pick the best mutually exclusive alternative is to maximize the perpendicular distance from the alternative's point to the B/C = 1. Explain why this is true. *Contributed by Hector Medina, Liberty University*

9-40 A Cornell has two options for upgrading their athletic facilities. The off-campus option costs only $2.5 million, but it will require frequent bus service to those facilities at an annual cost that starts at $200,000 and increases by 5% per year (buses, drivers' and mechanics' salaries, maintenance, road wear, etc.). Improving the on-campus facilities will cost $5.0 million, but no extra transportation costs are required. Both options involve an estimated annual maintenance cost of $1 million for about 30 years before new facilities will again be needed. Using benefit–cost ratio analysis, determine which option is more economically efficient. Use an interest rate of 10% per year. *Contributed by D. P. Loucks, Cornell University*

A do-nothing and two mutually exclusive alternatives are being considered for reducing traffic congestion. User benefits come from reduced congestion once the project is complete, while user disbenefits are due to increased congestion during construction. The interest rate is 8%, and the life of each alternative is 15 years. Which alternative should be chosen?

9-41

Alternative	*A*	*B*
User benefits ($M/yr)	2.1	2.6
User disbenefits ($M)	1.2	2.1
First cost ($M)	6.9	9.9
Operations and maintenance ($M/yr)	0.75	0.825

(*a*) Use the benefit–cost ratio.
(*b*) Use the modified benefit–cost ratio.
(*c*) Use the public/government version of the B/C ratio.
(*d*) Assume these numbers apply to a private firm and use a present worth index.
(*e*) Are your recommendations for (*a*) through (*d*) consistent? Which measure gives the largest value? Why?

9-42 E A school is overcrowded and there are three options. The do-nothing alternative corresponds to continuing to use modular classrooms. The school can be expanded, or a new school can be built to "split the load" between the schools. User benefits come from improvements in school performance for the expanded or new schools. If a new school is built, there are more benefits because more students will be able to walk to school, the average distance for those who ride the school buses will be shorter, and the schools will be smaller and more "student friendly." The disbenefits for the expanded school are due to the impact of the construction process during the school year. The interest rate is 8%, and the life of each alternative is 20 years. Which alternative should be chosen? What is the incremental ratio for the preferred alternative?
(*a*) Use the benefit–cost ratio.
(*b*) Use the modified benefit–cost ratio.
(*c*) Use the public/government version of the B/C ratio.
(*d*) Assume these numbers apply to a private firm and use a present worth index.
(*e*) Are your recommendations for (*a*) through (*d*) consistent? Which measure gives the largest value? Why?

(*f*) Describe the ethical issues involved in the overcrowded schools dilemma in terms of stakeholders and impacts.

Payback Period and Exact Methods

9-43 A project has the following costs and benefits. What is the payback period?

Year	Costs	Benefits
0	$65,000	
1–2	15,000	
3	5,000	$50,000
4–10		$10,000 in each year

9-44 A Able Plastics, an injection-molding firm, has negotiated a contract with a national chain of department stores. Plastic pencil boxes are to be produced for a 2-year period. If the firm invests $62,000 for special removal equipment to unload the completed pencil boxes from the molding machine, one machine operator can be eliminated saving $32,000 per year. The removal equipment has no salvage value and is not expected to be used after the 2-year production contract is completed. The equipment would be serviceable for about 15 years. What is the payback period? Should Able Plastics buy the removal equipment?

9-45 A car dealer leases a small computer with software for $5000 per year. As an alternative he could buy the computer for $7500 and lease the software for $3500 per year. Any time he would decide to switch to some other computer system he could cancel the software lease and sell the computer for $500.
(*a*) If he buys the computer and leases the software, what is the payback period?
(*b*) If he kept the computer and software for 8 years, what would be the benefit–cost ratio, based on a 5% interest rate?

9-46 Tom Sewel has gathered data on the relative costs of a solar water heater system and a conventional electric water heater. The data are based on statistics for a mid-American city and assume that during cloudy days an electric heating element in the solar heating system will provide the necessary heat.

The installed cost of a conventional electric water tank and heater is $850. A family of four uses an average of 300 liters of hot water a day, which takes $280 of electricity per year. The glass-lined tank has a 20-year guarantee. This is probably a reasonable estimate of its actual useful life.

A
G

The installed cost of two solar panels, a small electric pump, and a storage tank with auxiliary electric heating element is $2200. It will cost $85 a year for electricity to run the pump and heat water on cloudy days. The solar system will require $225 of maintenance work every 5 years. Neither the conventional electric water heater nor the solar water heater will have any salvage value at the end of its useful life.

Using Tom’s data, what is the payback period if the solar water heater system is installed, rather than the conventional electric water heater?

9-47 A cannery is considering installing an automatic case-sealing machine to replace current hand methods. If they purchase the machine for $5000 in June, at the beginning of the canning season, they will save $500 per month for the 4 months each year that the plant is in operation. Maintenance costs of the case-sealing machine are expected to be negligible. The case-sealing machine is expected to be useful for five annual canning seasons and then will have no salvage value. What is the payback period? What is the nominal annual rate of return?

A large project requires an investment of $210 million. The construction will take 3 years: $55 million will be spent during the first year, $85 million during the second year, and $70 million during the third year of construction. Two project operation periods are being considered: 10 years with the expected net profit of $50 million per year and 20 years with the expected net profit of $35 million per year. For simplicity of calculations it is assumed that all cash flows

9-48 occur at end of year. The company minimum required return on investment is 15%.Calculate for each alternative:

A

(*a*) The payback period

(*b*) The total equivalent investment cost at the end of the construction period

(*c*) The equivalent uniform annual worth of the project (use the operation period of each alternative)

Which operation period should be chosen?

9-49 Two alternatives with identical benefits are being considered:

	A	*B*
Initial cost	$500	$800
Uniform annual cost	200	150
Useful life, in years	8	8

(*a*) Compute the payback period if Alt. *B* is purchased rather than Alt. *A*.

(*b*) Use a MARR of 10% and benefit–cost ratio analysis to identify the alternative that should be selected.

9-50 Consider four mutually exclusive alternatives:

A

	A	*B*	*C*	*D*
Cost	$65	$55	$25	$80
Uniform annual benefit	16.3	15.1	5.2	21.3

Each alternative has a 6-year useful life and no salvage value. The MARR is 9%. Which alternative should be selected, based on

(*a*) The payback period

(*b*) Future worth analysis

(*c*) Benefit–cost ratio analysis

Consider three alternatives:

	A	*B*	*C*
First cost	$50	$150	$110

Uniform annual benefit	30	45	45
Useful life, in years*	3	9	6

9-51 * At the end of its useful life, an identical alternative (with the same cost, benefits, and useful life) may be installed.

All the alternatives have no salvage value. If the MARR is 10%, which alternative should be selected?
(*a*) Solve the problem by payback period.
(*b*) Solve the problem by future worth analysis.
(*c*) Solve the problem by benefit–cost ratio analysis.
(*d*) If the answers in parts (*a*), (*b*), and (*c*) differ, explain why this is the case.

9-52 A

Consider three mutually exclusive alternatives. The MARR is 10%.

Year	*X*	*Y*	*Z*
0	− $125	− $60	− $65
1	40	15	20
2	40	15	20
3	40	15	20
4	40	15	20

(*a*) For Alt. *X*, compute the benefit–cost ratio.
(*b*) Based on the payback period, which alternative should be selected?
(*c*) Determine the preferred alternative based on an exact economic analysis method.

9-53

You are an investor who wants to make your investment back as quickly as possible. There are four potential projects that you can invest in. Which project should you choose?

	Project 1	Project 2	Project 3	Project 4

Initial Cost	$50,000	$60,000	$65,000	$125,000
Annual Revenues	8500	12,500	8,500	18,000
Length of Ownership	6	4	9	10

Contributed by Gana Natarajan, Oregon State University

9-54 A

Three mutually exclusive alternatives are being considered:

	A	*B*	*C*
Initial cost	$750	$485	$350
Benefit at end of the first year	250	200	185
Uniform benefit at end of subsequent years	160	112	83
Useful life, in years	6	5	4

At the end of its useful life, an alternative is *not* replaced. If the MARR is 10%, which alternative should be selected

(*a*) Based on the payback period?

(*b*) Based on benefit–cost ratio analysis?

9-55

Year	*E*	*F*	*G*	*H*
0	− $90	− $110	− $100	− $120
1	20	35	0	0
2	20	35	10	0
3	20	35	20	0
4	20	35	30	0
5	20	0	40	0
6	20	0	50	180

(*a*) Based on the payback period, which alternative is preferred?

(*b*) Based on future worth analysis, which of the four alternatives is preferred at 5% interest?

(*c*) Based on future worth analysis, which alternative is preferred at 20% interest?
(*d*) At 10% interest, what is the benefit–cost ratio for Alt. *G*?

9-56 A new piece of laboratory equipment costing $10,000 promises to save $4000 per year in materials and overtime pay. If the cost of money is 12% and projects must have a 3-year discounted payback period, should the equipment be purchased?

9-57 G A new high-efficiency motor is being considered for a large compressor. The new motor will cost $20,000 but will save $8000 per year in electricity. If the firm’s MARR is 15%, what is the discounted payback period?

9-58 Two alternative pumps are being compared. Pump A costs $5200, will save $1275 per year in operating and maintenance expenses, and is expected to last for 6 years. Pump B costs $6000, will save $1550 per year, and is also expected to last for 6 years. If the cost of capital is 12%, which has the better discounted payback period?

9-59 Two equipment investments are estimated as follows:

Year	A	B
0	−$15,000	−$18,000
1	5,000	6,500
2	5,000	6,500
3	5,000	6,500
4	5,000	6,500
5	5,000	6,500

Which investment has the better discounted payback period if i = 14%?

Sensitivity

If the MARR is 12%, compute the value of *X* that makes the two alternatives equally desirable.

9-60 A

	A	*B*
Cost	$800	$1000
Uniform annual benefit	230	230
Useful life, in years	5	*X*

9-61 G

Analyze Problem 9–60 again with the following changes:
(*a*) What if *B*'s first cost is $1200?
(*b*) What if *B*'s annual benefit is $280?
(*c*) What if the MARR is 10% annually?
(*d*) What if (*a*), (*b*), and (*c*) happen simultaneously?

9-62 A

Consider two alternatives:

	A	*B*
Cost	$650	$425
Uniform annual benefit	95	95
Useful life, in years	Infinity	*X*

Assume that Alt. *B* is not replaced at the end of its useful life. If the MARR is 10%, what must be the useful life of *B* to make Alternatives *A* and *B* equally desirable?

9-63

If the MARR is 5%, compute the value of *X* that makes the two alternatives equally desirable.

	A	*B*
Cost	$150	$ *X*
Uniform annual benefit	40	65
Salvage value	100	200
Useful life, in years	6	6

9-64

Ithaca is considering a new $45,000 snowplow that will save the city $400 per day of use compared to the existing one. It should last 12 years and have a resale value of $2500.
(*a*) To obtain a 14% rate of return what is the minimum number of

A
G

days per year on average it will have to be used.
(*b*) Research the environmental impact of road salt. What are other options?
Contributed by D. P. Loucks, Cornell University

9-65

Victoria is choosing between a standard Honda Civic for $20,000 or a hybrid Civic for $25,000. She calculates her annual cost of ownership including payments but not including gasoline to be $5000 for the standard and $5800 for the hybrid. The standard Civic will cost Victoria 18/mile for gasoline, while the hybrid will cost her only 12/mile. How many miles must Victoria drive in a year before the hybrid vehicle becomes more cost efficient to her?
Contributed by Paul R. McCright, University of South Florida

9-66
A

Midwest Airlines flies a short nonstop with 148-passenger planes. Considering all the costs of owning each plane plus the salaries for their crews and the fuel costs and landing fees, the fixed cost for a single flight is $16,500. If the costs associated with each passenger (reservations cost, check-in cost, baggage handling cost, snack cost, etc.) total $75 per passenger and the average ticket price is $214 (before the various taxes are added), what percentage of seats must be filled for the flight to break even?
Contributed by Paul R. McCright, University of South Florida

9-67

Fence posts for a particular job cost $18.03 each to install, including the labor cost. They will last 10 years. If the posts are treated with a wood preservative, they can be expected to have a 15-year life. Assuming an 8% interest rate, how much could one afford to pay for the wood preservative treatment?

9-68
A
G

A piece of property is purchased for $25,000 and yields a $1500 yearly net profit. The property is sold after 10 years. What is its minimum price to break even with interest at 8%?

9-69

Analyze Problem 9–68 again with the following changes:
(*a*) What if the property is purchased for $12,000?
(*b*) What if the yearly net profit is $925?
(*c*) What if it is sold after 7 years?
(*d*) What if (*a*), (*b*), and (*c*) happen simultaneously?

9-70 A Midwest Airlines (MWA) is planning to expand its fleet of jets to replace some old planes and to expand its routes. It has received a proposal to purchase 112 small jets over the next 4 years. What annual net revenue must each jet produce to break even on its operating cost? The analysis should be done by finding the EUAC for the 10-year planned ownership period. MWA has a MARR of 12%, purchases the jet for $22 million, has operating and maintenance costs of $3.2 million the first year, increasing 8% per year, and performs a major maintenance upgrade costing $4.5M at end of Year 5. Assume the plane has a salvage value at end of Year 10 of $13 million.
Contributed by Paul R. McCright, University of South Florida

9-71 Plan *A* requires a $30,000 investment now. Plan *B* requires an $28,700 investment now and an additional $10,000 investment at a later time. At 12% interest, compute the breakeven point for the timing of the $10,000 investment.

9-72 A A low-carbon-steel machine part, operating in a corrosive atmosphere, lasts 8 years and costs $250 installed. If the part is treated for corrosion resistance, it will cost $325 installed. How long must the treated part last to be the preferred alternative, assuming 9% interest?

9-73 Analyze Problem 9–72 again with the following changes:
(*a*) What if the installed cost of the corrosion-treated part is $400?
(*b*) What if the untreated part will last only 4 years?
(*c*) What if the MARR is 12% annually?
(*d*) What if (*a*), (*b*), and (*c*) happen simultaneously?

9-74 A Tyrella Jackson is buying a used car. Alternative *A* is an American-built compact. It has an initial cost of $8900 and operating costs of 9c/km, excluding depreciation. From resale statistics, Tyrella estimates the American car can be resold at the end of 3 years for $1700. Alternative *B* is a foreign-built Fiasco. Its initial cost is $8000, the operating cost, also excluding depreciation, is 8¢/km. How low could the resale value of the Fiasco be to provide equally economical transportation? Assume Tyrella will drive 12,000 km/year and considers 8% as an appropriate interest rate.

9-75 Analyze Problem 9–74 again with the following changes:
(*a*) What if the Fiasco is more reliable than expected, so that its operating cost is $0.075/km?
(*b*) What if Tyrella drives only 9000 km/year?
(*c*) What if Tyrella's interest rate is 6% annually?
(*d*) What if (*a*), (*b*), and (*c*) happen simultaneously?

9-76 A A car company has decided to spend $140M on a museum for exhibiting its classic cars. Land can be purchased for $650,000. The museum building will require 50,000 square feet of general space, while each car displayed will require an additional 1000 square feet. The design and planning process will cost $80,000, which should be paid immediately. The construction of the building will cost $550 per square foot, and the building will be completed within the next 2 years, while the cost of construction will be distributed evenly between the 2 years of construction. All cars will be purchased during the second year of construction at an average cost of $95,000 per car. The annual operation of the museum will cost $1,750,000 plus $25,000 per car. If the funds are invested at 8% per year and the museum is to exist forever, how many cars can the trustees purchase? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*

9-77 G A road can be paved with either asphalt or concrete. Concrete costs $20,000/km and lasts 20 years. Assume the annual maintenance costs are $700 for concrete and $1000 for asphalt per kilometer per year. Use an interest rate of 5% per year.
Contributed by D. P. Loucks, Cornell University
(*a*) What is the maximum that should be spent for asphalt if it lasts only 10 years?
(*b*) Assume the asphalt road costs $8500 per kilometer. How long must it last to be the preferred alternative?
(*c*) Research and summarize conclusions of two scholarly articles on the environmental comparison of asphalt versus concrete.

9-78 A

Christina Cook studied the situation described in Problem 9-46 and decided that the solar system will *not* require the $180 of maintenance every 4 years. She believes future replacements of either the conventional electric water heater, or the solar water heater system can be made at the same costs and useful lives as the initial installation. Based on a 10% interest rate, what must be the useful life of the solar system to make it no more expensive than the electric water heater system?

9-79

A newspaper is considering buying locked vending machines to replace open newspaper racks in the downtown area. The vending machines cost $75 each. It is expected that the annual revenue from selling the same quantity of newspapers will increase $12 per vending machine. The useful life of the vending machine is unknown.

(*a*) To determine the sensitivity of rate of return to useful life, prepare a graph for rate of return versus useful life for lives up to 10 years.

(*b*) If the newspaper requires a 15% rate of return, what minimum useful life must it obtain from the vending machines?

(*c*) What would be the rate of return if the vending machines were to last indefinitely?

9-80 A

Rental equipment is for sale for $110,000. A prospective buyer estimates he would keep the equipment for 12 years and spend $6000 a year on maintaining it. Estimated annual net receipts from equipment rentals would be $14,400. It is estimated the rental equipment could be sold for $80,000 at the end of 12 years. If the buyer wants a 7% rate of return on his investment, what is the maximum price he should pay for the equipment?

9-81 G

Neither of the following machines has any net salvage value.

	A	*B*
Original cost	$55,000	$75,000
Annual expenses		
Operation	9,500	7,200
Maintenance	5,000	3,000

Taxes and insurance 1,700 2,250

At what useful life are the machines equivalent if
(*a*) 10% interest is used in the computations?
(*b*) 0% interest is used in the computations?

9-82 A

Jane Chang is making plans for a summer vacation. She will take \$1000 with her in the form of traveler's checks. From the newspaper, she finds that if she purchases the checks by May 31, she will not have to pay a service charge. That is, she will obtain \$1000 worth of traveler's checks for \$1000. But if she waits to buy the checks until just before starting her summer trip, she must pay a 1% service charge. (It will cost her \$1010 for \$1000 of traveler's checks.)
Jane can obtain a 13% interest rate, compounded weekly, on her money. How many weeks after May 31 can she begin her trip and still justify buying the traveler's checks on May 31?

9-83

A motor with a 200-horsepower output is needed in the factory for intermittent use. A Graybar motor costs \$7000 and has an electrical efficiency of 90%. A Blueball motor costs \$6000 and has an 85% efficiency. Neither motor would have any salvage value, since the cost to remove it would equal its scrap value. The annual maintenance cost for either motor is estimated at \$500 per year. Electric power costs \$0.12/kWh (1 hp = 0.746 kW). If an 18% interest rate is used in the calculations, what is the minimum number of hours the higher initial cost Graybar motor must be used each year to justify its purchase?

9-84 A

A machine costs \$5240 and produces benefits of \$1000 at the end of each year for 8 years. Assume an annual interest rate of 10%.
(*a*) What is the payback period (in years)?
(*b*) What is the breakeven point (in years)?
(*c*) Since the answers in (*a*) and (*b*) are different, which one is "correct"?

The Financial Advisor is a weekly column in the local newspaper. Assume you must answer the following question. "I recently retired at age 65, and I have a tax-free retirement annuity coming

9-85 due soon. I have three options. I can receive (*A*) \$30,976 now, (*B*) \$359.60 per month for the rest of my life, or (*C*) \$513.80 per month for the next 10 years. What should I do?” Ignore the timing of the monthly cash flows and assume that the payments are received at the end of year. Assume the 10-year annuity will continue to be paid to loved heirs if the person dies before the 10-year period is over.

Contributed by D. P. Loucks, Cornell University

(*a*) If i = 6%, develop a choice table for lives from 5 to 30 years. (You do not know how long this person or other readers may live.)

(*b*) If i = 10%, develop a choice table for lives from 5 to 30 years. (You do not know how long this person or other readers may live.)

(*c*) How does increasing the interest rate change your recommendations?

Minicases

9-86G A proposed steel mill may include a co-generation electrical plant. This plant will add \$2.3M in first cost with net annual savings of \$0.27M considering operating costs and electrical bills. The plant will have a \$0.4M salvage value after 25 years. The firm uses an interest rate of 12% and present worth index (PWI) in its decision making.

The public utility offers a subsidy for co-generation facilities because it will not have to invest as much in new capacity. This subsidy is calculated as 20% of the co-generation facility’s first cost, but it is paid annually. The utility calculates the subsidy using a benefit–cost ratio at 8% and a life of 20 years.

(*a*) Is the plant economically justifiable to the firm without the subsidy? What is the PWI?

(*b*) What is the annual subsidy?

(*c*) Is the plant economically justifiable to the firm with the subsidy? Now what is the PWI?

(*d*) How important is the difference in interest rates, and how does it affect these results?

(*e*) How important is the difference in horizons, and how does it affect these results?

(*f*) What is the "co" aspect of a co-generation power plant? What are the primary benefits of this system, and who accrues those benefits? Why aren't all power plants designed in this fashion?

9-87 Assume a cost improvement project has only a first cost of $100,000 and a monthly net savings, *M*. There is no salvage value. Graph the project's IRR for payback periods from 6 months to the project's life of *N* years. The firm accepts projects with a 2-year payback period or a 20% IRR. When are these standards consistent and when are they not?
(*a*) Assume that *N* = 3 years.
(*b*) Assume that *N* = 5 years.
(*c*) Assume that *N* = 10 years.
(*d*) What recommendation do you have for the firm about its project acceptance criteria?

9-88 The Louisiana Department of Transportation and Development (LaDOTD) in 2009 approved the feasibility analysis for upgrading 6 miles of US-167 South starting at the intersection with US-80 (California Avenue). An existing two-lane highway between is to be converted to a four-lane divided freeway. The proposed new freeway is projected to average 25,000 vehicles per day over the next 20 years. Truck volumes represent 6.25% of the total traffic. Annual maintenance on the existing highway is $1875 per lane mile. The existing accident rate is 5.725 per million vehicle miles (MVM). Capital improvement investment money can be secured at 6.25%. Which alternative is preferred (use benefit/cost ratio analysis)?
Plan 1: Add two adjacent lanes for $562,500 per mile. This will reduce auto travel time by 2.5 minutes and truck travel time by 1.25 minutes. It will reduce the accident rate to 3.125 per MVM. Annual maintenance is estimated to be $1560 per lane mile.
Plan 2: Make grade improvements while adding two adjacent lanes at a cost of $812,500 per mile. This would reduce auto and truck travel time by 3.75 minutes each. The accident rate is estimated to be 3.10 per MVM. Annual maintenance is estimated to be $1250 per lane mile.
Plan 3: Construct a new freeway on new alignment at a cost of $1,000,000 per mile. This would reduce auto travel time by 6.25 minutes and truck travel time by 5 minutes. Plan 3 is 0.5 miles longer than the others. The estimated accident rate is 3.00 per MVM. Annual

maintenance is estimated to be $1250 per lane mile. Plan 3 abandons the existing highway with no salvage value.
Additional data:

Operating cost—autos:	15¢ per mile
Operating cost—trucks:	22.5¢ per mile
Time saving—autos:	3.75 per vehicle minute
Time saving—trucks:	18.75¢ per vehicle minute
Average accident cost:	$1500

Contributed by Benedict Nwokolo, Grambling State University

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 16	**Great White Hall** Proposal comparison using B/C analysis for RFP with unclear specifications.
CASE 17	**A Free Lunch?** Is proposal too good to be true from two perspectives? Realistic (unordered) statement of facts.

APPENDIX 9A
INVESTING FOR RETIREMENT AND OTHER FUTURE NEEDS

Key Words

defined benefit

defined contribution

risk

volatility

DEFINED CONTRIBUTION AND DEFINED BENEFIT PLANS

Firms and individuals often borrow to buy capital items like buildings/homes, land, and vehicles. The money is needed now, and often the item is some kind of security for the loan. However, pension funds, retirement accounts, college savings, and down payments are accumulated by saving and investing. Because pensions and retirement have the longest horizons, compound interest plays a larger role than in shorter horizon problems.

Planning for retirement has become even more important to individuals because of a shift in how it is handled. As recently as 2000, 60% of Fortune 500 firms offered **defined benefit** retirement plans—also called pensions. Today it is estimated that about 90% offer new employees only **defined contribution** plans. Many governmental units have also shifted from traditional pensions to defined contribution plans.

A defined benefit plan is overseen by a firm or a government body for its employees. The employer is responsible for managing the fund and ensuring that there is enough money to cover all obligations. In many cases, both the employee and the employer contribute to the fund. Most payouts involve a formula, whereby the employee can calculate his or her monthly or annual benefit, which continues until death. As an example, Equation 9A–1 is applied to a 30-year employee who earned $80,000 yearly in the 3 years before retirement.

$$\begin{aligned}\text{Annual benefit} &= \text{average salary}_{\text{last 3 years}} \times 2\% \times \text{years}_{\text{employment}} \qquad (9A\text{–}1)\\ &= 80{,}000 \times 2\% \times 30 = \$48{,}000/\text{yr}\end{aligned}$$

(9A–1)

The benefit is predictable, and thus makes retirement planning much easier.

With a defined contribution plan, the employee designates the amount to be directed toward retirement and is responsible for its management. The employer may match part of an employee's contribution, and may offer a limited set of fund plans. Based on the choices available, employees select how much they deposit annually, for how many years, and what funds they invest in. Employees choose how safely or aggressively the funds are invested. The total in the account is the **defined contribution**; that total determines how much can be withdrawn and for how long. Thus, employees must also choose the withdrawal rate for the funds after they retire. Employees must estimate how long they will live.

Retirement planning has become more difficult and more necessary. Social security provides a safety net, but it is income replacement only for those who work half-time at the minimum wage. For those retiring in 2016 who contributed at the maximum taxed level for 35 years, the annual social security benefit is $31,668—much less than engineers typically earn.

WHAT RETURNS ARE REASONABLE TO EXPECT AND WHAT RISKS GO WITH THEM

Table 9A–1 summarizes the performance of three types of investments over the last 60 years: the U.S. stock market, U.S. government bonds with typical maturities of 10 to 30 years, and U.S. treasury bills (T-bills) with typical maturities of 1 to 6 months. Since these are market or nominal values, Table 9A–1 also includes the inflation rate (see Chapter 14 for more information). Over the past 60 years, stocks have returned about 7.3% over inflation, long-term treasury bonds about 2.5% above inflation, and T-bills about 0.8% above inflation. These are real rates of return.

The standard deviation and high and low values show that stocks have higher **risks** than bonds and much higher risks than T-bills. The annual returns vary much more for stocks, so stocks have a higher **volatility** than bonds or T-bills.

Note that the geometric mean is the correct average rather than the arithmetic average. The geometric rate of return is the interest rate that would be calculated using the present worth, the future value, and the number of years. It can also be calculated using Equation 9A–2.

$$\text{Geometric mean} = \left[\prod_{i=1}^{n} (1 + r_i)\right]^{1/n} - 1$$
$$\text{where } r_i = \text{return in year } i \qquad (9A–2)$$

The arithmetic average of annual returns overstates expected returns (especially over long intervals with positive and negative annual values). A quick example is gaining 50% one year and losing 50% the other year in either order. The arithmetic average is 0% change. The geometric average using Equation 9A-2 is −13.4% $(= \sqrt{(0.5 \times 1.5)} - 1$. You are not back where you started, as an arithmetic average of 0% would indicate. Instead, you have lost 25%, as the geometric average indicates. You have 75% (= 1.5 × 0.5) or (= 0.866 × 0.866) of what you started with. Example 9A–1 is another illustration of average geometric and arithmetic returns.

EXAMPLE 9A-1

What is the average monthly return for a firm that had these stock prices?

Date	Stock Price
03/01/2018	$14.02
04/02/2018	13.12
05/01/2018	14.05

What is the arithmetic average monthly return? What is the geometric average monthly return? Which correctly measures average monthly returns? What are the equivalent nominal annual returns?

SOLUTION

(i) The monthly return for March is (13.12 − 14.02)/14.02 = −0.0642

(ii) The monthly return for April is (14.05 − 13.12)/13.12 = 0.0709

$$\text{Arithmetic average} = \frac{-0.0642 + 0.0709}{2} = 0.00335 = \frac{0.335\%}{\textit{month}}$$

(iii) Nominal annual return = 0.335% × 12 = 4.02*%*

$$\text{Geometric return} = \sqrt{(1 - 0.0642)(1 + 0.0709)} - 1 = 0.00107$$
$$= 0.107\% \text{ per month}$$

(iv) Nominal annual return = 0.107% × 12 = 1.28*%*

The geometric average is the correct solution, as detailed by using the initial and final prices in the next solution.

5 BUTTON SOLUTION

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	
2	Exp. 9A-1		2	0	14.02	14.05	RATE	0.107%	monthly
3								1.28%	nominal

The solution using RATE is the average monthly return, and it is the geometric average.

The presence of a standard deviation demonstrates that returns vary from year to year. On average, the investments in Table 9A–1 will increase, but returns may be positive or negative in any given year. All investments carry risk, and this risk is often characterized by the standard deviation. Investments that have a higher risk are expected to deliver a higher return in order to compensate for that risk.

Figure 9A–1 plots the geometric means and standard deviations of the three investment types. The arrows emphasize that both higher returns and lower risks are preferred. Thus the best mix of investments depends on how the investor evaluates the trade-off between risk and return. Chapter 10 includes more on probabilities in economic models. Appendix 10A explains why a combination of stocks and bonds is expected to perform better than either alone; this is also why a diversified portfolio of stocks is expected to perform better than one with only a few stocks.

Table 9A-1 Returns and Standard Deviations for Investments and Inflation

	Common Stocks	Treasury Bonds	Treasury Bills	Inflation Rate
Geometric mean	11.0%	6.2%	4.5%	3.7%
Standard deviation	17.5%	10.8%	2.8%	2.9%
Maximum 1950–2012	52.6%	40.4%	11.6%	13.3%
Minimum 1950–2012	− 35.5%	− 12.2%	0.0%	−0.5%

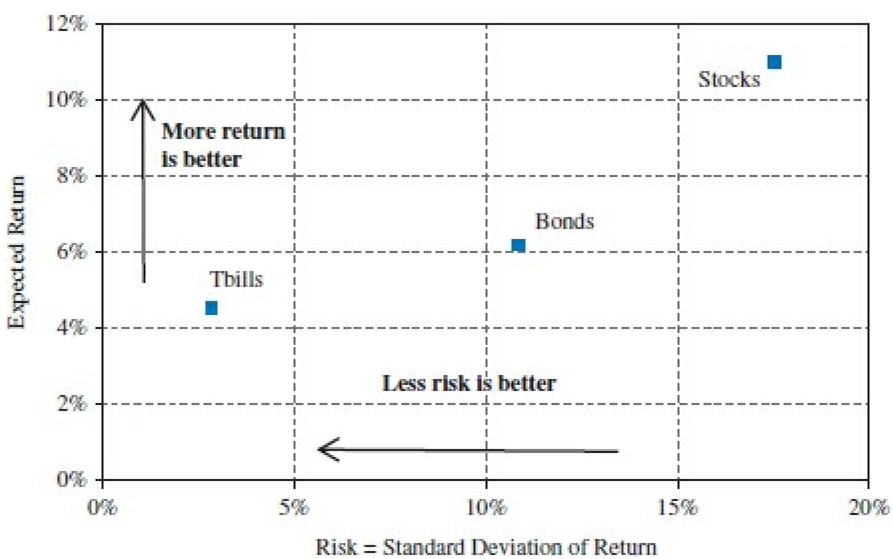


FIGURE 9A-1 Returns and risks for investments.

EXAMPLE 9A-2

An engineer has just finished paying off a student loan and is ready to start saving for retirement. Her current annual salary is $63,000. She expects salary increases to exceed inflation, but to be safe she wants to assume that salary increases will be matched with inflation. She expects to work for another 35 years. If she invests 15% of her salary, how much can be expected to be in her account if she invests in T-bills, bonds, or stocks?

SOLUTION

The first step is to determine the interest rates. Since this problem is in constant-value dollars, like nearly all of the text (except for bonds) up to Chapter 14, the interest rates are the market rates minus inflation. From the first paragraph of this section, stocks have returned about 7.3% over inflation, long-term treasury bonds about 2.5% above inflation, and T-bills about 0.8% above inflation.

To make the solution more flexible, the spreadsheet starts with a data block.

	A	B	C	D	E	F	G	H
1	$63,000	Salary						
2	15%	% saved						
3								
4	Exp. 9A-1	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
5	T-bill	0.8%	35	-9,450	0		FV	$379,957
6	Bonds	2.5%	35	-9,450	0		FV	$519,072
7	Stocks	7.3%	35	-9,450	0		FV	$1,394,949

EXAMPLE 9A-3 Example 9A–2 Revisited

The engineer in Example 9A–2 has decided that a retirement goal of $1 million is adequate. What fraction of her salary must be saved if she invests in T-bills, bonds, or stocks?

SOLUTION

The first step is to determine the annual deposits at the different interest rates. As in Example 9A–2, the interest rates are the market rates minus inflation: stocks have returned about 7.3% over inflation, long-term treasury bonds about 2.5% above inflation, and T-bills about 0.8% above inflation.

To make the solution more flexible, the spreadsheet starts with a data block.

	A	B	C	D	E	F	G	H	I
1	$63,000	Salary							
2									
3	Exp. 9A-3	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	% salary
4	T-bill	0.8%	35		0	1,000,000	PMT	-$24,871	39.5%
5	Bonds	2.5%	35		0	1,000,000	PMT	-$18,206	28.9%
6	Stocks	7.3%	35		0	1,000,000	PMT	-$6,774	10.8%

An investor who chooses to invest safely before retirement should also invest safely after retirement. If the $1M is invested at a lower rate, then less can be withdrawn each year to live on. It is common to suggest that *all* investors should invest more safely once they are approaching retirement or are already retired.

EXAMPLE 9A-4 Examples 9A–2 and 9A–3 Revisited

The engineer in Examples 9A–2 and 9A–3 wants to know how many years she can live on her savings before they are exhausted. Assume the common guideline of retirement spending equals 80% of pre-retirement net income. Also assume the same investments and returns before and after retirement. How long after retirement before her $1 million is exhausted?

SOLUTION

The average returns were determined for the earlier examples: stocks have returned about 7.3% over inflation, long-term treasury bonds about 2.5% above inflation, and T-bills about 0.8% above inflation. The required savings at each rate were determined in Example 9A–2. Note that the annual amount spent before retirement is $\$63,000 \times (1 - \%_{\text{saved}})$. The expected spending level after retirement is 80% of that. So the spending level with T-bills is $30,503 (= (63, 000 − 24, 871) × 80%).

	A	B	C	D	E	F	G	H	I
1	$63,000	Salary							
2	80%	% retirement spending							
3	Exp. 9A-3	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	% salary
4	T-bills	0.8%	35		0	1,000,000	PMT	-$24,871	39.5%
5	Bonds	2.5%	35		0	1,000,000	PMT	-$18,206	28.9%
6	Stocks	7.3%	35		0	1,000,000	PMT	-$6,774	10.8%
7									
8	Exp. 9A-4	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	
9	T-bills	0.8%		30,503	1,000,000	0	NPER	38.2	
10	Bonds	2.5%		35,836	1,000,000	0	NPER	48.4	
11	Stocks	7.3%		44,980	1,000,000	0	NPER	#NUM!	

No answer is returned for stocks. The reason is that an annual return of 7.3% on $1 million is $73,000. This is more than the expected retirement spending. It is also more than the pre-retirement salary!

EXAMPLE 9A-5 Example 9A–2 Revisited

An engineer has just finished paying off a student loan and is ready to start saving for retirement. Her current annual salary is $63,000. Assume that her salary increases 2% faster than inflation. She expects to work for another 35 years. If she invests 15% of her salary, how much can be expected to be in her account if she invests in T-bills, bonds, or stocks?

SOLUTION

The interest rates were identified in Example 9A–2, but the annuity functions used in Examples 9A–2 through 9–4 cannot be used here. The annual deposit increases with the salary, so tables of cash flows must be created. Each year a deposit equal to a percentage of the income is made. The balance in year *t* is that deposit plus (1 + return) × the previous year's balance. There is no interest in year 1 because the deposit is assumed to be made at the end of the year.

	A	B	C	D	E	F
1	$63,000	Salary				
2	15%	% saved				
3	2%	annual salary increase				
4				T-bills	Bonds	Stocks
5	Year	Salary	Deposit	0.8%	2.5%	7.3%
6	1	63,000	9,450	9,450	9,450	9,450
7	2	64,260	9,639	19,165	19,325	19,779
8	3	65,545	9,832	29,150	29,640	31,054
9	4	66,856	10,028	39,411	40,410	43,350
10	5	68,193	10,229	49,956	51,649	56,743
40	35	123,523	18,528	534,108	705,567	1,743,063
41						
42			=$C40+D39*(1+D$5)			
43					=$C40+F39*(1+F$5)	

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

An initial investment of $1000 in Miracle Plastics has annual returns that have varied a lot. For those returns find the arithmetic average and the geometric mean. Which one is the proper measure?

9A-1

Year	Annual Return
1	30%
2	-25%
3	-15%
4	35%

SOLUTION

9A-2 A 45-year-old engineer earning $120,000 per year wants to retire at age 65 with $2 million. The engineer has nothing saved and expects to earn 7% annually on the investment?

(a) How much money must be invested each year?
(b) If the employer does a 100% match of retirement savings up to 4% of the employee's salary, how much money must each invest annually?

SOLUTION

9A-3 A 45-year-old engineer earning $110,000 per year wants to retire at age 70 with $1.75 million. The engineer has nothing saved and expects to earn 6% annually on the investment? What fraction of the salary must be invested each year to reach the goal?

SOLUTION

9A-4 A 25-year old engineer wants to spend $40,000 per year traveling as long as possible before switching to saving for retirement. The engineer plans to retire at 60 with $1.5 million and then resume traveling. The engineer expects to earn 8% annually on the investment.

(a) For how many years must the engineer save for retirement?
(b) How long can the engineer travel before beginning to save for retirement?

SOLUTION

9A-5 An engineering manager retired with investments of $1,200,000 safely invested at 4%. She is 62 and needs $60,000 per year for living expenses, in addition to her social security benefit.

(a) How long will her investment last if it remains invested at 4%?
(b) How much can she spend if it must last until she is 86?

SOLUTION

9A-6 A new employee puts 10% of his salary of $72,000 into a retirement account. He expects his salary to increase 3% per year. The money is invested in a mutual fund that he expects to average a 6.5% return. How much is in the fund after 10 years?

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

For these problems assume that stocks, bonds, and T-bills return, respectively, 7.3%, 2.5%, and 0.8% over inflation.

9A-1 A 25-year-old engineer earning $65,000 per year wants to retire at age 55 with $2 million, and plans to invest in a stock fund.
(*a*) How much money must be invested each year?
(*b*) If the employer does a 100% match of retirement savings up to 3% of the employee's salary, how much money must each invest annually?

9A-2 A An engineer earning $70,000 per year wants to retire in 30 years with $1.5 million and plans to invest in a treasury bond fund.
(*a*) How much money must be invested each year?
(*b*) If this person works 5 additional years, how much money must be invested each year?
(*c*) If the employer does a 100% match of retirement savings up to 4.25% of the employee's salary, and the engineer wants to retire in 30 years, how much money must each invest annually?

9A-3 A risk-averse 25-year-old engineer earning $70,000 per year wants to retire at age 65 with $2 million and plans to safely invest in treasury bills.

(*a*) How much money must be invested each year?
(*b*) If the employer does a 100% match of retirement savings up to 5% of the employee's salary, how much money must each invest annually?

A-4 A
A new employee earning $48,000 annually has set a retirement goal of $1 million. She plans to work for 40 years, and will invest in stocks.
(*a*) What fraction of her salary must be saved?
(*b*) If the employer does a one-to-one match of retirement savings up to 4% of the employee's salary, what fraction of her salary must be saved?

9A-5
An employee earning $63,000 per year has set a goal of retiring in 40 years with $1 million. She will invest in treasury bonds.
(*a*) What fraction of her salary must be saved?
(*b*) If the employer does a one-to-one match of retirement savings up to 5% of the employee's salary, what fraction of her salary must be saved?

9A-6 A
A new engineer started her first job earning $60,000 annually, and wants to retire in 40 years with $1 million. She will invest in treasury bills.
(*a*) What fraction of her salary must be saved?
(*b*) If the employer does a one-to-one match of retirement savings up to 4% of the employee's salary, what fraction of her salary must be saved?

9A-7
A manager just retired at age 62, with his retirement savings of $800,000 invested in stocks. If he needs $60,000 per year for living expenses, how long will his savings last?

9A-8 A
A manager retired at age 65, and has her retirement savings of $1,500,000 invested in treasury bonds. She needs $65,000 per year for living expenses, in addition to her social security benefit. How long will her investment last?

9A-9
A person worked for many years with the same company and has accumulated a retirement account of $900,000. If $30,000 per year needs to be withdrawn from this account, how long will the savings

last if invested in treasury bills?

9A-10 A A new employee puts 4% of his salary of $65,000 into a retirement account, and his employer matches this, also putting 3% into the account. The money is invested in a diversified stock fund. His salary increases 2.5% per year.

(*a*) What is the value of the account after 10 years?

(*b*) What is the value of the employer’s matching funds?

(*c*) How much will be in the account after 40 years?

9A-11 An engineer changed jobs and is signing up for benefits. The company 401(k) includes a low cost treasury bond fund. The engineer will put 3% of her salary of $70,000 into the account, and her employer will match half this amount. Her salary is expected to increase 2.8% per year.

(*a*) What is the value of the account after 10 years?

(*b*) If she expects to work for 30 years, how much will be in the account?

9A-12 A A long-term employee is nearing retirement and will adjust his retirement account. The account has $500,000 in it. The employee will put that money and all new money in a treasury bill (T-bill) account. He puts 6% of his $135,000 salary into the account, and his employer matches half of this amount. His salary increases 2% per year. What is the value of the account after 10 years?

9A-13 Mike just changed jobs, leaving a company after 6 years. He is fully vested, and can keep the money his employer deposited in his retirement account. His employer has been contributing $200 per month into a diversified stock fund.

(*a*) Using average market rates, how much money has accumulated in the account?

(*b*) If this money is “rolled over” into another retirement account with an 8% annual return, how much will this be worth after 30 years?

9A-14 A Aonica became concerned about the stock market due to recent losses, and wants safe investments in treasury bonds. She moved $350,000 in her retirement account into an ETF specializing in government bonds, which average 2.5% above inflation. Her salary is currently $92,000 per year, and she expects raises of 2.0% each year.

	Her employer will match retirement savings up to 5% of her salary. If she wants $450,000 in her account in 5 years, what % of her salary does she need to save?
9A-15	Jorge was recently laid off, and may not be able to find another full-time job. Capital preservation is his primary concern, so he wants to invest in treasury bills. His retirement fund has $1.3 million. He can receive social security, so he will only need $38,000 per year from his savings to live well. Jorge is 64 years old. How long does he expect his savings to last? Should he look for a job or enjoy his retirement?

[1] *Happy Money* by Dunn and Norton notes that individuals can *decrease* spending and *increase* happiness by switching a daily habit to a weekly treat.

CHAPTER 10

UNCERTAINTY IN FUTURE EVENTS

Moving the Load

Most organizations hire third-party carriers to move freight. Some choose to outsource all of their transportation requirements to carriers, thereby avoiding the expense of acquiring, operating, and maintaining their own private fleet. Even organizations that have their own fleets, such as Walmart and Sysco, also rely on third-party carriers for flexible transportation capacity, which is critical in the face of uncertain demand. Such blended operations are quite common, with carriers handling roughly one-third of the outbound freight for U.S. firms with private fleets.

Carriers typically offer full-truckload (FTL) and less-than-truckload (LTL) shipping. FTL shipping is ideal for large loads—an FTL shipment via an 18-wheel semi-truck with a 53-foot trailer can typically accommodate up to 30 pallets and 50,000 pounds of freight. Scheduling an FTL shipment with a carrier tends to be a straightforward process and is convenient for the shipper. Furthermore, transit times are relatively short—an entire truck is dedicated to the shipment directly from the shipper to a single destination. However, the shipper must pay for the entire truck, which is not cost-effective when shipping small volumes.

By contrast, LTL shipping is typically for loads weighing more than 5,000 pounds on more than 10 pallets. With LTL shipping, multiple shippers share space on the same truck, and each pays only for the capacity that it uses. However, LTL shipments can cost up to 10 times more per pallet than a fully loaded FTL shipment. Organizing LTL shipments can be complex and time-consuming, as carriers attempt to fill truck capacity, reduce empty backhauls, and minimize the total distance traveled between multiple shippers and destinations.

Shippers have traditionally hired freight brokers to serve as intermediaries to connect them with carriers. Recently, however, some shippers have started to replace their brokers with digital freight apps. This method, known as "crowdshipping" or "crowd logistics," operates similarly to ride-share and food-delivery apps but is focused on commercial carriers and LTL freight. Digital freight apps, such as Uber Freight, allow carriers to view available shipments, including real-time pricing, destinations, and shipper requirements (e.g., hazmat, time windows), via a smartphone.

From the carrier's perspective, crowdshipping offers advantages over traditional brokerage services: it gives drivers the flexibility to choose and organize their own loads and routes, the pricing is clear, and crowdshipping providers tend to pay drivers more quickly than brokers. For LTL shippers, crowdshipping provides a cost-effective avenue for increasing logistics capacity as needed and eliminating brokerage fees.

Despite these advantages, adoption of digital freight apps has been slow. Crowdshipping providers have struggled to consistently source sufficient carriers to meet shippers' demand. Many carriers are accustomed to working with brokers, with whom they have developed trust-based relationships over time, and they prefer to outsource the burden of load sourcing to these brokers. Some shippers also have concerns about the safety and reliability of using a digital app to select carriers. ■■■

Contributed by Caroline C. Krejci, University of Texas at Arlington

QUESTIONS TO CONSIDER

1. Is crowdshipping likely to be more beneficial for large shippers (e.g., Walmart) or small-scale organizations?
2. For what sources of uncertainty in a supply chain is crowdshipping likely to be most effective?
3. What are the ethical implications of replacing human brokers with digital freight apps?

After Completing This Chapter ...

The student should be able to:

• Use a range of estimated variables to evaluate a project.
• Describe possible outcomes with probability distributions.
• Combine probability distributions for individual variables into joint probability distributions.
• Use expected values for economic decision making.
• Use economic decision trees to describe and solve more complex problems.
• Measure and consider risk when making economic decisions.
• Understand how simulation can be used to evaluate economic decisions.
• Understand why and how diversification reduces risk for investments and project portfolios. (Appendix 10A)

Key Words

beta distribution

chance node

decision node

decision tree

discrete probability distribution

dominated projects

efficient frontier

expected value

joint probability distribution

most likely

optimistic

outcome node

pessimistic

pruned branch

real options

risk

scenario

simulation

standard deviation

statistically independent

SUMPRODUCT

An assembly line is built after the engineering economic analysis has shown that the anticipated product demand will generate profits. A new motor, heat exchanger, or filtration unit is installed after analysis has shown that future cost savings will economically justify current costs. A new road, school, or other public facility is built after analysis has shown that the future demand

and benefits justify the present cost to build. However, future performance of the assembly line, motor, and so on is uncertain, and demand for the product or public facility is more uncertain.

Engineering economic analysis is used to evaluate projects with long-term consequences when the time value of money matters. Thus, it must concern itself with future consequences; but describing the future accurately is not easy. In this chapter we consider the problem of evaluating the future. The easiest way to begin is to make a careful estimate and a breakeven analysis. Then we examine the possibility of predicting a range of possible outcomes. Finally, we consider what happens when the probabilities of the various outcomes are known or may be estimated. We will show that the tools of probability are quite useful for economic decision making.

ESTIMATES AND THEIR USE IN ECONOMIC ANALYSIS

Economic analysis requires evaluating the future consequences of an alternative. In practically every chapter of this book, there are cash flow tables and diagrams that describe precisely the costs and benefits for future years. We don't really believe that we can exactly foretell a future cost or benefit. Instead, our goal is to select a single value representing the *best* estimate that can be made.

Breakeven analysis, as shown in Examples 9–11 and 12, is one means of examining the impact of the variability of some estimate on the outcome. It helps by answering the question, How much variability can a parameter have before the decision will be affected? While the preferred decision depends on whether the salvage value is above or below the breakeven value, the economic difference between the alternatives is small when the salvage value is "close" to breakeven.

What-if analysis, which was detailed in Example 9–13, is another way of creating and evaluating **scenarios** that describe future uncertainties. Breakeven and what-if analyses do not solve the basic problem of how to take the inherent variability of parameters into account in an economic analysis. This is the task of this chapter.

A RANGE OF ESTIMATES

It is usually more realistic to describe parameters with a range of possible values, rather than a single value. A range could include an **optimistic** estimate, the **most likely** estimate, and a **pessimistic** estimate. Then, the economic analysis can determine whether the decision is sensitive to the range of projected values. Sets of such optimistic, most likely, and pessimistic estimates are often used to form optimistic, most likely, and pessimistic **scenarios**.

EXAMPLE 10-1

A firm is considering an investment. The most likely data values were found during the feasibility study. Analyzing past data of similar projects shows that optimistic values for the first cost and the annual benefit are 5% better than most likely values. Pessimistic values are 15% worse.

The firm's most experienced project analyst has estimated the values for the useful life and salvage value. Note that 5% better and 15% worse is not +5% and −15% for cost. A lower cost is better and a higher cost is worse.

	Optimistic	Most Likely	Pessimistic
Cost	$950	$1000	$1150
Net annual benefit	210	200	170
Salvage value	100	0	0

Useful life, in years 12 10 8

Compute the rate of return for each estimate. If a 10% before-tax minimum attractive rate of return is required, is the investment justified under all three estimates? If it is justified only under some estimates, how can these results be used?

SOLUTION

	A	B	C	D	E	F	G	H
1	Scenario	*i*	*n*	*PMT*	*PV*	*FV*	*Solve for*	Answer
2	Optimistic		12	210	-950	100	*i*	19.80%
3	Most Likely		10	200	-1000	0	*i*	15.10%
4	Pessimistic		8	170	-1150	0	*i*	3.90%

To solve with tabulated factors, the equations for the respective cases are:

Optimistic

$$\text{PW} = 0 = -\$950 + 210(P/A, \text{IRR}_{\text{opt}}, 12) + 100(P/F, \text{IRR}_{\text{opt}}, 12)$$

Most Likely

$$\text{PW} = 0 = -\$1000 + 200(P/A, \text{IRR}_{\text{most likely}}, 10)$$
$$(P/A, \text{IRR}_{\text{most likely}}, 10) = 1000/200 = 5$$

Pessimistic

$$\text{PW} = 0 = -\$1150 + 170(P/A, \text{IRR}_{\text{pess}}, 8)$$
$$(P/A, \text{IRR}_{\text{pess}}, 8) = 1150/170 = 6.76$$

From the calculations we conclude that the rate of return for this investment is most likely to be 15.1%, but might range from 3.9% to 19.8%. The investment meets the 10% MARR criterion for two of the estimates. These estimates can be considered to be scenarios of what may happen with this project. Since one scenario indicates that the project is not attractive, we need to have a method of weighting the scenarios or considering how likely each is.

Example 10–1 made separate calculations for the sets of optimistic, most likely, and pessimistic values. The range of scenarios is useful. However, if there are more than a few uncertain variables, it is unlikely that all will prove to be optimistic (best case) or most likely or pessimistic (worst case). It is more likely that many parameters are the most likely values, while some are optimistic and some are pessimistic.

This can be addressed by using Equation 10-1 to calculate average or mean values for each parameter. Equation 10-1 puts four times the weight on the most likely value than on the other two. This equation has a long history of use in project management to estimate activity completion times. It is an approximation with the **beta distribution**.

$$\text{Mean value} = \frac{\text{Optimistic value} + 4(\text{Most likely value}) + \text{Pessimistic value}}{6} \quad (10\text{-}1)$$

This approach is illustrated in Example 10–2.

EXAMPLE 10-2

Solve Example 10–1 by using Equation 10-1. Compute the resulting mean rate of return.

SOLUTION

Compute the mean for each parameter:

$$\begin{aligned}
\text{Mean cost} &= [950 + 4(1000) + 1150]/6 = \$1016.7 \\
\text{Mean net annual benefit} &= [210 + 4(200) + 170]/6 = \$196.7 \\
\text{Mean useful life} &= [12 + 4(10) + 8]/6 = 10.0 \\
\text{Mean salvage life} &= 100/6 = \$16.7
\end{aligned}$$

Compute the mean rate of return:

	A	B	C	D	E	F	G
1	Scenario	*weight*	*n*	*PMT*	*PV*	*FV*	Solve for
2	Optimistic	1	12	210	-950	100	*i*
3	Most Likely	4	10	200	-1000	0	*i*
4	Pessimistic	1	8	170	-1150	0	*i*
5	weighted values		10	196.7	-1016.7	16.7	*i*
6			=SUMPRODUCT($B$2:$B$4,C2:C4)/SUM($B$2:$B$4				

Note that the **SUMPRODUCT** function is an easy way to multiply the set of weights by each of *n*, *PMT*, *PV*, and *FV*. The resulting 14.3% rate of return is lower than the return of the most likely case, because the pessimistic values for *PV* and *FV* are "further away" from the most likely values than the optimistic values are.

Example 10–1 gave a most likely rate of return (15.1%) that differed from the mean rate of return (14.3%) computed in Example 10–2. These values are different because the former is based exclusively on the most likely values and the mean considers other possible values.

In examining the data, we see that the pessimistic values are further away from the most likely values than are the optimistic values. This is common. For example, a savings of 10–20% may be the maximum possible, but a cost overrun can be 50%, 100%, or even more. This causes the weighted mean values to be less favorable than the most likely values. As a result, the mean rate of return is usually less than the rate of return based on the most likely values.

PROBABILITY

We all have used probabilities. For example, what is the probability of getting a "head" when flipping a coin? Using a model that assumes that the coin is fair, both the head and tail outcomes occur with a probability of 50%, or 1/2. This probability is the likelihood of an event in a single trial. It also describes the long-run relative frequency of getting heads in many trials (out of 50 coin flips, we expect to average 25 heads).

Probabilities can also be based on data, expert judgment, or a combination of both. Past data on weather and climate, on project completion times and costs, and on highway traffic are combined with expert judgment to forecast future events. These examples can be important in engineering economy.

Another example based on long-run relative frequency is the PW of a flood-protection dam that depends on the probabilities of different-sized floods over many years. This would be based on data from past floods and would include many years of observation. An example of a single event that may be estimated by expert judgment is the probability of a successful outcome for a research and development project, which will determine its PW.

All the data in an engineering economy problem may have some level of uncertainty. However, small uncertainties may be ignored, so that more analysis can be done with the large uncertainties. For example, the price of an off-the-shelf piece of equipment may vary by only ± 5%. The price could be treated as a known or deterministic value. On the other hand, demand over the next 20 years will have more uncertainty. Demand should be analyzed as a random or stochastic variable. We should establish probabilities for different values of demand.

There are also logical or mathematical rules for probabilities. If an outcome can never happen, then the probability is 0. If an outcome will certainly happen, then the probability is 1, or 100%. This means that probabilities cannot be negative or greater than 1; in other words, they must be within the interval [0, 1], as indicated shortly in Equation 10-2.

Probabilities are defined so that the sum of probabilities for all possible outcomes is 1 or 100% (Equation 10-3). Summing the probability of 0.5 for a head and 0.5 for a tail leads to a total of 1 for the possible outcomes from the coin flip. An exploration well drilled in a potential oil field will have three outcomes (dry hole, noncommercial quantities, or commercial quantities) whose probabilities will sum to one.

Equations 10-2 and 10-3 can be used to check that probabilities are valid. If the probabilities for all but one outcome are known, the equations can be used to find the unknown probability for that outcome (see Example 10–3).

$$0 \leq \text{Probability} \leq 1 \qquad (10\text{-}2)$$

$$\sum_{j=1 \text{ to } K} P(\text{outcome}_j) = 1, \quad \text{where there are } K \text{ outcomes} \qquad (10\text{-}3)$$

In a probability course many probability distributions, such as the normal, uniform, and beta, are presented. These continuous distributions describe a large population of data. However, for engineering economy it is more common to use a **discrete probability distribution** with 2 to 5 outcomes—even though the 2 to 5 outcomes only represent or approximate the range of possibilities. Quite often 3 outcomes are used—optimistic, most likely, and pessimistic.

This is done for two reasons. First, the data often are estimated by expert judgment, so that using 7 to 10 outcomes would be false accuracy. Second, each outcome requires more analysis. In most cases the 2 to 5 outcomes represents the best trade-off between representing the range of possibilities and the amount of calculation required. Example 10–3 illustrates these calculations.

EXAMPLE 10-3

What are the probability distributions for the annual benefit and life for the following project?

The annual benefit's most likely value is \$8000 with a probability of 60%. There is a 30% probability that it will be \$5000 and the highest likely value is \$10,000. A life of 6 years is twice as likely as a life of 9 years.

SOLUTION

For the annual benefit, probabilities are given for only two of the possible outcomes. The third value is found from the fact that the probabilities for the three outcomes must sum to 1 (Equation 10-3).

$$1 = P(\text{Benefit is } \$5000) + P(\text{Benefit is } \$8000) + P(\text{Benefit is } \$10{,}000)$$
$$P(\text{Benefit is } \$10{,}000) = 1 - 0.6 - 0.3 = 0.1$$

The probability distribution can then be summarized in a table. The histogram or relative frequency diagram is Figure 10–1.

Annual benefit	$5000	$8000	$10,000
Probability	0.3	0.6	0.1

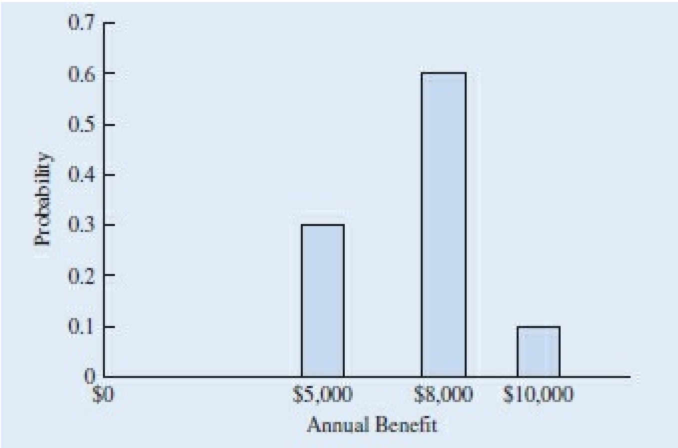


FIGURE 10-1 Probability distribution for annual benefit.

For the life's probability distribution, the problem statement tells us

$$P(\text{life is 6 years}) = 2P(\text{life is 9 years})$$

Equation 10-3 can be applied to write a second equation for the two unknown probabilities:

$$P(6) + P(9) = 1$$

Combining these, we write

$$
\begin{aligned}
2P(9) + P(9) &= 1 \\
P(9) &= 1/3 \\
P(6) &= 2/3
\end{aligned}
$$

The probability distribution for the life is $P(6) = 66.7\%$ and $P(9) = 33.3\%$.

JOINT PROBABILITY DISTRIBUTIONS

Example 10–3 constructed the probability distributions for a project's annual benefit and life. These examples show how likely each value is for the input data. We would like to construct a similar probability distribution for the project's present worth. This is the distribution that we can use to evaluate the project. That present worth depends on both input probability distributions, so we need to construct the **joint probability distribution** for the different combinations of their values.

For this introductory text, we assume that two random variables such as the annual benefit and life are unrelated or **statistically independent**. This means that the *joint* probability of a combined event (Event A defined on the first variable and Event B on the second variable) is the product of the probabilities for the two events. This is Equation 10-4:

$$\text{If A and B are independent, then } P(\text{A and B}) = P(\text{A}) \times P(\text{B}) \qquad (10\text{-}4)$$

For example, flipping a coin and rolling a die are statistically independent. Thus, the probability of {flipping a head and rolling a 4} equals the probability of a {heads} = 1/2 times the probability of a {4} = 1/6, for a joint probability = 1/12.

The number of outcomes in the joint distribution is the product of the number of outcomes in each variable's distribution. Thus, for the coin and the die, there are 2 times 6, or 12 combinations. Each of the 2 outcomes for the coin is combined with each of the 6 outcomes for the die.

Some variables are not statistically independent, and the calculation of their joint probability distribution is more complex. For example, a project with low revenues may be terminated early and one with high revenues may be kept operating as long as possible. In these cases annual cash flow and project life are not independent. While this type of relationship can sometimes be modeled with economic decision trees (covered later in this chapter), we will limit our coverage in this text to the simpler case of independent variables.

Example 10–4 uses the three values and probabilities for the annual benefit and the two values and probabilities for the life to construct the six possible combinations. Then the values and probabilities are constructed for the project's PW.

EXAMPLE 10-4

The project described in Example 10–3 has a first cost of $25,000. The firm uses an interest rate of 10%. Assume that the probability distributions for annual benefit and life are unrelated or statistically independent. Calculate the probability distribution for the PW.

SOLUTION

Since there are three outcomes for the annual benefit and two outcomes for the life, there are six combinations. The first four columns of the following table show the six combinations of life and annual benefit. The probabilities in columns 2 and 4 are multiplied to calculate the joint probabilities in column 5. For example, the probability of a low annual benefit and a short life is 0.3 × 2/3, which equals 0.2 or 20%.

The PW values include the $25,000 first cost and the results of each pair of annual benefit and life. For example, the PW for the combination of high benefit and long life is

$$PW_{\$10{,}000,9} = -25{,}000 + 10{,}000(P/A, 10\%, 9) = -25{,}000 + 10{,}000(5.759) = \$32{,}590$$

Annual Benefit	Probability	Life (years)	Probability	Joint Probability	PW
$ 5,000	30%	6	66.70%	20.00%	-$ 3,224

8,000	60	6	66.7	40	9,842
10,000	10	6	66.7	6.7	18,553
5,000	30	9	33.3	10	3,795
8,000	60	9	33.3	20	21,072
10,000	10	9	33.3	3.3	32,590
				100.0%	

Figure 10–2 shows the probabilities for the PW in the form of a histogram for relative frequency distribution, or probability distribution function.

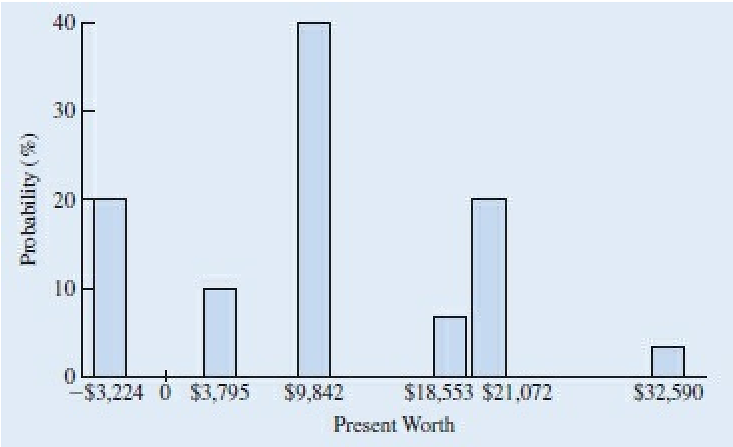


FIGURE 10-2 Probability distribution function for PW.

This probability distribution function shows that there is a 20% chance of having a negative PW. It also shows that there is a small (3.3%) chance of the PW being $32,590. The three values used to describe possible annual benefits for the project and the two values for life have been combined to describe the uncertainty in the project's PW.

Creating a distribution, as in Example 10–4, gives us a much better understanding of the possible PW values along with their probabilities. The three possibilities for the annual benefit and the two for the life are representative of the much broader set of possibilities that really exist. Optimistic, most likely, and pessimistic values are a good way to represent the uncertainty about a variable.

Similarly the six values for the PW represent the much broader set of possibilities. The 20% probability of a negative PW is one measure of risk that we will talk about later in the chapter.

Some problems, such as Examples 10–1 and 10–2, have so many variables or different outcomes that constructing the joint probability distribution is arithmetically burdensome. If the values in Equation 10-1 are treated as a discrete probability distribution function, the probabilities are 1/6, 2/3, 1/6. With an optimistic, most likely, and pessimistic outcome for each of 4 variables, there are $3^4 = 81$ combinations. In Examples 10–1 and 10-2, the salvage value has only two distinct values, so there are still $3 \times 3 \times 3 \times 2 = 54$ combinations.

When the problem is important enough, the effort to construct the joint probability distribution is worthwhile. It gives the analyst and the decision maker a better understanding of what may happen. It is also needed to calculate measures of a project's risk. While spreadsheets can automate the arithmetic, simulation (described at the end of the chapter) can be a better choice when there are a large number of variables and combinations.

EXPECTED VALUE

For any probability distribution we can compute the **expected value (EV)** or weighted arithmetic average (mean). To calculate the EV, each outcome is weighted by its probability, and the results are summed. This is NOT the simple average or unweighted mean. When the class average on a test is computed, this is an unweighted mean. Each student's test has the same weight. This simple "average" is the one that is shown by the button $\bar{x}$ on many calculators.

The expected value is a weighted average, like a student's grade point average (GPA). To calculate a GPA, the grade in each class is weighted by the number of credits. For the expected value of a probability distribution, the weights are the probabilities.

This is described in Equation 10-5. We saw in Example 10–2 that these expected values can be used to compute a rate of return. They can also be used to calculate a present worth as in Example 10–5.

$$\text{Expected value} = \text{Outcome}_A \times P(A) + \text{Outcome}_B \times P(B) + \cdots \qquad (10\text{-}5)$$

EXAMPLE 10-5

The first cost of the project in Example 10–3 is $25,000. Use the expected values for annual benefits and life to estimate the present worth. Use an interest rate of 10%.

SOLUTION

$$\begin{aligned} EV_{\text{benefit}} &= 5000(0.3) + 8000(0.6) + 10{,}000(0.1) = \$7300 \\ EV_{\text{life}} &= 6(2/3) + 9(1/3) = 7 \text{ years} \end{aligned}$$

The PW using these values is

$$PW(EV) = -25{,}000 + 7300(P/A, 10\%, 7) = -25{,}000 + 6500(4.868) = \$10{,}536$$

[*Note:* This is the present worth of the expected values, PW(EV), not the expected value of the present worth, EV(PW). It is an easy value to calculate that approximates the EV(PW), which will be computed from the joint probability distribution found in Example 10–4.]

Example 10–5 is a simple way to approximate the project's expected PW. But the true expected value of the PW is somewhat different. To find it, we must use the joint probability distribution for benefit and life, and the resulting probability distribution function for PW that was derived in Example 10–4. Example 10–6 shows the expected value of the PW or the EV(PW).

EXAMPLE 10-6

Use the probability distribution function of the PW that was derived in

Example 10–4 to calculate the EV(PW). Does this indicate an attractive project?

SOLUTION

The table from Example 10–4 can be reused with one additional column for the weighted values of the PW (= PW × probability). Then, the expected value of the PW is calculated by summing the column of present worth values that have been weighted by their probabilities.

Annual Benefit	Probability	Life (years)	Probability	Joint Probability	PW	PW×Joint Probability
$ 5,000	30%	6	66.70%	20.00%	-$ 3,224	-$ 645
8,000	60	6	66.7	40	9,842	3,937
10,000	10	6	66.7	6.7	18,553	1,237
5,000	30	9	33.3	10	3,795	380
8,000	60	9	33.3	20	21,072	4,214
10,000	10	9	33.3	3.3	32,590	1,086
				100.0%	EV(PW) = $10,209	

With an expected PW of $10,209, this is an attractive project. While there is a 20% chance of a negative PW, the possible positive outcomes are larger and more likely. Having analyzed the project under uncertainty, we are much more knowledgeable about the potential result of the decision to proceed.

The $10,209 value is more accurate than the approximate value calculated in Example 10–5. The values differ because PW is a nonlinear function of the life. The more accurate value of $10,209 is lower because the annual benefit values for the longer life are discounted by $1/(1 + i)$ for more years.

In Examples 10–5 and 10–6, the question was whether the project had a positive PW. With two or more alternatives, the criterion would have been to maximize the PW. With equivalent uniform annual costs (EUACs) the goal is to minimize the EUAC. Example 10–7 uses the criterion of minimizing the EV of the EUAC to choose the best height for a dam.

EXAMPLE 10-7

A dam is being considered to reduce river flooding. But if a dam is built, what height should it be? Increasing the dam's height will (1) reduce a flood's probability, (2) reduce the damage when floods occur, and (3) cost more. Which dam height minimizes the expected total annual cost? The state uses an interest rate of 5% for flood protection projects, and all the dams should last 50 years.

Dam Height (ft)	First Cost
No dam	$ 0
20	700,000
30	800,000
40	900,000

SOLUTION

The easiest way to solve this problem is to choose the dam height with the lowest equivalent uniform annual cost (EUAC). Calculating the EUAC of the first cost requires multiplying the first cost by $(A/P, 5\%, 50)$. For example, for the dam 20 ft high, this is $700,000(A/P, 5\%, 50) = \$38,344$.

Calculating the annual expected flood damage cost for each alternative is simplified because the term for the P(no flood) is zero, because the damages for no flood are $0. Thus we need to calculate only the term for flooding. This is done by multiplying the P(flood) times the damages if a flood happens. For example, the expected annual flood damage cost with no levee is $0.25 \times \$800,000$, or $\$200,000$.

Then the EUAC of the first cost and the expected annual flood damage are added together to find the total EUAC for each height. The 30 ft dam is somewhat cheaper than the 40 ft dam.

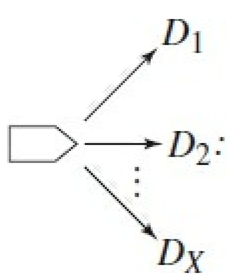
Dam Height	EUAC of First Cost	Annual *P* (flood)>	Damages if Flood	Expected Annual Flood	Total Expected

(ft)		Height	Occurs	Damages	EUAC
No dam	$ 0	25%	× $800,000	= $200,000	$200,000
20	38,344	0.05	× 5,00,000	= 25,000	63,344
30	43,821	0.01	× 3,00,000	= 3,000	46,821
40	49,299	0.002	× 2,00,000	= 400	49,699

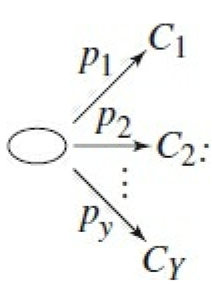
ECONOMIC DECISION TREES

Some engineering projects are more complex, and evaluating them properly is correspondingly more complex. For example, consider a new product with potential sales volumes ranging from low to high. If the sales volume is low, then the product may be discontinued early in its potential life. On the other hand, if sales volume is high, additional capacity may be added to the assembly line and new product variations may be added. This can be modeled with a **decision tree**.

The following symbols are used to model decisions with decision trees:



Decision node: Decision maker chooses 1 of the available paths.



Chance node: *:* Represents a probabilistic (chance) event. Each possible outcome (C_1, C_2, …, C_Y) has a probability (p_1, p_2, …, p_y) associated with it.

Outcome node: *:* Shows result for a particular path through the decision tree.

Pruned branch: *:* The double hash mark indicates that a branch has been pruned because another branch has been chosen. This can happen only at decision nodes, not at chance nodes. The term “pruned” is chosen to correspond to the gardener’s practice of trimming or pruning off branches to make a tree or bush healthier.

Figure 10–3 illustrates how decision nodes, chance nodes, and outcome nodes can be used to describe a problem’s structure. Details such as the probabilities and costs can be added on the branches that link the nodes. With the branches from decision and chance nodes, the model becomes a decision tree.

Figure 10–3 illustrates that decision trees describe the problem by starting at the decision that must be made and then adding chance and decision nodes in the proper logical sequence. Thus describing the problem starts at the first step and goes forward in time with sequences of decision and chance nodes.

To make the decision, calculations begin with the final nodes in the tree. Since they are the final nodes, enough information is available to evaluate them. At decision nodes the criterion is either to maximize PW or to minimize EUAC. At chance nodes an expected value for PW or EUAC is calculated.

Once all nodes that branch from a node have been evaluated, the originating node can be evaluated. If the originating node is a decision node, choose the branch with the best PW or EUAC and place that value in the node. If the originating node is a chance node, calculate the expected value and place that value in the node. This process “rolls back” values from the terminal nodes in the tree to the initial decision. Example 10–8 illustrates this process.

EXAMPLE 10-8

What decision should be made on the new product summarized in Figure 10–3? What is the expected value of the product's PW? The firm uses an interest rate of 10% to evaluate projects. If the product is terminated after one year, the capital equipment has a salvage value of $550,000 for use with other new products. If the equipment is used for 8 years, the salvage value is $0.

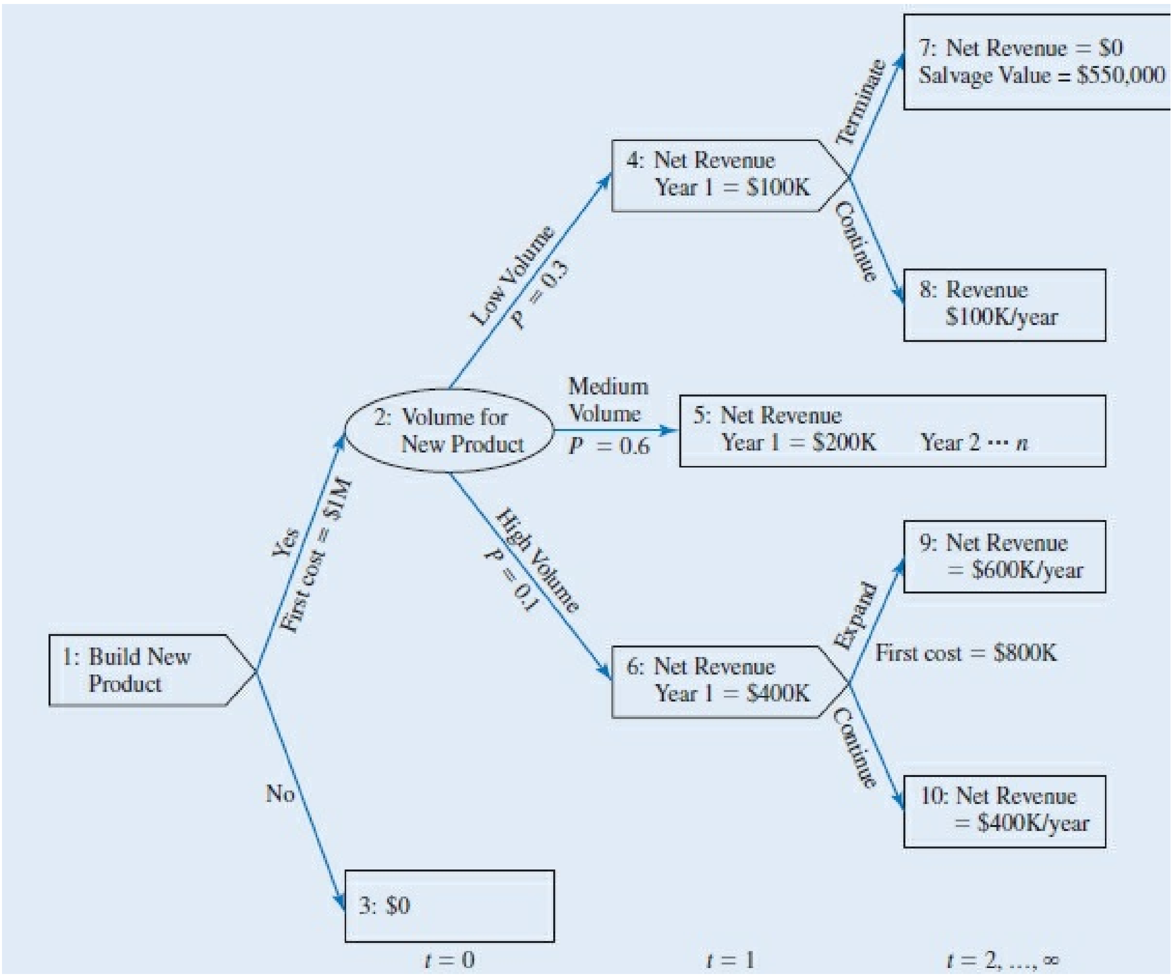


FIGURE 10-3 Economic decision tree for new product.

SOLUTION

Evaluating decision trees is done by starting with the end outcome nodes and the decisions that lead to them. In this case the decisions are whether to terminate after 1 year if sales volume is low and whether to expand after 1 year if sales volume is high.

The decision to terminate the product depends on which is more valuable, the equipment's salvage value of \$550,000 or the revenue of \$100,000 per year for 7 more years. The worth (PW_1) of the salvage value is \$550,000. The worth (PW_1) of the revenue stream at the end of Year 1 shown in node 8 is

$$\begin{aligned} PW_1 \text{ for node } 8 &= 100{,}000(P/A, 10\%, 7) \\ &= 100{,}000(4.868) = \$486{,}800 \end{aligned}$$

Thus, terminating the product and using the equipment for other products is better. We enter the two "present worth" values at the end of Year 1 in nodes 7 and 8. We make the *arc to node 7 bold* to indicate that it is our preferred choice at node 4. We use a *double hash mark* to show that we're *pruning the arc to node 8* to indicate that it has been rejected as an inferior choice at node 4.

The decision to expand at node 6 could be based on whether the \$800,000 first cost for expansion can be justified based on increasing annual revenues for 7 years by \$200,000 per year. However, this is difficult to show on the tree. It is easier to calculate the "present worth" values at the end of Year 1 for each of the two choices. The worth (PW_1) of node 9 (expand) is

$$\begin{aligned} PW_1 \text{ for node } 9 &= -800{,}000 + 600{,}000(P/A, 10\%, 7) \\ &= -800{,}000 + 600{,}000(4.868) \\ &= \$2{,}120{,}800 \end{aligned}$$

The value of node 10 (continue without expanding) is

$$\begin{aligned} PW_1 \text{ for node } 10 &= 400{,}000(P/A, 10\%, 7) \\ &= 400{,}000(4.868) \\ &= \$1{,}947{,}200 \end{aligned}$$

This is \$173,600 less than the expansion node, so the expansion should happen if volume is high. Figure 10–4 summarizes what we know at this stage of the process.

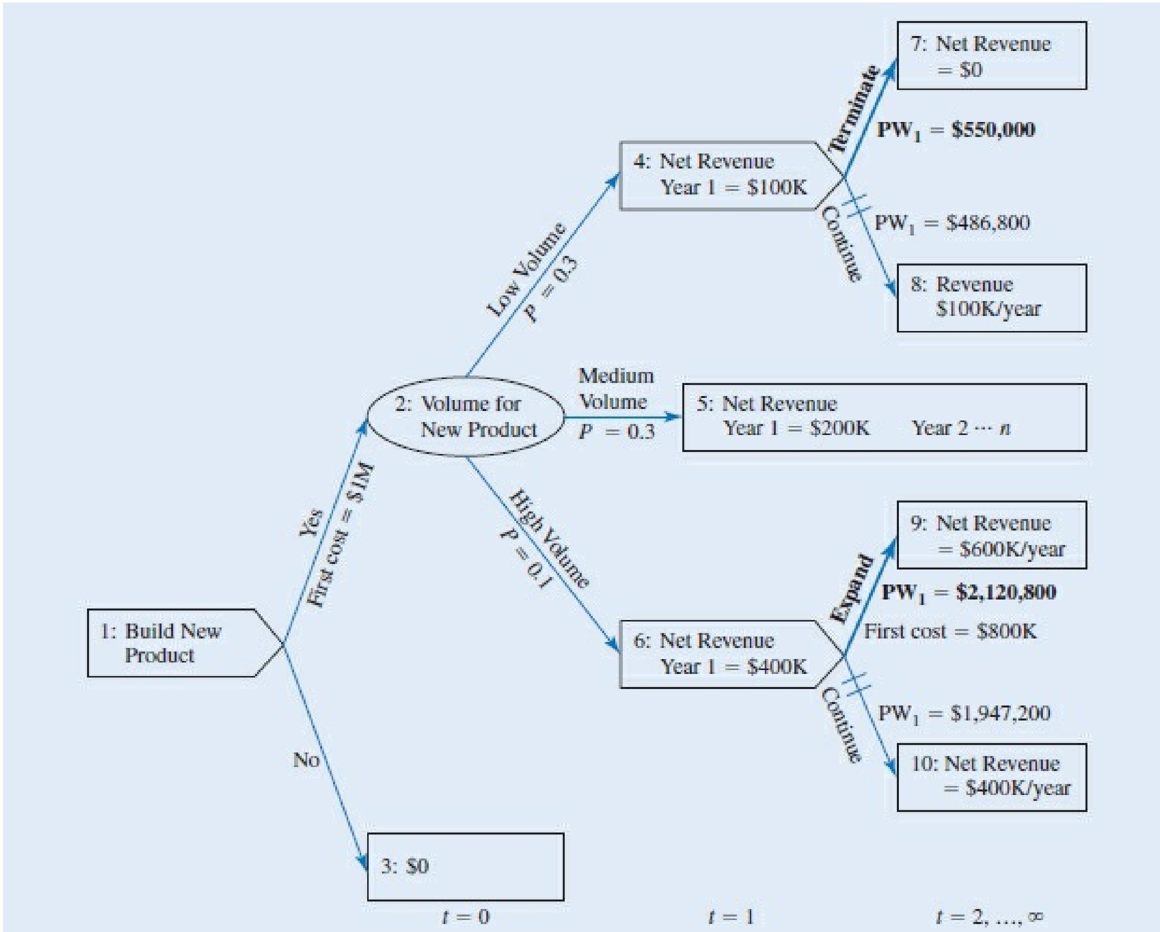


FIGURE 10-4 Partially solved decision tree for new product.

The next step is to calculate the PW (Time 0) at nodes 4, 5, and 6.

$$\text{PW at node 4} = (100{,}000 + 550{,}000)(P/F, 10\%, 1) = 650{,}000(0.9091) = \$590{,}915$$

$$\text{PW at node 5} = (200{,}000)(P/A, 10\%, 8) = 200{,}000(5.335) = \$1{,}067{,}000$$

$$\begin{aligned}\text{PW at node 6} &= [400{,}000 - 800{,}000 + 600{,}000(P/A, 10\%, 7)]\,(P/F, 10\%, 1)\\ &= [-400{,}000 + 600{,}000(4.868)](0.9091) = \$2{,}291{,}660\end{aligned}$$

Now the expected value at node 2 can be calculated:

$$\text{EV at node 2} = 0.3(590{,}915) + 0.6(1{,}067{,}000) + 0.1(2{,}291{,}660) = \$1{,}046{,}640$$

The cost of selecting node 2 is $1,000,000, so proceeding with the product has an expected PW of $46,640. This is greater than the $0 for not building the project. So the decision is to build. Figure 10–5 is the decision tree at the final stage.

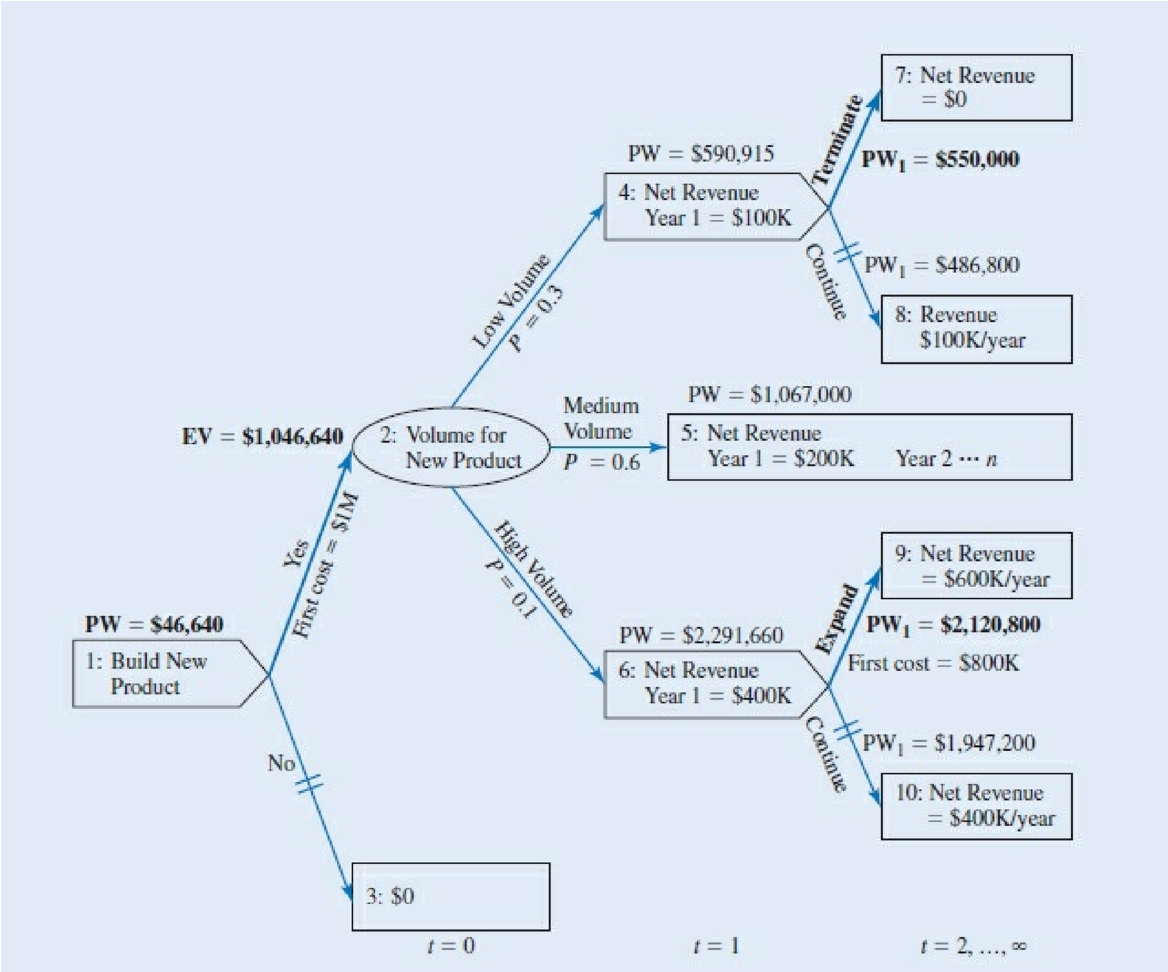


FIGURE 10-5 Solved decision tree for new product.

Example 10–8 is representative of many problems in engineering economy. The main criterion is maximizing PW or minimizing EUAC. However, as shown in Example 10–9, other criteria, such as risk, are used in addition to expected value.

EXAMPLE 10-9

Consider the economic evaluation of collision and comprehensive (fire, theft, etc.) insurance for a car. This insurance is typically required by lenders, but once the car has been paid for, this insurance is not required. (Liability insurance *is* a legal requirement.)

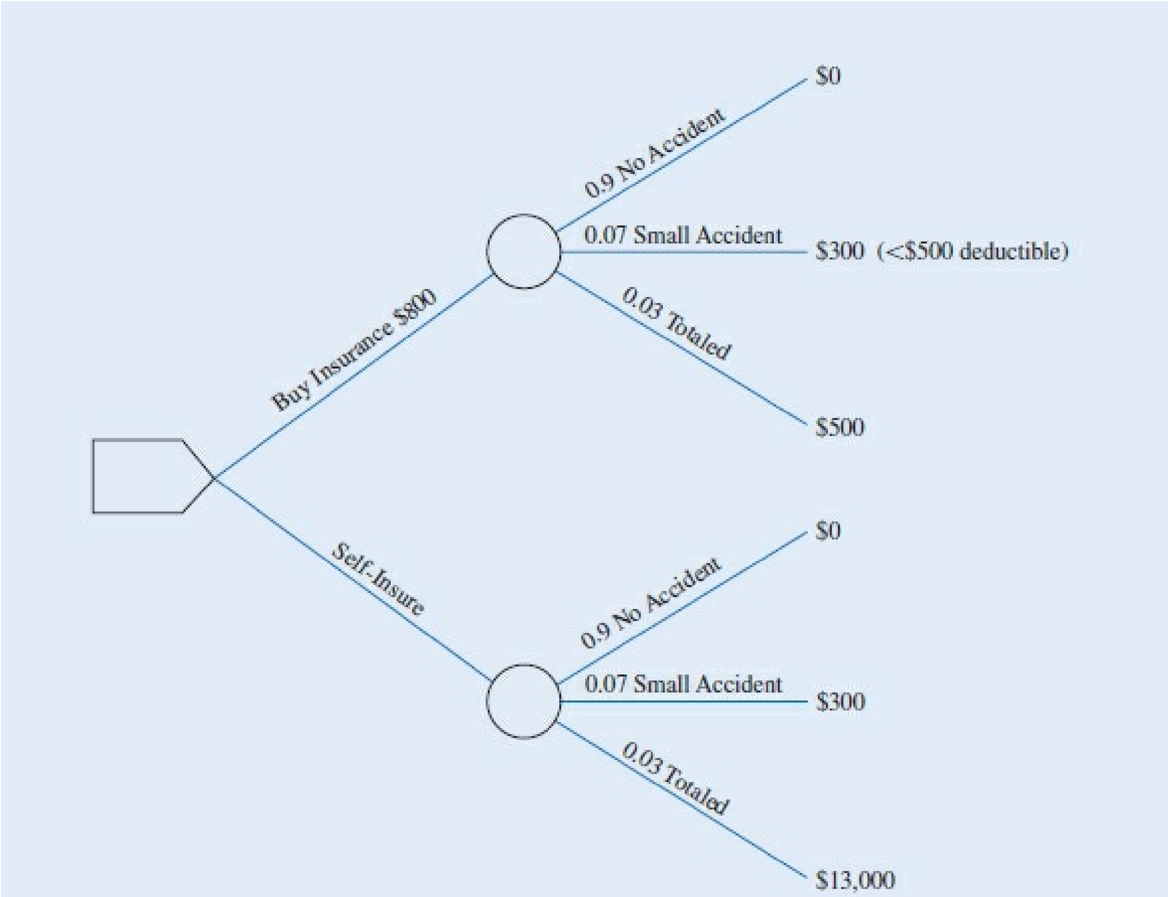


FIGURE 10-6 Decision tree for buying auto collision insurance.

Figure 10–6 begins with a decision node with two alternatives for the next year. Insurance will cost $800 per year with a $500 deductible if a loss occurs. The other option is to self-insure, which means to go without buying collision and comprehensive insurance. Then if a loss occurs, the owner must replace the vehicle with money from savings or a loan, or do without a vehicle until the owner can afford to replace it.

Three accident severities are used to represent the range of possibilities: a 90% chance of no accident, a 7% chance of a small accident (at a cost of $300, which is less than the deductible), and a 3% chance of totaling the $13,000 vehicle. Since our driving habits are likely to be the same with and without insurance, the accident probabilities are the same for both chance nodes.

Even though this is a text on engineering economy, we have simplified the problem and ignored the difference in timing of the cash flows. Insurance payments are made at the beginning of the covered period, and accident costs occur during the covered period. Since car insurance is usually paid semiannually, the results of the economic analysis are not changed significantly by the simplification. We focus on the new concepts of expected value, economic decision trees, and risk.

What are the expected values for each alternative, and what decision is recommended?

SOLUTION

The expected values are computed by using Equation 10-5. If insured, the maximum cost equals the deductible of \$500. If self-insured, the cost is the cost of the accident.

$$EV_{\text{accident w/ins.}} = (0.9)(0) + (0.07)(300) + (0.03)(500) = \$36$$
$$EV_{\text{accident w/o ins.}} = (0.9)(0) + (0.07)(300) + (0.03)(13{,}000) = \$411$$

Thus, buying insurance lowers the expected cost of an accident by \$375. To evaluate whether we should buy insurance, we must also account for the cost of the insurance. Thus, these expected costs are combined with the \$0 for self-insuring (total \$411) and the \$800 for insuring (total \$836). Thus self-insuring has an expected value cost that is \$425 less per year (= \$836 − \$411). This is not surprising, since the premiums collected must cover both the costs of operating the insurance company and the expected value of the payouts.

This is also an example of *expected values alone not determining the decision*. Buying insurance has an expected cost that is \$425 per year higher, but that insurance limits the maximum loss to \$500 rather than \$13,000. The \$425 may be worth spending to avoid that risk.

RISK

Risk can be thought of as the chance of getting an outcome other than the expected value—with an emphasis on something negative. One common measure of risk is the probability of a loss (see Example 10–4). The other common measure is the **standard deviation** (σ), which measures the dispersion of outcomes about the expected value. For example, many students have used the normal distribution in other classes. The normal distribution has 68% of its probable outcomes within ± 1 standard deviation of the mean and 95% within ± 2 standard deviations of the mean.

Appendix 9A introduced risk as measured by the standard deviation. In this section the standard deviation is calculated. The next section will cover considering risk and return together. Appendix 10A shows how diversification reduces risk, with a focus on investing for retirement. Risk reduction through diversification is one reason that firms form portfolios of projects, products, and business lines.

Calculating the Standard Deviation

Mathematically, the standard deviation is defined as the square root of the variance. This term is defined as the weighted average of the squared difference between the outcomes of the random variable *X* and its mean. Thus the larger the difference between the mean and the values, the larger are the standard deviation and the variance. This is Equation 10-6:

$$\text{Standard deviation } (\sigma) = \sqrt{[\text{EV}(X - \text{mean})^2]} \qquad (10\text{-}6)$$

Squaring the differences between individual outcomes and the EV ensures that positive and negative deviations receive positive weights. Consequently, negative values for the standard deviation are impossible, and they instantly indicate arithmetic mistakes. The standard deviation equals 0 if only one outcome is possible. Otherwise, the standard deviation is positive.

This is not the standard deviation formula built into most calculators, just as the weighted average is not the simple average built into most calculators. The calculator formulas are for *N* equally likely data points from a randomly drawn sample, so that each probability is 1/*N*. In economic analysis we will use a weighted average for the squared deviations since the outcomes may not be equally likely.

The second difference is that for calculations (by hand or the calculator), it is easier to use Equation 10-7, which is shown to be equivalent to Equation 10-6 in introductory probability and statistics texts.

$$\text{Standard deviation } (\sigma) = \sqrt{\{\text{EV}(X^2) - [\text{EV}(X)]^2\}} \qquad (10\text{-}7)$$

$$= \sqrt{\{\text{Outcome}_A^2 \times P(\text{A}) + \text{Outcome}_B^2 \times P(\text{B}) + \cdots - \text{expected value}^2\}} \qquad (10\text{-}7')$$

This equation is the square root of the difference between the average of the squares and the square of the average. The standard deviation is used instead of the *variance* because the standard deviation is measured in the same units as the expected value. The variance is measured in “squared dollars”—whatever they are.

The calculation of a standard deviation by itself is only a descriptive statistic of limited value. However, as shown in the next section on risk/return trade-offs, it is useful when the standard deviation of each alternative is calculated and these results are compared. But first, some examples of calculating the standard deviation.

EXAMPLE 10-10 (Example 10–9 continued)

Consider the economic evaluation of collision and comprehensive (fire, theft, etc.) insurance for an auto. One example was described in Figure 10–6. The probabilities and outcomes are summarized in the calculation of the expected values, which was done using Equation 10–5.

$$\text{EV}_{\text{accident w/ins.}} = (0.9)(0) + (0.07)(300) + (0.03)(500) = \$36$$
$$\text{EV}_{\text{accident w/o ins.}} = (0.9)(0) + (0.07)(300) + (0.03)(13{,}000) = \$411$$

Calculate the standard deviations for insuring and not insuring.

SOLUTION

The first step is to calculate the EV(outcome^2) for each.

$$\text{EV}^2_{\text{accident w/ins.}} = (0.9)(0^2) + (0.07)(300^2) + (0.03)(500^2) = 13{,}800$$
$$\text{EV}^2_{\text{accident w/o ins.}} = (0.9)(0^2) + (0.07)(300^2) + (0.03)(13{,}000^2) = 5{,}076{,}300$$

Then the standard deviations can be calculated.

$$\begin{aligned}\sigma_{\text{w/ins.}} &= \sqrt{\text{EV}^2_{\text{w/ins.}} - (\text{EV}_{\text{w/ins.}})^2} \\ &= \sqrt{(13{,}800 - 36^2)} = \sqrt{12{,}504} = \$112 \\ \sigma_{\text{w/o ins.}} &= \sqrt{\text{EV}^2_{\text{w/o ins.}} - (\text{EV}_{\text{w/o ins.}})^2} \\ &= \sqrt{(5{,}076{,}300 - 411^2)} = \sqrt{4{,}907{,}379} = \$2215\end{aligned}$$

As described in Example 10–9, the expected value cost of insuring is \$836 (= \$36 + \$800) and the expected value cost of self-insuring is \$411. Thus the expected cost of not insuring is about half the cost of insuring. But the standard deviation of self-insuring is 20 times larger. It is clearly riskier.

Which choice is preferred depends on how much risk one is comfortable with.

As stated before, this is an example of *expected values alone not determining the decision*. Buying insurance has an expected cost that is \$425 per year higher, but that insurance limits the maximum loss to \$500 rather than \$13,000. The \$425 may be worth spending to avoid that risk.

EXAMPLE 10-11 (Example 10–4 continued)

Using the probability distribution for the PW from Example 10–4, calculate the PW's standard deviation.

SOLUTION

The following table adds a column for $PW^2 \times$Probability to calculate the $EV(PW^2)$.

Annual Benefit	Probability	Life (years)	Probability	Joint Probability	PW	PW× Probability	$PW^2\times$ Probabil
$ 5,000	30%	6	66.7%	20.0%	−$ 3,224	−$ 645	2,079,48
8,000	60	6	66.7	40.0	9,842	3937	38,747,9
10,000	10	6	66.7	6.7	18,553	1237	22,950,0
5,000	30	9	33.3	10.0	3,795	380	1,442,10
8,000	60	9	33.3	20.0	21,072	4214	88,797,4
10,000	10	9	33.3	3.3	32,590	1086	35,392,7

EV=$10,209 189,409,

$$\text{Standard deviation} = \sqrt{\{\text{EV}(X^2) - [\text{EV}(X)]^2\}}$$

$$\sigma = \sqrt{\{189{,}405{,}745 - [10{,}209]^2\}} = \sqrt{85{,}182{,}064} = \$9229$$

For those with stronger backgrounds in probability than this chapter assumes, let us consider how the standard deviation in Example 10–11 depends on the assumption of independence between the variables. While exceptions exist, a positive statistical dependence between variables often increases the PW's standard deviation. Similarly, a negative statistical dependence between variables often decreases the PW's standard deviation.

RISK VERSUS RETURN

A graph of risk versus return is one way to consider these items together. Figure 10–7 in Example 10–12 illustrates the most common format. Risk measured by standard deviation is placed on the *x* axis, and return measured by expected value is placed on the *y* axis. This is usually done with internal rates of return of alternatives or projects.

EXAMPLE 10-12

A large firm is discontinuing an older product; thus some facilities are becoming available for other uses. The following table summarizes eight new projects that would use the facilities. Considering expected return and risk, which projects are good candidates? The firm believes it can earn 4% on a risk-free investment in government securities (labeled as Project *F*).

Project	IRR	Standard Deviation
1	13.1%	6.5%

2	12.0	3.9
3	7.5	1.5
4	6.5	3.5
5	9.4	8.0
6	16.3	10.0
7	15.1	7.0
8	15.3	9.4
F	4.0	0.0

SOLUTION

Answering the question is far easier if we use Figure 10–7. Since a larger expected return is better, we want to select projects that are as "high up" as possible. Since a lower risk is better, we want to select projects that are as "far left" as possible. The graph lets us examine the trade-off of accepting more risk for a higher return.

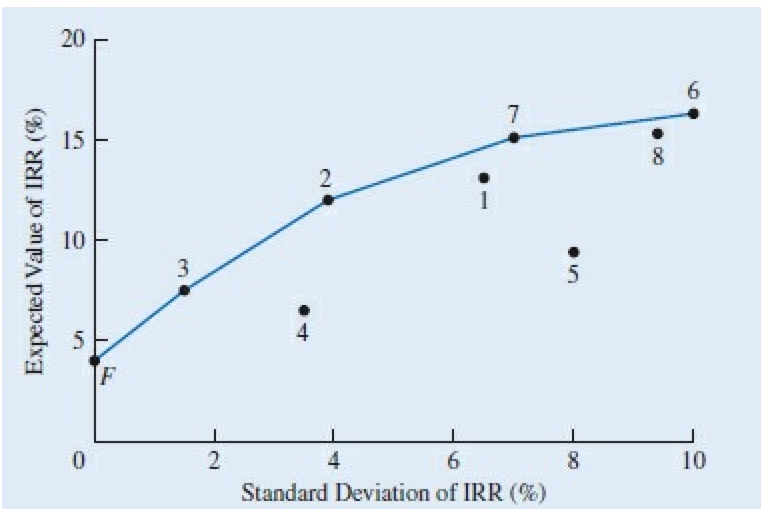


FIGURE 10-7 Risk-versus-return graph.

First, we can eliminate Projects 4 and 5. They are **dominated projects.** Dominated alternatives are no better than another alternative on all measures and inferior on at least one measure. Project 4 is dominated by Project 3, which has a higher expected return and a lower risk. Project 5 is dominated by Projects 1, 2, and 7. All three have a higher expected return and a lower risk.

Second, we look at the **efficient frontier.** This is the blue line in Figure 10–7 that connects Projects *F*, 3, 2, 7, and 6. Depending on the trade-off that we want to make between risk and return, any of these could be the best choice.

Project 1 appears to be inferior to Projects 2 and 7. Project 8 appears to be inferior to Projects 7 and 6. Projects 1 and 8 are inside and not on the efficient frontier.

There are models of risk and return that can allow us to choose between Projects *F*, 3, 2, 7, and 6; but those models are beyond what is covered here.

There is a simple rule of thumb for comparing a project's risk and return for which we would like to thank Joe Hartman. If the expected present worth is at least double the standard deviation of the present worth, then the project is relatively *safe*. For comparison, remember that a normal distribution has about 2.5% of its values less than 2 standard deviations below the mean.

Risk and Return as Multiple Objectives

To combine the effects of risk and return a single measure can be created by using the additive multiple objective model introduced in Chapter 1. Scores are calculated on a scale of 0 to 10 (worst to best). Since higher IRR values are better, Equation 10-8 is used for return. Since lower standard deviations are better, Equation 10-9 is used for risk. After scoring the alternatives, the next step is to estimate reasonable weights that add to 100% for the objectives, in this case return and risk. Then a total weighted score for each alternative can be calculated.

$$\text{When higher values are better} \qquad \text{Score} = (\text{Value} - \text{minimum}) \times 10/\text{range} \tag{10-8}$$

$$\text{When lower values are better} \qquad \text{Score} = 10 - (\text{Value} - \text{minimum}) \times 10/\text{range} \tag{10-9}$$

For examples with many alternatives like Example 10–13, the maximum and minimum values are calculated. If there are only a few alternatives, it is probably better to estimate theoretical or reasonably achievable maximums and minimums. Otherwise many, most, or even all scores will be 0 or 10.

EXAMPLE 10-13 (Example 10–12 Revisited)

For the projects in Example 10–12, apply a weighted multiple objective model using a weight of 60% for the IRR and a weight of 40% for the standard deviation. Which project is the best for this model?

SOLUTION

Whether by hand or spreadsheet, the first step is to calculate the minimum value, the maximum value, and the range of values for both the IRR and standard deviation data.

	IRR	Standard Deviation
Maximum	16.3%	10.0%
Minimum	4.0	0.0
Range	12.3	10.0

The solution will be by spreadsheet, but it is best to start with a hand calculation to ensure understanding and as a check on the later calculations. What are the scores for Project 1?

$$\begin{aligned}
\text{IRR}_1 &= (\text{Value} - \text{minimum}) \times 10/\text{range} && \text{using (10-8)}\\
&= (13.1\% - 4.0\%) \times 10/12.3\% = 7.4 &&\\
\text{Standard deviation}_1 &= 10 - (\text{Value} - \text{minimum}) \times 10/\text{range} && \text{using (10-9)}\\
&= 10 - (6.5\% - 0.0\%) \times 10/10.0\% = 3.5 &&\\
\text{Score}_1 &= 60\% \times 7.4 + 40\% \times 3.5 = 5.8 &&
\end{aligned}$$

It is worth noting that the three highest-scoring projects, 7, 2, and 6, are all on the efficient frontier in Figure 10–7. However, projects 1 and 8, which are not on the efficient frontier, have a weighted score higher than Project 3, which is on the efficient frontier.

	A	B	C	D	E	F
1				60%	40%	Weight
2	Project	IRR	Std.Dev.	IRR score	SD score	Total
3	1	13.1%	6.5%	7.4	3.5	5.84
4	2	12.0%	3.9%	6.5	6.1	6.34
5	3	7.5%	1.5%	2.9	8.6	5.14
6	4	6.5%	3.5%	2.0	6.5	3.82
7	5	9.4%	8.0%	4.4	2.0	3.43
8	6	16.3%	10.0%	10.0	0.0	6.00
9	7	15.1%	7.0%	9.0	3.0	6.61
10	8	15.3%	9.4%	9.2	0.6	5.75
11	F	4.0%	0.0%	0.0	10.0	4.00
12	min	4.0%	0.0%			
13	max	16.3%	10.0%			
14	range	12.3%	10.0%			

analysis is another approach for evaluating projects

SIMULATION

Simulation is a more advanced approach to considering risk in engineering economy problems. As such, the following discussion focuses on what it is. As the examples show, spreadsheet functions and add-in packages make simulation easier to use for economic analysis.

Economic **simulation** uses random sampling from the probability distributions of one or more variables to analyze an economic model for many iterations. For each iteration, all variables with a probability distribution are randomly sampled. These values are used to calculate the PW, IRR, or EUAC. Then the results of all iterations are combined to create a probability distribution for the PW, IRR, or EUAC.

Simulation can be done by hand, using a table of random numbers—if there are only a few random variables and iterations. However, results are more reliable as the number of iterations increases, so in practice this is usually computerized. This can be done in Excel using the RAND() function to generate random numbers, as shown in Example 10–14.

Because we were analyzing each possible outcome, the probability distributions earlier in this chapter (and in the end-of-chapter problems) used two or three discrete outcomes. This limited the number of combinations that we needed to consider. Simulation makes it easy to use continuous probability distributions like the uniform, normal, exponential, log normal, binomial, and triangular. Examples 10–14 and 10–15 use the normal and the discrete uniform distributions.

EXAMPLE 10-14

ShipM4U is considering installing a new, more accurate scale, which will reduce the error in computing postage charges and save $250 a year. The scale's useful life is believed to be uniformly distributed over 12, 13, 14, 15, and 16 years. The initial cost of the scale is estimated to be normally distributed with a mean of $1500 and a standard deviation of $150.

Use Excel to simulate 25 random samples of the problem and compute the rate of return for each sample. Construct a graph of rate of return versus frequency of occurrence.

SOLUTION

	A	B	C	D
1	250	Annual Savings		
2		Life	First Cost	
3	Min	12	1500	Mean
4	Max	16	150	Std dev
5				
6	Iteration	Life	First Cost	IRR
7	1	12	1277	16.4%
8	2	15	1546	13.9%
9	3	12	1523	12.4%
10	4	16	1628	13.3%
11	5	14	1401	15.5%
12	6	12	1341	15.2%
13	7	12	1683	10.2%
14	8	14	1193	19.2%
15	9	15	1728	11.7%
16	10	12	1500	12.7%
17	11	16	1415	16.0%
18	12	12	1610	11.2%
19	13	15	1434	15.4%
20	14	12	1335	15.4%
21	15	14	1468	14.5%
22	16	13	1469	13.9%
23	17	14	1409	15.3%
24	18	15	1484	14.7%
25	19	14	1594	12.8%
26	20	15	1342	16.8%
27	21	14	1309	17.0%
28	22	12	1541	12.1%
29	23	16	1564	14.0%
30	24	13	1590	12.2%
31	25	16	1311	17.7%
32				
33	Mean	14	1468	14.4%
34	Std dev	2	135	2.2%

FIGURE 10-8 Excel spreadsheet for simulation (N = 25).

Excel equations:

In B7:B31	Life = \$B\$3 + INT(RAND() * 5)
In C7:C31	Initial Cost = NORMINV(RAND(),\$C\$3, \$C\$4)
In D7:D31	IRR = RATE(Life, \$A\$1, -First Cost)

This problem is simple enough to allow us to construct a table with each iteration's values of the life and the first cost. From these values and the annual savings of $250, the IRR for each iteration can be calculated using the RATE function. These are shown in Figure 10–8. The IRR values are summarized in a relative frequency diagram in Figure 10–9.

Note: Each time Excel recalculates the spreadsheet, different values for all the random numbers are generated. Thus the results depend on the set of random numbers, and your results will be different if you create this spreadsheet.

Note for students who have had a course in probability and statistics: Creating the random values for life and first cost is done as follows. Select a random number in [0, 1] using Excel's RAND function. This is the value of the cumulative distribution function for the variable. Convert this to the variable's value by using an inverse function from Excel, or build the inverse function. For the discrete uniform life, the function is = min life + INT(range*RAND()). For the normally distributed first cost, the function is = NORMINV(RAND(), mean, standard deviation).

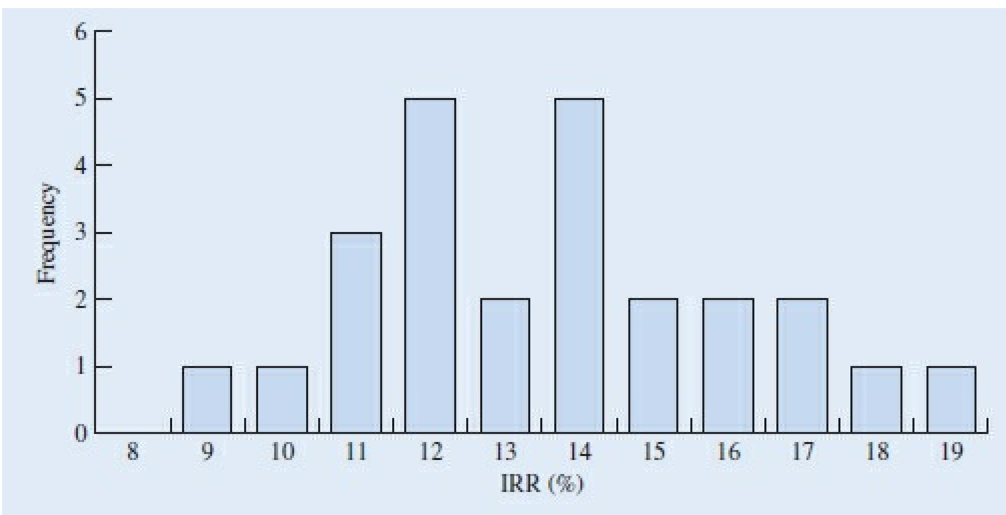


FIGURE 10-9 Graph of IRR values.

Stand-alone simulation programs and commercial spreadsheet add-in packages such as @Risk and Crystal Ball provide probability distribution functions to use for each input variable. In Example 10–15 the functions

RiskUniform and RiskNormal are used. The packages also collect values for the output variables, such as the IRR for Example 10–15. In other problems the PW or EUAC could be collected. These values form a probability distribution for the PW, IRR, or EUAC. From this distribution the simulation package can calculate the expected return, *P*(loss), and the standard deviation of the return.

Example 10–15 uses @Risk to simulate 1000 iterations of PW for the data in Example 10–14. A simulation package makes it easy to do more iterations. More important, accurate models can be built because it is much easier to use different probability distributions and parameters. Because the models are easier to build, they are less likely to contain errors.

EXAMPLE 10-15 (Example 10–14 Revisited)

Consider the scale described in Example 10–14. Generate 1000 iterations and construct a frequency distribution for the scale's rate of return.

SOLUTION

The first IRR (cell A7) of 14.01% that is computed in Figure 10–10 is based on the average life and the average first cost. The second IRR (cell A11) of 14.01% is computed by @Risk using the average of each distribution. The cell content in A11 is the RATE formula, referencing cells that are input cells to the simulation.

There are two input cells that contain variables that will change with each iteration. Cell A9 is the value of life, which uses a uniform distribution between 12 and 16, showing a static value of 14. Cell A10 is the first cost, using a normal distribution with a mean of −1500 and a standard deviation of 150, showing a static value of −1500. @Risk will use the static values to compute the value in the output cell (A11). Performing a simulation of 1000 iterations provides the graph, showing the distribution of the IRR. The graph will change each time the simulation is performed. The graph in Figure 10–

10 with 1000 iterations is much smoother than Figure 10–9, the graph from Example 10–14, where 25 iterations were done.

	A	B	C	D	E	F
1	−1500	average first cost				
2	150	standard deviation of first cost				
3	12	minimum value of life				
4	16	maximum value of life				
5	250	annual benefit				
6	IRR computed using averages					
7	14.01%	= RATE ((A3+A4)/2,A5,A1)				
8	Simulation					
9	14	=RiskUniform(12,16,RiskStatic(14))				
10	−1500	=RiskNormal(-1500,150,RiskStatic(-1500))				
11	14.01%	=RiskOutput()+RATE(A9,A5,A10)				

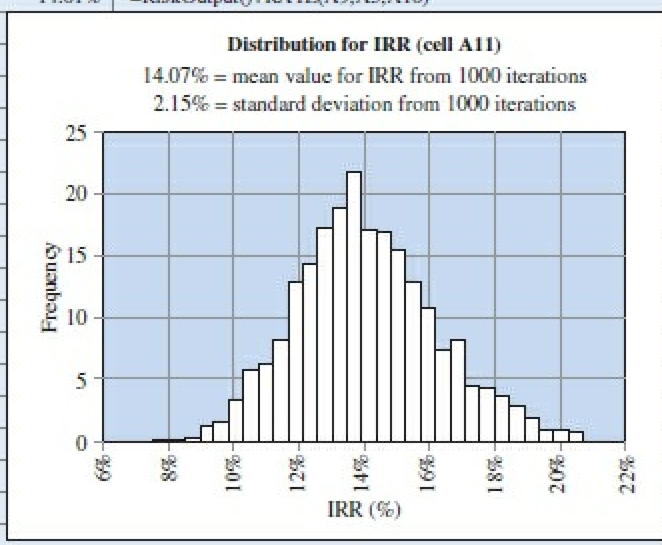


FIGURE 10-10 Simulation spreadsheet for Examples 10–14 and 10–15.

REAL OPTIONS

Real options analysis is another approach for evaluating projects with significant future uncertainties. It is derived from the theory underlying financial options in the stock market. Rather than buying or selling a share of stock, a financial option gives you the right to buy or sell the stock at a given price for a given time period. Since the option costs less than the stock, a $1000 investment in options has more potential gain than the same investment in the stock. It also has more risk. For engineering projects, a real option is an alternative that places a value on the ability to delay or change the decision in the future.

For an intuitive understanding of why real options make sense, consider the example of a lease for gas and oil development rights. Many such projects are never developed because they are not “economic.” Nevertheless, it may make sense to renew the lease for a noneconomic project because of the possibility that the project will *become* economically profitable. For example, prices may rise, or pipelines and facilities may be built to support a nearby tract. The lease is an option to pursue the project in the future.

The models and assumptions of real options are beyond the scope of this text. Interested readers are referred to the third edition of the advanced text *Economic Analysis of Industrial Projects* by Eschenbach, Lewis, Hartman, and Bussey.

These options may include delaying, abandoning, expanding, shrinking, changing, and replicating a project—with different models for different actions. However, some suggested guidelines may help you decide when you should consider applying real options to your project analysis.

• The value of exercising an option cannot be negative, since you can choose not to exercise it.
• An option to delay a project is valuable when the project is not currently economic but may become economic. Thus, if a project’s present worth is positive or very negative, a delay option is unlikely to be worthwhile.
• Unlike projects, options become more valuable as risk (described as volatility) increases. For projects, we want more return but less risk. However, options are more likely to be worthwhile and to result in action if the risk increases.
• Volatility is the key parameter in real option valuation, but it is difficult to calculate. In addition, much of the risk associated with a project may not be *actionable* volatility, and only actionable volatility can create value for a real option [Lewis, Eschenbach, and Hartman, 2008].
• Many published examples of real option analysis ignore the cost of waiting, but this is rarely appropriate for engineering projects [Eschenbach, Lewis, and Hartman, 2009]. For example, if an R&D project, a new product, or a

new building is delayed, then it will later face more competition and during the period of delay there are no revenues or cost savings since the project is not yet operating.

• In many cases decision trees and simulation are required to properly develop and describe the real option, and by then the real option analysis may add little value to the decision-making process.

SUMMARY

Estimating the future is required for economic analysis, and there are several ways to do this. Precise estimates will not ordinarily be exactly correct, but they are considered to be the best single values to represent what we think will happen.

A simple way to represent uncertainty is through a range of estimates for each variable, such as optimistic, most likely, and pessimistic. The full range of prospective results may be examined by using the optimistic values to solve the problem and then using the pessimistic values. Solving the problem with the most likely values is a good single value estimate. However, the extremes with all optimistic values or all pessimistic values are less likely—it is more likely that a mix of optimistic, most likely, and pessimistic values will occur.

One approach taken from project management uses the following weighted values instead of a range of estimates.

Estimate	Optimistic	Most Likely	Pessimistic
Relative weight	1	4	1

The most commonly used approach for decision making relies on **expected values.** Here, known or estimated probabilities for future events are used as weights for the corresponding outcomes.

$$\text{Expected value} = \text{Outcome}_A \times \text{Probability}_A + \text{Outcome}_B \times \text{Probability}_B + \cdots$$

Expected value is the most useful and the most frequently used technique for estimating a project's attractiveness.

However, risk as measured by standard deviation and the probability of a loss is also important in evaluating projects. Since projects with higher expected returns also frequently have higher risk, evaluating the trade-offs between risk and return is useful in decision making.

More complicated problems can be summarized and analyzed by using decision trees, which allow logical evaluation of problems with sequential chance, decision, and outcome nodes.

Where the elements of an economic analysis are stated in terms of probability distributions, a repetitive analysis of a random sample is often done. This simulation-based approach relies on the premise that a random sampling of increasing size becomes a better and better estimate of the possible outcomes. The large number of computations means that simulation is usually computerized.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

10-1	Replacement equipment is being purchased for $40,000, and is expected to reduce costs by 10,000 per year. Estimates of the expected life range from optimistic (12 years) to pessimistic (4 years), with a most likely value of 5 years. There is no salvage value. Determine the optimistic, pessimistic, and most likely rates of return.

SOLUTION

Annual savings in labor costs due to an automation project have a most

10-2 likely value of $35,000. The optimistic value is $45,000 and the pessimistic value is 30,000. What is the expected value for the annual savings?

SOLUTION

10-3 An outdoor construction project is to be completed in the fall. Records show that the chance of sunny weather is 25%, and the chance of early snow is 30%. With sunny weather, the project will cost $22,500. With early snow, the project will cost $30,000. If the weather is moderate, the cost will be $25,000. What is the probability distribution for the construction cost?

SOLUTION

10-4 Light rail service between Springfield and Old Saybrook is being considered and will likely cost $900,000 per kilometer. Costs are not certain, and there is a 30% probability that costs will be 40% higher. There is a 20% probability that costs could be 25% lower. What is the probability distribution for the cost per kilometer?

SOLUTION

10-5 Moore Science uses a discount rate of 15% to evaluate projects. Should the following project be undertaken if its life is 10 years and it has no salvage value?

First Cost	P	Net Revenue	P
$200,000	0.1	$70,000	0.3
300,000	0.6	80,000	0.6
500,000	0.3	100,000	0.1

SOLUTION

Tee-to-Green Golf, Inc., is considering the purchase of new automated club assembly equipment. The industrial engineer for TGG thinks that

10-6 she has determined the “best” choice. However, she is uncertain how to evaluate the equipment because of questions concerning the actual annual savings and salvage value at the end of the expected life. The equipment will cost $500,000 and is expected to last for 8 years. The engineer has the following information concerning the savings and salvage value estimates and the projected probabilities.

	p = .20	*p* = .50	*p* = .25	*p* = .05
Savings per year	$65,000	$82,000	$90,000	$105,000
Salvage value	40,000	55,000	65,000	75,000

Determine the NPW if the MARR is 6%.

SOLUTION

10-7 Acme Insurance offers an insurance policy that pays $1000 in reimbursement for luggage lost on a cruise. Historically the company pays this amount in 1 out of every 200 policies it sells. What is the minimum amount Acme must charge for such a policy if the company’s goal is to make at least $10 dollars per policy?

SOLUTION

10-8 A roulette wheel consists of 18 black slots, 18 red slots, and 2 green slots. If a $100 bet is placed on black, what is the expected gain or loss? (A bet on black or red pays even money.)

SOLUTION

10-9 Krispy Kookies is considering the purchase of new dough-mixing equipment. From the estimated NPW and probabilities of the four possible outcomes given, calculate the expected annual worth of the equipment if the life of the equipment is 6 years and i = 8%.

Outcome	NPW	Probability
1	$34,560	.15

2	38,760	.25
3	42,790	.40
4	52,330	.20

SOLUTION

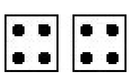
10-10 The probability that a machine will last a certain number of years is tabulated as follows.

Years of Life	Probability of Obtaining Life
10	.15
11	.20
12	.25
13	.20
14	.15
15	.05

What is the expected life of the machine?

SOLUTION

10-11 In the game of craps, two dice are tossed. One of the many bets available is the "Hard-way 8." A $1 bet will return to the bettor $4 if in the game the two dice come up 4 and 4 prior to one of the other ways of totaling eight. For the $1 bet, what is the expected result?



SOLUTION

Consolidated Edison Power is evaluating the construction of a new electric generation facility. The two choices are a coal-burning plant (CB) and a gaseous diffusion (GD) plant. The CB plant will cost $160 per megawatt to construct, and the GD plant will cost $180 per megawatt. Owing to uncertainties concerning fuel availability and the impact of future regulations related to air and water quality, the useful life of each plant is unknown, but the following probability estimates have been made.

10-12

	Probability	
Useful Life (years)	CB Plant	GD Plant
10	.10	.05
20	.50	.25
30	.30	.50
40	.10	.20

a. Determine the expected life of each plant.
b. Based on the ratio of construction cost per megawatt to expected life, which plant would you recommend that Con Ed build?

SOLUTION

Crush Cola Company must purchase a bottle-capping machine. The following is known about the machine and about possible cash flows.

10-13

	p = .30	p = .50	p = .20
First cost	$40,000	$40,000	$40,000
Annual savings	2,000	3,500	5,000
Annual costs	7,000	5,000	4,000
Actual salvage value	4,000	5,000	6,500

The machine is expected to have a useful life of 10 years. Crush has a MARR of 6%. Determine the NPW of the machine.

SOLUTION

10-14 The two finalists in a tennis tournament are playing for the championship. The winner will receive $60,000 and the runner-up $35,000. Determine the expected winnings for each participant if the players are considered to be evenly matched. What would the expected winnings be if one player were favored by 4-to-1 odds?

SOLUTION

A new heat exchanger must be installed by CSI, Inc. Alternative *A* has an initial cost of $33,400, and Alternative *B* has an initial cost of $47,500. Both alternatives are expected to last 10 for years. The annual cost of operating the heat exchanger depends on ambient temperature in the plant and on energy costs. The estimate of the cost and probabilities for each alternative is given. If CSI has a MARR of 8% and uses rate of return analysis for all capital decisions, which exchanger should be purchased?

10-15

	Annual Cost	Probability, *p*
Alternative *A*	$4500	.10
	7000	.60
	8000	.25
	9250	.05
Alternative *B*	$4000	.20
	5275	.60
	6450	.15
	8500	.05

SOLUTION

A dam is being considered to reduce river flooding in the Forked River Basin. Information concerning the possible alternatives is given.

10-16

Dam Height, *H* (ft)	First Cost	Annual Probability of Flood if Height =	Damages if Flooding Occurs
0	$ 0	0.25	$800,000
20	700,000	0.05	500,000
30	800,000	0.01	300,000
40	900,000	0.002	200,000

Which dam height minimizes the expected total annual cost? The state uses an interest rate of 5% for flood protection projects, and all dams must last 50 years.

SOLUTION

A new product’s chief uncertainty is its annual net revenue. Money has been spent, but an additional $30,000 is required to license a patent. The firm’s interest rate is 10%. What is the expected PW and standard deviation for deciding whether to proceed?

10-17

		State	
	Bad	OK	Great
Probability	0.4	0.5	0.1
Net revenue	−$3,000	$10,000	$25,000
Life &years)	5	5	10

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Range of Estimates

Telephone poles exemplify items that have varying useful lives. Telephone poles, once installed in a location, remain in useful service until one of a variety of events occur.
(a) Name three reasons why a telephone pole might be removed from useful service at a particular location.
10- (b) You are to estimate the total useful life of telephone poles. If the pole

1 G is removed from an original location while it is still serviceable, it will be installed elsewhere. Estimate the optimistic life, most likely life, and pessimistic life for telephone poles. What percentage of all telephone poles would you expect to have a total useful life greater than your estimated optimistic life?

(c) What is an environmental life cycle assessment (LCA)? How do treated wood, metal, and concrete poles compare?

10-2 A The purchase of a used pickup for $22,000 is being considered. Records for other vehicles show that costs for oil, tires, and repairs about equal the cost for fuel.

Fuel costs are $1950 per year if the truck is driven 12,500 miles. The salvage value after 6 years of use drops about 15¢ per mile. Find the equivalent uniform annual cost if the interest rate is 5%. How much does this change if the annual mileage is 15,000? 10,000?

10-3 For the data in Problem 10-2 assume that the 10,000, 12,500, and 15,000 mileage values are, respectively, pessimistic, most likely, and optimistic estimates. Use a weighted estimate to calculate the equivalent annual cost.

10-4 A G A heat exchanger is being installed as part of a plant modernization program. It costs $80,000, including installation, and is expected to reduce the overall plant fuel cost by $20,000 per year. Estimates of the useful life of the heat exchanger range from an optimistic 12 years to a pessimistic 4 years. The most likely value is 5 years. Assume the heat exchanger has no salvage value at the end of its useful life.

(*a*) Determine the pessimistic, most likely, and optimistic rates of return.

(*b*) Use the range of estimates to compute the mean life and determine the estimated before-tax rate of return.

10-5 A new engineer is evaluating whether to use a higher-voltage transmission line. It will cost $250,000 more initially, but it will reduce transmission losses. The optimistic, most likely, and pessimistic projections for annual savings are $25,000, $20,000, and $13,000. The interest rate is 6%, and the transmission line should have a life of 30 years.

(*a*) What is the present worth for each estimated value?

(*b*) Use the range of estimates to compute the mean annual savings, and then determine the present worth.

(*c*) Does the answer to *(b)* match the present worth for the most likely value? Why or why not?

10-6 A A new 2-lane road is needed in a part of town that is growing. At some point the road will need 4 lanes to handle the anticipated traffic. If the city's optimistic estimate of growth is used, the expansion will be needed in 3 years. For the most likely and pessimistic estimates, the expansion will be needed in 6 and 10 years, respectively. The expansion will cost $4.9 million. Use an interest rate of 7%.

(a) What is the PW for each scenario, and what is the range of values?

(b) Use Equation 10-1 to find the mean value of the expansion's PW.

Probabilities

10-7 When a pair of dice is tossed, the results may be any whole number from 2 through 12. In the game of craps one can win by tossing either a 7 or an 11 on the first roll. What is the probability of doing this? (*Hint:* There are 36 ways that a pair of six-sided dice can be tossed. What portion of them result in either a 7 or an 11?)

10-8 A The construction time for a bridge depends on the weather. The project is expected to take 250 daysif the weather is dry and hot. If the weather is damp and cool, the project is expected to take 350 days. Otherwise, it is expected to take 300 days. Historical data suggest that the probability of cool, damp weather is 40% and that of dry, hot weather is 10%. Find the project's probability distribution.

10-9 Over the last 12 years, the hurdle or discount rate for projects from the firm's research and development division has been 15% three times, 18% five times, and 23% the rest of the time. There is no recognizable pattern. Calculate the probability distribution for next year's discount rate.

10-10 A A new product's sales and profits are uncertain. The marketing department has predicted that sales might be as high as 9000 units per year with a probability of 15%. The most likely value is 6000 units annually. The pessimistic value is estimated to be 3500 units annually with a probability of 25%. Manufacturing and marketing together have estimated the most likely unit profit to be $33. The pessimistic value of $26 has a probability of 0.25, and the optimistic value of $37 has a probability of 0.3. Construct the probability distributions for sales and unit profits.

10-11 A road between Fairbanks and Nome, Alaska, will have a most likely construction cost of $7 million per mile. Doubling this cost is considered to have a probability of 30%, and cutting it by 25% is considered to have a probability of 10%. The state's interest rate is 4%, and the road should last 25 years before major reconstruction. What is the probability distribution of the equivalent annual construction cost per mile?

10-12 A Al took a midterm examination in physics and received a score of 78. The mean was 65 and the standard deviation was 18. Bill received a score of 20 in mathematics, where the exam mean was 14 and the standard deviation was 12. Which student ranked higher in his class? Explain.

10-13 You recently had an auto accident that was your fault. If you have another accident or receive a another moving violation within the next 3 years, you will become part of the "assigned risk" pool, and you will pay an extra $1250 per year for insurance.
(a) If the probability of an accident or moving violation is 15% per year, what is the probability distribution of your "extra" insurance payments over the next 4 years? Assume that insurance is purchased annually and that violations register at the end of the year—just in time to affect next year's insurance premium.
(b) Would it be ethically questionable to pay out of your own pocket for a fender bender to avoid having it reported to your insurance company?

Joint Probabilities

10-14 A For the data in Problem 10-10, construct the probability distribution for the annual profit. Assume that the sales and unit profits are statistically independent.

A project has a life of 10 years, and no salvage value. The firm uses an interest rate of 12% to evaluate engineering projects. The project has an uncertain first cost and net revenue.

	First Cost	P	Net Revenue	P
	$300,000	0.2	$ 70,000	0.3
10-15	400,000	0.5	90,000	0.5
	600,000	0.3	100,000	0.2

(*a*) What is the joint probability distribution for first cost and net revenue?
(*b*) Define optimistic, most likely, and pessimistic scenarios by using both optimistic, both most likely, and both pessimistic estimates. What is the present worth for each scenario?

A robot has just been installed at a cost of $81,000. It will have no salvage value at the end of its useful life.

	Savings per Year	Probability	Useful Life (years)	Probability
	$18,000	0.2	12	1/6
10-16 A	20,000	0.7	5	2/3
	22,000	0.1	4	1/6

(*a*) What is the joint probability distribution for savings per year and useful life?
(*b*) Define optimistic, most likely, and pessimistic scenarios by using both optimistic, both most likely, and both pessimistic estimates. What is the rate of return for each scenario?

Modifying an assembly line has a first cost of $165,000, and its salvage value is $0. The firm's interest rate is 10%. The savings shown in the table depend on whether the assembly line runs one, two, or three shifts and on whether the product is made for 4 or 6 years.

10-17

Shifts/day	Savings/year	Probability	Useful Life (years)	Probability
1	$25,000	0.25	4	0.65
2	30,000	0.40	6	0.35
3	35,000	0.35		

(*a*) Give the joint probability distribution for savings per year and the useful life.
(*b*) Define optimistic, most likely, and pessimistic scenarios by using both optimistic, both most likely, and both pessimistic estimates. Use a life of 4 years as the most likely value. What is the present worth for each scenario?

Expected Value

10-18 A For the data in Problem 10-8, compute the project's expected completion time.

10-19 For the data in Problem 10-9, compute the expected value for the next year's discount rate.

A man wants to decide whether to invest $1000 in a friend's speculative venture. He will do so if he thinks he can get his money back in one year. He believes the probabilities of the various outcomes at the end of one year are as follows:

Result	Probability
$2000 (double his money)	0.3

10-20 A

1500	0.1
1000	0.2
500	0.3
0 (lose everything)	0.1

What would be his expected outcome if he invests the $1000?

10-21 For the data in Problem 10-11 calculate the expected value of the equivalent annual construction cost per mile.

10-22 A Two instructors announced that they "grade on the curve," that is, give a fixed percentage of each of the various letter grades to each of their classes. Their curves are as follows:

Grade	Instructor A	Instructor B
A	8%	10%
B	12	15
C	50	50
D	20	15
F	10	10

If a random student came to you and said that his object was to enroll in the class in which he could expect the higher grade point average, which instructor would you recommend?

10-23 G

Annual savings due to an energy efficiency project have a most likely value of $30,000. The high estimate of $40,000 has a probability of .25, and the low estimate of $20,000 has a probability of .35.

(a) What is the expected value for the annual savings?

(b) What types of tax incentives are available to firms for green projects?

10-24 A

In the New Jersey and Nevada gaming casinos, craps is a popular gambling game. One of the many bets available is the “Hard-way 8.” A $1 bet in this fashion will win the player $4 if in the game the pair of dice come up 4 and 4 before one of the other ways of totaling 8. For a $1 bet, what is the expected result?

10-25

A man went to Atlantic City with $2000 and placed 100 bets of $20 each, one after another, on the same number on the roulette wheel. There are 38 numbers on the wheel, and the gaming casino pays 35 times the amount bet if the ball drops into the bettor’s numbered slot in the roulette wheel. In addition, the bettor receives back the original $20 bet. Estimate how much money the man is expected to win or lose in Atlantic City.

10-26 A

Assume that the pessimistic and optimistic estimates in Problem 10-6 have 35% and 15% probabilities, respectively.

(*a*) What is the expected PW the expansion costs?

(*b*) What is the expected number of years until the expansion?

(*c*) What is PW of the expansion cost using the expected number of years until the expansion?

(*d*) Do your answers to (*a*) and (*c*) match? If not, why not?

10-27

For the data in Problems 10-2 and 10-3, assume that the optimistic probability is 20%, the most likely is 50%, and the pessimistic is 30%.

(*a*) What is the expected value of the equivalent uniform annual cost?

(*b*) Compute the expected value for the number of miles, and the corresponding equivalent uniform annual cost.

(*c*) Do the answers to (*a*) and (*b*) match? Why or why not?

10-28 A

If your interest rate is 7%, what is the expected value of the present worth of the “extra” insurance payments in Problem 10-13?

10-29 For the data in Problem 10-5, assume that the optimistic probability is 20%, the most likely is 50%, and the pessimistic is 30%.
(*a*) What is the expected value of the present worth?
(*b*) Compute the expected value for annual savings, and the corresponding present worth.
(*c*) Do the answers to (*a*) and (*b*) match? Why or why not?

10-30 A

An industrial park is being planned for a tract of land near the river. To prevent flood damage to the industrial buildings that will be built on this low-lying land, an earthen embankment can be constructed. The height of the embankment will be determined by an economic analysis of the costs and benefits. The following data have been gathered.

Embankment Height Above Roadway (m)	Initial Cost
2.0	$100,000
2.5	165,000
3.0	300,000
3.5	400,000
4.0	550,000

Flood Level Above Roadway (m)	Average Frequency That Flood Level Will Exceed Height in Col. 1
2.0	Once in 3 years

2.5	Once in 8 years
3.0	Once in 25 years
3.5	Once in 50 years
4.0	Once in 100 years

The embankment can be expected to last 50 years and will require no maintenance. Whenever the flood water flows over the embankment, $300,000 of damage occurs. Should the embankment be built? If so, to which of the five heights above the roadway? A 12% rate of return is required.

10-31 G An energy efficiency project has a first cost of $400,000, a life of 10 years, and no salvage value. Assume that the interest rate is 10%. The most likely value for annual savings is $50,000. The optimistic value for annual savings is $80,000 with a probability of 0.2. The pessimistic value is $40,000 with a probability of 0.25.
(*a*) What is the expected annual savings and the expected PW?
(*b*) Compute the PW for the pessimistic, most likely, and optimistic estimates of the annual savings. What is the expected PW?
(*c*) Do the answers for the expected PW match? Why or why not?

The MSU football team has 10 games scheduled for next season. The business manager wishes to estimate how much money the team can be expected to have left over after paying the season's expenses, including any postseason "bowl game" expenses. From records for the past season and estimates by informed people, the business manager has assembled the following data:

Situation	Probability Situation	Net Income
Regular season	Regular season	

	Win 3 games	0.10	Win 5 or	$250,000
	Win 4 games	0.15	fewer	
	Win 5 games	0.20	games	
10-32 A	Win 6 games	0.15	Win 6 to 8	ß400,000
	Win 7 games	0.15	games	
	Win 8 games	0.10		
	Win 9 games	0.07	Win 9 or 10	ß600,000
	Win 10 games	0.03	games	
	Postseason		Postseason	Additional
	Bowl game	0.10	Bowl game	income of
				$100,000

What is the expected net income for the team next season?

10-33 For the data in Problem 10-4, assume that the optimistic probability is 15%, the most likely is 80%, and the pessimistic is 5%.

(*a*) What is the expected value of the rate of return?

(*b*) Compute the expected value for the life, and the corresponding rate of return.
(*c*) Do the answers to (*a*) and (*b*) match? Why or why not?

EV Joint Distribution

10-34 A	For the data in Problem 10-10, calculate the expected value of sales and unit profits. For the data in Problem 10-14, calculate the expected value of annual profit. Are these results consistent?
10-35 G	The energy efficiency project described in Problem 10-23 has a first cost of $150,000, a life of 10 years, and no salvage value. Assume that the interest rate is 8%. (*a*) What is the equivalent uniform annual worth for the expected annual savings? (*b*) Compute the equivalent uniform annual worth for the pessimistic, most likely, and optimistic estimates of the annual savings. What is the expected value of the equivalent uniform annual worth? (*c*) Do the answers to (*a*) and (*b*) match? Why or why not?
10-36 A	Should the project in Problem 10-15 be undertaken if the firm uses an expected value of present worth to evaluate engineering projects? (*a*) Compute the PW for each combination of first cost and revenue and the corresponding expected worth. (*b*) What are the expected first cost, expected net revenue, and corresponding present worth of the expected values? (*c*) Do the answers for (*a*) and (*b*) match? Why or why not?
10-37	A new engineer is evaluating whether to use a larger-diameter pipe for a water line. It will cost $600,000 more initially, but it will reduce pumping costs. The optimistic, most likely, and pessimistic projections for annual savings are $80,000, $50,000, and $7500, with respective probabilities of 25%, 45%, and 30%. The interest rate is 8%, and the water line should have a life of 50 years. (*a*) What is the PW for each estimated value? What is the expected PW? (*b*) Compute the expected annual savings and expected PW. (*c*) Do the answers for the expected PW match? Why or why not?

10-38 A

For the data in Problem 10-16:
(*a*) What are the expected savings per year, life, and corresponding rate of return for the expected values?
(*b*) Compute the rate of return for each combination of savings per year and life. What is the expected rate of return?
(*c*) Do the answers for (*a*) and (*b*) match? Why or why not?

10-39

For the data in Problem 10-17:
(*a*) What are the expected savings per year, life, and corresponding present worth for the expected values?
(*b*) Compute the present worth for each combination of savings per year and life. What is the expected present worth?
(*c*) Do the answers for (*a*) and (*b*) match? Why or why not?

Decision Trees

10-40 A

The tree in Figure P10-40 has probabilities after each chance node and values for each terminal node. What decision should be made? What is expected value?

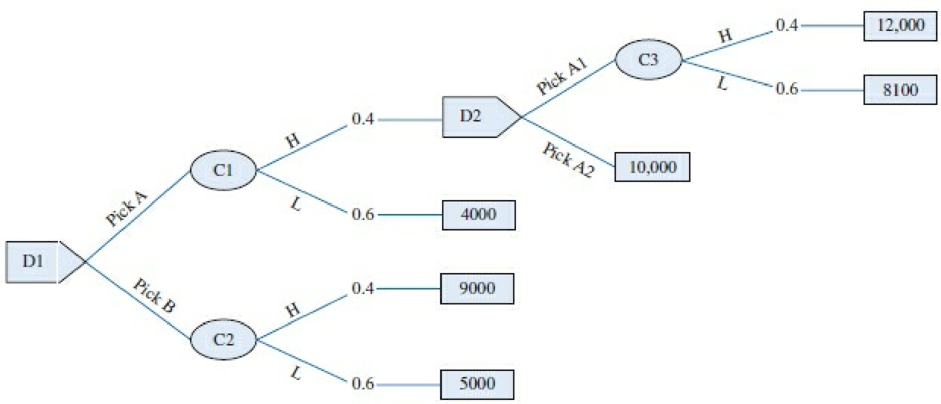


FIGURE P10-40

The tree in Figure P10-41 has probabilities after each chance node and values for each terminal node. What decision should be made? What is expected value?

10-41

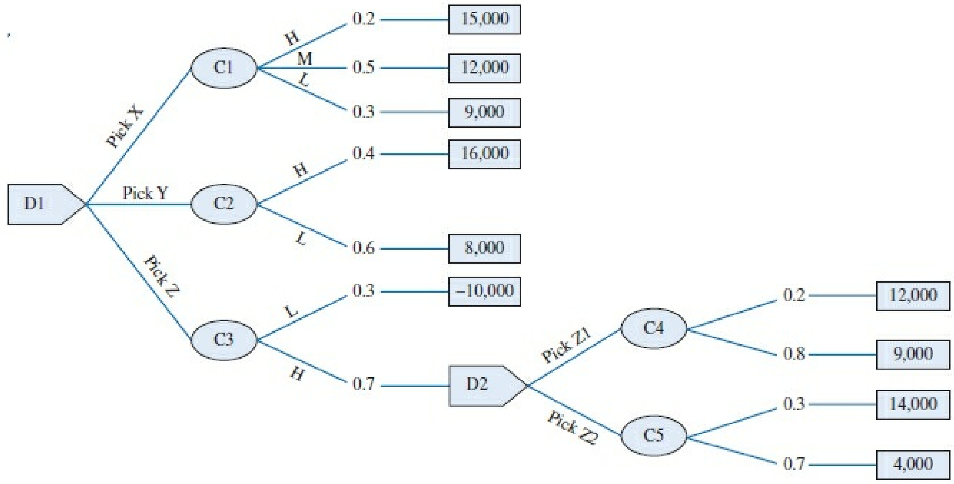


FIGURE P10-41

10-42 [A]

A decision has been made to perform certain repairs on the outlet work a small dam. For a particular 36-inch gate valve, there are three availab alternatives:

A. Leave the valve as it is.

B. Repair the valve.

C. Replace the valve.

If the valve is left as it is, the probability of a failure of the valve seats, over the life of the project, is 60%; the probability of failure of the valv stem is 50%; and of failure of the valve body is 40%.

If the valve is repaired, the probability of a failure of the seats, over the of the project, is 40%; of failure of the stem is 30%; and of failure of th body is 20%. If the valve is replaced, the probability of a failure of the seats, over the life of the project, is 30%; of failure of the stem is 20%; of failure of the body is 10%.

The present worth of cost of future repairs and service disruption of a failure of the seats is $10,000; the present worth of cost of a failure of stem is $20,000; the present worth of cost of a failure of the body is $30,000. The cost of repairing the valve now is $10,000; and of replac it is $20,000. If the criterion is to minimize expected costs, which alternative is best?

A factory building is located in an area subject to occasional flooding b nearby river. You have been brought in as a consultant to determine whether flood-proofing of the building is economically justified. The alternatives are as follows:

A. Do nothing. Damage in a moderate flood is $10,000 and in a severe flood, $25,000.

10-43 *B.* Alter the factory building at a cost of $15,000 to withstand moderat flooding without damage and to withstand severe flooding with $10,0C damages.

C. Alter the factory building at a cost of $20,000 to withstand a severe flood without damage.

In any year the probability of flooding is as follows: 0.70, no flooding the river; 0.20, moderate flooding; and 0.10, severe flooding. If interes 15% and a 15-year analysis period is used, what do you recommend?

10-44 A G Five years ago a dam was constructed to impound irrigation water and provide flood protection for the area below the dam. Last winter a 100- flood caused extensive damage both to the dam and to the surrounding area. This was not surprising, since the dam was designed for a 50-yea flood.

The cost to repair the dam now will be $250,000. Damage in the valle below amounts to $750,000. If the spillway is redesigned at a cost of $250,000 and the dam is repaired for another $250,000, the dam may b expected to withstand a 100-year flood without sustaining damage. However, the storage capacity of the dam will not be increased and the probability of damage to the surrounding area below the dam will be unchanged. A second dam can be constructed up the river from the exi dam for $1 million. The capacity of the second dam would be more th adequate to provide the desired flood protection. If the second dam is b redesign of the existing dam spillway will not be necessary, but the $250,000 of repairs must be done.

The development in the area below the dam is expected to be complet 10 years. A new 100-year flood in the meantime would cause a $1 mil loss. After 10 years the loss would be $2 million. In addition, there wo be $250,000 of spillway damage if the spillway is not redesigned. A 5(year flood is also likely to cause about $200,000 of damage, but the spillway would be adequate. Similarly, a 25-year flood would cause ab $50,000 of damage.

There are three alternatives: (1) repair the existing dam for $250,000 b make no other alterations, (2) repair the existing dam ($250,000) and redesign the spillway to take a 100-year flood ($250,000), and (3) repa the existing dam ($250,000) and build the second dam ($1 million). Ba on an expected annual cash flow analysis, and a 7% interest rate, whicl alternative should be selected? Draw a decision tree to clearly describe problem.

10-45 In Problems 10-17 and 10-39, how much is it worth to the firm to be al to extend the product's life by 3 years, at a cost of $50,000, at the end the product's initial useful life?

Risk

10-46 A

An engineer decided to make a careful analysis of the cost of fire insurance for his $200,000 home.
From a fire rating bureau he found the following risk of fire loss in any year.

Outcome	Probability
No fire loss	0.986
$ 10,000 fire loss	0.010
40,000 fire loss	0.003
200,000 fire loss	0.001

(*a*) Compute his expected fire loss in any year.
(*b*) He finds that the expected fire loss in any year is less than the $550 annual cost of fire insurance. In fact, an insurance agent explains that this is always true. Nevertheless, the engineer buys fire insurance. Explain why this is or is not a logical decision.

10-47 For the data in Problems 10-9 and 10-19, compute the standard deviation of the interest rate.

10-48 A The Graham Telephone Company may invest in new switching equipment. There are three possible outcomes, having net present worth of $6570, $8590, and $9730. The outcomes have probabilities of 0.3, 0.5, and 0.2, respectively. Calculate the expected return and risk measured by the standard deviation associated with this proposal.

10-49 For the data in Problems 10-11 and 10-21, compute the standard deviation of the equivalent annual cost per mile.

10-50 A What is your risk associated with Problem 10-28?

10-51 For the data in Problem 10-31, compute the standard deviation of the present worth.

10-52 A For the data in Problem 10-37, compute the standard deviation of the present worth.

10-53 A new machine will cost $50,000. The machine is expected to last 10 years and have no salvage value. If the interest rate is 15%, determine the return and the risk associated with the purchase.

P	0.3	0.4	0.3
Annual savings	$9000	$10,500	$12,000

10-54 A A new product's chief uncertainty is its annual net revenue. So far, $35,000 has been spent on development, but an additional $30,000 is required to finish development. The firm's interest rate is 10%.
(*a*) What is the expected PW for deciding whether to proceed?
(*b*) Find the *P*(loss) and the standard deviation for proceeding.

	State		
	Bad	OK	Great

Probability	0.3	0.5	0.2
Net revenue	−$15, 000	$15,000	$20,000
Life, in years	5	5	10

10-55 (*a*) In Problem 10-54, how much is it worth to the firm to terminate the product after 1 year if the net revenues are negative?
(*b*) How much does the ability to terminate early change the *P*(loss) and the standard deviation?

10-56 A Measure the risk for Problems 10-15 and 10-36 using the *P*(loss), range of PW values, and standard deviation of the PWs.

10-57 (*a*) In Problems 10-17 and 10-39, describe the risk using the *P*(loss) and standard deviation of the PWs.
(*b*) How much do the answers change if the possible life extension in Problem 10-45 is allowed?

Risk Versus Return

10-58 A A firm wants to select one new research and development project. The following table summarizes six possibilities. Considering expected return and risk, which projects are good candidates? The firm believes it can earn 5.5% on a risk-free investment in government securities (labeled as Project *F*).

Project	IRR	Standard Deviation
1	14.5%	7.0%
2	6.8	4.1
3	16.4	10.0
4	12.8	8.2
5	10.3	3.1
6	9.5	6.0
F	5.5	0.0

10-59 A firm is choosing a new product. The following table summarizes six new potential products. Considering expected return and risk, which products are good candidates? The firm believes it can earn 4% on a risk-free investment in government securities (labeled as Product *F*).

Product	IRR	Standard Deviation
1	10.4%	3.2%
2	9.8	2.3
3	6.0	1.6
4	12.1	3.6
5	12.2	8.0
6	13.8	6.5
F	4.0	0.0

Risk and Return as Multiple Objectives

10-60 Which project in Problem 10-58 should be selected if the rate of return has a weight of 65% and the standard deviation has a weight of 35%? How much difference in the weighted score is there between the best and the worst of projects 1 to 6?

10-61 Which project in Problem 10-58 should be selected if the rate of return has a weight of 35% and the standard deviation has a weight of 65%? How much difference in the weighted score is there between the best and the worst of projects 1 to 6?

10-62 Which product in Problem 10-59 should be selected if the rate of return has a weight of 75% and the standard deviation has a weight of 25%? How much difference in the weighted score is there between the best and the worst of projects 1 to 6?

10-63 Which product in Problem 10-59 should be selected if the rate of return has a weight of 30% and the standard deviation has a weight of 70%? How much difference in the weighted score is there between the best and the worst of projects 1 to 6?

Simulation

10-64 A	A project's first cost is $25,000, and it has no salvage value. The interest rate for evaluation is 7%. The project's life is from a discrete uniform distribution that takes on the values 7, 8, 9, and 10. The annual benefit is normally distributed with a mean of $4400 and a standard deviation of $1000. Using Excel's RAND function, simulate 25 iterations. What are the expected value and standard deviation of the present worth?
10-65	A factory's power bill is $55,000 a year. The first cost of a small geothermal power plant is normally distributed with a mean of $150,000 and a standard deviation of $50,000. The power plant has no salvage value. The interest rate for evaluation is 8%. The project's life is from a discrete uniform distribution that takes on the values 3, 4, 5, 6, and 7. (The life is relatively short due to corrosion.) The annual operating cost is expected to be about $10,000 per year. Using Excel's RAND function, simulate 25 iterations. What are the expected value and standard deviation of the present worth?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 13	**Guaranteed Return** Risk versus expected return and choice of interest rate.
CASE 18	**Gravity-Free High** New product with development stages and probability of failure.
CASE 19	**Crummy Castings** Decision tree problem with modest ambiguity. Options create more challenging problems.
CASE 21	**Glowing in the Dark** Expected return and variance for facility of minimal size, of minimal size plus preparation for later expansion, and of the full size now.
CASE 22	**City Car** Decision tree analysis. Includes discussion of strategy and risk. Some assumptions must be made.

CASE 23	**Washing Away** Levee height and probability of flood damage.
CASE 24	**Sinkemfast** Decision tree with assumptions required for realistic comparisons. Info supports creation of new, better alternatives.
CASE 39	**Uncertain Demand at WM** [3] Includes inflation, taxes, and uncertainty.
CASE 44	**Sunnyside—Up or Not?** Uncertain growth rates over 30 years and setting utility rates.
CASE 50	**Capital Planning Consultants** Capital budgeting including mutually exclusive alternatives. Includes uncertainties in first cost, annual benefit, and lives.

Appendix 10A Diversification Reduces Risk

Key Words

capital allocation line

diversification

PORTFOLIOS OF STOCKS AND BONDS

Appendix 9A described the expected returns and risks for stocks, treasury bonds, and treasury bills (T-bills). The returns listed were for portfolios of only stocks and portfolios of only bonds. Figure 10A–1 adds portfolios of combined stocks and bonds to Figure 9A–1. It shows that due to **diversification**, portfolios of bonds and stocks can offer better combinations of risk and return than either alone. We cannot draw a curve for more than two investments (funds or individual bonds and stocks), but the reduction of risk through diversification applies.

Table 10A–1 details the numbers, but this effect is easier to see in Figure

10A–1. For example, a portfolio that is invested 50% in each of stocks and bonds has the same risk (10.8%) as investing only in bonds. However, the 50/50 portfolio's expected return is 2.4% higher (= 8.6% − 6.2%).

The availability and performance of the 50/50 portfolio reduces the attractiveness of the all-stock portfolio. From Table 10A–1, the stock portfolio's risk is 6.7% (= 17.5% − 10.8%) higher, but its expected return is now only 2.4% higher than the 50/50 portfolio. The *efficient frontier* is the set of choices that should be considered. Other available choices are a worse combination of risk and return.

If portfolios of a low-risk stock and a higher-risk stock were formed, the performance would follow a similar curve. The math behind this is based on Equations 10A-1 and 10A-2. The expected return of a portfolio is simply a weighted average of the returns for the two choices. However, the standard deviation or risk of the portfolio is reduced, as long as the two choices are not perfectly correlated (ρ is the correlation coefficient in Equation 10A-2).

Expected return $$\mathrm{E}(R_P) = w_1 \mathrm{E}(R_1) + w_2 \mathrm{E}(R_2) \quad \text{(10A-1)}$$

Standard deviation $$\sigma_P = \sqrt{(w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2)} \quad \text{(10A-2)}$$

The equations can be generalized for portfolios of tens, hundreds, or even thousands of stocks. The reduction in risk through diversification is routine for large investors; thus an investor who buys only one or a few stocks has a much higher risk than the buyers and sellers that set the market price.

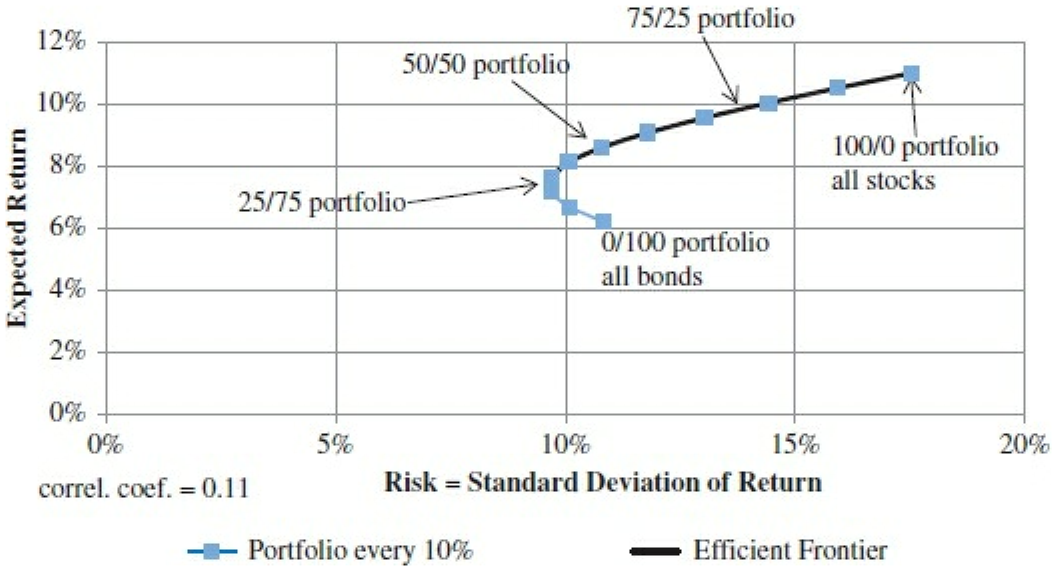


FIGURE 10A-1 Risk and return for portfolios of stocks and bonds.

Table 10A-1 Risk and Return Data for Portfolios of Stocks and Bonds

Stock	Treasury Bond	Portfolio Std Dev	Portfolio Return
100%	0%	17.5%	11.0%
75%	25%	13.7%	9.8%
50%	50%	10.8%	8.6%
25%	75%	9.6%	7.4%
0%	100%	10.8%	6.2%

The implication for individuals with a defined contribution retirement program is that the best strategy is to take advantage of all available company match dollars and invest in a diverse set of firms and bonds through an exchange traded fund (ETF) or a low-cost mutual fund. It is difficult for individuals to compete with institutional investors (who have millions to spend on how to invest billions). The key to long-term success seems to be keeping transaction costs down and investing regularly. Fortunately, ETFs offer the individual investor a way to hold very well diversified portfolios of stocks and/or bonds that are designed to match major market indices. We also note that these ETFs include returns to investors (dividends on stocks and interest on bonds) that go beyond the simple price indices. Both price and total return versions of the Dow Jones and the S&P 500 indices are available, but only total return versions should be used. We note that the S&P 500 and the Russell 1000 are generally regarded as better measures than the Dow Jones, as they include more firms and they weight them by their size, rather than simply averaging the stock market prices of 30 firms, as does the Dow Jones (with a divisor calculated to maintain historical continuity).

MAKING THE TRADE-OFF BETWEEN RISK AND RETURN

If U.S. Treasury bill data is added to Figure 10A–1, there is a more complete investment model with a *better* efficient frontier. T-bills typically have maturities of 3, 6, or 12 months. Because these are issued by the U.S. government and have short maturities, they are generally considered to be free of default risk. The new efficient frontier line in Figure 10A–2 connects the T-bill point with the point of tangency to the efficient frontier of the stock/bond portfolio. The point of tangency is typically near the 50/50 portfolio of stocks and bonds. While points of tangency and line slopes could be mathematically calculated, we suggest that the accuracy of the data is better matched to simply eyeballing the line as it is added. The efficient frontier is commonly called the **capital allocation line** because it describes the risk/return relationship that drives how capital is allocated. The risk and return combination that is preferred by an investor determines how that investor's capital should be allocated between T-bills, bonds, and stocks.

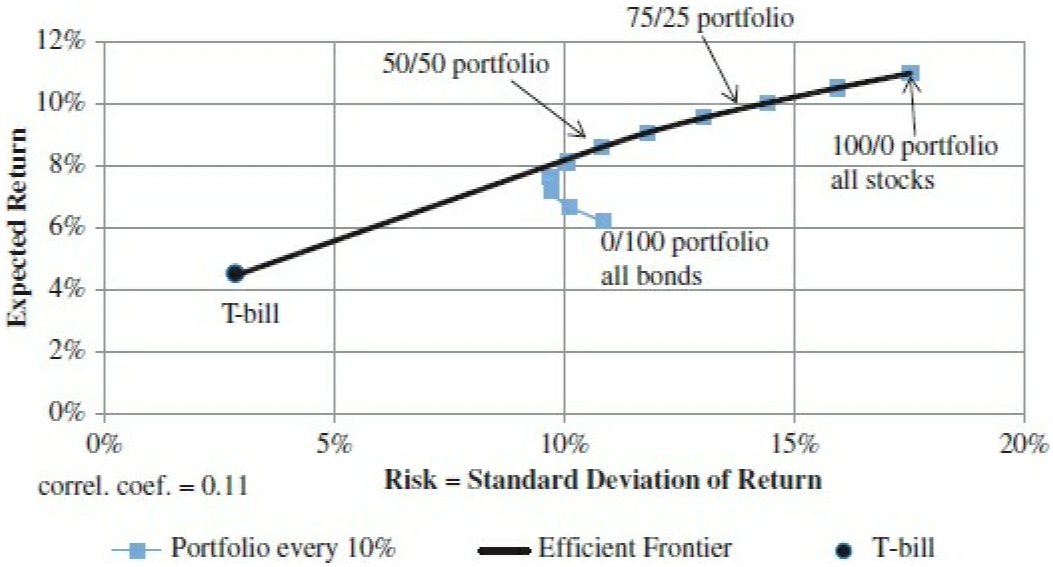


FIGURE 10A-2 Capital allocation line.

EXAMPLE 10A-1 (Example 9A–1 Revisited)

An engineer has just finished paying off a student loan and is ready to start saving for retirement. Her current annual salary is $63,000. She expects salary increases to exceed inflation, but to be safe she wants to assume that salary increases will be matched with inflation. She expects to work for another 35 years. If she invests 15% of her salary, how much can be expected to be in her account if she invests only in bonds or in a 50/50 portfolio of stocks and bonds?

SOLUTION

The first step is to recognize that both choices have a standard deviation of 10.8%—the same level of risk. Next we determine the expected returns or interest rates. Since this problem is in constant-value dollars, as is all of the text up to Chapter 14, the interest rates are the market rates minus inflation. Long-term treasury bonds have returned about 2.5% above the average inflation rate of 3.7% (Table 9A–1). For the 50/50 portfolio we must subtract average inflation from the 8.6% expected return shown in Tables A–1 and A–2. Thus the rate of return for the 50/50 portfolio is 4.9%.

To make the solution more flexible, the spreadsheet starts with a data block.

	A	B	C	D	E	F	G	H
1	$63,000	Salary						
2	15%	% saved						
3								
4	Exp. 10A-1	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
5	Bonds	2.5%	35	-9450	0		FV	$519,072
6	50/50	4.9%	35	-9450	0		FV	$836,054

Table 10A-2 Risk and Return Data for Capital Allocation Line

	Treasury		Portfolio	
T-bill	Stock	Bond	Std Dev	Return
	100%	0%	17.5%	11.0%
	90%	10%	15.9%	10.5%
	80%	20%	14.4%	10.0%
	70%	30%	13.0%	9.6%
	60%	40%	11.8%	9.1%

0%	50%	50%	10.8%	8.6%
20%	40%	40%	9.2%	7.8%
40%	30%	30%	7.6%	7.0%
60%	20%	20%	6.0%	6.2%
80%	10%	10%	4.4%	5.3%
100%	0%	0%	2.8%	4.5%

EXAMPLE 10A-2 (Example 10A–1 Revisited)

The engineer in Example 10A–1 has decided that a retirement goal of $1 million is adequate. What fraction of her salary must be saved if she invests only in bonds or in a 50/50 portfolio of stocks and bonds?

SOLUTION

The first step is to determine the annual deposits at the different interest rates. As in Example 10A–1, the interest rates are the market rates minus inflation: long-term treasury bonds about 2.5% above inflation and a 50/50 portfolio about 4.9% above inflation.

To make the solution more flexible, the spreadsheet starts with a data block.

	A	B	C	D	E	F	G	H	1
1	$63, 000	Salary							
2									
3	Exp. 10A-2	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	% salary
4	Bonds	2.5%	35		0	1.000.000	PMT	-$18,206	28.9%
5	50/50	4.9%	35		0	1.000.000	PMT	-$11,303	17.9%

An investor who chooses to invest safely before retirement should also invest safely after retirement. If the $1M is invested at a lower rate, then less can be drawn out each year to live on. It is common to suggest that *all* investors should invest more safely once they are approaching retirement or are already retired.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

An initial investment of $1000 in Hi-Tech Chain has annual returns that have varied a lot. For those returns find the arithmetic average and the geometric mean. Which one is the proper measure?

10A-1

Year	Annual Return
1	27%
2	-25%
3	30%
4	-33%
5	31%

SOLUTION

A 35-year-old engineer earning $98,000 per year wants to retire at age

10A-2 60 with $2.5 million. The engineer has nothing saved and expects to earn 9.6% annually on a 70% stock/30% bond portfolio? How much must be saved each year to reach the goal?

SOLUTION

10A-3 A 25-year old engineer wants to save $15,000 per year until retiring at 65. Her plan is that after 5 years she will spend the savings on buying a house and later savings will be for retirement. As her income increases she wants to spend more on herself and her family, which is why she chose a fixed dollar amount to save. Her investing mix will become more conservative over the years so she expects her investment returns to average 5.5% over inflation. How much will she have saved for the house? How much will she have saved at retirement?

SOLUTION

10A-4 An engineer nearing retirement at 66 with $750,000 invested is planning to shift to a portfolio of 20% T-bills, 40% Treasury bonds, and 40% stocks and expects to earn 4% over inflation. How much can the engineer withdraw each year if dying at 100 is the plan?

SOLUTION

10A-5 An engineer changed jobs and is signing up for benefits. The company 401(k) includes a low-cost fund that is expected to earn 5.3% annually. The engineer's employer will contribute up to 2% by matching half the employee contribution. So she will save at least 4% of her salary of $80,000 into the account. She expects her salary to increase 2.5% per year. What is the value of the account after 15 years if she deposits the 4% minimum?

SOLUTION

An engineer changed jobs and is signing up for benefits. The company 401(k) includes a low-cost fund that is expected to earn 5.3% annually.

10A-6 The engineer's employer will contribute up to 2% by matching half the employee contribution. What fraction of her salary (now $80,000) must she save if she wants $250,000 in the account in 15 years? She expects her salary to increase 2.5% per year.

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

As in the examples, assume constant value dollars and use real rates of return above inflation.

10A-1 A 25-year-old engineer earning $65,000 per year wants to retire at age 55 with $2 million, and plans to invest in a fund made up of 60% stocks and 40% bonds.
(a) How much money must be invested each year?
(b) If the employer does a 100% match of retirement savings up to 3% of the employee's salary, how much money must each invest annually?

10A-2 A An engineer earning $72,500 per year wants to retire in 30 years with $1.5 million and plans to invest in a fund containing 30% stocks, 30% bonds, and 40% treasury bills.
(a) How much money must be invested each year?
(b) If this person works 5 additional years, how much money must be invested each year?
(c) If the employer does a 100% match of retirement savings up to 3% of the employee's salary, and the engineer wants to retire in 30 years, how much money must each invest annually?

A new employee earning $60,000 annually set a retirement goal of $2 million. She plans to work for 40 years, and will invest in a fund having a portfolio of 80% stocks and 20% bonds.

10A-3

(a) What fraction of her salary must be saved?

(b) If the employer does a one-to-one match of retirement savings up to 3% of the employee's salary, what fraction of her salary must be saved?

10A-4 A

An employee earning $70,000 per year set a goal of retiring in 35 years with $1 million. She will invest in a fund investing 70% in stocks and 30% in treasury bonds.

(a) What fraction of her salary must be saved?

(b) If the employer does a one-to-one match of retirement savings up to 4% of the employee's salary, what fraction of her salary must be saved?

10A-5

A manager just retired at age 62, with his retirement savings of $800,000 invested in a portfolio of 90% stocks and 10% bonds. If he continues with this portfolio, and needs $85,000 per year for living expenses, how long will his savings last?

10A-6 A

A manager retired at age 65, and has her retirement savings of $650,000 invested in a mutual fund composed of 30% treasury bonds and 70% stocks. She needs $65,000 per year for living expenses, in addition to her social security benefit. How long will her investment last if there is no inflation?

10A-7

A new employee puts 4% of his salary of $60,000 into a retirement account, and his employer matches this, also putting 4% into the account. The money is invested into a diversified fund of 50% stocks and 50% bonds. His salary increases 3% per year.

(a) What is the value of the account after 10 years?

(b) What is the value of the employer's matching funds after 10 years?

(c) How much will be in the account after 40 years?

10A-8 A

An engineer changed jobs and is signing up for benefits. The company 401(k) includes a low-cost fund that invests 40% in stocks, 40% in bonds, and 20% in T-bills. The engineer will put 3% of her salary of $70,000 into the account, and her employer will match half this amount. Her salary is expected to increase 2.8% per year.

(a) What is the value of the account after 10 years?

	(b) If she expects to work for 30 years, how much will be in the account?
10A-9	Monica became concerned about the stock market due to recent losses, and moved $350,000 into a fund of 60% T-bills, 20% stocks, and 20% bonds. Her salary is currently $92,000 per year, and she expects raises of 2.0% each year. Her employer will match retirement savings up to 5% of her salary. If she wants $500,000 in her account in 5 years, what percentage of her salary does she need to save?
10A-10 A	Jorge was recently laid off, and may not be able to find another full-time job. Capital preservation is his primary concern, so he wants to invest in a portfolio of 60% treasury bills, 20% stocks, and 20% bonds. His retirement fund has $1.2 million. He can receive social security, so he will only need $85,000 per year from his savings to live well. Jorge is 64 years old. How long does he expect his savings to last? Should he look for a job or enjoy his retirement?
10A-11 E	Discuss the ethical considerations of a firm’s retirement savings plan. The firm will match retirement savings up to 10% of an employee’s salary, but only if the employee selects the option to invest in the firm’s stock. The employee’s contribution can be withdrawn or invested in another fund at any time. However, the matching funds have a 5-year vesting period, and the match goes away if the employee’s contribution is shifted out of the firm’s stock before vesting.

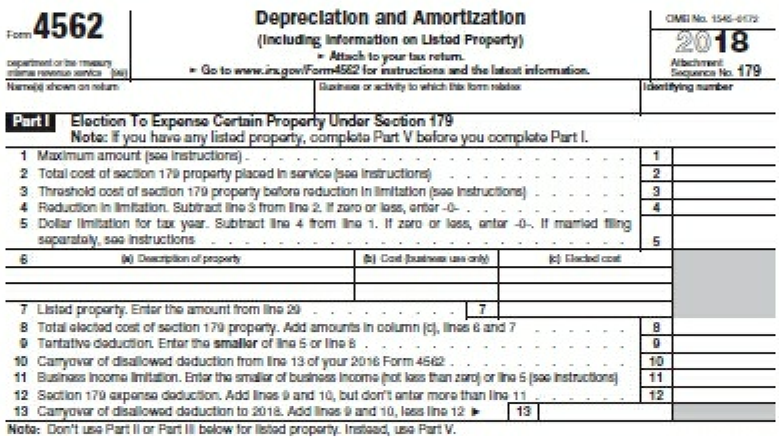
CHAPTER 11

DEPRECIATION

Depreciation and Intangible Property

The U.S. government allows firms to subtract many business expenses from their gross income in determining taxes due. This process is relatively straightforward for some expenses, such as labor and materials, which are "consumed" in the process of producing goods and services. This chapter on depreciation presents what happens when a business purchases a piece of durable equipment, such as a forklift, crane, or computer, which will be used over many years. This equipment is not directly consumed but does deteriorate with time and is clearly a business expense. The government has devised depreciation rules that allow firms to recoup durable equipment and other durable property value over time, much like other business expenses.

Firms can also apply depreciation to durable intangible assets, such as patents, trademarks, or even the estimated value of customer relationships. Like durable equipment, these types of durable intangible property provide value to the business over time, rather than being consumed during production, and also degrade in value or usefulness over time. For instance, patents and most customer relationships have a limited lifespan. While trademarks do not, the goods and services they are associated with are not generally expected to have indefinite appeal. When depreciation is applied to an intangible asset, this process is typically referred to as *amortization*.


Form 4562

Department of the Treasury Internal Revenue Service (99)

Depreciation and Amortization

(Including Information on Listed Property)

► Attach to your tax return.

► Go to www.irs.gov/Form4562 for instructions and the latest information.

OMB No. 1545-0172

2018

Attachment Sequence No. 179

Name(s) shown on return	Business or activity to which this form relates	Identifying number

Part I Election To Expense Certain Property Under Section 179

Note: If you have any listed property, complete Part V before you complete Part I.

1	Maximum amount (see instructions)	1	
2	Total cost of section 179 property placed in service (see instructions)	2	
3	Threshold cost of section 179 property before reduction in limitation (see instructions)	3	
4	Reduction in limitation. Subtract line 3 from line 2. If zero or less, enter -0-	4	
5	Dollar limitation for tax year. Subtract line 4 from line 1. If zero or less, enter -0-. If married filing separately, see instructions	5	

6	(a) Description of property	(b) Cost (business use only)	(c) Elected cost

7	Listed property. Enter the amount from line 29	7	
8	Total elected cost of section 179 property. Add amounts in column (c), lines 6 and 7	8	
9	Tentative deduction. Enter the **smaller** of line 5 or line 8	9	
10	Carryover of disallowed deduction from line 13 of your 2016 Form 4562	10	
11	Business income limitation. Enter the smaller of business income (not less than zero) or line 5 (see instructions)	11	
12	Section 179 expense deduction. Add lines 9 and 10, but don't enter more than line 11	12	
13	Carryover of disallowed deduction to 2018. Add lines 9 and 10, less line 12 ►	13	

Note: Don't use Part II or Part III below for listed property. Instead, use Part V.

While amortization of intangible assets may seem like it would be a minor concept for most businesses, the value of a Coca-Cola or Nike brand, a major drug or hardware patent, or the customer base of an acquired firm can be in the millions or billions of dollars. For instance, Apple and Samsung have been engaged in an ongoing and highly publicized multinational legal battle over patents, trademarks, and other intangible assets with damages sought totaling in the billions of dollars. While not every firm will have intangible assets worth quite this much, intangible assets are a critical property class in many firms. Recent data indicate that intangible property accounts for around 80% of the total market value of the “typical” U.S. firm. For example, intangible property comprised about 78% of the market value of Alphabet, Inc. (the parent company of Google) circa 2015. Thus, all firms should consider their intangible property in investment decisions, including correctly evaluating tax implications over time through the application of the appropriate amortization procedures.

Contributed by Jennifer A. Cross, Texas Tech University

QUESTIONS TO CONSIDER

1. Besides Apple and Samsung, what other examples of legal battles over intangible assets can you identify?
2. If you had to develop a method for amortizing an intangible asset such as a patent, how would you go about doing this? What sorts of parameters would you need to consider in developing this method?
3. Does the percentage of market value tied to intangible assets in U.S. firms surprise you? Why or why not? Do you think this percentage differs in other countries? Why or why not?

After Completing This Chapter…

The student should be able to:

• Distinguish between depreciation and a cash flow and between depreciation for taxes and for valuation.
• Distinguish various types of depreciable property and differentiate between depreciation expenses and other business expenses.
• Use classic depreciation methods to calculate the *annual depreciation charge* and *book value* over the asset's life.
• Fully account for *capital gains/losses, ordinary losses,* and *depreciation recapture* due to the disposal of a depreciated business asset.
• Use bonus depreciation, the modified accelerated cost recovery system (MACRS), and bonus plus MACRS to calculate allowable *annual depreciation charge* and *book value* over the asset's life for various cost bases, property classes, and recovery periods.
• Compare commonly used depreciation methods.
• Use the *units of production* and *depletion* depreciation methods as needed in engineering economic analysis problems.
• Use spreadsheets to calculate depreciation.

Key Words

annual depreciation

depreciation

percentage depletion

assets

depreciation recapture

personal property

bonus depreciation

double declining balance

real property

book value

expensed item

recovery period

capital gain

income statement

straight-line depreciation

cost basis

intangible property

tangible property

cost depletion

loss

Tax Cuts and Jobs Act (TCJA)

declining balance depreciation

loss on disposal

unit-of-production

depletion

MACRS

depreciable life

obsolescence

We have so far dealt with a variety of economic analysis problems and many techniques for their solution. In the process we have avoided income taxes, which are an important element in many private sector economic analyses. Now, we move to more realistic—and more complex—situations.

Governments tax individuals and businesses to support their processes—lawmaking, domestic and foreign economic policy making, even the making and issuing of money itself. The omnipresence of taxes requires that they be included in economic analyses, which means that we must understand the *way* taxes are imposed. For capital equipment, knowledge about depreciation is required to compute income taxes. Chapter 11 examines depreciation, and Chapter 12 illustrates how depreciation and other effects are incorporated in income tax computations. The goal is to support decision making on engineering projects, not to support final tax calculations.

The focus of this chapter is computing depreciation in order to find after-tax cash flows in Chapter 12. However, depreciation is also part of computing the value of what the firm owns: its **assets**. Many people do not realize that firms often use different depreciation methods for the two tasks. For tax purposes firms want the tax deductions as soon as possible but a slower depreciation method is preferred to increase the firm’s valuation.

INCOME, DEPRECIATION, CASH FLOW, AND VALUATION

The role of depreciation is most easily understood by starting with a firm’s

income statement or profit and loss statement. Revenue or sales may be a single line item, or it may be broken out into products and services. As shown in Figure 11–1, costs are often broken down into far more detail. All of the revenues and costs shown are cash flows—except for depreciation.

Depreciation is not a cash flow; it is an accounting entry that allocates a portion of the cost of machines, buildings, etc., in each year. However, because depreciation is one of the costs subtracted from revenue to determine taxable income, it does change the cash that flows when taxes are paid.

Now some items—wages, materials, and the like—are paid for shortly after they are used. Other items—like office rent and insurance premiums—are paid for monthly, quarterly, or yearly. But most items are paid within the period of a year and so it is reasonable to compare revenues for the year with expenses for the year.

However, capital **assets**—such as land, buildings, equipment, machinery—last longer than one year and typically cost more. It would be misleading to charge for all of their costs at the time of purchase. Instead, their costs are typically depreciated over time, except for land, which doesn't wear out and thus can't be depreciated.

Simplified Income Statement for XYZ Company for year ending December 31, 2019 (all amounts in $M)

Revenue	*Sales of products and services*	**Total Revenue**	**$184**
Costs	**Cost of Goods Sold**		
	Wages, materials, and utilities		110
	Depreciation		35
	Selling, Administration, and Financing Costs		19
		Total Costs	**$164**
		Net Income Before Taxes	**20**
Taxes			**7**
Profit			**13**

FIGURE 11-1 The Income Statement.

The $35M for depreciation shown in Figure 11–1 demonstrates how the value of capital assets decreases over time. By U.S. Generally Accepted Accounting Principles (GAAP), there are only four depreciation methods that are permitted for asset valuation: straight-line, declining balance, units of production, and sum-of-years'-digits. Straight-line is the most commonly used, and it slowly reduces the remaining value of the firm's assets. Declining balance may be chosen because a constant rate of decline in the assets' value may more accurately reflect true salvage values. The straight-line and declining balance methods are time- and value-based, and they are presented first.

However, the $7M in taxes shown in Figure 11–1 is calculated using (in the U.S.) a *different* depreciation method to calculate taxable income. Allowable depreciation methods are specified by the federal and state governments. To stimulate economic activity, these methods may allow a faster rate of depreciation. Firms generally choose the fastest depreciation legally allowed to maximize the present value of tax savings. The depreciation methods for taxation—bonus depreciation, modified accelerated cost recovery system (MACRS), and bonus plus MACRS—are presented after straight-line and declining balance depreciation.

BASIC ASPECTS OF DEPRECIATION

The word *depreciation* is defined as a "decrease in value." This is somewhat ambiguous because *value* has several meanings. In economic analysis, value may refer to either *market value* or *value to the owner*. For example, an assembly line is far more valuable to the manufacturing firm that it was designed for, than it is to a used equipment market. Thus, we now have two definitions of depreciation: a decrease in value to the market or a decrease to the owner.

Deterioration and Obsolescence

A machine may depreciate because it is deteriorating, or wearing out and no longer performing its function as well as when it was new. Many kinds of machinery require increased maintenance as they age, reflecting a slow but continuing failure of individual parts. Anyone who has worked to maintain a car has observed deterioration due to failure of individual parts (such as fan belts, mufflers, and batteries) and the wear on mechanical components. In other types of equipment, the quality of output may decline due to wear on components and resulting poorer mating of parts.

Depreciation is also caused by **obsolescence**. A machine that is in excellent working condition, and serving a needed purpose, may still be obsolete. Newer models are more capable with new features. Generations of computers have followed this pattern. The continuing stream of newer models makes older ones obsolete.

The accounting profession defines depreciation in yet another way, as allocating an asset's cost over its **depreciable life**. This may be done by one depreciation method for valuation and another depreciation method for taxation. Thus, we now have *three distinct definitions of depreciation:*

1. Decline in market value of an asset.
2. Decline in value of an asset to its owner.
3. Systematic allocation of an asset's cost over its depreciable life.

Depreciation and Expenses

It is the third (accountant's) definition that is used to compute depreciation for business assets. Business costs are generally either **expensed** or *depreciated*. Expensed items, such as labor, utilities, materials, and insurance, are part of regular business operations, and for tax purposes they are subtracted from business revenues when they occur. Expensed costs reduce income taxes because businesses are able to *write off* their full amount when they occur. We will see that Section 179 of the tax code allows profitable *small* businesses, and 100% bonus depreciation allows *all* firms, to expense capital asset purchases.

With the exceptions above, business costs due to capital assets (buildings, forklifts, chemical plants, etc.) are not fully written off when they occur. Capital assets lose value gradually and must be written off or *depreciated* over an extended period. For instance, consider an injection-molding machine used to produce the plastic beverage cups found at sporting events. The plastic pellets melted into the cup shape lose their value as raw material directly after manufacturing. The raw material cost for production material (plastic pellets) is expensed immediately. On the other hand, the injection-molding machine itself will lose value over time, and thus its costs (purchase price and installation expenses) are written off (or depreciated) over its **depreciable life** or **recovery period**. This is often different from the asset's useful or most economic life. Depreciable life is determined by the depreciation method used to spread out the cost—depreciated assets of many types operate well beyond their depreciable life.

Depreciation is a *noncash* cost that requires no exchange of dollars. Companies do not write a check to someone to *pay* their depreciation expenses. Rather, these are business expenses that are allowed by the government to offset the loss in value of business assets. Remember, the company has paid for assets up front; depreciation is simply a way to claim these "business expenses" over time. Depreciation deductions reduce the taxable income of businesses and thus reduce the amount of taxes paid. Since taxes are cash flows, depreciation must be considered in after-tax economic analyses.

In general, business assets can be depreciated only if they meet the following basic requirements:

1. The property must be used for business purposes to produce income.
2. The property must have a useful life that can be determined, and this life must be longer than one year.
3. The property must be an asset that decays, gets used up, wears out, becomes obsolete, or loses value to the owner from natural causes.

EXAMPLE 11-1

Consider the costs that are incurred by a local pizza business. Identify each cost as either *expensed* or *depreciated* and describe why that classification applies.

- Cost for pizza dough and toppings
- Cost to pay wages for janitor
- Cost of new baking oven
- Cost of new delivery van

• Cost of furnishings in dining room
• Utility costs for soda refrigerator

SOLUTION

Cost Item	Type of Cost	Why
Pizza dough and toppings	Expensed	Life < 1 year; lose value immediately
New delivery van	Depreciated	Meets 3 requirements for depreciation
Wages for janitor	Expensed	Life < 1 year; lose value immediately
Furnishings in dining room	Depreciated	Meet 3 requirements for depreciation
New baking oven	Depreciated	Meets 3 requirements for depreciation
Utilities for soda refrigerator	Expensed	Life < 1 year; lose value immediately

Types of Property

The rules for depreciation depend on the type of business property.

Tangible property can be seen, touched, and felt.
Real property includes land, buildings, and all things growing on, built upon, constructed on, or attached to the land.
Personal property includes equipment, furnishings, vehicles, office machinery, and anything that is tangible that is not *real property*.
Intangible property is all property that has value to the owner but cannot be directly seen or touched. Examples include patents, copyrights, trademarks, trade names, and franchises. This chapter's opening vignette is about intangible property and depreciation.

Many different types of property that wear out, decay, or lose value can be depreciated as business assets. This wide range includes copy machines, helicopters, buildings, interior furnishings, production equipment, and computer networks. Almost all tangible property can be depreciated.

One important exception is land, which is *never* depreciated. Land does not wear out, lose value, or have a determinable useful life and thus does not qualify as a depreciable property. Rather than decreasing in value, most land becomes more valuable as time passes. In addition to the land itself, expenses for clearing, grading, preparing, planting, and landscaping are not generally depreciated because they have no fixed useful life. Other tangible property that *cannot* be depreciated includes factory inventory, containers considered as inventory, and leased property. The leased property exception highlights that only the owner of property may claim depreciation expenses.

Tangible properties used in *both* business and personal activities, such as a vehicle used in a consulting engineering firm and to take one's kids to school, can be depreciated, but only in proportion to the business use.

Depreciation Calculation Fundamentals

To understand the complexities of depreciation, the first step is to examine the fundamentals of depreciation calculations. Figure 11–2 illustrates the general depreciation problem of allocating the total depreciation charges over the asset's depreciable life. The vertical axis is labeled **book value**. At time zero the curve of book value starts at the **cost basis** (= the first cost plus installation cost). Over time, the book value declines to the salvage value. Thus, at any point in time:

$$\text{Book value} = \text{Cost basis} - \text{Depreciation charges made to date}$$

Looked at another way, book value is the asset's remaining unallocated cost.

In Figure 11–2, *book value* goes from a value of *B* at time zero in the recovery period to a value of *S* at the end of Year 5. Thus, book value is a *dynamic* variable that changes over an asset's recovery period. Equation 11–1 is used to calculate an asset's book value over time.

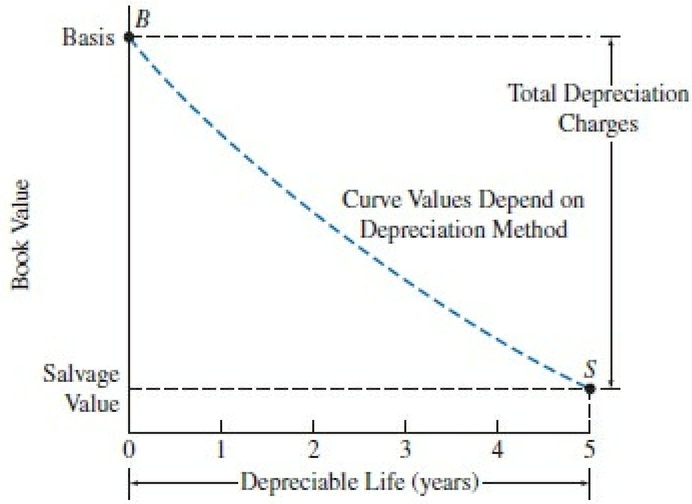


FIGURE 11-2 General depreciation.

$$BV_t = \text{Cost basis} - \sum_{j=1}^{t} d_j \quad (11\text{-}1)$$

where BV_t = book value of the depreciated asset at the end of time t

Cost basis = B = dollar amount that is being depreciated; this includes the asset's purchase price as well as any other costs necessary to make the asset "ready for use"

$\sum_{j=1}^{t} d_j$ = sum of depreciation deductions taken from time 0 to time t, where d_j is the depreciation deduction in Year j

Depreciation and Asset Disposal

When a depreciated asset is disposed of, the key question is, Which is larger, the asset's *book value*, *BV*, or the asset's *market value*, *MV*? If the book value is lower than the market value, then excess depreciation will be recaptured and taxed. On the other hand, if the book value is higher than the market value, there is a **loss on the disposal**. In either case, the level of taxes owed changes.

This calculation is needed whenever the asset is disposed of. The disposal might occur shortly after purchase if the asset is unsatisfactory. It might occur in the middle of the asset's life if it is no longer needed. Or, disposal may be long after the asset has been fully depreciated.

Depreciation recapture *(ordinary gains)*: Depreciation recapture, also called ordinary gains, is necessary when an asset is sold for more than an asset's current book value. If more than the original cost basis is received, only the amount up to the original cost basis is recaptured depreciation.
Losses: A *loss* occurs when less than book value is received for a depreciated asset. If we sell an asset for a dollar amount less than its book value, that is a loss.
Capital gains: Capital gains occur when we sell an asset for more than the asset's original cost basis. The excess over the original cost basis is the *capital gain*. As described in Chapter 12, the tax rate on such gains is sometimes lower than the rate on ordinary income, but this depends on how long the investment has been held ("short," ≤ 1 year; "long," ≥ 1 year). In most engineering economic analyses capital gains are very uncommon because business and production equipment and facilities almost always *lose* value over time. Capital gains are much more likely to occur for nondepreciated assets like stocks, bonds, real estate, jewelry, art, and collectibles.

The relationship between depreciation recapture, loss, and capital gain is illustrated in Figure 11–3. Each case given is at a point in time in the life of the depreciated asset, where the original cost basis is $10,000 and the book value is $5000. Case (a) represents depreciation recapture (ordinary gain), Case (b) represents a loss, and in Case (c) both recaptured depreciation and a capital gain are present.

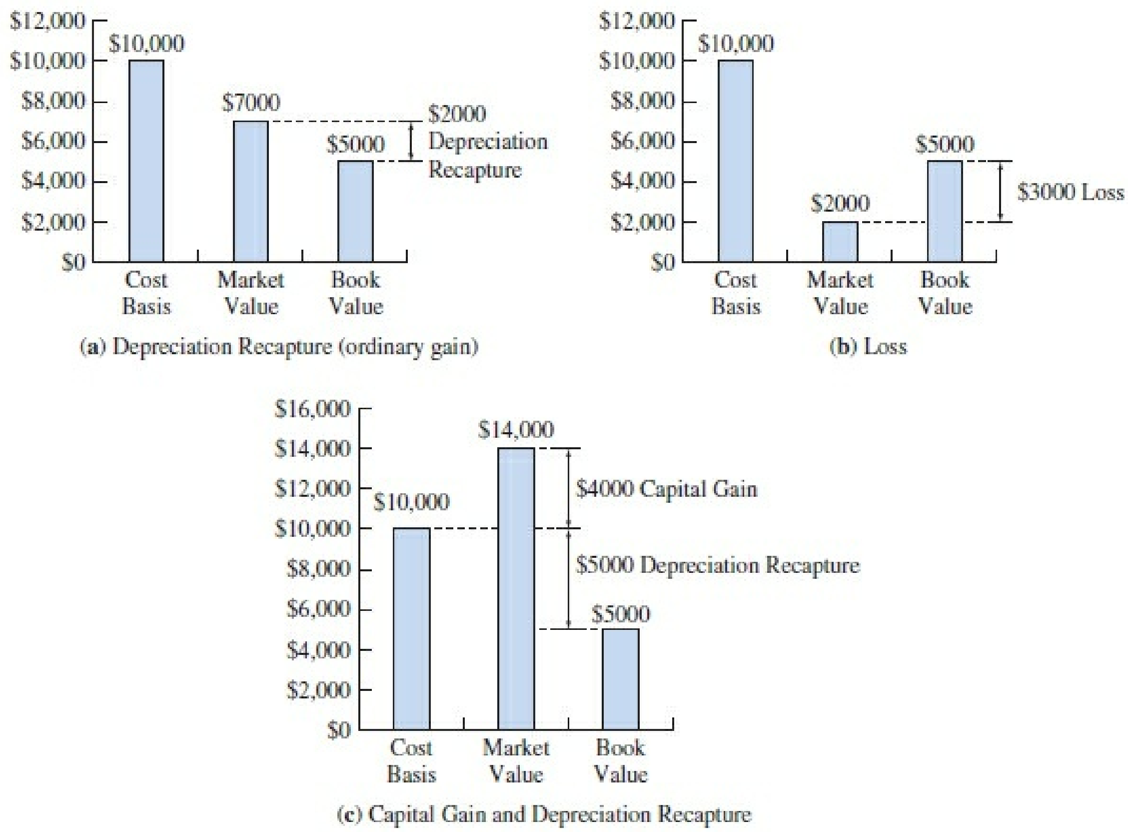


FIGURE 11-3 Recaptured depreciation, loss on sale, and capital gain.

TIME- AND VALUE-BASED DEPRECIATION

Straight-line and *declining balance* depreciation methods require estimates of an asset's useful life and salvage value. These methods are used in the U.S. for valuation and in constructing the tables of MACRS depreciation percentages. Most countries allow one or more of the time- and value-based techniques to be used for tax purposes. Many firms are multinational and pay taxes in multiple jurisdictions. *Sum-of-the-years'-digits* and *sinking fund* depreciation methods are time- and value-based, but are used less often—so they are not included here.

Straight-Line Depreciation

The simplest and best known depreciation method is **straight-line depreciation**. To calculate the constant **annual depreciation charge**, the total amount to be depreciated, $B - S$, is divided by the depreciable life, in years, N:[1]

$$\text{Annual depreciation charge} = d_t = \frac{B - S}{N} \quad (11\text{-}2)$$

EXAMPLE 11-2

Consider the following (in $1000):

Cost of the asset, *B*	$900
Depreciable life, in years, *N*	5
Salvage value, *S*	$70

Compute the straight-line depreciation schedule.

SOLUTION

$$\text{Annual depreciation charge} = d_t = \frac{B - S}{N} = \frac{900 - 70}{5} = \$166$$

Year	Depreciation for Year *t* ($1000)	Sum of Depreciation Charges Up to Year *t* ($1000)	Book Value at the End of Year *t* ($1000)
t	d_t	$\sum_{j=1}^{t} d_j$	$BV_t = B - \sum_{j=1}^{t} d_j$
1	$166	$166	900 − 166 = 734
2	166	332	900 − 332 = 568
3	166	498	900 − 498 = 402
4	166	664	900 − 664 = 236
5	166	830	900 − 830 = 70 = *S*

This situation is illustrated in Figure 11–4. Notice the constant $166,000 d_t each year for 5 years and that the asset has been depreciated down to a book value of $70,000, which was the estimated salvage value.

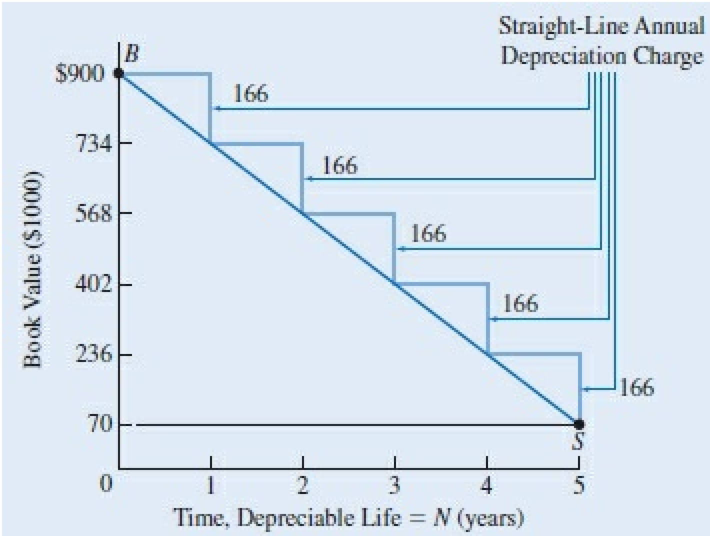


FIGURE 11-4 Straight-line depreciation.

The straight-line (SL) method is often used for intangible property. For example, Veronica's firm bought a patent in April that was not acquired as part of purchasing a business. She paid $6800 for this patent and must use the straight-line method to depreciate it over 17 years with no salvage value. The annual depreciation is $400 (= $6800/17). Since the patent was purchased in April, the first year's deduction must be prorated over the 9 months of ownership. This year the deduction is $300 (= $400 × 9/12), and then next year she can begin taking the full $400 per year.

Declining Balance Depreciation

Declining balance depreciation applies a *constant depreciation rate* to the property's declining book value. For longer life assets a rate of 150% may be used, but the most common rate is 200% of the straight-line rate. Since 200% is twice the straight-line rate, it is called **double declining balance**, or DDB; the general equation is

$$\text{Double declining balance} \qquad d_t = \frac{2}{N}(\text{Book value}_{t-1}) \qquad (11\text{-}3a)$$

Since book value equals cost *minus* depreciation charges to date, DDB $\quad d_t = \frac{2}{N}(\text{Cost} - \text{Depreciation charges to date})$

or

$$d_t = \frac{2}{N}\left(B - \sum_{j=1}^{t-1} d_j\right) \quad (11\text{-}3b)$$

EXAMPLE 11-3

Compute the DDB depreciation schedule for the situation in Example 11–2 ($1000):

Cost of the asset, *B*	$900
Depreciable life, in years, *N*	5
Salvage value, *S*	$70

SOLUTION

Year	Depreciation for Year *t* Using Equation 11-4a ($1000)	Sum of Depreciation Charges Up to Year *t* ($1000)	Book Value at End of Year *t* ($1000)
t	d_t	$\sum_{j=1}^{t} d_j$	$BV_t = B - \sum_{j=1}^{t} d_j$
1	$(2/5)900 = 360$	$360	$900 - 360 = 540$
2	$(2/5)540 = 216$	576	$900 - 576 = 324$
3	$(2/5)324 = 130$	706	$900 - 706 = 194$
4	$(2/5)194 = 78$	784	$900 - 784 = 116$
5	$(2/5)116 = 46$	830	$900 - 830 = 70 = S$

Figure 11–5 illustrates the situation.

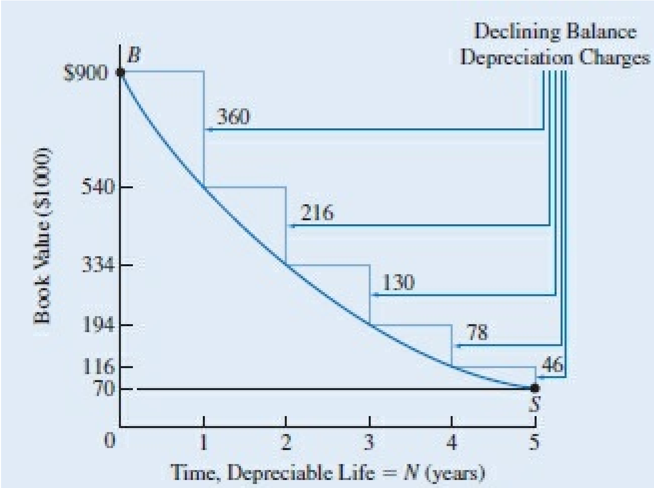


FIGURE 11-5 Declining balance depreciation.

The final salvage value of $70,000 for Examples 11–2 and 11–3 was chosen to match the ending value for the double declining balance method. This does not normally happen. If the final salvage value of Example 11–3 had not been $70,000, the double declining balance method would have had to be modified. One modification stops further depreciation once the book value has come to equal the salvage value—this prevents taking too much depreciation. The other modification would switch declining balance depreciation to straight line—this ensures taking enough depreciation.

DEPRECIATION FOR TAXES—BONUS DEPRECIATION

Bonus depreciation in the U.S. became available for the 2001 tax year. When available, it allows the expensing or immediate write-off of a specified percentage of an asset’s cost basis. This percentage may be as high as 100%. From the firm’s perspective, using the largest bonus depreciation allowed means the PW of tax savings is maximized. This is *very* attractive. When bonus depreciation is less than 100%, the remaining book value is depreciated with other methods.

From the government’s perspective, bonus depreciation stimulates business activity and investment, and thus the economy. Figure 11–6 illustrates that the bonus percentage rate has fluctuated over time. When first introduced, bonus depreciation was viewed as a temporary special case. It appears that it has become a permanent part of our tax code.

Tax Cuts & Jobs Act

The Tax Cuts & Jobs Act (TCJA), approved in December 2017, made significant changes to rules governing asset depreciation for taxes. In particular, the TCJA expanded bonus depreciation. Specifically, the rate through 2022 was increased to 100% (from planned declines to 40%, 30%, and 0% in 2018, 2019, and 2020). When 100% bonus depreciation is used, book value becomes zero, and recaptured depreciation occurs if the asset is sold.

Under the TCJA, 100% bonus depreciation is available from 2018 to 2022; then it is scheduled to begin a 5-year phase-out. As shown in Figure 11–6, in 2023, bonus depreciation will apply to 80% of an asset's value. In each following year, bonus depreciation will apply to 60%, then 40%, then 20%, and finally 0% of an asset's value. The portion of an asset's value not subject to bonus depreciation uses MACRS.

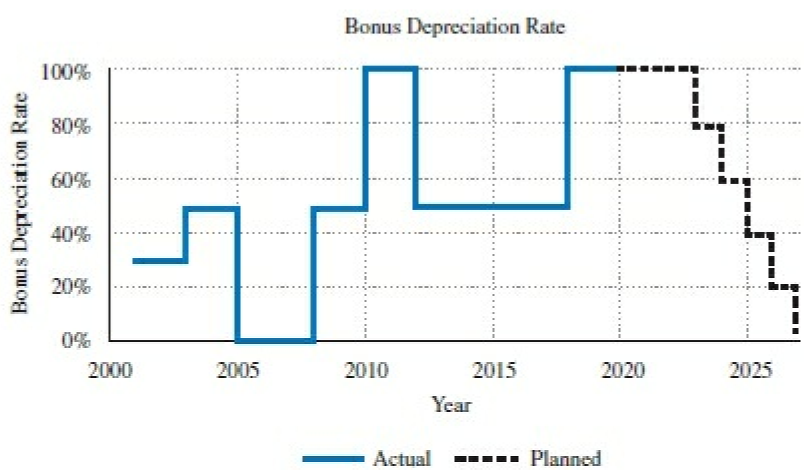


FIGURE 11-6 Historical and planned bonus depreciation rates.

For the first time, TCJA also allowed bonus depreciation to be applied to *used* assets purchased and put into service by a new owner. Bonus depreciation is not available in electrical energy, water, sewage disposal, and gas or steam distribution utilities. Tangible property with recovery periods over 20 years and passenger vehicles do not qualify for bonus depreciation. In addition, a number of states have disallowed bonus depreciation for computing state taxes.

The Tax Cuts and Jobs Act also expanded the Section 179 deduction for small businesses. The Act allows complete expensing of up to $1 million of capital expenditures in the year of purchase. While this is more limited than the current 100% bonus depreciation, for eligible businesses it has allowed immediate expensing of capital costs every year since 1987 with a maximum expensing limit that has been steadily increasing.

EXAMPLE 11-4

A personal property asset has a cost basis of $80,000 and an expected salvage of $20,000. It is not subject to special limitations for bonus depreciation. How much bonus depreciation was, is, or will be allowed

(a) in 2015?
(b) in 2019?
(c) in 2023?
(d) in 2027?

SOLUTION

Bonus depreciation is calculated on the cost basis. The salvage value is part of the recapture/loss calculations when the asset is disposed of.

(a) $40,000 = 50% × $80,000
(b) $80,000 = 100% × $80,000
(c) $64,000 = 80% × $80,000
(d) $0 = 0% × $80,000

EXAMPLE 11-5 (Examples 11–2

and 11–3 revisited)

Determine the depreciation schedule with 100% bonus depreciation for the situations in Examples 11–2 and 11–3 (in $1000):

Cost of the asset, *B*	$900
Depreciable life, in years, *n*	5
Salvage value, *S*	$70

SOLUTION

Depreciation, year 1	$900
Depreciation, years 2–5	0

The asset is 100% depreciated in its first year of use. Any salvage value that occurs after year 1 is taxed as recaptured depreciation or regular income.

EXAMPLE 11-6

Consider an asset with a cost basis of $10,000 that has been depreciated using 100% bonus depreciation. What is the gain or loss if the asset is disposed of after 5 years of operation for (**a**) $7000, (**b**) $0, and (**c**) a cost of $2000?

SOLUTION

To find *gain* or *loss* at disposal we compare *market* and *book value*. Since 100% bonus depreciation fully depreciates an asset during its first year, the book value equals $0.

(a) Recaptured depreciation = $7000.
(b) Since book value equals salvage value, there is no recaptured depreciation or loss.
(c) Since the money is paid for disposal, this is less than the book value, and there is a loss of $2000.

This general method for calculating recaptured depreciation or loss applies to all of the depreciation methods described in this chapter.

Because 100% bonus depreciation treats capital investments as expenses, after-tax analysis may no longer be needed for prospective decision making. As will be shown in Chapter 12, the before-tax and after-tax rates of return are the same. Depreciation and tax determination are only required for present or annual worth calculations.

DEPRECIATION FOR TAXES—MACRS DEPRECIATION

The modified accelerated cost recovery system (MACRS) depreciation method, introduced by the Tax Reform Act of 1986, is still the most consistently and broadly allowed method to compute depreciation for U.S. income taxes. Three major advantages of MACRS are that (1) the "property class lives" are less than the "actual useful lives," (2) salvage values are assumed to be zero, and (3) tables of annual percentages simplify computations. Use of MACRS focuses on the general depreciation system (GDS), which is based on declining balance with switch to straight-line depreciation. Since MACRS assumes $S = 0$ for its annual calculations, MACRS often has recaptured depreciation at disposal.

Property Class and Recovery Period

Each depreciated asset is placed in a *MACRS property class*, which defines the **recovery period** and the depreciation percentage for each year. The recovery period language was carefully chosen, because those periods are *shorter* than the asset's expected life. There are also two property classes for real estate, but those are usually not used by engineers.

The MACRS GDS property classes are described in Table 11–1. The proper MACRS property class can be found several different ways and are available from the Internal Revenue Service (IRS). Of the three approaches listed, the first one that works should be used.

1. Property class given in the problem.
2. Asset is named in Table 11–1 under a property class.
3. Depreciate as 7-year property for “all other property not assigned to another class.”

Table 11-1 MACRS GDS Property Classes

Property Class	Personal Property (all property except real estate)
3-year property	Special handling devices for food and beverage manufacture. Special tools for the manufacture of finished plastic products, fabricated metal products, and motor vehicles
5-year property	Automobiles and trucks Aircraft (of non-air-transport companies) Equipment used in research and experimentation Computers Petroleum drilling equipment
7-year property	All other property not assigned to another class Office furniture, fixtures, and equipment
10-year property	Assets used in petroleum refining and certain food products Vessels and water transportation equipment
15-year property	Telephone distribution plants Municipal sewage treatment plants
20-year property	Municipal sewers

For more detail, see Tables B1 & B2 in IRS Publication 946, *How to Depreciate Property* (www.irs.gov).

Once the MACRS property class is known the year-to-year depreciation deductions can be calculated for assets over their recovery period using

$d_t = B \times r_t$ (11-4)

where d_t = depreciation deduction in year t

B = cost basis being depreciated

r_t = appropriate MACRS percentage rate

Percentage Tables

The IRS has prepared tables to assist in calculating depreciation charges when MACRS depreciation is used. Table 11–2 gives the yearly depreciation percentages (r_t) that are used for the six personal property classes (3-, 5-, 7-, 10-, 15-, and 20-year property classes). Notice that the values are given in *percentages*—thus, for example, the value of 33.33% (given in Table 11–2 for Year 1 for a 3-year MACRS property) is 0.3333.

Table 11-2 MACRS Depreciation for Personal Property: Half-Year Convention

	Applicable Percentage for Property Class					
Recovery Year	3-Year Property	5-Year Property	7-Year Property	10-Year Property	15-Year Property	20-Year Property
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81*	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52*	12.49	11.52	7.70	6.177
5		11.52	8.93*	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55*	5.90*	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462*
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461

15	5.91	4.462
16	2.95	4.461
17		4.462
18		4.461
19		4.462
20		4.461
21		2.231

Computation method

• The 3-, 5-, 7-, and 10-year classes use 200% and the 15- and 20-year classes use 150% declining balance depreciation.
• All classes convert to straight-line depreciation in the optimal year, shown with asterisk (*).
• A half-year of depreciation is allowed in the first and last recovery years.

Notice in Table 11–2 that the depreciation percentages continue for *one year beyond* the property class life. For example, a MACRS 10-year property has an r_t value of 3.28% in Year 11. This is due to the *half-year convention* that also halves the percentage for the first year. The half-year convention assumes that all assets are placed in service at the midpoint of the first year.

Another characteristic of the MACRS percentage tables is that the r_t values in any column sum to 100%. This means that assets depreciated using MACRS are *fully depreciated* at the end of the recovery period. This assumes a salvage value of zero.

Calculating and Using MACRS Percentage Rates (r_t)

MACRS is based on declining balance with a switch to straight line. That combined method is used with three further assumptions that are detailed at the bottom of Table 11–2.

As shown in Example 11–7, the MACRS percentage rates can be derived from the declining balance and straight-line methods. However, it is obviously much easier to simply use the r_t values from Table 11–2.

EXAMPLE 11-7

Consider a 5-year MACRS property asset with an installed and “made ready for use” cost basis of $100. (*Note*: The $100 value used here is for illustration purposes in developing the rates. One would not depreciate an asset with a cost basis of only $100.) Develop the MACRS percentage rates (r_t) for the asset based on the underlying depreciation methods.

SOLUTION

To develop the 5-year MACRS property percentage rates, we use the 200% declining balance method, switching over to straight line at the optimal point. Since the assumed salvage value is zero, the entire cost basis of $100 is depreciated. Also the $100 basis mimics the 100% that is used in Table 11–2.

Let’s explain the accompanying table year by year. In Year 1 the basis is $\$100 - 0$, and the d_t values are halved for the initial half-year assumption. Double declining balance has a rate of 40% for 5 years (= 2/5). This is larger than straight line for Year 1. So one-half of the 40% is used for Year 1. The rest of the declining balance computations are simply 40% ×(basis minus the cumulative depreciation).

In Year 2 there are 4.5 years remaining for straight line, so 4.5 is the denominator for dividing the remaining $80 in book value. Similarly in Year 3 there are 3.5 years remaining. In Year 4 the DDB and SL calculations happen to be identical, so the switch from DDB to SL can be done in either Year 4 or Year 5. Once we know that the SL depreciation is 11.52 at the switch point, then the only further calculation is to halve that for the last year.

Notice that the DDB calculations get smaller every year, so that at some point the straight-line calculations lead to faster depreciation. This point is the optimal switch point, and it is built into Table 11–2 for MACRs.

Year	DDB Calculation	SL Calculation	MACRS r_t	Cumulative

			(%) Rates	Depreciation (%)
1	(1/2)(2/5)(100 − 0) =**20.00**	1/2(100 − 0)/5 =**10.00**	20.00 (DDB)	20.00
2	(2/5)(100 − 20.00) =**32.00**	(100 − 20)/4.5 = 17.78	32.00 (DDB)	52.00
3	(2/5)(100 − 52.00) =**19.20**	(100 − 52)/3.5 = 13.71	19.20 (DDB)	71.20
4	(2/5)(100 − 71.20) =**11.52**	(100 − 71.20)/2.5 =**11.52**	11.52 (either)	82.72
5		**11.52**	11.52 (SL)	94.24
6		(1/2) (11.52) = 5.76	5.76 (SL)	100.00

The values given in this example match the r_t percentage rates given in Table 11–2 for a 5-year MACRS property.

EXAMPLE 11-8

Use the MACRS to calculate the yearly depreciation allowances and book values for a firm that has purchased $150,000 worth of office equipment that does not qualify for 100% bonus depreciation. The equipment is estimated to have a salvage (market) value of $30,000 (20% of the original cost) after 10 years.

SOLUTION

1. The assets qualify as depreciable property.
2. The cost basis is given as $150,000.
3. The assets are being placed in service in Year 1 of our analysis.
4. MACRS applies.
5. The salvage value is not used with MACRS to calculate depreciation or book value.

Office equipment is listed in Table 11–1 as a 7-year property. We now use the MACRS 7-year property percentages from Table 11–2 and Equation 11-4 to calculate the year-to-year depreciation allowances. We use Equation 11-1 to calculate the book value of the asset.

Year, t	MACRS, r_t		Cost Basis	d_t	Cumulative d_t	$BV_t = B - Cum.d_t$
1	14.29%	×	\$150,000	\$ 21,435	\$ 21,435	\$128,565
2	24.49		150,000	36,735	58,170	91,830
3	17.49		150,000	26,235	84,405	65,595
4	12.49		150,000	18,735	103,140	46,860
5	8.93		150,000	13,395	116,535	33,465
6	8.92		150,000	13,380	129,915	20,085
7	8.93		150,000	13,395	143,310	6,690
8	4.46		150,000	6,690	150,000	0
	100.00%			\$150,000		
10				−\$30,000	recaptured	depreciation

Notice in this example several aspects of the MACRS depreciation method: (1) the sum of the r_t values is 100.00%, (2) this 7-year MACRS property is depreciated over 8 years (= property class life + 1), (3) the book value after 8 years is \$0, and (4) the salvage value is recaptured depreciation in year 10.

If the asset is in the middle of its depreciable life, then recaptured depreciation and losses are calculated in a similar manner—compare the *market* and *book values* at the time of disposal. However, in computing the book value with MACRS depreciation, a special rule must be applied for assets disposed of before the end of the recovery period. The rule is to *take one half of the allowable depreciation deduction for that year.* This rule assumes that disposals take place on average halfway through the year. Thus for a 5-year asset disposed of in the middle of Year 4, the rate allowed for MACRS depreciation is half of 11.52% or 5.76%. If the asset is disposed of in Year 6, it is already past the recovery period, and a half-year assumption has already been built into the MACRS schedule. Thus, the full r_6 is taken.

However, Example 11–11 illustrates that economic analyses will arrive at the same taxable income whether 0%, 50%, or 100% of the normal depreciation is claimed in the year of disposal.

Thus, correct economic analyses can assume the year of disposal is *just like* every other year and claim 100% of that year's depreciation.

EXAMPLE 11-9

Consider a $10,000 asset that does not qualify for bonus depreciation. Using MACRS and a 3-year recovery period, calculate the effect of disposal if this asset is sold during Year 2 for $5000 and

1. 50% depreciation is claimed in Year 2.
2. 0% depreciation is claimed in Year 2.
3. 100% depreciation is claimed in Year 2.

SOLUTION

The first effect of the disposal is a before-tax cash flow of $5000. This is not affected by the amount of depreciation claimed. The second effect of the disposal is the total deduction from taxable income for that year.

In every case

$$\begin{aligned} MV_2 &= \$5000 \text{ (market value in year 2)} \\ BV_1 &= 10{,}000 - 10{,}000 r_1 \\ &= 10{,}000 - 10{,}000 \times 0.3333 \\ &= 6667 \end{aligned}$$

1. If 50% depreciation is claimed in year 2:

$$\begin{aligned} D_2 &= 10{,}000(r_2/2) = 10{,}000\ (0.4445/2) \\ &= \$2222.50 \\ BV_2 &= BV_1 - D_2 = 6667 - 2222.50 \\ &= \$4444.50 \end{aligned}$$

This is less than MV_2, so excess depreciation must be recaptured.

$$\begin{aligned} \text{Recaptured depreciation} &= 5000 - 4444.50 = 555.50 \\ \text{Total deduction from taxable income} &= D_2 - \text{Recaptured depreciation} \\ &= 2222.50 - 555.50 = \$1667 \end{aligned}$$

2. If 0% depreciation is claimed in year 2: No depreciation, but total deduction from taxable income is the loss because the market value of $5000 is $1667 less than the book values of $6667.
3. If 100% depreciation is claimed in year 2:

$$D_2 = 10{,}000\, r_2 = 10{,}000 \times 0.4445 = \$4450$$
$$BV_2 = BV_1 - D_2 = 6667 - 4450 = \$2217 < MV_2 \Rightarrow \text{depreciation recapture}$$

$$\begin{aligned} \text{Recaptured depreciation} &= 5000 - 2217 = \$2783 \\ \text{Total deduction from taxable income} &= D_2 - \text{recapture} \\ &= 4450 - 2783 = \$1667 \end{aligned}$$

In all three cases, the total deduction from taxable income is $1667. The first approach follows the tax language. The second, with 0% claimed, is the easiest for hand calculations. The third approach with 100% claimed is the easiest for spreadsheet calculations, because it treats the year of disposal like any other year.

DEPRECIATION FOR TAXES—BONUS PLUS MACRS

When bonus depreciation is less than 100%, or for assets for which only partial bonus depreciation is allowed, the rest of the cost basis must be depreciated by another method. That is done with MACRS, as demonstrated in the following examples.

EXAMPLE 11-10

An organic foods company engaged in the farm-to-table market purchased

$1,300,000 of new 7-year MACRS equipment. This equipment and firm meet the requirements for full Section 179 and 60% bonus depreciation deductions. What is the total deduction from taxable income for the first year?

SOLUTION

Deductions from taxable income in year 1:

• Section 179: The firm claims the $1,000,000 limit, which reduces the equipment's cost basis to $300,000 (= $1,300,000 – $1,000,000).
• Bonus depreciation: At a 60% rate, an additional $180,000 (= $300,000 × 0.60) is claimed, which lowers the equipment's cost basis to $120,000 (= $300,000 – $180,000).
• MACRS depreciation: For 7-year MACRS equipment, the first-year deduction is $17,148 (= $120,000 × 0.1429). The remaining years of MACRS depreciation will use a cost basis of $120,000.

The total deduction from taxable income in year 1 is $1,197,148 (= $1,000,000 + $180,000 + $17,148).

EXAMPLE 11-11 (Examples 11–2, 11–3, and 11–5 revisited)

Determine the depreciation schedule with 60% bonus depreciation for the situations in Examples 11–2, 11–3, and 11–5 (in $1000):

Cost of the asset, *B*	$900
Depreciable life, in years, *N*	MACRS 5-year class
Salvage value, *S*	$70

SOLUTION

With 60% bonus depreciation, 0.6 × 900,000 = $540,000 is immediately depreciated (expensed). This leaves 40% of the asset value ($360,000) as the cost basis to be depreciated using MACRS.

Year, t	MACRS, r_t		Cost Basis	d_t	Cumulative d_t	$BV_t = B - Cum.d_t$
1	20.00%	×	$360,000	$72,000	$72,000	$288,000
2	32.00		360,000	115,200	187,200	172,800
3	19.20		360,000	69,120	256,320	103,680
4	11.52		360,000	41,472	297,792	62,208
5	11.52		360,000	41,472	339,264	20,736
6	5.76		360,000	20,736	360,000	0
	100.00%			$360,000		

The total first-year depreciation is $612,000 (= 540,000 + 72,000), leaving a book value of $288,000. The total second-year depreciation is $115,200, the MACRS depreciation only. At the end of six years, the book value is zero. If the asset is sold at any time, then depreciation recapture and perhaps capital gains applies.

EXAMPLE 11-12 (Example 11–8 revisited)

Determine the depreciation schedule for the problem in Example 11–8 with 40% bonus depreciation.

Cost of the asset, B	$150,000
Depreciable life, in years, N	MACRS 7-year class

SOLUTION

With 40% bonus depreciation, 0.4 × 150,000 = $60,000 is immediately depreciated (expensed). This leaves 60% of the asset value ($90,000) as the cost basis to be depreciated using MACRS.

Year, t	MACRS, r_t		Cost Basis	d_t	Cumulative d_t	$BV_t = B - Cum.d_t$
1	14.29%	×	$90,000	$12,861	$12,861	$77,139

2	24.49	90,000	22,041	34,902	55,098
3	17.49	90,000	15,741	50,643	39,357
4	12.49	90,000	11,241	61,884	28,116
5	8.93	90,000	8,037	69,921	20,079
6	8.92	90,000	8,028	77,949	12,051
7	8.93	90,000	8,037	85,986	4,014
8	4.46	90,000	4,014	90,000	0
	100.00%		$90,000		

The total first year depreciation is $72,861 (= 60,000 + 12,861), leaving a book value of $77,139 at the end of the first year. The second-year depreciation is $22,041, the MACRS depreciation.

COMPARING DEPRECIATION METHODS

In Examples 11–2, 11–3, and 11–5 we used the straight-line, declining balance and bonus depreciation methods to illustrate how the book value of an asset that cost $900,000 and had a salvage value of $70,000 changed over its 5-year depreciation life. Figures 11–2 through 11–4 provided a graphical view of book value over the 5-year depreciation period using these methods. Example 11–13 compares straight-line, double-declining, MACRS, and bonus plus MACRS depreciation methods.

EXAMPLE 11-13 (Examples 11–8 and 11–12 revisited)

Consider the equipment that was purchased in Examples 11–8 and 11–12. Calculate the asset's depreciation deductions, book values, and present worth of the deductions. Do this for 100% bonus, straight-line, double-declining, MACRS, and 40% bonus plus MACRS depreciation methods.

SOLUTION

Depreciation deductions *benefit* a firm after taxes because they reduce taxable income and taxes. The time value of money ensures that it is better to take these deductions as soon as possible. Because 100% bonus depreciation is the fastest it has the highest PW. In year 1 there is depreciation of \$150,000 and in year 10 there is recaptured depreciation of \$30,000. PW = $150{,}000/1.1^{1} - 30{,}000/1.1^{10}$ = \$124,797.

Table 11-3 Comparison of MACRS, Bonus Depreciation, and Classic Methods for Asset in Example 11–7

Year, t	Straight Line		Double Declining		MACRS		40% Bonus Depreciation Plus MACRS	
	d_t	BV_t	d_t	BV_t	d_t	BV_t	d_t	BV_t
1	12,000	138,000	30,000	120,000	21,435	128,565	72,861	77,1
2	12,000	126,000	24,000	96,000	36,735	91,830	22,041	55,0
3	12,000	114,000	19,200	76,800	26,235	65,595	15,241	39,3
4	12,000	102,000	15,360	61,440	18,735	46,860	11,241	28,1
5	12,000	90,000	12,288	49,152	13,395	33,465	8,037	20,0
6	12,000	78,000	9,830	39,322	13,380	20,085	8,028	12,0
7	12,000	66,000	7,864	31,457	13,395	6,690	8,037	4,01
8	12,000	54,000	1,457	30,000	6,690	0	4,014	0
9	12,000	42,000	0	30,000	0	0	0	0
10	12,000	30,000	0	30,000	0	0	0	0
Recaptured depr.	0		0		−30,000		−30,000	
$PW_{10\%}$	\$62,169		\$78,351		\$96,651		\$107,910	

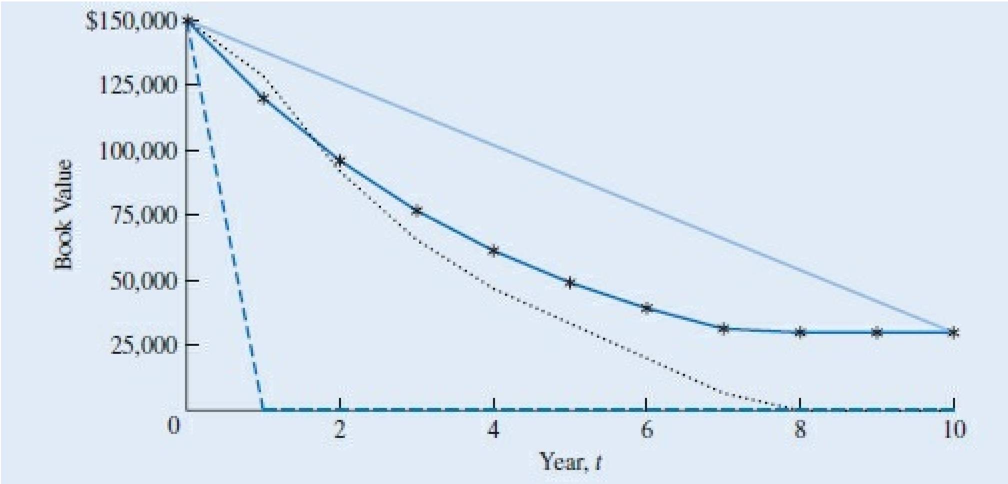


FIGURE 11-7 Comparing straight line (———), double declining balance (—*—), MACRS (..........), and 100% bonus depreciation (– – – – –).

When computing cash flows and income taxes, the 100% bonus depreciation schedule results in the highest PW and thus it will minimize the PW of taxes. However, for valuing the business the straight-line deductions have the lowest PW, thus maximizing the firm's reported value.

UNIT-OF-PRODUCTION DEPRECIATION

At times the recovery of depreciation on a particular asset is more closely related to use than to time. In these few situations (and they are rare), the **unit-of-production** (**UOP**) **depreciation** in any year is

$$\text{UOP depreciation in any year} = \frac{\text{Production for year}}{\text{Total lifetime production for asset}}(B - S) \qquad (11\text{-}5)$$

This method might be useful for machinery that processes natural resources if the resources will be exhausted before the machinery wears out. Historically, this method was sometimes used for construction equipment that had very heavy use in some years and very light use in others. It is not considered an acceptable method for general use in depreciating industrial equipment.

EXAMPLE 11-14

For numerical similarity with previous examples, assume that equipment costing $900,000 has been purchased for use in a sand and gravel pit. The pit will operate for 5 years, while a nearby airport is being reconstructed and paved. Then the pit will be shut down, and the equipment removed and sold for $70,000. Compute the unit-of-production (UOP) depreciation schedule if the airport reconstruction schedule calls for 40,000 m^3 of sand and gravel as follows:

Year	Required Sand and Gravel (m^3)
1	4,000
2	8,000
3	16,000
4	8,000
5	4,000

SOLUTION

The cost basis, *B*, is $900,000. The salvage value, *S*, is $70,000. The total lifetime production for the asset is 40,000 m^3 of sand and gravel. From the airport reconstruction schedule, the first-year UOP depreciation would be

$$\text{First-year UOP depreciation} = \frac{4000\ \text{m}^3}{40{,}000\ \text{m}^3}(900{,}000 - 70{,}000) = \$83{,}000$$

Similar calculations for the subsequent 4 years give the complete depreciation schedule:

Year	UOP Depreciation (in $1000)
1	$ 83
2	166
3	332

4	166
5	83
	$830

It should be noted that the actual unit-of-production depreciation charge in any year is based on the actual production for the year rather than the scheduled production.

DEPLETION

Depletion is the exhaustion of natural resources as a result of their removal. Since depletion covers such things as mineral properties, oil and gas wells, and standing timber, removal may take the form of digging up metallic or nonmetallic minerals, producing petroleum or natural gas from wells, or cutting down trees.

Depletion is recognized for income tax purposes for the same reason depreciation is—capital investment is being consumed or used up. Thus a portion of the gross income should be considered to be a return of the capital investment. The calculation of the depletion allowance is different from depreciation because there are two distinct methods of calculating depletion: *cost depletion* and *percentage depletion*. Except for standing timber and most oil and gas wells, depletion is calculated by both methods and the larger value is taken as depletion for the year. For standing timber and most oil and gas wells, only cost depletion is permissible.

Cost Depletion

Depreciation relied on an asset's cost, depreciable life, and salvage value to apportion the cost *minus* salvage value *over* the depreciable life. In some cases, where the asset is used at fluctuating rates, we might use the unit-of-production (UOP) method of depreciation. For mines, oil wells, and standing timber, fluctuating production rates are the usual situation. Thus, **cost depletion** is computed like unit-of-production depreciation using:

1. Property cost, less cost for land.
2. Estimated number of recoverable units (tons of ore, cubic meters of gravel, barrels of oil, million cubic feet of natural gas, thousand board-feet of timber, etc.).
3. Salvage value, if any, of the property.

EXAMPLE 11-15

A small lumber company bought a tract of timber for $35,000, of which $5000 was the land's value and $30,000 was the value of the estimated 1.5 million board-feet of standing timber. The first year, the company cut 100,000 board-feet of standing timber. What was the year's depletion allowance?

SOLUTION

$$\text{Depletion allowance per 1000 board-ft} = \frac{\$35{,}000 - \$5000}{1500 \times 1000 \text{ board-ft}} = \$20 \text{ per 1000 board-ft}$$

The depletion allowance for the year would be

$$100{,}000 \text{ board-ft} \times \$20 \text{ per 1000 board-ft} = \$2000$$

Percentage Depletion

Percentage depletion is an alternate method for mineral property. The allowance is a certain percentage of the property's gross income during the year. This is an entirely different concept from depreciation. Unlike depreciation, which allocates cost *over* useful life, the **percentage depletion** allowance (see Table 11–4) is based on the property's gross income.

Table 11-4 Percentage Depletion Allowances for Selected Deposits

Deposits	Rate
Sulfur, uranium, and, if from deposits in the U.S., asbestos, lead ore,	22%

zinc ore, nickel ore, and mica	
Gold, silver, copper, iron ore, and certain oil shale, if from deposits in the U.S.	15%
Borax, granite, limestone, marble, mollusk shells, potash, slate, soapstone, and carbon dioxide produced from a well	14%
Coal, lignite, and sodium chloride	10%
Clay and shale used or sold for use in making sewer pipe or bricks or used or sold for use as sintered or burned lightweight aggregates	$7\,^1/_2\%$
Clay used or sold for use in making drainage and roofing tile, flower pots, and kindred products, and gravel, sand, and stone (other than stone used or sold for use by a mine owner or operator as dimension or ornamental stone)	5%

Source: Internal Revenue Service, Publication 535, Chapter 9. Section 613(b) of the Internal Revenue Code gives a complete list of minerals and their percentage depletion rates.

Since percentage depletion is computed on the *income* rather than the property's cost, the total depletion *may exceed the cost of the property*. In computing the *allowable percentage depletion* on a property in any year, the *percentage depletion allowance* cannot exceed 50% of the property's taxable income computed without the depletion deduction. The percentage depletion calculations are illustrated by Example 11–16.

EXAMPLE 11-16

A coal mine has a gross income of $250,000. Mining expenses equal $210,000. Compute the allowable percentage depletion deduction.

SOLUTION

From Table 11–4, coal has a 10% depletion allowance based on gross mining income. The allowable percentage depletion deduction is also limited to a maximum of 50% of taxable income.

Computed Percentage Depletion	
Gross income from mine	$250,000
Depletion percentage	× 10%
Computed percentage depletion	$25,000

Taxable Income Limitation	
Gross income from mine	$250,000
Less: Expenses other than depletion	−210,000
Taxable income from mine	40,000
Deduction limitation	× 50%
Taxable income limitation	$20,000

Since the taxable income limitation ($20,000) is less than the computed percentage depletion ($25,000), the allowable percentage depletion deduction is $20,000. If the cost depletion were higher, it could be claimed instead.

As previously stated, on mineral property the depletion deduction can be based on either cost or percentage depletion. Each year, depletion is computed by cost and percentage depletion methods, and the allowable depletion deduction is the larger of the two amounts.

SPREADSHEET FUNCTIONS FOR DEPRECIATION

The spreadsheet functions for depreciation are summarized in Table 11–5. These functions find the value of the depreciation in year *t*. They include parameters for *cost* (initial book value), *salvage* (final salvage value), and *life* (depreciation period). If the depreciation amounts change, a *period* (year) must be specified. DDB and VDB include a *factor* which has a default value of 2 for 200% or double declining balance, but another commonly used value is 1.5 for 150%. The more complicated final VDB function returns MACRS depreciation amounts for the class life.

Table 11-5 Spreadsheet Functions for Depreciation

Depreciation Technique	Excel
Straight line	SLN(cost, salvage, life)
Double declining balance	DDB(cost, salvage, life, period, factor)
Sum of years' digits	SYD(cost, salvage, life, period)
Variable declining balance	VDB(cost, salvage, life, start_period, end_period, factor, no_switch)
MACRS for period *t*	VDB(cost, 0, life, max(0, *t*-1.5), min (life, *t*-.5), factor)

SUMMARY

From the perspective of engineering economy, depreciation matters even though it is *not* a cash flow. It is part of determining taxable income and the cash flow of taxes.

Depreciation is part of computing income taxes in economic analysis. There are three distinct definitions of depreciation:

1. Decline in asset's market value.
2. Decline in asset's value to its owner.
3. Allocating the asset's cost *less* its salvage value *over* its recovery period or depreciable life.

While the first two definitions are used in valuing an asset or a firm, it is the third definition that is used in tax computations and thus the focus of this chapter. Book value is the remaining unallocated cost of an asset, or

$$\text{Book value} = \text{Asset cost} - \text{Depreciation charges made to date}$$

This chapter describes how depreciable assets are *written off* (or claimed as a business expense) over a period of years instead of *expensed* in a single period (like wages, material costs, etc.). The depreciation methods described include the classic methods: *straight-line* and *declining balance*. These methods required estimating the asset's salvage value and depreciable life.

The current tax law specifies use of 100% bonus depreciation and the modified accelerated capital recovery system (MACRS). Most new assets may now be expensed in their first year of use through 100% bonus depreciation. Those assets that do not qualify for bonus depreciation continue to be depreciated using MACRS, which is economically more attractive than the classic methods. While historical from a U.S. tax perspective, the classic methods are often used in the U.S. when valuing assets and firms and internationally for both valuation and tax purposes.

The MACRS system assumes a salvage value of zero. This is in contrast with historical methods, which ensured the final book value would equal the predicted salvage value. When using 100% bonus depreciation and often when using MACRS, it is necessary to consider recaptured depreciation. This is the excess of salvage value over book value, and it is taxed as ordinary income. Similarly, losses on sale or disposal are taxed as ordinary income.

Unit-of-production (UOP) depreciation relies on usage to quantify the loss in value. UOP is appropriate for assets that lose value based on the number of units produced, the tons of gravel moved, and so on (vs. number of years in service). However, this method is not considered to be acceptable for most business assets.

Depletion is the exhaustion of natural resources like minerals, oil and gas, and standing timber. The owners of the natural resources are consuming their investments as the natural resources are removed and sold. Cost depletion is computed based on the fraction of the resource that is removed or sold. For minerals and some oil and gas wells, an alternate calculation called percentage depletion is allowed. Percentage depletion is based on income, so the total allowable depletion deductions may *exceed* the invested cost.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

11-1 Equipment that qualifies for 100% bonus depreciation was purchased for $250,000. Determine the depreciation schedule.

SOLUTION

11-2 Production equipment has a cost basis of $200,000 and an expected salvage value of $20,000. This equipment qualifies for bonus depreciation. How much bonus depreciation is allowed?

a. in 2020?
b. in 2024?
c. in 2026?

SOLUTION

11-3 A piece of machinery costs $5000 and has an anticipated $1000 resale value at the end of its 5- year useful life. Compute the depreciation schedule for the machinery by the straight-line method.

SOLUTION

11-4 Seed-cleaning equipment was purchased in 2018 for $8500 and was depreciated by the double declining balance (DDB) method for an expected life of 12 years. What is the book value of the equipment at the end of 2023? The original salvage value was estimated to be $2500 at the end of 12 years.

SOLUTION

11-5 Suds-n-Dogs just purchased new automated bun-handling equipment for $12,000. The salvage value of the equipment is anticipated to be $1200 at the end of its 5-year life. Use MACRS to determine the depreciation schedule using a three-year property class.

SOLUTION

To meet increased sales, a large dairy is planning to purchase 10 new

11-6 delivery trucks. Each truck will cost $18,000. Compute the depreciation schedule for each truck, using the modified accelerated cost recovery system (MACRS) method; the recovery period is 5 years.

SOLUTION

11-7 An asset is purchased for $100,000. The asset is depreciated by using MACRS depreciation and a 5-year recovery period. At the end of the third year of use, the business changes its product mix and disposes of the asset. Determine the depreciation allowed in the third year.

SOLUTION

11-8 A firm is purchasing office furniture worth $200,000 that has an expected salvage value of $20,000. The furniture is a MACRS 7-year property, but is also eligible for 60% bonus depreciation. Calculate the depreciation schedule.

SOLUTION

11-9 Computers worth $50,000 are eligible for 20% bonus depreciation plus MACRS. Determine the depreciation schedule.

SOLUTION

11-10 An asset will cost $1750 when purchased this year. It is further expected to have a salvage value of $250 at the end of its 5-year depreciable life. Calculate complete depreciation schedules giving the depreciation charge, $D(n)$, and end-of-year book value, $B(n)$, for straight-line (SL), double declining balance (DDB), 100% bonus depreciation, and modified accelerated cost recovery (MACRS) depreciation methods. Assume a MACRS recovery period of 5 years.

SOLUTION

11- Your company is considering the purchase of a secondhand scanning microscope at a cost of $10,500, with an estimated salvage value of $500

11 and a projected useful life of 4 years. Determine the straight-line (SL), and double declining balance (DDB) depreciation schedules.

SOLUTION

11-12 A new machine costs $12,000 and has a $1300 salvage value at the end of its 8-year useful life. Prepare a year-by-year depreciation schedule by the double declining balance (DDB) method.

SOLUTION

11-13 A used piece of depreciable property was bought for $20,000. If it has a useful life of 10 years and a salvage value of $5000, and you use the 150% declining balance schedule, how much will it be depreciated in the 3rd year?

SOLUTION

11-14 A front-end loader costs $70,000 and has a depreciable salvage value of $10,000 at the end of its 5-year useful life. Use MACRS depreciation to compute the depreciation schedule and book value of the equipment.

SOLUTION

11-15 A pump cost $1000 and has a salvage value of $100 after a life of 5 years. Using the straight-line depreciation method, determine:

a. The depreciation in the first year.
b. The book value after 5 years.
c. The book value after 4 years if the salvage was only $50.

SOLUTION

11-16 Nuts-R-Us, Inc. purchased nut-shelling equipment at a total cost of $80,000. The equipment was depreciated by using MACRS with a recovery class of 3 years and an anticipated end-of-useful-life value of $8000. The company has decided the equipment is no longer needed after two years and wishes to determine the minimum value it can accept for the equipment (that is, the lowest value that will result in no loss on the sale). Find the minimum selling price for the equipment.

SOLUTION

11-17 Thick Trunk Sawmill purchases a new automated log planer for $95,000. The asset is depreciated by using straight-line depreciation over a useful life of 10 years to a salvage value of $5000. Find the book value at the end of Year 6.

SOLUTION

11-18 Adventure Airlines recently purchased a new baggage crusher for $50,000. It is in a MACRS 7-year property class with estimated salvage value of $8000. Use 40% bonus depreciation with MACRS to determine the depreciation charge on the crusher for the third year of its life and the book value at the end of 3 years.

SOLUTION

11-19 Hoppy Hops, Inc., purchased hop-harvesting machinery for $150,000 four years ago. Owing to a change in the method of harvesting, the machine was recently sold for $37,500. Determine the MACRS deprecation schedule for the machinery for the 4 years of ownership. Assume a 5-year property class. What is the recaptured depreciation or loss on the sale of the machinery?

SOLUTION

11-20 Equipment costing $100,000 was bought in early 2019 and sold three years later for $20,000. Determine the depreciation recapture, ordinary losses, or capital gains associated with selling the equipment. Consider two cases:

a. 100% bonus depreciation
b. 5-year MACRS

SOLUTION

11-21 A lumber company purchased a tract of timber for $70,000. The value of the 25,000 trees on the tract was established to be $50,000. The value of the land was established to be $20,000. In the first year of operation, the lumber company cut down 5000 trees. What was the depletion allowance for the year?

SOLUTION

11-22 In the production of beer, a final filtration is accomplished by the use of kieselguhr, or diatomaceous earth, which is composed of the fossil remains of minute aquatic algae a few microns in diameter, and pure silica. A company has purchased a property for $840,000 that contains an estimated 60,000 tons of kieselguhr. Compute the depreciation charges for the first 3 years, given that production (or extraction) of 3000 tons, 5000 tons, and 6000 tons is planned for Years 1, 2, and 3, respectively. Use the cost-depletion methods, assuming no salvage value for the property.

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Assume that the depreciation methods listed in each problem can be used for tax and valuation purposes.

Bonus Depreciation

11-1 A metal fabrication company is buying a CNC machine for $600,000. After 20 years of use, the machine should have a salvage value of $35,000.
(a) Under 100% bonus depreciation, what depreciation can be claimed in year 1?
(b) Under 100% bonus depreciation, what depreciation can be claimed in year 2?

11-2 A A manual press costs $16,000, and it will be scrapped after 10 years. Compute the depreciation and book value for the first two years using 100% bonus depreciation.

11-3 Using 100% bonus depreciation, determine the depreciation schedule for $375,000 worth of equipment that was purchased by a small design firm in 2018. The firm had no other capital expenditures.

11-4 A A small, profitable construction contractor purchased equipment costing a total of $1,600,000 in 2018. Using Section 179 expensing first, and then 100% bonus depreciation, determine the depreciation schedule for the equipment.

11-5 Machinery for an assembly line has a cost basis of $150,000 and an expected salvage of $25,000. It is not subject to special limitations for bonus depreciation. How much bonus depreciation was, is, or will be allowed
(a) in 2014?
(b) in 2018?
(c) in 2014?
(d) in 2016?

11-6 A Machinery for an assembly line has a cost basis of $150,000 and an expected salvage of $25,000. It is not subject to special limitations for bonus depreciation. How much bonus depreciation was, is, or will be allowed
(a) in 2014?
(b) in 2018?
(c) in 2024?
(d) in 2026?

Depreciation Schedules

11-7	For an asset that fits into the MACRS "all property not assigned to another class" designation, show in a table the depreciation and book value over the asset's 10-year life of use. The cost basis of the asset is $20,000.
11-8 A	A company that manufactures food and beverages in the vending industry has purchased some handling equipment that cost $75,000 and will be depreciated using either 100% bonus depreciation or 3-year MACRS. Show in a table for each method the yearly depreciation amount and book value of the asset over its depreciable life.
11-9	A new machine tool is being purchased for $25,000 and is expected to have a zero salvage value at the end of its 5-year useful life. Compute its DDB depreciation schedule. Assume any remaining depreciation is claimed in the last year.
11-10 A	Gamma Cruise, Inc. purchased a new tender (a small motorboat) for $35,000. Its salvage value is $7500 after its useful life of 5 years. Calculate the depreciation schedule using (*a*) MACRS and (*b*) 100% bonus depreciation methods.
11-11 G	A $5 million oil drilling rig has a 5-year depreciable life and a $250,000 salvage value at the end of that time. (a) Determine which one of the following methods provides the preferred depreciation schedule: 100% bonus depreciation or MACRS. (b) Show the depreciation schedule for the preferred method. (c) Search for new oil rig technologies and describe three that improve environmental impact.
11-12 A	The RX Drug Company has just purchased a capsulating machine for $76,000. The plant engineer estimates the machine has a useful life of 5 years and no salvage value. Compute the depreciation schedule using: (a) Straight-line depreciation (b) Double declining balance depreciation (assume any remaining depreciation is claimed in the last year) (c) 100% bonus depreciation (d) MACRS
	Some special handling devices can be obtained for $20,000. At the

end of 5 years, they can be sold for $2000. Compute the depreciation schedule for the devices using the following methods:

11-13 (a) Straight-line depreciation
(b) Double declining balance depreciation
(c) 100% bonus depreciation
(d) MACRS depreciation

11-14 A The company treasurer must determine the best depreciation method for office furniture that costs $50,000 and has a zero salvage value at the end of a 10-year depreciable life. Compute the depreciation schedule using:
(a) Straight line
(b) Double declining balance
(c) 100% bonus depreciation
(d) Modified accelerated cost recovery system

11-15 The Acme Chemical Processing Company paid $50,000 for research equipment, which it believes will have zero salvage value at the end of its 5-year life. Compute the depreciation schedule using:
(a) Straight line
(b) Double declining balance
(c) 100% bonus depreciation
(d) Modified accelerated cost recovery system
(e) What is the U.S. EPA's *Presidential Green Chemistry Challenge*? What impact has this initiative had on green chemical processing?

11-16 A Units-of-production depreciation is being used for a machine that, based on usage, has an allowable depreciation charge of $6500 the first year and increasing by $1000 each year until complete depreciation. If the machine's cost basis is $110,000, set up a depreciation schedule that shows depreciation charge and book value over the machine's 10-year useful life.

Consider a $6500 piece of machinery, with a 5-year depreciable life and an estimated $1200 salvage value. The projected utilization of the machinery when it was purchased, and its actual production to date, are as follows:

Year	Projected Production (tons)	Actual Production (tons)
1	3500	3000

	2	4000	5000
	3	4500	[Not
11-17	4	5000	yet
	5	5500	known]

Compute the depreciation schedule using:

(a) Straight line
(b) Double declining balance
(c) 100% bonus depreciation
(d) MACRS
(e) Unit of production (for first 2 years only)

11-18 A

Al Jafar Jewel Co. purchased a crystal extraction machine for $50,000 that has an estimated salvage value of $10,000 at the end of its 8-year useful life. Compute the depreciation schedule using:
(a) Straight-line depreciation
(b) Double declining balance depreciation
(c) 100% bonus depreciation
(d) MACRS depreciation

11-19

The depreciation schedule for a machine has been arrived at by several methods. The estimated salvage value of the equipment at the end of its 6-year useful life is $600. Identify the resulting depreciation schedules.

Year	*A*	*B*	*C*	*D*
1	$8000	$2000	$1600	$1233
2	0	1500	2560	1233
3	0	1125	1536	1233
4	0	844	922	1233
5	0	633	922	1233
6	0	475	460	1233

Consider five depreciation schedules:

Year	*A*	*B*	*C*	*D*	*E*
1	$135.00	$35.00	$29.00	$58.00	$43.50
2	0.00	20.00	46.40	34.80	30.45

3	0.00	30.00	27.84	20.88	21.32
4	0.00	30.00	16.70	12.53	14.92
5	0.00	20.00	16.70	7.52	10.44
6			8.36		

11-20 A

They are based on the same initial cost, useful life, and salvage value. Identify each schedule as one of the following:

- Straight-line depreciation
- 100% bonus depreciation
- 150% declining balance depreciation
- Double declining balance depreciation
- Unit-of-production depreciation
- Modified accelerated cost recovery system

11-21

The depreciation schedule for an asset, with a salvage value of $90 at the end of the recovery period, has been computed by several methods. Identify the depreciation method used for each schedule.

Year	*A*	*B*	*C*	*D*	*E*
1	$1060.0	$212.0	$424.0	$194.0	$107.0
2	0	339.2	254.4	194	216
3	0	203.5	152.6	194	324
4	0	122.1	91.6	194	216
5	0	122.1	47.4	194	107
6		61.1			
	1060	1060	970	970	970

11-22 A

A heavy construction firm has been awarded a contract to build a large concrete dam. It is expected that a total of 8 years will be required to complete the work. The firm will buy $600,000 worth of special equipment for the job. During the preparation of the job cost estimate, the following utilization schedule was computed for the special equipment:

Year	Utilization (hr/yr)	Year	Utilization (hr/yr)
1	6000	5	800
2	4000	6	800
3	4000	7	2200

4	1600	8	2200

At the end of the job, it is estimated that the equipment can be sold at auction for $60,000.

(a) Compute the straight-line depreciation schedule.
(b) Compute the unit-of-production depreciation schedule.

11-23 A profitable company making earthmoving equipment is considering an investment of $150,000 on equipment that will have a 5-year useful life and a $50,000 salvage value. Use a spreadsheet to compute the 60% bonus depreciation with MACRS depreciation schedule. Show the total depreciation taken (=sum( )) as well as the PW of the depreciation charges discounted at 10%.

11-24 A A custom-built production machine is being depreciated using the units-of-production method. The machine costs $65,000 and is expected to produce 1.5 million units, after which it will have a $5000 salvage value. In the first 2 years of operation the machine was used to produce 140,000 units each year. In the 3rd and 4th years, production went up to 400,000 units. After that time annual production returned to 135,000 units. Use a spreadsheet to develop a depreciation schedule showing the machine's depreciation allowance and book value over its depreciable life.

11-25 You are equipping an office. The total office equipment will have a first cost of 2.0M and a salvage value of $200,000. You expect the equipment will last 10 years. Use a spreadsheet to compute the 40% bonus depreciation with MACRS depreciation schedule.

11-26 A Office equipment whose initial cost is $100,000 has an estimated actual life of 6 years, with an estimated salvage value of $10,000. Prepare tables listing the annual costs of depreciation and the book value at the end of each 6 years, based on straight-line, double declining balance, 100% bonus depreciation, and MACRS depreciation. Use a spreadsheet to show the depreciation methods.

Comparing Depreciation Methods

The XYZ Block Company purchased a new office computer and other depreciable computer hardware for $12,000. During the third year, the computer is declared obsolete and is donated to the local community college. Using an interest rate of 10%, calculate the PW of the depreciation deductions. Assume that no salvage value was initially declared and that the machine was expected to last 5 years.

11-27 E

(a) Straight-line depreciation
(b) Double declining balance depreciation
(c) 100% bonus depreciation
(d) MACRS depreciation
(e) Which method is preferred for determining the firm's taxes?
(f) Which method is preferred for determining the firm's value?
(g) Is using two accounting methods ethical?

11-28 A

Some equipment that costs $1000 has a 5-year depreciable life and an estimated $50 salvage value at the end of that time. You have been assigned to determine whether to use straight-line or double declining balance depreciation. If a 10% interest rate is appropriate, which is the preferred depreciation method for this profitable corporation? Use a spreadsheet to show your computations of the difference in present worths.

11-29

The FOURX Corp. has purchased $50,000 of experimental equipment. The anticipated salvage value is $5000 at the end of its 5-year depreciable life. This profitable corporation is considering two methods of depreciation: straight-line and double declining balance. If it uses 10% interest in its comparison, which method do you recommend? Use a spreadsheet to develop your solution.

The White Swan Talc Company paid $120,000 for mining equipment for a small talc mine. The mining engineer's report indicates the mine contains 40,000 cubic meters of commercial-quality talc. The company plans to mine all the talc in the next 5 years as follows:

Year	Talc Production (m^3)
1	15,000
2	11,000
3	4,000
4	6,000

11-30 G

5	4,000

At the end of 5 years, the mine will be exhausted and the mining equipment will be worthless. The company accountant must now decide whether to use double declining balance depreciation or unit-of-production depreciation. The company considers 15% to be an appropriate time value of money.

(a) Which would you recommend? How much better is the present worth for the recommended choice?
(b) What is talc and how is it used? As the softest mineral, are there special health and environmental issues/risks that are present in the mining, processing, and use of talc?

11-31 A small used delivery van can be purchased for $20,000. At the end of its useful life (8 years), the van can be sold for $3000. Determine the PW of the depreciation schedule based on 15% interest using:
(a) Straight-line depreciation
(b) Double declining balance depreciation
(c) 100% bonus depreciation
(d) MACRS depreciation

11-32 A E Loretta Livermore Labs purchased R&D equipment costing $200,000. The interest rate is 5%, salvage value is $20,000, and expected life is 10 years. Compute the PW of the depreciation deductions assuming:
(a) Straight-line depreciation
(b) Double declining balance depreciation
(c) 100% bonus depreciation
(d) MACRS depreciation
(e) Which method is preferred for determining the firm’s taxes?
(f) Which method is preferred for determining the firm’s value?
(g) Is using two accounting methods ethical?

Depreciation and Book Value

Explain in your own words the difference between capital gains and

11-33 ordinary gains. In addition, explain why it is important to our analysis as engineering economists. Do we see capital gains much in industry-based economic analyses or in our personal lives?

11-34 The MACRS depreciation percentages for 7-year personal property are given in Table 11–2. Make the necessary computations to determine if the percentages shown are correct.

11-35 The MACRS depreciation percentages for 10-year personal property are given in Table 11–2. Make the necessary computations to determine if the percentages shown are correct.

11-36 A For its fabricated metal products, the Able Corp. is paying $10,000 for special tools that have a 4-year useful life and no salvage value. Compute the depreciation charge for the *second* year by each of the following methods:

(a) Straight-line
(b) Double declining balance
(c) 100% bonus depreciation
(d) Modified accelerated cost recovery system

11-37 E Global Fitters, an international clothing company, has purchased material handling equipment that cost $100,000 and a salvage value of $18,000 after 10 years. Determine the book value of the equipment after 3 years using:

(a) Straight-line depreciation
(b) 150% declining balance depreciation
(c) 100% bonus depreciation
(d) 7-year MACRS depreciation
(e) Global Fitters uses low-cost labor in emerging world economies to manufacture its products. List three potential ethical issues that are associated with the use of this labor pool.

11-38 A A pump in an ethylene production plant costs $15,000. After 9 years, the salvage value is declared at $0.

(a) Determine depreciation charge and book value for Year 9 using straight-line, 100% bonus depreciation, and 7-year MACRS depreciation.
(b) Find the PW of each depreciation schedule if the interest rate is 5%.

A used drill press costs $60,000, and delivery and installation

11-39 charges add $5000. The salvage value after 10 years is $10,000. Compute the accumulated depreciation through Year 5 using
(a) Straight-line depreciation
(b) Double declining balance depreciation
(c) 60% bonus depreciation with the balance using 7-year MACRS

11-40 [A] Metal Stampings, Inc., can purchase a new forging machine for $100,000. After 20 years of use the forge should have a salvage value of $15,000. What depreciation is allowed for this asset in Year 3 for
(a) Straight-line depreciation?
(b) Double declining balance depreciation?
(c) 40% bonus depreciation with the balance using MACRS?

11-41 Muddy Meadows Earthmoving can purchase a bulldozer for $150,000. After 7 years of use, the bulldozer should have a salvage value of $50,000. What depreciation is allowed for this asset in Year 4 for
(a) Straight-line depreciation?
(b) 150% declining balance depreciation?
(c) 40% bonus depreciation with the balance using 5-year MACRS?

11-42 [A] An asset costs $150,000 and has a salvage value of $15,000 after 10 years. What is the depreciation charge for the 4^{th} year, and what is the book value at the end of the 8^{th} year with
(a) Straight-line depreciation?
(b) Double declining balance depreciation?
(c) 60% bonus depreciation with the balance using 7-year MACRS?

11-43 A precision five-axis CNC milling machine costs $200,000, and it will be scrapped after 10 years. Compute the book value and depreciation for the first 3 years using
(a) straight-line depreciation
(b) double declining balance depreciation
(c) 150% declining balance depreciation
(d) 100% bonus depreciation
(e) MACRS depreciation

A company is considering buying a new piece of machinery. A 10% interest rate will be used in the computations. Two models of the machine are available.

11-44 A

	Machine I	Machine II
Initial cost	$80,000	$100,000
End-of-useful-life salvage value, *S*	20,000	25,000
Annual operating cost	18,000	15,000 first 10 years 20,000 thereafter
Useful life, in years	20	25
MACRS class	7 yr	7 yr

(a) Determine which machine should be purchased, based on equivalent uniform annual cost.
(b) What is the capitalized cost of Machine I?
(c) Machine I is purchased and a fund is set up to replace Machine I at the end of 20 years. Compute the required uniform annual deposit.
(d) Machine I will produce an annual saving of material of $28,000. What is the rate of return if Machine I is installed?
(e) What will be the book value of Machine I after 2 years, based on 60% bonus depreciation with the balance using MACRS?
(f) What will be the book value of Machine II after 3 years, based on straight-line depreciation?
(g) What would be the MACRS depreciation in the third year for Machine II?

Gain/Loss on Disposal

11-45 Equipment costing $20,000 that is a MACRS 5-year property is disposed of during the second year for $15,000. Calculate any depreciation recapture, ordinary losses, or capital gains associated with disposal of the equipment.

11-46 A An asset costing $50,000 was purchased on January 1, 2019. Calculate any depreciation recapture, ordinary losses, or capital gains associated with selling the equipment on December 31, 2021, for $15,000, $25,000, and $60,000. Consider two cases of depreciation for the problem: if 5-year MACRS is used, and if 100%

bonus depreciation is used.

11-47 A purchased machine cost \$320,000 with delivery and installation charges amounting to \$30,000. The declared salvage value was \$50,000. Early in Year 3, the company changed its product mix and found that it no longer needed the machine. One of its competitors agreed to buy the machine for \$180,000. Determine the loss, gain, or recapture if (*a*) 100% bonus depreciation is used, and (*b*) if MACRS depreciation is used. Use a 7-year MACRS class.

11-48 A

O'Leary Engineering Corp. has been depreciating a \$50,000 machine for the last 3 years. The asset was just sold for 60% of its first cost. What is the size of the recaptured depreciation or loss at disposal using the following depreciation methods?

(a) Straight-line with $N = 8$ and $S = 2000$

(b) Double declining balance with $N = 8$

(c) 40% bonus depreciation with the balance using 7-year MACRS

11-49 A \$150,000 asset has been depreciated with the straight-line method over a 10-year life. The estimated salvage value was \$30,000. At the end of the 7th year the asset was sold for \$38,000. From a tax perspective, what is happening at the time of disposal, and what is the dollar amount?

11-50 A

A numerically controlled milling machine was purchased for \$95,000. The estimated salvage value was \$15,000 after 15 years. What is the machine's book value after 5 years of depreciation? If the machine is sold for \$20,000 early in Year 7, how much gain on sale or recaptured depreciation is there? Assume

(a) Straight-line depreciation

(b) 150% declining balance depreciation

(c) 80% bonus depreciation with the balance using 7-year MACRS depreciation

(d) 7-year MACRS depreciation

11-51 G

A computer costs \$3500 and its salvage value in 5 years is negligible. What is the book value after 3 years? If the machine is sold for \$1500 in Year 5, how much gain or recaptured depreciation is there? Assume

(a) Straight-line depreciation

(b) Double declining balance depreciation

(c) 20% bonus depreciation with the balance using 5-year MACRS
(d) MACRS depreciation
(e) There are two important considerations when disposing of old computers—one environmental and one personal. What are they, and how can you lessen the effect of each?

11-52 A

A belt-conveyor purchased for $140,000 had shipping and installation costs of $20,000. It was expected to last 6 years, when it would be sold for $25,000 after paying $5000 for dismantling. Instead, it lasted 4 years, and several workers were permitted to take it apart on their own time for reassembly at a private technical school. How much gain, loss, or recaptured depreciation is there? Assume
(a) Straight-line depreciation
(b) 150% declining balance depreciation
(c) 40% bonus depreciation with the balance using 7-year MACRS
(d) 7-year MACRS

Depletion and Unit-of-Production

11-53

A piece of machinery has a cost basis of $50,000. Its salvage value will be $5000 after 9000 hours of operation. With units-of-production depreciation, what is the allowable depreciation rate per hour? What is the book value after 4000 hours of operation?

11-54 A

When a major highway was to be constructed nearby, a farmer realized that a dry streambed running through his property might be a valuable source of sand and gravel. He shipped samples to a testing laboratory and learned that the material met the requirements for certain low-grade fill material. The farmer contacted the highway construction contractor, who offered 65¢ per cubic meter for 45,000 cubic meters of sand and gravel. The contractor would build a haul road and would use his own equipment. All activity would take place during a single summer.

The farmer hired an engineering student for $2500 to count the truckloads of material hauled away. The farmer estimated that 2 acres of streambed had been stripped of the sand and gravel. The 640-acre farm had cost him $300 per acre, and the farmer felt the property had not changed in value. He knew that there had been no use for the sand and gravel prior to the construction of the highway, and he could foresee no future use for any of the remaining 50,000 cubic meters of sand and gravel.

Determine the farmer's depletion allowance.

11-55 During the construction of a highway bypass, earthmoving equipment costing $40,000 was purchased for use in transporting fill from the borrow pit. At the end of the 4-year project, the equipment will be sold for $20,000. The schedule for moving fill calls for a total of 100,000 cubic feet during the project. In the first year, 40% of the total fill is required; in the second year, 30%; in the third year, 25%; and in the final year, the remaining 5%. Determine the units-of-production depreciation schedule for the equipment.

11-56 A Mr. H. Salt purchased an $\frac{1}{8}$ interest in a producing oil well for $45,000. Recoverable oil reserves for the well were estimated at that time at 15,000 barrels, $\frac{1}{8}$ of which represented Mr. Salt's share of the reserves. During the subsequent year, Mr. Salt received $12,000 as his $\frac{1}{8}$ share of the gross income from the sale of 1000 barrels of oil. From this amount, he had to pay $3000 as his share of the expense of producing the oil. Compute Mr. Salt's depletion allowance for the year.

11-57 G The Piney Copper Company purchased an orebearing tract of land for $10.0M. The geologist for Piney estimated the recoverable copper reserves to be 500,000 tons. During the first year, 50,000 tons were mined and 40,000 tons were sold for $5.0M. Expenses (not including depletion allowances) were $3.0M.

(a) What are the percentage depletion and the cost depletion allowances?

(b) Copper mining and production are subject to high levels of regulation, including control of air and water quality as well as materials handling and disposal practices. What are the primary environmental risks, and how has regulation lessened those risks?

What are the most significant health risks, and how have these been lessened?

11-58 A

American Pulp Corp. (APC) has entered into a contract to harvest timber for $450,000. The total estimated available harvest is 150 million board-feet.
(a) What is the depletion allowance for Years 1 to 3, if 42, 45, and 35 million board-feet are harvested by APC in those years?
(b) After 3 years, the total available harvest for the original tract was reestimated at 180 million board-feet. Compute the depletion allowances for Years 4 and beyond.

11-59

An automated assembly line is purchased for $2,500,000. The company has decided to use units-of-production depreciation. At the end of 8 years, the line will be scrapped for an estimated $500,000. Using the following information, determine the depreciation schedule for the assembly line.

Year	Production Level
1	5,000 units
2	10,000 units
3	15,000 units
4	15,000 units
5	20,000 units
6	20,000 units
7	10,000 units
8	5,000 units

11-60 A G

Western Carolina Coal Co. expects to produce 125,000 tons of coal annually for 15 years. The deposit cost $3M to acquire; the annual gross revenues are expected to be $9.50 per ton, and the net revenues are expected to be $4.25 per ton.
(a) Compute the annual depletion on a cost basis.
(b) Compute the annual depletion on a percentage basis.
(c) What are some of the primary environmental impacts of both surface and underground mining? What health risks do underground miners face? What is being done to make mining safer and more environmentally friendly?

11-61 Eastern Gravel expects to produce 60,000 tons of gravel annually for 5 years. The deposit cost $150K to acquire; the annual gross revenues are expected to be $9 per ton, and the net revenues are expected to be $4 per ton.
(a) Compute the annual depletion on a cost basis.
(b) Compute the annual depletion on a percentage basis.

11-62 A A 2500-acre tract of timber is purchased by the Houser Paper Company for $1,200,000. The acquisitions department at Houser estimates the land will be worth $275 per acre once the timber is cleared. The materials department estimates that a total of 5 million board-feet of timber are available from the tract. The harvest schedule calls for equal amounts of the timber to be harvested each year for 5 years. Determine the depletion allowance for each year.

11-63 Mining recently began on a new deposit of 10 million metric tons of ore (2% nickel and 4% copper). Annual production of 350,000 metric tons begins this year. The market price of nickel is $3.75 per pound and $0.65 for copper. Mining operation costs are expected to be $0.50 per pound. XYZ Mining Company paid $600 million for the deposits. What is the maximum depletion allowance each year for the mine?

11-64 A The Red River oil field will become less productive each year. Rojas Brothers is a small company that owns Red River, which is eligible for percentage depletion. Red River costs $2.5M to acquire, and it will be produced over 15 years. Initial production costs are $4 per barrel, and the wellhead value is $10 per barrel. The first year's production is 90,000 barrels, which will decrease by 6000 barrels per year.
(a) Compute the annual depletion (each year may be cost-based or percentage-based).
(b) What is the PW at i = 12% of the depletion schedule?

CASES

The following case from *Cases in Engineering Economy* (www.oup.com/us/newnan) is suggested as matched with this chapter.

CASE 26 **Molehill & Mountain Movers**

Compare depreciation methods with option for inflation.

[1] *N* is used for the depreciation period because it may be shorter than *n*, the horizon (or project life).

CHAPTER 12

INCOME TAXES FOR CORPORATIONS

Corporate Taxes Around the World: How to Compare?

In 2018, the U.S. lowered its federal statutory corporate income tax rate from 35% to 21%. For two decades, U.S. corporate income taxes had been cited as the highest among developed nations. Thus, it was argued that U.S. companies were moving their investments internationally to avoid paying such high taxes. Is the statutory tax rate the appropriate rate for comparison? If not, how does the U.S. actually compare with other developed nations?

There are three types of tax rates that are commonly used for comparisons.

• The statutory rate is the nominal rate set by law. Attention is focused on the highest rate for higher incomes.
• The average rate is what firms actually pay. It equals taxes paid divided by profits. This includes the various tax benefits that reduce a firm's taxable

income.
• The marginal corporate rate is a measure of a firm's tax burden on returns from new projects.

All three measures influence firms' decisions. The average rate is useful when determining whether to invest long-term in a particular country. The marginal rate is useful when determining whether to expand ongoing projects. The statutory rate is useful when developing legal and accounting strategies to shift income to countries with lower taxes.

When comparing U.S. statutory corporate tax rates with those of other developed countries, prior to 2018 the differences were indeed striking. U.S. rates exceeded average rates of other developed nations by almost 10%. However, those differences are far less striking for actual taxes that firms in the U.S. paid after accounting for tax breaks. These include subsidies for investments in research and development, production activities deduction, green energy credits, and faster depreciation schedules, among others. A 2017 report by the *Congressional Budget Office* suggested that when comparing average corporate income taxes in the U.S. and other developed countries, the U.S. firms actually pay, on average, closer to 19% in income taxes, less than some other developed countries.

With the new statutory corporate tax rate of 21% introduced in 2018, experts from the *Economic Policy Institute* estimate that the effective corporate tax rate in the U.S. will likely be between 10% and 15%, which puts the U.S. well below other developed nations.

Contributed by Eva Andrijcic, Rose-Hulman Institute of Technology

QUESTIONS TO CONSIDER

1. Given that different metrics are available for comparing corporate income taxes across nations, which metric is more objective and should be used?
2. Why is it more meaningful to compare U.S. corporate income taxes to those paid in other large, high-income nations, rather than to taxes paid in low-income nations?

3. Firms invest significant resources in tax planning to shift profits overseas to lower their corporate tax payments in their home countries. These actions are legal, but are they ethical?

After Completing This Chapter…

The student should be able to:

- Calculate *taxes due* or *taxes owed* for corporations.
- Calculate a combined income tax rate for state and federal income taxes and select an appropriate tax rate for engineering economic analyses.
- Utilize an *after-tax tax table* to find the after-tax cash flows for a prospective investment project.
- Calculate after-tax measures of merit, such as present worth, annual worth, payback period, internal rate of return, and benefit–cost ratio, from after-tax cash flows.
- Evaluate investment alternatives on an after-tax basis including asset disposal.
- Calculate personal incomes taxes and make choices about student loans, retirement accounts, insurance, and personal budgeting. (Appendix 12A)

Key Words

after-tax cash flow table

capital expenditures

capital gain

capital loss

combined incremental tax rate

expensed

incremental tax rates

investment tax credit

taxable income

As Benjamin Franklin said, two things are inevitable: death and taxes. There are many types of taxes and structures for taxation in the U.S., including sales taxes, gasoline taxes, property taxes, and state and federal income taxes. In this chapter we will concentrate our attention on federal income taxes for corporations. Income taxes are part of most real problems and often have a substantial impact that must be considered.

First, we must understand the way in which taxes are imposed. Depreciation is an integral part of this analysis, so the principles covered in Chapter 11 must be well understood. Then, having understood the mechanism, we will see how federal income taxes affect our economic analysis.

A PARTNER IN THE BUSINESS

Probably the most straightforward way to understand the role of federal income taxes is to consider the U.S. government as a partner in every business activity. As a partner, the government shares in the profits from every successful venture. In a somewhat more complex way, the government shares in the losses of unprofitable ventures too. The tax laws are complex, and it is not our purpose to fully explain them. Instead, we will examine the fundamental concepts of federal income tax for corporations—we emphasize at the start that there are exceptions and variations to almost every statement we shall make!

CALCULATION OF TAXABLE INCOME

At the mention of income taxes, one can visualize dozens of elaborate and complex calculations. There is some truth to that vision, for there can be complexities in computing income taxes. Yet incomes taxes are just another type of disbursement or cash outflow that affects profitability. Our economic analysis calculations in prior chapters have dealt with all sorts of disbursements: operating costs, maintenance, labor and materials, and so forth. Now we simply add one more prospective disbursement to the list—

income taxes.

Classification of Business Expenditures

When an individual or a firm operates a business, there are three distinct types of business expenditure:

1. For depreciable assets.
2. For nondepreciable assets.
3. All other business expenditures.

Expenditures for Depreciable Assets: When facilities or productive equipment with useful lives in excess of one year are acquired, the firm will normally recover the investment through depreciation charges. Chapter 11 detailed how to allocate an asset's cost over its useful life.
Expenditures for Nondepreciable Assets: Land is considered to be a nondepreciable asset, for there is no finite life associated with it. Other nondepreciable assets are properties *not* used either in a trade, in a business, or for the production of income. The final category of nondepreciable assets comprises those subject to *depletion*, rather than *depreciation*. Since business firms generally acquire assets for use in the business, their only nondepreciable assets normally are land and assets subject to depletion.
All Other Business Expenditures: This category is probably the largest of all, for it includes all the ordinary and necessary expenditures of operating a business. Labor costs, materials, all direct and indirect costs, and facilities and productive equipment with a useful life of one year or less are part of routine expenditures. They are charged as a business expense—*expensed*—when they occur.

Business expenditures in the first two categories—that is, for either depreciable or nondepreciable assets—are called **capital expenditures.** In the accounting records of the firm, they are **capitalized**; all ordinary and necessary expenditures in the third category are **expensed.**

Taxable Income of Business Firms

The starting point in computing a firm's taxable income is *gross income*. All ordinary and necessary expenses to conduct the business—*except* capital expenditures—are deducted from gross income. Capital expenditures may *not* be deducted from gross income. Except for land, business capital expenditures are allowed on a period-by-period basis through depreciation or depletion charges.

For business firms, taxable income is computed as follows:

$$\textbf{Taxable income} = \text{Gross income} - \text{All expenditures except capital expenditures} - \text{Depreciation and depletion charges} \quad (12\text{-}1)$$

Because of the treatment of capital expenditures for tax purposes, the taxable income of a firm may be quite different from the actual cash flows.

EXAMPLE 12-1

During a 3-year period, a firm had the following cash flows (in millions of dollars):

	Year 1	Year 2	Year 3
Gross income from sales	$200	$200	$200
Purchase of special tooling	− 60	0	0
(useful life: 3 years)			
All other expenditures	− 140	− 140	− 140
Cash flows for the year	$ 0	$ 60	$ 60

Compute the taxable income for each of the 3 years.

SOLUTION

The cash flows for each year would suggest that Year 1 was a poor one, while Years 2 and 3 were very profitable. A closer look reveals that the firm's cash flows were adversely affected in Year 1 by the purchase of special tooling. Since the special tooling has a 3-year useful life, it is a capital expenditure with its cost normally allocated over the useful life. If we assume that straight-line depreciation applies with no salvage value, we use Equation 11-2 to find the annual charge:

$$\text{Annual depreciation charge} = \frac{B-S}{N} = \frac{60-0}{3} = \$20 \text{ million}$$

Applying Equation 12-1, we write

$$\text{Taxable income} = 200 - 140 - 20 = \$40 \text{ million}$$

In each of the 3 years, the taxable income is $40 million.

If we assume 100% bonus depreciation, the entire cost of the tooling is depreciated in Year 1. The taxable income is then the same as the “Cash flows for the year” shown in the table.

Taxes are levied at the firm level—not at the project level. A project analysis may consider only the costs, it may have a large bonus depreciation deduction in year 1, or a large MACRS deduction in year 2. In each case the taxable income may be negative for some or all years of the project. The negative taxes from the project’s economic analysis will reduce the taxes paid at the firm level. The negative taxes can be thought of as a tax credit. Even if a firm has an unprofitable year, the tax law includes carry-forward and -backward provisions to transfer tax reductions to profitable years.

INCOME TAX RATES

Corporate Tax Rates

The Tax Cuts and Jobs Act simplified and reduced federal corporate income tax rates (see Table 12–1) from eight tax brackets to one flat tax rate of 21% beginning in 2018.

Table 12-1 Corporate Income Tax Rates

Taxable Income	2017 Tax Rate	2018 Tax Rate
Not over $50,000	15%	21%
$50,000–75,000	25%	
$75,000–100,000	34%	
$100,000–335,000	39%	
$335,000–10 million	34%	
$10 million–15 million	35%	
$15 million–18,333,333	38%	
≥$18,333,333	35%	

EXAMPLE 12-2

A firm expanded one of its manufacturing operations inside an existing building. New processing and packaging equipment was purchased for $800,000. Sales revenue for the year was $1.25 million. Operating expenses for that year, not including the capital expenditures, were $360,000. The new equipment qualifies for 100% bonus depreciation.

(i) What is the first-year depreciation charge?
(ii) What is the first-year taxable income?
(iii) What are the federal income taxes for the year?

SOLUTION

The new equipment qualifies for 100% bonus depreciation, so

$$\begin{aligned} \text{First-year depreciation} &= \$800{,}000 \\ \text{Taxable income} &= \text{Gross income} - \text{operating expenses} - \text{depreciation} \\ &= \$1{,}250{,}000 - 360{,}000 - 800{,}000 = \$90{,}000 \\ \text{Federal income tax} &= \$90{,}000 \times 0.21 = \$18{,}900 \end{aligned}$$

EXAMPLE 12-3

The equipment in Example 12–2 qualifies for 40% bonus depreciation instead of 100% bonus depreciation. The equipment is in the MACRS 7-year property class.

(a) What is the first-year depreciation charge?
(b) What is the first-year taxable income?
(c) What are the federal income taxes for the year?

SOLUTION TO PART a

The new equipment qualifies for 40% bonus depreciation, so the first 40% of the capital is expensed in the first year.

$$\text{Bonus depreciation} = \$800{,}000 \times 0.4 = \$320{,}000.$$

In addition, MACRS depreciation is used to depreciate the remaining $480,000 (60% of the capital cost).

$$\begin{aligned}\text{First-year MACRS depreciation} &= 0.1429 \times 480{,}000 = \$68{,}592.\\ \text{Total first-year depreciation} &= 320{,}000 + 68{,}592 = \$388{,}592\end{aligned}$$

SOLUTION TO PARTS b AND c

$$\begin{aligned}\text{Taxable income} &= \text{Gross income} - \text{operating expenses} - \text{depreciation}\\ &= \$1{,}250{,}000 - 360{,}000 - 388{,}592 = \$501{,}408\\ \text{Federal income tax} &= \$501{,}408 \times 0.21 = \$105{,}296\end{aligned}$$

Federal income tax rates are now fixed at 21%, but state income tax rates can vary depending on the level of taxable income, as demonstrated in Example 12–4. Many states use **incremental tax rates** to determine corporate and personal taxes.

EXAMPLE 12-4

A company in Kentucky has taxable income of $160,000. The Commonwealth of Kentucky taxes corporate income with a series of marginal rates. Taxable income up to $50,000 is taxed at 4%, income over $50,000 up to $100,000 is taxed at 5%, and income over $100,000 is taxed at 6%.

(i) How much income tax is owed?
(ii) What is the marginal (incremental) tax rate?
(iii) What is the average tax rate?

SOLUTION

$$\text{Tax} = 0.04 \times 50{,}000 + 0.05 \times (100{,}000 - 50{,}000) + 0.06 \times (160{,}000 - 100{,}000) = \$8100$$

Marginal tax rate is the rate on the next \$1 of income = 6%.

$$\text{Average tax rate} = 8100/160{,}000 = 0.0506 = 5.06\%$$

Combined Federal and State Income Taxes

With the decrease in federal tax rates, state taxes are of increased importance. It would be convenient if we could derive a single tax rate to represent both the state and federal incremental tax rates. In the computation of taxable income for federal taxes, the amount of state taxes paid is one of the allowable itemized deductions. Federal income taxes are not deductible in the computation of state taxable income—except in a handful of states. Therefore, the state income tax is applied to a *larger* taxable income than is the federal income tax rate. As a result, the combined incremental tax rate will not be the sum of two tax rates.

For an increment of income (ΔIncome) and tax rate on incremental income (ΔTax rate):

$$\begin{aligned}\text{State income taxes} &= (\Delta\text{State tax rate})(\Delta\text{Income})\\ \text{Federal taxable income} &= (\Delta\text{Income})(1 - \Delta\text{State tax rate})\\ \text{Federal income taxes} &= (\Delta\text{Federal tax rate})(\Delta\text{Income}) \times (1 - \Delta\text{State tax rate})\end{aligned}$$

The total of state and federal income taxes is

$$[\Delta\text{State tax rate} + (\Delta\text{Federal tax rate})(1 - \Delta\text{State tax rate})](\Delta\text{Income})$$

The term in the brackets gives the combined incremental tax rate.

$$\textbf{Combined incremental tax rate} = \Delta\text{State tax rate} + (\Delta\text{Federal tax rate})(1 - \Delta\text{State tax rate}) \quad (12\text{-}2)$$

EXAMPLE 12-5

A small design firm is located in Pennsylvania where the state tax rate is 9.99%. Income taxes need to be considered as the firm bids for new projects. What is the firm's combined federal and state income tax rate on new income?

SOLUTION

Using Equation 12–2, the combined incremental tax rate is:

$$= 0.0999 + 0.21(1 - 0.0999) = 0.2889 = 28.89\%.$$

Since state income tax rates may vary with the level of taxable income one must decide which tax rate to use in a particular situation. The simple answer is that the tax rate to use is the incremental tax rate that applies to the expected change in taxable income projected in the economic analysis.

ECONOMIC ANALYSIS TAKING INCOME TAXES INTO ACCOUNT

An important step in economic analysis has been to resolve the consequences of alternatives into a cash flow. Because income taxes have been ignored, the result has been a *before-tax cash flow*. This before-tax cash flow is an essential component in economic analysis that also considers the consequences of income tax. Decision making is often done on a before-tax basis. With 100% bonus depreciation before-tax and after-tax rates of return can be the same, as will be shown in Example 12–8. The principal elements in an *after-tax analysis* are as follows:

- Before-tax cash flow
- Depreciation
- Taxable income (Before-tax cash flow − Depreciation)
- Income taxes (Taxable income × Incremental tax rate)
- After-tax cash flow (Before-tax cash flow − Income taxes)

These elements are usually arranged to form an **after-tax cash flow table**. This is illustrated by Example 12–6.

EXAMPLE 12-6

A profitable company buys $25,000 of equipment that qualifies for 100% bonus depreciation. The equipment is expected to save $8000 per year over its 5-year life, when it will be sold for $6000.

(a) What is the before-tax present worth and rate of return? Use a MARR of 15%.
(b) What is the after-tax present worth and rate of return? The after-tax MARR is usually lower, but the same 15% is used so that the impact of taxes can be clearly seen.

TABLE SOLUTION TO PART a

The first step is to compute the before-tax cash flows.

Year	Before-Tax Cash Flow
0	–$25,000
1	8,000
2	8,000
3	8,000
4	8,000
5	{ 8,000 6,000

Solve for the before-tax present worth:

$$\begin{aligned} \text{PW} &= -25{,}000 + 8000(P/A, 15\%, 5) + 6000(P/F, 15\%, 5) \\ &= -25{,}000 + 8000(3.352) + 6000(.4972) = \$4799 \end{aligned}$$

Solve for the before-tax rate of return:

$$\text{At } i = 20\%, \text{PW} = -25{,}000 + 8000(P/A, 20\%, 5) + 6000(P/F, 20\%, 5)$$
$$= -25{,}000 + 8000(2.991) + 6000(.4019) = \$1339.4$$
$$\text{At } i = 25\%, \text{PW} = -25{,}000 + 8000(P/A, 25\%, 5) + 6000(P/F, 25\%, 5)$$
$$= -25{,}000 + 8000(2.689) + 6000(.3277) = -\$1521.8$$
$$\text{IRR}_{\text{BT}} = 20\% + 5\% \left(\frac{1339.4 - 0}{1339.4 - (-1521.8)} \right) = 0.223 = 22.3\%$$

5-BUTTON SOLUTION TO PART a

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 12-6	15%	5	8000		6000	PV	29,800
3					–25,000		PV	$4,800
4								
5			5	8000	–25,000	6000	RATE	22.2%

TABLE SOLUTION TO PART b

For an after-tax present worth, set up an after-tax cash flow table (Table 12–2). The first element is the before-tax cash flows (BTCF). The depreciation schedule for the equipment is the 100% bonus depreciation.

$$\text{Bonus depreciation} = \$25{,}000 \text{ in Year 1}$$

The $25,000 in bonus depreciation is greater than year 1’s $8000 BTCF. The −$17,000 (= 8000 – 25,000) in taxable income from this equipment will result in tax savings for the firm as a whole. At a 21% tax rate this will save the firm $3570 in taxes—which is shown as a positive cash flow in column d.

The last row in Table 12–1 shows the $6000 salvage value as a positive cash flow. It is taxable as recaptured depreciation, since the equipment was depreciated to a book value of $0.

Table 12-2 After-Tax Cash Flow Table for Example 12–6

Year	(a) Before-Tax Cash Flow	(b) Bonus Depreciation	(c) Taxable Income (a)−(b)	(d) 21% Income Taxes −0.21 (c)	(e) After-Tax Cash Flow (a) + (d)
0	−$25,000				−$25,000
1	8,000	$25,000	−$17,000	$3570	11,570
2	8,000	0	8,000	−1680	6,320
3	8,000	0	8,000	−1680	6,320
4	8,000	0	8,000	−1680	6,320
5	{ 8,000	0	8,000	−1680	{ 6,320
	6,000		6,000	−1260	4,740

SPREADSHEET SOLUTION TO PART b

We now use the after-tax cash flows to find the after-tax present worth and IRR.

	A	B	
1	MARR	15%	
2	Year	After-Tax Cash Flow	
3	0	–$25,000	
4	1	11,570	
5	2	6,320	
6	3	6,320	
7	4	6,320	
8	5	11,060	
9	NPV	$3,107	=B3+NPV(B1,B4:B8)
10	IRR	20.34%	=IRR(B3:B8)

The after-tax present worth is $3107 and the rate of return is 20.3%. This is a decrease in both present worth and rate of return from the before-tax results.

Per the Tax Cuts and Jobs Act, 100% bonus depreciation is scheduled to be phased out after five years (2023), at which time bonus depreciation will apply to only a portion of the investment. The balance of the depreciation will be governed by MACRS. This scenario is demonstrated in Example 12–7.

EXAMPLE 12-7

Repeat Example 12–6 using 40% bonus depreciation. The purchase price was \$25,000 with annual savings of \$8000 over its 5-year life, when it will be sold for \$6000. What is the before-tax and after-tax rate of return?

SOLUTION

For an after-tax present worth, set up an after-tax cash flow table (Table 12–3). Bonus depreciation will be 0.40 × 25,000 = \$10,000, leaving \$15,000 to be depreciated using MACRS. So Year 1 depreciation is \$10,000 from bonus depreciation, plus (0.20 × 15, 000 =) \$3000 MACRS depreciation. Depreciation for years 2–5 are from the MACRS depreciation table. Note that the equipment is sold before it is fully depreciated, so the final year depreciation is a half year's. The book value at the end of Year 5 is then \$1728, leaving recaptured depreciation (a taxable gain) of (6, 000 − 1728 =) \$4272.

The federal income tax rate is 21%, so taxes are 21% of taxable income. The recaptured depreciation is taxed as regular income. The after-tax cash flow equals the before-tax cash flow (column a) minus income taxes (column d).

Table 12-3 After-Tax Cash Flow Table for Example 12–7

Year	(a) Before-Tax Cash Flow	(b) Depreciation	(c) Taxable Income (a)−(b)	(d) 21% Income Taxes −0.21(c)	(e) After-Tax Cash Flow (a) + (d)
0	−\$25,000				−\$25,000
1	8,000	\$13,000	−\$5000	\$1050	9,050
2	8,000	4,800	3200	−672	7,328
3	8,000	2,880	5120	−1075	6,925
4	8,000	1,728	6272	−1317	6,683
5	{ 8,000	864*	7136	−1499	{ 6,501
	6,000	BV=1,728	4272	−897	5,103

The \$864 in year 5 depreciation is based on claiming only 1/2 of the MACRS depreciation in the year of sale. As shown in Example

11–9, 0%, 50%, or 100% of the final year depreciation can be claimed, and the total taxable income in that year does not change.

We now use the cash flows to find the before-tax and after-tax IRR.

	A	B	C
1	Year	BTCF	ATCF
2	0	–$25,000	–$25,000
3	1	8,000	9,050
4	2	8,000	7,328
5	3	8,000	6,925
6	4	8,000	6,683
7	5	14,000	11,604
8	IRR	22.2%	19.1%

The tax impact is to lower the internal rate of return from 22.2% to 19.1%.

Investment Tax Credit

Another tax policy technique used historically in the U.S. to promote capital investments is the **investment tax credit (ITC)**. Under the ITC, businesses are able to deduct a percentage of the purchase price of equipment as a *tax credit*. Depending on the provisions of the ITC law in effect, this credit might be subtracted from the asset's basis for depreciation, or perhaps not. In recent years investment tax credits have only been allowed in some specialized cases such as historic building preservation and in the development of alternate energy sources. The general ITC for all assets may be allowed at some future time.

THE AFTER-TAX RATE OF RETURN

After-Tax Rate of Return with 100% Bonus Depreciation Can Equal Before-Tax Rate

In practice, firms pay taxes quarterly, not annually, and thus depreciation is not calculated at the end of Year 1. Rather, it can be reported at Month 3, much closer to the beginning of the year than the end. As a result, it can be more accurate to assume that Year 1 depreciation is recorded in Year 0. This convention would increase a project's after-tax rate of return to *equal* its before-tax rate of return, as shown in Example 12–8. This example is shown for illustration purposes—we assume the end-of-year convention for depreciation elsewhere in the text.

EXAMPLE 12-8

Recalculate the after-tax rate of return on the project in Example 12–5, assuming the 100% bonus depreciation is reported in Year 0.

SOLUTION

An after-tax cash flow table (Table 12–4) is set up similar to Table 12–3, but with 100% bonus depreciation moved to Year 0. Bonus depreciation "expenses" the capital investment, making the $25,000 investment a negative taxable income.

Taxable income in Year 0 is negative, so the tax in Year 0 is a credit. The after-tax cash flows are calculated as before.

Table 12-4 After-Tax Cash Flow Table for Example 12–8

Year	(a) Before-Tax Cash Flow	(b) Depreciation	(c) Taxable Income (a)−(b)	(d) 21% Income Taxes −0.21(c)	(e) After-Tax Cash Flow (a) + (d)
0	−$25,000		−$25,000*	$5250	−$19,750
1	8,000	0	8,000	−1680	6,320
2	8,000	0	8,000	−1680	6,320
3	8,000	0	8,000	−1680	6,320
4	8,000	0	8,000	−1680	6,320
5	8,000	0	8,000	−1680	6,320
	6,000		6,000	−1260	4,740

* 100% Bonus depreciation "expenses" the $25,000 investment.

The before-tax and after-tax IRRs are calculated from the before-tax and after-tax cash flows.

	A	B	C
1	Year	BTCF	ATCF
2	0	–$25,000	–$19,750
3	1	8,000	6,320
4	2	8,000	6,320
5	3	8,000	6,320
6	4	8,000	6,320
7	5	14,000	11,060
8	IRR	22.2%	22.2%

The after-tax IRR is exactly the same as the before-tax IRR. Bonus depreciation has made the return on investment unaffected by federal tax. *Observation contributed by Jeff Wolstad of South Dakota School of Mines and Technology.*

The after-tax IRR in Example 12–8 is exactly the same as the before-tax IRR when using 100% bonus depreciation. This example is not unique. Current tax law decreases the after-tax present worth of a project, but it may not affect the after-tax IRR at all. This is a huge tax break for businesses and thus has implications for decision making.

When using 100% bonus depreciation, firms do not need to perform an after-tax analysis when evaluating eligible capital expenditures. If that depreciation is claimed at or near time 0, the before-tax IRR and the after-tax IRR are the same (based on federal taxes). As a result, taxes would need to be considered only when state taxes have a significant influence.

Estimating a Project's After-Tax IRR

In most cases, there is no shortcut method for computing the after-tax rate of return from the before-tax rate of return when 100% bonus depreciation is not used. One possible exception to this statement is in the situation of nondepreciable assets. In this special case, we have

After-tax rate of return = (1 − Incremental tax rate) × (Before-tax rate of return)

This relationship may be helpful for selecting a trial after-tax rate of return when the before-tax rate of return is known. It must be emphasized, however, this relationship is only a rough approximation in almost all situations, and does not hold when using bonus depreciation.

Firm's After-Tax Minimum Attractive Rate of Return

While a project's IRR can sometimes be estimated by applying the incremental tax rate to the before-tax rate of return, the firm's after-tax MARR cannot be estimated this way. The reason is that virtually all firms are financed by a mix of debt and equity, and only the interest payments on debt are tax deductible. The dividends paid on stock and other returns to equity holders are not tax deductible.

This will be demonstrated in Example 15–1 where the pre-tax MARR is 9.8%, and the after-tax MARR is 8.98%. Even though the tax rate is 26%, the tax shield only applies to the 40% of the capital structure that is debt. If the tax shield is incorrectly applied to the pre-tax MARR, the after-tax MARR is incorrectly estimated as 7.25% (= 9.8% × (1 − 0.26)).

CAPITAL GAINS AND LOSSES FOR NONDEPRECIATED ASSETS

When a nondepreciated capital asset is sold or exchanged, appropriate entries are made in the firm's accounting records. If the selling price of the capital asset exceeds the original cost basis, the excess is called a **capital gain.** If the selling price is less than the original cost basis, the difference is a **capital loss.** Examples of nondepreciated assets include stocks, land, art, and collectibles.

$$\text{Capital}\begin{Bmatrix}\text{Gain}\\ \text{Loss}\end{Bmatrix} = \text{Selling price} - \text{Original cost basis}$$

It is not uncommon for capital gains tax rates to be different from those for ordinary income—historically these have been set at a *lower* rate. This is in contrast to recaptured depreciation, which is taxed at the same rate as other (ordinary) income. The tax treatment of capital gains and losses for nondepreciated assets is shown in Table 12–5.

Table 12-5 Tax Treatment of Capital Gains and Losses for Corporations

Capital gain	Taxed as ordinary income.
Capital loss	Corporations may deduct capital losses only to the extent of capital gains. Any capital loss in the current year that exceeds capital gains can be carried back 3 years and forward for up to 5 years.

SUMMARY

Since income taxes are part of most problems, no realistic economic analysis can ignore their consequences. Income taxes make the U.S. government a partner in every business venture. Thus the government benefits from all profitable ventures and shares in the losses of unprofitable ventures.

For corporations, taxable income equals gross income *minus* all ordinary and necessary expenditures (except capital expenditures) and depreciation and depletion charges. The income tax computation (whether for an individual or a corporation) is relatively simple, with rates ranging from 10 to 39%. The proper rate to use in an economic analysis is the incremental tax rate applicable to the increment of taxable income being considered.

Most corporations pay state income taxes in addition to federal income taxes. Since state income taxes are an allowable deduction in computing federal taxable income, it follows that the taxable income for the federal computation is lower than the state taxable income.

$$\text{Combined state and federal incremental tax rate} \\ = \Delta\text{State tax rate} + (\Delta\text{Federal tax rate})(1 - \Delta\text{State tax rate})$$

To introduce the effect of income taxes into an economic analysis, the starting point is a before-tax cash flow. Then the depreciation schedule is deducted from appropriate parts of the before-tax cash flow to obtain taxable income. Income taxes are obtained by multiplying taxable income by the proper tax rate. Before-tax cash flow less income taxes equals the after-tax cash flow. This data is all captured in an after-tax cash flow table.

Taxes are considered even if only the costs of a project are known. The firm that does an engineering project must generate profits—or go out of business. Even if a firm has an unprofitable year, the tax law includes carry-forward and -backward provisions to transfer deductions to profitable years.

Governments can encourage corporate capital investment using tax policy, giving companies an incentive to make investments now versus deferring or not investing at all. Section 179 deductions, 100% bonus depreciation, and investment tax credits are examples of such policies.

Most new capital expenditures have been depreciated using some level of bonus depreciation since 2001 (see Figure 11–6), and the Tax Cuts and Jobs Act continues this. When using 100% bonus depreciation and claiming the depreciation deduction in the first quarter, the after-tax rate of return matches the before-tax rate of return. This is a large tax break for businesses. It also demonstrates that prospective decision making can be done using before-tax analysis. The additional calculation of after-tax rate of return (based on federal taxes) will yield precisely the same result. Only state and local taxes will be of concern.

When dealing with *nondepreciable* assets, there is a nominal relationship between before-tax and after-tax rate of return. It is

$$\text{After-tax rate of return} = (1 - \Delta\text{Tax rate})(\text{Before-tax rate of return})$$

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

12-1 Determine the income taxes for a firm with the following results:

Income from sales	$10.0 million
Total expenses	$ 2.4 million
Depreciation	$1.6 million

SOLUTION

12-2 A small firm has a taxable income of $560,000. They operate in a state that has a corporate income tax rate of 6.5%. What is the total federal and state tax that they must pay? What is their total incremental tax rate?

SOLUTION

12-3 A company, whose earnings put it in the 25% marginal tax bracket, is considering the purchase of a new piece of equipment for $25,000. The equipment will be depreciated by using the straight-line method over a 4-year depreciable life to a salvage value of $5000. It is estimated that the equipment will increase the company's earnings by $8000 for each of the 4 years it is used. Should the equipment be purchased? Use an interest rate of 10%.

SOLUTION

12-4 A corporation expects to receive $32,000 each year for 15 years if a particular project is undertaken. There will be an initial investment of $150,000. The expenses associated with the project are expected to be $7500 per year. Assume straight-line depreciation, a 15-year useful life, and no salvage value. Use a combined state and federal 28% marginal tax rate, and determine the project's after-tax rate of return.

SOLUTION

12-5 A corporation's marginal tax rate is 28%. An outlay of $35,000 is being considered for a new asset. Estimated annual receipts are $20,000 and annual disbursements $10,000. The useful life of the asset is 5 years, and it has no salvage value.

a. What is the prospective rate of return before income tax?
b. What is the prospective rate of return after taxes, assuming straight-line depreciation?

SOLUTION

12-6 A project under consideration by PHI Inc. is summarized. The company uses straight-line depreciation, pays taxes at the 30% marginal rate, and requires an after-tax MARR of 12%. Use net present worth to determine whether the project should be undertaken.

First cost	$75,000
Annual revenues	26,000
Annual costs	13,500
Salvage value	15,000
Useful life	10 years

SOLUTION

12-7 PARC, a large profitable firm, has an opportunity to expand one of its production facilities at a cost of $375,000. The equipment is expected to have an economic life of 10 years and to have a resale value of $25,000 after 10 years of use. If the expansion is undertaken, PARC expects that income will increase by $60,000 for Year 1, and then increase by $5000 each year through Year 10. The annual operating cost is expected to be $5000 for the first year and to increase by $250 per year thereafter. If the equipment is purchased, PARC will depreciate it by using the straight-line method to a zero salvage value at the end of Year 8 for tax purposes. The applicable marginal tax rate is 28%.

If PARC's minimum attractive rate of return (MARR) is 15%, should the firm undertake this expansion?

SOLUTION

The Salsaz-Hot manufacturing company must replace a machine used to crush tomatoes for its salsa. The industrial engineer favors a machine called the Crusher. Information concerning the machine is given.

First cost	$95,000
Annual productivity savings	19,000
Annual operating costs	6,000
Annual insurance cost*	1,750
Property taxes equal to 5% of the first cost are payable at the end of each year.	
*Payable at the beginning of each year	

12-8

Depreciable salvage value	$10,000
Actual salvage value	14,000
Depreciable life	6 years
Actual useful life	10 years
Depreciation method	SL

Relevant financial information for Salsaz-Hot:

Marginal tax rate	26%
MARR	10%

Determine the net present worth.

SOLUTION

A company bought an asset at the beginning of 2018 for $100,000. The company now has an offer to sell the asset for $60,000 at the end of 2019. Using double declining balance, determine the capital loss or recaptured depreciation that would be realized for 2018.

12-

9

Depreciable	Salvage	Recaptured	Capital
Life (years)	Value*	Depreciation	Loss
6	0		

*Assumed for depreciation purposes.

SOLUTION

12-10 Scallop Corporation purchased oil exploration equipment for $600,000 that will be depreciated over 10 years using the double declining balance method. Combined state and federal tax rate is 24%. The equipment may be rented each year for $330,000, and will then be sold after 5 years for $200,000. What is the after-tax rate of return?

SOLUTION

12-11 New equipment costing $30,000 has a 5-year life and no salvage value. Benefits are expected to be $8000 per year. The equipment qualifies for 100% bonus depreciation. The firm has a 28% combined marginal income tax rate. What is the after-tax rate of return?

SOLUTION

12-12 Macoupin Mining bought $60,000 of equipment that qualified for 40% bonus depreciation, with the balance using 5-year MACRS depreciation. The equipment saved $16,000 each year, and was sold for $20,000 after 5 years of use. The combined incremental tax rate is 26%, and the firm's after-tax MARR is 12%. Was this a wise investment?

SOLUTION

12-13 An asset with 5-year MACRS life will be purchased for $10,000. It will produce net annual benefits of $2000 per year for 6 years, after which time it will have a net salvage value of zero and will be retired. The company's marginal tax rate is 24%. Calculate the after-tax cash flows.

SOLUTION

12-14 A large and profitable company, in the 34% marginal tax bracket, is considering the purchase of a new piece of machinery that will yield benefits of $10,000 for Year 1, $15,000 for Year 2, $20,000 for Year 3, $20,000 for Year 4, and $20,000 for Year 5. The machinery is to be depreciated by using the modified accelerated cost recovery system (MACRS) with a 3-year recovery period. The MACRS percentages are 33.33, 44.45, 14.81, 8.41, respectively, for Years 1, 2, 3, and 4. The company believes the machinery can be sold at the end of 5 years of use for 25% of the original purchase price. If the company requires a 12% after-tax rate of return, what is the maximum purchase cost it can pay?

SOLUTION

12-15 An office is looking at enhancing their ERP system to replace 5 people. Each person earns $40,000 per year, with benefits counting for an additional 40% of salaries. The office uses 10% as an after-tax interest rate. Maintenance is assumed to be 5% of cost, and property taxes are 2% of cost. Depreciation is straight-line over 5 years, and income taxes are 24%. How much of a computer investment can be justified?

SOLUTION

2-16 A developer is deciding whether to purchase some heavy-duty landscaping equipment. The alternative is to rent the equipment. One set of equipment will cost $40,000 with no salvage value, depreciated using straight-line over 5 years. Operating and maintenance costs are $320 per day. Similar equipment can be rented for $600 per day (weekends included). The analysis needs to be on an after-tax basis. The combined marginal income tax rate is 26%, and the after-tax MARR is 10%. How many days per year must the equipment be used in order to justify its purchase?

SOLUTION

12-17 ACME Coyote Products bought processing equipment for $275,000, eligible for the Section 179 depreciation. The equipment saves the firm $70,000 per year and is good for 5 years, when it will have a negligible salvage value. ACME has a 24% tax rate. What is the after-tax rate of return on this purchase?

SOLUTION

12-18 A company has purchased a major piece of equipment that has a useful life of 20 years. An analyst trying to decide on a maintenance program has narrowed the choices to two alternatives. Alternative *A* is to perform $1000 of major maintenance every year. Alternative *B* is to perform $5000 of major maintenance only every fourth year. In either case, maintenance will be performed during the last year so that the equipment can be sold for $10,000. If the MARR is 12%, which maintenance plan should be chosen? Is it possible that the decision would change if income taxes were considered? Why or why not?

SOLUTION

12-19 A large company must build a bridge to have access to land for expansion of its manufacturing plant. The bridge could be fabricated of normal steel for an initial cost of $30,000 and should last for 15 years. Maintenance will cost $1000 per year. If the steel used were more corrosion resistant, the annual maintenance cost would be only $100 per year, although the life would be the same. In 15 years there would be no salvage value for either bridge. The company pays combined federal and state taxes at the 32% marginal rate and uses straight-line depreciation. If the minimum attractive after-tax rate of return is 12%, what is the maximum amount that should be spent on the corrosion-resistant bridge?

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

These problems can be solved by hand, but most will be solved much more easily with a spreadsheet.

Corporate Taxes

12-1 From the perspective of taxes paid, describe the difference between capital asset costs and other costs-of-operation that are expensed.

12-2 E (*a*) If permitted to choose between depreciating a cost over several years versus expensing it in a single year, which would you choose for your company? What factors might come into play in your recommendation?
(*b*) Firms can reduce the taxes they pay in the U.S. by setting internal transfer prices so the “profit” is earned in countries with low tax rates or by selling themselves to an international firm. What are the ethical pros and cons of these practices?

12-3 G Earth Powered Oil Company has purchased green engineering technology equipment for algae farms that turn sunlight into automotive biofuel. Two sets of equipment, each costing $200,000, are needed. One set is being depreciated using MACRS depreciation and the other is being depreciated with bonus depreciation with zero salvage value. Assume the company pays taxes annually and the tax rate is constant. Does the firm save more on income taxes with either one of the equipment sets? If so, which one?

12-4 A Compute income taxes owed for a firm with the following data:

Gross income from sales	$ 20 million
Accumulated business expenses	$ 5.5 million
Depreciation charges on assets	$ 3.5 million

A firm’s annual revenues are $850,000. Its expenses for the year are

12-5 $615,000, and it claims $135,000 in depreciation expenses. What does it pay in taxes, and what is its after-tax income?

12-6 A

A major industrialized state has a state corporate tax rate of 9.6% of taxable income. If a corporation has a state taxable income of $275,000, what is the total state and federal income tax it must pay? Also, compute its combined incremental state and federal income tax rate.

12-7 A company wants to set up a new office in a country where the corporate tax rate is: 15% of first $50,000 profits, 25% of next $25,000, 34% of next $25,000, and 39% of everything over $100,000. Executives estimate that they will have gross revenues of $500,000, total costs of $300,000, $30,000 in allowable tax deductions, and a one-time business start-up credit of $8000. What is taxable income for the first year, and how much should the company expect to pay in taxes?

12-8 A

To increase its market share, Sole Brother Inc. decided to borrow $50,000 for more advertising for its shoe retail line. The loan is to be paid in four equal annual payments with 15% interest. The loan is discounted 12 points. The first 6 "points" are an additional interest charge of 6% of the loan, deducted immediately from what is received from the $50,000 loan. Another 6 points or $3000 of additional interest is deducted as four $750 additional annual interest payments. What is the after-tax interest rate on this loan, if the firm's combined tax rate is 28%?

12-9 Concepcion Industries paid $308,000 in federal taxes last year. If business expenses and depreciation charges were $345,000, what were their gross sales for the year?

Straight-Line Depreciation

12-10 A salad oil bottling plant can either buy caps for the glass bottles at 5¢ each or install $500,000 worth of plastic molding equipment and manufacture the caps at the plant. The manufacturing engineer estimates the material, labor, and other costs would be 3¢ per cap.
(*a*) If 11 million caps per year are needed and the molding

A

equipment is installed, what is the payback period?
(*b*) The plastic molding equipment would be depreciated by straight-line depreciation using a 6-year useful life and no salvage value. Assuming a combined 26% income tax rate, what is the after-tax payback period, and what is the after-tax rate of return?

12-11

A computer-controlled milling machine will cost Ajax Manufacturing $65,000 to purchase plus $4700 to install.
(*a*) If the machine would have a salvage value of $6600 at EOY 20, how much could Ajax charge annually to depreciation of this equipment? Ajax uses straight-line depreciation.
(*b*) What is the book value of the machine at EOY 3?
(*c*) Ajax Manufacturing earns a net profit before tax (also called a taxable income) of $28,800,000. How much tax would Ajax owe for this year?

12-12
A

Contributed by Paul R. McCright, University of South Florida
The effective combined tax rate in a firm is 28%. An outlay of $2 million for certain new assets is under consideration. Over the next 9 years, these assets will be responsible for annual receipts of $650,000 and annual disbursements (other than for income taxes) of $225,000. After this time, they will be used only for stand-by purposes with no future excess of receipts over disbursements.
(*a*) What is the prospective rate of return before income taxes?
(*b*) What is the prospective rate of return after taxes if straight-line depreciation can be used to write off these assets for tax purposes in 9 years?
(*c*) What is the prospective rate of return after taxes if it is assumed that these assets must be written off for tax purposes over the next 20 years, using straight-line depreciation?

12-13
G

A project using passive heating/cooling design concepts to reduce energy costs requires an investment of $125,000 in equipment (straight-line depreciation with a 10-year depreciable life and $0 salvage value), and $30,000 in labor (not depreciable). At the end of 10 years, the project will be terminated. Assuming a combined tax rate of 26% and after-tax MARR of 15%, determine the project’s after-tax present worth.

12-14 A Florida Construction Equipment Rentals (FCER) purchases a new 10,000-pound-rated crane for rental to its customers. This crane costs $1,125,000 and is expected to last for 25 years, at which time it will have an expected salvage value of $147,000. FCER earns $195,000 before-tax cash flow each year in rental income from this crane, and its total taxable income each year is between $10M and $15M. If FCER uses straight-line depreciation and a MARR of 15%, what is the present worth of the after-tax cash flow for this equipment? Should the company invest in this crane? *Contributed by Paul R. McCright, University of South Florida*

12-15 A firm manufactures padded shipping bags. A cardboard carton should contain 100 bags, but machine operators fill the cardboard cartons by eye, so a carton may contain anywhere from 98 to 123 bags (average = 105.5 bags). Each padded bag costs $0.03. Management realizes that they are giving away 5 1/2% of their output by overfilling the cartons. One solution is to automate the filling of shipping cartons. This should reduce the average quantity of bags per carton to 100.3, with almost no cartons containing fewer than 100 bags.
The equipment would cost $18,600 and straight-line depreciation with a 10-year depreciable life and a $3600 salvage value would be used. The equipment costs $16,000 annually to operate. 200,000 cartons will be filled each year. This large profitable corporation has a 28% combined federal-plus-state incremental tax rate. Assume a 10-year study period for the analysis and an after-tax MARR of 15%.
Compute:
(*a*) The after-tax present worth
(*b*) The after-tax internal rate of return
(*c*) The after-tax simple payback period

12-16 ACDC Company is considering the installation of a new machine that costs $150,000. The machine is expected to lead to net income of $44,000 per year for the next 5 years. Using straight-line depreciation, $0 salvage value, and an effective income tax rate of 28%, determine the after-tax rate of return for this investment. If the company's after-tax MARR rate is 12%, would this be a good investment or not?

Contributed by Mukasa Ssemakula, Wayne State University

12-17 An old duplex was bought for \$200,000 cash. Both sides were rented for \$2500 per month. The total annual expenses for property taxes, repairs, gardening, and so forth are estimated at \$200 per month. For tax purposes, straight-line depreciation over a 20-year remaining life with no salvage value is used. Of the total \$200,000 cost of the property \$50,000 is the value of the lot. Assume 38% incremental income tax bracket (combined state and federal taxes) applies throughout the 20 years.

If the property is held for 20 years, what after-tax rate of return can be expected?

(*a*) Assume the building and the lot can be sold for the lot's \$50,000 estimated value.

(*b*) A more optimistic estimate of the future value of the building and the lot is that the property can be sold for \$150,000 at the end of 20 years.

Declining Balance Depreciation

12-18 A

A firm is considering the following investment project:

Year	Before-Tax Cash Flow (thousands)
0	−\$1000
1	500
2	340
3	244
4	100
5	100 125 Salvage value

The project has a 5-year useful life with a \$125,000 salvage value, as shown. Double declining balance depreciation will be used, assuming the \$125,000 salvage value. The combined income tax rate is 24%. If the firm requires a 10% after-tax rate of return, should the project be undertaken?

12-19 The Shellout Corp. owns a piece of petroleum drilling equipment that costs $300,000 and will be depreciated over 10 years by double declining balance depreciation. There is a combined 30% tax rate. Shellout will lease the equipment to others and each year receive $165,000 in rent. At the end of 5 years, the firm will sell the equipment for $80,000. What is the after-tax rate of return Shellout will receive from this equipment investment?

12-20 A

An automaker is buying some special tools for $100,000. The tools are being depreciated by double declining balance depreciation using a 4-year depreciable life and a $6250 salvage value. It is expected the tools will actually be kept in service for 6 years and then sold for $6250. The before-tax benefit of owning the tools is as follows:

Year	Before-Tax Cash Flow
1	$30,000
2	30,000
3	35,000
4	40,000
5	10,000
6	10,000
	6,250 Selling price

Compute the after-tax rate of return for this investment situation, assuming a 30% incremental tax rate.

12-21 This is the continuation of Problem 12-20. Instead of paying $100,000 cash for the tools, the corporation will pay $20,000 now and borrow the remaining $80,000. The depreciation schedule will remain unchanged. The loan will be repaid by 4 equal end-of-year payments of $25,240.

Prepare an expanded cash flow table that takes into account both the special tools and the loan.

(*a*) Compute the after-tax rate of return for the tools, taking into

account the $80,000 loan.
(*b*) Explain why the rate of return obtained in part (*a*) is different from the rate of return obtained in Problem 12-20.

12-22 A

A firm may invest in equipment that will be depreciated by double declining balance depreciation with conversion to straight-line depreciation in year 5. For depreciation purposes a $700,000 salvage value at the end of 6 years is assumed. But the actual value is thought to be $1,000,000, and it is this sum that is shown in the before-tax cash flow.

Year	Before-Tax Cash Flow (in $1000)
0	−$12,000
1	1,727
2	2,414
3	2,872
4	3,177
5	3,358
6	1,997
	1,000 Salvage value

If the firm wants a 9% after-tax rate of return and its combined incremental income tax rate is 24%, determine by annual cash flow analysis whether the investment is desirable.

Bonus Depreciation

Assume all bonus depreciation occurs in Year 1 unless otherwise stated.

12-23 A firm has invested $60,000 in machinery with a 5-year useful life. The machinery will have no salvage value, as the cost to remove it will equal its scrap value. The uniform annual benefits from the machinery are $15,000. For a combined 45% income tax rate, and 100% bonus depreciation, compute the after-tax rate of return.

12-24 A

A farmer bought a new harvester for $120,000. The harvester's operating expenses averaged $10,000 per year but the harvester saved $40,000 per year in labor costs. It was depreciated over a life of 5 years using the 100% bonus depreciation method, assuming a salvage value of $30,000. The farmer sold the harvester for only $10,000 at the end of the fifth year. Given an income tax rate of 30% and a MARR rate of 5% per year, determine the after-tax net present worth of this investment. *Contributed by Mukasa Ssemakula, Wayne State University*

12-25 G

A firm has invested $400,000 in car-washing equipment. They will depreciate the equipment by 60% bonus depreciation with the balance using 5-year MACRS, assuming a $50,000 salvage value at the end of the 5-year useful life. The firm is expected to have a before-tax cash flow, after meeting all expenses of operation (except depreciation), of $165,000 per year. The firm's combined corporate tax rate is 28%.

(*a*) If the projected income is correct, and the equipment can be sold for $100,000 at the end of 5 years, what after-tax rate of return would the corporation receive from this venture?

(*b*) Summarize the environmental impact of commercial car washing. Include how wash water drainage is directed, how discharge sources are treated (or not), the impacts of sediment, detergents, waxes, and heavy metals, and local/state policies/practices.

12-26 A

A mining corporation purchased $120,000 of production machinery and depreciated it using 40% bonus depreciation with the balance using 5-year MACRS depreciation, a 5-year depreciable life, and zero salvage value. The corporation is a profitable one that has a 22% combined incremental tax rate.

At the end of 5 years the mining company changed its method of operation and sold the production machinery for $40,000. During the 5 years the machinery was used, it reduced mine operating costs by $32,000 a year, before taxes. If the company MARR is 12% after taxes, was the investment in the machinery a satisfactory one?

Zeon, a large, profitable corporation, is considering adding some automatic equipment to its production facilities. An investment of

12-27 $120,000 will produce an annual benefit of $40,000. If the firm uses 60% bonus depreciation with the balance using 7-year MACRS depreciation, an 8-year useful life, and $12,000 salvage value, will it obtain the desired 12% after-tax rate of return? Assume that the equipment can be sold for its $12,000 salvage value at the end of the 8 years. Also assume a 28% income tax rate for state and federal taxes combined.

MACRS Depreciation

12-28 A A special power tool for plastic products costs $400,000 and has a 4-year useful life, no salvage value, and a 2-year before-tax payback period. Assume uniform annual end-of-year benefits.

(*a*) Compute the before-tax rate of return.

(*b*) Compute the after-tax rate of return, based on MACRS depreciation and a 24% combined corporate income tax rate.

12-29 G (*a*) The BVM Corp., a construction company, purchased a used hybrid electric pickup truck for $30,000 and used MACRS depreciation in the income tax return. During the time the company had the truck, they estimated that it saved $9500 a year. At the end of 4 years, BVM sold the truck for $9000. The combined federal and state income tax rate for BVM is 28%. Compute the after-tax rate of return for the truck.

(*b*) Hybrid electric vehicles (HEVs), such as the truck purchased by BVM Corp., are recognized for their fuel efficiency and earth-friendliness. However, no major technology is without negative environmental impacts. Summarize the key environmental issues, such as greenhouse emissions in use, life-cycle impacts of battery sources (such as lithium-ion and nickel metal hydride), vehicle production impacts, and other life-cycle impacts.

12-30 A A chemical company bought a small vessel for $550,000; it is to be depreciated by 10-year MACRS. When requirements changed suddenly, the chemical company leased the vessel to an oil company for 6 years at $100,000 per year. The lease also provided that the oil company could buy the vessel at the end of 6 years for $350,000. At the end of the 6 years, the oil company exercised its option and bought the vessel. The chemical company has a 24% combined incremental tax rate. Compute its after-tax rate of return on the vessel.

12-31 E

Xon, a small oil equipment company, purchased a new petroleum drilling rig for $2,000,000. Xon will depreciate it using MACRS depreciation. The drilling rig has been leased to a firm, which will pay Xon $750,000 per year for 8 years. After 8 years the drilling rig will belong to the firm. Xon has a 28% combined incremental tax rate and a 20% after-tax MARR

(*a*) Does the investment appear to be satisfactory?

(*b*) Some claim that the coal and/or oil industries are inherently unsustainable and harmful to the environment. Develop a short position summary to support and challenge this perspective.

12-32 A

The profitable Palmer Golf Cart Corp. is considering investing $300,000 in special tools for some of the plastic golf cart components. The present golf cart model will continue to be manufactured and sold for 5 years, after which a new cart design will be needed, together with a different set of special tools. The saving in manufacturing costs, owing to the special tools, is estimated to be $150,000 per year for 5 years. Assume MACRS depreciation for the special tools and a 27% combined income tax rate.

(*a*) What is the after-tax payback period for this investment?

(*b*) If the company wants a 12% after-tax rate of return, is this a desirable investment?

12-33

Granny's Butter and Egg Business is such that she pays an effective tax rate of 25%. Granny is considering the purchase of a new Turbo Churn for $25,000. This churn is a special handling device for food manufacture and has an estimated life of 4 years and a salvage value of $5000. The new churn is expected to increase net income by $8000 per year for each of the 4 years of use. If Granny works with an after-tax MARR of 10% and uses 3-year MACRS depreciation, should she buy the churn?

An engineer is working on the layout of a new research and experimentation facility. Two plant operators will be required. If,

12-34 A
however, an additional $100,000 of instrumentation and remote controls were added, the plant could be run by a single operator. The total before-tax cost of each plant operator is projected to be $35,000 per year. The instrumentation and controls will be depreciated by means of the modified accelerated cost recovery system (MACRS). If this corporation (22% combined corporate tax rate) invests in the additional instrumentation and controls, how long will it take for the after-tax benefits to equal the $100,000 cost? In other words, what is the after-tax payback period?

12-35
A corporation with a 27% combined income tax rate is considering the following investment in research equipment.

Year	Before-Tax Cash Flow
0	−$7,500,000
1	650,000
2	950,000
3	2,750,000
4	1,900,000
5	800,000
6	450,000

Prepare an after-tax cash flow table assuming MACRS depreciation.
(*a*) What is the before-tax rate of return?
(*b*) What is the after-tax rate of return?

12-36 A
Specialty Machining, Inc. bought a new multi-turret turning center for $250,000. The machine generated new revenue of $80,000 per year. Operating costs for the machine averaged $10,000 per year. Following IRS regulations, the machine was depreciated using the MACRS method, with a recovery period of 7 years. The center was sold for $75,000 after 5 years of service. The company uses an after-tax MARR rate of 12% and is in the 23% tax bracket. Determine the after-tax net present worth of this asset over the 5-year service period. *Contributed by Mukasa Ssemakula, Wayne State University*

ABC Co. is contemplating an $18,000 investment in a methane gas generator. They estimate gross income will be $4500 the first year

and increase by $500 each year over the next 10 years. Expenses of $300 the first year would increase by $250 each year over the next 10 years. ABC will depreciate the generator by MACRS
12-37 depreciation, assuming a 7-year property class. A 10-year-old
G methane generator has no market value. The combined income tax rate is 31%. (Remember that recaptured depreciation is taxed at the same 31% rate.)

(*a*) Construct the after-tax cash flow for the 10-year project life.

(*b*) Determine the after-tax rate of return on this investment. ABC Co. thinks it should be at least 8%.

(*c*) If ABC Co. could sell the generator for $10,000 at the end of the fifth year, what would the rate of return be?

12-38 A
Fleet Fleet rental car company purchased 10 new cars for a total cost of $180,000. The cars generated income of $150,000 per year and incurred operating expenses of $60,000 per year. The company uses MACRS depreciation and its marginal tax rate is 28% (*Note:* Per IRS regulations, cars have a class life of 5 years). The 10 cars were sold at the end of the third year for a total of $75,000. Assuming a MARR of 10% and using NPW, determine if this was a good investment on an after-tax basis. *Contributed by Mukasa Ssemakula, Wayne State University*

12-39
Mid-America Shipping is considering purchasing a new barge for use on its Ohio River routes. The new barge will cost $13.2 million and is expected to generate an income of $7.5 million the first year (growing $1M each year), with additional expenses of $2.6 million the first year (growing $400,000 per year). If Mid-America uses MACRS, is in the 26% tax bracket, and has a MARR of 12%, what is the present worth of the first 4 years of after-tax cash flows from this barge? Would you recommend that Mid-America purchase this barge? Does your answer change at 5 or 6 or 7 or …years? *Contributed by Paul R. McCright, University of South Florida*

12-40 A
An investor bought a racehorse for $1 million. The horse's average winnings were $700,000 per year and expenses averaged $200,000 per year. The horse was retired after 3 years, at which time it was sold to a breeder for $175,000. Assuming MACRS depreciation, a class life of 3 years, and an income tax rate of 28%, determine the investor's after-tax rate of return on this investment. *Contributed by*

Mukasa Ssemakula, Wayne State University

12-41 Tampa Electric Company (TECO) is planning a major upgrade in its computerized demand management system. In order to accommodate this upgrade, a building will be constructed on land already owned by the company. The building is estimated to cost $1.8M and will be opened in August of this year. The computer equipment for the building will cost $2.75M, and all office equipment will cost $225,000. Annual expenses for operating this facility (labor, materials, insurance, energy, etc.) are expected to be $325,000 for the rest of this year. Use of the new demand management system is expected to decrease fuel and other costs for the company by $1.8M this year. If the company expects to earn 9% on its investments, is in the 23% tax bracket, and uses a 20-year planning horizon, determine the estimated after-tax cash flow from this project for this year. *Contributed by Paul R. McCright, University of South Florida*

Solving for Unknowns

12-42 A

A store owner, Jing Lang, believes his business has suffered from the lack of adequate customer parking space. He may buy an old building and lot next to his store. He would demolish the old building and make off-street parking on the lot. Jing estimates that the new parking would increase his before-income-tax profit by $7000 per year. It would cost $2500 to demolish the old building.

Mr. Lang's accountant advised that both costs (the property and demolishing the old building) would be considered to comprise the total value of the land for tax purposes, and it would not be depreciable.

Mr. Lang would spend an additional $3000 right away to put a light gravel surface on the lot. He believes this may be charged as an operating expense immediately. His combined state and federal incremental income tax rate will average 28%. If Jing wants a 15% after-tax rate of return from this project, how much could he pay to purchase the adjoining land with the old building?

Assume that the analysis period is 10 years and that the parking lot could always be sold to recover the costs of buying the property and demolishing the old building.

12-43 The management of a private hospital is considering automating some back office functions. This would replace five personnel that currently cover three shifts per day, 365 days per year. Each person earns $35,000 per year. Company-paid benefits and overhead are 45% of wages. Money costs 8% after income taxes. Combined federal and state income taxes are 28%. Annual property taxes and maintenance are $2\frac{1}{2}$ and 4% of investment, respectively. Depreciation is 15-year straight line. Disregarding inflation, how large an investment in the automation project can be economically justified?

12-44 A A house and lot are for sale for $155,000. It is estimated that $45,000 is the land's value and $110,000 is the value of the house. The net rental income would be $12,000 per year after taking all expenses, except depreciation, into account. The house would be depreciated by straight-line depreciation using a 27.5-year depreciable life and zero salvage value.

Mary Silva, the prospective purchaser, wants a 10% after-tax rate of return on her investment after considering both annual income taxes and a capital gain when she sells the house and lot. At what price would she have to sell the house at the end of 10 years to achieve her objective? Assume that Mary has an incremental income tax rate of 24% in each of the 10 years and a capital gain rate of 20%.

12-45 A contractor has to choose one of the following alternatives in performing earthmoving contracts:

A. Buy a heavy-duty truck for $35,000. Salvage value is expected to be $8000 at the end of the vehicle's 7-year depreciable life. Maintenance is $2500 per year. Daily operating expenses are $200.

B. Hire a similar unit for $550 per day.

Based on a 10% after-tax rate of return, how many days per year must the truck be used to justify its purchase? Base your calculations on straight-line depreciation and a 28% income tax rate.

12-46 A large profitable company, in the 28% combined federal/state tax bracket, is considering the purchase of a new piece of equipment that will yield benefits of $10,000 in Year 1, $15,000 in Year 2, $20,000 in Year 3, and $20,000 in Year 4. The equipment is to be depreciated using

A 5-year MACRS depreciation starting in the year of purchase (Year 0). It is expected that the equipment will be sold at the end of Year 4 at 20% of its purchase price. What is the maximum equipment purchase price the company can pay if its after-tax MARR is 10%?

12-47 The Able Corporation is considering the installation of a small electronic testing device for use in conjunction with a government contract the firm has just won. The testing device will cost $20,000 and will have an estimated salvage value of $5000 in 5 years when the government contract is finished. The firm will depreciate the instrument by MACRS using 5 years as the class life. Assume that Able pays 27% federal and state corporate income taxes and uses 8% *after-tax* in economic analysis. What minimum equal annual benefit must Able obtain *before taxes* in each of the 5 years to justify purchasing the electronic testing device?

12-48 A

A sales engineer has the following alternatives to consider in touring his sales territory.

A. Buy a 2-year old used car for $14,500. Salvage value is expected to be about $5000 after 3 more years. Maintenance and insurance cost is $1000 in the first year and increases at the rate of $500/year in subsequent years. Daily operating expenses are $50/day.

B. Rent a similar car for $80/day.

Based on a 12% after-tax rate of return, how many days per year must he use the car to justify its purchase? You may assume that this sales engineer is in the 23% incremental tax bracket. Use MACRS depreciation.

12-49 G

For many firms, environmental regulation and oversight are converting external “social costs” to internal “real costs.” TDE Industries is considering investing in equipment to reduce emissions. Without the investment they expect to incur fines and fees of $750,000 per year. TDE uses a 28% combined tax rate, an after-tax MARR of 20%, and will depreciate such equipment as a 5-year MACRS property.

(*a*) Management has asked you to determine the maximum amount they should pay for equipment to eliminate these fines. Use a 6-year study period.

(*b*) What argument would you add to the economic analysis to justify the investment?

(*c*) If your manager decides to purchase $1M in equipment, what level of decreased fees would justify the investment? Use a 6-year study period.

Section 179, Bonus Depreciation, and Tax Credits

12-50 A Christopher wants to add a solar photovoltaic system to his home. He plans to install a 2-kW system and has received a quote from an installer who will install this unit for $19,750. The federal government will give him a tax credit of 30% of the cost, and the state will give a 10% tax credit. State law requires the utility company to buy back all excess power generated by the system. Christopher's annual power bill is estimated to be $2000, and this will be eliminated by the solar system. Christopher expects to receive a check from the power company for $600 each year for his excess production. If the tax credits are received at EOY 1 and Christopher receives a $2000 savings plus a $600 income at the end of each year, use present worth to determine if the system pays for itself in 8 years. Assume that Christopher earns 3% on all investments. *Contributed by Paul R. McCright, University of South Florida*

12-51 Rework Problem 12-30 assuming that a $1,000,000 Section 179 deduction and a 60% bonus depreciation deduction apply.

12-52 A Rework Problem 12-31 assuming that a $1,000,000 Section 179 deduction applies along with 40% bonus depreciation.

12-53 Rework Problem 12-33 assuming that a 60% bonus depreciation applies.

12-54 A Rework Problem 12-36 assuming that 40% bonus depreciation applies.

12-55 Rework Problem 12-10 assuming that a 10% Investment Tax Credit (ITC) that does not reduce the depreciable basis applies.

12-56 A Rework Problem 12-12 assuming that a 10% Investment Tax Credit (ITC) that does not reduce the depreciable basis applies.

12-57 Rework Problem 12-25 assuming that a 10% Investment Tax Credit (ITC) that does reduce the depreciable basis applies.

12-58 Rework Problem 12-26 assuming that a 10% Investment Tax Credit (ITC) that does reduce the depreciable basis applies.
A

Multiple Alternatives

12-59 A small-business corporation is considering whether to replace some equipment in the plant. An analysis indicates there are five alternatives in addition to the do-nothing option, Alt. *A*. The alternatives have a 5-year useful life with no salvage value. Straight-line depreciation would be used.

Alternatives	Cost (thousands)	Before-Tax Uniform Annual Benefits (thousands)
A	$0	$0
B	25	8
C	10	5
D	5	2
E	15	5

The firm has a combined federal and state income tax rate of 26%. If the corporation expects a 8% after-tax rate of return for any new investments, which alternative should be selected?

A corporation with $7 million in annual taxable income and no state tax is considering two alternatives:

	Before-Tax Cash Flow ($1000)	
Year	Alt. 1	Alt. 2
0	−$10,000	−$20,000
1–10	4,500	4,500
11–20	0	4,500

12-60

A Both alternatives will be depreciated by 40% bonus depreciation taken in year 0 plus 10-year MACRS depreciation. Neither alternative is to be replaced at the end of its useful life. If the corporation has a minimum attractive rate of return of 10% *after taxes,* which alternative should it choose? Solve the problem by:

(*a*) Present worth analysis
(*b*) Annual cash flow analysis
(*c*) Rate of return analysis
(*d*) Future worth analysis
(*e*) Benefit–cost ratio analysis

12-61 If a firm's after-tax minimum attractive rate of return is 10% and its combined incremental income tax rate is 28%, which alternative should be selected? Use incremental IRR.

	Alt. *A*	Alt. *B*
Initial cost	$11,000	$33,000
Uniform annual benefit	3,000	9,000
End-of-depreciable-life salvage value	2,000	3,000
Depreciation method	40% bonus plus MACRS	40% bonus plus MACRS
End-of-useful-life salvage value obtained	2,000	5,000
Depreciable life, in years	5	5
Useful life, in years	5	5

12-62 Use the after-tax IRR method to evaluate the following three alternatives using 60% bonus depreciation in Year 0 and 3-year MACRS, and offer a recommendation. The after-tax MARR is 25%, the project life is 5 years, and the firm has a combined incremental tax rate of 31%.

A

Alt.	First Cost	Annual Costs	Salvage Value
A	$14,000	$2500	$ 5,000
B	18,000	1000	10,000
C	10,000	5000	0

12-63 VML Industries has need of specialized yarn manufacturing equipment for operations over the next 3 years. The firm could buy the machinery for $95,000 and depreciate it using MACRS. Annual maintenance would be $7500, and it would have a salvage value of $25,000 after 3 years. Another alternative would be to lease the same machine for $45,000 per year on an "all costs" inclusive lease (maintenance costs included in lease payment). These lease payments are due at the beginning of each year. VML Industries uses an after-tax MARR of 18% and a combined tax rate of 28%. Do an after-tax present worth analysis to determine which option is preferred.

12-64 Padre Pio owns a small business and has taxable income of $150,000 and a combined tax rate of 28%. He is considering four mutually exclusive alternative models of machinery. Which machine should be selected on an after-tax basis? The after-tax MARR is 15%. Assume that each machine qualifies for bonus depreciation, which is taken in Year 1, and can be sold for a market value that is 25% of the purchase cost, and the project life is 10 years.

A

Model	I	II	III	IV
First cost	$9000	$8000	$7500	$6200
Annual costs	25	200	300	600

12-65 LoTech Welding can purchase a machine for $175,000 and depreciate it as 5-year MACRS property. Annual maintenance would be $9800, and its salvage value after 8 years is $15,000. The machine can also be leased for $35,000 per year on an "all costs" inclusive lease (maintenance costs included). Lease payments are due at the beginning of each year, and they are tax-deductible. The firm's combined tax rate for state and federal income taxes is 28%. If the firm's after-tax interest rate is 28%, which alternative has the lower EAC and by how much?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 26	**Molehill & Mountain Movers**
	Compares depreciation methods with option for inflation.
CASE 28	**Olives in Your Backyard**
	Emphasizes taxes and sensitivity analysis.
CASE 29	**New Fangled Manufacturing**
	Emphasizes taxes and sensitivity analysis.
CASE 36	**Brown's Nursery (Part A)**
	After-tax analysis of expansion opportunity.
CASE 38	**West Muskegon Machining and Manufacturing**
	More complex inflation and tax problem with sunk cost and leverage.
CASE 53	**Problems in Pasta Land**
	Long case statement. Includes taxes and limited uncertainty.

APPENDIX 12A
Taxes and Personal Financial Decision

Making

Key Words

adjusted gross income

buy term invest difference

cash value benefit

death benefit

deductible

defined benefit

defined contribution

gross income

individual retirement account

insurance

liability insurance

needs/savings/wants

Perkins loans

premiums

Stafford loans

taxable income

tax refund

withholding

401(k) plan

403(b) plan

In this appendix our goal is to focus on decisions involving personal income taxes and likely financial scenarios that students will face. We first describe how taxes are computed for individuals. As is the case with corporations, individuals should consider decisions that will be affected by taxes on an *after-tax* basis. We detail several example applications of this knowledge to common personal finance scenarios including student loans and retirement accounts. Lastly, we provide details on decisions involving insurance and personal budgeting.

INCOME TAXES FOR INDIVIDUALS

Calculating Taxes Due

An important expense for individuals each year is federal taxes. The dollar amount due depends on both the level of taxable income and the applicable income tax rates. Calculating annual federal taxes for individuals involves the following steps:

1. Calculate **gross income**, which is the sum of all wages and income streams:

$$\text{Gross income} = \text{Wages, salary, etc.} + \text{Interest income} + \text{Dividends} + \text{Capital gains} + \text{Unemployment compensation} + \text{Other income} \quad (12A\text{-}1)$$

2. Determine **adjusted gross income (AGI)** from gross income. AGI includes allowable adjustments such as retirement plan contributions.

$$\text{Adjusted gross income} = \text{Gross income} - \text{allowable tax adjustments} \quad (12A\text{-}2)$$

3. Calculate **taxable income** from AGI and allowable exemptions and deductions. Historically one exemption was allowed for each person who depends on the gross income. Taxpayers are allowed a standard deduction to

account for various expenses. However, a person may itemize deductions if they will exceed the standard deduction. These itemized deductions (with limits) include medical expenses, state and local income taxes, real estate and property taxes, home mortgage interest, charitable contributions, theft and losses, job expenses, and other categories.

$$\text{Taxable income} = \text{Adjusted gross income} - \text{personal exemption(s)} - \text{Itemized deductions or standard deduction} \qquad (12A\text{-}3)$$

4. Determine income tax from taxable income and applicable tax rates.

$$\text{Income tax} = \sum (\text{Taxable income in bracket}) \times (\text{Tax rate for bracket})$$
$$\text{or} = \text{Tax from lower brackets} + (\text{Taxable income in bracket}) \qquad (12A\text{-}4)$$

$\times$ (Tax rate for bracket)(Tables 12A-1a & 1b)

Withholding Taxes

The federal government uses **withholding** to collect taxpayer payments on a regular basis throughout the year. This includes income, social security, and Medicare taxes. In most cases, employers withhold some portion of wages from each employee each pay period based on income and number of dependents claimed. In the case where more taxes are withheld than are due, the taxpayer receives a **tax refund**. If tax withholdings are less than taxes due, the difference must be paid—which may involve interest and penalties if too large.

Tax Cuts and Jobs Act and Personal Income Taxes

The Tax Cuts and Jobs Act (2017) simplified tax calculations by eliminating personal exemptions. It also nearly doubled the standard deduction and added limits to the itemized deduction for state and local taxes—so itemizing deductions makes sense for fewer taxpayers. Equation 12A–3 becomes:

$$\text{Taxable income} = \text{Adjusted gross income} - \text{Standard deduction or itemized deductions} \qquad (12A\text{-}5)$$

The standard deduction for 2018 tax returns is $12,000 for single taxpayers and $24,000 if married and filing jointly. Important **itemized deductions** include:

• Medical and dental expenses exceeding 7.5% of AGI
• State and local taxes (SALT) up to $5000 on single returns and $10,000 on joint returns. This includes income or sales taxes (but *not* both) and property taxes.
• Home mortgage interest (limit of $750,000 in debt)
• Charitable contributions (limit of 60% of AGI)

Tables 12A–1a and 12A–1b show the brackets and tax rates for single and married filing jointly. Example 12A–1 illustrates how the values in these tables are constructed.

Table 12A–1a 2018 Tax Rate Schedule, Filing Status Is Single

Taxable Income	Tax Rate
$0 to $9525	10%
$9526 to $38,700	$952.50 plus 12% of the amount over $9525
$38,701 to $82,500	$4453.50 plus 22% of the amount over $38,700
$82,501 to $157,500	$14,089.50 plus 24% of the amount over $82,500
$157,501 to $200,000	$32,089.50 plus 32% of the amount over $157,500
$200,001 to $500,000	$45,689.50 plus 35% of the amount over $200,000
$500,001 or more	$150,689.50 plus 37% of the amount over $500,000

Table 12A–1b 2018 Tax Rate Schedule, Filing Status Is Married Filing Jointly

Taxable Income	Tax Rate
$0 to $19,050	10%

\$19,051 to \$77,400	\$1905.00 plus 12% of the amount over \$19,050
\$77,401 to \$165,000	\$8907.00 plus 22% of the amount over \$77,400
\$165,001 to \$315,000	\$28,179.00 plus 24% of the amount over \$165,000
\$315,001 to \$400,000	\$64,179.00 plus 32% of the amount over \$315,000
\$400,001 to \$600,000	\$91,379.00 plus 35% of the amount over \$400,000
\$600,001 or more	\$161,379.00 plus 37% of the amount over \$600,000

Note: Tables are also available for unmarried individual with dependent relatives (“head of household”) and married taxpayers filing separately.

EXAMPLE 12A-1

A young engineer earned \$85,000 in wages and other income in the last tax year. She is unmarried and from company records knows that \$13,000 has been withheld for taxes. If she had \$8000 in charitable donations last year, calculate if she will receive a refund or owe taxes.

SOLUTION

$$
\begin{aligned}
\textbf{Taxable Income} &= \text{Adjusted Gross Income} - \text{Standard or Itemized Deductions} \\
&= 85{,}000 - 12{,}000 \text{ (standard deduction was greater)} \\
&= \$73{,}000
\end{aligned}
$$

$$
\textbf{Federal Income Tax} = 4453.50 + (0.22)(73{,}000 - 38{,}700) = \$11{,}999.50
$$

$$
\textbf{The \$4453.50 = 10\% of \$9525 + 12\% of (\$38,700 − \$9525)}
$$

Since her company withheld \$13,000 in taxes, she will receive a refund of \$1000.50(=13, 000 − 11, 999.50) from the federal government.

Combined Federal and State Income Taxes

Individuals, like corporations, are often subject to both federal and state income taxes. However, when state taxes are deductible (a person must be itemizing deductions) Equation 12-2 applies. If the standard deduction is used then Equation 12A-6 applies.

$$\text{Combined incremental tax rate} = \Delta\text{State tax rate} + \Delta\text{Federal tax rate} \quad (12A\text{-}6)$$

EXAMPLE 12A-2

A single engineer's expected taxable income is expected to be $92,000. Instead of taking a planned end-of-year vacation, the engineer could accept assignment to a project with a year-end deadline. The expected overtime would increase earnings by $8000. The marginal rate for state income tax at either total income is 8%. What is the engineer's combined marginal tax rate?

1. If deductions are itemized.
2. If the standard deduction is used.

SOLUTION

In either case the marginal tax rate at the federal level is 24%.

1. If deductions are itemized then Eq. 12-2 applies.

$$\text{Combined rate} = 0.08 + 0.24(1 - 0.08) = 0.08 + 0.221 = 30.1\%$$

2. If the standard deduction is used then Eq. 12A-6 applies.

$$\text{Combined rate} = 0.08 + 0.24 = 32\%$$

Capital Gains/Losses for Individuals

As with corporations, individual taxpayers incur capital gains or losses when selling houses, land, stocks, art, jewelry, and other collectables. Depending on tax bracket, type of asset, and duration of ownership, the capital gain tax rate can range from 0% to 37%. The most important capital gain provision for many is the exemption of up to a $500,000 gain from sale of a principal residence. Because this Appendix is only an introduction to personal income taxes, the regulations for the sale of a house and capital gains tax rates are not detailed here.

Tax Credits vs. Tax Deductions

This chapter and appendix have already included examples of *tax credits* and *deductions from taxable income*. Comparing the choices available in “writing off” a portion of college tuition expenses allows us to directly compare the value of a tax deduction vs. a tax credit. Table 12A–2 summarizes tuition-related federal tax deductions and credits for single filers. Income levels are doubled for joint filers, and so is the education expense deduction. The credits are not changed.

Table 12A–2 Federal tax deductions and credits for single filers (2015)

Modified Adjusted		
Provision for Education Expenses	Gross Income (MAGI)	Detai
Education Expenses	$0–65,000	100%
Deduction	$65,000–80,000	100%
American Opportunity	$0–80,000	100%
Tax Credit	$80,000–90,000	Phase
Lifetime Learning	$0–57,000	20%
Credit	$57,000–67,000	Phase

The reduction in allowable values is proportional to percentage of phase out bracket. Income levels and the Education Expenses Deduction are doubled for joint filers.

Student Loan Interest Deduction

The interest on most consumer loans is not tax deductible, but interest on student loans (up to the limits shown in Table 12A–3) and home mortgages is tax deductible.

EXAMPLE 12A-3

An independent first-year college student has an adjusted gross income of $18,000 in 2018 and pays $10,000 in tuition. Which choice described in Table 12A–2 is the most valuable?

SOLUTION

The student files a single return, so the limits in Table 12A–2 apply, and the income is within the brackets where the full value of each deduction or credit is available. The student’s taxable income is $6000 (=18,000−12,000), which is in the 10% tax bracket.

Only $4000 of the tuition payments is deductible under the education expenses deduction. Since deductions reduce taxable income, the reduction in taxes would be $400 (= 4000 × 0.10).

While deductions reduce *taxable income*, credits reduce *income taxes*. Thus, both credits are much more attractive than the deduction. The American Opportunity Credit at $2500 allows $500 (= 25% of next $2000) more than the Lifetime Learning Credit, which is limited to $2000.

Table 12A–3 Federal tax deduction for student loan interest (2018)

Modified Adjusted Gross Income (MAGI)	Details

$ 0–65,000 (single)	
$ 0–135,000 (joint)	$2500 or total paid
$ 65,000–80,000 (single)	
$ 135,000–160,000 (joint)	Phase out bracket

EXAMPLE 12A-4

A recent engineering graduate is single and earning $60,000 annually with a government labs group. What is the after-tax cost of the $4500 in student loan interest that the engineer paid last year?

SOLUTION

Only $2500 of the interest is deductible from taxable income. With $60,000 in income, the engineer will be in a 22% tax bracket. Thus, there is a reduced tax bill (savings) of $550 (= 2500 × 0.22). The after-tax cost of the interest is $3950 (= 4500 − 550).

EXAMPLE 12A-5

Two engineers are married and filing a joint return. Last year their MAGI was $155,000. What is the amount of allowable deduction if they paid $800 interest on their student loans? If they paid $2750 in student loan interest?

SOLUTION

With $800 interest the deduction must be reduced because their MAGI falls in the $135,000 to 165,000 phase out range.

$$Reduce_{800} = \$800 \times \frac{155{,}000 - 135{,}000}{165{,}000 - 135{,}000} = \$553$$
$$Deduction_{800} = 800 - 553 = \$267$$

With $2750 in interest, first the $2500 limit is applied and then the phase out reduction.

$$Reduce_{2750} = \$2500 \times \frac{155{,}000 - 135{,}000}{165{,}000 - 135{,}000} = \$1667$$
$$Deduction_{2750} = 2500 - 1667 = \$833$$

STUDENT LOANS

If you are borrowing money to attend college, you are part of a very large group. In 2017 nearly 20 million people attended colleges or universities in the U.S., and over 70% borrowed from the federal government or private sources. An estimated 44 million current and former students owe nearly $1.5 trillion collectively—with an average student debt of about $39,400.

This is not an overview of the different programs with their rates, limits, and regulations. Instead Examples 12A–6 and 7 illustrate the economic analysis that is part of better decision making. Some of that decision making is easy. Subsidized student loans with low rates of interest are better than loans with more expensive rates. Other decisions, such as the choice between working more, borrowing less, and perhaps taking longer to graduate, are more complex. On-the-job learning, building a resume, and balancing life and school are difficult to include in an economic analysis—but they may be the decision-making drivers.

EXAMPLE 12A-6

An undergraduate is comparing the financial aid packages offered by two institutions. Both packages include $20,000 in Stafford loans over 4 years but one is subsidized and one is not. If the student pays back the loans in 5 years after graduation, how much interest is paid on each loan? To simplify the calculations for an easier to follow example assume annual tuition and loan payments and ignore reductions in taxable income. To further simplify the calculations, ignore the loan fees (see Chapter 7 to include). Assume there is one loan for 4 years rather than a separate loan at a possibly different rate for each year. For 2018–19, direct subsidized and unsubsidized undergraduate

Stafford loans have a fixed interest rate of 5.05%.

5-BUTTON SOLUTION

For the subsidized Stafford loan the interest is paid by the government while the student is in school (0% compounding for the student). Thus for the subsidized loan only $20,000 is owed at the end of 4 years (the government pays the interest), and thus the payment is calculated at $4626. The total interest paid in the 5 years after graduation is $3129.

	A	B	C	D	E	F	G	H	I	J
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for		Total Paid	Interest Paid
2	Subsidized	5.05%	5		–20,000	0	PMT	4,626	23,129	3,129
3				begin						
4	Unsubsidized	5.05%	4	5,000	0		FV	–22,656		
5		5.05%	5		–22,656	0	PMT	5,240	26,201	6,201

For the unsubsidized loan the amount due at graduation is $22,656 (over 13% higher than $20,000). The payments after graduation of $5240 are also over 13% higher than the $4626. The total interest paid is $6201, which is nearly double the interest paid for the subsidized scenario.

Example 12A–6 illustrates that even if student loans are paid off quickly, rates and subsidies can dramatically change what must be repaid. Many students owe more and take much longer to repay the loan, which can interfere with their ability to buy a house, save for retirement, or even take vacations. It is also worth noting that student loans can be discharged in bankruptcy only if undue hardship can be proven.

EXAMPLE 12A-7

Find the effective rate of interest the student pays for the subsidized loan in Example 12A–6, assuming the loan rate is 5.05%.

SPREADSHEET SOLUTION

The timing of the $5000 tuition payments and the loan payment amounts and timing do not change. The future value of the tuition payments and the present value of the loan payments are equal, and the interest rate is the unknown. For visibility and convenience the total payment value for the subsidized loan is entered to establish a goal. Then GOAL SEEK is used to find the effective interest rate of 2.68%.

	A	B	C	D	E	F	G	H	I	J
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for		Total Paid	
2				begin						
3	Subsidized	2.68%	4	5,000	0		FV	-21,378		
4		2.68%	5		-21,378	0	PMT	4,626	23,129	=C4*H4
5		=B3			=H3	Subsidized Exp. 12A-6			23,129	
6								difference	0	

EXAMPLE 12A-8

A student has borrowed $27,500 in Perkins loans (available to students with exceptional need). The rate on the loan is 5% and the government has paid the interest while the student has been in school. To simplify the calculations assume annual tuition and loan payments. What are the differences in the payment amount and the total paid if the student pays the loan back in 5 years and in 20 years? Is the availability of the student loan interest likely to reduce the after-tax cost of the loan?

5-BUTTON SOLUTION

The first year the interest is 5% of the loan amount or $1375. This is less than the maximum deduction for student loan interest, so it all would be deductible. However, it is also *much* less than the standard deduction. Few students buy a home right out of school, and student loan debt is forcing some to postpone it longer than they would like. It is mortgage interest and property taxes that often make itemizing deductions attractive.

	A	B	C	D	E	F	G	H	I	J
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Annual Payment	Total Paid	Interest Paid
2	5-year	5%	5		-27,500	0	PMT	6352	31,759	4259
3	20-year	5%	20		-27,500	0	PMT	2207	44,133	16,633

Using the longer repayment term reduces the payment by 65%, but it increases the total paid by 39% and the interest paid by 391%.

RETIREMENT ACCOUNTS

Appendix 9A introduced **defined benefit** and **defined contribution** retirement plans. The shift to defined contribution plans by firms and governments means that students graduating today must pay more attention than their parents' generation to saving enough for retirement. Examples 12A–9 and 10 illustrate a few of the options for retirement savings. Good planning starts with understanding the power of compound interest. It includes the analytical techniques used here. It also includes keeping up with changing investment details as your personal situation evolves so that you can make the best choices.

401(k) plans are available to employees of corporations. Similar retirement plans exist for employees of other types of organizations (403(b) for public education and some nonprofit employees, 401(a) plans for some government workers, 457(b) and (f) plans for other government, nongovernment, and nonprofit employees).

These plans allow qualified employees to direct part of each paycheck into a designated retirement account. Human resource (HR) departments have staff and resources dedicated to explaining the options and establishing these accounts. When choosing a plan, any fees paid by the employee will need to be considered. (In some cases expected returns before fees are quoted.)

Dollars are directed into 401(k) accounts on a before-tax basis. Deposits are subtracted from salary and wages when reporting taxable income so taxes are also reduced. In 2018 the federal limit (the section 402(g) limit) on pre-tax dollar investments and/or expenses was $18,500 per individual (up to $24,500 if over 50, via a “catch-up contribution”).

EXAMPLE 12A-9

A 25-year-old with a dual BS/MS engineering degree accepts employment with a national firm which has a required retirement age of 65. For simplicity assume that the engineer stays through retirement and that the average annual salary over the next 40 years is $96,000. The firm offers a *$1 firm-for-$2 employee 401(k) matching plan* (up to a maximum employee contribution of 6%). The chosen plan offers a 3% expected rate of return after subtracting fees. What will the engineer have in the plan at retirement?

SOLUTION

Pre-tax monthly salary	= $96,000/12	= $8000
Monthly contributions from employee	= $8000 × 6%	= $480
Monthly contributions from company	= $8000 × 3%	= $240 (1-for-2 match)
Total monthly contribution	= $480 + 240	= $720
Future value in 40 years?	= $720($F/A$, 0.25%, 480)[1]	= $666,763
	[1] (F/A, 3%/12, 40 × 12)	

Without the company match the accumulated amount in the account would have been 1/3 less. All distributions from this account to individuals will be taxable income. But marginal tax rates after retirement should be much lower.

EXAMPLE 12A-10

For the data in Example 12A–9 compute the after-tax difference in income with and without the 401(k) deduction. Assume a single taxpayer and standard deduction.

SOLUTION

	Choose 401(k) Plan	No 401(k) Plan	Notes
Wages	$90,240	$96,000	$96,000*.06=$5760 contribution
Std. deduction	12,000	12,000	
Taxable income	78,240	84,000	marginal tax at 22% and 24%
Taxes (from table)	13,152	14,449	
After-tax wages	77,088	81,551	= Wages – taxes Difference is $4463

Tax computations:

$$401(k)\ \$13,152 = 4453.50 + (0.22)(78,240 - 38,700)$$
$$\text{No } 401(k)\ \$14,449 = 14,089.50 + (0.24)(84,000 - 82,500)$$

By choosing the 401(k) plan the engineer is putting $8640 ($5760 employee + 2880 company match) into the account and losing only $4463 in after-tax wages.

INSURANCE

Insurance helps individuals manage risk. Policyholders pay monthly or annual **premiums** to insure their life, home, vehicle, and so on. Renters insurance may cover your possessions and your potential liability if someone is hurt in your apartment. For trip insurance or an extended warranty, the purchase is for *a* trip or *a* product, where the insurance company pays in the event of an insured loss. With insurance, the benefit payment is often reduced by a **deductible** that the individual is responsible for. Because the insurance company has expenses to operate, to market, and to limit fraudulent claims, the payout is on average less than the premiums collected—sometimes less than 50% and sometimes higher than 85%. This means that for a properly rated individual, the policy's expected value is negative.

So how do you decide which types of insurance you want? Sometimes you may not have a choice. If you finance your vehicle or purchase a home with a mortgage, the lender will require coverage to protect itself. If your employer does not offer medical insurance, you may be required by the government to purchase it. In most states, if you license a vehicle you are required to have **liability insurance**. Once you decide to purchase insurance, in some cases you will also have to choose how much coverage to carry.

For example, most students and engineers starting out have few assets, so bankruptcy may be an alternative to higher liability insurance levels; however, they may have young children and a nonworking spouse, so life insurance may be critical. In contrast, an engineer approaching retirement may have substantial assets to protect and no dependents.

Automobile Insurance

Buying a vehicle usually means an insurance policy must be chosen. Coverage and deductible limits, state requirements, and optional choices can quickly overwhelm a car buyer. Policy buyers must choose property damage and bodily injury liability levels, medical payments protection/personal injury protection levels, collision versus comprehensive coverage, uninsured motorist protection, roadside assistance, and rental car options.

EXAMPLE 12A-11

The state of New Hampshire is the one state that does not require drivers to carry liability insurance. Assume you just received your dream job in the Granite State. Should you purchase optional automotive insurance?

SOLUTION

The answer is that it depends! What can you afford and what is your risk tolerance? Let's start by looking at the financial outcomes of three scenarios:

1. No insurance,* no accident:
• Annual insurance premium = $0 **Total Annual Expense = $0**
2. No insurance, 1 accident: Assume the driver is responsible for 1 two-car accident with these costs:
• Own car is totaled = $5000 net replacement value
• Damage to other car = 4000
• Own medical bills = 2500
• Other driver, medical = 8000
• Property damage = 1500 replace traffic post/sign

Due others = $13,500
<u>Cost to self = 7,500</u>
Total = $21,000

3. Insurance, 1 accident: Assume the driver has the minimum liability coverage policy for a driver in NH** and collision coverage with a $500 deductible and causes the scenario 2 accident:
•Annual insurance premium = $150/month (estimate)
•Deductible expenses = $500

Total Annual Expense = $2300

Scenario 2 shows the risk of not having at least minimum auto insurance. But there are other costs that are harder to quantify that are part of choosing insurance types and levels. Scenario 2 could easily have included legal costs that are part of the insurance firm's expertise and cost of doing business, but that would be expensive in time and money for you to buy.

Less expensive coverage without collision insurance for the Scenario 2 accident would have meant having to replace the $5000 vehicle. Do you have the funds to do so? Do you need the car to get to work, or just for weekend activities? How much is the time that you would need to spend resolving issues with the insurance company worth to you?

[*]Currently 20 of the 49 states that require liability minimums also require uninsured motorists coverage.

[**]2018 NH minimums for those electing to insure their vehicles are 25/50/25 in $1000 for bodily liability per person and per accident, and property per accident.

Most policyholders are willing to trade off the expense of a periodic premium for the peace of mind that insurance provides, as well as protection against potentially catastrophic financial circumstances. To reduce premiums: make firms compete for your business, consider bundling different insurance coverages with the same company; make one annual payment versus monthly payments; increase your deductible levels; and ask for available low mileage, good driver, and student discounts.

Life Insurance

Life insurance policies provide a **death benefit** that is paid to the survivors (beneficiaries) when the insured person dies. This benefit is generally not subject to income tax, but there may be estate taxes. Some types of life insurance also offer a **cash value benefit** to policyholders. The policy's cash portion can be withdrawn or borrowed against, often after some initial accumulation period.

EXAMPLE 12A-12

A recent engineering graduate started her new job with a required session in the firm's human resources (HR) office. She needs to explore the *life insurance* options that were discussed. What should she know to assist her choice? Her options are:

(1) Do not purchase any additional life insurance.
(2) Purchase additional *term* life insurance.
(3) Purchase a *permanent* life insurance policy.

SOLUTION

(1) As allowed in IRS Code Section 79, her employer provides a $50,000 term life policy to all employees free of charge and not subject to income tax.
(2) *Term* life insurance policies provide a death benefit for the specified 5-, 10-, 20-, . . . year term as long as premiums are paid on time. The younger and healthier the insured and the shorter the term, the lower the premium. Once the term ends, some policies can be renewed at a higher cost and others require a new policy.
(3) *Permanent* life insurance comes in many types that include whole, ordinary, variable, universal, index-universal, and variable universal life. Most have both a cash value and a death benefit. The cash value accumulation acts as a tax-free investment instrument over the policy's life. In many cases the policyholder has the right to keep the policy for his or her *whole* life until death, when the death benefit is paid.

There is no one right answer. Life insurance is a complex decision based on one's life circumstances, goals, and perspectives. Decisions are often influenced by number and age of dependents and one's desire to provide financial security for survivors.

Example 12A-13 (Example 12A–12 Revisited)

The engineer in Example 12A–12 recently returned from an investment seminar that recommended the **Buy Term and Invest the Difference** (BTID) strategy. Should she use this approach?

SOLUTION

Permanent life policies offer more than death benefits for a defined term, and thus the premiums are higher than for term life policies. If term insurance is purchased, this *difference* in premiums can be invested.

The main disadvantage of BTID is that the insured must actually *invest* the difference (versus spend it!). In addition, investments in permanent life policies grow tax free and other BTID investments might not. Advantages of the BTID strategy include control of the investment portfolio, higher potential investment gains, lower investment fees, greater access to funds, and the fact that the investor's nest egg (cash value) passes on to beneficiaries after death—rather than being kept by the insurance company, as is the case for permanent life insurance policies.

The biggest advantage of the BTID strategy may be that the amount of insurance purchased can be better matched to the insured's need (which varies over the insured's life) to provide for dependents in the case of the insured's death.

PERSONAL BUDGETING

At its essence, money management employs a simple concept: *live within your means*. Yet, this concept is challenged by the abundance of heavily marketed consumer choices, as well as the spending habits of many people. A simple way to organize a personal budget divides spending into:

1. **Needs:** spending on the basics such as housing, groceries, transportation, insurances, energy/utilities, clothing, and other essentials. This includes loan payments that must be made. Since income taxes are usually withheld by the employer, it is common to base other calculations on after-tax income that has subtracted the withholding.
2. **Savings:** building an emergency fund (target 6 months of needs), paying off high-interest loans faster, and saving for the future—retirement, home down payment, a new vehicle, or major travel.

3. **Wants:** discretionary spending on entertainment, meals out, clothing beyond basics, hobbies, recreational travel, and the like.

A common guideline (the 50/20/30 rule) allocates after-tax income as 50% or less for needs, 20% or more for savings, and 30% or less for wants. These percentages are not always achievable. When income is low, all spending may be on needs, but even then the guidelines represent a goal as income increases.

Recognizing and naming types of spending forces individuals to be thoughtful about where their money goes each month. This text has many examples of how a few dollars here and a few dollars there add up to significant sums. Be aware of where your expenses occur and prioritize your spending goals. It is incredibly easy to let wants become needs. For example, taking out a loan to buy a more expensive vehicle or incurring credit card debt on electronics, meals out, or travel converts a want into a required payment. Be intentional about your spending.

EXAMPLE 12A-14

Dave Parish is a mid-career civil engineer with an annual after-tax income of $90,000. His monthly expenses are:

Category	**Monthly expense**	**Category**	**Monthly expense**
House payment	$1250	Groceries	$400
Gasoline (automobile)	180	Utilities (gas, water, electric)	250
Communications (cable, phone, Internet)	255	Car payment	535
Restaurants	350	Hobbies	175
Home essentials	230	All savings	1550
Insurance and taxes	565	Vacations	550
Child support	1000	Entertainment	210

What percentage of Dave’s expenses fit into each of the three categories? What adjustments should he consider if his goal is 50/20/30%?

SOLUTION

Categorizing Dave’s annual expenses:

Category	**Expenses**	**Total $/year**	**Percentage of budget**
Needs	Home payment, groceries, gas, utilities, car, home, insurances/taxes, child support	$52,920	58.8%
Savings	Retirement account	18,600	20.7
Wants	Communications, restaurants, hobbies, vacations, entertainment	18,480	20.5

Like many of us, Dave’s monthly budget is heavier than he’d prefer in the needs category. However, he is meeting his goal for savings! Looking into the future, he could have more money for wants if he keeps his car after it is paid off. That removes the car loan payment from the needs category. In general, this budget could be right for Dave at this life stage.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

12A-1 A young entry-level engineer, Kelly Green, earned $43,000 last year. Kelly is single and will not itemize his tax return. How much federal income tax does Kelly owe? Assume no other income adjustments.

SOLUTION

12A- Tammie pays federal income taxes at the incremental rate of 22% and state income taxes at the incremental rate of 3.4%. What is her

2 combined incremental income tax rate?

SOLUTION

12A-3 As a senior engineer for a large consulting firm, Ray earned $113,000 last year. Ray is married with two children. His wife had no income and they will not itemize their tax return. What is the effective federal tax rate the couple will pay if they file a joint return? Assume no other income adjustments.

SOLUTION

12A-4 Lexie earned $29,995 last year while also attending the University of New London as a graduate student. She paid tuition of $4100 during the year and had additional educational expenses of $900. Lexie is single and will not itemize her tax return. Assuming no other income adjustments, how much federal tax will Lexie owe?

SOLUTION

12A-5 A student pays her own tuition and fees. Her income is $45,000 and her education expenses are $5200. Using the American Opportunity Tax Credit, what is the maximum tax credit she is due?

SOLUTION

12A-6 A 1-year savings certificate that pays 15% is purchased for $10,000. If the purchaser pays taxes at the 27% incremental income tax rate, the after-tax rate of return on this investment is

SOLUTION

12A- A newly hired engineer wants to take advantage of the firm's generous 401k plan, where the company will do a 1:1 match up to 3% of the person's salary. The new engineer is making $60,000 per year. If the

7 engineer is paid twice per month, how much is the person putting into the 401k each pay period? How much is being added each pay period in total?

SOLUTION

12A-8 A whole life policy worth $100,000 can be purchased for $1200 per year, payable at the beginning of the year. The insurance company uses 12% of this to fund insurance, and the remaining is put into a savings plan that guarantees a 1.9% rate of return. What is the cash value of the plan after 10 years, when there is no penalty for cashing the policy?

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Income Taxes

12A-1 Miriam Anne is a single taxpayer. What federal taxes does she pay if her taxable income is
(*a*) $5000,
(*b*) $20,000,
(*c*) $95,000,
(*d*) $350,000,
(*e*) $1 million?

12A-2 A John Adams has a $95,000 adjusted gross income from Apple Corp. and allowable itemized deductions of $7200. Mary Eve has a $75,000 adjusted gross income and $3000 of allowable itemized deductions. Compute the total tax they would pay as unmarried individuals. Then compute their tax as a married couple filing a joint return.

12A-3 An unmarried taxpayer with no dependents expects an adjusted gross income of $87,000 in a given year. His nonbusiness deductions are expected to be $7000.
(*a*) What will his federal income tax be?
(*b*) He is considering an additional activity expected to increase his adjusted gross income. If this increase should be $15,000 and there should be no change in nonbusiness deductions or exemptions, what will be the increase in his federal income tax?

12A-4 A Bill Jackson had a total taxable income of $3000. Bill's employer wants him to work another month during the summer, but Bill had planned to spend the month hiking. If an additional month's work would increase Bill's taxable income by $2000, how much more money would he have after paying the income tax?

12A-5 A Amara and her husband, Mosi, are both employed. Amara will have an adjusted gross income this year of $130,000. Mosi has an adjusted gross income of $5000 a month. Amara and Mosi have agreed that Mosi should continue working only until the federal income tax on their joint income tax return becomes $30,700. On what date should Mosi quit his job?

12A-6 A married couple filing jointly have a combined total adjusted gross income of $110,000. They have computed that their allowable itemized deductions are $5000. Compute their federal income tax.

12-A7 An unmarried individual in California with a taxable income of about $75,000 has a federal incremental tax rate of 25% and a state incremental tax rate of 9.3%. What is his combined incremental tax rate?
(*a*) Assume standard deduction taken.
(*b*) Assume deductions are itemized.

12A-8 A Veronica Marie has an income that places her in the 24% federal and 6.5% state incremental tax brackets. What is her combined tax on a $1000 honorarium that she received recently for a speech?
(*a*) Assume standard deduction taken.
(*b*) Assume deductions are itemized.

12A-9 Given the following data, compute your combined income tax rate (CTR) assuming you deduct allowable expenses on your income tax forms: a before-tax MARR of 5%, an inflation rate of 3%, a federal income tax rate of 32%, a state income tax rate of 6%, a local city income tax of 3%, and a capital gains tax rate of 15%, as applicable.
Contributed by D. P. Loucks, Cornell University

12A-10 A A $10,000 commercial bond that has a 6% bond rate and matures in 5 years can be purchased for $11,000. Interest is paid at the end of each year for the next 5 years. Find the annual after-tax rate of return of this investment. Assume a 35% tax rate applies.
Contributed by D. P. Loucks, Cornell University

12A-11 Jane Shay operates a management consulting business. The business has been successful and now produces a taxable income of $100,000 per year after all "ordinary and necessary" expenses and depreciation have been deducted. At present the business is operated as a proprietorship; that is, Jane pays personal federal income tax on the entire $100,000. For tax purposes, it is as if she had a job that pays her a $100,000 salary per year.
As an alternative, Jane is considering incorporating the business. If she does, she will pay herself a salary of $40,000 a year from the corporation. The corporation will then pay taxes on the remaining $60,000 and retain the balance of the money as a corporate asset. Thus Jane's two alternatives are to operate the business as a proprietorship or as a corporation. Jane is single and has $3500 of itemized personal deductions. Which alternative will result in a smaller total payment of taxes to the government?

12A-12 Gains from non-business-related investment assets held by individuals that increase in value are subject to taxes. What distinguishes assets taxed at the ordinary rate from those taxes at the capital gains rate?

12A-13 Juan DeBaptist purchased $10,000 in corporate stock on June 1 and sold the stock when its value reached $13,000 on October 26. Ignoring stock transaction fees, what federal taxes did Juan pay on this stock investment if his taxable income is $90,000? Assume a capital gains tax rate of 15%.

A married couple (filing jointly) bought an antique armoire at an

12A-14 [A]	estate sale, then sold it 6 months later for twice what they paid for it. If their federal taxable income is \$80,000 and they paid \$750 in taxes on this transaction, how much did they pay for the armoire? Assume capital gains are taxed at a rate of 15%.
12A-15	An investor bought investment property at the beach for \$35,000 per acre. Twenty years later she sold the 100-acre lot to a developer for a profit, and paid \$1.05 million in taxes as a result of the sale. If capital gains are taxed at 15% and her marginal tax bracket is 35%, what was the price paid by the developer for the lot?
12A-16 [A]	You recently bought a mini-supercomputer for \$10,000 to allow for tracking and analysis of real-time changes in stock and bond prices. Assume you plan on spending half your time tending to the stock market with this computer and the other half as personal use. Also assume you can depreciate your computer by 20% per year over 5 years (straight line rate). How much tax savings will you have in each of those 5 years, if any? Use a tax rate of 28%. *Contributed by D. P. Loucks, Cornell University*
12A-17	Calculate the taxable income and federal taxes paid for each of the following: (*a*) Single filer; AGI = \$65,000; itemized deductions = \$2500 (*b*) Single filer; AGI = \$110,000; itemized deductions = \$15,000 (*c*) Married filing jointly; AGI = \$150,000; itemized deductions = \$8000 (*d*) Two people are married, but they are filing as two single people; AGI(1) = \$80,000, AGI(2) = \$70,000; itemized deductions(1) = \$5000, itemized deductions(2) = \$3000; compare the combined answers here to the answer in part (c); now find the new tax brackets for “married filing separately” and do the same comparison.
12A-18 [A]	Professor M. Grace has federal taxable income of \$90,000 and she falls into the 5.0% state incremental income tax bracket. She recently received an offer to consult in her area of technical expertise. What is the after-tax amount on a stipend of \$8000 for this work? Her filing status is single, and she is able to itemize deductions, including her state income tax.
	A royalty check arrived for \$500, boosting the annual income of you and your spouse to \$84,500. You are married, filing jointly, with

12A-19 itemized deductions of $12,800. Your state taxes income at a marginal rate of 6.5%. How much of your check will you be able to spend?

12A-20 A

Ms. Ima N. Jinere is looking over the federal tax return that her accountant has produced for her review. Consider the analysis below and either confirm the accountant's report, or quantify the effect of any errors. Her filing status is single.

Given Data:	
Adjusted gross income	$100,000
Itemized deductions:	
Medical expenses	12,000
Property taxes	2,000
Mortgage interest*	7,500
Charitable gifts	3,500
Federal taxes withheld	14,000

*on $400,000 home mortgage debt

Accountant Calculations:	
Taxable income	$= 100,000 - 12,000$**
	$= \$88,000$
Taxes	$= 14,059.50 + .24(88,000 - 82,500)$
	$= \$15,379.50$
Due IRS	$= 15,379.50 - 14,000$
	$= \$1379.50$

**standard deduction

12A-21	Two married engineers filing jointly, and using the standard deduction, do not want to pay more than $80,000 in federal taxes this year. The combined income on their regular salaries and investment income is $380,000. What is the maximum that they can earn on a new venture and meet their goal?

Student Loans

12A-22 A	An independent student has a modified adjusted gross income of $25,000 and qualifying educational expenses of $18,000. If the American Opportunity Tax Credit is used, what is the amount of the tax credit?
12A-23	Joint filing parents have $30,000 in educational expenses as the first of their three children goes off to college. Assuming the family has a modified adjusted gross income of $80,000, what effect would each of the education deductions/credits listed in Table 12A-3 have on the family's taxes?
12A-24 A	A single MS engineer started a new job at $70,000 and paid $3000 in interest on her student loans last year. If she uses the federal tax deduction on student loan interest paid, what tax savings does this represent?
12A-25	Federal subsidized Stafford student loans historically had interest calculated at 3.4%, and the government pays the interest while the student is in school. Using the historical rates, what are a student's annual payments if she borrows $2500 per year in subsidized Stafford loans for 4 years and then pays off the loan in 3 years after graduation?
12A-26 A	Referring to Problem 12A-25 above, compare the total amount paid by the student over the term of school and repaying the loan if the borrowing was a nonsubsidized 8% bank loan.

Retirement

An engineering professor has contributed $300 per month into her 403b tax-deferred retirement account for the past 24 years. She is

12A-27 now eligible to retire and wants to know her after-tax account balance (she is contemplating a lump sum distribution) if it has earned 0.5% per month over the 24 years. Assume that she is currently in the 24% marginal tax bracket.

12A-28 A A newly hired engineer signed up for the 401k plan at her new job. She decided to contribute the maximum amount allowed (assume that the annual federal limit is $18,000 of tax-exempted income) and to take advantage of the match offered by her company ($1 firm-for-$3 employee). What is the size of total monthly contribution to her retirement?

12A-29 Mike just changed jobs, leaving a company after 6 years. He is fully vested, and thus can keep the money his employer deposited in his retirement account. His employer has been contributing $200 per month into a diversified stock fund.
(*a*) Using average market rates of 7.3%, how much money has accumulated in the account?
(*b*) If this money is "rolled over" into another retirement account with an 8% annual return today, how much will this be worth after 30 years?

Insurance

12A-30 The required automobile insurance varies by state. Investigate the laws in your state and identify the minimum required coverage for bodily injury per person and per accident, and property per accident.

12A-31 E A newly hired professional bought a used car to get to the new job. Automobile insurance is expensive, so he decided to not get any. It is against the law to drive without insurance, but he figures that he won't be fined if he isn't caught. If he is in an accident, he might get sued, but he doesn't have anything so he has nothing to lose. Comment on the ethics of this situation.

12A-32 A A $500,000 whole life policy is available for $5150 per year, payable at the beginning of the year. If 10% of this amount pays for insurance, and 90% goes into a savings plan, what is the cash value of the policy after 10 years? The insurance company guarantees a rate of 2.4% per year.

12A-33 A newly hired employee has a choice regarding life insurance. A $250,000 whole life policy is available for an annual premium of $1180, payable at the beginning of each year. At the end of 10 years, the policy has a cash value of $11,600. A term life policy for the same amount is available for an annual premium of $140, with the premium increasing 1% each year. There is no cash value to the term policy.
(*a*) Which policy would you choose, and why?
(*b*) If the whole life policy were cashed in after 10 years, what is the rate of return on the savings portion of the whole life policy?

Personal Budgeting

12A-34 A

Your neighbors have a household after-tax income of $70,000. Their monthly expenses are:

Category	Monthly expense	Category	Monthly expense
House payment	$1500	Groceries	$650
Gasoline	200	Utilities	250
Phone/Internet/TV	250	Car payment	450
Eating out	200	Savings	1000
Home maintenance	233	Vacation & entertainment	500
Insurance and taxes	600		

(*a*) What percentage of the family's expenses fit into the *Needs* category?
(*b*) What percentage of the family's expenses fit into the *Savings* category?
(*c*) What percentage of the family's expenses fit into the *Wants*

category?
(*d*) They are paying $1050 per year for fire insurance and $3990 per year in taxes on the house as part of their house payment. What is the monthly mortgage payment?
(*e*) If the family pays 15% of their total income in taxes (combined federal and state), what is their before-tax income?

12A-35 Your neighbors have a household after-tax income of $120,000. Their monthly expenses are:

Category	Monthly expense	Category	Monthly expense
House payment	$2100	Groceries	$550
Gasoline	200	Utilities	250
Phone/Internet/TV	250	Car payment	750
Eating out	800	Savings	900
Home maintenance	333	Vacation & entertainment	800
Insurance and taxes	800		

(*a*) What percentage of the family’s expenses fit into the *Needs* category?
(*b*) What percentage of the family’s expenses fit into the *Savings* category?
(*c*) What percentage of the family’s expenses fit into the *Wants* category?
(*d*) They are paying $1600 per year for fire insurance and $17,500 in mortgage payments as part of their house payment. What is their annual property tax that is paid as a part of their house payment?
(*e*) If the family pays 18.5% of their total income in taxes (combined federal and state), what is their before-tax income?

12A-36 Many people graduate from college with large student loan debt. What category of a personal budget does this debt fall in? Why?

A couple wants to purchase a $260,000 house, and they have the required 20% down payment and money for other closing costs. The bank is offering a 30-year mortgage at 4.625% interest, compounded monthly. The couple has an annual after-tax income of $55,000 and other debts totaling $650 per month. *Contributed by Kate Abel, Stevens Institute of Technology*

12A-37 (*a*) If the maximum debt-to-income ratio (total monthly debt divided by after-tax monthly income) is 43%, can the couple afford to purchase the home?

(*b*) If the couple lives in the house for 30 years, what is the total amount paid for the house, including down payment, principal, and interest?

12A-38 A

A couple wants to purchase a $170,000 house, and they have enough saved for a 5% down payment and money for other closing costs. The bank is offering a 30-year mortgage at 5.35% interest, compounded monthly. The couple has an annual after-tax income of $85,000 and other debts totaling $850 per month. Because their down payment is less than 20%, they are required to pay for private mortgage insurance, which costs 1% of the loan amount each year. *Contributed by Kate Abel, Stevens Institute of Technology*

(*a*) If the maximum debt-to-income ratio (total monthly debt divided by after-tax monthly income) is 43%, can the couple afford to purchase the home?

(*b*) If the couple lives in the house for 30 years, what is the total amount paid for the house, including down payment, principal, interest, and private mortgage insurance?

12A-39

The couple in Problem 12A-37 has only enough money for a 10% down payment and other closing costs. Thus the bank is offering a loan at a higher rate of 5.15% and requiring private mortgage insurance, which costs 1% of the loan amount each year. *Contributed by Kate Abel, Stevens Institute of Technology*

(*a*) If the maximum debt-to-income ratio (total monthly debt divided by after-tax monthly income) is still 43%, can the couple afford to purchase the home?

(*b*) If the couple lives in the house for 30 years, what is the total amount paid for the house, including down payment, principal, interest, and private mortgage insurance?

Why is how long you expect to live in a house important to the choice between buying and renting a home? *Contributed by Kate Abel, Stevens Institute of Technology*

12A-40 (*a*) Can you find credible guidelines for how long you should expect to own a home to justify buying it? What are some of the central concepts?

(*b*) What are the key factors that would lengthen or shorten this period?

(*c*) If a couple will be moving in 3 years, what will be the key factors in the decision of whether or not to buy now?

CHAPTER 13

ECONOMIC LIFE AND REPLACEMENT ANALYSIS

Mike Siegel/The Seattle Times. Copyright 2013, Seattle Times Company. Used with Permission.

Aging Bridges

On May 23, 2013, a 160-foot span of the Skagit River Bridge on I-5 north of Seattle collapsed moments after upper bridge supports were struck by a tractor-trailer with an oversized load. The truck made it safely across, but two other vehicles fell into the water 24 feet below. Three people were rescued without major injuries. The bridge was constructed in 1955 and designed for an expected life of 50 years.

The Skagit River Bridge is rated by the Federal Highway Administration (FHWA) as *functionally obsolete*—it is not designed to today's standards, but it is not necessarily unsafe. The steel through-truss bridge has a fracture critical design, which means that the failure of a single element could cause collapse. There are about 18,000 fracture critical bridges throughout the

United States, built mostly between the mid-1950s and late 1970s. Modern construction methods are much more resilient to damage.

In 2007 the I-35W bridge carrying traffic over the Mississippi River between Minneapolis and St. Paul collapsed suddenly during rush hour, killing 13 people and injuring 145. The Minnesota bridge, completed in 1967, was also a fracture critical bridge and was classified as *structurally deficient* by the FHWA. Structural deficiency indicates that the bridge has one or more defects in its support structure or deck and therefore requires maintenance, repair, and eventual rehabilitation or replacement.

The nation's 615,000 bridges have an average age of 43 years, and 8.9% are rated as structurally deficient. The FHWA calculates that almost 40% of U.S. bridges exceed their 50-year design life. The required fiscal investment for reconstruction and renovation poses a significant challenge for federal, state, and local governments—but some progress is being made.

Contributed by Letitia M. Pohl, University of Arkansas

QUESTIONS TO CONSIDER

1. Decisions on how to allocate funding to upgrade and replace deficient bridges are influenced by both economic and non-economic factors. List three of each.
2. The Skagit River Bridge carries an estimated 71,000 vehicles a day and is a main commercial route between the United States and Canada. How would you calculate the economic impact of the catastrophic failure of the bridge? Compare the economic impact to commuters versus commercial traffic.
3. What factors should be considered when engineers determine whether to either rehabilitate or replace a deficient bridge?
4. The Federal Highway Administration released $1 million in federal emergency funding to the state of Washington the day after the I-5 bridge collapse and almost a month later allocated $15.6 million in federal funding to help rebuild the bridge. Discuss the ethical dilemma of state and local governments that have aging infrastructure to repair before tragedy strikes, but insufficient funding to make the repairs.

5. Both of the bridges described here were routinely inspected and deemed safe for use. Discuss how this inability to predict structural failures complicates the job of transportation officials.

After Completing This Chapter…

The student should be able to:

• Understand why economic life may be shorter than an asset's useful life.
• Calculate the *economic* or *minimum cost life* of alternatives.
• Calculate and use *marginal cost data for* an existing asset to decide when to replace.
• Perform economic life and replacement analysis on an after-tax basis.

Key Words

cash flow approach

economic life

deterioration

marginal costs

minimum cost life

obsolescence

opportunity first cost

replacement analysis

Examples and problems in previous chapters have assumed that an asset's life is known. However, just because an asset *could* be used for 10, 20, or 50 years—does not mean it *should* be. What asset life minimizes the asset's cost of ownership (EUAC)? Similarly, how soon should an existing asset be replaced?

Firms, agencies, and individuals all face economic decisions with very long and indefinite horizons. For example, firms buy robots, school and transit systems buy buses, individuals buy vehicles, and all buy computers. In each case, the assets have lives shorter than the needs they fill. An asset's **economic life** minimizes the EUAC of ownership, and thus is at the optimal replacement interval.

Determining an item's economic life requires choosing between different length lives and thus using the annual economic measures presented in Chapter 6. Often only costs are considered, but differences in benefits can be as well. Near the end of an asset's life, there is another economic question: how soon will the replacement happen? This comparison of an aging asset with a new replacement is **replacement analysis**.

ECONOMIC LIFE OF A NEW ASSET

Drivers Behind the Economic Life

Why is there an economic life that minimizes the EUAC? If assets are kept for only a few years, then the EUAC is high because capital costs are spread over only a few years. If assets are kept for too long, then the EUAC is high because maintenance and operating costs become large. The economic life strikes a balance that minimizes the total EUAC.

The costs to maintain and operate an asset include the costs of planned maintenance and unplanned repair or replacement. Compare the low cost of replacing a worn tire in a scheduled appointment when the tires are on sale, versus replacing the tire when it fails on the way to work or on a weekend trip to the mountains. Failures requiring unplanned repairs can shut down assembly lines, pipelines, and other processes until a key part or subassembly arrives. Failures can be catastrophic for airplanes and nuclear power plants. In some contexts a 10% chance of an unplanned repair or replacement may be just fine; in others, a one-in-a-thousand chance may be too high.

Example 13–1 shows that to calculate the minimum cost life of an asset, we determine the EUAC for each possible life less than or equal to the useful life. Example 13–1 and Figure 13–1 show that the EUAC tends to be high if the asset is kept only a few years; then it decreases to some minimum value, and then increases again as the asset ages. By identifying the number of years at which the EUAC is a minimum and then keeping the asset for that number of years, we are minimizing the yearly cost of ownership. It is cheaper to buy another asset with the same or better costs than to continue use of this one.

Example 13–2 shows that even with complicated cost structures, it is still relatively easy to find the minimum cost EUAC with the power of spreadsheets. Example 13–3 also illustrates the assumptions common in analysis of economic lives.

• Salvage values decline over time.
• Annual costs for operations, maintenance, risk of breakdowns, and so on increase over time.

EXAMPLE 13-1

A machine costs \$6800 to buy, \$700 to install, and has no salvage value. Maintenance costs are expected to be \$0 the first year, but will increase by \$900 every year after that. Operating costs are expected to be \$1200 every year. If the machine will last 10 years and the interest rate is 8%, compute the machine's economic life that minimizes the EUAC.

SOLUTION

In earlier chapters, when we were given a life of 10 years we would have calculated the EUAC of ownership over 10 years.

$$\begin{aligned}\text{EUAC}_{10} &= 7500(A/P, 8\%, 10) + 900(A/G, 8\%, 10) + 1200 = \\ &= 7500(0.1490) + 900(3.871) + 1200 = \$5801\end{aligned}$$

Now, this calculation is needed for every year.

If Retired at the End of Year *n*

Year, t	EUAC of Capital Recovery Costs: $7500(*A*/*P*, 8%, *t*)	EUAC of Maintenance and Repair Costs: $900(*A*/*G*, 8%, *t*)	EUAC of Operating Costs: $1200	EUAC Total
1	$8100	$ 0	$1200	$9300
2	4206	433	1200	5838
3	2910	854	1200	4964
4	2264	1264	1200	**4728** ←
5	1878	1661	1200	4739
6	1622	2048	1200	4870
7	1440	2425	1200	5065
8	1305	2789	1200	5294
9	1200	3142	1200	5542
10	1117	3484	1200	5801

The total EUAC data are plotted in Figure 13–1. From either the table or the figure, we see that the machinery's minimum cost life is at 4 years, with a minimum EUAC of $4728 for each of those 4 years.

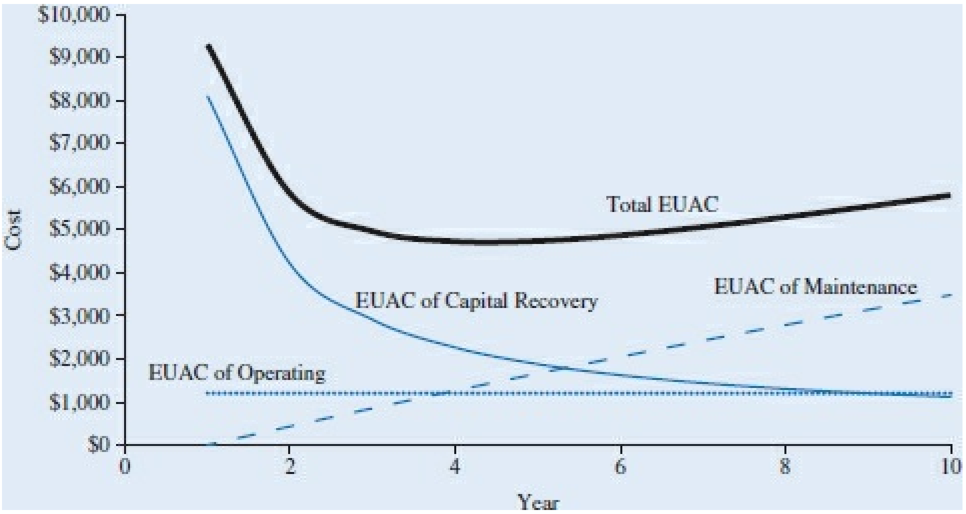


FIGURE 13-1 Plot of costs for Example 13–1.

Looking at Figure 13–1 more closely, we see the effects of each of the individual cost components on total EUAC (capital recovery, maintenance/repair, and operating expense EUACs) and how they behave over time. The total EUAC curve of most assets tends to follow this concave "bathtub" shape—high for short lives due to capital recovery costs and high for long lives due to increased maintenance/repair and operating expenses. The minimum EUAC occurs somewhere between these high points.

Spreadsheets for Economic Life

The amount of arithmetic in computing the economic life is best done with spreadsheets. The formulas are easier to build if cash flows are separated into two groups.

• The first group includes all costs to own, operate, maintain, and repair an asset. Equations 13–1 and 13–1′ find the present worth of these costs, PWcost_t, of cash flows through year *t* for this group.

$\text{PWcost}_t = CF_0 + \sum_{1=0}^{t} \text{cost}_t/(1+i)^j$ (13-1)

$\text{PWcost}_t = \text{CF}_0 + \text{NPV}(i, \text{CF}_1 : \text{CF}_t)$ (13-1')

When writing Eq. 13–1′ as a spreadsheet formula, the cell for CF_1 should be specified with an absolute address, but the cell for CF_t with a relative address that can change when the formula is copied. This ensures that the range is from year 1 to year *t*.

• The second group includes all cash flows linked to asset disposal. These occur in year *t* only if that is the year of disposal. Equation 13–2 for S_t is a more precisely defined salvage value for year *t*.

$S_t = \text{selling price}_t - \text{removal costs} - \text{selling costs}$ (13-2)

• Then Equation 13–3 or 13–3′ is used to calculate the EUAC for a life of *t* years from a time 0 value of PWcost_t and a final value of *S*.

$\text{EUAC}_t = \text{PWcost}_t(A/P, i, t) - S_t(A/F, i, t)$ (13-3)

$\text{EUAC}_t = -\text{PMT}(n, i, \text{PWcost}_t, -S_t)$ (13-3′)

EXAMPLE 13-2 Example 13–1 Revisited

Use the data from Example 13–1 and spreadsheets to model and solve for the minimum EUAC.

SOLUTION

The spreadsheet is a little bit simpler if the operating and maintenance costs (O&M) are combined into a single cash flow series that starts at $900 the first year and increases by $1300 per year. The salvage value column is 0 for every year.

	A	B	C	D	E
1	$7,500	first cost			
2	0	salvage value			
3	$1,200	O&M			
4	$900	O&M gradient			
5	10	life (years)			
6	8%	interest rate			
7			=$B$9+NPV($A$6,$B$10:B10)		
8	year	cost_t	PWcost_t	S_t	EUAC_t
9	0	$7,500			
10	1	1,200	$8,611	0	$9,300
11	2	2,100	10,412	0	5,838
12	3	3,000	12,793	0	4,694
13	4	3,900	15,660	0	4,728
14	5	4,800	18,926	0	4,740
15	6	5,700	22,518	0	4,871
16	7	6,600	26,369	0	5,065
17	8	7,500	30,421	0	5,294
18	9	8,400	34,624	0	5,543
19	10	9,300	38,931	0	5,802
20					
21			=-PMT($A$6,A19,C19,-D19)		
22		=$B$9+NPV($A$6,$B$10:B19)			
23	=B18+$A$4				

FIGURE 13-2 Spreadsheet for Example 13–1 and 13–2.

The EUAC values for each year match those in Example 13–1, and the economic life is still 4 years with a minimum EUAC of $4728.

Including Salvage Values and Overhauls

Like many pieces of installed equipment, the machine in Examples 13–1 and 13–2 had no salvage value. To match the specific requirements of unique processes and configurations, most industrial assets are purchased new. Unfortunately, once installed the salvage value of most is near zero.

For example, a manufacturing facility was once built to support a new line of snack foods, with multiple manufacturing lines to support the expected sales volume. Each line had equipment costing well over $1 million. However, the product did not sell as well as expected, and the decision was made to shut down all but one production line after only two years. All unneeded equipment was sold as scrap metal—at ten cents per pound!

There are used assets, like a car, that can easily be sold for a value that depends on age and condition. Nevertheless, even if the asset is easy to sell, there are still costs, and valuable time must be spent. These reduce the net salvage value of disposing of the asset. As noted in Equation 13–2, there may also be removal costs as well as selling costs. The costs to acquire and install an asset and the costs to remove and sell it increase an asset's capital cost, which may increase the minimum cost life.

Example 13–3 has a declining selling price, which equals its salvage value, so only one column in the spreadsheet is required. If there were removal and selling costs, then multiple columns would be used as they are for the O&M and overhaul costs in Example 13–3.

Overhaul expenses may be required at the start of year *t*, so that use can continue in year *t* and beyond. Multiplying the overhaul expenses by (1 + *i*) as in Equation 13–4 converts the overhaul expenses to an end-of-year value. Thus, the overhaul expenses will only be incurred for service lives of *t* or longer.

$$\text{Overhaul}_t = \text{Overhaul}_{\text{beginning } t}(1+i) \quad (13\text{-}4)$$

EXAMPLE 13-3

A machine costs \$18,000 to buy and install. Its operating and maintenance (O&M) costs start at \$900 per year and then climb \$500 per year. Its selling price if sold used is \$5000 at the end of year 1 and then it declines 10% per year. If the machine is to be used more than 5 years, an overhaul costing \$3000 is needed at the end of year 5. The machine's useful life is 10 years and interest is 8%. Use a spreadsheet to compute the machinery's economic life that minimizes the EUAC.

SOLUTION

To ensure the cost calculations are correct, the O&M costs are in column B, the overhaul cost using Equation 13–4 is in column C, and column D is the total of each year's costs.

	A	B	C	D	E	F	G
1	$18,000	first cost					
2	$5000	salvage value					
3	–10%	salvage gradient					
4	$900	O&M					
5	$500	O&M gradient					
6	$3,000	overhaul end of year 5					
7	10	life (years)					
8	8%	interest rate					
9					=$D$11+NPV($A$8,$D$12:D12)		
10	year	O&M	overhaul	cost_t	PWcost_t	S_t	EUAC_t
11	0			$18,000			
12	1	$900		900	$18,833	$5000	$15,340
13	2	1,400		1,400	20,034	4500	9,071
14	3	1,900		1,900	21,542	4050	7,111
15	4	2,400		2,400	23,306	3645	6,228
16	5	2,900		2,900	25,280	3281	5,772
17	6	3,400	$3,240	6,640	29,464	2952	5,971
18	7	3,900		3,900	31,740	2657	5,799
19	8	4,400		4,400	34,117	2391	5,712
20	9	4,900		4,900	36,568	2152	**5,681**
21	10	5,400		5,400	39,069	1937	5,689
22							
23					=-PMT($A$8,A21,E21,-F21)		
24			=$D$11+NPV($A$8,$D$12:D21)				
25	=B20+$A$5						

FIGURE 13-3 Spreadsheet for [Example 13–3](#).

A 5-year life has a lower cost than a 6- or 7-year life, but the lowest cost or economic life is 9 years. Even though the overhaul cost means that plotted curves would not be as smooth as [Figure 13–1](#), it is still possible to calculate the minimum EUAC of $5681 at the asset's 9-year economic life.

Economic Life, Unplanned Repairs, and the Opportunity Cost of New Alternatives

In both [Example 13–1](#) and 13–3, lives close to the economic life had EUACs that were close to the minimum cost. This is common, but not guaranteed. The bathtub curve of EUACs like [Figure 13–1](#) tends to be flat near the minimum cost life—like a bathtub. This is adequate accuracy for current decision making.

[Examples 13–1](#) and [13–3](#) were about unspecified machinery, but economic life problems are all around us. How long should you keep your car? Your computer? In both cases, the probability of an unplanned repair or the capabilities of newer cars and computers may be the critical quantity to be estimated. How could these be included?

In Example 13–3, the O&M costs climbed $500 per year. Suppose this machinery was critical to an assembly line and an unplanned repair would cost $50,000 while the line was shut down. Then a 1% increase each year in the probability of an unplanned repair would have a $500 per year gradient.

Suppose for each year that passes, a newer car or computer has features worth $400 per year to you. Thus, using a 2-year old asset is costing you $800 in the opportunity cost of forgone benefits. This can be included just like increasing O&M costs.

While operating and maintenance costs may or may not have large gradients, we all know that aging vehicles and computers are more prone to failure. We know that newer versions have newer or improved features that have value to us. Economic life guides the choice of how long to use an asset that may have a longer physical life.

REPLACEMENT ANALYSIS

Up to this point in our economic analysis we have considered the evaluation and selection of *new* alternatives. Which new car or production machine should we buy? What is the economic life of the planned material handling system or vehicle? However, economic analysis often weighs *existing* versus *new* facilities. The key choice is not which new way to perform the desired task. Instead, we have equipment performing the task, and the question is: Should the existing equipment be retained or replaced?

This question may arise because the asset is approaching the end of its economic life. In other cases, earlier replacement may be considered because needs have changed or better alternatives have become available. In all cases, the answer depends on an economic analysis based on current information. This is the domain of **replacement analysis.**

The Replacement Problem

Replacement analysis considers the same factors that were part of finding the economic life that minimizes the EUAC—obsolescence and deterioration due to aging.

Obsolescence: occurs when an asset’s technology is surpassed by newer and/or different technologies.
Deterioration due to aging: costs for maintenance often increase as production machinery and other business assets age. Aging equipment often has a greater risk of breakdowns. Planned replacements can be scheduled to minimize the time and cost of disruptions. Unplanned replacements can be very costly or even, as with an airplane engine, potentially catastrophic.

There are variations of the replacement problem: An existing asset may be abandoned or retired, augmented by a new asset but kept in service, or overhauled to reduce its operating and maintenance costs. These variations are most easily considered as potential new alternatives—just as you might consider buying a used car as a “new” way to meet your needs.

Replacement problems are normally analyzed by looking only at the *costs* of the existing and replacement assets. Since the assets typically perform the same function, the value of using the vehicle, machine, or other equipment can be ignored. If the new asset has new features or better performance, these can be included as a cost savings for the new asset or as an opportunity cost for the existing asset.

Alternatives in a replacement problem almost always have *different lives*. This is because an existing asset will often be kept for at most a few years longer, while the potential replacements may have lives of any length. Because the alternatives have differing lives, economic comparisons use annual values—annual marginal costs and EUACs. We can calculate present worth of costs, but only as a step in calculating EUAC values.

Because there is an existing asset, replacement analysis must often deal with sunk costs and management attitudes. Examples include:

• This machine cost $12,000. We cannot replace it until we have gotten our money's worth out of it.
• If it isn't broken, don't fix it. What we have now is good enough.
• We would rather deal with the problem we know (current aging machine) than a solution we do not understand (new technology).

Even if no recommendation to replace now is made, such a recommendation may be made in a year or more. At *some* point, existing equipment will be replaced, either when it is no longer necessary or when better equipment is available. Thus, the question is not *if* the existing asset will be replaced, but *when* it will be replaced. If we do decide to keep the asset for another year, we will often reanalyze the problem next year. The operating environment and costs may change, or new alternatives with lower costs or better performance may emerge.

Replacement Analysis Common Scenario

The most common replacement scenario for firms is one in which an existing implemented asset is aging and nearing the end of its economic life. Its past purchase, installation, repair, and maintenance costs are all *sunk* costs. Its realizable salvage value is a selling price minus removal and sales costs. This value is usually only a small fraction of its initial cost. Costs for repair and maintenance are rising, as are the risks of equipment breakdowns, which can cause unscheduled production shutdowns (and are often the largest cost). The asset may be obsolete (or becoming so).

This type of aging asset is evaluated by asking, what is the cost of using this asset for one more year? For a second year? For a third year? How soon does a new alternative have a lower cost? In replacement analysis, such questions are evaluated formally, with the following rule applied for decision-making purposes:

Keep the existing asset in place for the present year if and until the marginal cost of one more year of ownership exceeds the minimum EUAC of a new asset available at that time.

The replacement rule for comparing an existing old asset with a potential new one follows this process:

1. Calculate the marginal cost (MC) of keeping the existing old asset one more year (this year).
2. Calculate the minimum EUAC of the best available new asset.
3. Decision rule:

If $MC_{old} > EUAC_{new}$, replace now;

otherwise, keep the old asset one more year and re-evaluate next year.

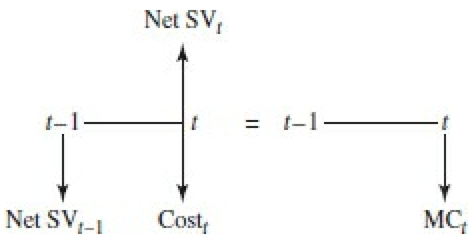
Examples 13–1 to 13–3 illustrate the common case where costs of a current old asset are rising each year. These increasing annual costs (marginal costs) are detailed in Examples 13–4 to 13–6. Lastly, in Example 13–7 the replacement rule is used to compare currently implemented old and new alternatives.

MARGINAL COSTS

Marginal costs, as opposed to an EUAC, are the year-by-year costs of keeping an asset. Therefore, the "period" of any yearly marginal cost of ownership is always *1 year*. In our analysis, marginal cost is compared with EUAC, which is an end-of-year cash flow. Therefore, the marginal cost is also calculated as end-of-year cash flow. The marginal cost of ownership for any year in an asset's life is the cost for *that year only*. Example 13–4 illustrates total marginal costs for all years of an asset.

Choosing to keep an asset for a given year ensures that the marginal cost for that year is incurred. At the beginning of the year, the net salvage value from the previous year is "invested" by keeping the asset. Then, at the end of the year, the new net salvage value is available. Other costs for that year are assumed to be end-of-year cash flows. This is formalized in Equation 13–5 or the equivalent Equation 13–6.

$$MC_t = \text{Net SV}_{t-1}(1+i) - \text{Net SV}_t + \text{Cost}_t \quad (13\text{-}5)$$



Examples 13–4 and 13–5 illustrate calculating marginal costs over a new asset’s life using Equation 13-6. This calculates capital costs as a loss in net salvage value plus interest on the capital tied up for the year.

$$MC_t = (\text{Net SV}_{t-1} - \text{Net SV}_t) + \text{Net SV}_{t-1}(i) + \text{Cost}_t \quad (13\text{-}6)$$

EXAMPLE 13-4

A new piece of production machinery has the following costs.

Investment cost =	\$25,000
Annual operating and maintenance cost =	\$2000 in Year 1 and then increasing by \$500 per year
Annual cost for risk of breakdown =	\$5000 per year for 3 years, then increasing by \$1500 per year
Useful life =	7 years
MARR =	15%

Calculate the marginal cost of keeping this asset over its useful life.

SOLUTION

From the problem data we can easily find the marginal costs for O&M and risk of breakdowns. However, to calculate the marginal capital recovery cost, we need estimates of each year’s market or salvage value.

Year	Salvage Value
1	\$18,000
2	13,000
3	9,000

4	6,000
5	4,000
6	3,000
7	2,500

We can now calculate the machinery's *marginal cost* (year-to-year cost of ownership) over its 7-year useful life.

Year, t	Loss in Salvage Value in Year t	Interest in Year t	O&M Cost inYear t	Cost of Breakdown Riskin Year t	Total Marginal Cost in Year t
1	$\$25,000 - 18,000 = \$7000$	$\$25,000(0.15) = \$3750$	$2000	$ 5,000	$17,750
2	$18,000 - 13,000 = 5000$	$18,000(0.15) = 2700$	2500	5,000	15,200
3	$13,000 - 9,000 = 4000$	$13,000(0.15) = 1950$	3000	5,000	13,950
4	$9,000 - 6,000 = 3000$	$9,000(0.15) = 1350$	3500	6,500	14,350
5	$6,000 - 4,000 = 2000$	$6,000(0.15) = 900$	4000	8,000	14,900

6	4, 000 − 3, 000 = 1000	4, 000(0.15) = 600	4500	9,500	15,600
7	3, 000 − 2, 500 = 500	3, 000(0.15) = 450	5000	11,000	16,950

Notice that each year's total marginal cost includes loss in market value, interest, O&M cost, and cost for risk of breakdowns. For example, the Year-5 marginal cost of $14,900 is calculated as 2000 + 900 + 4000 + 8000.

In each year the marginal cost *is* the cost to keep the asset for one more year. The pattern in Example 13–4 is typical. In early years the marginal costs are decreasing because the capital costs are being spread over more years. In later years the marginal costs are increasing because yearly costs are increasing. As the asset ages it will be replaced when its marginal cost exceeds the EUAC of the best new alternative.

Example 13–5 shows how EUAC's can be calculated from the marginal cost data in Example 13–4.

EXAMPLE 13-5 Example 3–4 Revisited

Taking the machinery in Example 13–4 as a new asset, calculate the $EUAC_t$ for each year from the marginal costs.

SOLUTION

The EUAC of keeping this asset for each year of its useful life is worked out as follows.

Year, t	New Asset Total Marginal Cost in Year t	Present Cost if Kept Through Year t ($PWcost_t$)	EUAC if Kept Through Year t
1	\$17,750	[17, 750(P/F, 15%, 1)]	× (A/P, 15%, 1) = \$17,750
2	15,200	PC_1 + 15, 200(P/F, 15%, 2)	× (A/P, 15%, 2) = 16,560
3	13,950	PC_2 + 13, 950(P/F, 15%, 3)	× (A/P, 15%, 3) = 15,810
4	14,350	PC_3 + 14, 350(P/F, 15%, 4)	× (A/P, 15%, 4) = 15,520
5	14,900	PC_4 + 14, 900(P/F, 15%, 5)	× (A/P, 15%, 5) = 15,430 ←
6	15,600	PC_5 + 15, 600(P/F, 15%, 6)	× (A/P, 15%, 6) = 15,450
7	16,950	PC_6 + 16, 950(P/F, 15%, 7)	× (A/P, 15%, 7) = 15,580

A minimum EUAC of \$15,430 is attained for the new asset at Year 5, which is the asset's *economic* or *minimum cost life.* As long as the marginal cost for another year is less than the EUAC, the EUAC continues to decline. When the marginal cost in year 6 exceeds the EUAC, then the EUAC begins to increase.

We can see that marginal costs increase in each subsequent year of ownership. As before the asset is replaced when its marginal cost exceeds the minimum EUAC of the best new alternative.

Unfortunately, the $13,250 marginal cost of keeping the 5-year old asset for one more year cannot be repeated. Even if used assets were available the costs to find one, install it, and remove it 1 year later would far exceed the $1000 loss in value for the existing asset.

EXAMPLE 13-6

An asset purchased 5 years ago for $75,000 can be sold today for $15,000. Operating expenses will be $10,000 this year, but these will increase by $1500 per year. It is estimated that the asset's salvage value will decrease by $1000 per year over the next 5 years. If the MARR used by the company is 15%, calculate the total marginal cost of ownership of this old asset (that is, the currently implemented asset) for each of the next 5 years.

SOLUTION

We calculate the total marginal cost of maintaining the old asset for the next 5-year period as follows:

Year, *t*	Loss in Salvage Value in Year *t*	Interest in Year *t*	Operating Cost in Year *t*	Marginal Cost in Year *t*
1	$15, 000 − 14, 000 = $1000	$15, 000(0.15) = $2250	$10,000	$13,250
2	14, 000 − 13, 000 = 1000	14, 000(0.15) = 2100	11,500	14,600

3	13, 000 − 12, 000 = 1000	13, 000(0.15) = 1950	13,000	15,950
4	12, 000 − 11, 000 = 1000	12, 000(0.15) = 1800	14,500	17,300
5	11, 000 − 10, 000 = 1000	11, 000(0.15) = 1650	16,000	18,650

EXAMPLE 13-7

The machine in Example 13–6 is the existing asset, and the machine in Examples 13–4 and 13–5 is a new alternative. When should the existing asset be replaced.

SOLUTION

The new asset has a minimum EUAC of $15,430 for its economic life of 5 years. When that is less than the marginal cost of the existing machine, it is time for the replacement.

Year, *t*	Existing Asset Total Marginal Cost in Year *t*	Example 13–4 Alternative Minimum EUAC	Comparison Result and Recommendation
1	$13,250	$15,430	Since $13,250 is *less than* $15,430, keep existing asset.
			Since $14,600 is *less than*

2	14,600	15,430	$15,430, keep existing asset.
3	15,950	15,430	Since $15,950 is *greater than* $15,430, replace with alternative of Example 13–4.
4	17,300		
5	18,650		

COMPLICATIONS IN REPLACEMENT ANALYSIS

Other Scenarios for Replacement Analysis

Sometimes an existing asset is evaluated for early replacement. Perhaps estimating and analysis errors were made when it was purchased and have been discovered. Requirements may have changed, or better alternatives may have now become available. In such cases, the existing alternative’s new economic life and new minimum EUAC are calculated. From this, a comparison to available alternatives may be made. Does a new alternative have a lower minimum EUAC?

Another scenario is when the existing asset will be replaced now or left in place until the next major plant overhaul. The EUAC for each choice must be calculated and compared with that of newer alternatives over the same lives. This can also apply for the duration of a multi-year construction project.

Defining First Costs for Existing and Replacement

Assets

Because the existing asset is already in service, analysts often misunderstand what first cost to assign it. Example 13–8 demonstrates this problem.

EXAMPLE 13-8

A model SK-30 was purchased 2 years ago for \$1600; it has been depreciated by straight-line depreciation using a 4-year life and zero salvage value. Because of recent innovations, the current price of the SK-30 is \$995. An equipment firm has offered a trade-in allowance of \$350 for the SK-30 on a new \$1200 model EL-40. Some discussion revealed that without a trade-in, the EL-40 can be purchased for \$1050. Thus, the originally quoted price of the EL-40 was overstated to allow a larger trade-in allowance. The true current market value of the SK-30 is probably only \$200. In a replacement analysis, what value should be assigned to the SK-30?

SOLUTION

In the example, five different dollar amounts relating to the SK-30 have been outlined:

1. *Original cost:* It cost \$1600 2 years ago.
2. *Present cost:* It now sells for \$995.
3. *Book value:* The original cost less 2 years of depreciation is $1600 - 2/4(1600 - 0) = \$800$.
4. *Trade-in value:* The offer was \$350.
5. *Market value:* The estimate was \$200.

We know that an economic analysis is based on the current situation, not on the past. We refer to past costs as *sunk* costs to emphasize this. These costs cannot be altered, and they are not relevant. (There is one exception: past costs may affect present or future income taxes.)

We want to use actual cash flows for each alternative. Here the question is, What value should be used in an economic analysis for the SK-30? The relevant cost is the equipment's present market value of $200. Neither the original cost, the present cost, the book value, nor the trade-in value is relevant.

Example 13–8 illustrated that of the several different values that can be assigned to the existing asset, the correct one is the present market value less any costs to remove and to sell the asset.

Determining the first cost of a new alternative usually includes the purchase price, sales tax, installation costs, and other items that occur initially on a one-time basis. These values are usually rather straightforward to obtain. One must not arbitrarily subtract the existing asset's salvage value from the new alternative's first cost. This **cash flow approach**, can lead to an incorrect analysis.

As described in Example 13–8, the correct first cost to assign to the existing SK-30 is its $200 current market value. This is the present economic benefit that we would be *forgoing* if we keep the SK–30. This can be called our **opportunity first cost.** If we assume it is a *cash benefit* to the new asset, a potential error arises.

The error is using the *cash flow* perspective when the lives of the new and existing assets are not equal, which is usually the case.

The *opportunity cost* perspective will always lead to the correct answer, so it is the one that should be used.

For example, consider the SK-30 and EL-40 from Example 13–8. It is reasonable to assume that the 2-year-old SK-30 has 3 years of life left and that the new EL-40 would have a 5-year life. Assume that neither will have any salvage value at the end of its life. Compare the difference in their annual capital costs with the correct *opportunity cost* perspective and the incorrect *cash flow* perspective.

SK-30		EL-40	
Market value	$200	First cost	$1050
Remaining life	3 years	Useful life	5 years

Looking at this from an *opportunity cost* perspective, the annual cost comparison of the first costs is

$$\text{Annualized first cost}_{\text{SK-30}} = \$200(A/P, 10\%, 3) = \$80$$
$$\text{Annualized first cost}_{\text{EL-40}} = \$1050(A/P, 10\%, 5) = \$277$$

The *difference* in annualized first cost between the SK-30 and EL-40 is

$$\text{AFC}_{\text{EL-40}} - \text{AFC}_{\text{SK-30}} = \$277 - \$80 = \$197$$

Now using an incorrect *cash flow* perspective to look at the first costs, we can calculate the *difference* due to first cost between the SK-30 and EL-40.

$$\text{Annualized first cost}_{\text{SK-30}} = \$0(A/P, 10\%, 3) = \$0$$
$$\text{Annualized first cost}_{\text{EL-40}} = (\$1050 - 200)(A/P, 10\%, 5) = \$224$$
$$\text{AFC}_{\text{EL-40}} - \text{AFC}_{\text{SK-30}} = \$224 - \$0 = \$224$$

When the existing asset's remaining life (3 years) differs from the new asset's useful life (5 years), the two perspectives yield different results. The correct difference of \$197 is shown by using the *opportunity cost* approach.

A Closer Look at Future Alternatives

Over time, the best available alternative can indeed change. And given the trend in our technological society, it seems likely that future alternatives will be better than the present ones. If so, the prospect of improved future alternatives may affect the present decision between the existing asset and the best alternative today.

Figure 13–4 illustrates two possible estimates of future alternatives. In many technological areas it seems likely that the equivalent uniform annual costs associated with future alternatives will decrease by a constant amount each year. In other fields, however, a rapidly changing technology will produce a sudden and substantially improved alternative—with decreased costs or increased benefits. The uniform decline curve of Figure 13–4 reflects the assumption that each future alternative has a minimum EUAC that is a fixed amount less than the previous year's alternative. This assumption, of course, is only one of many possible assumptions that could be made regarding future alternatives.

If future alternatives will be better what impact will this have on an analysis now? The prospect of better future alternatives may make it more desirable to delay replacement.

As engineering economic analysts, we must familiarize ourselves with potential technological advances in assets targeted for replacement. This part of the decision process is much like the search for all available alternatives, from which we select the best. Upon finding out more about what alternatives and technologies are emerging, we will be better able to understand the repercussions of investing in the current best available alternative. Selecting the current best alternative can be particularly risky when (1) the costs are very high and/or (2) the useful minimum cost life is relatively long (5–10 years or more). When one or both of these conditions exist, it may be better to keep or even augment the existing asset until better future alternatives emerge.

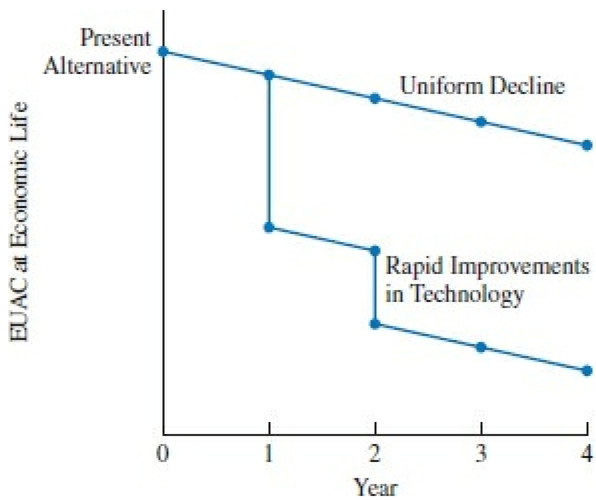


FIGURE 13-4 Two possible ways the EUAC of future alternatives may decline.

AFTER-TAX REPLACEMENT ANALYSIS

As described in Chapter 12, an after-tax analysis provides greater realism and insight. Tax effects can alter before-tax recommendations. Consequently, one should always perform or check these analyses on an after-tax basis.

Economic or Minimum Cost Life Problems

Here we illustrate the effect of taxes on the calculation of the minimum cost life of the existing and new assets. The after-tax minimum EUAC depends on both the depreciation method used and changes in the asset's market value over time. Using an accelerated depreciation method (like MACRS) tends to reduce the after-tax costs early in the asset's life. This alters the shape of the total EUAC curve—the concave shape can be shifted and the minimum EUAC changed. Example 13–9 illustrates the effect that taxes can have.

EXAMPLE 13-9

Some new production machinery has a first cost of $100,000 and a useful life of 10 years. Its estimated operating and maintenance (O&M) costs are $10,000 the first year, which will increase annually by $4000. The asset's before-tax market value will be $50,000 at the end of the first year and then will decrease by $5000 annually. This property is a 7-year MACRS property. The company uses a 6% after-tax MARR and is subject to a combined federal/state tax rate of 28%.

Calculate the after-tax cash flows.

SOLUTION

To find this new production machinery's minimum cost life, we first find the after-tax cash flow (ATCF) effect of the O&M costs and depreciation (Table 13–1). Then, we find the ATCFs of disposal if the equipment is sold in each of the 10 years (Table 13–2). Finally in the closing section on spreadsheets, we combine these two ATCFs (in Figure 13–5) and choose the minimum cost life.

In Table 13–1, the O&M expense starts at $10,000 and increases at $4000 per year. The depreciation entries equal the 7-year r_t MACRS depreciation values given in Table 11–2 multiplied by the $100,000 first cost. The taxable income, which is simply the O&M costs minus the depreciation values, is then multiplied by minus the tax rate to determine the tax savings. The O&M expense plus the tax savings is the Table 13–1 portion of the total ATCF.

Regarding the market value data in this problem, it should be pointed out that the initial decrease of $50,000 in Year 1 is not uncommon. This is especially true for custom-built equipment for a particular and unique application at a specific plant. Such equipment would not be valuable to others in the marketplace. Also, the $100,000 first cost (cost basis) could have included costs due to installation, facility modifications, or removal of old equipment. The $50,000 may be optimistic for the market value of one-year-old equipment.

The next step is to determine the ATCFs that would occur in each possible year of disposal. (The ATCF for Year 0 is easy; it is − $100, 000.) For example, as shown in Table 13–2, in Year 1 there is a $35,710 loss as the book value exceeds the market value. The tax savings from this loss are added to the salvage (market) value to determine the ATCF (*if the asset is disposed of during Year 1*).

Table 13-1 ATCF for O&M and Depreciation for Example 13–9

Year, t	O&M Expense	MACRS Depreciation, d_t	Taxable Income	Tax Savings (at 28%)	O&M Depreciation ATCF
1	−$10,000	$14,290	−$24,290	$6,801	−$3,199
2	−14,000	24,490	−38,490	10,777	−3,223

3	−18,000	17,490	−35,490	9,937	−8,063
4	−22,000	12,490	−34,490	9,657	−12,343
5	−26,000	8,930	−34,930	9,780	−16,220
6	−30,000	8,920	−38,920	10,898	−19,102
7	−34,000	8,930	−42,930	12,020	−21,980
8	−38,000	4,460	−42,460	11,889	−26,111
9	−42,000	0	−42,000	11,760	−30,240
10	−46,000	0	−46,000	12,880	−33,120

Table 13-2 ATCF in Year of Disposal for Example 13–9

Year, *t*	Market Value	Book Value	Gain/Loss Tax	Gain or Loss (at 28%)	ATCF if Disposed of
1	$50,000	$85,710	−$35,710	$9,999	$59,999
2	45,000	61,220	−16,220	4,542	49,542

3	40,000	43,730	−3,730	1,044	41,044
4	35,000	31,240	3,760	−1,053	33,947
5	30,000	22,310	7,690	−2,153	27,847
6	25,000	13,390	11,610	−3,251	21,749
7	20,000	4,460	15,540	−4,351	15,649
8	15,000	0	15,000	−4,200	10,800
9	10,000	0	10,000	−2,800	7,200
10	5,000	0	5,000	−1,400	3,600

These tables assume that depreciation is taken during the year of disposal and then calculates the recaptured depreciation (gain) or loss on the year-end book value.

	A	B	C	D	E	F
1		**Table 13-1**		**Table 13-2**	6%	Interest Rate
2		O&M & Depr.	PW_t	if disposed of		
3	Year	ATCF	without S	ATCF	$EUAC_t$	
4	0		-$100,000			
5	1	−3,200	−103,018	$59,999	$49,200	=PMT($E$1,A5,C5,D5)
6	2	−3,223	−105,886	49,542	33,705	
7	3	−8,062	−112,656	41,044	29,253	
8	4	−12,341	−122,432	33,947	27,573	
9	5	−16,221	−134,553	27,847	27,002	
10	**6**	**−19,101**	−148,019	**21,749**	**26,984**	**optimal life**
11	7	−21,981	−162,637	15,649	27,270	
12	8	−26,111	−179,019	10,800	27,737	
13	9	−30,240	−196,918	7,200	28,325	
14	10	−33,120	−215,412	3,600	28,994	=PMT($E$1,A14,C14,D14)
15						
16		=$C$4+NPV($E$1,$B$5:B14)				
17		= year 0 + NPV(*i*, B column)				

FIGURE 13-5 Spreadsheet for life with minimum after-tax cost.

Spreadsheets are very useful in nearly all after-tax calculations. However, they are absolutely required for optimal life calculations in after-tax situations, because bonus plus MACRS is the tax law, and after-tax cash flows are different in every year. Thus, the NPV function and the PMT function are both needed to find the minimum EUAC after taxes. Figure 13–5 illustrates the calculation of the minimum cost life for Example 13–9.

In Figure 13–5, the NPV finds the present worth of the irregular cash flows from Period 1 through Period *t* for *t* = 1 to life. Then PMT can be used to find the EUAC over each potential life. Before-tax replacement analysis was done this way in Example 13–2. The spreadsheet block function NPV is used to find the PW of cash flows from Period 1 to Period *t*. Note that the cell for Period 1 is an absolute address and the cell for period *t* is a relative address. This allows the formula to be copied.

Marginal Costs on an After-Tax Basis

Marginal costs on an after-tax basis represent the cost that would be incurred through ownership of the existing asset *in each year*. On an after-tax basis we must consider the effects of ordinary taxes as well as gains and losses due to asset disposal. Consider Example 13–10.

EXAMPLE 13-10

Refer to Example 13–4, where we calculated the before-tax marginal costs for a new piece of production machinery. Calculate the asset's after-tax marginal costs considering this additional information.

• Depreciation is by the straight-line method, with S = \$0 and n = 5 years, so d_t = (\$25,000−\$0)/5 = \$5000.
• Ordinary income, recaptured depreciation, and losses on sales are taxed at a combined state and federal rate of 28%.
• The after-tax MARR is 10%.

Some classes skip or have not yet covered Chapter 10's explanation of expected value. Thus, the expected cost for risk of breakdowns is described here as an insurance cost.

SOLUTION

The after-tax marginal cost of ownership will involve the following elements: incurred or forgone loss or recaptured depreciation, interest on invested capital, tax savings due to depreciation, and annual after-tax operating/maintenance and insurance. Figure 13–6 shows example cash flows for the marginal cost detailed in Table 13–3.

As a refresher of the recaptured depreciation calculations in Chapter 12:

The market value in Year 0 = 25,000.
The market value decreases to \$18,000 at Year 1.
The book value at Year 1 = 25,000 − 5000 = \$20,000.
So loss on depreciation = 20,000 − 18,000 = \$2000.
This results in a tax savings of (2000)(0.28) = \$560.

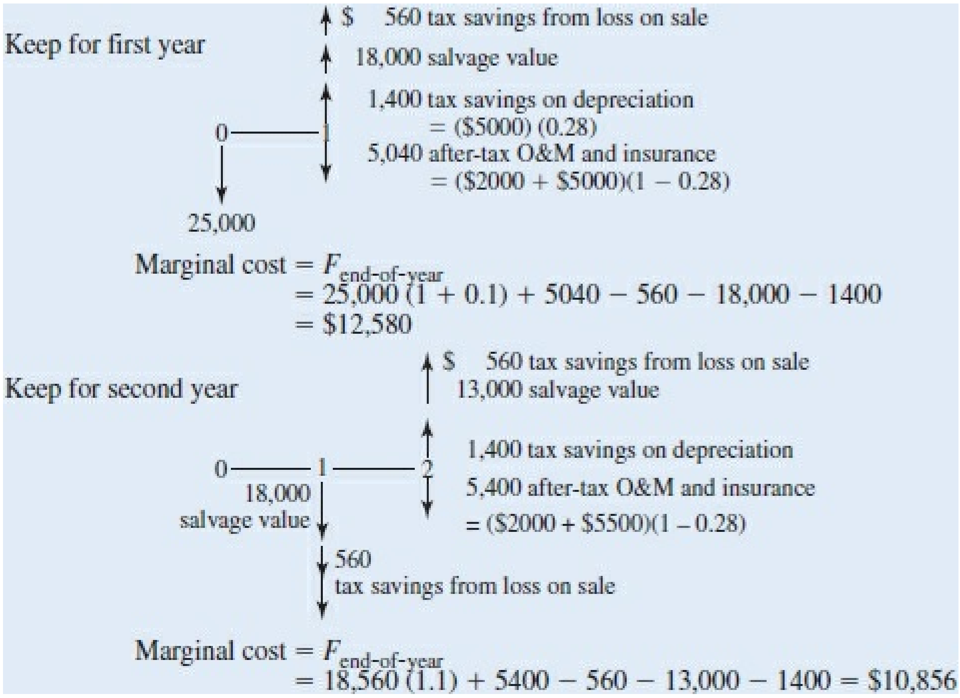


FIGURE 13-6 Cash flow diagrams and calculations for marginal cost.

Table 13-3 Marginal Costs of Ownership

	Market	Book	Recaptured	Taxes or	After-Tax	
Year	Value	Value	Depr. or Loss	Tax Savings	Market Value	
0	$25,000	$25,000			$25,000	
1	18,000	20,000	–$2000	–$560	18,560	
2	13,000	15,000	–2000	–560	13,560	

3	9,000	10,000	–1000	–280	9,280	
4	6,000	5,000	1000	280	5,720	
5	4,000	0	4000	1120	2,880	
6	3,000		3000	840	2,160	
7	2,500		2500	700	1,800	
	Col. B	**Col. C**	**Col. D**		**Col. F**	**= C + D + F – B**
	After-Tax	**Beg. Yr**	**Tax Savings**	**O&M and**	**After-Tax**	**Marginal**
Year	**Market Value**	**Value × (1 + *i*)**	**from Depr. Deduct.**	**Insurance Cost**	**Annual Expense**	**Cost**
0	$25,000					
1	18,560	$27,500	–$1400	$7,000	$5,040	$12,580
2	13,560	20,416	–1400	7,500	5,400	10,856

3	9,280	14,916	–1400	8,000	5,760	9,996
4	5,720	10,208	–1400	10,000	7,200	10,288
5	2,880	6,292	–1400	12,000	8,640	10,652
6	2,160	3,168	0	14,000	10,080	11,088
7	1,800	2,376	0	16,000	11,520	12,096

The marginal cost in each year is much lower after taxes than the pretax numbers shown in Example 13–4. This is true because depreciation and expenses can be subtracted from taxable income. However, the pattern of declining and then increasing marginal costs is the same, and Year 3 is still the year of lowest marginal costs.

SUMMARY

This chapter has shown that an asset's **economic life** may be shorter than its physical life. The economic life minimizes the EUAC of ownership that is calculated for all possible lives. Economic life balances the dynamic of spreading capital costs over more years with costs that are increasing over time—operating, maintenance, risk of unplanned repair or replacement, and the opportunity cost of not taking advantage of better alternatives.

The question in selecting new equipment is: *Which alternative will be most economical and with what economic life?* However, when there is an existing machine, the question is: *Shall we replace it now, or shall we keep it for one or more years?* When we already have equipment, a common mistake is to use past or sunk costs in the replacement analysis. Only present and future costs are relevant.

Replacement analysis must consider the **marginal costs** that are the year-to-year costs of ownership. These are different from EUAC in that they represent costs *as they occur* in each year. In the most common case, where the existing asset's costs are increasing year to year, the decision rule is:

Keep the existing asset in place for the present year if and until the marginal cost of one more year of ownership exceeds the minimum EUAC of a new asset available at that time.

An important concept when calculating the EUAC of an existing or new alternative is the first cost to be assigned to each alternative for calculation purposes. When the lives of the two alternatives match, either an **opportunity cost** or a **cash flow approach** may be used. However, in the more common case of different useful lives, only the opportunity cost approach accurately assigns first costs to the existing and new assets.

It is important when performing engineering economic analyses to include the effects of taxes. This is much easier to accomplish with spreadsheets. Spreadsheets also make it easy to compute the optimal **economic life** of vehicles and equipment—even when this includes complex patterns of declining salvage values, warranty periods, increasing repair costs, and overhaul costs.

Replacement analyses are vastly important, yet are often ignored by companies as they invest in equipment and facilities. Investments in business and personal assets should not be forgotten once an initial economic evaluation has produced a "buy" recommendation. It is important to continue to evaluate assets over their respective life cycles to ensure that invested monies continue to yield the greatest benefit. Replacement analyses help us to ensure this.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

13-1 An engineer is trying to determine the economic life of a new metal press. The press costs $10,000 initially. First-year maintenance costs are $1000. Maintenance costs are forecast to increase $1000 per year for each year after the first. Fill in the table and determine the economic life of the press. Consider only maintenance and capital recovery in your analysis. Interest is 5%.

	Maintenance Cost	EUAC of Capital Recovery	EUAC of Maintenance	Total EUAC
1	$1000			
2	2000			
…	…			
8	8000			

SOLUTION

13-2 A robot's first cost is $40,000, and its market value declines by 20% annually. The operating and maintenance costs start at $2000 per year and climb by $1500 each year. If the MARR is 8%, find the minimum EUAC and the machine's economic life.

SOLUTION

13-3 A machine's first cost is $60,000 with salvage values over the next 5 years of are $50K, $40K, $32K, $25K, and $12K. The annual operating and maintenance costs are the same every year. Determine the machine's economic life and its minimum EUAC, if the interest rate is 7%.

SOLUTION

13-4 A petroleum company, whose minimum attractive rate of return is 10%, needs to paint the vessels and pipes in its refinery periodically to prevent rust. Tuff-Coat, a durable paint, can be purchased for $8.05 a gallon, while Quick-Cover, a less durable paint, costs $3.25 a gallon. The labor cost of applying a gallon of paint is $6.00. Both paints are equally easy to apply and will cover the same area per gallon. Quick-Cover is expected to last 5 years. How long must Tuff-Coat promise to last to justify its use?

SOLUTION

13-5 A hospital is considering the purchase of a new $40,000 diagnostic machine that will have no salvage value after installation, as the cost of removal equals any resale value. Maintenance is estimated to be $2000 per year as long as the machine is owned. After 10 years the machine must be scrapped because the radioactive ion source will have caused so much damage to machine components that safe operation is no longer possible. The most economic life of this machine is

a. One year, since it will have no salvage after installation.
b. Ten years, because maintenance doesn't increase.
c. Less than 10 years, but more information is needed to determine the economic life.

SOLUTION

13-6 A graduate of an engineering economy course has compiled the following set of estimated costs and salvage values for a proposed machine with a first cost of $15,000; however, he has forgotten how to find the most economic life. Your task is to show him how to do this by calculating the equivalent annual cost (EUAC) for n = 8, given a MARR of 12%.

Life (n) Years	Estimated End-of-Year Maintenance	Estimated Salvage if Sold in Year n
1	$ 0	$10,000
2	$ 0	9,000
3	300	8,000
4	300	7,000
5	800	6,000
6	1300	5,000

7	1800	4,000
8	2300	3,000
9	2800	2,000
10	3300	1,000

Remember: Calculate only one EUAC (for n = 8). You are not expected to actually find the most economical life.

SOLUTION

13-7 A truck salesperson is quoted as follows:

"Even though our list price has gone up to $42,000, I'll sell you a new truck for the old price of $40,000, an immediate savings of $2000, and give you a trade-in allowance of $21,000, so your cost is only ($40,000 – 21,000) = $19,000. The book value of your old truck is $12,000, so you're making an additional ($21,000 – 12,000) = $9000 on the deal." The salesperson adds, "Actually I am giving you more trade-in for your old truck than the current market value of $19,500, so you are saving an extra ($21,000 – 19,500) = $1500."

a. In a proper replacement analysis, what is the first cost of your current truck?
b. In a proper replacement analysis, what is the first cost of the new truck?

SOLUTION

13-8 A car was purchased 4 years ago for $25,000. Its estimated salvage value after 7 years was $8000. The car can be sold for $14,000 now or $10,000, $7000, or $5000 in each later year. The annual maintenance cost will be $1800 for this year and increasing by $400 per year. What are the relevant cash flows for choosing how long to keep the car?

SOLUTION

Ten years ago Hyway Robbery installed a conveyor system for $8000.

The conveyor system has been fully depreciated to a zero salvage value. The company is considering replacing the conveyor because maintenance costs have been increasing. The estimated end-of-year maintenance costs for the next 5 years are:

13-9

Year	Maintenance
1	$1000
2	1250
3	1500
4	1750
5	2000

At any time, the cost of removal just equals the value of the scrap metal recovered from the system. The replacement the company is considering has an EUAC of $1028 at its most economic life. The company has a minimum attractive rate of return (MARR) of 10%.

a. Should the conveyor be replaced now? Show the basis used for your decision.
b. If the old conveyor could be sold at any time as scrap metal for $500 more than the cost of removal and all other data remain the same, should the conveyor be replaced now?

SOLUTION

13-10 Ten years ago, the Cool Chemical Company installed a heat exchanger for $10,000. Maintenance costs have been increasing, and they will be $1000 this year. The cost of removal will be $1500 more than the heat exchanger is worth as scrap metal. The replacement the company is considering has an EUAC of $800 at its most economic life. If the company’s minimum attractive rate of return (MARR) is 10%, should the heat exchanger be replaced now?

SOLUTION

A firm uses a MARR of 12%. A crane was purchased 4 years ago for

13-11 $180,000 and it has a current market value of $60,000. Expected operating and maintenance costs and market values follow. Data for a new crane have been analyzed. Its most economic life is 8 years with a minimum EUAC of $38,000. When should the existing crane be replaced?

Year	O&M costs	Market value
1	17,000	50,000
2	20,000	42,000
3	25,000	35,000
4	30,000	30,000
5	35,000	24,000

SOLUTION

13-12 One year ago, Machine *A* was purchased for $15,000, to be used for 5 years. The machine has not performed as expected, and it costs $750 per month for repairs, adjustments, and downtime. Machine *B*, designed to perform the same functions, can be purchased for $25,000 with monthly costs of $75. The expected life of machine *B* is 5 years. Operating costs are substantially equal for the two machines, and salvage values for both are negligible. If 6% is used, the incremental annual net equivalent of Machine *B* is nearest to

a. $2165
b. $2886
c. $4539
d. $5260

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; ☐ = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Economic Life

13-1 An injection-molding machine has a first cost of $1,050,000 and a salvage value of $225,000 in any year. The maintenance and operating cost is $235,000 with an annual gradient of $75,000. The MARR is 10%. What is the most economic life?

13-2 A A machine has a first cost of $50,000. Its market value declines by 25% annually. The operating and maintenance costs start at $2000 per year and climb by $3000 per year. The firm's MARR is 9%. Find the minimum EUAC for this machine and its economic life.

13-3 A machine has a first cost of $24,000. Its market value declines by 20% annually. The repair costs are covered by the warranty in Year 1, and then they increase $900 per year. The firm's MARR is 12%. Find the minimum EUAC for this machine and its economic life.

13-4 A A vehicle has a first cost of $22,500. Its market value declines by 20% annually. It is used by a firm that estimates the effect of older vehicles on the firm's image. A new car has no "image cost." But the image cost of older vehicles climbs by $950 per year. The firm's MARR is 10%. Find the minimum EUAC for this vehicle and its economic life.

13-5 The Clap Chemical Company needs a large insulated stainless steel tank to expand its plant. A recently closed brewery has offered to sell their tank for $15,000 delivered. The price is so low that Clap believes it can sell the tank at any future time and recover its $15,000 investment.

Installing the tank will cost $9000 and removing it will cost $5000. The outside of the tank is covered with heavy insulation that requires considerable maintenance. This will cost $3500 in year 1 and increase by $1000 per year.

(*a*) Based on a 12% before-tax MARR, what life of the insulated tank has the lowest EUAC?

(*b*) When the insulated tank is replaced by another tank is the replacement's economic life likely to be shorter or larger? Explain.

13-6 A

An electric oil pump's first cost is $47,500, and the interest rate is 11%. The pump's end-of-year salvage values over the next 5 years are $43K, $38K, $35K, $32K, and $26K. Determine the pump's economic life.

13-7

A chemical process in your plant leaves scale deposits on the inside of pipes. The scale cannot be removed, but increasing the pumping pressure maintains flow through the narrower diameter. The pipe costs $35 per foot to install, and it has no salvage value when it is removed. The pumping costs are $8 per foot of pipe initially, and they increase annually by $7.50 per year starting in Year 2. What is the economic life of the pipe if the interest rate is 12%?

13-8 A

A $40,000 machine will be purchased by a company whose interest rate is 12%. The installation cost is $10K, and removal costs are insignificant. What is its economic life if its salvage values and O&M costs are as follows?

(*a*)

Year	1	2	3	4	5
S	$25K	$20K	$15K	$10K	$5K
O&M	$4K	$7.5K	$11K	$14.5K	$1.5K/year

(*b*) For lives of 6 years or more, an overhaul costing $15 K is required of the end of year 5.

13-9

A $50,000 machine will be purchased by a company whose interest rate is 10%. The installation cost is $8K, and removal costs are insignificant. What is its economic life if its salvage values and O&M costs are as follows?

(*a*)

Year	1	2 and later

S	$35K drop $5K per year				
O&M	$8K up $1.65K per year				

(*b*) An overhaul costing $9K is needed after 5 years service.

13-10 A

A $25,000 machine will be purchased by a company whose interest rate is 12%. It will cost $5000 to install, but its removal costs are insignificant. What is its economic life if its salvage values and O&M costs are as follows?

Year	1	2	3	4	5
S	$16K	$13K	$11K	$10K	$9.5K
O&M	$5K	$8K	$11K	$14K	$7K

13-11 Mytown's street department repaves a street every 8 years. Potholes cost $15,000 per mile beginning at the end of Year 3 after construction or repaving. The cost to fix potholes generally increases by $15,000 each year. Repaving costs are $200,000 per mile. Mytown uses an interest rate of 8%. What is the EUAC for Mytown's policy? What is the EUAC for the optimal policy? What is the optimal policy?

13-12 A

The plant manager may purchase a piece of unusual machinery for $10,000. Its resale value after 1 year is estimated to be $3000. Because the device is sought by antique collectors, resale value is rising $500 per year.

The maintenance cost is $300 per year for each of the first 3 years, and then it is expected to double each year. Thus the fourth-year maintenance will be $600; the fifth-year maintenance, $1200, and so on. Based on a 15% before-tax MARR, what life of this machinery has the lowest EUAC?

13-13 J&E Fine Wines recently purchased a new grape press for $150,000. The annual operating and maintenance costs for the press are estimated to be $4000 the first year. These costs are expected to increase by $5000 each year after the first. The market value is expected to decrease by $25,000 each year to a value of zero. Installation and removal of a press each cost $3500. Using an interest rate of 9%, determine the economic life of the press.

13-14 A

A machine's initial maintenance costs are covered by a 3-year warranty. Its initial installed cost is $22,000. Salvage value in any year is zero. Assume a 10% interest rate and ignore income taxes.

Annual maintenance	Year
None	1–3
$6500	4–5
$2500	6
+ $3500/yr	7–10

(*a*) Compute the economic life with the lowest EUAC.
(*b*) How does the economic life and minimum EUAC change if an overhaul costing $8000 at the end of year 5 is required for lives of 6 years or greater.

13-15 A

A new $100,000 bottling machine will have no salvage value when it is removed. The plant manager has asked you to estimate the machine's economic service life, ignoring income taxes. He estimates that the annual maintenance cost will be constant at $2500 per year. What service life will result in the lowest equivalent uniform annual cost?

13-16 A

A 2000-pound, counterbalanced, propane forklift can be purchased for $30,000. Due to the intended service use, the forklift's market value drops 20% of its prior year's value in Years 1 and 2 and then declines by 15% until Year 10 when it will have a scrap/market value of $1000. Maintenance of the forklift is $500 per year during Years 1 and 2 while the warranty is in place. In Year 3 it jumps to $750 and increases $750 per year thereafter. What is the optimal life of the forklift using i = 10%?

Demonstrate how one would calculate the economic life of a truck costing \$40,000 initially, and at the end of this and each following year (y) costing OMR_y in operation, maintenance and repair costs.

13-17 The truck is depreciated using the straight-line method over 4 years (i.e., $\$40,000/4 = D_y$). Its salvage value each year equals its book value. Develop an expression to show how to determine the truck's economic life—that is, the year when the truck's uniform equivalent annual cost is a minimum.

Contributed by D. P. Loucks, Cornell University

Bill's father read that each year a car's value declines by 10%. He also read that a new car's value declines by 12% as it is driven off the dealer's lot. Maintenance costs and the costs of "car problems" are only \$200 per year during the 2-year warranty period. Then they jump to \$750 per year, with an annual increase of \$500 per year.

Bill's dad wants to keep his annual cost of car ownership low. The car he prefers cost \$30,000 new, and he uses an interest rate of 8%. For this car, the new vehicle warranty is transferrable.

13-18

(*a*) If he buys the car new, what is the minimum cost life? What is the minimum EUAC?
(*b*) If he buys the car after it is 2 years old, what is the minimum cost life? What is the minimum EUAC?
(*c*) If he buys the car after it is 4 years old, what is the minimum cost life? What is the minimum EUAC?
(*d*) If he buys the car after it is 6 years old, what is the minimum cost life? What is the minimum EUAC?
(*e*) What strategy do you recommend? Why?

Replacement Problems

13-19 Typically there are two alternatives in a replacement analysis. One alternative is to replace the existing asset now. The other alternative is which one of the following?

A. Keep the existing asset for its remaining useful life.
B. Keep the existing asset for another year and then reexamine the situation.

C. Keep the existing asset until there is an improved new asset that is better than the present new asset.

13-20 The existing asset's economic life can be found if certain estimates about it can be made. Assuming those estimates prove to be exactly correct, one can accurately predict the year when the existing asset should be replaced, even if nothing is known about potential new assets. True or false? Explain.

13-21 A proposal has been made to replace a large heat exchanger (3 years ago, the initial cost was $85,000) with a new, more efficient unit at a cost of $120,000. The existing heat exchanger is being depreciated by the MACRS method. Its present book value is $20,400, but its scrap value just equals the cost to remove it from the plant. In preparing the before-tax economic analysis, should the $20,400 book value of the old heat exchanger be

A. *added* to the cost of the new exchanger?

B. *subtracted* from the cost of the new exchanger?

C. *ignored* in this before-tax economic analysis?

13-22 Which one of the following is the proper dollar value of existing equipment to use in replacement analysis?

A. Original cost

B. Present market value if sold

C. Present trade-in value

D. Present book value

E. Present market value if sold minus removal and selling expenses

13-23 A drill press was purchased 4 years ago for $40,000. Its estimated salvage value after 7 years was $5000. The press can be sold for $15,000 today, or for $12,000, $9000, or $6000 at the ends of each of the next 3 years. The annual operating and maintenance cost for the next 3 years will be $2700 for this year and then will increase by $800 per year. What are the relevant cash flows for this machine?

13-24 E Describe an example in a replacement analysis scenario where the replacement is being considered due to

(*a*) reduced performance of the existing equipment.

(*b*) altered requirements.

(*c*) obsolescence of the existing equipment.

(*d*) risk of catastrophic failure or unplanned replacement of the existing

equipment.
(*e*) a previous equipment choice that was incorrectly analyzed by your boss. What ethical and pragmatic challenges exist in this case?

13-25 A pulpwood-forming machine was purchased and installed 6 years ago for $50,000. The declared salvage value was $5000, with a useful life of 10 years. The machine can be replaced with a more efficient model that costs $90,000, including installation. The old machine can be sold on the open market for $25,000. The cost to remove the old machine is $4000. Which are the relevant costs for the old machine?

13-26 A A $25,000 machine that has been used for one year has a salvage value of $16,000 now, which will drop by $4000 per year. The maintenance costs for the next 4 years are $1250, $1450, $1750, and $2250. When the machine is sold, it will cost $2000 to remove and sell. When the machine was purchased, the estimated salvage value in 5 years was $3500. What are the relevant costs for the machine?

Marginal Costs

13-27 In Problem 13-23, what is the marginal cost for each year?

13-28 A In Problem 13-26, what is the marginal cost for each year?

13-29 In Problem 13-18, what is the marginal cost for each year?

13-30 SHOJ Enterprises has asked you to look at the following data. The interest rate is 10%.

Year, *n*	Marginal Cost Data Existing Asset	EUAC if Kept *n* Years New Asset
1	$3000	$6500
2	3150	4150
3	3400	3200

A

4	3800	3100
5	4250	3150
6	4950	3400

(*a*) What is the new asset's economic life?
(*b*) When, if at all, should we replace the existing asset with the new asset?

Should NewTech's pollution testing system be replaced this year? The system has a salvage value now of $5000, which will fall to $4000 by the end of the year. The cost of lower productivity linked to the older system is $3000 this year. NewTech uses an interest rate of 15%. What is the cost advantage of the best system? A potential new system costs $12,000 and has the following salvage values and lost productivity for each year.

13-31 G

Year	S	Lost Productivity
0	$12,000	
1	9,000	$ 0
2	7,000	1000
3	5,000	2000
4	3,000	3000

13-32 A	Five years ago, Thomas Martin installed production machinery that had a first cost of \$25,000. At that time initial yearly costs were estimated at \$1250, increasing by \$500 each year. The market value of this machinery each year would be 90% of the previous year's value. There is a new machine available now that has a first cost of \$27,900 and no yearly costs over its 5-year minimum cost life. If Thomas Martin uses an 8% before-tax MARR, when, if at all, should he replace the existing machinery with the new unit?
13-33	Consider Problem 13-32 involving Thomas Martin. When, if at all, should the old machinery be replaced with the new, given the following changes in the data. The old machine retains only 70% of its value in the market from year to year. The yearly costs of the old machine were \$3000 in Year 1 and increase at 10% thereafter.
13-34 A	In evaluating projects, LeadTech's engineers use a rate of 15%. One year ago a robotic transfer machine was installed at a cost of \$38,000. At the time, a 10-year life was estimated, but the machine has had a downtime rate of 28%, which is unacceptably high. A \$12,000 upgrade should fix the problem, or a labor-intensive process costing \$3500 in direct labor per year can be substituted. The plant estimates indirect plant expenses at 60% of direct labor, and it allocates front office overhead at 40% of plant expenses (direct and indirect). The robot has a value in other uses of \$15,000. What is the difference between the EUACs for upgrading and switching to the labor-intensive process?
	Mary O'Leary's company ships fine wool garments from County Cork, Ireland. Five years ago she purchased some new automated packing equipment having a first cost of \$125,000. The annual costs for operating, maintenance, and insurance, as well as market value data for each year of the equipment's 10-year useful life are as follows.

Year *n* Annual Costs in Year ***n*** for Market Value in Year *n*

Operating Maintenance Insurance

1	$16,000	$5,000	$17,000	$80,000
2	20,000	10,000	16,000	78,000
3	24,000	15,000	15,000	76,000
4	28,000	20,000	14,000	74,000
5	32,000	25,000	12,000	72,000
6	36,000	30,000	11,000	70,000
7	40,000	35,000	10,000	68,000
8	44,000	40,000	10,000	66,000
9	48,000	45,000	10,000	64,000
10	52,000	50,000	10,000	62,000

Now Mary is looking at the remaining 5 years of her investment in this equipment. What is the marginal cost for each of the remaining 5 years? When, if at all, should Mary replace this packing equipment with a new asset that has a minimum EUAC of $110,000?

13-35

Eight years ago, the Blank Block Building Company installed an automated conveyor system for $38,000. When the conveyor is replaced, the net cost of removal will be $2500. The minimum

13-36 EUAC of a new conveyor is $6500. When should the conveyor be replaced if BBB's MARR is 12%? The O&M costs for the next 5 years are $5K, $6K, $7K, $8K, and $9K.
A

13-37 Big-J Construction Company, Inc. is conducting a routine periodic review of existing field equipment. They use a MARR of 20%. This includes a replacement evaluation of a paving machine now in use. The machine was purchased 5 years ago for $200,000, The paver's current market value is $65,000, and yearly operating and maintenance costs are as follows.

Year, *n*	Operating Cost in Year *n*	Maintenance Cost in Year *n*	Market Value if Sold in Year *n*
1	17,000	12,000	50,000
2	20,000	18,000	40,000
3	25,000	20,000	35,000
4	30,000	25,000	30,000
5	35,000	30,000	25,000

Data for a new paving machine have been analyzed. Its most economic life is at 8 years, with a minimum EUAC of $62,000.

When should the existing paving machine be replaced?

13-38 A

VMIC Corp. has asked you to look at the following data. The interest rate is 10%.

Year, *n*	Marginal Cost Data Existing Asset	EUAC if Kept *n* Years New Asset
1	$2000	$6500
2	2200	4200
3	2300	3000
4	2550	2650
5	2900	2700
6	3400	2800
7	4000	3000

(*a*) What is the new asset's minimum cost life?
(*b*) When, if at all, should we replace the existing asset with the new asset?

13-39 A

The existing business asset has the tabulated total marginal cost data. What is the maximum first cost for a new asset for it to be preferred over the existing asset today? With no salvage value and no differential annual costs from the existing asset, the new asset's minimum cost life is 5 years. Use i = 20%.

	Year	1	2	3	4
	Total Marginal Cost ($1000s)	100	250	300	400

Complications in Analysis

13-40 A G	You are considering the purchase of a new high-efficiency machine to replace older machines now. The new machine can replace four of the older machines, each with a current market value of $600. The new machine will cost $5000 and will save the equivalent of 10,000 kWh of electricity per year. After a period of 10 years, neither option (new or old) will have any market value. If you use a before-tax MARR of 25% and pay $0.075 per kilowatt-hour, would you replace the old machines today with the new one?
13-41 E	A professor of engineering economics owns an older car. In the past 12 months, he has paid $2000 to replace the transmission, bought two new tires for $160, and installed a music system for $110. He wants to keep the car for 2 more years because he invested money 3 years ago in a 5-year certificate of deposit, which is earmarked to pay for his dream machine, a red European sports car. Today the old car's engine failed. The professor has two alternatives. He can have the engine overhauled at a cost of $1800 and then most likely have to pay another $800 per year for the next 2 years for maintenance. The car will have no salvage value at that time. Alternatively, a colleague offered to make the professor a $5000 loan to buy another used car. He must pay the loan back in two equal installments of $2500 due at the end of Year 1 and Year 2, and at the end of the second year he must give the colleague the car. What interest rate is the professor paying on the loan from his colleague, if the vehicle will be worth $3000 after 2 years? Is this an ethical interest rate?

The “new” used car has an expected annual maintenance cost of $300. If the professor selects this alternative, he can sell his current vehicle to a junkyard for $500. Interest is 6%. Using present worth analysis, which alternative should he select and why?

13-42 A

Sacramento Cab Company owns several taxis that were purchased for $25,000 each 4 years ago. The cabs’ current market value is $12,000 each, and if they are kept for another 6 years they can be sold for $2000 per cab. The annual maintenance cost per cab is $1000 per year. Sacramento Cab has been approached about a leasing plan that would replace the cabs. The leasing plan calls for payments of $6000 per year. The annual maintenance cost for each leased cab is $750 per year. Should the cabs be replaced if the interest rate is 10%?

13-43

The local telephone company purchased four special pole hole diggers 8 years ago for $14,000 each. Owing to an increased workload, additional machines will soon be required.

Recently an improved model of the digger was announced. The new machines have a higher production rate and lower maintenance expense than the old machines but will cost $32,000 each with a service life of 8 years and salvage value of $750 each. The four original diggers have an immediate salvage of $2000 each and an estimated salvage value of $500 each 8 years hence. The average annual maintenance expense of each old machine is about $1500, compared with $600 each for the new machines.

The workload would require three additional new machines if the old machines continue in service. However, if the old machines were all retired from service, the workload could be carried by six new machines with an annual savings of $12,000 in operation costs. A training program to prepare employees to run the machines will be necessary at an estimated cost of $1200 per new machine. If the MARR is 8% before taxes, what should the company do?

JMJ Inc. bought a manufacturing line 5 years ago for $35M (million). At that time it was estimated to have a service life of 10

13-44 A	years and salvage value at the end of its service life of $10M. JMJ's CFO recently proposed to replace the old line with a modern line expected to last 15 years and cost $95M. This line will provide $5M savings in annual O&M costs, increase revenues by $2M, and have a $15M salvage value. The seller of the new line is willing to accept the old line as a trade-in for its current fair market value, which is $12M. The CFO estimates that if the old line is kept for 5 more years, its salvage value will be $6M. If the MARR is 8% per year, should the company keep the old line or replace it with the new line? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*
13-45	A couple bought their house 10 years ago for $165,000. At the time of purchase, they made a $35,000 down payment, and the balance was financed by a 30-year mortgage with monthly payments of $988.35. They expect to live in this house for 20 years, after which time they plan to sell the house and move to another state. Alternatively, they can sell the house now and live in a rental unit for the next 20 years. The house can be sold now for $210,000, from which a 6% real estate commission and $110,000 remaining loan balance and miscellaneous expenses will be deducted. If they stay in the house, the house can be sold after 20 years for $320,000, from which a 6% real estate commission and $10,000 miscellaneous expenses will be deducted. A comparable rental unit rents for $960 payable at the beginning of every month. No security deposit will be required of them to rent the unit, and the rent will not increase if they maintain a good payment record. They use an interest rate of 0.5% per month for analyzing this financial opportunity. Should they stay in the house or should they sell it and move into a rental unit? *Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology*
	A used car can be kept for two more years and then sold for an estimated value of $3000, or it can be sold now for $7500. The average annual maintenance cost over the past 7 years has been $500 per year. However, if the car is kept for two more years, this cost is expected to be $1800 the first year and $2000 the second year. As an

13-46 alternative, a new car can be purchased for $22,000 and be used for
A 4 years, after which it will be sold for $8,000. The new car will be under warranty the first 4 years, and no extra maintenance cost will be incurred during those years. If the MARR is 15% per year, what is the better option?

Contributed by Hamed Kashani, Saeid Sadri, and Baabak Ashuri, Georgia Institute of Technology

13-47 The Quick Manufacturing Company, a large profitable corporation, may replace a production machine tool. A new machine would cost $37,000, have an 8-year useful life, and have no salvage value. For tax purposes, 3-year MACRS depreciation would be used. The existing machine tool cost $40,000 4 years ago, and it has been completely depreciated. The tool could be sold now to a used equipment dealer for $10,000 or be kept in service for another 8 years. It would then have no salvage value. The new machine tool would save about $9000 per year in operating costs compared to the existing machine.

Assume a 20% combined state and federal tax rate. Compute the **before**-tax rate of return on the replacement proposal of installing the new machine rather than keeping the existing machine.

After-Tax Replacement

Fifteen years ago the Acme Manufacturing Company bought a propane-powered forklift truck for $4800. The company depreciated the forklift using straight-line depreciation, a 12-year life, and zero salvage value. Estimated end-of-year maintenance costs for the next 10 years are as follows:

Year	Maintenance Cost
1	$ 400

	2	600
	3	800
13-48 A	4	1000
	5–10	1400/year

The old forklift has no present or future net salvage value (scrap value equals disposal costs). A modern unit can be purchased for $6500. It has an economic life equal to its 10-year depreciable life. Straight-line depreciation will be employed, with zero salvage value at the end of the 10-year depreciable life. Maintenance on the new forklift is estimated to be a constant $50 per year for the next 10 years, after which maintenance is expected to increase sharply. Should Acme Manufacturing keep its old forklift truck for the present or replace it now with a new one? The firm expects an 8% after-tax rate of return on its investments. Assume a 28% combined state-and-federal tax rate.

13-49

State the advantages and disadvantages with respect to after-tax benefits of the following options for a major equipment unit:
A. Buy new equipment.
B. Trade in and buy a similar, rebuilt equipment from the manufacturer.
C. Have the manufacturer rebuild your equipment with all new available options.
D. Have the manufacturer rebuild your equipment to the original specifications.
E. Buy used equipment.

A new employee at CLL Engineering Consulting Inc., you are asked to join a team performing an economic analysis for a client. The client has a combined federal/state tax rate of 25% on ordinary

income, depreciation recapture, and losses.
Existing Asset: This asset was placed in service 7 years ago. At that time the $50,000 cost basis was set up on a straight-line depreciation schedule with an estimated salvage value of $15,000 over its 10-year life. This asset has a present market value of $30,000.

13-50 A

New Asset: The new asset has a first cost of $85,000 and will be depreciated by MACRS depreciation over its 10-year class life. This asset qualifies for a 10% investment tax credit.
(*a*) Your task is to find the Time-0 ATCFs.
(*b*) How would your calculations change if the present market value of the existing asset is $25,500?
(*c*) How would your calculations change if the present market value of the existing asset is $18,000?

Machine *A* has been completely overhauled for $9000 and is expected to last another 12 years. The $9000 was treated as an expense for tax purposes last year. Machine *A* can be sold now for $30,000 net after selling expenses, but will have no salvage value 12 years hence. It was bought new 9 years ago for $54,000 and has been depreciated since then by straight-line depreciation using a 12-year depreciable life.

13-51 Because less output is now required, Machine *A* can be replaced with a smaller machine: Machine *B* costs $42,000, has an anticipated life of 12 years, and would reduce operating costs $2500 per year. It would be depreciated by straight-line depreciation with a 12-year depreciable life and no salvage value.

The income tax rate is 25%. Compare the after-tax annual costs and decide whether Machine *A* should be retained or replaced by Machine *B*. Use a 10% after-tax rate of return.

A firm is concerned about the condition of some of its plant machinery. Bill James, a newly hired engineer, reviewed the situation and identified five feasible, mutually exclusive alternatives.
Alternative A: Spend $44,000 now repairing various items. The $44,000 can be charged as a current operating expense (rather than capitalized) and deducted from other taxable income immediately.

These repairs will keep the plant functioning for 7 years with current operating costs.

13-52 A

Alternative B: Spend $49,000 to buy general-purpose equipment. Depreciation would be 5-year MACRS. The equipment has no salvage value after 7 years. The new equipment will reduce annual operating costs by $7000.

Alternative C: Spend $56,000 to buy new specialized equipment. This equipment would be depreciated by 5-year MACRS. This equipment would reduce annual operating costs by $12,000. It will have no salvage value.

Alternative D: This is the “do nothing” alternative, with annual operating costs $8000 above the present level.

This profitable firm pays 28% corporate income taxes and uses a 10% after-tax rate of return. Which alternative should the firm adopt?

13-53 Compute the after-tax rate of return on the replacement proposal for Problem 13-47.

13-54 A BC Junction purchased some embroidering equipment for their Denver facility 3 years ago for $15,000. This equipment qualified as MACRS 5-year property. Maintenance costs are estimated to be $1000 this next year and will increase by $1000 per year thereafter. The market (salvage) value for the equipment is $10,000 at the end of this year and declines by $1000 per year in the future. If BC Junction has an after-tax MARR of 30%, a marginal tax rate of 28% on ordinary income, depreciation recapture, and losses, what after-tax life of this previously purchased equipment has the lowest EUAC?

Minicases

13-55 Reconsider the acquisition of packing equipment for Mary O’Leary’s business, as described in Problem 13-35. Given the data tabulated there use an after-tax MARR of 25% and a tax rate of 35% on ordinary income to evaluate the investment. Determine the lowest after-tax EUAC of the equipment at its initial purchase.

A 2000-pound, counterbalanced, electric forklift can be purchased for $25,000 plus $3000 for the charger and $3000 for a battery. The forklift's market value is 15% less for each of its first 6 years of service. After this period the market value declines at the rate of 7.5% for the next 6 years.

13-56 A The battery has a life of 12 years and a salvage value of $300. The charger has a 12-year life and a $100 salvage value. The charger's market value declines 20% per year of use. The battery's market value declines by 15% of its purchase price each year. Maintenance of the charger and battery are minimal. The battery will most likely not work with a replacement forklift.

Maintenance of the forklift is $400 per year during Years 1 and 2 while the warranty is in place. In Year 3 it jumps to $800 and increases $500 per year thereafter. What is the optimal ownership policy using $i = 10\%$?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 27 **To Use or Not to Use?**

Focus is treatment of sunk costs. More complicated than most. Some discoveries in the data gathering process. Solution uses equation rather than cash flow table.

CASE 31 **Freeflight Superdiscs**

Inflation and sensitivity analysis for three alternatives. Includes taxes.

CASE 32 **Mr. Speedy**

Includes two memos using different inappropriate financial comparisons. Choose optimal life for replacement of vehicles.

CASE 33 **Piping Plus**

Data from case intro, three memos, income statement, and balance sheet. Computer improvement in a professional services firm. Assumptions will lead to an instructive variety of results.

CASE 34 **R&D Device at EBP**

Equipment replacement cost comparison with unequal lives. Continuing demand requires careful analysis statement by the student because of detailed cost data. Before or after taxes.

CHAPTER 14

INFLATION AND PRICE CHANGE

Price Trends in Solar Technologies

I In spite of inflation increasing most prices, the price of solar power is declining. Also in contrast to fossil fuels, the cost of generating electricity from solar energy is driven by the infrastructure costs instead of the cost of the natural resource. Therefore the costs and prices are more stable, particularly for large-scale electricity generation.

Historically, solar technologies have had high upfront infrastructure costs but low operating costs. The SunShot Initiative launched in 2011 by the U.S. Department of Energy (DOE) seeks to make solar energy economically competitive by 2020.

The U.S. DOE has reported that cumulative adoption of solar technologies has increased over tenfold since 2008. It is expected that achieving the price reduction set by the SunShot Initiative could lead to solar representing 14% of the electricity demand in the U.S. by 2030 and 27% by 2050.

According to a study by the National Renewable Energy Laboratory and the Lawrence Berkeley National Laboratory, reported prices of both residential and commercial photovoltaic systems decreased on average from 6% to 12% annually between 1998 and 2014. At the end of 2014, photovoltaic prices ranged from $4.27/W for residential systems with a median installed capacity of 6kW to $2.08/W for utility-scale systems with a median installed capacity of 14MW.

In 2017, the SunShot Initiative successfully met its 2020 cost target of $0.06 per kilowatt-hour for utility-scale power, three years ahead of schedule. The SunShot Initiative also set new goals for 2030 which reduce the cost of residential, commercial, and utility-scale solar electricity by an additional 50% between 2020 and 2030. It is expected that within the next two decades, the cost of solar technologies will be lower than the costs of conventional fossil fuel electricity technologies. These lower electricity prices can decrease inflation and contribute to increased economic activity and growth.

Interestingly, the deployment of solar technologies continues to increase despite the 2015 plunge in oil prices. Solar was the third most added electricity-generation capacity in 2015, after wind and natural gas. In 2017, about 30% of new U.S. electricity generating capacity came from solar. However, solar power represented just under 2% of total annual generation in 2017. So it remains to be seen if the goals of the SunShot Initiative will be met.

Contributed by Ona Egbue, University of South Carolina Upstate

QUESTIONS TO CONSIDER

1. Deflation decreases the general prices of goods and services and increases purchasing power. How does this differ from the declining price of solar technologies?

2. Currently, most electricity generation in the U.S. is from fossil fuels, including petroleum, coal, and natural gas. In what ways has the decline in oil prices since 2014 affected the adoption of solar technologies?
3. Use Internet resources to compare the changes in the prices of renewable energy technologies including wind, solar, and geothermal over the last 5 years. Which of these three technologies has achieved the greatest cost decline during that period?
4. In addition to cost, what are other barriers to the adoption of photovoltaic systems?

After Completing This Chapter…

The student should be able to:

• Describe inflation, explain how it happens, and list its effects on purchasing power.
• Define real and actual dollars and interest rates.
• Conduct constant dollar and nominal dollar analyses.
• Define and use composite and commodity-specific price indexes.
• Develop and use cash flows that inflate at different interest rates and cash flows subject to different interest rates per period.
• Incorporate the effects of inflation in before-tax and after-tax calculations.
• Develop spreadsheets to incorporate the effects of inflation and price change.

Key Words

actual dollar

base year

composite cost index

constant value dollars

cost-push inflation

deflation

demand-pull inflation

exchange rate

inflation rate

market interest rate

money supply

nominal dollars

price index

purchasing power

real dollar

real interest rate

Thus far we have used constant-value dollars in our analyses, thus they were unaffected by inflation or price change. However, this is not always valid or realistic. In this chapter we develop several key concepts and illustrate how inflation and price changes may be explicitly modeled.

MEANING AND EFFECT OF INFLATION

Inflation is an important concept because the purchasing power of money used in most world economies rarely stays constant. Rather, over time the amount of goods and services that can be bought with a fixed amount of money tends to change. Inflation causes money to lose **purchasing power.** That is, when prices inflate we can buy less with the same amount of money. *Inflation makes future dollars less valuable than present dollars.* Think about examples in your own life, or for an even starker comparison, ask your grandparents how much a loaf of bread or a new car cost 50 years ago. Then compare those prices with what you would pay today for the same items.

This exercise will reveal the effect of inflation: as time passes, goods and services cost more, and more monetary units are needed to buy the same goods and services.

Because of inflation, dollars in one period of time are not equivalent to dollars in another. We know that engineering economic analysis requires that comparisons be made on an equivalent basis. So, it is important for us to be able to incorporate the effects of inflation.

When the purchasing power of a monetary unit *increases* rather than decreases as time passes, the result is **deflation.** Deflation, very rare in the modern world, nonetheless can exist. Deflation has the opposite effect of inflation—one can buy *more* with money in future years than can be bought today. Thus, deflation makes future dollars more valuable than current dollars.

How Does Inflation Happen?

Economists generally believe that inflation depends on the following, either in isolation or in combination.

Money supply: The amount of money in our national economy has an effect on its purchasing power. If there is too much money in the system (the Federal Reserve controls the flow of money) versus goods and services to purchase with that money, the value of dollars tends to decrease. When there are fewer dollars in the system, they become more valuable. The Federal Reserve attempts to influence economic growth and employment by controlling the amount of money in the system.
Exchange rates: The strength of the dollar in world markets affects the profitability of international companies. Prices may be adjusted to compensate for the dollar's relative strength or weakness in the world market. As corporations' profits are weakened or eliminated in some markets owing to fluctuations in exchange rates, prices may be raised in other markets to compensate.
Cost–push inflation: This cause of inflation develops as producers of goods and services "push" their increasing operating costs along to the customer through higher prices. These operating costs include

fabrication/manufacturing, marketing, and sales.
Demand–pull inflation: This cause is realized when consumers spend money freely on goods and services. As more and more people demand certain goods and services, the prices of those goods and services will rise (demand exceeding supply).

A further consideration in analyzing how inflation works is the usually different rates at which prices and wages rise. Do workers benefit if, as their wages increase, the prices of goods and services increase? To determine the net effect of differing rates of inflation, we must be able to make comparisons and understand costs and benefits from an equivalent perspective. In this chapter we will learn how to make such comparisons.

Definitions for Considering Inflation in Engineering Economy

The following definitions are used throughout this chapter to illustrate how inflation and price change affect two quantities: interest rates and cash flows.

Inflation rate (*f*): The inflation rate captures the effect of goods and services costing more—a decrease in the purchasing power of dollars. More money is required to buy a good or service whose price has inflated. The inflation rate is measured as the annual rate of increase in the number of dollars needed to pay for the same amount of goods and services.
Real interest rate (i'): This interest rate measures the "real" value of money excluding the effect of inflation. Because it does not include inflation, it is sometimes called the *inflation-free interest rate*.
Market interest rate (i): This is the rate of interest that one obtains in the general marketplace. For instance, the interest rates on passbook savings, checking plus, and certificates of deposit quoted by banks are all market rates. The lending interest rate for autos and boats is also a market rate. This rate is sometimes called the *combined interest* rate because it incorporates the effect of both real interest **and** inflation. We can view i as follows:

Market interest rate	**has in it**	**"Real" value of money**	**and**	**Effect of inflation**

The mathematical relationship between the inflation, real and market interest rates is given as

$i = i' + f + i'f$ (14-1)

This is the first point where we have defined a real interest rate and a market or combined interest rate. This naturally leads to the question of what meaning should be attached to the interest rate *i*, which is found throughout the text. In fact, both meanings have been used.

• In problems about savings accounts and loans, the interest rate is usually a market rate.
• In problems about engineering projects where costs and benefits are often estimated as $*x* per year, the interest rate is a real rate.

EXAMPLE 14-1

Suppose a professional golfer wants to invest some recent golf winnings in her hometown bank for one year. Currently, the bank is paying a rate of 5.5% *compounded annually*. Assume inflation is expected to be 2% per year. Identify *i*, *f*, and *i′*. Repeat for inflation of 8% per year.

SOLUTION

If Inflation Is 2% per Year

The bank is paying a *market rate* (*i*). The *inflation rate* (*f*) is given. What then is the *real interest rate* (*i′*)?

$$i = 5.5\%, \qquad f = 2\%, \qquad i' = ?$$

Solving for *i′* in Equation 14-1, we have

$$\begin{aligned} i &= i' + f + i'f \\ i - f &= i'(1 + f) \\ i' &= (i - f)/(1 + f) \\ &= (0.055 - 0.02)/(1 + 0.02) = 0.034 \quad \text{or} \quad \mathbf{3.4\%\ per\ year} \end{aligned}$$

This means that the golfer will have 3.4% **more** purchasing power than she had a year ago. At the end of the year she can buy 3.4% more goods and services than she could have at the beginning of the year. For example, assume she was buying golf balls that cost \$5 each and that she had invested \$1000.

At the *beginning* of the year she could buy

$$\text{Number of balls purchased today} = \frac{\text{Dollars today available to buy balls}}{\text{Cost of balls today}}$$
$$= 1000/\$5 = 200 \text{ golf balls}$$

At the *end* of the year she could buy

$$\text{Number of balls bought at end of year} = \frac{\text{Dollars available for purchase at end of year}}{\text{Cost per ball at end of year}}$$
$$= \frac{(\$1000)(F/P, 5.5\%, 1)}{(\$5)(1+0.02)^1} = \frac{\$1055}{\$5.10} = 207 \text{ golf balls}$$

The golfer can, after one year, buy 3.4% more golf balls than she could before. With rounding, this is 207 balls.

If Inflation Is 8%

As with the lower inflation rate, we would solve for *i′*:

$$i' = (i - f)/(1 + f)$$
$$= (0.055 - 0.08)/(1 + 0.08)$$
$$= -0.023 \quad \text{or} \quad \mathbf{-2.3\% \text{ per year}}$$

In this case the real growth in money has *decreased* by 2.3%, so that the golfer can now buy 2.3% fewer balls with the money she had invested. Even though she has more money year-end, it is worth less, so she can purchase less.

Regardless of how inflation behaves over the year, the bank will pay the golfer \$1055 at the end of the year. However, as we have seen, inflation can greatly affect the “real” growth of dollars over time. In a presidential speech, inflation has been called “that thief” because it steals real purchasing power from our dollars.

Let us continue the discussion of inflation by focusing on cash flows. We define dollars of two types:

Actual dollars (*A$*): This is the type of dollar that we ordinarily think of when we think of money. They circulate in our economy and are used for investments and payments. We can touch these dollars and often keep them in our purses and wallets—they are “actual” and exist physically. Sometimes they are called *inflated dollars* because they carry any inflation that has reduced their worth. These are also the dollars shown or **nominal dollars** on paychecks, credit card receipts, and normal financial transactions.
Real dollars (*R$*): This type of dollar is a bit harder to define. Real dollars are always expressed in terms of some constant purchasing power “base” year, for example, 2017-based dollars. Real dollars are sometimes called **constant value dollars or constant purchasing power dollars,** and because they have been adjusted for the effects of inflation, they are also known as *inflation-free dollars*.

Having defined *market, inflation,* and *real interest rates* as well as *actual* and *real dollars,* let us describe how these quantities relate. Figure 14–1 illustrates the relationship between these quantities.

Figure 14–1 illustrates the following principles:

When dealing with actual dollars (*A$*), use a market interest rate (i), and when discounting *A$* over time, also use i.

When dealing with real dollars (*R$*), use a real interest rate (i'), and when discounting *R$* over time, also use i'.

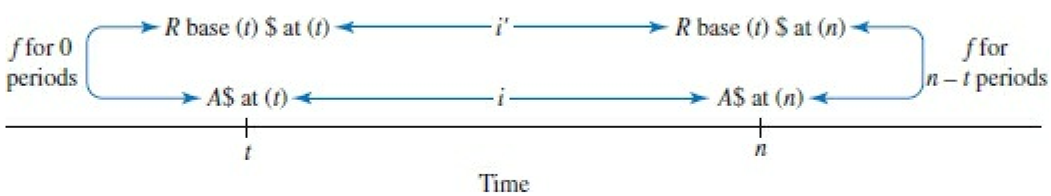


FIGURE 14-1 Relationship between i, f, i', *A$*, and *R$*.

Figure 14–1 shows the relationships between *A$* and *R$* that occur *at the same period of time.* Actual and real dollars are related by the *inflation rate*, in this case, over the period of years defined by $n - t$. To translate between dollars of one type to dollars of the other (*A$* to *R$* or *R$* to *A$*), use the inflation rate for the right number of periods. The following example illustrates many of these relationships.

EXAMPLE 14-2

When the university's stadium was completed in 1965, the total cost was $1.2 million. At that time a wealthy alumnus gifted the university with $1.2 million to be used for a future replacement. University administrators are now considering building the new facility in the year 2020. Assume that:

• Inflation is 6.0% per year from 1965 to 2020.
• In 1965 the university invested the gift at a market interest rate of 8.0% per year.
(a) Define i, i', f, and *A$*.
(b) How many actual dollars in the year 2020 will the gift be worth?
(c) How much would the actual dollars in 2020 be in terms of 1965 *purchasing power?*
(d) How much better or worse should the new stadium be?

SOLUTION TO PART a

Since 6.0% is the inflation rate (f) and 8.0% is the market interest rate (i), we can write

$$i' = (0.08 - 0.06)/(1 + 0.06) = 0.01887, \quad \text{or} \quad 1.887\%$$

The building's cost in 1965 was $1,200,000, which were the actual dollars (*A$*) spent in 1965.

SOLUTION TO PART b

From Figure 14–1 we are going from *actual dollars at t, in 1965,* to *actual dollars at n, in 2020*. To do so, we use the *market interest rate* and compound this amount forward 55 years, as illustrated in Figure 14–2.

$$\begin{aligned}\text{Actual dollars in 2020} &= \text{Actual dollars in 1965 }(F/P, i, 55\text{ years})\\ &= \$1{,}200{,}000(F/P, 8\%, 55)\\ &= \$82{,}701{,}600\end{aligned}$$

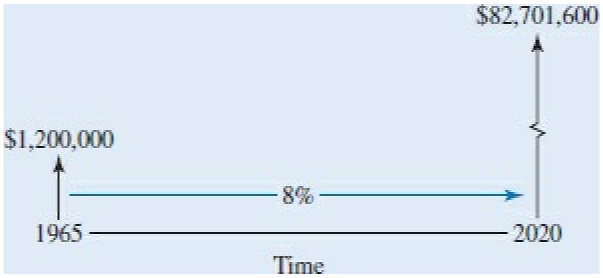


FIGURE 14-2 Compounding *A$* in 1965 to *A$* in 2020.

SOLUTION TO PART c

Now we want to determine the amount of *real 1965 dollars that occur in the year 2020,* which are equivalent to the $82.7 million from the solution to part **b**. Let us solve this problem two ways.

1. Translate *actual dollars in the year 2020 to real 1965 dollars in the year 2020.* From Figure 14–1 we can use the inflation rate to **strip 55 years of inflation** from the actual dollars. We do this by using the *P*/*F* factor for 55 years at the inflation rate. We are not physically moving the dollars in time; rather, we are simply removing inflation from these dollars one year at a time —the *P*/*F* factor does that for us. This is illustrated in the following equation and Figure 14–3.

$$\begin{aligned}\text{Real 1965 dollars in 2020} &= (\text{Actual dollars in 2020})(P/F, f, 55)\\ &= (\$82{,}701{,}600)(P/F, 6\%, 55)\\ &= \$3{,}357{,}000\end{aligned}$$

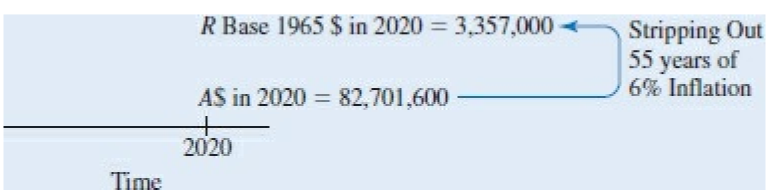


FIGURE 14-3 Translation of *A$* in 2020 to *R* 1965-based dollars in

2020.

2. Translate real 1965 dollars in 1965 to real 1965 dollars in 2020. The $1.2 million can also be said to be *real 1965 dollars that circulated in 1965*. So, let us translate those real dollars from 1965 to the year 2020 (Figure 14–4). Since they are *real dollars,* we use the *real interest rate.*

$$\begin{aligned}\text{Real 1965 dollars in 2020} &= (\text{Real 1965 dollars in 1965})(P/F, i', 55)\\ &= (\$1{,}200{,}000)(F/P, 1.887\%, 55)\\ &= \$3{,}355{,}000\end{aligned}$$

Note: The answers differ due to rounding the market interest rate to 1.887% versus carrying it out to more significant digits. The difference due to this rounding is less than 0.1%. If i' and the factors have enough digits, the answers to the two parts would be identical.

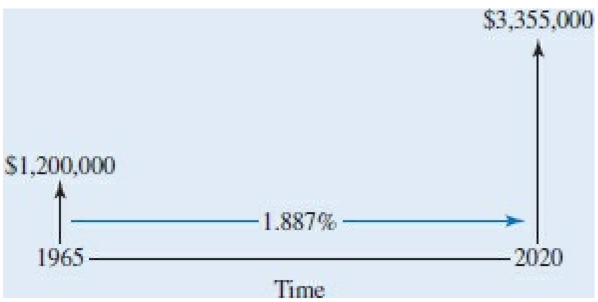


FIGURE 14-4 Translation of *R* 1965 dollars in 1965 to *R* 1965 dollars in 2020.

SOLUTION TO PART d

Assuming that construction costs increased at the rate of 6% per year, then the amount available for the project *in terms of 1965 dollars* is almost $3.4 million. This means that the new stadium will be about 3.4/1.2 or approximately 2.8 times “better” than the original one using *real dollars*.

EXAMPLE 14-3

In 1924 Mr. O’Leary buried $1000 worth of quarters in his backyard. Over the years he had always thought that the money would be a nice nest egg to give to his first grandchild. His first granddaughter, Gabrielle, arrived in 1994. From 1924 to 1994, inflation averaged 4.5%, the stock market increased an average of 15% per year, and investments in government bonds averaged 6.5% return per year. What was the relative purchasing power of the jar of quarters that Mr. O'Leary gave to his granddaughter Gabrielle at birth? What might have been a better choice for his "backyard investment"?

SOLUTION

Mr. O'Leary's $1000 in quarters was *actual dollars* both in 1924 **and** in 1994.

To obtain the *real 1924 dollar equivalent* of the $1000 that Gabrielle received in 1994, we would **strip 70 years of inflation out of those dollars.** As it turned out, Gabrielle's grandfather gave her $45.90 worth of 1924 purchasing power. Because inflation has "stolen" purchasing power from his stash of quarters during the 70-year period, Mr. O'Leary gave his grandaughter much less than the amount he first spaded underground. This loss of purchasing power caused by inflation can be calculated as follows:

$$\begin{aligned}\text{Real 1924 dollars in 1994} &= (\text{Actual dollars in 1994})(P/F,\ f, 1994 - 1924)\\ &= \$1000(P/F, 4.5\%, 70) = \$45.90\end{aligned}$$

On the other hand, if Mr. O'Leary had put his $1000 in the stock market in 1924, he would have made baby Gabrielle an instant multimillionaire by giving her $17,735,000. We calculate this as follows:

$$\begin{aligned}\text{Actual dollars in 1994} &= (\text{Actual dollars in 1924})(F/P, i, 1994 - 1924)\\ &= \$1000(F/P, 15\%, 70) = \$17{,}735{,}000\end{aligned}$$

At the time of Gabrielle's birth that $17.7 million translates to $814,069 in 1924 purchasing power. This is quite a bit different from the $45.90 in 1924 purchasing power calculated for the unearthed jar of quarters.

$$\text{Real 1924 dollars in 1994} = \$17{,}735{,}000(P/F, 4.5\%, 70) = \$814{,}069$$

Mr. O'Leary was never a risk taker, so it is doubtful he would have chosen the stock market for his future grandchild's nest egg. If he had chosen government bonds instead of his backyard, by 1994 the investment would have grown to \$59,076 (actual dollars)—the equivalent of \$2712 in 1924 purchasing power.

$$\begin{aligned}\text{Actual dollars in 1994} &= (\text{Actual dollars in 1924})(F/P, i, 1994 - 1924)\\ &= \$1000(F/P, 6\%, 70) = \$59{,}076\\ \text{Real 1924 dollars in 1994} &= \$59{,}076(P/F, 4.5\%, 70) = \$2712\end{aligned}$$

Obviously, either option would have been better than the choice Mr. O'Leary made. This example illustrates the effects of inflation and purchasing power, as well as the power of compound interest. However, in Mr. O'Leary's defense, if the country had experienced 70 years of *deflation* instead of *inflation,* he might have had the last laugh!

There are in general two ways to approach an economic analysis problem after the effects of inflation have been recognized. The first is to ignore these effects in conducting the analysis, as we've done so far in the text.

Ignoring inflation in the analysis: Use **real dollars** and a **real interest rate** that is adjusted for inflation.

The second approach is to systematically include the effects of inflation, as studied in this chapter.

Incorporating inflation in the analysis: Use a **market interest rate** and **actual dollars** that include inflation.

Since inflation is so common, why do many economic analyses of engineering projects and most of this text choose *not* to explicitly address inflation? This question is best answered by referring to the many examples and problems that contain statements like "Operations and maintenance costs are expected to be \$30,000 annually for the equipment's 20-year life."

Does such a statement mean that accounting records for the next 20 years will show constant costs? Obviously not. Instead, it means that in constant-dollar terms the O&M costs are not expected to increase. In real dollars, O&M costs are uniform. In actual or inflated dollars, we will pay more each year, but each of those dollars will be worth less.

Most costs and benefits in the real world and in this text have prices that increase at about the same rate of inflation as the economy as a whole. In most analyses these inflation increases are addressed by simply stating everything in real dollar terms and using a real interest rate.

There are specific items, such as computers and depreciation deductions, where inflation is clearly expected to differ from the general rate of inflation. It is for these cases, that this chapter is included in this text.

ANALYSIS IN CONSTANT DOLLARS VERSUS THEN-CURRENT DOLLARS

Performing an analysis requires that we distinguish cash flows as being either constant dollars (real dollars, expressed in terms of some purchasing power base) or then-current dollars (actual dollars that are then-current when they occur). As previously stated, constant (real) dollars require the use of a *real interest rate* for discounting; then-current dollars require a *market (or combined) interest rate*. We must not mix these two dollar types when performing an analysis. If both types are stated in the problem, one type must be converted to the other, so that a consistent comparison can be made.

One of the challenges in inflation analysis is that cash flows are stated in a variety of ways. The three most common are (1) then-current or actual dollars, (2) today’s or time-0 dollars, and (3) year-1 dollars. This is combined with the normal assumption that first costs occur at time 0 and other costs occur at the end of the year. Very careful reading of the examples, homework problems, and test questions is recommended. If a simple statement of constant-value dollars is all you are given, the normal assumption is that they are time-0 dollars and that the values have been chosen to match normal time 0 and end-of-period assumptions.

EXAMPLE 14-4

The Waygate Corporation is interested in contracting out a testing function.

Two firms have submitted bids. Waygate believes that both companies will be able to deliver equivalent services for the 5-year period. Determine which one Waygate should choose if the corporate MARR (investment market rate) is 25% and price inflation is assumed to be 3.5% per year over the next 5 years.

Company Alpha costs: Costs will be $150,000 in year-1 dollars the first year and will increase at a rate of 5% over the 5-year period.
Company Beta costs: Costs will be a constant $150,000 per year in terms of today's dollars over the 5-year period.

SOLUTION

The costs for each of the two alternatives are as follows:

	Alpha	Beta
Year	Then-Current Costs	Time-0 Dollar Costs
1	$\$150,000 \times 1.05^{0} = \$150,000$	$150,000
2	$150,000 \times 1.05^{1} = 157,500$	150,000
3	$150,000 \times 1.05^{2} = 165,375$	150,000
4	$150,000 \times 1.05^{3} = 173,644$	150,000
5	$150,000 \times 1.05^{4} = 182,326$	150,000

We inflate (or escalate) the stated yearly cost given by Company Alpha by 5% per year to obtain the then-current (actual) dollars each year.

Using a Constant Dollar Analysis

Here we must convert the then-current costs given by Company Alpha to constant today-based dollars. We do this by stripping the number of years of general inflation from each year's cost using $(P/F, f, n)$ or $(1+f)^{-n}$.

	Alpha	Beta
Year	Time-0 Dollar Costs	Time-0 Dollar Costs
1	$\$150,000 \times 1.035^{-1} = \$144,928$	$150,000

2	$157,500 \times 1.035^{-2} = 147,028$	150,000
3	$165,375 \times 1.035^{-3} = 149,159$	150,000
4	$173,644 \times 1.035^{-4} = 151,321$	150,000
5	$182,326 \times 1.035^{-5} = 153,514$	150,000

We use the *real interest rate* (i') calculated from Equation 14-1 to calculate the present worth of costs for each alternative:

$$i' = (i - f)/(1 + f) = (0.25 - 0.035)/(1 + 0.035) = 0.208$$

$$\begin{aligned}\text{PW of cost (Alpha)} &= 144{,}928(P/F, 20.8\%, 1) + 147{,}028(P/F, 20.8\%, 2)\\ &\quad + 149{,}159(P/F, 20.8\%, 3) + 151{,}321(P/F, 20.8\%, 4)\\ &\quad + 153{,}514(P/F, 20.8\%, 5) = \$436{,}000\\ \text{PW of cost (Beta)} &= \$150{,}000(P/A, 20.8\%, 5) = \$150{,}000(2.9387) = \$441{,}000\end{aligned}$$

Using a Then-Current Dollar Analysis

Here we must convert the constant dollar costs of Company Beta to then-current dollars. We do this by using (F/P, f, n) or $(1+f)^n$ to "add in" the appropriate number of years of general inflation to each year's cost.

	Alpha	Beta
Year	Then-Current Costs	Then-Current Costs
1	$\$150,000 \times 1.05^0 = \$150,000$	$\$150,000 \times 1.035^1 = \$155,250$
2	$150,000 \times 1.05^1 = 157,500$	$150,000 \times 1.035^2 = 160,684$
3	$150,000 \times 1.05^2 = 165,375$	$150,000 \times 1.035^3 = 166,308$
4	$150,000 \times 1.05^3 = 173,644$	$150,000 \times 1.035^4 = 172,128$
5	$150,000 \times 1.05^4 = 182,326$	$150,000 \times 1.035^5 = 178,153$

Calculate the present worth of costs for each alternative using the *market interest rate (i)*.

$$\begin{aligned}\text{PW of cost (Alpha)} &= 150{,}000(P/F, 25\%, 1) + 157{,}500(P/F, 25\%, 2)\\ &\quad + 165{,}375(P/F, 25\%, 3) + 173{,}644(P/F, 25\%, 4)\\ &\quad + 182{,}326(P/F, 25\%, 5) = \$436{,}000\end{aligned}$$

$$\text{PW of cost (Beta)} = 155{,}250(P/F, 25\%, 1) + 160{,}684(P/F, 25\%, 2) + 166{,}308(P/F, 25\%, 3) + 172{,}128(P/F, 25\%, 4) + 178{,}153(P/F, 25\%, 5) = \$441{,}000$$

Using either a constant dollar or then-current dollar analysis, Waygate should chose Company Alpha's offer, which has the lower present worth of costs. There may, of course, be intangible elements in the decision that are more important than a 1% difference in the costs.

Example 14–5 confirms that analyses done in constant-value dollars with real interest rates will lead to the same results as analyses done in nominal dollars with market interest rates—if both are done correctly. It also illustrates another very important point. When costs are estimated to be uniform over time, this is virtually always consistent with assuming constant-value dollars. The only common exception is bonds, where the interest payments are uniform and stated in nominal dollar terms.

EXAMPLE 14-5

A new heat exchanger will cost $220,000, and it will save $50,000 annually. After 10 years it will have no salvage value. The firm's real interest rate is 15%. If inflation is 5%, is this project worth doing? Analyze in actual and constant value dollars.

SOLUTION

While not stated explicitly, the $50,000 in annual savings should be assumed to be constant value dollars and a constant amount of energy. Only coincidentally would decreases in volume exactly match increases in prices. It is also assumed that the constant value dollar is a time-0 dollar and that the values are chosen to match the beginning and end-of-period assumptions.

Thus, one way to analyze the problem is to use the real interest rate of 15% with the constant value dollar annual savings of $50,000. Inflation has been correctly included by using constant value dollars and a real rate.

Alternatively, the inflation rate of 5% can be explicitly included. The $50,000 is treated as a time 0 value and inflation is explicitly applied. Now the interest rate must also include inflation. Using Equation 14-1 we find that the appropriate interest rate is 20.75%.

$$i = i' + f + i'f$$
$$i = 0.15 + 0.05 + 0.15 \times 0.05$$
$$i = 0.2075 = 20.75\%$$

	A	B	C	D
1	$220,000	First cost ($M)		
2	$50,000	Annual savings		
3	5%	Inflation rate		
4	Interest rate	15%	20.75%	
5	Year	Constant value $s	Actual $s	
6	0	-$220,000	-$220,000	
7	1	50,000	52,500	=$A$2*(1+$A$3)^A7
8	2	50,000	55,125	
9	3	50,000	57,881	
10	4	50,000	60,775	
11	5	50,000	63,814	
12	6	50,000	67,005	
13	7	50,000	70,355	
14	8	50,000	73,873	
15	9	50,000	77,566	
16	10	50,000	81,445	
17	Present worth	$30,938	$30,938	=C6+NPV(C4,C7:C16)

Both approaches calculate the same present worth of $30,938. The project is attractive.

Inflation and Uniform Flow Equivalence

Can we convert the PWs in Example 14–5 to meaningful EUAWs? For the constant dollar case, the answer is obviously yes. The $6164 calculated below is a constant value dollar EUAW. However, the $7568 that is calculated for the actual dollar case is a time and discount rate weighted average of a non-uniform series of actual dollars that is hard to correctly interpret. Thus analyses that state conclusions as an equivalent uniform annual worth or cost may only make sense in conjunction with a constant value dollar assumption.

	A	B	C	D	E	F	G	H
1	Alternative	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Constant $	15%	10		30,938	0	PMT	6164
3	Actual $	20.75%	10		30,938	0	PMT	7568 ??

PRICE CHANGE WITH INDEXES

We have already described the effects that inflation can have on money over time. Also, several definitions and relationships regarding dollars and interest rates have been given. It would not be correct to compare the benefits of an investment in 2010-based dollars with costs in 2020-based dollars. This is like comparing apples and oranges. Such comparisons of benefits and costs can be meaningful only if a standard purchasing power base of money is used. So we ask, "How do I know what inflation rate to use in my studies? and How can we measure price changes over time?"

What Is a Price Index?

Price indexes (introduced in Chapter 2) describe the relative price fluctuation of goods and services. They provide a *historical* record of prices over time. Price indexes are tracked for *specific commodities* as well as for *bundles (composites) of commodities.* As such, price indexes can be used to measure historical price changes for individual cost items (like labor and material costs) as well as general costs (like consumer products). We use *past* price fluctuations to predict *future* prices.

Table 14–1 lists the historic prices of sending a first-class letter in the U.S. from 1970 to 2019. The cost is given both in terms of dollars (cents) and a letter cost index (LCI) that we have created to show how indexes are constructed.

Notice two important aspects of the LCI. First, as with all cost or price indexes, there is a **base year,** which is assigned a value of 100. Our LCI has a base year of 1970—thus for 1970, LCI = 100. Values for subsequent years are stated in relation to the 1970 value. Second, the LCI changes only when the cost of first-class postage changes. In years when this quantity does not change, the LCI is not affected. These general observations apply to all price indexes.

Table 14-1 Historic Prices of First-Class Mail, 1970–2019, and Letter Cost Index

Year, *n*	Cost of First-ClassMail	LCI	Annual Increase for n	Year, *n*	Cost of First-Class Mail	LCI	Annual Increase for *n*
1970	$0.06	100		1995	0.32	533	10.34%
1971	0.08	133	33.33%	1996	0.32	533	0
1972	0.08	133	0	1997	0.32	533	0
1973	0.08	133	0	1998	0.33	550	3.13
1974	0.1	166	25	1999	0.33	550	0
1975	0.13	216	30	2000	0.33	550	0
1976	0.13	216	0	2001	0.34	567	3.03
1977	0.13	216	0	2002	0.37	617	8.82
1978	0.15	250	15.74	2003	0.37	617	0
1979	0.15	250	0	2004	0.37	617	0
1980	0.15	250	0	2005	0.37	617	0
1981	0.2	333	33.33	2006	0.39	650	5.41
1982	0.2	333	0	2007	0.41	683	5.13
1983	0.2	333	0	2008	0.42	700	2.44
1984	0.2	333	0	2009	0.42	700	0
1985	0.22	367	10	2010	0.44	733	4.76
1986	0.22	367	0	2011	0.45	750	2.27
1987	0.22	367	0	2012	0.45	750	0

1988	0.25	417	13.64	2013	0.46	767	2.27
1989	0.25	417	0	2014	0.49	817	6.52
1990	0.25	417	0	2015	0.49	817	0
1991	0.29	483	16	2016	0.47	783	–4.08
1992	0.29	483	0	2017	0.49	817	4.34
1993	0.29	483	0	2018	0.5	833	1.96
1994	0.29	483	0	2019	0.55	917	10.08

In general, engineering economists are the "users" of cost indexes such as our LCI. That is, cost indexes are calculated or tabulated by some other party, and our interest is in assessing what the index tells us about the historical prices and how these may affect our estimate of future costs. However, we should understand how the LCI in Table 14–1 was calculated.

In Table 14–1, the LCI is assigned a value of 100 because 1970 serves as our base year. In the following years the LCI is calculated on a year-to-year basis based on the annual percentage increase in first-class mail. Equation 14-2 illustrates the arithmetic used.

$$\mathrm{LCI}_n = \frac{\text{cost }(n)}{\text{cost 1970}} \times 100 \quad (14\text{-}2)$$

For example, consider the LCI for the year 2018. We calculate the LCI as follows.

$$\mathrm{LCI}_{2018} = \frac{0.50}{0.06} \times 100 = 833$$

As mentioned, engineering economists often use cost indexes to project future cash flows. As such, our first job is to use a cost index to **calculate** the *year-to-year* percentage increase (or *inflation*) of prices tracked by an index. We can use Equation 14-3:

$$\text{Annual percentage increase}_n = \frac{\text{Index}_n - \text{Index}_{n-1}}{\text{Index}_{n-1}} \times 100\% \quad (14\text{-}3)$$

To illustrate, let us look at the percent change from 2012 to 2013 for the LCI.

$$\text{Annual percentage increase}_{2013} = \frac{767 - 750}{750} \times 100\% = 2.27\%$$

For 2013 the price of mailing a first-class letter increased by 2.27% over the previous year.

An engineering economist often wants to know how a particular cost quantity changes over time. Often we are interested in calculating the *average* rate of price increases over a period of time. For instance, we might want to know the average yearly increase in postal prices from 2000 to 2010. If we generalize Equation 14-3 to calculate the percent change from 2000 to 2010, we obtain

$$\%\ \text{Increase}_{2000\ \text{to}\ 2010} = \frac{700 - 550}{550} \times 100\% = 273\%$$

How do we use this to obtain the *average* rate of increase over those 10 years? Should we divide 273% by 10 years (273/10 = 27.3%)? Of course not! Inflation, like interest, compounds. Such a simple division treats inflation like simple interest—without compounding. So the question remains: How do we calculate an *equivalent average rate of increase* in postage rates over a period of time? If we think of the index numbers as cash flows, we have

$$P = 550, \qquad F = 700, \qquad n = 10,\ \text{years}, \qquad i = ?$$

$$\text{Using } F = P(1+i)^n$$

$$700 = 500(1+i)^{10}$$

$$i = (700/550)^{1/10} - 1 = 0.0244 = 2.44\%$$

We can use a cost index to calculate the average rate of increase over any period of years, which should provide insight into how prices may behave in the future.

Composite Versus Commodity Indexes

Cost indexes come in two types: commodity-specific indexes and composite indexes. Commodity-specific indexes measure the historical change in price for specific items—such as construction labor or iron ore. Commodity indexes, like our letter cost index, are useful when an economic analysis includes individual cost items that are tracked by such indexes. For example, if we need to estimate the direct-labor cost portion of a construction project, we could use an index that tracks the inflation, or escalation, of labor costs. The U.S. Departments of Commerce and Labor track many cost quantities through the Department of Economic Analysis and Bureau of Labor Statistics. Example 14–6 uses data from the California Construction Cost Index (CCCI) to demonstrate using a commodity index. This data is compiled from the *Engineering News-Record*.

EXAMPLE 14-6

In January 2016 bids were opened for a new building in Los Angeles. The low bid and the final construction cost were $52.5 million. Another building of the same size, quality, and purpose is planned with a bid opening in January 2018. Estimate the new building's low bid and cost using the CCCI.

SOLUTION

In January 2016 the California Construction Cost Index (CCCI) had a value of 6106 and in January 2000 the value was 3746. If we wanted a cost estimate for January 2016, we could simply use the ratio of these values and Equation 14-2. But we want a value for January 2018, which is outside our data set. (This is true for all future estimates.)

The solution is to estimate the average annual rate of increase and then to apply this to the January 2016 cost.

$$F = 6106, P = 3746, n = 16, \text{ find } f$$
$$F = P(1+f)^n$$
$$f = (6106/3746)^{1/16} - 1 = 3.10\% \text{ per year}$$

Now we can apply the inflation rate for n = 2 years to the building's cost in 2016.

$$F = 52.5 \text{ million } \times 1.0310^2 = \$55.8 \text{ million in 2018}$$

Composite cost indexes do not track historical prices for individual classes of items. Instead, they measure the historical prices of *bundles* or *market* *baskets* of assets. Examples of composite indexes include the *Consumer Price Index* (CPI) and the *Producer Price Index* (PPI). The CPI measures prices for consumers in the U.S. marketplace, and each PPI measures prices for categories of producers in the U.S. economy.

The CPI, an index calculated by the Bureau of Labor Statistics, tracks the cost of a standard *bundle of consumer goods* from year to year. This "consumer bundle" or "market basket" includes housing, clothing, food, transportation, and entertainment. The CPI enjoys popular identification as the "inflation" indicator. Table 14–2 gives yearly index values and annual percent increases in the CPI. Figure 14–5 charts the CPI inflation rate for the same period.

Table 14-2 CPI Index Values and Yearly Percentage Increases, 1955–2015

Year	CPI Value*	CPI Increase	Year	CPI Value*	CPI Increase
1955	26.8	–	1994	148.2	2.6%
1960	29.6	–	1995	152.4	2.8
1965	31.5	–	1996	156.9	2.9
1970	38.8	–	1997	160.5	2.3
1975	53.8	9.1%	1998	163.0	1.6
1976	56.9	5.8	1999	166.6	2.2

1977	60.6	6.5	2000	172.2	3.4
1978	65.2	7.6	2001	177.1	2.8
1979	72.6	11.3	2002	179.9	1.6
1980	82.4	13.5	2003	184.0	2.3
1981	90.9	10.3	2004	188.9	2.7
1982	96.5	6.2	2005	195.3	3.4
1983	99.6	3.2	2006	201.6	3.2
1984	103.9	4.3	2007	207.3	2.8
1985	107.6	3.6	2008	215.3	3.8
1986	109.6	1.9	2009	214.5	−0.4
1987	113.6	3.6	2010	218.1	1.6
1988	118.3	4.1	2011	224.9	3.2
1989	124.0	4.8	2012	229.6	2.1

1990	130.7	5.4	2013	233.0	1.5
1991	136.2	4.2	2014	236.7	1.6
1992	140.3	3.0	2015	237.0	0.1
1993	144.5	3.0	2016	240.0	1.3
			2017	245.1	2.1

*Reference base: 1982–1984 = 100.

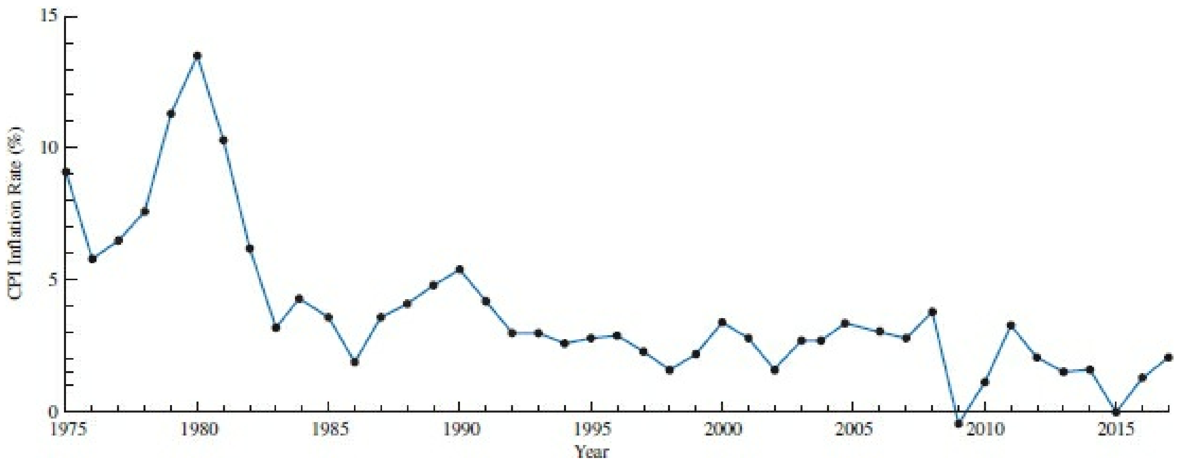


FIGURE 14-5 CPI inflation rate: 1975–2015.

Composite indexes are used the same way as commodity-specific indexes. That is, we can pick a single value from the table if we are interested in measuring the historic price for a single year, or we can calculate an *average inflation rate* or *average rate of price increase* as measured by the index over a time period extending several years.

How to Use Price Indexes in Engineering Economic

Analysis

One may question the usefulness of *historical* data (as provided by price indexes) when engineering economic analysis deals with economic effects projected to occur in the *future*. However, historical index data are often better predictors of future prices than official government predictions, which may be influenced by political considerations. The engineering economist can use *average historical percentage increases (or decreases)* from commodity-specific and composite indexes, along with data from market analyses and other sources, to estimate future costs and benefits.

When the estimated quantities are items that are tracked by commodity specific indexes, then those indexes should be used to calculate *average historical percentage increases (or decreases)*. If no commodity-specific indexes are kept, use an appropriate composite index to make this calculation.

For example, to estimate electric usage costs for a turret lathe over a 5-year period, you would first want to refer to a commodity-specific index for electric power in your area. If such an index does not exist, you might use a specific index for a very closely related commodity—perhaps an index of electric power costs nationally. In the absence of such substitute or related commodity indexes, you could use a composite index for national energy prices. The key point is that you should try to identify and use the price index that most closely relates to the quantity being estimated.

CASH FLOWS THAT INFLATE AT DIFFERENT RATES

Engineering economic analysis requires the estimation of various parameters. Over time, it is not uncommon for these parameters to *inflate* or *increase* (or even decrease) at different rates. For instance, one parameter might *increase* 5% per year and another 15% per year, and a third might *decrease* 3.5% per year. Since we are looking at the behavior of cash flows over time, we must have a way of handling this effect.

In Example 14–7 several commodity prices change at different rates. By using the respective individual inflation rates, the *actual dollar* amounts for each commodity are obtained in each year. Then we use a market interest rate to discount these actual dollar amounts.

EXAMPLE 14-7

On your first assignment as an engineer, your boss asks you to develop the utility cost estimate for a new manufacturing facility. After some research you define the problem as finding the present worth of utility costs given the following data:

• Your company uses a minimum attractive rate of return (MARR) = 35% as *i* (not adjusted for inflation.)
• The project has a useful life of 25 years.
• Utilities to be estimated include electricity, water, and natural gas.
• The 35-year historical data reveal:
Electricity costs increase at 8.5% per year
Water costs increase at 5.5% per year
Natural gas costs increase at 6.5% per year
• First-year estimates of the utility costs (in today's dollars) are as follows:
Electricity will cost $55,000
Water will cost $18,000
Natural gas will cost $38,000

SOLUTION

For this problem we will take each of the utilities used in our manufacturing facility and inflate them independently at their respective historical annual rates. Once we have these actual dollar amounts (*A$*), we can total them and then discount each year's total at 35% back to the present.

The formulas shown calculate each cost using the appropriate inflation rate—calculated from time 0. The formulas can also be constructed by applying the inflation rate from one year to the next. This approach is better if inflation rates are changing over time.

$$CF_{t+1} = CF_t(1 + f_{commodity})$$

Year	Electricity	Water	Natural Gas	Total
1	$55,000(1.085)^0 =$ \$55, 000	$18,000(1.055)^0 =$ \$18, 000	$38,000(1.065)^0 =$ \$38, 000	\$111,000
2	$55,000(1.085)^1 =$ 59, 675	$18,000(1.055)^1 =$ 18, 990	$38,000(1.065)^1 =$ 40, 470	119,135
3	$55,000(1.085)^2 =$ 64, 747	$18,000(1.055)^2 =$ 20, 034	$38,000(1.065)^2 =$ 43, 101	127,882
4	$55,000(1.085)^3 =$ 70, 251	$18,000(1.055)^3 =$ 21, 136	$38,000(1.065)^3 =$ 45, 902	137,289
5	$55,000(1.085)^4 =$ 76, 222	$18,000(1.055)^4 =$ 22, 299	$38,000(1.065)^4 =$ 48, 886	147,407
6	$55,000(1.085)^5 =$ 82, 701	$18,000(1.055)^5 =$ 23, 525	$38,000(1.065)^5 =$ 52, 063	158,290
7	$55,000(1.085)^6 =$ 89, 731	$18,000(1.055)^6 =$ 24, 819	$38,000(1.065)^6 =$ 55, 447	169,997
8	$55,000(1.085)^7 =$ 97, 358	$18,000(1.055)^7 =$ 26, 184	$38,000(1.065)^7 =$ 59, 051	182,594

.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
24	$55,000(1.085)^{23} =$ 359, 126	$18,000(1.055)^{23} =$ 61, 671	$38,000(1.065)^{23} =$ 161, 743	582,539
25	$55,000(1.085)^{24} =$ 389, 652	$18,000(1.055)^{24} =$ 65, 063	$38,000(1.065)^{24} =$ 172, 256	626,970

The present worth of the total yearly utility costs is

$$\begin{aligned}\text{PW} &= 111{,}000(P/F, 35\%, 1) + 119{,}135(P/F, 35\%, 2) + \cdots + 626{,}970(P/F, 35\%, 25)\\ &= \$5{,}540{,}000\end{aligned}$$

DIFFERENT INFLATION RATES PER PERIOD

In this section we address the situation of inflation rates that are changing over the study period. Rather than different inflation rates for different cash flows, in Example 14–8 the *inflation rate* for the same cash flow is changing over time. A method for handling this situation is much like that of the preceding section. We can simply apply the inflation rates in the years in which they are projected to occur. We would do this for each cash flow. Once we have all these actual dollar amounts, we can use the market interest rate to apply PW or other measures of merit.

Example 14–8 provides another example of how the effect of changes in inflation rates over time can affect an analysis.

EXAMPLE 14-8

If general price inflation is estimated to be 5% for the next 5 years, 7.5% for the 3 years after that, and 3% the following 5 years, at what market interest rate (i) would you have to invest your money to maintain a real purchasing power growth rate (i') of 10% during those years?

SOLUTION

In Years 1–5 you must invest at $0.10 + 0.050 + (0.10)(0.050) = 0.1150 =$ 11.50% per year.
In Years 6–8 you must invest at $0.10 + 0.075 + (0.10)(0.075) = 0.1825 =$ 18.25% per year.
In Years 9–13 you must invest at $0.10 + 0.030 + (0.10)(0.030) = 0.1330 =$ 13.30% per year.

Note: Most interest-bearing investments have fixed, up-front rates that the investor well understands going in. On the other hand, inflation is not quantified, and its effect on real return is not measured until the end of the year. Therefore the real investment return (i') may not turn out to be what was originally required.

INFLATION EFFECT ON AFTER-TAX CALCULATIONS

Earlier we noted the impact of inflation on before-tax calculations. If the future benefits keep up with the rate of inflation, the rate of return will not be adversely affected by the inflation. Unfortunately, we are not so lucky when we consider a situation with income taxes, as illustrated by Example 14–9. The value of the depreciation deduction is diminished by inflation.

EXAMPLE 14-9

A $12,000 investment with no salvage value will return annual benefits for 6 years. Assume straight-line depreciation and a 28% combined state and federal income tax rate. Solve for both before- and after-tax rates of return for two situations:

1. *No inflation:* the annual benefits are constant at $2918 per year.
2. *Inflation equal to 5%:* the benefits from the investment increase at this same rate, so that they continue to be the equivalent of $2918 in Year-0 dollars. Assume the $2918 is a time-0 value.

The benefit schedules are as follows:

Year	Annual Benefit for Both Situations (Year-0 dollars)	No Inflation, Actual Dollars Received	5% Inflation Factor*	5% Inflation, Actual Dollars Received
1	$2918	$2918	1.05^1	$3064
2	2918	2918	1.05^2	3217
3	2918	2918	1.05^3	3378
4	2918	2918	1.05^4	3547
5	2918	2918	1.05^5	3724
6	2918	2918	1.05^6	3910

*May be read from the 5% compound interest table as (*F*/*P*, 5%, *n*).

SOLUTIONS

Before-Tax Rate of Return

Since both situations (no inflation and 5% inflation) have an annual benefit, stated in Year-0 dollars of $2918, they have the same before-tax rate of return.

$$\text{PW of cost} = \text{PW of benefit}$$
$$12{,}000 = 2918(P/A, i, 6), \qquad (P/A, i, 6) = \frac{12{,}000}{2918} = 4.11$$

The before-tax rate of return equals 12%.

After-Tax Rate of Return, No Inflation

Year	Before-Tax Cash Flow	Straight-Line Depreciation	Taxable Income	28% Income Taxes	Actual Dollars, and Year-0 Dollars, After-Tax Cash Flow
0	−$12,000				−$12,000
1–6	2,918	$2000	$918	−$257	2661

$$\text{PW of cost} = \text{PW of benefit}$$
$$12{,}000 = 2661(P/A, i, 6), \qquad (P/A, i, 6) = \frac{12{,}000}{2661} = 4.51$$

The after-tax rate of return equals 8.8%.

After-Tax Rate of Return, 5% Inflation

Year	Before-Tax Cash Flow	Straight-Line Depreciation	Taxable Income	28% Income Taxes	Actual Dollars, After-Tax Cash Flow
0	−$12,000				−$12,000

1	3,064	$2000	$1064	$298	2,766
2	3,217	2000	1217	341	2,876
3	3,378	2000	1378	386	2,992
4	3,547	2000	1547	433	3,114
5	3,724	2000	1724	483	3,241
6	3,910	2000	1910	535	3,375

Converting to Year-0 Dollars and Solving for Rate of Return

Year	Actual Dollars, After-Tax Cash Flow	Conversion Factor	Year-0 Dollars, After-Tax Cash Flow	Present Worth at 7%	Present Worth at 8%
0	− $12, 000		−$12,000	−$12,000	− $12, 000
1	2, 766 ×	1.05^{-1} =	2,634	2,462	2,439
2	2, 876 ×	1.05^{-2} =	2,609	2,279	2,237
3	2, 992 ×	1.05^{-3} =	2,585	2,110	2,052
4	3, 114 ×	1.05^{-4} =	2,562	1,954	1,883

5	3, 241 ×	1.05^{-5} =	2,540	1,811	1,729
6	3, 375 ×	1.05^{-6} =	2,519	1,678	1,587
				+ 294	− 74

Linear interpolation between 7 and 8%:

After-tax rates of return = 7% + 1%[294/(294 + 74)] = 7.8%

From Example 14–9, we see that the before-tax rate of return for both situations (no inflation and 5% inflation) is the same. Equal before-tax rates of return are expected because the benefits in the inflation situation increased in proportion to the inflation. No special calculations are needed in before-tax calculations when future benefits are expected to respond to inflation or deflation rates.

The after-tax calculations illustrate that equal before-tax rates of return do not produce equal after-tax rates of return considering inflation.

	Rate of Return	
Situation	Before Taxes	After Taxes
No inflation	12%	8.8%
5% inflation	12	7.8

Inflation reduces the after-tax rate of return, even though the benefits increase at the same rate as the inflation. A review of the cash flow table reveals that while benefits increase, the depreciation schedule does not. Thus, the inflation results in increased taxable income and, hence, larger income tax payments.

The result is that while the after-tax cash flow in actual dollars increases, the augmented amount is not high enough to offset *both* inflation and increased income taxes. The Year-0-dollar after-tax cash flow is smaller with inflation than the Year-0-dollar after-tax cash flow without inflation.

USING SPREADSHEETS FOR INFLATION CALCULATIONS

Spreadsheets are the perfect tool for incorporating consideration of inflation into analyses of economic problems. For example, next year's labor costs are likely to be estimated as equal to this year's costs times $(1 + f)$, where f is the inflation rate. Thus each year's value is different, so we can't use factors for uniform flows, A. Also the formulas that link different years are easy to write. As a result, problems that are tedious to do by hand are easy with a spreadsheet.

Example 14–10 illustrates two different ways to write the equation for inflating costs. In Figure 14–6 the labor costs (column B) are calculated by applying the inflation rate to the previous year's labor costs. The transportation costs (column D) vary each year, so each year's costs are calculated by applying the inflation rate for the number of years since time-0. Example 14–11 illustrates that inflation reduces the after-tax rate of return because inflation makes the depreciation deduction less valuable.

EXAMPLE 14-10

Two costs for construction of a small, remote mine are for labor and transportation. Labor costs are expected to be \$350,000 the first year, with inflation of 6% annually. Unit transportation costs are expected to inflate at 5% annually, but the volume of material being moved changes each year. In Time-0 dollars, the transportation costs are estimated to be \$40,000, \$60,000, \$50,000, and \$30,000 in Years 1 through 4. The inflation rate for the value of the dollar is 3%. If the firm uses an i' of 7%, what is the equivalent annual cost in Time-0 dollars for this 4-year project?

SOLUTION

The data for labor costs can be stated so that no inflation needs to be applied in Year 1: the cost is $350,000. In contrast, the transportation costs for Year 1 are determined by multiplying $40,000 by 1.05 (= 1 + f).

Also in later years the labor cost$_t$ = labor cost$_{t-1}(1 + f)$, while each transportation cost must be computed as the Time-0 value times $(1+f)^t$. In Figure 14–6, the numbers in the Year-0 (or real) dollar column equal the values in the actual dollars column divided by $(1.03)^t$.

	A	B	C	D	E	F	G	H
1						7%	Inflation-Free Interest	
2	Inflation Rate	6%		5%		3%		
3			Transportation Costs		Total	Total		
4	Year	Labor Costs	Year 0 $s	Actual $s	Actual $s	Real $s		
5	1	120,000	40,000	42,000	162,000	157,282	= E5/(1+$F$2)^A5	
6	2	127,200	60,000	66,150	193,350	182,251		
7	3	134,832	50,000	57,881	192,713	176,360		
8	4	142,922	30,000	36,465	179,387	159,383		
9						$571,732	= NPV(F1,F5:F8)	
10					=B8+D8	$168,791	= −PMT(F1,4,F9)	
11	=B7*(1+$B$2)		=C8*(1+$D$2)^A8					

FIGURE 14-6 Spreadsheet for inflation.

The equivalent annual cost equals $168,791 in Time-0 dollars. As noted earlier the meaning of an EUAC in actual dollars in unclear.

EXAMPLE 14-11 Example 14–9 Revisited

For the data of Example 14–9, calculate the IRR with and without inflation with MACRS depreciation. How are the results affected by inflation by comparison with the earlier results.

SOLUTION

Most of the formulas for this spreadsheet are given in rows 11 and 12 of Figure 14–7 for the data in Year 6. The benefits received are computed from the base value in cell B5. The depreciation is the MACRS percentage times the $12,000 spent in Year 0. This value is not influenced by inflation, so the depreciation deduction is less valuable as inflation increases. The tax paid equals the tax rate times the taxable income, which equals dollars received minus the depreciation charge. Then ATCF (after-tax cash flow) equals the before-tax cash flow minus the tax paid.

In Figure 14–7, notice that in Year 2 the depreciation charge is large enough to cause this project to pay “negative” tax. For a firm, this means that the deduction on this project will be used to offset income from other projects.

	A	B	C	D	E	F	G	H
1		0%	= Inflation Rate		28%	= Tax Rate		
2		Actual $s	MACRS	Actual $s	Actual $s	Actual $s	Real $s	
3	Year	Received	Deprec. %	Deprec.	Tax	ATCF	ATCF	
4	0	−12000				−12000	−12000	
5	1	2918	20.00%	2400	145	2773	2773	= F5/(1+$B$1)^A5
6	2	2918	32.00%	3840	−258	3176	3176	
7	3	2918	19.20%	2304	172	2746	2746	
8	4	2918	11.52%	1382	430	2488	2488	
9	5	2918	11.52%	1382	430	2488	2488	
10	6	2918	5.76%	691	624	2294	2294	
11	Formulas		= −$B$4*C10			=B10−E10		
12	for Yr 6	=$B$5*(1+$B$1)^A10		=(B10−D10)*$E$1			9.25%	= IRR
13								
14		5%	= Inflation Rate		28%	= Tax Rate		
15		Actual $s	MACRS	Actual $s	Actual $s	Actual $s	Real $s	
16	Year	Received	Deprec. %	Deprec.	Tax	ATCF	ATCF	
17	0	−12000				−12000	−12000	
18	1	3064	20.00%	2400	186	2878	2741	
19	2	3217	32.00%	3840	−174	3392	3076	
20	3	3378	19.20%	2304	301	3077	2658	
21	4	3547	11.52%	1382	606	2941	2419	
22	5	3724	11.52%	1382	656	3068	2404	
23	6	3910	5.76%	691	901	3009	2245	
24							8.33%	= IRR

FIGURE 14-7 After-tax IRRs with MACRS and inflation.

The IRRs are higher in this example (9.25% without inflation vs. 8.8% with straight-line depreciation in Example 14–9, and 8.33% with inflation vs. 7.8%) because MACRS supports faster depreciation, so the depreciation deductions are more valuable. Also because the depreciation is faster, the results are affected somewhat less by inflation. Specifically, 5% inflation lowers the IRR by 0.92% with MACRS and by 1.0% with straight-line depreciation.

SUMMARY

Inflation is characterized by rising prices for goods and services, whereas deflation produces a fall in prices. An inflationary trend makes future dollars have less **purchasing power** than present dollars. Deflation has the opposite effect. If money is borrowed over a period of time in which deflation is occurring, then debt will be repaid with dollars that have **more** purchasing power than those originally borrowed. Inflation and deflation have opposite effects on the purchasing power of a monetary unit over time.

To distinguish and account for the effect of inflation in our engineering economic analysis, we define *inflation*, *real*, and *market* interest rates. These interest rates are related by the following expression:

$$i = i' + f + i'f$$

Each rate applies in a different circumstance, and it is important to apply the correct rate to the correct circumstance. Cash flows are expressed in terms of either *actual* or *real dollars*. The *market interest* rate should be used with *actual dollars* and the *real interest rate* should be used with *real dollars*. Virtually always when costs are estimated to be uniform over time, this is consistent with assuming constant-value dollars and a real interest rate must be used. This may be the easiest way to correctly account for inflation. In addition, when results are stated as an EUAW, this may only be meaningful in constant value dollar terms.

The different cash flows in our analysis may inflate or change at different interest rates when we look over the life cycle of the investment. Also, a single cash flow may inflate or deflate at different rates over time. These two circumstances are handled easily by applying the correct inflation rates to each cash flow over the study period to obtain the actual dollar amounts occurring in each year. After the actual dollar quantities have been calculated, the analysis proceeds using the market interest rate to calculate the measure of merit of interest.

Historical price change for single commodities and bundles of commodities are tracked with price indexes. The Consumer Price Index (CPI) is an example of a composite index formed by a bundle of consumer goods. The CPI serves as a surrogate for general inflation in our economy. Indexes can be used to calculate the *average annual increase* (or decrease) of the costs and benefits in our analysis. The historical data provide valuable information about how economic quantities may behave in the future over the long run.

The effect of inflation on the computed rate of return for an investment depends on how future benefits respond to the inflation. Usually the costs and benefits increase at the same rate as inflation, so the before-tax rate of return will not be adversely affected by the inflation. This outcome is not found when an after-tax analysis is made because the allowable depreciation schedule does not increase. The result will be increased taxable income and income tax payments, which reduce the available after-tax benefits and, therefore, the after-tax rate of return. The important conclusion is that estimates of future inflation or deflation may be important in evaluating capital expenditure proposals.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

14-1	A company requires a real MARR of 12%. What unadjusted MARR should be used if inflation is expected to be 2%?

SOLUTION

14-2 The real interest rate is 4%. The inflation rate is 3%. What is the apparent interest rate?

SOLUTION

14-3 A project that was analyzed under the assumption of 3% inflation was found to have a unadjusted internal rate of return (IRR) of 18%. What is the real IRR for the project?

SOLUTION

14-4 An electronic device cost $1250 in 2011. If inflation has averaged 2% each year, what is the price of the device in 2018?

SOLUTION

14-5 The apparent interest rate is 9.18% and the real interest rate is 6%. What is the inflation rate?

SOLUTION

14-6 A government agency has predicted 3% inflation for the next 5 years. How much will an item that presently sells for $1000 cost in 5 years?

SOLUTION

14-7 If prices have increased 50% over 10 years, what is the inflation rate?

SOLUTION

14-8 An automobile that cost $24,500 in 2018 had an equivalent model 4 years later that cost $29,250. If the increase is attributed to inflation, what was the average annual rate of inflation?

SOLUTION

14-9 Jorge purchases a lot for $40,000 cash and plans to sell it after 5 years. What should he sell it for if he wants a 15% before-tax rate of return, after taking the 3% annual inflation rate into account?

SOLUTION

14-10 The apparent interest rate is 9.18% and the real interest rate is 6%. What is the inflation rate?

SOLUTION

14-11 The cost of a wastewater treatment plant for a small town of 6000 people was estimated to be about $85/person in 2006. If a modest estimate of the rate of inflation is 2.5% for the period to 1984, what will be the per capita cost of the treatment plant in 2021?

SOLUTION

14-12 An investor is considering the purchase of a bond. The bond has a face value of $1000 and an interest rate of 6%; it pays interest once a year and matures in 8 years. This investor's real MARR is 25%. If the investor expects an inflation rate of 4% per year for the next 8 years, how much should he be willing to pay for the bond?

SOLUTION

14-13 A bond that pays no interest is called a zero-coupon bond. A $10,000 zero-coupon bond that matures in 10 years can be purchased today. If the expected annual rate of inflation is 3% and the buyer's unadjusted MARR is 8%, what is the maximum that should be paid for the bond?

SOLUTION

14-14 Sylvia B. bought an 8% tax-free municipal bond. The cost of the bond was $1000, and it will pay $80 each year for 20 years. The bond will mature at that time, returning the original $1000. If inflation is expected to average 3% during the period, what is the inflation-adjusted rate of return?

SOLUTION

14-15 An inventor expects to receive $75,000 per year from royalties on a patent for 12 years. If the inventor's interest rate is 9% and the inflation rate is 4%, determine the present value of the cash flows.

SOLUTION

14-16 A vacant lot is purchased for $20,000. After 5 years the lot is to be offered for sale. If the buyer requires a before-tax return on investments of 15% and inflation has averaged 4% per year over the 5-year period, what is the required selling price?

SOLUTION

14-17 A product has sales of $7M this year, but sales are expected to decline at 10% per year until it is discontinued after year 5. If the firm's interest rate is 15%, calculate the PW of the revenues.

SOLUTION

14-18 How much life insurance should a person buy if he wants to leave enough money to ensure that his family through generations will receive $25,000 per year in interest, of constant Year-0-value dollars? The interest rate expected from banks is 11%, while the inflation rate is expected to be 4% per year.

SOLUTION

14- An electronics store offers two options to buy a new laptop computer that has a price of $440. A customer can pay cash and immediately receive a discount of $49, or she can pay for the computer on an installment plan. The installment plan has a nominal rate of 12% compounded bi-yearly and requires an initial down payment of $44, 19 followed by four equal payments (principal and interest) every 6 months for 2 years. If for the typical customer the cost of money is 5%, what is the maximum effective annual inflation rate for the next 2 years that would make paying cash preferable to paying installments? All figures given are quoted in Time-0 dollars.

SOLUTION

14-20 A machine has a first cost of $100,000 (in today's dollars) and a salvage value of $20,000 (in then-current dollars) at the end of its 10-year life. Each year the machine is used will eliminate the job of one full-time worker. A worker costs $30,000 (today's dollars) in salary and benefits. Labor costs are expected to escalate at 10% per year. Operating and maintenance costs will be $10,000 per year (today's dollars) and will escalate at 7% per year.

Construct a table showing before-tax cash flows in current dollars and in today's dollars. The inflation rate is 7%.

SOLUTION

14-21 On January 1, 2005 the National Price Index was 208.5, and on January 1, 2015, it was 516.71. What was the inflation rate, compounded annually, over that 10-year period? Assuming that the same rate continued to hold for the next 10 years, what would the National Price Index be on January 1, 2025?

SOLUTION

14-22 A firm's internal price index for the cost of the raw materials for its most important product was 175 four years ago. Those prices have averaged an annual increase of 7% since. Calculate the current value of the index.

SOLUTION

Your savings account pays 4% interest on the $40,000 you deposited at

14-23 time 0. Inflation was 3% for 4 years and then 2% for 6 years. How much is in the account after 10 years in year-10 dollars? How much is that in year-0 dollars? What has been your real rate of return?

SOLUTION

14-24 A European investor lives near one of his country's borders. In Country *A* (where he lives), banks are offering an 8% interest rate and the inflation rate is 3%. Country *B*, on the other hand, has an inflation rate of 23%, and banks are offering 26% interest on deposits.

a. What real or effective interest rate does the investor earn when investing in his own country?
b. The investor believes that the currency of Country *B* will not change in its value relative to the value of the currency of Country *A* during this year. In which country would he get a larger effective interest rate?
c. Suppose that he invests in a bank in Country *B* and that his prediction turns out to be wrong: the currency of Country *B* was devaluated 20% with respect to the exchange value of Country *A*'s currency. What effective interest rate would he obtain in this case?

SOLUTION

14-25 Property, in the form of unimproved land, is purchased at a cost of $8000 and is held for 6 years, at which time it is sold for $32,600. Each during the six years of ownership $220 is paid in property tax and may be accounted for at an interest of 12%. The income tax rate on the long-term capital gain is 15% of the gain. Inflation during the period is 4% per year. What is the annual rate of return for this investment?

SOLUTION

14-26 A lot purchased for $4500 is held for 5 years and sold for $13,500. The average annual property tax is $45 and may be accounted for at an interest rate of 12%. The income tax rate on the long- term capital gain is 15% of the gain. What is the rate of return on the investment if the allowance for inflation is treated at an average annual rate of 6%?

SOLUTION

14-27 Undeveloped property near the planned site of an interstate highway is estimated to be worth $48,000 in 6 years when the construction of the highway will be completed. Consider a 15% capital gains tax on the gain, an annual property tax of 0.85% of the purchase price, an annual inflation rate of 7%, and an expected return of 15% on the investment. What is the indicated maximum purchase price now?

SOLUTION

14-28 A solar heating system costs $6500 initially and qualifies for a federal tax credit (40% of cost, not to exceed $4000). The cost of money is 10%, and inflation is expected to be 7% during the life of the system. The expected life of the system is 15 years with zero salvage value. The homeowner is in the 40% income tax bracket. The initial (first-year) fuel saving, estimated at $500, is expected to increase in response to inflation. The annual maintenance cost of the system is established at 5% of the annualized cost of the system. What is the time required for the payback condition to be reached for this investment?

SOLUTION

14-29 The net cost of a home solar heating system, expected to last for 20 years, is $8000. If the value of money is 10%, inflation is expected to be 8%, and the initial annual fuel saving is $750, what is the time for the payback condition to be reached for the system? Assume that the homeowner is in the 30% income tax bracket.

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Meaning and Effect

14-1	Define inflation in terms of the purchasing power of dollars.
14-2	Define and describe the relationships between the following: actual and real dollars, inflation, and real and market (combined) interest rates.
14-3 E	How does inflation happen? (*a*) Describe a few circumstances that cause prices in an economy to increase. (*b*) Is it common for governments with large debts to promote inflation, in order to pay off their debts more easily? Is this ethical?
14-4 A	Is it necessary to account for inflation in an engineering economy study? What are the two approaches for handling inflation in such analyses?
14-5	In Chapters 5 (Present Worth Analysis) and 6 (Annual Cash Flow Analysis) it is assumed that prices are stable and a machine purchased today for $5000 can be replaced for the same amount many years hence. In fact, prices have generally been rising, so the stable price assumption tends to be incorrect. Under what circumstances is it correct to use the “stable price” assumption when prices actually are changing?
14-6 A	An economist has predicted 2% inflation during the next 10 years. How much will an item that presently sells for $100 cost in 10 years?
14-7	Explain how high inflation in a booming real estate market could benefit an engineer who sells a home 5 years after she buys it.
14-8 A	A newspaper reports that in the last 8 years, prices have increased a total of 45%. This is equivalent to what annual inflation rate, compounded annually?
14-9	An investor wants a real rate of return i' of 6% per year. If the expected annual inflation rate for the next several years is 2.5%,

	what interest rate *i* should be used in project analysis calculations?
14-10 A	A man wishes to set aside some money for his daughter's college education. His goal is to have a bank savings account containing an amount equivalent to $20,000 in today's dollars at the girl's 18th birthday. The estimated inflation rate is 8%. If the bank pays 5% compounded annually, what lump sum should he deposit today on the child's 4th birthday?
14-11 E	A man bought a 5% tax-free municipal bond. It cost $1000 and will pay $50 interest each year for 20 years. At maturity the bond returns the original $1000. (*a*) If there is 2% annual inflation, what real rate of return will the investor receive? (*b*) What ethical questions are linked to governments issuing negative interest rate bonds?
14-12 A G	An automaker has a car that gets 12 kilometers per liter of gasoline. Gas prices will increase 2.5% per year, compounded annually, for the next 8 years. The manufacturer believes that the fuel consumption for its new cars should decline as fuel prices increase to keep the fuel costs constant. (*a*) To achieve this, what must be the fuel rating, in kilometers per liter, of the cars 8 years from now? (*b*) What limit to average kilometers per liter is likely?
14-13	Inflation has been a reality for the general economy of the U.S. in many years. Given this assumption, calculate the number of years it will take for the purchasing power of today's dollars to equal *one-third* of their present value. Assume that inflation will average 2.5% per year.
14-14 A	Sally Johnson loaned a friend $10,000 at 15% interest, compounded annually. The loan will be paid in four equal end-of-year payments. Sally expects the inflation rate to be 3%. After taking inflation into account, what rate of return is Sally receiving on the loan? Compute your answer to the nearest 0.1%.
14-15	If inflation averages 2.5% each year from 2015 to 2025, what is the purchasing power in 2015 dollars of $75,000 in 2025? *Contributed by Paul R. McCright, University of South Florida*

14-16 A If inflation is currently 2.55% and a bank is lending money at 7.75% interest, what is the real interest rate the bank is earning on its loans? *Contributed by Paul R. McCright, University of South Florida*

14-17 A South American country has had a high rate of inflation. Recently, its exchange rate was 15 cruzados per dollar; that is, one dollar will buy 15 cruzados. It is likely that the country will continue to experience a 25% inflation rate and that the U.S. will continue at a 3% inflation rate. Assume that the exchange rate will vary the same as the inflation. In this situation, one dollar will buy how many cruzados 5 years from now?

14-18 A An economist has predicted that for the next 5 years, the U.S. will have a 2.5% annual inflation rate, followed by 5 years at a 3.5% inflation rate. This is equivalent to what average price change per year for the entire 10-year period?

14-19 An economist has predicted that during the next 12 years, prices in the U.S. will increase 55%. He expects a further increase of 25% in the subsequent 8 years. Compute the annual inflation rate, f, for the entire 20-year period.

14-20 A You may pay $15,000 for an annuity that pays $2500 per year for the next 10 years. You want a real rate of return of 5%, and you estimate inflation will average 6% per year. Should you buy the annuity?

14-21 A homebuilder's advertising had the caption, "Inflation to Continue for Many Years." The ad explained that if one buys a home now for $297,000, and construction cost inflation continues at 7%, the home will be worth $819,400 in 15 years. Thus, by buying a new home now, one can realize a profit of $522,400 in 15 years. Do you find this logic persuasive? Explain.

14-22 A The average cost of a certain model car was $22,000 ten years ago. This year the average cost is $35,000.

(*a*) Calculate the average monthly inflation rate (f_m) for this model.

(*b*) Given the monthly rate f_m, what is the effective annual rate, f, of inflation for this model?

(*c*) Estimate what these will sell for 10 years from now, expressed in today's dollars.

Contributed by D. P. Loucks, Cornell University

14-23 You were recently looking at the historical prices paid for homes in a neighborhood that interests you. Calculate on a year-to-year basis how home prices in this neighborhood have inflated (*a*–*e* in the table).

Year	Average Home Price	Inflation Rate for That Year
5 years ago	$265,000	
4 years ago	267,000	(*a*)
3 years ago	272,000	(*b*)
2 years ago	280,000	(*c*)
Last year	283,000	(*d*)
This year	288,000	(*e*)

(*f*) What is your estimate of the inflation rate for next year?

14-24 A

ShaNey saw that the campus bookstore is having a special on pads of computation paper normally priced at $3.75 a pad, now on sale for $3.50 a pad. This sale is unusual and ShaNey assumes the paper will not be put on sale again. On the other hand, she expects that there will be no increase in the $3.75 regular price, even though the inflation rate is 0.75% every 3 months. ShaNey believes that competition in the paper industry will keep wholesale and retail prices constant. She uses a pad of computation paper every 3 months. ShaNey considers 9% a suitable minimum attractive rate of return. ShaNey will buy one pad of paper for her immediate needs. How many extra pads of computation paper should she buy?

14-25 Samantha receives a starting salary offer of $60,000 for Year 1. If inflation is 3% each year, what must her salary be to have the same purchasing power in Year 10? Year 20? Year 30? Year 40?

14-26 A

Inflation is 3.2%. If $1000 is invested in an account paying 8% compounded quarterly, what is the Year-0 dollar value of the account at the end of the 10 years?

Assume that Samantha (Problem 14-25) receives an annual 5% raise.

14-27 How much more, in Year-1 dollars, is her salary in Year 10? Year 20? Year 30? Year 40?

14-28 A Assume your salary is $55,000 in 2015 and $160,000 in 2045. If inflation has averaged 2% per year, what is the real or differential inflation rate of salary increases?

14-29 In the 1920 Sears Roebuck catalog, an oak chest of drawers cost $8 plus freight. In 1990 this same chest of drawers, in good condition, was $1200. If the average rate of inflation over that 70-year period was 3%, what was the average yearly rate of appreciation, adjusted for inflation?

14-30 A Assume general inflation is 2.5% per year. What is the price tag in 8 years for an item that has an inflation rate of 4.5% that costs $700 today?

14-31 The expected rise in prices due to inflation over the next 6 years is expected to be 30%. Determine the average annual inflation rate over the 6-year period.

14-32 A The price of a HeeHaw Model BR549 computer is presently $2200. If deflation of 2% per quarter is expected on this computer, what will its price be in nominal dollars at the end of 1 year? If inflation is 4.5% per year, what will the price be in Year-0 dollars?

14-33 You place $4000 into an account paying 8% compounded annually. Inflation is 2.5% during each of the next 3 years. What is the account's value at the end of the 3 years in Year-0 dollars?

14-34 A Felix Jones, a recent engineering graduate, expects a starting salary of $65,000 per year. His future employer has averaged 5% per year in salary increases for the last several years. If inflation is estimated to be 4% per year for the next 3 years, how much, in Year-1 dollars, will Felix be earning each year? What is the real growth rate in Felix's salary?

14-35 If $25,000 is deposited in a 5% savings account and inflation is 3%, what is the value of the account at the end of Year 20 in Year-0 dollars? If the time value of money is 4%, what is the present worth?

14-36 The cost of garbage pickup in Green Gulch is $4,500,000 for Year 1. The population is increasing at 6%, the nominal cost per ton is increasing at 5%, and the general inflation rate is estimated at 4%.

A
G

(*a*) Estimate the cost in Year 4 in Year-1 dollars and in nominal dollars.
(*b*) Reference a data source for trends in volume of garbage per person. How does including this change your answer?

14-37

The following series from the *Historical Statistics of the United States* can be combined with data in Table 14–2 to construct a long-term measure of inflation. What is the average inflation rate from to 2015?
(*a*) 1955
(*b*) 1940
(*c*) 1910
(*d*) 1800
(*e*) 1779

1910 = 100 (base)

Year	1779	1785	1800	1803	1830	1850	1864	1880
Index	226	92	129	118	91	84	193	100

1927 = 100 (base)

Year	1890	1910	1920	1921	1932	1940	1950	1955
Index	56	90	154	98	65	79	162	174

14-38
A

Your beginning salary is $68,000. You deposit 8% each year in a savings account that earns 3% interest. Your salary increases by 5% per year and inflation is 2.5% per year. What value does your savings account show after 30 years? What is the value in Year-1 dollars?

14-39 The market for widgets is increasing by 15% per year from current profits of $2,000,000. Investing in a design change will allow the profit per widget to stay steady; otherwise the price will drop by 3% per year. If inflation in the economy is 4%, what is the present worth in Year-1 dollars of the savings over the next 5 years? 10 years? The interest rate is 10%.

14-40 A G A homeowner is considering an upgrade of a fuel-oil-based furnace to a natural gas unit. The investment in the fixed equipment, such as a new boiler, will be $2500 installed. The cost of the natural gas will average $60 per month over the year, instead of the $145 per month that the fuel oil costs. If funds cost 9% per year and cost inflation in fossil fuels will be 3% per year, how long will it take to recover the initial investment? Solve on a monthly basis.

14-41 Enrollment at City University is increasing 3% per year, its cost per credit hour is increasing 8% per year, and state funds are decreasing by 4% per year. State funds currently pay a third of the costs for City U., while tuition pays the rest. What annual increase in tuition is required?

14-42 Joan earns a salary of $110,000 per year, and she expects to receive increases at a rate of 4% per year for the next 30 years. She is purchasing a home for $380,000 at 7% for 30 years (under a special veterans, preference loan with 0% down). She expects the home to appreciate at a rate of 3% per year. She will also save 10% of her gross salary in savings certificates that earn 5% per year. Assume that her payments and deposits are made annually. If inflation is assumed to have a constant 5% rate, what is the value (in Year-1 dollars) of each of Joan’s two investments at the end of the 30-year period? Use a before-tax analysis.

Before-Tax Problems

14-43 Using a market interest rate of 6% and an inflation rate of 2.5%, calculate the future equivalent in Year 15 of $10,000 today:
(*a*) Having today’s purchasing power.
(*b*) Having then-current purchasing power.

Auntie Frannie wants to help with tuition for her nieces to attend a private school. She intends to send a check for $14,000 at the end of each of the next 8 years.

14-44 [A](#)

(*a*) If general price inflation, as well as tuition price inflation, is expected to average 5% per year for those 8 years, calculate the present worth of the gifts. Assume that the real interest rate will be 3.5% per year.

(*b*) If Auntie Frannie wants her gifts to keep pace with inflation, what would be the present worth of her gifts? Again assume inflation is 5% and the real interest rate is 3.5%.

14-45

Ima Luckygirl recently found out that her grandfather had passed away and left her his Rocky Mountain Gold savings account. The only deposit was 75 years ago, when Ima's grandfather deposited $5000. If the account has earned an average rate of 10% and inflation has been 4%, answer the following:

(*a*) How much money is now in the account in *actual dollars*?

(*b*) Express the answer to part (*a*) in terms of the purchasing power of dollars from 75 years ago.

14-46 [A](#)

The ABC Block Company anticipates receiving $50,000 per year from its investments (with no change) over the next 10 years. If ABC's interest rate is 8% and the inflation rate is 3%, determine the present value of the cash flows.

14-47 G

Pollution control equipment must be purchased to remove the suspended organic material from liquid being discharged from a vegetable packing plant. Two alternative pieces of equipment are available that would accomplish the task. A Filterco unit costs $7000 and has a 5-year useful life. A Duro unit, on the other hand, costs $10,000 but will have a 10-year useful life.

With inflation, equipment costs are rising at 5% per year, compounded annually, so when the Filterco unit needed to be replaced, the cost would be much more than $7000. Based on a 10-year analysis period, and a 20% minimum attractive rate of return, which pollution control equipment should be purchased?

14-48 [A](#)

A 30-year mortgage for $180,000 has been issued. The interest rate is 5% and payments are made annually. If your time value of money is 10%, what is the PW of the payments in Year-1 dollars if inflation is 0%? 3%? 6%? 9%?

14-49 A group of students decided to lease and run a gasoline service station. The lease is for 10 years. Almost immediately the students were confronted with the need to alter the gasoline pumps to read in liters. The Dayton Company has a conversion kit available for $3000 that may be expected to last 10 years. The firm also sells a $1000 conversion kit that has a 5-year useful life. The students believe that any money not invested in the conversion kits may be invested elsewhere at a 10% interest rate. Income tax consequences are to be ignored in this problem.

(*a*) Assuming that future replacement kits cost the same as today, which alternative should be selected?

(*b*) If one assumes a 4% inflation rate, which alternative should be selected?

14-50 A If McDonnell Manufacturing has a MARR of 20%, inflation is 2.75%, and the company uses present worth analysis with a planning horizon of 15 years in making economic decisions, which of the following alternatives would be preferred?

	Alternative *A*	Alternative *B*
Initial costs	$236,000	$345,000
Annual operating costs	64,000	38,000
Annual maintenance costs	4,000	5000
Salvage value (EOY 20)	23000	51000

Contributed by Paul R. McCright, University of South Florida

14-51 Due to competition from a new polycarbon, revenues for the mainstay product of Toys-R-Polycarbon are declining at 7.5% per year. Revenues will be $5M for this year. The product will be discontinued at the end of Year 6. If the firm's interest rate is 20%, calculate the PW of the revenue stream.

14-52 The City of Columbia is trying to attract a new manufacturing business. It has offered to install and operate a water pumping plant to provide service to the proposed plant site. This would cost $50,000 now, plus $5000 per year in operating costs for the next 10 years, all measured in Year-0 dollars.

A To reimburse the city, the new business must pay a fixed uniform annual fee, *A*, at the end of each year for 10 years. In addition, it is to pay the city $50,000 at the end of 10 years. It has been agreed that the city should receive a 3% rate of return, after taking an inflation rate, *f*, of 7% into account.
Determine the amount of the uniform annual fee.

14-53 The Wildwood Widget Company needs a milling machine for its new assembly line. The machine presently costs $85,000, but has a cost inflation rate of 2%. Widget will not need to purchase the machine for 3 years. If general inflation is expected to be 4% per year during those 3 years, determine the price of the machine. What is the present worth of the machinery if the market rate of interest for Widget is 25%?

14-54 A As a recent graduate, you are considering employment offers from three different companies. However, in an effort to confuse you and perhaps make their offers seem better, each company has used a different *purchasing power base* for expressing your annual salary over the next 5 years. If you expect inflation to be 2.5% for the next 5 years and your personal (real) MARR is 5.0%, which plan would you choose?
Company A: A constant $60,000 per year in terms of today's purchasing power.
Company B: $50,000 the first year, with increases of $5,500 per year thereafter.
Company C: A constant $70,000 per year in terms of Year-5-based purchasing power.

14-55 Bob has lost his job and had to move back in with his mother. She agreed to let Bob have his old room back on the condition that he pay her $250 rent per month, and an additional $1500 every other year to pay for her biannual jaunt to Florida. Since he is down on his luck, she will allow him to pay his rent at the end of the year. If inflation is 3% and Bob's interest rate is 5%, how much is the present cost (in Year-1 dollars) for a 5-year contract with mom? (*Note*: Mom's trips are in Years 2 and 4).

A firm is having a large piece of equipment overhauled. It expects that the machine will be needed for the next 12 years. The firm has an 8% minimum attractive rate of return. The contractor has suggested three alternatives:

14-56 A

A. A complete overhaul for $6000 that should permit 12 years of operation.

B. A major overhaul for $4500 that can be expected to provide 8 years of service. At the end of 8 years, a minor overhaul would be needed.

C. A minor overhaul now. At the end of 4 and 8 years, additional minor overhauls would be needed.

If minor overhauls cost $2500, which alternative should the firm select? If minor overhauls, which now cost $2500, increase in cost at + 5% per year, but other costs remain unchanged, which alternative should the firm select?

14-57 Sam bought a house for $150,000 with some creative financing. The bank, which agreed to lend Sam $120,000 for 6 years at 15% interest, took a first mortgage on the house. The Joneses, who sold Sam the house, agreed to lend Sam the remaining $30,000 for 6 years at 12% interest. They received a second mortgage on the house. Thus Sam became the owner without putting up any cash. Sam pays $1500 a month on the first mortgage and $300 a month on the second mortgage. In both cases these are "interest only" loans, and the principal is due at the end of the loan.
Sam rented the house to Justin and Shannon, but after paying the taxes, insurance, and so on, he had only $800 left, so he was forced to put up $1000 a month to make the monthly mortgage payments. At the end of 3 years, Sam sold the house for $205,000. After paying off the two loans and the real estate broker, he had $40,365 left. After taking an 8% inflation rate into account, what was his before-tax rate of return?

Indexes

14-58 What is the Consumer Price Index (CPI)? What is the difference between commodity-specific and composite price indexes? Can each be used in engineering economic analysis?

14-59 A composite price index for the cost of vegetarian foods called *Eggs, Artichokes, and Tofu* (EAT) was 350 ten years ago and has averaged an annual increase of 5% since. Calculate the current value of the index.

14-60 A

From the data in Table 14–1 in the text, calculate the *overall rate change* of first-class postage as measured by the LCI for the following years:

(*a*) The 1970s (1970–1979)
(*b*) The 1980s (1980–1989)
(*c*) The 1990s (1990–1999)
(*d*) The 2000s (2000–2009)
(*e*) (2010–2018)

14-61 From the data in Table 14–1 in the text, calculate the *average annual inflation rate* of first class postage as measured by the LCI for the following years:

(*a*) End of 1970 to end of 1979
(*b*) End of 1980 to end of 1989
(*c*) End of 1990 to end of 1999
(*d*) End of 2000 to end of 2009
(*e*) (2010–2018)

14-62 A

From the data in Table 14–2 in the text, calculate the *average annual inflation rate* as measured by the CPI for the following years:

(*a*) End of 1970 to end of 1982
(*b*) End of 1980 to end of 1989
(*c*) End of 1985 to end of 2002
(*d*) End of 2000 to end of 2017

14-63 Redo Problem 14-13, but estimate the annual inflation rate using the period from 2000 to 2010 and the CPI values in Table 14–2.

14-64 A E

(*a*) Compute the equivalent annual inflation rate, based on the Consumer Price Index, for the period from 1990 to 2017.
(*b*) Using the equivalent annual inflation rate computed in part (*a*), estimate the consumer price index for 2022.
(*c*) Is it more practical or more ethical to use an approach like (*b*) rather than relying on government forecasts of inflation?

14-65 Below is 10 years' data on the Manager's Salary Index (MSI).

Year	MSI	Change in MSI
2008	93	3.9%
2009	100	*a*
2010	*b*	5.0
2011	115	*c*
2012	126	*d*
2013	*e*	1.2
2014	138	8.3
2015	145	5.1
2016	*f*	12.0
2017	170	4.5

(*a*) Calculate the unknown quantities *a, b, c, d, e, and f* in the table. Review Equation 14-3.
(*b*) What is the *base year* of the MSI? How did you determine it?
(*c*) Given the data for the MSI, calculate the *average annual price increase* in salaries paid to managers between 2009 and 2013 and between 2013 and 2017.
(*d*) Assume the average rate from 2008 to 2017 continues. What is the MSI for 2018, 2019, and 2020?

14-66 Homeowner Henry is building a fireplace for his house. The fireplace will require 2500 bricks.
(*a*) If the cost of a Chimney Brick in 2007 was $2.10, calculate the material cost of Henry's project in 2020. The Chimney Brick Index (CBI) was 442 in 2007 and is expected to be 651 in 2020.
(*b*) Estimate the material cost of a similar fireplace to be built in the year 2023. What assumption did you make?

Different Rates

14-67 General price inflation is estimated to be 3% for the next 5 years, 5% the 5 years after that, and 2.5% the following 5 years. If you invest $25,000 at a rate of 4% for those 15 years, what is the future worth of your investment in actual dollars at that time and in Year-0 dollars at that time?

14-68 A

Due to cost structures, trade policies, and corporate changes, three big automakers have different inflation rates for the purchase prices of their vehicles. Which car should Mary Clare buy at graduation 4 years from now, assuming everything but purchase price is equivalent?

Automaker	Current Price	Price Will Inflate x% per Year
X	$27,500	5.0%
Y	30,000	2.5
Z	25,000	6.0

14-69

Granny Viola has been saving money in the Bread & Butter mutual fund for 15 years. She has been a steady depositor over those years and has a pattern of putting $100 into the account every 3 months. If her original investment 15 years ago was $500 and interest in the account has varied as shown, what is the current value of her savings?

Years	Interest Earned in the Account
1–5	12% compounded quarterly
6–10	8 compounded quarterly
10–15	4 compounded quarterly

14-70 A

Andrew just bought a new boat for $15,000 to use on the river near his home. He has taken delivery of the boat and agreed to the terms of the following loan: all principal and interest is due in 3 years (balloon loan), first-year annual interest (on the purchase price) is set at 5%, this is to be adjusted up 1.5% per year for each of the following years of the loan.

(*a*) How much does Andrew owe to pay off the loan in 3 years?

(*b*) If inflation is 4%, what is the payment in Year-0 dollars?

Given the following data, calculate the present worth of the investment.

First cost = \$60,000 Project life = 10 years

Salvage value = \$15,000 MARR = 25%

14-71

General price inflation = 4% per year

Annual cost 1 = \$4500 in Year 1 and inflating at 2.5% per year

Annual cost 2 = \$7000 in Year 1 and inflating at 6% per year

Annual cost 3 = \$10,000 in Year 1 and inflating at 4.5% per year

Annual cost 4 = \$8500 in Year 1 and inflating at −2.5% per year

14-72 A

As the owner of Beanie Bob's Basement Brewery, you are interested in a construction project to increase production to offset competition from your crosstown rival, Bad Brad's Brewery and Poolhall. Construction cost percentage increases, as well as current cost estimates, are given for a 3-year period. Use a market interest rate of 25%, and assume that general price inflation is 5% over the 3-year period.

	Cost if Incurred	Cost Percentage Increase		
Item	Today	Year 1	Year 2	Year 3
Structural metal/concrete	\$120,000	4.3%	3.2%	6.6%
Roofing materials	14,000	2.0	2.5	3.0
Heating/plumbing equipment/fixtures	35,000	1.6	2.1	3.6

Insulation material	9,000	5.8	6.0	7.5
Labor	85,000	5.0	4.5	4.5

(*a*) What would the costs be for labor in Years 1, 2, and 3?
(*b*) What is the *average percentage increase* of labor cost over the 3-year period?
(*c*) What is the present worth of the insulation cost of this project?
(*d*) Calculate the future worth of the labor and insulation material cost portion of the project.
(*e*) Calculate the present worth of the total construction project for Beanie Bob.

14-73 Philippe Marie wants to race in the Tour de France 10 years from now. He wants to know what the cost of a custom-built racing bicycle will be 10 years from today.

Item	Current Cost	Cost Will Inflate x% per Year
Frame	$3500	4.5%
Wheels	1200	7
Gearing system	1800	5
Braking system	1250	3
Saddle	250	2.5
Finishes	400	8

After-Tax Problems

14-74 A Ruralville is suffering a 1% annual loss of population and property values. Even so, Ruralville must maintain its tax collections at a constant value of $3.2 million. If the inflation rate is 4.5%, what inflation rate in taxes for the remaining taxpayers is required for Ruralville to attain its goal? (*Note*: Although Ruralville uses a rate of 6% for discounting purposes, that rate is irrelevant to this problem.)

14-75 The U.S. tax laws provide for the depreciation of equipment based on original cost. Yet owing to substantial inflation, the replacement cost of equipment is often much greater than the original cost. What effect, if any, does this have on a firm's ability to buy new equipment to replace old equipment?

14-76 A Tom Ward put $10,000 in a 5-year certificate of deposit that pays 3.5% interest per year. At maturity he will receive his $10,000 back. Tom's marginal income tax rate is 24%. If the inflation rate is 2% per year, find his

(*a*) before-tax rate of return, ignoring inflation

(*b*) after-tax rate of return, ignoring inflation

(*c*) after-tax rate of return, after taking inflation into account

14-77 Sally Seashell bought a lot at the Salty Sea for $50,000 cash. She does not plan to build on the lot, but instead will hold it as an investment for 25 years. She wants a 10% after-tax rate of return after taking the 6% annual inflation rate into account. If income taxes amount to 15% of the capital gain, at what price must she sell the lot at the end of the 10 years?

14-78 A Assume that your private university's tuition is $28,000.

(*a*) If the inflation rate for tuition is 5% per year, calculate what the tuition will cost 20 years from now.

(*b*) If the general inflation rate for the economy is 3% per year, express that future tuition in today's dollars.

(*c*) Calculate the amount you would have to invest today to pay for tuition costs 20, 21, 22, and 23 years from now. Assume you can invest at 7% per year, your income tax rate is 28% per year, and the tuition has to be paid at the beginning of the year.

Contributed by D. P. Loucks, Cornell University

14-79 Dick DeWolf and his wife have a total taxable income of $60,000 this year and file a joint federal income tax return. If inflation continues for the next 20 years at a 5% annual rate, Dick wonders what their taxable income must be to provide the same purchasing power after taxes. Assuming the federal income tax rate table is unchanged, what must their taxable income be 20 years from now?

You must decide when to go on a vacation. One option is right after graduation. The other option is to wait and go after spending 2 years with the Peace Corps. Assume the vacation costs $2500 now, and your annual income tax rate is 12% now and is expected to continue to be 12% during the next 2 years. Also assume the annual inflation rate for a week's trip to Hawaii (hotel included) is 4%.

14-80 A

(*a*) Calculate the additional money you could spend on your vacation, after taxes, by putting your vacation money ($2500) into a taxable investment at 6% per year (before taxes) and waiting 2 years until after you come out of the Peace Corps compared to taking your well-deserved vacation now.

(*b*) If your tax rate drops to 0% while you're in the Peace Corps, how much additional money will you have for your vacation?

Contributed by D. P. Loucks, Cornell University

14-81

A small research device can be purchased for $10,000 and depreciated by 5-year MACRS depreciation. The net benefits from the device, before deducting depreciation, are $2500 at the end of the first year and increasing $1250 per year after that, until the device is hauled to the junkyard at the end of 10 years. During the 10-year period there will be an inflation rate *f* of 7%.

This profitable corporation has a 28% combined federal and state income tax rate. If it requires a real 15% after-tax rate of return on its investment, should the device be purchased?

14-82 A

When there is little or no inflation, a homeowner can expect to rent an unfurnished home for 12% of its market value. About 1/8 of the rental income is paid out for property taxes, insurance, and other operating expenses. Thus the net annual income to the owner is 10.5% of the market value. Since prices are relatively stable, the future selling price of the property often equals the original price paid by the owner.

For a $150,000 property (where the land is estimated at $46,500 of the $150,000), compute the after-tax rate of return, assuming the selling price 59 months later (in December) equals the original purchase price. Use modified accelerated cost recovery system depreciation beginning January 1. Also, assume a 35% income tax rate.

14-83 (This continues Problem 14-82.) As inflation has increased throughout the world, the rental income of homes has decreased and a net annual rental income of 8% of the market value is common. On the other hand, the market value of homes tends to rise about 2% per year more than the inflation rate. As a result, both annual net rental income and the resale value of the property rise faster than the inflation rate. Consider the following situation.

A $150,000 property (with the house valued at $103,500 and the land at $46,500) is purchased for cash in Year 0. The market value of the property increases at a 12% annual rate. The annual rental income is 8% of the beginning-of-year market value of the property. Thus the rental income also increases each year. The general inflation rate *f* is 10%.The individual who purchased the property has an average income tax rate of 35%.

(*a*) Use MACRS depreciation, beginning January 1, to compute the actual dollar after-tax rate of return for the owner, assuming he sells the property 59 months later (in December).

(*b*) Similarly, compute the after-tax rate of return for the owner, after taking the general inflation rate into account, assuming he sells the property 59 months later.

14-84 A

A couple in Ruston, Louisiana, must decide whether it is more economical to buy a home or to continue to rent during an inflationary period. The couple rents a one-bedroom duplex for $450 a month plus $139 a month in basic utilities (heating and cooling). These costs have a projected inflation rate of 5%, so the couple's monthly costs per year over a 10-year planning horizon are:

n =	1	2	3	4	5	6	7	8	9	10
Rent	450	473	496	521	547	574	603	633	665	698
Utilities	139	146	153	161	169	177	186	196	205	216

The couple would like to buy a home that costs $75,000. A local mortgage company will provide a loan that requires a down payment of 5% plus estimated closing costs of 1% cash. The couple prefers a 30-year fixed-rate mortgage with an 8% interest rate. The couple falls in the 28% marginal income tax rate (federal plus state), and as such, buying a home will provide them some tax write-off. It is estimated that the basic utilities for the home inflating at 5% will cost $160 per month; insurance and maintenance also inflating at 5% will cost $50 per month. The home will appreciate in value about 6% per year. Assuming a nominal interest rate of 15.5%, which alternative should the couple select? Use a present worth analysis. Realtor's sales commission is 5%.

14-85 Consider two mutually exclusive alternatives stated in Year-0 dollars. Both alternatives have a 3-year life with no salvage value. Assume the annual inflation rate is 5%, a combined income tax rate of 34%, and straight-line depreciation. The minimum attractive after-tax rate of return (MARR) is 8%. Use rate of return analysis to determine which alternative is preferable.

Year	*A*	*B*
0	−$420	−$300
1	200	150
2	200	150
3	200	150

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 4	**Northern Windows**
	Tax affected energy conservation project for a home with simplifying assumptions made clear. Includes inflation.
CASE 37	**Brown's Nursery (Part B)**

	Adds inflation to Case 36.
CASE 38	**West Muskegon Machining and Manufacturing**
	More complex inflation and tax problem with sunk cost and leverage.
CASE 39	**Uncertain Demand at WM^3**
	Includes inflation, taxes, and uncertainty.
CASE 41	**Freeflight Superdiscs**
	Inflation and sensitivity analysis for three alternatives. Includes taxes.

CHAPTER 15

SELECTION OF A MINIMUM ATTRACTIVE RATE OF RETURN

What's the Rate of Return on a Dam?

The broadly defined "energy" industry has often done well in recent years even when the U.S. and world economies have not. It's now quite evident that humans are becoming increasingly dependent on various forms of energy to power our world. Some forms, especially oil and coal, pose environmental threats. Thus government and businesses alike are scrambling to come up with "green" solutions to the world's energy needs. *But is the economic investment in renewable forms of energy (solar, wind, water) worth the risks? Do private investors need to reconsider their rates of return when evaluating risky investments?*

Consider Tajikistan, a former Soviet republic with a population of about 8.7

million people. While the country is lacking in most natural resources and has few major industries (cotton and aluminum), it is the source of over 40% of Central Asia's water. Tajikistan's government is relying on hydroelectric power for the country's economic and political future. The Rogun Dam is an unfinished dam across the Vakhsh River in southern Tajikistan with a planned capacity of 3600 megawatts of power. Construction began in 1976 but was halted after the collapse of the Soviet Union. In 2010, hoping to revitalize this project and complete the dam, Tajikistan made a public offering of bonds worth $1.4 billion. Because this is an extremely expensive undertaking, the country must rely heavily on foreign investment, and the investors are likely to expect significant rates of returns. The project was restarted in 2017 when an Italian firm won the contract to complete it.

Upstream and downstream countries have different perspectives on how dam projects will affect their own economies and water supplies. The more hydroelectric projects Tajikistan develops, the more control they will have over water in the region; thus political tensions are a significant issue. The Nurek Dam, downstream on the Vakshsh River, is one of the world's highest. Its associated 9-unit power plant provides about 3000 megawatts of power, and the water impounded irrigates about 300 square miles of farmland.

Uzbekistan, Tajikistan's downstream neighbor, strongly opposed the Rogun Dam. In addition, Tajikistan is in an earthquake zone, and any dam projects must be built to withstand significant seismic shocks, not to mention the effects that these projects will have on the plant and animal life in the region. Despite neighboring countries' objections and recent earthquakes in the Rogun region, construction continues and some electricity could flow in 2018. This kind of investment is very risky for any business or government.

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QUESTIONS TO CONSIDER

1. What are the advantages of relying on hydroelectric power in a country such as Tajikistan?

2. What are the disadvantages of relying on hydroelectric power in a country such as Tajikistan? What are the possible consequences of new dam construction?
3. How would you balance the benefits against the costs in order to determine whether a new hydroelectric generating facility should be built in a given location?
4. What are some of the ethical issues that arise when one is considering investment in foreign countries?
5. What risks are associated with the Rogun Dam for Tajikistan? For its neighboring countries downstream? For foreign investors?
6. How would a private company determine the MARR to be used in evaluating such a project?

After Completing This Chapter

The student should be able to:

• Define various sources of capital and the costs of those funds to the firm.
• Discuss the impact of inflation and the cost of borrowed money.
• Select a firm's MARR based on the opportunity cost approach for analyzing investments.
• Adjust the firm's MARR to account for risk and uncertainty.
• Use spreadsheets to develop cumulative investments and the opportunity cost of capital.

Key Words

capital budgeting

equity

risk

composite value

opportunity cost

treasury stock

cost of capital

ownership

uncertainty

debt

prime rate

weighted average cost of capital

Selecting the interest rate or minimum attractive rate of return that is suitable for use in a particular situation is complex, and no single answer is always appropriate. A discussion of what interest rate to use must inevitably begin by examining the sources of capital, followed by looking at the prospective investment opportunities and risk. Only in this way can an interest rate or minimum attractive rate of return (MARR) be chosen intelligently.

MARR FOR INDIVIDUALS

This chapter focuses on business firms, but the concepts that apply to firms also make sense for individuals. For example, the *minimum attractive rate of return* is the highest rate of several possibilities in both cases. For individuals the *cost of capital* becomes the highest interest rate loan or credit card balance that the individual is paying interest on. If a credit card payment has been late, these rates may reach nearly 30% in some cases! On the other hand, if credit cards are paid off every month, the interest rate on them is typically 0%. Interest rates on vehicle, student, or home loans may be the highest interest rates for an individual.

The *opportunity cost* for investing for most individuals is the expected rate of return on the individual's investment portfolio. Historical returns above inflation were calculated in Appendix 9A for stocks, long-term treasury bonds, and treasury bills (T-bills). Then Appendix 10A showed that portfolios can reduce risk by including a mix of bonds, stocks, and T-bills. Other common investments include corporate bonds, which offer a higher return than treasury bonds, but at a higher risk of default—bankrupt firms may make no or only partial payment on their bonds. Some individuals earn higher or possibly lower returns by investing in a small business, real estate, or high-growth firms.

SOURCES OF CAPITAL

In broad terms there are three sources of capital available to a firm: money generated from the firm's operation, borrowed money, and money from selling stock.

Money Generated from the Firm's Operations

A major source of capital investment money is retained profits from the firm's operation. Overall, industrial firms retain about half of their profits and pay out the rest to stockholders. In addition to profit, the firm generates money equal to the annual depreciation charges on existing capital assets. In other words, a profitable firm will generate money equal to its depreciation charges *plus* its retained profits. Even a firm that earns zero profit will still generate money from operations equal to its depreciation charges. (A firm with a loss, of course, will have still less funds.)

External Sources of Money

When a firm requires money for a few weeks or months, it typically borrows from banks. Longer-term unsecured loans (of, say, 1–4 years) may also be arranged through banks. While banks undoubtedly finance a lot of capital expenditures, regular bank loans cannot be considered a source of permanent financing.

Longer-term financing is done by selling bonds (see Chapter 5) to banks, insurance firms, pension funds, and the public. A wide variety of bonds exist, but most are interest-only loans, where interest is paid at a *coupon* rate every 6 months or once a year and the principal is due at the bond's maturity. Common maturities are 10 to 30 years, although some extend to 100 years and a few even longer. Chapters 5 and 7 include examples of how to calculate the interest rates.

A firm can also raise funds by issuing new stock (shares of ownership in the firm). Many firms have also repurchased their own stock in the past, which is called **treasury stock**. Another way firms can raise funds is to sell this treasury stock.

One of the finance questions each firm must address is maintaining an appropriate balance between **debt** (loans and bonds) and **equity** (stock and retained earnings). The debt has a maturity date, and there are legal obligations to repay it unless the firm declares bankruptcy. On the other hand, stockholders expect a higher rate of return to compensate them for the risks of ownership. Those who are interested in models used to calculate the cost of equity capital are referred to *The Economic Analysis of Industrial Projects*, 3rd edition, by Eschenbach, Lewis, Hartman, and Bussey, published by Oxford University Press.

Choice of Sources of Funds

Choosing the source of funds for capital expenditures is a decision for the firm's top executives, and it may require approval of the board of directors. When internal operations generate adequate funds for the desired capital expenditures, external sources of money are not likely to be used. But when the internal sources are inadequate, external sources must be employed or the capital expenditures will have to be deferred or canceled.

COST OF FUNDS

Cost of Borrowed Money

A first step in deciding on a minimum attractive rate of return might be to determine the interest rate at which money can be borrowed. Longer-term loans or bonds may be obtained from banks, insurance companies, or the variety of places in which substantial amounts of money accumulates (for example, the sovereign wealth funds of the oil-producing nations).

A large, profitable corporation might be able to borrow money at the **prime rate**, that is, the interest rate that banks charge their best and most sought-after customers. All other firms are charged an interest rate that is higher by one-half to several percentage points. In addition to the firm's financial strength and ability to repay the debt, the interest rate will depend on the debt's duration and on whether the debt has collateral or is unsecured.

Cost of Capital

Another relevant interest rate is the **cost of capital.** This is also called the **weighted average cost of capital (WACC)**. This is the rate from *all* sources of funds in the firm's overall capitalization. The mechanics for computing the cost of capital or WACC are given in Example 15–1.

EXAMPLE 15-1

For a particular firm, the purchasers of common stock require an 11% rate of return, bonds are sold at a 7% interest rate, and bank loans are available at 9%. Compute the cost of capital or WACC for the following capital structure:

		Rate of Return
$ 20 million	Bank loan	9%
20	Bonds	7
60	Common stock and retained earnings	11
$100 million		

SOLUTION

The weighted cost of capital "weights" the return on each source of capital by the fraction of the total capital it represents. In this case, 20% of the total capital is from the bank loan, 20% of the capital is from bonds, and 60% of the capital is from equity sources—that is, common stock and retained earnings.

$$\begin{aligned} \text{WACC}_{\text{before-taxes}} &= (0.2)(9\%) + (0.2)(7\%) + (0.6)(11\%) \\ &= 1.8\% + 1.4\% + 6.6\% = 9.8\% \end{aligned}$$

Note that since this is an *average*, the result must be between the lowest rate of return (7%) and the largest (11%). Since it is a *weighted* average, the return with the largest weight (60%) has the most impact on the final average. We recommend some "mental" arithmetic to approximate the expected answer, which will help catch any errors with your calculator.

The cost of capital is also computed after considering that interest payments on debt, like bank loans and bonds, are tax-deductible business expenses. Thus,

$$\text{After-tax interest cost} = (\text{Before-tax interest cost}) \times (1 - \text{Tax rate})$$

If we assume that the firm pays 26% income taxes, the computations become

Bank loan	After-tax interest cost = 9%(1 − 0.26) = 6.7%
Bonds	After-tax interest cost = 7%(1 − 0.26) = 5.2%

Dividends paid on the ownership in the firm (common stock + retained earnings) are not tax deductible. The cost of debt capital can also be computed by dividing the total amount of interest by the total amount of debt capital. Combining the three components, the after-tax interest cost for the $100 million of capital is

$$\$20 \text{ million } (6.7\%) + \$20 \text{ million } (5.2\%) + \$60 \text{ million } (11\%) = \$8.98 \text{ million}$$

$$\text{WACC}_{\text{after-taxes}} = \frac{\$8.98 \text{ million}}{\$100 \text{ million}} = 8.98\%$$

In practical situations, the cost of capital is often difficult to compute. The fluctuation in the price of common stock, for example, makes it difficult to pick a cost, and because of the fluctuating prospects of the firm, it is even more difficult to estimate the future benefits that purchasers of the stock might expect to receive. Given the fluctuating costs and prospects of future benefits, what rate of return do stockholders require? There is no precise answer, but we can obtain an approximate answer. As described in the next section, inflation complicates the task of finding the *real* interest rate.

FIGURE 15-1 The real interest rate. The interest rate on 20-year Treasury bonds *minus* the inflation rate, *f*, as measured by changes in the Consumer Price Index.

Inflation and the Cost of Borrowed Money

As inflation varies, what is its effect on the cost of borrowed money? A widely held view has been that interest rates on long-term borrowing, like 20-year Treasury bonds, will be about 3% more than the inflation rate. For borrowers this is the real—that is, after-inflation—cost of money, and for lenders the real return on loans. If inflation rates were to increase, it would follow that borrowing rates would also increase. All this suggests a rational and orderly situation, about as we might expect.

Unfortunately, things have not worked out this way. Figure 15–1 shows that the real interest rate has not always been about 3% and, in fact, there have been long periods during which the real interest rate was negative. Can this be possible? Would anyone invest money at an interest rate several percentage points below the inflation rate? Well, consider this: when the U.S. inflation rate was 12%, savings banks were paying $5\frac{1}{2}\%$ on regular passbook deposits—and there was a lot of money in those accounts. While there must be a relationship between interest rates and inflation, Figure 15–1 suggests that it is complex.

The relationship between the inflation rate and the rate of return on investments is quite important, because inflation reduces the real rate of return (as shown in Appendix 9A and Chapter 14). In addition, many interest rates are reported without adjusting for inflation. For example, interest rates on loans (auto, home, and student) and investment returns (savings accounts, etc.) are all stated without adjusting for inflation.

INVESTMENT OPPORTUNITIES

An industrial firm has larger amounts of money, which allows investments that are unavailable to individuals, with their more limited investment funds. More important, however, is the fact that a firm conducts a business, which itself offers many investment opportunities. While exceptions can be found, a good generalization is that the opportunities for investment of money within the firm are superior to the investment opportunities outside the firm. Consider the available investment opportunities for a particular firm as outlined in Table 15–1. The cumulative investment required for all projects at or above a given rate of return is given in Figure 15–2.

Figure 15–2 illustrates that a firm may have a broad range of investment opportunities available at varying rates of return and with varying lives and uncertainties. It may take some study and searching to identify the better investment projects available to a firm. Typically, the good projects will almost certainly require more money than the firm budgets for capital investment projects.

Opportunity Cost

There are two aspects of investing that are basically independent. One factor is the source and quantity of money available for capital investment projects.

The other aspect is the firm’s investment opportunities.

Investment opportunities typically exceed the available money supply. Thus some investment opportunities can be selected and others must be rejected. Obviously, we want to ensure that *all the selected projects are better than the best rejected project.* To do this, we must know something about the rate of return on the best rejected project. The best rejected project is the best opportunity forgone, and this in turn is called the **opportunity cost.**

Opportunity cost = Cost of the best opportunity forgone

= Rate of return on the best rejected project

Table 15-1 A Firm’s Available Investment Opportunities

Project Number	Project	Cost **($\times 10^3$)**	Estimated Rate of Return
Investment Related to Current Operations			
1	New equipment to reduce labor costs	$150	30%
2	Other new equipment to reduce labor costs	50	45
3	Overhaul particular machine to reduce material costs	50	38
4	New test equipment to reduce defective products produced	100	40

New Operations			
5	Manufacture parts that previously had	200	35
	been purchased		
6	Further processing of products	100	28
	previously sold in semifinished form		
7	Further processing of other products	200	18
New Production Facilities			
8	Relocate production to new plant	250	25
External Investments			
9	Investment in a different industry	300	20
10	Other investment in a	300	10

	different industry		
11	Overseas investment	400	15
12	Purchase of Treasury bills	Unlimited	0.8

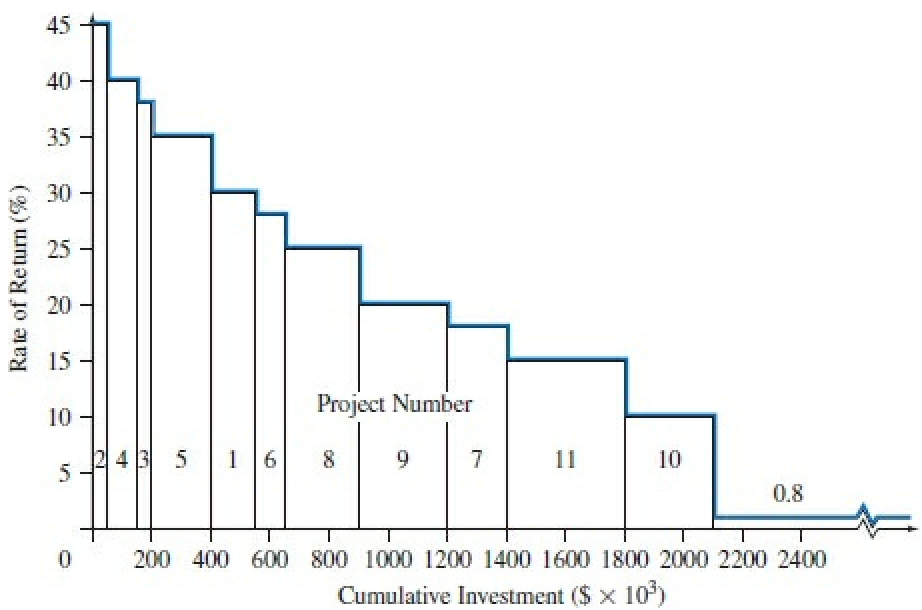


FIGURE 15-2 Cumulative investment required for all projects at or above a given rate of return.

If one could predict the opportunity cost for some future period (like the next 12 months), this rate of return could be one way to judge whether to accept or reject any proposed capital expenditure. Examples 15–2 and 15–3 illustrate this.

EXAMPLE 15-2

Consider the situation represented by Table 15–1 and Figure 15–2. For a capital expenditure budget of $1.2 million ($\$1.2 \times 10^6$), what is the opportunity cost?

SOLUTION

From Figure 15–2 we see that the eight projects with a rate of return of 20% or more require a cumulative investment of \$1.2 ($\times 10^6$). We would take on these projects and reject the other four (7, 11, 10, and 12) with rates of return of 18% or less. The best rejected project is 7, and it has an 18% rate of return. Thus the opportunity cost is 18%.

EXAMPLE 15-3

Nine independent projects are being considered. Figure 15–3 may be prepared from the following data.

Project	Cost (thousands)	Uniform Annual Benefit (thousands)	Useful Life (years)	Salvage Value (thousands)	Computed Rate of Return
1	\$100	\$23.85	10	\$0	20%
2	200	39.85	10	0	15
3	50	34.72	2	0	25
4	100	20.00	6	100	20
5	100	20.00	10	100	20
6	100	18.00	10	100	18

7	300	94.64	4	0	10
8	300	47.40	10	100	12
9	50	7.00	10	50	14

If a capital budget of $650,000 is available, what is the opportunity cost of capital? With this model, which projects should be selected?

SOLUTION

Looking at the nine projects, we see that some are expected to produce a larger rate of return than others. It is natural that if we are to select from among them, we will pick those with a higher rate of return. When the projects are arrayed by rate of return, as in Figure 15–3, Project 2 is the last one funded. With Project 2 the cumulative first cost of $650,000 matches the budget. Thus, the opportunity cost of capital is 14% from Project 9, the highest ranked unfunded project. This model implies funding Projects 3, 1, 4, 5, 6, and 2.

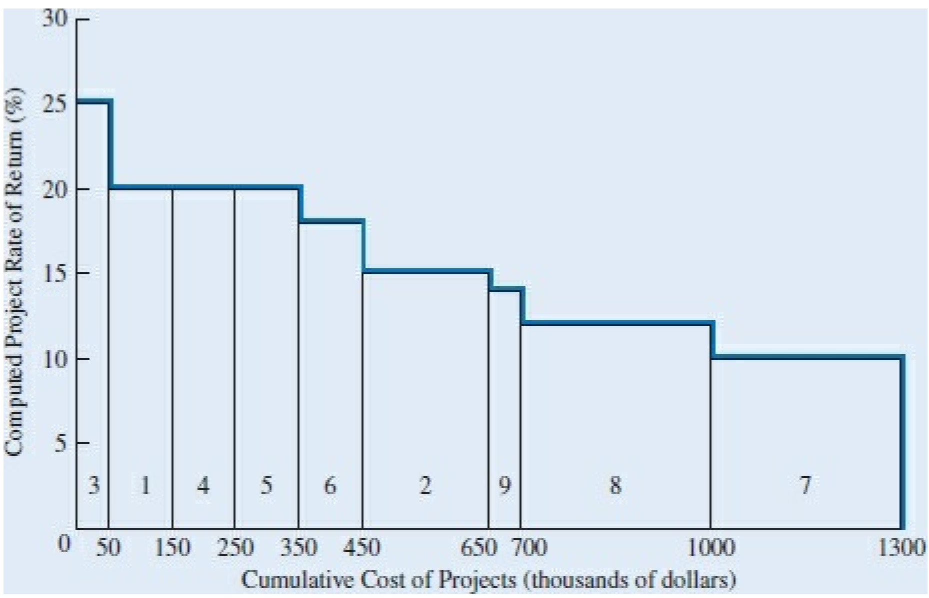


FIGURE 15-3 Cumulative cost of projects versus rate of return.

Spreadsheets can be used to sort the projects by rate of return and then calculate the cumulative first cost. This is accomplished through the following steps.

1. Enter or calculate each project's rate of return.
2. Select the data to be sorted. Do *not* include headings, but do include all information on the row that goes with each project.
3. Select the SORT tool (found in the menu under DATA), identify the rate of return column as the first key, and a sort order of descending. Also ensure that row sorting is selected. Sort.
4. Add a column for the cumulative first cost. This column is compared with the capital limit to identify the opportunity cost of capital and which projects should be funded.

Example 15–4 illustrates these steps. Example 15–5 outlines the more complicated case of independent projects with mutually exclusive alternatives.

EXAMPLE 15-4

A firm has a budget of $800,000 for projects this year. Which of the following projects should be accepted? What is the opportunity cost of capital?

Project	First Cost	Annual Benefit	Salvage Value	Life (years)
A	$200,000	$25,000	$50,000	15
B	250,000	47,000	−25,000	10
C	150,000	17,500	20,000	15

D	100,000	20,000	15,000	10
E	200,000	24,000	25,000	20
F	300,000	35,000	15,000	15
G	100,000	18,000	0	10
H	200,000	22,500	15,000	20
I	350,000	50,000	0	25

SOLUTION

The first step is to use the RATE function to find the rate of return for each project. The results of this step are shown in the top portion of Figure 15–4. Next the projects are sorted in descending order by their rates of return. Finally, the cumulative first cost is computed. Projects *D*, *I*, *B*, and *G* should be funded. The opportunity cost of capital is 10.6% the rate for the first project rejected.

	A	B	C	D	E	F	G	H
1	Project	First Cost	Annual Benefit	Salvage Value	Life	IRR		
2	A	200,000	25,000	50,000	15	10.2%	=RATE(E2,C2,−B2,D2)	
3	B	250,000	47,000	−25,000	10	12.8%		
4	C	150,000	17,000	20,000	15	8.6%		
5	D	100,000	20,000	15,000	10	16.0%		
6	E	200,000	24,000	25,000	20	10.6%		
7	F	300,000	35,000	15,000	15	8.2%		
8	G	100,000	18,000	0	10	12.4%		
9	H	200,000	22,500	15,000	20	9.6%		
10	I	350,000	50,000	0	25	13.7%		
11	Projects Sorted by IRR						Cumulative First Cost	
12	D	100,000	20,000	15,000	10	16.0%	100,000	
13	I	350,000	50,000	0	25	13.7%	450,000	
14	B	250,000	47,000	−25,000	10	12.8%	700,000	
15	G	100,000	18,000	0	10	12.4%	800,000	
16	E	200,000	24,000	25,000	20	10.6%	1,000,000	
17	A	200,000	25,000	50,000	15	10.2%	1,200,000	
18	H	200,000	22,500	15,000	20	9.6%	1,400,000	
19	C	150,000	17,500	20,000	15	8.6%	1,550,000	
20	F	300,000	35,000	15,000	15	8.2%	1,850,000	

FIGURE 15-4 Spreadsheet for finding opportunity cost of capital.

In Example 15–5, each of the more expensive mutually exclusive alternatives (1*B*, 1*C*, and 3*B*) has lower rates of return than the less expensive alternatives (1*A* and 3*A*). This is common because many projects exhibit decreasing returns to scale. As in Example 15–5, when it occurs, then it makes the ranking of increments easy to use. Sometimes more expensive mutually exclusive alternatives have higher rates of return; then several combinations or more advanced techniques must be tried.

EXAMPLE 15-5

A company is preparing its capital budget for next year. The amount has been set at $250,000 by the board of directors. Rank the following project proposals for the board's consideration and recommend which should be funded.

Project Proposals	Cost (thousands)	Uniform Annual Benefit (thousands)	Salvage Value (thousands)	Useful Life (years)
Proposal 1				
Alt. *A*	$100	$23.85	$0	10
Alt. *B*	150	32.20	0	10
Alt. *C*	200	39.85	0	10
Proposal 2	50	14.92	0	5
Proposal 3				
Alt. *A*	100	18.69	25	10
Alt. *B*	150	19.42	125	10

SOLUTION

For project proposals with two or more alternatives, incremental rate of return analysis is required.

					Incremental Analysis		
Combination of	Cost ($1000s)	Uniform Annual Benefit	Salvage Value	Rate of	Cost ($1000s)	Uniform Annual Benefit	Salvage Value

Alternatives		($1000s)	($1000s)	Return		($1000s)	($1000s)
Proposal 1							
A	$100	$23.85	$0	20.0%			
B − *A*					$50	8.35	$0
B	150	32.20	0	17.0			
C − *B*					50	7.65	0
C − *A*					100	16.00	0
C	200	39.85	0	15.0			
Proposal 2	50	14.92	0	15.0			
Proposal 3							
A	100	18.69	25	15.0			
B − *A*					50	0.73	100
B	150	19.42	125	12.0			

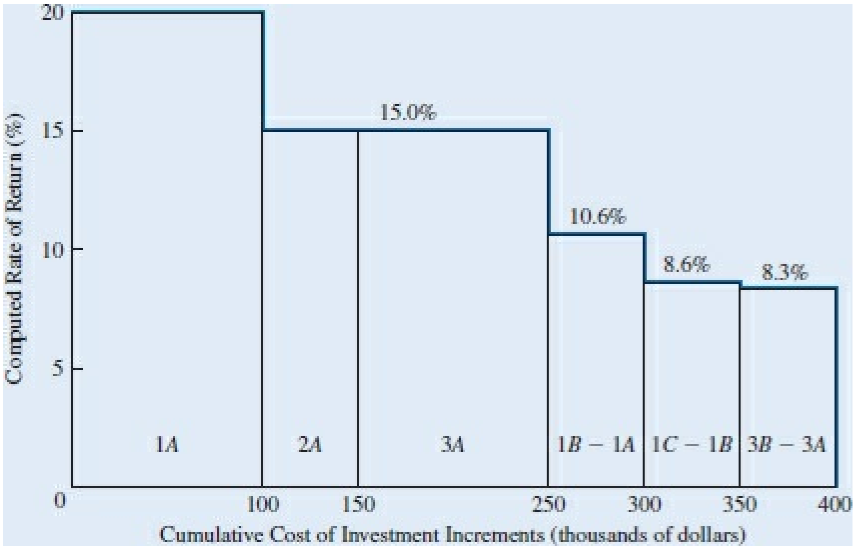


FIGURE 15-5 Cumulative cost versus incremental rate of return.

The various separable increments of investment may be ranked by the rate of return. They are plotted in a graph of cumulative cost versus rate of return in Figure 15–5. The ranking of projects by rate of return gives the following:

Project	Rate of Return
1*A*	20.0%
2	15.0%
3*A*	15.0%
1*B* in place of 1*A*	10.6%
1*C* in place of 1*B*	8.6%
3*B* in place of 3*A*	8.3%

For a budget of $250,000 the selected projects are 1*A*, 2, and 3*A*. Note that if a budget of $300,000 were available, 1*B* would replace 1*A*, making the proper set of projects 1*B*, 2, and 3*A*. At a budget of $400,000 1*C* would replace 1*B*; and 3*B* would replace 3*A*, making the selected projects 1*C*, 2, and 3*B*.

SELECTING A MINIMUM

ATTRACTIVE RATE OF RETURN

Focusing on the three concepts on the cost of money (the cost of borrowed money, the cost of capital, and opportunity cost), which, if any, of these values should be used as the minimum attractive rate of return (MARR) in economic analyses?

Fundamentally, we know that unless the benefits of a project exceed its cost, we cannot add to the profitability of the firm. A lower boundary for the minimum attractive rate of return must be the cost of the money invested in the project. It would be unwise, for example, to borrow money at 8% and invest it in a project yielding a 6% rate of return.

Further, we know that no firm has an unlimited ability to borrow money. Bankers—and others who evaluate the limits of a firm's ability to borrow money—look at both the profitability of the firm and the relationship between the components in the firm's capital structure. This means that increases in a firm's debt will usually be matched by proportionate increases in equity (stock and retained earnings from operations). This maintains an acceptable ratio between **ownership** (equity) and debt. In other words, borrowing for a particular investment project is only a block of money from the overall capital structure of the firm. This suggests that the MARR should not be less than the cost of capital. Finally, we know that the MARR should not be less than the rate of return on the best opportunity forgone. Stated simply,

Minimum attractive rate of return should be greater than or equal to the largest of: cost of borrowed money, cost of capital, or opportunity cost.

ADJUSTING MARR TO ACCOUNT FOR RISK AND UNCERTAINTY

We know from our study of estimating the future that what actually occurs is often different from the estimate. When we are fortunate enough to be able to assign probabilities to a set of possible future outcomes, we call this a **risk** situation. We saw in Chapter 10 that techniques like expected value and simulation may be used when the probabilities are known.

Uncertainty is the term used to describe the condition when the probabilities are *not* known. Thus, if the probabilities of future outcomes are known, we have *risk*, and if the probabilities are unknown, we have *uncertainty*. With uncertainty, adjustments for risk are more subjective.

In projects accompanied by normal business risk and uncertainty, the MARR is used without adjustment. For projects with greater than average risk or uncertainty, most firms increase the MARR. As reported in Block (2005), the percentage of firms using risk-adjusted rates varied from 66% in retail to 82% in health care. Some of the percentages for other industries are 70% for manufacturing, 73% for energy, and 78% for technology firms. Table 15–2 lists an example of risk-adjusted MARRs in manufacturing.

Some firms use the same rates for all divisions and groups. Other firms vary the rates by division for strategic reasons. There are even cases when a project-specific rate based on that project's financing may be justified. For example, a firm or joint venture may be founded to develop a specific mine, pipeline, or other resource development project.

However, as shown in Example 15–6, risk-adjusted rates may not work well. A preferable way deals explicitly with the probabilities using the techniques from Chapter 10. When the interest rate (MARR) used in economic analysis calculations is raised to adjust for risk or uncertainty, greater emphasis is placed on immediate or short-term results and less emphasis on longer-term results.

Table 15-2 Example Risk-Adjusted MARR Values in Manufacturing

Rate (%)	Applies to:
6	Equipment replacement
8	New equipment
10	New product in normal market
12	New product in related market

16	New product in new market
20	New product in foreign market

EXAMPLE 15-6

Consider the two following alternatives. The MARR of Alt. *B* has been raised from 10% to 15% to take into account the greater risk and uncertainty associated with that alternative. What is the impact of this change of MARR on the decision?

Year	Alt. *A*	Alt. *B*
0	− $80	− $80
1–10	10	13.86
11–20	20	10

SOLUTION

			NPW	
Year	Alt. *A*	At 14.05%	At 10%	At 15%
0	−$80	−$80.00	−$80.00	−$80.00
1–10	10	52.05	61.45	50.19
11–20	20	27.95	47.38	24.81
		0	+28.83	−5.00

			NPW	
Year	Alt. *B*	At 15.48%	At 10%	At 15%
0	−$80	−$80.00	−$80.00	−$80.00
1–10	13.86	68.31	85.14	69.56
11–20	10	11.99	23.69	12.41
		0	+28.83	+1.97

Computations at MARR of 10% Ignoring Risk and Uncertainty

Both alternatives have the same positive NPW (+$28.83) at a MARR of 10%. Also, the differences in the benefits schedules (*A* − *B*) produce a 10% incremental rate of return. (This calculation is not shown here.) This must be true if NPW for the two alternatives is to remain constant at a MARR of 10%.

Considering Risk and Uncertainty with MARR of 10%

At 10%, the alternatives are equally desirable. Since Alt. *B* is believed to have greater risk and uncertainty, a logical conclusion is to select Alt. *A* rather than *B*.

Increase MARR to 15%

At a MARR of 15%, Alt. *A* has a negative NPW and Alt. *B* has a positive NPW. Alternative *B* is preferred under these circumstances.

Conclusion

Based on a business-risk MARR of 10%, the two alternatives are equivalent. Recognizing some greater risk of failure for Alt. *B* makes *A* the preferred alternative. If the MARR is increased to 15%, to add a margin of safety against risk and uncertainty, the computed decision is to select *B*. Since Alt. *B* has been shown to be less desirable than *A*, the decision, based on a MARR of 15%, may be an unfortunate one. The difficulty is that the same risk adjustment (increase the MARR by 5%) is applied to both alternatives even though they have different amounts of risk.

The conclusion to be drawn from Example 15–6 is that increasing the MARR to compensate for risk and uncertainty is only an approximate technique and may not always achieve the desired result. Nevertheless, it is common practice in industry to adjust the MARR upward to compensate for increased risk and uncertainty.

REPRESENTATIVE VALUES OF MARR USED IN INDUSTRY

We argued that the minimum attractive rate of return should be established at the highest one of the following: cost of borrowed money, cost of capital, or the opportunity cost.

The cost of borrowed money will vary from enterprise to enterprise, with the lowest rate being the prime interest rate. The prime rate may change several times in a year; it is widely reported in newspapers and business publications. As we pointed out, the interest rate for firms that do not qualify for the prime interest rate may be 1/2 to several percentage points higher.

The cost of capital of a firm is an elusive value. There is no widely accepted way to compute it; we know that as a **composite value** for the capital structure of the firm, it conventionally is higher than the cost of borrowed money. The cost of capital must consider the market valuation of the shares (common stock, etc.) of the firm, which may fluctuate widely, depending on future earnings prospects of the firm. We cannot generalize on representative costs of capital.

Somewhat related to cost of capital is the computation of the return on total capital (long-term debt, capital stock, and retained earnings) actually achieved by firms. *Fortune* magazine, among others, does an annual analysis of the rate of return on total capital. The after-tax rate of return on total capital for individual firms ranges from 0% to about 40% and averages 8%. *Business Week* does a periodic survey of corporate performance. This magazine reports an after-tax rate of return on common stock and retained earnings. We would expect the values to be higher than the rate of return on total capital, and this is the case. The after-tax return on common stock and retained earnings ranges from 0% to about 65% with an average of 14%.

Higher values for the MARR are used by firms that are short of capital, such as high-technology start-ups. They are also used in industries, such as petroleum and mining, where volatile prices increase the risk of poor returns for projects. Rates of 25 to 30% are relatively common, and even higher rates are sometimes used. For companies with more normal levels of risk, rates of 12 to 15% are more typical.

Note that the values of MARR given earlier are approximations. But the values quoted appear to be opportunity costs, rather than cost of borrowed money or cost of capital. This indicates that firms cannot or do not obtain money to fund projects whose anticipated rates of return are nearer to the cost of borrowed money or cost of capital. One reason that firms operate as they do is that they can focus limited resources of people, management, and time on a smaller number of good projects.

One cannot leave this section without noting that the MARR used by enterprises is much higher than can be obtained by individuals. (Where can you get a 30% after-tax rate of return without excessive risk?) The reason appears to be that businesses are not obliged to compete with the thousands of individuals in any region seeking a place to invest $2000 with safety, whereas the number of people who could or would want to invest $500,000 in a business is far smaller. This diminished competition, combined with a higher risk, appears to explain at least some of the difference.

CAPITAL BUDGETING OR SELECTING THE BEST PROJECTS

The opportunity cost of capital approach of ranking projects by their rate of return introduced a new type of problem. Up to that point we'd been analyzing mutually exclusive alternatives, where only one could be chosen. Engineering design problems are this type of problem, where younger engineers use engineering economy to choose the best alternative design.

At higher levels in the organization, engineering economy is applied to solve a different problem. For example, 30 projects have passed initial screening and are being proposed for funding. Every one of the 30 meets the MARR. The firm can afford to invest in only some of them. So, which ones should be chosen and how? This is called the **capital budgeting** problem.

Examples 15–2 and 15–3 applied the opportunity cost of capital approach to the capital budgeting problem. Firms often use this approach as a starting point to rank the projects from best to worst. In some cases the ranking by rate of return is used to make the decision.

More often, managers then meet and decide which projects will be funded by obtaining a consensus, or a decision by the highest-ranking manager, which will modify the rate of return ranking. At this meeting, business units argue for a larger share of the capital budget, as do plants in the same business, groups at the same plants, and individuals within the groups. Some considerations, such as strategy, necessity, and the availability and capability of particular resources and people, are difficult to represent in the project's *numbers*, which are the subject of economic analysis.

Other firms rank projects using a benefit–cost ratio or present worth index. As shown in Example 15–7, the present worth index is the NPW divided by the cost's present value.

Anyone who has ever bought firecrackers probably used the practical ranking criterion of "biggest bang for the buck" in making a selection. This same criterion—stated more elegantly—is used by some firms to rank independent projects.

Rank independent projects according to their value of net present worth divided by the present worth of cost. The appropriate interest rate is MARR (as a reasonable estimate of the cutoff rate of return).

Some consider the present worth index to be a better measure, but this can be true only if PW is applied at the correct interest rate. It is more common for firms to simply rank on the rate of return. If independent projects can be ranked in their order of desirability, then the selection of projects to be included in a capital budget is a simple task. One may proceed down the list of ranked projects until the capital budget has been exhausted. The only difficulty with this scheme occurs, occasionally, when the capital budget is more than enough for *n* projects but too little for *n* + 1 projects.

EXAMPLE 15-7

Rank the following nine independent projects in their order of desirability, based on a 14.5% minimum attractive rate of return.

Project	Cost (thousands)	Uniform Annual Benefit (thousands)	Useful Life (years)	Salvage Value (thousands)	Computed Rate of Return	Computed NPW at 14.5% (thousands)	Co NP (th
1	$100	$23.85	10	$0	20%	$22.01	0.2
2	200	39.85	10	0	15	3.87	0.0
3	50	34.72	2	0	25	6.81	0.1
4	100	20.00	6	100	20	21.10	0.2

5	100	20.00	10	100	20	28.14	0.2
6	100	18.00	10	100	18	17.91	0.1
7	300	94.64	4	0	10	−27.05	−0
8	300	47.40	10	100	12	−31.69	−0
9	50	7.00	10	50	14	−1.28	−0

SOLUTION

Ranked NPW/PW of cost, the projects are listed as follows:

Project	NPW/PW of Cost	Rate of Return
5	0.2814	20%
1	0.2201	20
4	0.2110	20
6	0.1791	18
3	0.1362	25
2	0.0194	15
9	−0.0256	14
7	−0.0902	10
8	−0.1056	12

With a 14.5% MARR, Projects 1 to 6 are recommended for funding and 7 to 9 are not. However, they are ranked in a different order by the present worth index and by the rate of return approaches. For example, Project 3 has the highest ranking for the rate of return and is fifth by the present worth index.

In Example 15–7, suppose the capital budget is \$550,000. This is more than enough for the top five projects (sum = \$450,000) but not enough for the top six projects (sum = \$650,000). When we have this situation, it may not be possible to say with certainty that the best use of a capital budget of \$550,000 is to fund the top five projects. There may be some other set of projects that makes better use of the available \$550,000. While some trial-and-error computations may indicate the proper set of projects, more elaborate techniques are needed to prove optimality.

As a practical matter, a capital budget total or line items have some flexibility. If in Example 15–7 the capital budget is \$550,000, then a careful examination of Project 2 will dictate whether to it should be funded. If so, then the budget may be expanded or some or all projects may be approved with a lower funding level. Or perhaps Project 2 can be started in this budget year and finished next year.

SUMMARY

There are three general sources of capital available to a firm. The most important one is money generated from the firm’s operations. This has two components: there is the portion of profit that is retained in the business; in addition, funds equal to its depreciation charges are available for reinvestment.

The two other sources of capital are from outside the firm’s operations.

Debt: borrowed as loans from banks, insurance companies, and so forth.
Longer-term borrowing: from selling bonds.
Equity: sale of equity securities like common or preferred stock.

Retained profits and cash equal to depreciation charges are the primary sources of investment capital for most firms, and the only sources for many enterprises.

In selecting a value of MARR, three values are frequently considered:

1. Cost of borrowed money.

2. Cost of capital. This is a composite cost of the components of the firm's overall capitalization.
3. Opportunity cost, which is the rate of return on the best investment project that is rejected.

The MARR should be equal to the highest one of these three values.

When there is a risk aspect to the problem (probabilities are known or reasonably estimated), this can be handled by techniques like expected value and simulation. Where there is uncertainty (probabilities of the various outcomes are not known), there are analytical techniques, but they are less satisfactory. A method commonly used to adjust for risk and uncertainty is to increase the MARR. This method can distort the time-value-of-money relationship. The effect is to discount longer-term consequences more heavily than short-term consequences, which may or may not be desirable. Prior to this chapter we had assumed that all worthwhile projects are approved and implemented. But industrial firms, like individuals and governments, are typically faced with more good projects than can be funded with the money available. The task is to select the best projects and reject, or at least delay, the rest.

Capital may be rationed among competing investment opportunities by either rate of return or present worth methods. The results may not always be the same for these two methods in many practical situations.

If projects are ranked by rate of return, a proper procedure is to go down the list until the capital budget has been exhausted. The rate of return at this point is the cutoff rate of return. This procedure gives the best group of projects, but does not necessarily have them in the proper priority order.

It has been shown in earlier chapters that the usual business objective is to maximize NPW, and this is not necessarily the same as maximizing rate of return. One suitable procedure is to use the ratio (NPW/PW of cost) to rank the projects, letting the MARR equal the cutoff rate of return (which is the opportunity cost of capital). This present worth ranking method will order the projects so that, for a limited capital budget, NPW will be maximized. The MARR must equal the cutoff rate of return for the rate of return and present worth methods to yield compatible results.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

15-1 The capital structure of a firm is as follows.

Source of Capital	Share of Capitalization	Interest Rate
Loans	35	7%
Bonds	40	8%
Common stock	25	10%

The combined state and federal income tax rate for the firm is 42%. Find the after-tax and before-tax costs of capital to the firm.

SOLUTION

15-2 NuStuff Inc. has raised $8M by selling bonds at an average rate of 7%. NuStuff's stockholders expect a 14% rate of return, and there is $15M in common stock and retained earnings. NuStuff has $4M in loans at an average rate of 9%. What is the firm's cost of capital:

(*a*) Before taxes?
(*b*) After taxes with a combined tax rate of 26%?

SOLUTION

15-3 Abby Industries Inc., has the following capital structure:

Type	Amount	Average Minimum Return
Mortgages	$ 25,000,000	7%
Bonds	180,000,000	9%
Common stock	100,000,000	10%

Preferred stock	50,000,000	8%
Retained earnings	120,000,000	10%

Determine the weighted average cost of capital (WACC) for Abby.

SOLUTION

15-4 A new utility had \$10M in start-up capital, and it has sold \$45M in bonds at 7% to raise money for electric power generation and connecting to the existing power grid. There were some cash flow problems so it had to borrow \$2.5M at 11%. The stockholders expect a 15% rate of return, and the combined tax rate is 25%. What is the utility's before- and after-tax costs of capital?

SOLUTION

15-5 A loan has an interest rate of 8%, and the inflation rate is 2.4%. What is the loan's real interest rate adjusted for inflation?

SOLUTION

15-6 A country with an inflation rate of 20% is offering 5-year bonds at an interest rate of 30%. What is the real rate on the bond? Appendix 9A and Figure 13-1 use a common approximation for the real rate. What is that value in this case?

SOLUTION

15- A small surveying company identifies its available independent alternatives.

	Alternative	Initial Cost	Rate of Return
A:	Repair existing equipment	\$1000	30%
B:	Buy EDM instrument	2500	9%
C:	Buy a new printer	3000	11%
D:	Buy equipment for an additional crew	3000	15%

7	*E*: Buy programmable calculator	500	25%

The owner of the company has $5000 of savings currently invested in securities yielding 8% that could be used for the company.

a. Assuming that the funds are limited to the owner’s savings, what is the apparent cutoff rate of return?
b. If the owner can borrow money at 10%, how big a loan should she undertake?

SOLUTION

Barber Brewing is in the process of determining the capital budget for the coming year. The following projects are under consideration.

		A	*B*	*C*	*D*
15-8	First cost	$10,000	$13,000	$20,000	$33,000
	Annual income	10,000	9,078	16,000	16,455
	Annual cost	7,362	5,200	11,252	7,300

All projects have a 5-year useful life. If Barber’s budget is set at $50,000, which alternative(s) should be selected?

SOLUTION

A manufacturing plant has a capital budget of $350,000. Which of these projects should be done?

15-9

Project	First Cost	Annual Benefit	Life (years)
1	$200,000	$50,000	15
2	$50,000	$7,000	20
3	$100,000	$40,000	6
4	$50,000	$15,000	7
5	$150,000	$30,000	15

SOLUTION

Which projects should be done, if a parks and recreation department has a capital budget of $350,000?

15-10

Project	First Cost	Annual Benefit	Life (years)
1	$50,000	$12,000	15
2	$50,000	$9,000	20
3	$100,000	$15,000	20
4	$50,000	$12,000	10
5	$100,000	$30,000	10
6	$100,000	$20,000	15
7	$150,000	$25,000	15

SOLUTION

A small construction company identifies the following alternatives, which are independent except where noted.

		Alternative	Initial Cost	Incremental Rate of Return	On Investment Over
	1.	Repair bulldozer	$5,000	30.0%	0
	2.	Replace backhoe			
		with Model *A*	20,000	15.0%	0
		with Model *B*	25,000	10.5%	2*A*
15-11	3.	Buy new dump truck			
		Model *X*	20,000	20.0%	0
		Model *Y*	30,000	14.0%	3*X*
	4.	Buy computer			
		Model *K*	5,000	12.0%	0
		Model *L*	10,000	9.5%	4*K*

a. Assuming that the company has $55,000 available for investment and is not able to borrow money, what alternatives should be chosen, and what is opportunity coast of capital?

b. If, however, the company can borrow money at 10%, how much should be borrowed, and which alternatives should be selected?

SOLUTION

15-12 A variant of a firm's leading product was developed for a related market segment dominated by a competitor. The competitor's new product release is increasing its dominance in that segment. The revised plan is to release the product in a foreign market that the firm wants to enter. Should the firm go ahead or develop another plan? $45M has been spent on product development. The product will cost $50M to adapt and to introduce. The net sales should be $15M per year for 5 years. Good will from the product will be worth $20M in 5 years for new products in this market.

SOLUTION

15-13 A new product is being developed for the normal market for this product line. Using Table 15-2, how much higher do the annual net sales have to be in a related market to justify releasing the product there? In both cases another $5.4M is required, in neither case is there a salvage value, and in both cases sales are expected for 5 years.

SOLUTION

15-14 A city engineer calculated the present worth of benefits and costs of a number of possible projects, based on 10% interest and a 10-year analysis period.

Costs and Benefits ($1000s)

Project	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Present Worth of Costs	75	70	50	35	60	25	70
Present Worth of Benefits	105	95	63	55	76	32	100

If 10% is a satisfactory minimum attractive rate of return (MARR), and $180,000 is available for expenditure, which project(s) should be selected? Use a NPW/C ranking.

SOLUTION

The CFO of Republic Express has asked regional managers to submit capital projects for next year. The CEO has decided to fund each region's top request and to fund two additional requests. But no region may have more than two projects funded. Which projects should be funded and what is capital budget?

15-15

Region	Project	Cost	Annual Benefit	Life (years)
Southeastern (SE)	A	$ 90,000	$16,400	15
	B	40,000	15,000	5
	C	60,000	20,400	5
	D	120,000	27,600	20
Midwest (MW)	A	50,000	10,000	20
	B	120,000	36,700	15
	C	75,000	21,600	5
	D	50,000	16,200	5
Northeastern (NE)	A	50,000	16,700	20
	B	80,000	23,500	5
	C	75,000	26,100	10
Western (W)	A	60,000	16,900	15
	B	50,000	15,300	10

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Cost of Funds

15-1 Assume you have $2000 available for investment for a 5-year period. You wish to *invest* the money—not just spend it on fun things. There are obviously many alternatives available. You should be willing to assume a modest amount of risk of loss of some or all of the money if this is necessary, but not a great amount of risk (no investments in poker games or at horse races). How would you invest the money? What is your minimum attractive rate of return? Explain.

15-2 There are many venture capital syndicates that consist of a few (say, eight or ten) wealthy people who combine to make investments in small and (hopefully) growing businesses. Typically, the investors hire a young investment manager (often an engineer with an MBA) who seeks and analyzes investment opportunities for the group. Would you estimate that the MARR sought by this group is more or less than 12%? Explain.

15-3 Determine the current interest rate on the following securities, and explain why the interest rates are different for these different bonds.
(*a*) U.S. Treasury bond due in 10 years
(*b*) General obligation bond of a municipal district, city, or a state due in 10 years
(*c*) Corporate debenture bond of a U.S. industrial firm due in 10 years

15-4 A A firm's stockholders expect a 10% rate of return, and there is $15M in common stock and retained earnings. The firm has $4M in loans at an average rate of 8%. The firm has raised $9M by selling bonds at an average rate of 5%. What is the firm's cost of capital:
(*a*) Before taxes?
(*b*) After taxes with a tax rate of 21%?

15-5 An engineering firm has borrowed $1.0M at 6%. The stockholders have invested another $1.5M. The firm's retained earnings total $1.2M. The return on equity is estimated to be 12%. What is the firm's cost of capital:
(*a*) Before taxes?
(*b*) After taxes with a tax rate of 28%?

15-6 A small engineering firm has borrowed $125,000 at 8%. The partners have invested another $75,000. If the partners require a 12% rate of return, what is the firm's cost of capital:

A

(*a*) Before taxes?

(*b*) After taxes with a tax rate of 21%?

15-7 A public university system wants to apply the concept of the WACC to developing its interest rate for analyzing capital projects. It has an endowment of $850 million which is earning 6.3% interest. It is paying 4.5% interest on $300 million in bonds. It believes that $120 million in general funds from the taxpayers should be assigned an interest rate of 13% What is the university's cost of capital? Note that only new bonds or the interest on the endowment is available to fund capital projects.

15-8 A firm's stockholders expect an 12% rate of return, and there is $20M in common stock and retained earnings. The firm has $7M in loans at an average rate of 9%. The firm has raised $12M by selling bonds at an average rate of 5%. What is the firm's cost of capital:

A

(*a*) Before taxes?

(*b*) After taxes with a tax rate of 24%?

15-9 A firm has 40,000 shares whose current price is $80.75. Those stockholders expect a return of 15%. The firm has a 2-year loan of $900,000 at 6.4%. It has issued 12,500 bonds with a face value of 1000, 15 years left to maturity, semiannual compounding, a coupon interest rate of 6%, and a current price of $1090. Using market values for debt and equity, what is the firm's cost of capital:

(*a*) Before taxes?

(*b*) After taxes with a tax rate of 28%?

15-10 A firm has 60,000 shares whose current price is $45.90. Those stockholders expect a return of 14%. The firm has a 3-year loan of $1,900,000 at 7.3%. It has issued 22,000 bonds with a face value of 1000, 20 years left to maturity, semiannual compounding, a coupon interest rate of 7%, and a current price of $925. Using market values for debt and equity, what is the firm's cost of capital:

A

(*c*) Before taxes?

(*d*) After taxes with a tax rate of 21%?

Some countries have initiated negative interest rates on short-term

15-11 E bonds and savings accounts to spur economic growth. What is the impact of such policies on people trying to save money for their future?

15-12 E Payday loan companies make short-term loans to people with no credit or poor credit. These loans often need to be repaid the next time a person receives a paycheck. Fees and interest are added, and effective interest rates often exceed 100% on an annualized basis. Investigate the payday loan industry.
(*a*) Are these legal in all states?
(*b*) What is the legal limit on the interest rate in the state where you are in?
(*c*) Are the high interest rates justified?

Inflation

15-13 What is the interest rate on a 2-year certificate of deposit at a bank or credit union in your area? What is the most recent value of the Consumer Price Index (CPI)? If inflation matches that rate, what is the real rate of return on the 2-year CD? Include references for the sources of your data.

15-14 What is the interest rate on a 4- or 5-year new car loan at a bank or credit union in your area? What is the most recent value of the Consumer Price Index (CPI)? If inflation matches that rate, what is the real interest rate you would pay on the car loan? Include references for the sources of your data.

15-15 Over the last 10 years, what has the inflation rate been? Compare this with the rate of return on the "Dow" average over the same period. What has the real rate of return for investing in this mix of stocks been? Include references for the sources of your data.

15-16 Over the last 10 years, what has the inflation rate been? Compare this with the rate of return on the NASDAQ average over the same period. What has the real rate of return for investing in this mix of stocks been? Include references for the sources of your data.

Per the effect referenced in Figure 15-1, calculate the real rate of investment for a country that has the following data. Find (a), (b), (c), and (d). Then graph these results as in the figure.

15-17

Year	Rate on 20-Year Bonds	General Inflation Rate	Real Rate on Investment
0	15%	9.52%	5%
1	10%	22.22%	(a)
2	12%	1.82%	(b)
3	18%	12.38%	(c)
4	5%	10.53%	(d)

15-18 A The long-term borrowing rate has historically been approximately 3% higher than general inflation. What would the Long Term Borrowing Index (LTBI) be in 2017 if its index base is the year 2000, and inflation since 2000 has averaged 2.3%?

Opportunity Cost of Capital

15-19 A factory has a $100,000 capital budget. Determine which project(s) should be funded and the opportunity cost of capital.

Project	First Cost	Annual Benefits	Life (years)	Salvage Value
A	$50,000	$13,500	5	$5000

B	50,000	9,000	10	0
C	50,000	13,250	5	1000
D	50,000	9,575	8	6000

15-20
A

Chips USA is considering the following projects to improve their production process. Chips have a short life, so a 3-year horizon is used in evaluation. Which projects should be done if the budget is $70,000? What is the opportunity cost of capital?

Project	First Cost	Benefit
1	$20,000	$12,000
2	30,000	13,000
3	10,000	6,500
4	5,000	2,700
5	25,000	14,000
6	15,000	8,200
7	40,000	17,000

15-

A city has identified a series of projects to replace incandescent light bulbs with LED bulbs in traffic lights, office buildings, and other uses. Which projects should be done if the budget is (a) $500,000, and (b) $1.2 million? What is the opportunity cost of capital?

Project	First Cost	Annual Benefit	Life (years)
1	$200,000	$50,000	15
2	300,000	70,000	10

21 G	3	100,000	40,000	5
	4	50,000	12,500	10
	5	250,000	75,000	5
	6	150,000	32,000	20
	7	400,000	125,000	5

The Winthrop Company has decided to fund $1.8 million in new project proposals for the coming budget year. Which projects should be funded? What is the opportunity cost of capital?

	Project	First Cost	Annual Benefit	Life (years)
	A	$150,000	$42,000	5
	B	200,000	58,000	5
	C	300,000	87,000	5
15-22 A	*D*	250,000	79,000	6
	E	400,000	110,000	6

F	500,000	135,000	6
G	350,000	85,000	10
H	600,000	120,000	10
I	750,000	147,000	10

15-23 G

For the following projects what is the opportunity cost of capital if the budget is (*a*) $60,000, and (*b*) $120,000? If Project 4 has an external environmental cost of $1000 annually that is included, (*c*) how does this change the answer to (*a*)? (*d*) How does this change the answer to (*b*)?

Project	Life (years)	First Cost	Annual Benefit	Salvage Value
1	20	$20,000	$4000	
2	20	20,000	3200	$20,000
3	30	20,000	3300	10,000
4	15	20,000	4500	
5	25	20,000	4500	−20,000
6	10	20,000	5800	

7	15	20,000	4000	10,000

Risk-Adjusted MARR

15-24 A	Use the example risk-adjusted interest rates for manufacturing projects in Table 15–2. Assume Project *B* in Problem 15-19 is for new equipment. What is the interest rate for evaluating this project? Should it be done?
15-25	Use the example risk-adjusted interest rates for manufacturing projects in Table 15–2. Assume Project *E* in Problem 15-22 is a new product in an normal market. What is the interest rate for evaluating this project? Should it be done?
15-26 A	Use the example risk-adjusted interest rates for manufacturing projects in Table 15–2. Assume Project 1 in Problem 15-23 is a new product in a foreign market. What is the interest rate for evaluating this project? Should it be done?
15-27	Use the example risk-adjusted interest rates for manufacturing projects in Table 15–2 and the project data in Problem 15-22. Assume that each project is to launch a new product. New products for the normal market will have a life of 10 years. New products for a related market will have a life of 6 years. New products for a new market will have a life of 5 years. Which new products are recommended for funding? What is the total cost of these projects?

Capital Budgeting

Each of the following 10 independent projects has a 10-year life and no salvage value.

Project	Cost (thousands)	Uniform Annual Benefits (thousands)	Rate of Return
1	$ 50	$10.3	16%

2	150	32.2	17
3	100	17.7	12
4	300	48.8	10
5	50	11.9	20
6	200	38.3	14
7	50	10.0	15
8	200	36.9	13
9	50	11.5	19
10	100	22.3	18

15-28
A

The projects have been proposed by the staff of the Ace Card Company. The MARR of Ace has been 12% for several years.

(*a*) If there is ample money available, what projects should Ace approve?
(*b*) Rank-order all the acceptable projects according to desirability using NPW/cost.
(*c*) If only $450,000 is available, which projects should be approved?

(*d*) Are the results the same if the projects are ranked on IRR? What is the opportunity cost of capital?

15-29 Ten capital spending proposals have been made to the budget committee as the members prepare the annual budget for their firm. Each independent project has a 5-year life and no salvage value.

Project	Initial Cost (thousands)	Uniform Annual Benefit (thousands)	Rate of Return
A	$10	$2.98	15%
B	15	5.58	25
C	5	1.53	16
D	20	5.55	12
E	15	4.37	14
F	30	9.81	19
G	25	7.81	17
H	10	3.49	22
I	5	1.67	20
J	10	3.20	18

(*a*) Based on a MARR of 14%, which projects should be approved?
(*b*) Rank-order all the projects according to desirability.
(*c*) If only $70,000 is available, which projects should be approved?
(*d*) Are the results the same if the projects are ranked on IRR? What is the opportunity cost of capital?

15-30 A At Miami Products, four project proposals (three with mutually exclusive alternatives) are being considered. All the alternatives have a 10-year useful life and no salvage value.

Project Proposal	Cost (thousands)	Uniform Annual Benefits (thousands)	Rate of Return
Project 1			
Alt. *A*	$25	$4.61	13%
Alt. *B*	50	9.96	15
Alt. *C*	10	2.39	20
Project 2			
Alt. *A*	20	4.14	16
Alt. *B*	35	6.71	14
Project 3			

Alt. *A*	25	5.56	18
Alt. *B*	10	2.15	17
Project 4	10	1.70	11

(*a*) Use rate of return methods to determine which set of projects should be undertaken if the capital budget is limited to about $100,000.
(*b*) For a budget of about $100,000, what interest rate should be used in rationing capital by present worth methods? (Limit your answer to a value for which there is a compound interest table available in Appendix C).
(*c*) Using the interest rate determined in part (*b*), rank-order the eight different investment opportunities by means of the present worth method.
(*d*) For a budget of about $100,000 and the ranking in part (*c*), which of the investment opportunities should be selected?

Minicases

A financier has a staff of three people whose job it is to examine possible business ventures for him. Periodically they present their findings concerning business opportunities. On a particular occasion, they presented the following investment opportunities:
Project A: This is a project for the use of commercial land the financier already owns. There are three mutually exclusive alternatives.
• *A1*. Sell the land for $500,000.
• *A2*. Lease the property for a car-washing business. An annual income, after all costs (property taxes, etc.) of $98,700 would be received at the end of each year for 20 years. At the end of the 20 years, it is believed that the property could be sold for $750,000.
• *A3*. Construct an office building on the land. The building will cost $4.5 million to construct and will not produce any net income for the

first 2 years. The probabilities of various levels of rental income, after all expenses, for the subsequent 18 years are as follows:

Annual Rental Income	Probability
$1,000,000	0.1
1,100,000	0.3
1,200,000	0.4
1,900,000	0.2

The property (building and land) probably can be sold for $3 million at the end of 20 years.

15-31 *Project B:* An insurance company is seeking to borrow money for 90 days at 13$\frac{3}{4}$ % per annum, compounded continuously.
Project C: A financier owns a manufacturing company. The firm desires additional working capital to allow it to increase its inventories of raw materials and finished products. An investment of $2 million will allow the company to obtain sales that in the past the company had to forgo. The additional capital will increase company profits by $500,000 a year. The financier can recover this additional investment by ordering the company to reduce its inventories and to return the $2 million. For planning purposes, assume the additional investment will be returned at the end of 10 years.
Project D: The owners of *Sunrise* magazine are seeking a loan of $500,000 for 10 years at a 16% interest rate.
Project E: The Galveston Bank has indicated a willingness to accept a deposit of any sum of money over $100,000, for any desired duration, at a 14.06% interest rate, compounded monthly. It seems likely that this interest rate will be available from Galveston, or some other bank, for the next several years.
Project F: A car rental firm is seeking a loan of $2 million to expand its fleet. The firm offers to repay the loan by paying $1 million at the end of Year 1 and $1,604,800 at the end of Year 2.

• If there is $4 million available for investment now (or $4.5 million if the Project *A* land is sold), which projects should be selected? What is the MARR in this situation?

• If there is $9 million available for investment now (or $9.5 million if the Project *A* land is sold), which projects should be selected?

The Raleigh Soap Company has been offered a 5-year contract to manufacture and package a leading brand of soap for Taker Bros. It is understood that the contract will not be extended past the 5 years because Taker Bros. plans to build its own plant nearby. The contract calls for 10,000 metric tons (one metric ton equals 1000 kg) of soap a year. Raleigh normally produces 12,000 metric tons of soap a year, so production for the 5-year period would be increased to 22,000 metric tons. Raleigh must decide what changes, if any, to make to accommodate this increased production. Five projects are under consideration.

Project 1: Increase liquid storage capacity. Raleigh has been forced to buy caustic soda in tank truck quantities owing to inadequate storage capacity. If another liquid caustic soda tank is installed to hold 1000 cubic meters, the caustic soda may be purchased in railroad tank car quantities at a more favorable price. The result would be a saving of 0.1¢ per kilogram of soap. The tank, which would cost $83,400, has no net salvage value.

Project 2: Acquire another sulfonation unit. The present capacity of the plant is limited by the sulfonation unit. The additional 12,000 metric tons of soap cannot be produced without an additional sulfonation unit. Another unit can be installed for $320,000.

Project 3: Expand the packaging department. With the new contract, the
packaging department must either work two 8-hour shifts or have
15-32 another packaging line installed. If the two-shift operation is used, a 20%
wage premium must be paid for the second shift. This premium would
amount to $35,000 a year. The second packaging line could be installed
for $150,000. It would have a $42,000 salvage value at the end of 5
years.

Project 4: Build a new warehouse. The existing warehouse will be inadequate for the greater production. It is estimated that 400 square meters of additional warehouse is needed. A new warehouse can be built on a lot beside the existing warehouse for $225,000, including the land. The annual taxes, insurance, and other ownership costs would be $5000 a year. It is believed the warehouse could be sold at the end of 5 years for $200,000.

Project 5: Lease a warehouse. An alternative to building an additional warehouse would be to lease warehouse space. A suitable warehouse one mile away could be leased for $15,000 per year. The $15,000 includes taxes, insurance, and so forth. The annual cost of moving materials to this more remote warehouse would be $34,000 a year.

The contract offered by Taker Bros. is a favorable one, which Raleigh Soap plans to accept. Raleigh management has set a 15% before-tax minimum attractive rate of return as the criterion for any of the projects. Which projects should be undertaken?

Mike Moore's microbrewery is considering production of a new ale called Mike's Honey Harvest Brew. To produce this new offering, Mike is considering two independent projects. Each of these projects has two mutually exclusive alternatives, and each alternative has a useful life of 10 years and no salvage value. Mike's MARR is 8%. Information regarding the projects and alternatives is given in the following table.

Project/Alternative	Cost	Annual Benefit
Project 1. Purchase new		
fermenting tanks		
Alt. *A*: 5000-gallon tank	$ 5,000	$1192
Alt. *B*: 15,000-gallon tank	10,000	1992
Project 2. Purchase bottle filler		
and capper		
Alt. *A*: 2500-bottle/hour	15,000	3337
machine		
Alt. *B*: 5000-bottle/hour	25,000	4425
machine		

15-31 Use incremental rate of return analysis to complete the following worksheet.

Proj./Alt. Cost, P Annual Benefit, *A* (*A/P*, *i*, 10) IRR

1*A*	\$ 5,000	\$1192	0.2385	20%
1*B*–1*A*	5,000	800	0.1601	
2*A*	15,000	3337		
2*B*–2*A*	10,000			

Use this information to determine:

(*a*) which projects should be funded if only \$15,000 is available.
(*b*) the cutoff rate of return if only \$15,000 is available.
(*c*) which projects should be funded if \$25,000 is available.

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 46	**Aero Tech**
	Case budgeting with memos for three different approaches to ranking.
CASE 47	**Bigstate Highway Department**
	Capital budgeting including mutually exclusive alternatives. Includes uncertain value of a life.
CASE 48	**Dot Puff Project Selection**
	Public-sector capital budgeting with added constraint on man-years required.

CASE 49	**The Arbitrator**
	Includes two memos of suggested solutions for four projects with nine alternatives. Data presented in text discussion rather than tabulated.
CASE 50	**Capital Planning Consultants**
	Capital budgeting including mutually exclusive alternatives. Includes uncertainties in first cost, annual benefit, and lives.
CASE 51	**Refrigerator Magnet Company**
	Capital budgeting including mutually exclusive alternatives.

CHAPTER 16

ECONOMIC ANALYSIS IN THE PUBLIC SECTOR

From Waste to Power and Money

One human activity that contributes to global warming is the dumping of organic material into landfills. This material decomposes over time, producing a significant amount of landfill gas (LFG). About 50% of LFG is methane, which traps heat in the atmosphere with an effect that is twenty times greater than carbon dioxide. Landfills that do not control the escape of landfill gas are the largest single human source of methane emissions in the United States, as well as a source of air pollution and odors.

But humans must produce waste to live—it is one of the basic laws of thermodynamics. That is bad news all around—except that using engineering principles, bringing together various “constituencies,” and evaluating alternatives with engineering economy can bring benefits out of almost every one of these issues.

Landfill gas (LFG) can be an asset if it is extracted and properly utilized. LFG has about half of the heating value of natural gas. As a result, more and more owners of municipal solid waste landfills extract this gas and use it to generate electricity. Such electricity is used directly for their power needs or is sold to the general power grid.

Extracting and using a landfill to generate electricity is a “6 win” situation.

1. It benefits the environment by reducing unwanted gas emission;
2. It adds electrical power to the grid;
3. It produces cash flow for the landfill owner;
4. It lowers the use of non-renewable fossil fuels;
5. It reduces financial costs to the local population; and
6. It reduces the hazardous, noxious, and odorous gases for those nearby and downwind.

Contributed by William R. Truran, U.S. Department of Defense, and Peter A. Cerenzio, Cerenzio & Panaro Consulting Engineers

QUESTIONS TO CONSIDER

1. How can this topic best be summarized for a non-technical public audience? This is an important “skill” of an engineer—conveying sometimes complex and confusing scientific issues to a less informed, less scientific, and less interested audience with a short attention span.
2. What engineering economic principles would you apply in analyzing this application? Which measures would be most effective for the public audience? The important measures, and the metrics used to describe them, may be considered “key performance indicators.”
3. This waste to power application cannot be answered with a simple equation with a closed-ended answer that is either “correct” or “wrong.” This problem requires the cooperation of several (at least) agencies and communities of interest. Name some of these stakeholders and identify how their objectives are aligned or in conflict.
4. The Enron example—which is a “poster child” of what can go wrong in business—like this vignette is related to electric power generation. Discuss

ways in which some of the stakeholders may be "shady" in such situations. Discuss how you—as an engineer—could not only affect factual answers but could project perhaps an image that may not be real. As an engineer, you can be honored for (or guilty of) your representations.

After Completing This Chapter...

The student should be able to:

- Distinguish the unique objective and viewpoint of public decisions.
- Explain methods for determining the interest rates for evaluating public projects.
- Use the benefit–cost ratio to analyze projects.
- Distinguish between the conventional and modified versions of the benefit–cost ratio.
- Use an incremental benefit–cost ratio to evaluate a set of mutually exclusive projects.
- Discuss the impact of financing, duration, quantifying and valuing consequences, and politics in public investment analysis.

Key Words

benefit–cost ratio

conventional B/C ratio

design to cost

disbenefits

general obligation municipal bonds

government opportunity cost

incremental benefit–cost ratio

modified B/C ratio

net benefits to the users

project duration

project financing

project politics

promote the general welfare

revenue bonds

taxpayer opportunity cost

view-point

Federal, state, and local governments, port authorities, school districts, government agencies, and other public organizations make investment decisions. For these decision-making bodies, economic analysis is complicated by several factors that do not affect firms in the private sector. These factors include the overall purpose of investment, the viewpoint for analysis, and how to select the interest rate. Other factors include project financing sources, expected project duration, quantifying and valuing benefits and disbenefits, effects of politics, beneficiaries of investment, and the multipurpose nature of investments. The overall mission in the public sector is the same as that in the private sector—to make prudent investment decisions that promote the organization's overall objectives.

The primary economic measure used in the public sector is the benefit–cost (B/C) ratio, which was introduced in Chapter 9. This measure is calculated as a ratio of the equivalent worth of the project's benefits to the equivalent worth of the project's costs. If the B/C ratio is *greater than 1.0,* the project under evaluation is accepted; if not, it is rejected. The B/C ratio is used to evaluate both single investments and sets of mutually exclusive projects (where the incremental B/C ratio is used). The uncertainties of quantifying cash flows, long project lives, and low interest rates all tend to lessen the reliability of public sector engineering economic analysis. There are two versions of the B/C ratio: *conventional* and *modified*. Both provide consistent recommendations to decision makers for single investment decisions and for decisions involving sets of mutually exclusive alternatives. The B/C ratio is a widely used and accepted measure in government economic analysis and decision making.

INVESTMENT OBJECTIVE

Organizations exist to promote the overall goals of those they serve. In private sector firms, investment decisions are based on increasing the firm's wealth and economic stability. Beneficiaries of investments generally are clearly identified as the firm's owners and/or stockholders.

In the public sector, the purpose of investment decisions is sometimes ambiguous. In the United States of America, the Preamble to the Constitution establishes the overall theme:

We the People of the United States, in order to form a more perfect Union, establish justice, insure domestic tranquillity, provide for the common defense, *promote the general welfare* [italics supplied], and secure the blessings of liberty to ourselves and our posterity, do ordain and establish this Constitution for the United States of America.

The catch phrase **promote the general welfare** serves as a guideline for public decision making. But what does this phrase mean? At best it is a general guideline; at worst it is a vague slogan that can be used to justify any action. Projects some citizens want may be opposed by other citizens. In government economic analysis, it is not always easy to distinguish which investments promote the "general welfare" and which do not.

Consider the case of a dam construction project to provide water, electricity, flood control, and recreational facilities. Such a project might seem to be advantageous for a region's entire population. But on closer inspection, decision makers must consider that the dam will require the loss of land upstream due to backed-up water. Farmers will lose pastures or cropland, and nature lovers will lose canyon lands. Or perhaps the land to be lost is a pivotal breeding ground for protected species, and environmentalists will oppose the project. The project may also have a negative impact on towns, cities, and states downstream. How will it affect their water supply? Thus, a project initially deemed to have many benefits may also have many conflicts. Projects' conflicting aspects are characteristic of public-sector investment and decision making.

Public investment decisions are more difficult than those in the private sector owing to the many people, organizations, and political units potentially affected. Opposition to a proposal is more likely in public investment decisions than in those made by private-sector companies because for every group that benefits from a particular project, there is usually an opposing group. Many conflicts in opinion arise when the project involves the use of public lands, including industrial parks, housing projects, business districts, roadways, sewage and power facilities, and landfills. Opposition may be based on the belief that development of *any* kind is bad or that the proposed development should not be near "our" homes, schools, or businesses.

Consider the decision that a small town might face when considering whether to establish a municipal rose garden, seemingly a beneficial public investment with no adverse consequences. However, an economic analysis of the project must consider *all* effects of the project, including potential unforeseen outcomes. Where will visitors park their vehicles? Will increased travel around the park necessitate new traffic lights and signage? Will traffic and visitors to the park increase noise levels for adjacent homes? Will special varieties of roses create a disease hazard for local gardens? Will the garden require high levels of fertilizers and insecticides, and where will these substances wind up after they have been applied? Clearly, many issues must be addressed. Our simple rose garden illustrates how effects on *all parties involved* must be identified, even for projects that seem very useful.

The **Flood Control Act of 1936** specified that waterway improvements for flood control could be made as long as "the benefits *to whomsoever they accrue* [italics supplied] are in excess of the estimated costs." Perhaps the overall general objective of investment decision analysis in government should be a dual one: to promote the general welfare and to ensure that the value to those who can potentially benefit exceeds the overall costs to those who do not benefit.

VIEWPOINT FOR ANALYSIS

When governmental bodies do economic analysis, an important concern is the proper **view-point** of the analysis. A look at industry will help to explain how the viewpoint from which an analysis is conducted influences the final recommendation. A firm pays its costs and counts *its* benefits. Thus, both the costs and benefits are measured from the firm's perspective.

The discussion of internal and external costs in Chapter 2 demonstrated one approach to expanding the firm's perspective that allows some consideration of the external consequences depicted in Figure 16–1.

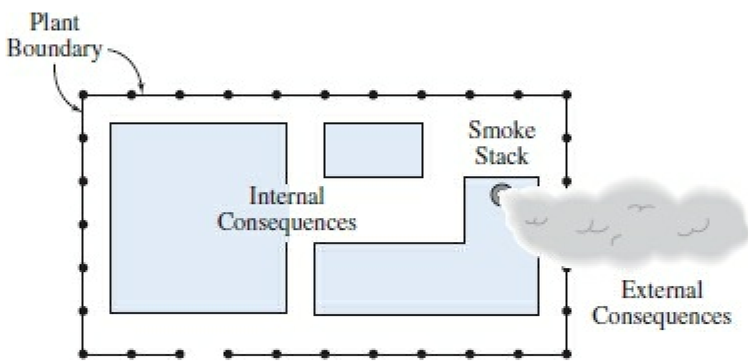


FIGURE 16-1 Internal and external consequences of an industrial plant.

The council members of a small town that levies taxes can be expected to take the "viewpoint of the town" in making decisions: unless it can be shown that the money from taxes can be used *effectively*, the town council is unlikely to spend it. But what happens when the money is contributed to the town by the federal government, as in "revenue sharing" or by means of some other federal grant? Often the federal government pays a share of project costs varying from 10 to 90%. Example 16–1 illustrates the viewpoint problem that is created.

EXAMPLE 16-1

A municipal project will cost $1 million. The federal government will pay 60% of the cost if the project is undertaken. Although the original economic analysis showed that the PW of benefits was $1.5 million, a subsequent detailed analysis by the city engineer indicates a more realistic estimate of the

PW of benefits is $750,000. The city council must decide whether to proceed with the project. What would you advise?

SOLUTION

From the viewpoint of the city, the project is still a good one. If the city puts up 40% of the cost ($400,000) it will receive all the benefits ($750,000). On the other hand, from an *overall* viewpoint, the revised estimate of $750,000 of benefits does not justify the $1 million expenditure. This illustrates the dilemma caused by varying viewpoints. For economic efficiency, one does not want to encourage the expenditure of money, regardless of the source, unless the benefits at least equal the costs.

Possible viewpoints that may be taken include those of an individual, a business firm or corporation, a town or city district, a city, a state, a nation, or a group of nations. The proper approach is to *take a viewpoint at least as broad as the larger of those who pay the costs and those who receive the benefits*. When the costs and benefits are totally confined to a town, for example, the town's viewpoint seems to be an appropriate basis for the analysis. But when the costs or the benefits are spread beyond the proposed viewpoint, then the viewpoint should be enlarged to this broader population.

Other than investments in defense and social programs, most of the benefits provided by government projects are realized at a regional or local level. Projects, such as dams for electricity, flood control, and recreation, as well as transportation facilities, such as roads, bridges, and harbors, all benefit most those in the region in which they are constructed. Even smaller-scale projects, such as the municipal rose garden, although funded by public monies at a local or state level, provide most benefit to those nearby. It is important to adopt an appropriate and *consistent* viewpoint and to designate all the costs and benefits that arise from the prospective investment from that perspective. To use different perspectives when quantifying costs and benefits could greatly skew the results of the analysis and subsequent decision, and thus different perspectives are inappropriate.

SELECTING AN INTEREST RATE

Several factors, not present for private-sector firms, influence selecting an interest rate for economic analysis in the government sector. Recall that for private-sector firms the overall objective is wealth maximization, and the interest rate is selected consistent with this goal. Most firms use *cost of capital* or *opportunity cost* concepts when setting an interest rate. How to set an interest rate is less clear-cut for public projects. Possibilities include no interest rate, cost of capital concepts, and an opportunity cost concept.

No Time-Value-of-Money Concept

In government, monies are obtained through taxation and spent about as quickly as they are obtained. Often, there is little time delay between collecting money from taxpayers and spending it. (Remember that the federal government and many states collect taxes every paycheck in the form of withholding tax.) The collection of taxes, like their disbursement, although based on an annual budget, is actually a continuous process. Using this line of reasoning, some would argue that there is little or no time lag between collecting and spending tax dollars. Thus, they would advocate the use of a 0% interest rate for economic analysis of public projects. Not surprisingly, this viewpoint is most often expressed by people who are *not* engineering economists and who are pushing a marginal project.

Cost of Capital Concept

Another approach in determining interest rates in public investments is that most levels of government (federal, state, and local) borrow money for capital expenditures in addition to collecting taxes. Where money is borrowed for a specific project, one line of reasoning is to use an interest rate equal to the *cost of borrowed money*. This argument is less valid for state and local governments than for private firms because the federal government, through the income tax laws, subsidizes state and local bonded debt. If a state, county, city, or special assessment district raises money by selling bonds, the interest paid on these bonds is exempt from federal taxes. In this way the federal government is *subsidizing* the debt, thereby encouraging investors to purchase such bonds.

These bonds, called municipal bonds, can be either general obligation or revenue bonds. **General obligation municipal bonds** pay interest and are retired (paid off) through taxes raised by the issuing government unit. A school district may use property taxes it receives to finance bond debt for construction of new language labs. **Revenue bonds** are not supported by the taxing authority of the government unit; rather, they are supported by revenues earned by the project being funded. As an example, a city could use toll revenues from a new bridge to retire debt on revenue bonds sold for the bridge's construction.

For those who purchase municipal bonds, the instruments' tax-free status means that the expected return on this investment is somewhat less than that required of fully taxed bond investments (of similar risk). As a rough estimate, when fully taxed bonds yield an 8% interest rate, municipal bonds might make interest payments at a rate of 6%. The difference of 2% is due to the preferred treatment for federal taxation and the federal subsidy on tax-free bonds. This can skew the *cost of capital* approach by lowering the apparent cost of long-term bonds. The true cost of the bonds should include the federal subsidy, which is paid by all taxpayers.

Opportunity Cost Concept

Opportunity cost, which is related to the interest rate on the best opportunity forgone, may be based on either the government's or the taxpayers' opportunity cost. The **government opportunity cost** is the interest rate of the best prospective project for which funding is not available. One disadvantage of the government opportunity cost concept is that different government agencies and subdivisions have different opportunities. Therefore, units could use different interest rates for economic analysis, and a project that is rejected in one branch might be accepted in another. Differing interest rates lead to inconsistent evaluation and decision making across government.

Dollars used for public investments are generally gathered by taxing the citizenry. The concept of **taxpayer opportunity cost** suggests that a correct interest rate for evaluating public investments is that which the *taxpayer* could have received if the government had not collected those dollars through taxation. This philosophy holds that through taxation the government is taking away the taxpayers’ opportunity to use the same dollars for investment. The interest rate that the government requires should not be less than what the taxpayer would have received. This concept argues that it is not economically desirable to take money from a taxpayer with a 12% opportunity cost, for example, and invest it in a government project yielding 4%.

The most widely followed standard is found in the Office of Management and Budget (OMB) A94 directive, which stipulates that a 7% interest rate be used in economic analysis for a wide range of federal projects.

Recommended Concept

The general rule of thumb in setting an interest rate for government investments has been to select the *largest* of the cost of capital, the government opportunity cost, or the taxpayer opportunity cost interest rates. However, as is the case in the private sector, there is no hard and fast rule, universally applied in all decision circumstances. Setting an interest rate for use in economic analysis is at the discretion of the government entity performing the analysis.

THE BENEFIT–COST RATIO

The **benefit–cost ratio** was described briefly in Chapter 9. This method is used almost exclusively in public investment analysis, and because of the magnitude of the amount of public dollars committed each year through such analysis, the benefit–cost ratio deserves our attention and understanding.

One of the primary reasons for using the benefit–cost ratio (B/C ratio) in public decision making is its simplicity. The ratio is formed by calculating the equivalent worth of the project’s benefits divided by the equivalent worth of the project’s costs. The benefit–cost ratio can be shown as follows:

$$\text{B/C ratio} = \frac{\text{Equivalent worth of net benefits}}{\text{Equivalent worth of costs}}$$

$$= \frac{\text{PW benefits}}{\text{PW costs}} = \frac{\text{FW benefits}}{\text{FW costs}} = \frac{\text{AW benefits}}{\text{AW costs}}$$

Notice that *any* of the equivalent worth methods (present, future, and annual) can be used to calculate this ratio. Each formulation of the ratio will produce an identical result, as illustrated in Example 16–2.

EXAMPLE 16-2

Demonstrate that for this highway expansion project, the same B/C ratio is obtained using the present, future, and annual worth formulations.

Initial costs of expansion	$1,500,000
Annual costs for operating/maintenance	65,000
Annual savings and benefits to travelers	225,000
Residual value of benefits after horizon	300,000
Useful life of investment	30 years
Interest rate	8%

SOLUTION

Using Present Worth

$$\text{PW benefits} = 225{,}000(P/A, 8\%, 30) + 300{,}000(P/F, 8\%, 30) = \$2{,}563{,}000$$

$$\text{PW costs} = 1{,}500{,}000 + 65{,}000(P/A, 8\%, 30) = \$2{,}232{,}000$$

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	change sign
2	Benefits	8.00%	30	225,000		300,000	PV	-$2,562,814	$2,562,814

3	Costs	8.00%	30	65,000	0	PV	-$731,756	$731,756
4					1,500,000		Total costs	$2,231,756
5							B/C ratio	1.15

Using Future Worth

$$\text{FW benefits} = 225{,}000(F/A, 8\%, 30) + 300{,}000 = \$25{,}790{,}000$$

$$\text{FW costs} = 1{,}500{,}000(F/P, 8\%, 30) + 65{,}000(F/A, 8\%, 30) = \$22{,}460{,}000$$

Using Annual Worth

$$\text{AW benefits} = 225{,}000 + 300{,}000(A/F, 8\%, 30) = \$227{,}600$$

$$\text{AW costs} = 1{,}500{,}000(A/P, 8\%, 30) + 65{,}000 = \$198{,}200$$

$$\text{B/C ratio} = \frac{2{,}563{,}000}{2{,}232{,}000} = \frac{25{,}790{,}000}{22{,}460{,}000} = \frac{227{,}600}{198{,}200} = 1.15$$

One can see that the ratio provided by each of these methods produces the same result: 1.15.

When one is using the B/C ratio, the decision rule is:

If the B/C ratio is > 1.0, then invest.

If the B/C ratio is < 1.0, then do not invest.

Cases of a B/C ratio equal to 1.0 are analogous to the case of a calculated net present worth of $0 or an IRR analysis that yields *i* = MARR%. In other words, the decision measure is at the breakeven criteria. In such cases a detailed analysis of the input variables and their estimates is necessary, and one should consider the merits of other available opportunities for the targeted funds. But, if the B/C ratio is clearly greater than or less than 1.0, the recommendation is clear.

The B/C ratio is a numerator/denominator relationship between the equivalent worths (EW) of *benefits* and *costs:*

$$\text{B/C ratio} = \frac{\text{EW of net benefits to whomsoever they may accrue}}{\text{EW of costs to the sponsors of the project}}$$

The numerator and denominator aspects of the ratio are sometimes interpreted and used in different fashions. For instance, the **conventional B/C ratio** (see Example 16–2) can be restated as follows:

$$\text{Conventional B/C ratio} = \frac{\text{EW of net benefits}}{\text{EW of initial costs} + \text{EW of operating and maintenance costs}}$$

However, there is also the **modified B/C ratio**, which subtracts the *annual operating and maintenance costs* in the numerator, rather than adding them as a cost in the denominator. This *modified B/C ratio* is mathematically similar to the present worth index defined in Chapter 9. The ratio becomes

$$\text{Modified B/C ratio} = \frac{\text{EW of net benefits} - \text{EW of operating and maintenance costs}}{\text{EW of initial costs}}$$

For decision making, the two versions of the benefit–cost ratio will produce the same recommendation on whether to *invest* or *not invest* in the project being considered. The *numeric values for the B/C ratio* will not be the same, but the recommendation will be. This is illustrated in Example 16–3.

EXAMPLE 16-3 Example 16–2 Revisited

Consider the highway expansion project from Example 16–2. Let us use the present worth formulation of *conventional* and *modified* versions to calculate the B/C ratio.

SOLUTION

Using the Conventional B/C Ratio

$$\text{B/C ratio} = \frac{225{,}000(P/A, 8\%, 30) + 300{,}000(P/F, 8\%, 30)}{1{,}500{,}000 + 65{,}000(P/A, 8\%, 30)} = 1.15$$

Using the Modified B/C Ratio

$$\text{B/C ratio} = \frac{225{,}000(P/A, 8\%, 30) + 300{,}000(P/F, 8\%, 30) - 65{,}000(P/A, 8\%, 30)}{1{,}500{,}000} = 1.22$$

We could calculate these same values using a spreadsheet, as in Figure 16–2. Figure 16–2 replicates these results with a 5-button solution.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Conventional							
3	Benefits	8.00%	30	225,000		300,000	PV	$2,562,814
4	Costs	8.00%	30	65,000		0	PV	$731,756
5					1,500,000		Total cost	$2,231,756
6							B/C ratio	1.15
7	Modified							
8	Benefits	8.00%	30	160,000		300,000	PV	$1,831,059
9	Costs	8.00%	30		1,500,000		Total cost	$1,500,000
10							B/C ratio	1.22

FIGURE 16-2

Whether the conventional or the modified ratio is used, the recommendation is to invest in the highway expansion project. The ratios are not identical in magnitude (1.15 vs. 1.22), but the decision is the same.

It is important when one is using the conventional and modified B/C ratios not to directly compare the magnitudes of the two versions. Evaluating a project with one version may produce a higher ratio than is produced with the other version, but this does not imply that the project is somehow better.

The **net benefits to the users** of government projects are the difference between the expected *benefits* from investment minus the expected *disbenefits*. Disbenefits are the negative effects of government projects felt by some individuals or groups. For example, consider the U.S. National Park System. Development projects by the skiing or lumber industries might provide enormous benefits to the recreation or construction sectors while creating simultaneous disbenefits for environmental groups.

INCREMENTAL BENEFIT–COST ANALYSIS

In Chapter 9 we demonstrated using the **incremental benefit–cost ratio** in economic decision analysis. As with the internal rate of return (IRR) measure, the incremental B/C ratio should be used in comparing *mutually exclusive alternatives.* Incremental B/C ratio analysis is consistent with maximizing the present worth of the alternatives. As with the incremental IRR method, it is *not* proper to simply calculate the B/C ratio for each alternative and choose the one with the highest value. Rather, an *incremental* approach is called for.

Elements of the Incremental Benefit–Cost Ratio Method

1. *Identify all relevant alternatives.* Decision rules or models can recommend a *best* course of action *only* from the set of identified alternatives. If a better alternative exists but is not considered, then it will never be selected, and when available, the solution will be suboptimal. For benefit–cost ratio problems, the “do-nothing” option is always the “base case” from which the incremental methodology proceeds.

2. *(Optional) Calculate the B/C ratio of each alternative*. Once the individual B/C ratios have been calculated, the alternatives with a ratio *less than 1.0* are eliminated from further consideration. This step gets the “poor performers” out of the way before the incremental procedure is initiated. This step may be omitted, however, because the incremental analysis method will eliminate the subpar alternatives in due time.

Note: There is a case where this step *must* be skipped. If doing nothing is not an alternative and if *all* alternatives have a B/C ratio less than 1.0, then the best of the unattractive alternatives will be selected through incremental analysis.

3. *Rank-order the projects.* The alternatives must be ordered from smallest to largest size of the *denominator of the B/C ratio.* (The rank order will be the same regardless of whether one uses the present worth, annual worth, or future worth of costs to calculate the *denominator.*) When available, the “do-nothing” alternative always becomes the first on the ordered list.

4. *Identify the increment under consideration.* The first increment considered is always that of going from the lowest cost alternative (when it is available, this is the do-nothing option) to the next higher cost alternative. As the analysis proceeds, any identified increment is always in reference to some previously justified alternative.

5. *Calculate the B/C ratio for the incremental cash flows.* First, calculate the *incremental benefits* and the *incremental costs.* This is done by finding the cash flows that represent the difference (Δ) between the two alternatives under consideration. For two alternatives *X* and *Y*, where *X* is the defender, or base case, and *Y* is the challenger, the increment can be written as ($X \rightarrow Y$) to signify *going from X to Y* or as (*Y* − *X*) to signify the *cash flows of Y* ***minus*** *cash flows of X*. Both modes identify the incremental costs (ΔC) and benefits (ΔB) of investing in Alternative *Y*, where *X* is a previously justified (or base) alternative. The *incremental B/C ratio* equals ΔB/ΔC.

6. *Use the incremental B/C ratio to decide which alternative is better*. If the incremental B/C ratio (ΔB/ΔC) calculated in Step 5 is greater than 1.0, then the increment is desirable or justified; if the ratio is less than 1.0, it is not desirable, or is not justified. If an increment is accepted, the alternative (the challenger) associated with that increment of investment becomes the base from which the next increment is formed. When the increment is not justified, the alternative associated with the additional increment is rejected and the previously justified alternative (the defender) is maintained as the base for formation of the next increment.

7. *Iterate to Step 4 until all increments (projects) have been considered.* The incremental method requires that the entire list of ranked feasible alternatives be evaluated. All pairwise comparisons are made such that the additional increment being considered is examined with respect to a previously justified alternative. The incremental method continues until all alternatives have been evaluated.

8. *Select the best alternative from the set of mutually exclusive competing projects.* After all alternatives (and associated increments) have been considered, the incremental B/C ratio method calls for selecting the alternative *associated with the last justified increment.* This assures that a maximum investment is made such that each ratio of equivalent worth of incremental benefits to equivalent worth of incremental costs is greater than 1.0. (A common error in applying the incremental B/C method is selecting the alternative with the *largest* incremental B/C ratio, which is inconsistent with the objective of maximizing investment size with incremental B/C ratios above 1.0.)

Both the conventional and modified versions of the B/C ratio can be used with the incremental B/C ratio methodology just described, but the two versions should not be mixed in the same problem. Such an approach could cause confusion and errors. Instead, *one* of the two versions should be *consistently* used throughout the analysis.

In Examples 16–4 and 16–5 the incremental B/C ratio (conventional and modified version) is used to evaluate a set of mutually exclusive alternatives.

EXAMPLE 16-4

A midwestern industrial state may construct and operate two coal-burning power plants and a distribution network to provide electricity to several state-owned properties. The following costs and benefits have been identified.

Primary costs: Construction of the power plant facilities; cost of installing the power distribution network; life-cycle maintenance and operating costs.
Primary benefits: Elimination of payments to the current electricity provider; creation of jobs for construction, operation, and maintenance of the facilities and distribution network; revenue from selling excess power to utility companies; increased employment at coal mines in the state.

There have been four competing designs identified for the power plants. Each has a life of 45 years. Use the *conventional* B/C ratio with an interest rate of 8% to recommend a course of action.

	Values (× $\$10^4$) for Competing Design Alternatives			
	I	II	III	IV

Project costs				
Plant construction cost	$12,500	$11,000	$12,500	$16,800
Annual operating and maintenance cost	120	480	450	145
Project benefits				
Annual savings from utility payments	580	700	950	1,300
Revenue from overcapacity	700	550	200	250
Annual effect of jobs created	400	750	150	500

SOLUTION

Alternatives I through IV and the do-nothing alternative are *mutually exclusive* choices because one and only one will be selected. Therefore, an incremental B/C ratio method is used to obtain the solution.

Step 1 *Identify alternatives*. The alternatives are do nothing and designs I, II, III, and IV.
Step 2 *Calculate B/C ratio for each alternative*.

B/C ratio (I) =(580 + 700 + 400) (P/A,8%,45)/[12,500 + 120 (P/A,8%,45)] = 1.46

B/C ratio (II) =(700 + 550 + 750) (P/A,8%,45)/[11,000 + 480 (P/A,8%,45)] = 1.44

B/C ratio (III) =(200 + 950 + 150) (P/A,8%,45)/[12,500 + 325 (P/A,8%,45)] = 0.96

B/C ratio (IV) =(1300 + 250 + 500)(P/A,8%,45)/[16,800 + 145(P/A,8%,45)] = 1.34

Alternatives I, II, and IV all have B/C ratios greater than 1.0 and thus merit further consideration. Alternative III does not meet the acceptability criterion and could be eliminated from further consideration. However, to illustrate that Step 2 is optional, all four design alternatives will be analyzed incrementally.

Step 3 *Rank-order projects*. Here we calculate the PW of costs for each alternative. The denominator of the *conventional* B/C ratio includes first cost and annual O&M costs, so the PW of costs for the alternatives are:

$$\text{PW costs (I)} = 12{,}500 + 120(P/A, 8\%, 45) = \$13{,}953$$
$$\text{PW costs (III)} = 12{,}500 + 325(P/A, 8\%, 45) = \$16{,}435$$
$$\text{PW costs (II)} = 11{,}000 + 480(P/A, 8\%, 45) = \$16{,}812$$
$$\text{PW costs (IV)} = 16{,}800 + 145(P/A, 8\%, 45) = \$18{,}556$$

The rank order from low to high value of the B/C ratio *denominator* is as follows: do nothing, I, III, II, IV.

Step 4 *Identify increment under consideration.*

Step 5 *Calculate B/C ratio.*

Step 6 *Which alternative is better?*

		1st Iteration	2nd Iteration	3rd Iteration	4th Iteration
Step 4	Increment Under Consideration	(Do Nothing → I)	(I → III)	(I → II)	(II → IV)
	ΔPlant construction cost	\$12,500	\$ 0	\$−1500	\$5800
	ΔAnnual O&M cost	120	205	360	−335

	PW of ΔCosts	13,953	2482	2859	1744
	ΔAnnual utility payment savings	580	370	120	600
	ΔAnnual overcapacity revenue	700	500	−150	−300
	ΔAnnual benefits of new jobs	400	−250	350	−250
	PW of ΔBenefits	20,342	−4601	3875	605
Step 5	ΔB/C ratio (PW ΔB)/(PW ΔC)	1.46	−1.15	1.36	0.35
Step 6	Is increment justified?	Yes	No	Yes	No

As an example of these calculations, consider the third increment (I $\rightarrow$ II).

ΔPlant construction cost	$= 11,000 - 12,500 = -\$1500$
ΔAnnual O&M cost	$= 480 - 120 = \$360$
PW of ΔCosts	$= -1500 + 360(P/A, 8\%, 45) = \$2859$
or	$= 16,812 - 13,953 = \$2859$
ΔAnnual utility payment savings	$= 700 - 580 = \$120$
ΔAnnual overcapacity revenue	$= 550 - 700 = -\$150$
ΔAnnual benefits of new jobs	$= 750 - 400 = \$350$
PW of ΔBenefits	$= (120 - 150 + 350)(P/A, 8\%, 45)$

	= $3875
ΔB/C ratio (PW ΔB)/(PW ΔC)	= 3875/2859 = 1.36

The analysis in the table proceeded as follows: do nothing to Alternative I was justified (Δ*B*/*C* ratio = 1.46), Alternative I became the new base; Alternative I to Alternative III was not justified (ΔB/C ratio = −1.15), Alternative I remained base; Alternative I to Alternative II was justified (ΔB/C ratio = 1.36), Alternative II became the base; Alternative II to Alternative IV was not justified (ΔB/C ratio = 0.35).

Step 7 *Select best alternative*. Alternative II became the recommended power plant design alternative because it is the one associated with the last justified increment. Notice that Alternative III did not affect the recommendation and was eliminated through the incremental method. Notice also that the first increment considered (do nothing → I) was not selected even though it had the *largest* ΔB/C ratio (1.45). The alternative associated with the *last justified increment* (in this case, Alt. II) should be selected.

EXAMPLE 16-5 Example 16–4 Revisited

Let us reconsider Example16–4, this time using the modified B/C ratio to analyze the alternatives. Again we will use the present worth method.

SOLUTION

Here we use the modified B/C ratio.

Step 1 *Identify alternatives*. The alternatives are still do nothing and designs I, II, III, and IV.

Step 2 *Calculate modified B/C ratio for each alternative*.B/C ratio (I) =(580 + 700 + 400 -120) (P/A, 8%, 45)/12,500 = 1.51B/C ratio (II) =(700 + 550 + 750 - 480) (P/A, 8%, 45)/11,000 = 1.67B/C ratio (III) =(200 + 950 + 150 - 325) (P/A, 8%, 45)/12,500 = 0.95B/C ratio (IV) =(1300 + 250 + 500 - 145) (P/A, 8%, 45)/16,800 = 1.37

Again Alternative III can be eliminated from further consideration because its

B/C ratio is *less than 1.0*. In this case we will eliminate it. The remaining alternatives are do nothing and alternative designs I, II, and IV.
Step 3 *Rank-order projects*. The PW of costs for each alternative:

PW Costs (I) = \$12,500
PW Costs (II) = \$11,000
PW Costs (IV) = \$16,800

The correct rank order is now do nothing, II, I, IV. Notice that the *modified* B/C ratio produces an order of comparison different from that yielded by the *conventional* version in Example 16–4.
Step 4 *Identify increment being considered*.
Step 5 *Calculate B/C ratio*.
Step 6 *Which alternative is better?*

		1st Iteration	2nd Iteration	3rd Iteration
Step 4	Incremental Effects	(Do Nothing → II)	(II → I)	(II → IV)
	ΔPlant construction cost	\$11,000	\$1500	\$5800
	PW of ΔCosts	11,000	1500	5800
	ΔAnnual utility payment savings	700	−120	600
	ΔAnnual overcapacity revenue	550	150	−300
	ΔAnnual benefits of new jobs	750	−350	−250

	ΔAnnua1 O&M disbenefit	480	−360	−335
	PW of ΔBenefits	18,405	484	4662
Step 5	ΔB/C ratio (PW ΔB)/(PW ΔC)	1.67	0.32	0.80
Step 6	Is increment justified?	Yes	No	No

As an example of the calculations in the foregoing table, consider the third increment (II → IV).

ΔPlant construction cost	= 16, 800 − 11, 000 = *$*5800
PW of ΔCosts	= *$*5800
ΔAnnual utility payment savings	= 1300 − 700 = *$*600
ΔAnnual overcapacity revenue	= 250 − 550 = −*$*300
ΔAnnual benefits of new jobs	= 500 − 750 = −*$*250
ΔAnnual O&M disbenefit	= 145 − 480 = −*$*335
PW of ΔBenefits	= (600 − 300 − 250 + 335)(P/A, 8*%*, 45) = *$*4662
ΔB/C ratio, (PW ΔB)/(PW ΔC)	= 4662/5800 = 0.80

When the modified version of the B/C ratio is used, Alt. II emerges as the recommended power plant design—just as it did when we used the conventional B/C ratio.

NOTE: A useful exercise for the student would be to develop a spreadsheet that can be used to solve this problem. Try it out!

OTHER EFFECTS OF PUBLIC PROJECTS

Four areas remain that merit discussion in describing the differences between government and nongovernment economic analysis: (1) financing government versus nongovernment projects, (2) the typical length of government versus nongovernment project lives, (3) quantifying and valuing benefits and disbenefits, and (4) the general effects of politics on economic analysis.

Project Financing

Governmental organizations and market-driven firms differ in the way investments in equipment, facilities, and other projects are financed. In general, firms rely on monies from investors (through stock and bond issuance), private lenders, and retained earnings from operations. These sources serve as the pool from which investment dollars for projects come. Management's job is to match financial resources with projects in a way that keeps the firm growing, results in an efficient and productive environment, and continues to attract investors and future lenders of capital.

On the other hand, the government sector often uses taxation and municipal bond issuance as the source of investment capital. In government, taxation and revenue from operations are adequate to finance only modest projects. However, public projects tend to be large in scale (roadways, bridges, etc.), which means that for many public projects 100% of the investment costs must be borrowed. To prevent excessive public borrowing and to assure timely debt repayment, the U.S. government, through constitutional and legislative channels, has restricted government debt. These restrictions include:

1. Local government bodies are limited in their borrowing to a specified percentage of the assessed property value in their taxation district.
2. For new construction, borrowed funds attained through the sale of bonds require the approval of local voters (sometimes by a two-thirds majority). For

example, a $20 million bond proposition for a new municipal jail might increase property taxes in the city's tax district by $1.50 for every $1000 of assessed property value. These added tax revenues would then be used to retire the debt on the bonds.
3. Repayment of public debt must be made following a specific plan over a preset period of time. For monies borrowed by issuing bonds, interest payments and maturity dates are set at the time of issuance.

Limitations on the use and sources of borrowed monies make funding public sector projects much different from in the private sector. Private sector firms are seldom able to borrow 100% of required funds for projects, as can be done in the public sector, but at the same time, private entities do not face restrictions on debt retirement or the uncertainty of voter approval. Passing the bond proposition is the public *go-ahead*.

When projects are financed by bonds, it may become very difficult to shift funds between projects or to add funds for a project. Thus public agencies sometimes use a **design to cost** methodology, rather than minimizing life-cycle costs (the approach that has been used throughout the text). The goal is to design the best possible school, road, bridge, and so on that can be built without exceeding the fixed budgeted cost.

Project Duration

Government projects often have longer lives than those in the private sector. In the private sector, projects most often have a projected or intended life ranging between 5 and 15 years. Some markets and technologies change more rapidly and some more slowly, but a majority of projects fall in this interval. Complex advanced manufacturing technologies, like computer-aided manufacturing or flexible automated manufacturing cells, typically have project lives at the longer end of this range.

Government projects typically have lives in the range of 20 to 50 years (or longer). Typical projects include federal highways, city water/sewer infrastructure, county dumps, and state libraries and museums. These projects, by nature, have a longer useful life than a typical private-sector project. There are exceptions to this rule because private firms invest in facilities and other long-range projects, and government entities also invest in projects with shorter-term lives. But, in general, investment duration in the government sector is longer.

Government projects, because they tend to be long range and large scale, usually require substantial funding in the early stages. Highway, water/sewer, and dam projects can require millions of dollars in design, surveying, and construction costs. Therefore, it is in the best interest of decision makers who are advocates of such projects to spread that first cost over as many years as possible to reduce the annual cost of capital recovery. Using longer project lives to downplay the effects of a large first cost increases the desirability of the project, as measured by the B/C ratio. Another aspect closely associated with managing the size of the capital recovery cost in a B/C ratio analysis is the interest rate used for discounting. Lower interest rates reduce the capital recovery cost of having money tied up in a project. Example 16–6 illustrates the effects that project life and interest rate can have on the analysis and acceptability of a project.

EXAMPLE 16-6

Consider a project that has been approved by local voters to build a new junior high school, needed because of increased (and projected) population growth. Analyze the project with interest rates of 3, 10, and 15% and with horizons of 15, 30, and 60 years.

Building first costs (design, planning, and construction)	$10,000,000
Initial cost for roadway and parking facilities	5,500,000
First cost to equip and furnish facility	500,000
Annual operating and maintenance costs	350,000
Annual savings from rented space	400,000
Annual benefits to community	1,600,000

SOLUTION

With this project we examine the effect that varying project lives and interest rates have on the economic value of a public project. In each case, the formula is

$$\text{Conventional B/C ratio} = \frac{1{,}600{,}000 + 400{,}000}{(10{,}000 + 5{,}500{,}000 + 500{,}000)(A/P, i, n) + 350{,}000}$$

The B/C ratio for each combination of project life and interest rate is tabulated as follows:

Conventional Benefit–Cost Ratio for Various Combinations of Project Life and Interest Rate

	Interest		
Project Life (years)	3%	10%	15%
15	1.24	0.86	0.69
30	1.79	1.03	0.76
60	2.24	1.08	0.77
	Build ⟵		⟶ Do not build

From these numbers one can see the effect of project life and interest on the analysis and recommendation. At the lower interest rate, the project has B/C ratios above 1.0 in all cases of project life, while at the higher rate the ratios are all less than 1.0. At an interest rate of 10% the recommendation to invest changes from *no* at a life of 15 years to *yes* at 30 and 60 years. A higher interest rate discounts the benefits in later years more heavily, so that they may not matter. In this case, the benefits from Years 31 to 60 add only 0.01 to the B/C ratio at 15% and 0.05 at 10%. At 3% those benefits add 0.45 to the ratio. By manipulating these two parameters (project life and interest rate), it is possible to reach entirely different conclusions regarding the desirability of the project.

EXAMPLE 16-7 Example 16–6 Revisited

Looking at the data in Example 16–6, develop a spreadsheet table and graph that illustrates changes to the B/C ratio as interest rate changes between 1% and 15% and project life varies from 5 to 60 years.

SOLUTION

Notice in Figure 16–3 the rightmost curve where values to the right and below represent a B/C ratio less than 1 and those to the left and above greater than 1. This line, called the efficient frontier, can be used to show decision makers the combinations of interest rate and project life that lead to a barely acceptable B/C ratio of 1. This figure provides much more information than the simple results table in Example 16–6. For instance, regardless of project life, the junior high school is not recommended if interest is above ~10.5%. Also, if the life of the school is less than 10 years, the B/C ratio is not favorable.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	$10,000	building first cost											
2	5,500	initial roadway cost											
3	500	first cost to equip											
4	350	annual costs											
5	400	annual savings											
6	1,600	annual benefit				Conventional B/C Ratio							
7						Project life (years)							
8	Interest	5	10	15	20	25	30	35	40	45	50	55	60
9	1%	0.50	0.98	1.43	1.86	2.27	2.66	3.03	3.39	3.72	4.04	4.35	4.64
10	2%	0.49	0.93	1.33	1.69	2.01	2.31	2.58	2.82	3.04	3.24	3.42	3.58
11	3%	0.47	0.88	1.23	1.53	1.80	2.02	2.22	2.38	2.53	2.65	2.76	2.85
12	4%	0.46	0.84	1.15	1.40	1.61	1.78	1.92	2.04	2.14	2.22	2.28	2.33
13	5%	0.45	0.80	1.07	1.29	1.45	1.59	1.69	1.77	1.83	1.88	1.92	1.95
14	6%	0.43	0.76	1.00	1.18	1.32	1.42	1.50	1.55	1.59	1.63	1.65	1.67
15	7%	0.42	0.72	0.94	1.09	1.20	1.28	1.34	1.37	1.40	1.42	1.44	1.45
16	8%	0.41	0.69	0.88	1.01	1.10	1.16	1.20	1.23	1.25	1.26	1.27	1.28
17	9%	0.40	0.66	0.83	0.94	1.01	1.06	1.09	1.11	1.12	1.13	1.14	1.14
18	10%	0.39	0.63	0.78	0.88	0.94	0.97	0.99	1.01	1.02	1.02	1.03	1.03
19	11%	0.38	0.61	0.74	0.82	0.87	0.90	0.91	0.92	0.93	0.93	0.93	0.94
20	12%	0.37	0.58	0.70	0.77	0.81	0.83	0.84	0.85	0.85	0.86	0.86	0.86
21	13%	0.36	0.56	0.67	0.72	0.76	0.77	0.78	0.79	0.79	0.79	0.79	0.79
22	14%	0.35	0.54	0.63	0.68	0.71	0.72	0.73	0.73	0.73	0.74	0.74	0.74

=-PV($A9,B$8,($A$6+$A$5-$A$4))/($A$3+$A$2+$A$1)
all others copied from this reference cell

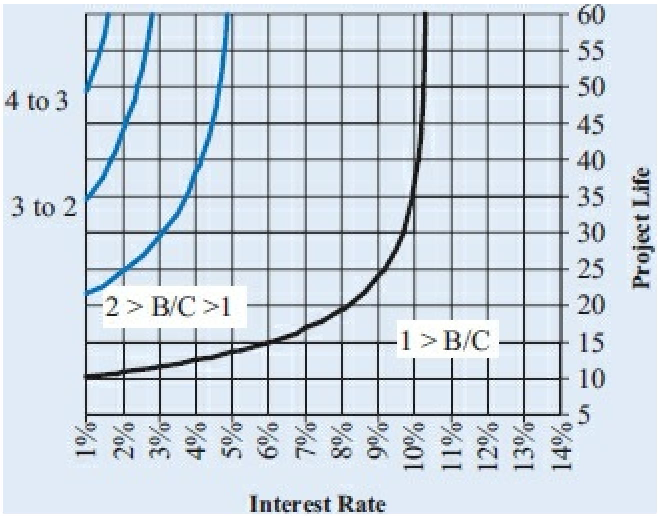


FIGURE 16-3 B/C ratios graph for Exp. 16–6

Examples 16–6 and 16–7 demonstrate that we must ensure that a long life and a low interest rate for a public project are truly appropriate and not chosen solely to make a marginal project look better.

Quantifying and Valuing Benefits and Disbenefits

The junior high school in Example 16–6 included annual benefits to the community of $1.6 million. If you were evaluating the school, how would you estimate this? Many public sector projects like the junior high school and the examples in Table 16–1 have consequences that are difficult to state in monetary terms. These include both project benefits and **disbenefits** (negative aspects of the project).

Table 16-1 Example Benefits and Disbenefits for Public Investments

Public Project	Primary Benefits	Primary Disbenefits
New airport outside city	More flights, new businesses	Increased travel time to airport, more traffic on outer belt
Interstate bypass	Quicker commute times, reduced	Lost sales to businesses on surface roads, lost

around town	congestion on surface roads	agricultural lands
New metro subway system	Faster commute times, less pollution	Lost jobs due to bus line closing, less access to service (fewer stops)
Creation of a city waste disposal facility versus sending waste out of state	Less costly; faster and more responsive to customers	Objectionable sight and smells, lost market value to homeowners, lost forestland
Construction of a nuclear power plant	Cheaper energy costs, new industry in area	Environmental risks

First the number of people affected by the project have to be counted—now and through the project's horizon. Then a dollar value for each person is required. For the junior high school it may be easy to estimate the number of students. But how much better will the educational outcomes be with the new school, and how valuable is that improvement?

On the other hand, consider the levees around New Orleans that needed to be rebuilt in the aftermath of Katrina. An economic evaluation of the different alternatives required estimating the number of residents that would be protected by improving the levees in New Orleans, which is extremely difficult. This required estimating not only how the rebuilding of the city might progress but also storm surges from future hurricanes, whose frequency and intensity may be changing. Once the number of people and homes had been estimated over the next 30 to 100 years, it was necessary to put a value on property, on disrupted lives, and on human lives.

While many individuals find it difficult to put a dollar value on a human life, there are many public projects whose main intent is to reduce the number of deaths due to floods, cancer, auto accidents, and other causes. Those projects are often justified by the value of preventing deaths. Thus, valuing human lives is an inescapable part of public-sector engineering economy.

Because the benefits and disbenefits of public projects are often difficult to quantify and value, the estimated values will have more uncertainty than is typical for private-sector projects. Thus those who favor a project and those who oppose it will often push to have values used that support their position.

Project Politics

To some degree political influences are felt in nearly every decision made in any organization. Predictably, some individual or group will support particular interests over competing views. This actuality exists in both firms and government organizations. In government the effects of politics are continuously felt at all levels because of the large-scale and multipurpose nature of projects, because government decision making involves the use of the citizenry's common pool of money, and because individuals and groups have different values and views. For example, how important is economic development relative to environmental protection?

The guideline for public decision making, as set forward in the Preamble to the Constitution, is to *promote the general welfare* of citizens. However, it is impossible to please everyone all the time. The term "general welfare" implies that the architects of this document understood that the political process would produce opposing viewpoints, but at the same time they empowered decision makers to act in a representative way.

Since government projects tend to be large in scale, the time required to plan, design, fund, and construct them is usually several years. However, the political process tends to produce government leaders who support short-term decision making (because many government terms of office, either elected or appointed, are relatively short). Therein lies another difference between firm and government decision making—short-term decision making, long-term projects.

Because government decision makers are in the public eye more than those in the private sector, governmental decisions are generally more affected by "politics." Thus, the decisions that public officials make may not always be the best from an *overall* perspective. If a particular situation exposes a public official to ridicule, he may choose an expedient action to eliminate negative exposure (whereas a more careful analysis might have been better). Or, such a decision maker may placate a small, but vocal, political group over the interest of the majority of citizens by committing funds to a favored project (at the expense of other better projects). Or, a public decision maker may avoid controversy by declining to make a decision on an important, but politically charged, issue (whereas it would be in the overall interest of the citizenry if action were taken). Indeed, the role of politics in government decision making is more complex and far ranging than in the private sector.

EXAMPLE 16-8 Example 16–4 Revisited

Consider again Example16–4, where we evaluated designs for two coal-burning power plants. What social, political, environmental, regulatory, or business considerations might come into play as the designed plants move toward construction and operation?

Solution

Remember that government projects are often opposed and supported by different groups in the populace. Thus, decision makers become very aware of potential political aspects when they are considering such projects.

For the electric power plant decision, several political considerations may affect any evaluation of funding this project.

- The governor has been a strong advocate of workers' rights and has received abundant campaign support from organized labor (which is especially important in an industrialized state). By championing this project, the governor should be seen as pro-labor, thereby benefiting his bid for reelection, even if the project is not funded.
- The regulated electric utility providers in the state are strongly against this project, claiming that it would directly compete with them and take away some of their biggest customers. The providers have a strong lobby and key contacts with the state's utilities commission. A senior state senator has already protested that this project is the first step toward "rampant socialism in this great state."

• Business leaders in the municipalities where the two facilities would be constructed favor the project because it would create more jobs and increase the tax base. These leaders promote the project as a win–win opportunity for government and industry, where the state can benefit by reducing costs, and the electric utilities can improve their service by focusing more effectively on residential customers and their needs.
• The lieutenant governor is promoting this project, proclaiming that it is an excellent example of “initiating proactive and creative solutions to the problems that this state faces.”
• Federal and state regulatory agencies are closely watching this project with respect to the Clean Air Act. Speculation is that the state plans to use a high-sulfur grade of local coal exclusively. Thus “stack scrubbers” would be required, or the high-sulfur coal would have to be mixed with lower-sulfur coal imported from other states to bring the overall air emissions in line with federal standards. The governor is using this opportunity to make the point that “the people of this state don’t need regulators to tell us if we can use our own coal!”
• The state’s coal operators and mining unions strongly support this project. They see the increased demands for coal and the governor’s pro-labor advocacy as very positive. They plan to lobby the legislature strongly in favor of the project.
• Land preservation and environmental groups are strongly opposing the proposed project. They have studied the potential negative impacts of this project on the land and on water and air quality, as well as on the ecosystem and wildlife, in the areas where the two facilities would be constructed. Environmentalists have started a public awareness campaign urging the governor to act as the “chief steward” of the state’s natural beauty and resources.

Will the project be funded? We can only guess. Clearly, however, we can see the competing influences that can be, and often are, part of decision making in the public sector.

SUMMARY

Economic analysis and decision making in government is notably different from these processes in the private sector because the basic objectives of the public and private sectors are fundamentally different. Government investments in projects seek to maximize benefits to the *greatest number of citizens,* while minimizing the *disbenefits to citizens* and *costs to the government*. Private firms, on the other hand, are focused primarily on maximizing stockholder wealth.

Several factors, not affecting private firms, enter into the decision-making process in government. The source of capital for public projects is limited primarily to taxes and bonds. Government bonds issued for project construction are subject to legislative restrictions on debt not required for private firms. Also, raising tax and bond monies involves sometimes long and politically charged processes not present in the private sector. In addition, government projects tend to be larger in scale than the projects of competitive firms and the government projects affect many more people and groups in the population. The benefits and disbenefits to the many people affected are difficult to quantify and value, which is unlike the private sector, where products and services are sold and the revenue to the firm is clearly defined. All these factors slow down the process and make investment decision analysis more difficult for government decision makers than for those in the private sector. Another difference between the public and private sectors lies in how the interest rate (MARR) is set for economic studies. In the private sector, considerations for setting the rate include the cost of capital and opportunity costs. In government, establishing the interest rate is complicated by uncertainty in specifying the cost of capital and the issue of assigning opportunity costs to taxpayers or to the government.

The benefit–cost ratio is widely used to evaluate and justify government-funded projects. This measure of merit is the ratio of the equivalent worth of benefits to the equivalent worth of costs. This ratio can be calculated by PW, AW, or FW methods. A B/C ratio *greater than 1.0* indicates that a project should be invested in if funding sources are available. For considering *mutually exclusive alternatives*, an incremental analysis is required. This method results in the recommendation of the project with the highest investment cost that can be incrementally justified. Two versions of the B/C ratio, the *conventional* and *modified* B/C ratios, produce identical recommendations. The conventional B/C ratio treats annual operating and maintenance costs as a cost in the denominator, while the modified B/C ratio subtracts those costs from the benefits in the numerator.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

16-1 A city's park department has a capital budget of $600,000. What is the government's opportunity cost of capital and which projects should be done?

Project	First Cost	Return	B/C
A	$300,000	23%	1.45
B	$100,000	22%	1.50
C	$100,000	17%	1.40
D	$200,000	19%	1.23
E	$100,000	18%	1.26

SOLUTION

16-2 Tumbleweed Junction bought new traffic enforcement equipment for $18,000. It will annually generate revenues of $25,000 and expenses of $15,000 for 5 years. There is no salvage value. Compute the benefit/cost ratio if i = 10%.

SOLUTION

16-3 Froggy University may buy a new garbage incinerator. The "best" alternative costs $55,000 and is expected to save $6000 in garbage fees during the first year, with the savings increasing by $750 each year thereafter. The incinerator will decrease air quality around campus, which is estimated to be worth $1000 per year. The incinerator will have no salvage value at the end of its 10-year useful life. If Froggy U.

evaluates all capital outlays with a 6% interest and requires B/C analysis, what would you recommend?

SOLUTION

16-4 A new electric generation plant is expected to cost $43,250,000 to complete. Expected revenues will be $3,875,000 per year, while operational expenses will be $2,000,000 per year. If the plant is expected to last 40 years and the electric authority uses 3% as its cost of capital, determine whether the plant should be built.

SOLUTION

16-5 At 10% interest, what is the benefit/cost ratio for the following government project?

Initial cost	$2,000,000
Additional costs at end of Years 1 & 2	30,000
Benefits at end of Years 1 & 2	0
Annual benefits at end of Years 3–10	90,000

SOLUTION

16-6 Gordon City is considering a new garbage dump on the outskirts of town. The land will cost $85,000. Earthwork and other site preparation will cost $250,000. Environmental inspection prior to use will cost $15,000. The upkeep and operating costs will be $50,000 per year for its 8-year life. The new dump will reduce annual disposal fees by $6 each for the 27,000 customers it will serve. Assume that the number of customers will remain relatively constant during the dump's life. Changes in the environmental conditions adjacent to the dump will result in an annual cost of $32,000. At the end of its useful life the dump must be "capped" at a cost of $75,000. Determine the benefit/cost ratio if Gordon City uses 4% as its cost of money.

SOLUTION

A new water treatment plant will cost Frogjump $2,000,000 dollars to
build and $100,000 per year to operate. After 20 years the salvage value
will be 0. The new plant is more efficient; the utility bills for each of
16-7 Frogjump’s 6000 customers will drop $50 per year. The annual cost of
reduced air quality is $5 per resident per year. The population of
Frogjump, which is currently 18,000, is expected to remain relatively
constant over the life of the plant. If 3% is used to evaluate public works
projects, should the water treatment plant be built?

SOLUTION

Drygulch may dam the nearby Twisted River to create a recreational
lake. The earthen dam will cost $1,000,000. Every 10 years dam
reworking will cost $100,000. Annual operating expenses will be
16-8 $20,000. Expected annual usage is 8000 persons with a monetary benefit
to each user of $7.75. A fee of $6 will be charged to each of the boats
that launch into the lake; an annual total of 1200 boats is estimated. At
an interest rate of 4%, determine whether Drygulch should build the
dam.

SOLUTION

The Tennessee Department of Highways (TDOH) is considering its first
“tolled bypass” around Greenfield. The initial cost of the bypass will be
$5.7 million. Installing tollbooths will cost $1.3 million. Annual bypass
maintenance will be $105,000 while annual tollbooth costs will be
$65,000. The bypass should increase tax revenues by $225,000 per year.
TDOH projects user benefits to be $100,000 more than the tolls each
16-9 year. Each user will pay a toll of $0.90. TDOH estimates that there will
be 500,000 users annually. Other relevant data are as follows:

Resurfacing cost (every 7 years)	4% of bypass initial cost
Shoulder grading/rework	90% of resurfacing cost

If the state uses an interest rate of 7%, should the “tolled bypass” be
constructed? Assume perpetual life.

SOLUTION

16-10 Mathis City may construct a municipal park. Land required would be bought as an initial \$62,000 purchase and a \$24,000 expansion 5 years later. Initial construction would take 2 years and cost \$250,000 each year. The expansion cost at the start of year 6 would be \$80,000. Park activities, such as putt-putt golf, would generate user fees of \$35,000 per year. The monetary value of citizens' enjoyment is \$26,000 per year. Maintenance and upkeep would be contracted out at a cost of \$12,000 per year. The park would be used indefinitely, and Mathis City uses an interest rate of 8%. Determine the park's B/C ratio.

SOLUTION

16-11 Two different water delivery methods are available to supply the town of Dry-Hole with much- needed water. Use an interest rate of 6% and the appropriate analysis for public projects to determine which should be chosen.

	Deep Well	Canal
First cost	\$435,000	\$345,000
Annual M&O costs	18,000	25,500
Useful life	20 years	20 years

SOLUTION

16-12 A city engineer is evaluating an irrigation system required for the city park's soccer fields. Which alternative should be selected, if the interest rate is 12% and there is no salvage value?

Year	A	B
0	–\$15,000	–\$25,000
1–10	+5,310	+7,900

SOLUTION

An impoundment may be built for the city's water supply. Two different sites are technically, politically, socially, and financially feasible. The city council has asked for a benefit/cost analysis at a 6% interest rate.

16-13

Year	Rattlesnake Canyon	Blue Basin
0	–$15,000,000	–$27,000,000
1–75	+2,000,000	+3,000,000

Which site should you recommend?

SOLUTION

16-14

A city engineer may accept one of two bids for new park equipment. Using benefit/cost analysis at a 10% interest rate should one be selected? If yes, which one?

Computer	Cost	Annual Benefits	Salvage	Useful Life
A	$48,000	$13,000	$0	6 years
B	40,000	12,000	0	6 years

SOLUTION

16-15

The hotel and conference center at Wicker Valley State Park may be expanded.

	Alternative *A*	Alternative *B*	Alternative *C*
Investment cost	$180,000	$100,000	$280,000
Annual operating costs	16,000	12,000	28,000
Annual benefits	53,000	35,000	77,000

Use a MARR of 10% and a 10-year life and complete a benefit/cost ratio analysis.

SOLUTION

At an interest rate of 12% and a horizon of 15 years use benefit/cost ratios to choose which design is preferred.

16-16

	A	B	C
Initial Investment	$12,000	$17,000	$18,000
Annual Savings	$3,500	$4,500	$9,000
Annual Costs	$1,000	$2,500	$6,500
Salvage Value	$5,000	$4,500	$11,000

SOLUTION

Use benefit/cost ratio analysis to determine which alternative should be selected; each has a 6-year useful life. Assume a 10% MARR.

16-17

	A	*B*	*C*	*D*
First cost	$880	$560	$700	$900
Annual benefit	240	130	110	250
Annual cost	80	20	0	40
Salvage value	300	200	400	110

SOLUTION

16-18 Four mutually exclusive alternatives, each with a 20-year life and no salvage value, have been presented to the city council of Anytown, U.S.A. Which alternative should be selected?

	A	*B*	*C*	*D*
PW of costs	$4000	$9,000	$6000	$2000
PW of benefits	6480	11,250	5700	4700

SOLUTION

The city council of Arson, Michigan, is debating whether to buy a new fire truck

Truck *A* Truck *B*

	First cost	$50,000	$60,000
	Annual maintenance	5,000	4,000
16-19	Salvage value	5,000	6,000
	Annual reduction in fire damage	20,000	21,000
	Useful life	7 years	7 years

a. Use the modified B/C ratio method to determine whether the city should buy a new truck, and if so, which one to buy, assuming that it will be paid for with money borrowed at an interest rate 7%.
b. How would the decision be affected if inflation were considered? Assume that maintenance cost, salvage value, and fire damage are responsive to inflation.

SOLUTION

PROBLEM

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Objective and Viewpoint

16-1 Public-sector economic analysis and decision making is often called “a multi-actor or multi-stakeholder decision problem.” Explain.

16-2 E Compare the general underlying objective of public decision making versus private decision making. List two ethical issues unique to each.

16-3 The text recommends a viewpoint that is appropriate in public decision making. What is suggested? What example is given to highlight the dilemma of viewpoint in public decision making? Provide another example.

16- In government projects, what is meant by the phrase “most of the benefits are local”? What conflict does this create for the federal

4 government in funding the projects from public monies?

16-5 List the potential costs, benefits, and disbenefits that should be considered in evaluating a potential nuclear power plant. What stakeholder viewpoints will need to be considered?

16-6 G A municipal landfill and incineration facility is planned.
(*a*) Name at least three benefits, three disbenefits, and three costs. What stakeholder viewpoints will need to be considered?
(*b*) Burning landfill materials produces environmental and health issues. List the primary issues in each case.

16-7 E An interstate bypass will completely circle a city.
(*a*) Name at least three benefits, three disbenefits, and three costs. What stakeholder viewpoints will need to be considered?
(*b*) Routes for the bypass use private land seized under *eminent domain*. Defend your position on the ethics of eminent domain.

16-8 A light rail system will connect the airport, the city center, and a cluster of high-density housing on the other side of the river. Name at least three benefits, three disbenefits, and three costs. What stakeholder viewpoints will need to be considered?

16-9 Improvements at a congested intersection require that the government acquire the properties on the four corners: two gas stations, a church, and a bank. Construction will take a year, and the costs will be shared 70% by the state and 30% by the city. Traffic through the intersection is mainly commuters, local residents, and deliveries to and by local businesses. There is some through traffic from other parts of the metropolitan area.
Identify the benefits, disbenefits, and costs that must be considered in evaluating this project. From the city's viewpoint, which must be included? From the state's viewpoint? What viewpoint should be used to evaluate this project?

The state may eliminate a railroad grade crossing by building an overpass. The new structure, together with the needed land, would cost \$5.5 million. The analysis period is assumed to be 30 years because either the railroad or the highway above it will be relocated by then. Salvage value of the bridge (actually, the net value of the land on either side of the railroad tracks) 30 years hence is estimated to be \$950,000. A

16-10 E	6% interest rate is to be used. About 1000 vehicles per day are delayed by trains at the grade crossing. Trucks represent 35%, and 65% are other vehicles. Time for truck drivers is valued at $25 per hour and for other drivers at $8 per hour. Average time saving per vehicle will be 2 minutes if the overpass is built. No time saving occurs for the railroad. The railroad spends $80,000 annually for crossing guards. During the preceding 10-year period, the railroad has paid out $1.5 million in settling lawsuits and accident cases related to the grade crossing. The proposed project will entirely eliminate both these expenses. The state estimates that the new overpass will save it about $12,000 per year in expenses directly due to the accidents. The overpass, if built, will belong to the state. Should the overpass be built? If the overpass is built, how much should the railroad be asked to contribute to the state as its share of the $5.5 million construction cost?

Selecting Rate

16-11	Discuss the alternative concepts that can be employed when setting the discounting rate for economic analysis in the public sector. What is the authors' final recommendation for setting this rate?
16-12 A	Is the 7% interest rate specified in OMB A94 a real or a nominal interest rate? Should it be used with costs expressed as constant-value dollars or with costs inflated using carefully selected inflation rates?
	The city's landfill department has a capital budget of $600,000. (*a*) What is the government's opportunity cost of capital if it has the following independent projects to consider? What does this indicate about which projects should be recommended? (*b*) If B/C ratios at 7% are the basis, what projects are recommended? (*c*) Which recommendation is better?

Project First Cost Rate of Return (%) B/C Ratio at 7%

A	$100,000	23	1.30
B	200,000	22	1.40
C	300,000	17	1.50
D	200,000	19	1.35
E	100,000	18	1.56

16-13

The state's fish and game department has a capital budget of $9 million. What is the government's opportunity cost of capital if it has the following projects to consider? What does this indicate about which projects should be recommended?

Project	First Cost	Rate of Return (%)	B/C Ratio at 7%
A	$2,000,000	13	1.40
B	1,000,000	14	1.25
C	2,000,000	15	1.36
D	3,000,000	10	1.15
E	2,000,000	13	1.23

16-14
A

F	3,000,000	15	1.30
G	3,000,000	11	1.10
H	1,000,000	14	1.42

16-15 A municipal bond has a face value of $1000. Interest of $35 is paid every 6 months. The bond has a life of 10 years. What is the effective rate of interest on this bond? Is this rate adjusted for inflation? What is the municipal government's cost of capital for this bond? Estimate the rate, considering all levels of government.

16-16 A E A proposed bridge would cost $4 million to build and $180,000 per year in maintenance. The bridge should last 40 years. Benefits to the driving public are estimated to be $900,000 per year. Damages (not paid) to adjacent property owners due to noise are estimated to be $250,000 per year. The interest rate that should be used to evaluate this project is unclear.

(*a*) Calculate the breakeven annual interest rate that results in a B/C ratio of 1.

(*b*) What is the breakeven interest rate if the noise disbenefits are included? Should they be included?

Contributed by D. P. Loucks, Cornell University

B/C and Modified Ratio

16-17 What is the essential difference between the *conventional* and *modified* versions of the benefit–cost ratio? Is it possible for these two measures to provide conflicting recommendations regarding invest/do-not-invest decisions?

Consider the following investment opportunity:

	Initial cost	$100,000
16-18 A	Additional cost at end of Year 1	150,000
	Benefit at end of Year 1	0
	Annual benefit per year at end of Years 2–10	20,000

With interest at 5%, what is the benefit–cost ratio for this project?

Calculate the conventional and modified benefit– cost ratios for the following project.

	Required first costs	$1,500,000
	Annual benefits to users	$500,000
16-19	Annual disbenefits to users	$25,000
	Annual cost to government	$180,000
	Project life	40 years
	Interest rate	4%

16-20 A

A government agency has estimated that a flood control project has costs and benefits that are parabolic, according to the equation

$$(\text{Present worth of benefits})^2 - 10(\text{Present worth of cost}) + 30 = 0$$

where both benefits and costs are stated in millions of dollars. What is the present worth of cost for the optimal size project?

16-21 For the data given in Problem 16-19, for handling benefits and costs, demonstrate that the calculated B/C ratio is the same using each of the following methods: present worth, annual worth, and future worth.

16-22 A G Chungyang Dam is being constructed across the Hungshui River in southern China. The dam will produce electricity to serve over 500,000 people in the region. The initial cost is 4 billion yuan and annual operating costs are 35 million yuan. A major overhaul of the electric generation facilities estimated to cost 700 million yuan will occur at the end of Year 25. The dam and generating plant have no salvage value, but must be torn down and removed at the end of Year 50 for 185 million yuan. Ishan Electric has a MARR of 12%.

(*a*) What annual benefit in yuan is needed for a B/C ratio of 1?

(*b*) Use the Internet and current exchange rates to find the annual benefit in U.S. dollars.

(*c*) What are the top 3 sources for electricity generated from renewable power in the U.S.? List three benefits and three disbenefits for each source.

Contributed by Paul R. McCright, University of South Florida

Incremental Analysis

16-23 The city engineer has prepared two plans for roads in the city park. Both plans meet anticipated requirements for the next 40 years. The city's minimum attractive rate of return is 7%.

Plan *A* is a three-stage development program: $300,000 is to be spent now followed by $250,000 at the end of 15 years and $300,000 at the end of 30 years. Annual maintenance will be $75,000 for the first 15 years, $125,000 for the next 15 years, and $250,000 for the final 10 years.

Plan *B* is a two-stage program: $450,000 is required now, followed

by $50,000 at the end of 15 years. Annual maintenance will be $100,000 for the first 15 years and $125,000 for the subsequent years. At the end of 40 years, this plan has a salvage value of $150,000.
Use a conventional benefit–cost ratio analysis to determine which plan should be chosen.

16-24 A

The Highridge Water District needs an additional supply of water from Steep Creek. The engineer has selected two plans for comparison:
Gravity plan: Divert water at a point 10 miles up Steep Creek and carry it through a pipeline by gravity.
Pumping plan: Divert water at a point near the district and pump it through 2 miles of pipeline. The pumping plant can be built in two stages: half now and half 10 years later.

	Gravity	Pumping
Initial investment	$2,500,000	$1,250,000
Investment in 10^{th} year	0	150,000
Operation, maintenance, replacements, per year	12,500	20,000
Average annual power cost		
First 10 years	0	40,000
Next 25 years	0	75,000

Annual benefits are $190,00 for both plans. Use a 35-year analysis period and 6% interest. Salvage values can be ignored. Use the conventional benefit–cost ratio method to select the more economical plan.

The Arkansas Department of Transportation may build a new highway between Texarkana and Fort Smith, currently a distance of 181 miles. Design 1 is a four-lane highway built entirely on the existing route. Design 2 includes a significant rerouting through a mountainous region that would reduce the mileage to 166 miles. Design 3 is a fully access-controlled interstate-quality highway with more rerouting, which would reduce the total mileage to 148 miles.Benefits for this project depend on mileage saved times the number of vehicles, plus the estimated value for the larger number of trips that will occur with the shorter and faster routes. The estimated benefits and costs of the three potential designs are shown in the table. Doing nothing yields no costs and no benefits. Using

16-25 incremental analysis for the B/C ratio, a planning horizon of 75 years, and a MARR of 6%, which design would you recommend?

	Initial Cost	Annual Maintenance Cost	Annual Benefit
Design 1	$ 456M	$17M	$107M
Design 2	810M	28M	198M
Design 3	1552M	58M	287M

Contributed by Paul R. McCright, University of South Florida

Two different routes are being considered for a mountain highway. The **high road** would require building several bridges and would navigate around the highest mountain points, thus requiring more roadway. The **low road** would construct several tunnels for a more direct route through the mountains. Projected travel volume for this new section of road is 3500 cars per day. Use the *modified* B/C ratio

16-26 to determine which alternative should be recommended. Assume

A that project life is 45 years and $i = 5\%$.

	High Road	Low Road
Construction cost/mile	$275,000	$425,000
Number of miles required	35	15
Annual benefit/car-mile	$0.02	$0.04
Annual O&M costs/mile	$2,000	$10,000

A 50-meter tunnel must be constructed for a new city aqueduct. One alternative is to build a full-capacity tunnel now for $500,000. The other alternative is to build a half-capacity tunnel now for $300,000 and then to build a second parallel half-capacity tunnel 20 years hence for $300,000. The cost to repair the tunnel lining every 10 years is $20,000 for the full-capacity tunnel and $16,000 for each half-capacity tunnel.

16-27 Determine whether the full-capacity tunnel or the half-capacity tunnel should be constructed now. Solve the problem by the conventional benefit–cost ratio analysis, using a 4% interest rate and a 50-year analysis period. There will be no tunnel lining repair at the end of the 50 years.

16-28 A

A two-lane highway between two cities, 10 miles apart, is to be converted to a four-lane divided freeway. The average daily traffic (ADT) on the new freeway is forecast to average 20,000 vehicles per day over the next 20 years. Trucks represent 5% of the total traffic. Annual maintenance on the existing highway is $1500 per lane-mile. The existing accident rate is 4.58 per million vehicle-miles (MVM). Three alternate plans are under consideration.

Plan A: Improve along the existing development by adding two lanes adjacent to the existing lanes at a cost of $450,000 per mile. This will reduce auto travel time by 2 minutes and truck travel time by 1 minute. The estimated accident rate is 2.50 per MVM. Annual maintenance is $1250 per lane-mile.

Plan B: Improve the highway along the existing alignment with grade improvements at a cost of $650,000 per mile. Plan *B* adds two lanes and would reduce auto and truck travel time by 3 minutes each. The accident rate on this improved road is estimated to be 2.40 per MVM. Annual maintenance is $1000 per lane-mile.

Plan C: Construct a new freeway on a new alignment at a cost of $800,000 per mile. This plan would reduce auto travel time by 5 minutes and truck travel time by 4 minutes. Plan *C* is 0.3 mile longer than *A* or *B*. The estimated accident rate for *C* is 2.30 per MVM. Annual maintenance is $1000 per lane-mile. Plan *C* includes abandoning the existing highway with no salvage value.

Incremental operating cost		
Autos		6¢ per mile
Trucks		18¢ per mile
Time saving		
Autos	3¢ per minute	
Trucks	15¢ per minute	
Average accident cost	$1200	

If a 5% interest rate is used, which of the three proposed plans should be adopted?

16-29 Evaluate these mutually exclusive alternatives with a horizon of 20 years and a MARR of 15%.

	A	*B*	*C*
Initial investment	$9500	$18,500	$22,000
Annual savings	3200	5,000	9,800
Annual costs	1000	2,750	6,400
Salvage value	6000	4,200	14,000

Use each of these approaches:

(*a*) *Conventional* B/C ratio
(*b*) *Modified* B/C ratio
(*c*) Present worth analysis
(*d*) Internal rate of return analysis
(*e*) Payback period

The Fishery and Wildlife Agency of Ireland is considering four mutually exclusive design alternatives for a major salmon hatchery.

This agency of the Irish government uses the following B/C ratio for decision making (EW = EUAW):

$$\text{B/C ratio} = \frac{\text{EW(Net benefits)}}{\text{EW(Capital recovery cost)} + \text{EW(O\&M cost)}}$$

Using an interest rate of 7% and a project life of 30 years, recommend which design is best.

(Values in 1000s)

Irish Fishery Design

16-30
A
G

Alternatives

	A	*B*	*C*	*D*
First cost	\$10,000	\$12,000	\$14,000	\$16,000
Annual benefits	2200	1,500	1,000	2400
Annual O&M costs	550	175	325	145
Annual disbenefits	350	150	75	500
Salvage value	1500	3000	4000	6000

Six mutually exclusive investments have been identified for evaluation by means of the benefit–cost ratio method. Assume a

MARR of 10% and an equal project life of 25 years for all alternatives.

Mutually Exclusive Alternatives

Annualized	1	2	3	4	5	6
Net costs to sponsor ($M)	15.5	13.7	16.8	10.2	17.0	23.3
Net benefits to users ($M)	20.0	16.0	15.0	13.7	22.0	25.0

16-31

(*a*) Use annual worth and the B/C ratio to identify the best alternative.
(*b*) Is there an easier way to select the best alternative?
(*c*) If this were a set of *independent* alternatives, how would you conduct a comparison?

A section of state highway needs repair. At present, the traffic volume is low and few motorists would benefit. However, traffic is expected to increase, with resulting increased motorist benefits. The repair work will produce benefits for 10 years after it is completed.
Should the road be repaired and, if so, when should the work be done? Use a 15% MARR.

(Costs in 1000s)

Year	Repair Now	Repair in 2 Years	Repair in 4 Years	Repair in 5 Years
0	−$150			

	1	5			
	2	10	−$150		
	3	20	20		
	4	30	30	−$150	
16-32 A	5	40	40	40	−$150
	6	50	50	50	50
	7	50	50	50	50
	8	50	50	50	50
	9	50	50	50	50
	10	50	50	50	50
	11	0	50	50	50
	12	0	50	50	50
	13	0	0	50	50

14	0	0	50	50
15	0	0	0	50

The local highway department is analyzing reconstruction of a mountain road. The vehicle traffic increases each year, hence the benefits to the motoring public also increase. Based on a traffic count, the benefits are projected as follows:

16-33 The reconstructed pavement will cost $265,000 when it is installed and will have a 15-year useful life. The construction period is short, hence a beginning-of-year reconstruction will result in the listed end-of-year benefits. Assume a 7% interest rate. The reconstruction, if done at all, must be done no later than 2024. When is the first year that this project is justified?

Year	End-of-Year Benefit
2019	$10,000
2020	12,000
2021	14,000
2022	16,000
2023	18,000
2024	20,000

The reconstructed pavement will cost $265,000 when it is installed and will have a 15-year useful life. The construction period is short, hence a beginning-of-year reconstruction will result in the listed end-of-year benefits. Assume a 7% interest rate. The reconstruction, if done at all, must be done no later than 2024. When is the first year that this project is justified?

Challenges for Public Sector

Describe how a decision maker can use each of the following to "skew" the results of a B/C ratio analysis in favor of his or her own position on funding projects.

16-34 E
(*a*) Conventional versus modified ratios
(*b*) Interest rates
(*c*) Project duration
(*d*) Benefits, costs, and disbenefits
(*e*) When presenting your results, is it ethical to pick a method that matches your preferred choice?

16-35
Briefly describe your sources and methods for estimating the value of
(*a*) a saved hour of commuting time.
(*b*) converting 15 miles of unused railroad tracks near a community of 300,000 into a new bike path.
(*c*) reducing annual flood risks from the Mississippi River for St. Louis by 5%.
(*d*) a new engineering or engineering technology major at your university.

16-36 If you favored Plan *B* in Problem 16-23, what value of MARR would you use in the computations? Explain.

16-37 Discuss potential data sources and methods for estimating each of the costs, benefits, and disbenefits identified in Problem 16-5 for the nuclear power plant.

16-38 For the municipal landfill and incineration facility in Problem 16–6, discuss potential data sources and methods for estimating each of the benefits, disbenefits, and costs.

16-39 For the interstate bypass in Problem 16-7, discuss potential data sources and methods for estimating each of the benefits, disbenefits, and costs.

16-40 For the light rail system in Problem 16-8, discuss potential data sources and methods for estimating each of the benefits, disbenefits, and costs.

16-41 Discuss potential data sources and methods for estimating each of the costs, benefits, and disbenefits identified in Problem 16-9 for the congested intersection.

Think about a major government construction project under way in

14-42 your state, city, or region. Are the decision makers who originally analyzed and initiated the project currently in office? How can politicians use "political posturing" with respect to government projects?

Big City Carl, a local politician, is pushing a new dock and pier system at the river to attract commerce. A committee appointed by the mayor (an opponent of Carl's) has developed the following estimates.

16-43

Cost to remove current facilities	$ 750,000
New construction costs	2,750,000
Annual O & M costs	185,000
Annual benefits from new commerce	550,000
Annual disbenefits to sportsmen	35,000
Project life	20 years
Interest rate	8%

(*a*) Using the *conventional* B/C ratio, determine whether the project should be funded.
(*b*) After studying the numbers given by the committee, Big City Carl argued that the project life should be *at least* 25 years and more likely closer to 30 years. How did he arrive at this estimate, and why is he making this statement?
(*c*) If Carl suggests that each number is 5% "better," then what B/C ratio would he support?

The federal government proposes to construct a multipurpose water project to provide water for irrigation and municipal use. In addition, there are flood control and recreation benefits. The benefits are given below. The annual benefits are one-tenth of the decade benefits. The operation and maintenance cost is $175,000 per year. Assume a 50-year analysis period with no net project salvage value.
(*a*) If an interest rate of 5% is used, and a benefit–cost ratio of unity, what capital expenditure can be justified to build the water project now?
(*b*) If the interest rate is changed to 8%, how does this change the justified capital expenditure?

(Benefits in 1000s)

16-44 A

	Decades				
Purpose	First	Second	Third	Fourth	Fifth
Municipal	$45	$48	$62	$75	$105
Irrigation	350	370	370	360	340
Flood control	150	150	150	150	150
Recreation	65	70	80	85	90
Totals	$610	$638	$662	$670	$685

Minicases

16-45 G Research and report on how an agency of your state evaluates public project proposals. What interest rate and what economic measures are used? Are factors that include life-cycle costs or environmental effects included?

Research and report on how an agency of your municipality evaluates public project proposals. What interest rate and what economic measures are used?

Data for 16-47 to 16-49 adds detail to the chapter-opening vignette. This material was contributed by *William R. Truran, U.S. Department of*

Defense, and Peter A. Cerenzio, Cerenzio & Panaro Consulting Engineers

Daily and intermediate cover material is 20% of the landfill's usable space. Density of solid waste is 1500 pounds per cubic yard. LFG recovery rates:

3000 cubic feet per ton for municipal solid waste (MSW)

1500 cubic feet per ton for construction and demolition waste (C&D)

16-46 Assume waste composition is 80% MSW and 20% C&D. Methane content in the LFG is 50% for MSW and 20% for C&D.

Heating value

For methane: 1,030,000 BTU/1000 cubic feet

For fuel oil: 138,800 BTU/gallon

Assumed furnace efficiency is 88% for methane and 82% for fuel oil.

Cost of fuel oil is $2.50/gallon.

Heating load for residential dwelling is 100 million BTU per year.

1.17×10^4 BTU per kilowatt-hours

Methane BTUs valued at equivalence to $0.05/kWh

An economic analysis is needed for a new municipal solid waste landfill. This includes determining the potential economic benefit from the gas that is generated when solid waste decomposes. The landfill is proposed at 14 acres, with a design capacity of 1 million cubic yards of capacity.
16- The final capping system will require a 3-foot layer over the 14 acres,
47 and the waste flow rate is 120,000 tons per year. Calculate:
G (*a*) The life of the landfill.
(*b*) The average annual methane production (assume all methane production has ceased by 15 years after landfill closure).

(*c*) The dollar value of annual methane converted to electricity (neglect collection and energy production costs).

16-48G A developer has proposed a 650-unit residential development adjacent to the landfill. The proposal includes using the gas generated from the landfill to heat the homes (and mitigate odors). To determine the economic feasibility of the proposal, the value of the gas for heating purposes must be determined. Determine whether the quantity is sufficient for heating the development. Is this economically more attractive than using fuel oil? Is it operationally feasible? What impact might the project have on the environment? Are there any disbenefits from the project to the homeowners?

16-49G A 4.6-acre landfill must be evaluated for economic viability. As part of the cost–benefit analysis, the cost to extract and treat the landfill gas must be determined. Using the following, design a landfill gas extraction system and estimate the cost to implement such a system:

Landfill dimensions are 1000 feet by 200 feet.

Area of influence of a landfill gas extraction well is a 50-foot radius.

Cost of well construction is $3000 each.

Cost of wellhead (necessary for each well) is $2500 each.

Cost of collection header piping 8" diameter

HDPE is $35/linear foot.

Cost of condensate knockouts (located at low points) is $5000 each.

Cost of blower/flare station is $500,000.

CASES

The following cases from *Cases in Engineering Economy*

(www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 16	**Great White Hall**
	Proposal comparison using B/C analysis for RFP with unclear specifications.
CASE 41	**Metropolitan Highway**
	Realistic variety of benefits and costs. Requires that assumptions be made.
CASE 40	**Olympic Bid Perspectives**
	Public sector with data and questions from three perspectives.
CASE 42	**Protecting the Public**
	P(injury) and fraction of contact wearers must be "guesstimated." Considers opportunity losses to other venues in visitor-days.
CASE 43	**Bigstate Bridging the Gap**
	Difficulty ranges from comparing three alternative bridges with different lives to considering growth in traffic and uncertain construction costs.
CASE 44	**Sunnyside—Up or Not?**
	Uncertain growth rates over 30 years and setting utility rates.
CASE 45	**Transmission Intertie**
	Electric power project with primary and secondary benefits.
CASE 47	**Bigstate Highway Department**
	Capital budgeting including mutually exclusive alternatives. Includes uncertain value of a life.
CASE	

48	**Dot Puff Project Selection**
	Public-sector capital budgeting with added constraint on man-years required.

CHAPTER 17

ACCOUNTING AND ENGINEERING ECONOMY

A Tale of Three Engineers

Accounting is more than bookkeeping. Accounting analyzes and reports on the amount and availability of an organization's or an individual's financial resources. Understanding basic accounting is important for your professional development as an engineer as well as the management of your personal finances. Three examples demonstrate how engineers use accounting in both their careers and personal lives.

Moving Up the Career Ladder. Our first engineer excelled technically and was quickly asked to accept project management and then line management duties. The engineer had the responsibility for both justifying engineering decisions and project budgets. Quantifying and accounting for financial performance and efficiency was an essential component of the position. Whether you work for a large, medium, or small firm, enter academia, or form your own company, knowledge of accounting can help you obtain a promotion, as it did for our first engineer.

Investing for the Future. Our second engineer uses knowledge of accounting to monitor and manage her personal finances—almost as if she were running a business. Not only are monthly and annual expenses tracked, but this engineer has created a budget and balance sheet. One of the key concepts utilized in this plan has been the fundamental accounting equation "Assets = Liabilities + Equity." This engineer initially had a student loan (a liability) immediately after graduation, and now has a mortgage (a liability) on a house (an asset). The engineer's savings plan includes the employer's defined contribution plan. Finances had been planned immediately after graduation and in 5-year increments. Personal expenses and income are regularly checked against a short- and long-term strategy. These changed when the engineer married and expected a child. Savings and investments included a down payment for a house, an emergency fund, savings for the child's education, and a long-term nest egg. Knowledge of accounting is being applied to manage, control, and secure a financial future. (Appendix 9A introduces investing and defined benefit and defined contribution plans. Appendix 10A discusses how diversification reduces risk. Appendix 12A introduces budgeting.)

Engineering Entrepreneurs. Our third engineer is actually a group of engineering students from the University of California, Davis. After the unexpected death of their research advisor, the students formed a small corporation and applied for funding from a government agency. They believed they had an excellent idea and submitted a proposal and budget justification. Two months later, the student business received an award letter for over $1 million. The key to their successful bid was an innovative idea, a sound technical plan and budget, and the accounting skills to manage the finances and required reporting to a government agency. As an engineering firm, their corporate tax rate on profits was recently reduced to a flat rate of 21%.

What happened to those students? They finished their engineering degrees. One of them became a faculty member and teaches engineering economics. She shares with students that engineers are inherently technical, but when armed with financial knowledge and management skills, engineering students can become great entrepreneurs.

Contributed by Jani Macari Pallis, University of Bridgeport and Cislunar Aerospace, Inc.

QUESTIONS TO CONSIDER

1. How can you develop accounting or financial analysis skills before you enter the workforce?
2. Accounting and ethics both played large roles in the last fiscal crisis and in many smaller scandals. How can knowledge of accounting help you guard against ethical lapses?
3. Create annual budgets for the next 5 years. Use these budgets to connect your current and planned assets, liabilities, and equity.

After Completing This Chapter...

The student should be able to:

• Describe the links between engineering economy and accounting.
• Describe the objectives of general accounting, explain what financial transactions are, and show how they are important.
• Use a firm’s balance sheet and associated financial ratios to evaluate the firm’s health.
• Use a firm’s income statement and associated financial ratios to evaluate the firm’s performance.
• Use traditional absorption costing to calculate product costs.
• Understand the greater accuracy in product costs available with activity-based costing (ABC).

Key Words

absorption costing

accounting data

assets

balance sheet

business transaction

cost accounting

current ratio

direct cost

equity

fundamental accounting equation

general accounting

income statement

indirect cost

interest coverage

liabilities

profit and loss statement

profit margin

quick assets

quick ratio

working capital

Engineering economy focuses on the financial aspects of projects, while accounting focuses on the financial aspects of firms. Thus the application of engineering economy is much easier if one has some understanding of accounting principles. In fact, one important accounting topic, depreciation, was the subject of an earlier chapter.

THE ROLE OF ACCOUNTING

Accounting data are used to value capital equipment, to decide whether to make or buy a part, to determine costs and set prices, to set indirect cost rates, and to make product mix decisions. Accounting is used in private-sector firms and public-sector agencies, but for simplicity this chapter uses "the firm" to designate both. Accountants track the costs of projects, products, and services, which are the basis for estimating future costs and revenues.

The engineering economy, accounting, and managerial functions are interdependent. As shown in Figure 17–1, data and communications flow between them. Whether carried out by a single person in a small firm or by distinct divisions in a large firm, all are needed.

• Engineering economy analyzes the economic impact of design alternatives and projects over their life cycles.
• Accounting determines the dollar impact of past decisions, reports on the economic viability of a unit or firm, and evaluates potential funding sources.
• Management allocates available investment funds to projects, evaluates unit and firm performance, allocates resources, and selects and directs personnel.

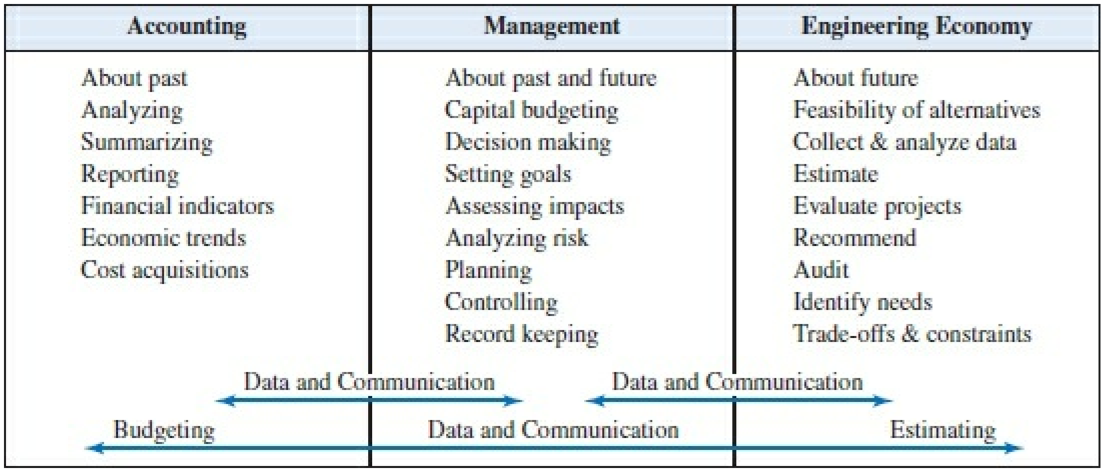


FIGURE 17-1 The accounting, managerial, and engineering economy functions.

Accounting for Business Transactions

A **business transaction** involves two parties and the exchange of dollars (or the promise of dollars) for a product or service. Each day, millions of transactions occur between firms and their customers, suppliers, vendors, and employees. Transactions are the lifeblood of the business world and are most often stated in monetary terms. The accounting function records, analyzes, and reports these exchanges.

Transactions can be as simple as payment for a water bill, or as complex as the international transfer of millions of dollars of buildings, land, equipment, inventory, and other assets. Also, with transactions, one business event may lead to another—all of which need to be accounted for. Consider, for example, the process of selling a robot or a bulldozer. This simple act involves several related transactions: (1) releasing equipment from inventory, (2) shipping equipment to the purchaser, (3) invoicing the purchaser, and finally (4) collecting from the purchaser.

Transaction accounting involves more than just reporting: it includes finding, synthesizing, summarizing, and analyzing data. For the engineering economist, historical data housed in the accounting function are the foundation for estimates of future costs and revenues.

Most accounting is done in nominal or *stable* dollars. Higher market values and costs due to inflation are less objective than cost data, and with a going concern, accountants have decided that objectivity should be maintained. Similarly, most assets are valued at their acquisition cost adjusted for depreciation and improvements. To be conservative, when market value is lower than this adjusted cost, the lower value is used. If a firm must be liquidated, then current market value must be estimated.

The accounting function provides data for **general accounting** and **cost accounting.** This chapter's presentation begins with the balance sheet and income statement, which are the two key summaries of financial transactions for general accounting. This discussion includes some of the basic financial ratios used for short- and long-term evaluations. The chapter concludes with a key topic in cost accounting—allocating indirect expenses.

THE BALANCE SHEET

The primary accounting statements are the balance sheet and the income statement. The balance sheet describes the firm's financial condition at a specific time, while the income statement describes the firm's performance over a period of time—usually a year.

The balance sheet lists the firm's assets, liabilities, and equity on a specified date. This is a picture of the organization's financial health or a snapshot in time. Usually, balance sheets are taken at the end of the quarter and fiscal year. The balance sheet is based on the **fundamental accounting equation:**

$$\text{Assets} = \text{Liabilities} + \text{Equity} \quad (17\text{-}1)$$

Figure 17–2 illustrates the basic format of the balance sheet. Notice in the balance sheet, as in Equation 17–1, that assets are listed on the left-hand side and liabilities and equity are on the right-hand side. The fact that the firm's resources are *balanced* by the sources of funds is the basis for the name of the balance sheet.

Balance Sheet for Engineered Industries, December 31, 2017 (all amounts in $1000s)

Assets		**Liabilities**	
Current assets		Current liabilities	
Cash	$ 1940	Accounts payable	$ 1150
Accounts receivable	950	Notes payable	80
Securities	4100		
Inventories	1860	Accrued expense	950
(*minus*) Bad debt provision	−80	Total current liabilities	2180
Total current assets	8770		
		Long-term liabilities	1200
		Total liabilities	**3380**
Fixed assets			
Land	335		
Plant and equipment	6500		
(*minus*) Accumulated depr.	−2350		
Total fixed assets	4485	**Equity**	
		Preferred stock	110
Other assets		Common stock	650
Prepays/deferred charges	140	Capital surplus	930
Intangibles	420	Retained earnings	8,745
Total other assets	560	Total equity	10,435
Total assets	**13,815**	**Total liabilities and equity**	**13,815**

FIGURE 17-2 Sample balance sheet.

Assets

In Equation 17–1 and Figure 17–2, **assets** are items owned by the firm and have monetary value. Liabilities are what the firm owes. **Equity** represents funding from the firm and its owners (the shareholders). In Equation 17–1, assets are always balanced by the sum of the liabilities and the equity. The value for retained earnings is set so that equity equals assets minus liabilities.

On a balance sheet, assets are listed in order of decreasing liquidity, that is, according to how quickly each one can be converted to cash. Thus, *current assets* are listed first, and within that category in order of decreasing liquidity are listed cash, receivables, securities, and inventories. *Fixed assets*, or *property, plant, and equipment*, are used to produce and deliver goods and/or services, and they are not intended for sale. Items such as prepayments and intangibles such as patents are listed last.

The term “receivables” comes from the manner of handling billing and payment for most business sales. Rather than requesting immediate payment for every transaction by check or credit card, most businesses record each transaction and then bill for all transactions. The total that has been billed less payments already received is called accounts receivable, or receivables.

Liabilities

On the balance sheet, **liabilities** are divided into two major classifications—short term and long term. The *short-term* or *current liabilities* are expenses, notes, and other payable accounts that are due within one year from the balance sheet date. *Long-term liabilities* include mortgages, bonds, and loans with later due dates. For Engineered Industries in Figure 17–2, total current and long-term liabilities are \$2,180,000 and \$1,200,000, respectively. Often in performing engineering economic analyses, the **working capital** for a project must be estimated. The total amount of working capital available may be calculated with Equation 17-2 as the difference between current assets and current liabilities.

$$\text{Working capital} = \text{Current assets} - \text{Current liabilities} \qquad (17\text{-}2)$$

For Engineered Industries, there would be $\$8,770,000 - \$2,180,000 = \$6,590,000$ available in working capital.

Equity

Equity is also called *owner’s equity* or *net worth.* It includes the par value of the owners’ stockholdings and the capital surplus, which are the excess dollars brought in over par value when the stock was issued. The capital surplus can also be called *additional paid-in capital*, or APIC. Retained earnings are dollars a firm chooses to retain rather than paying out as dividends to stockholders.

Retained earnings within the equity component is the dollar quantity that always brings the balance sheet, and thus the fundamental accounting equation, into balance. For Engineered Industries, *total equity* value is listed at \$10,435,000. From Equation 17–1 and the assets, liabilities, and equity values in Figure 17–2, we can write the balance as follows:

$$\text{Assets} = \text{Liabilities} + \text{Equity}$$
$$\text{Assets (current, fixed, other)} = \text{Liabilities (current and long-term)} + \text{Equity}$$
$$8{,}770{,}000 + 4{,}485{,}000 + 560{,}000 = 2{,}180{,}000 + 1{,}200{,}000 + 10{,}435{,}000$$
$$\$13{,}815{,}000 = \$13{,}815{,}000$$

An example of owner's equity is ownership of a home. Most homes are purchased by means of a mortgage loan that is paid off at a certain interest rate over 15 to 30 years. At any point in time, the difference between what is owed to the bank (the remaining balance on the mortgage) and what the house is worth (its appraised market value) is the *owner's equity*. In this case, the loan balance is the *liability*, and the home's value is the *asset*—with *equity* being the difference. Over time, as the house loan is paid off, the owner's equity increases.

The balance sheet is a very useful tool that shows one view of the firm's financial condition at a particular point in time.

Financial Ratios Derived from Balance Sheet Data

One common way to evaluate the firm's health is through ratios of financial quantities found on the balance sheet. Firms in a particular industry will typically have similar values, and exceptions will often indicate firms with better or worse performance. Two common ratios used to analyze the firm's current position are the current ratio and the quick (or acid-test) ratio.

A firm's **current ratio** is the ratio of current assets to current liabilities, as in Equation 17-3.

$$\text{Current ratio} = \text{Current assets}/\text{Current liabilities} \tag{17-3}$$

This ratio provides insight into the firm's solvency over the short term by indicating its ability to cover current liabilities. Historically, firms aim to be at or above a ratio of 2.0; however, this depends heavily on the industry as well as the individual firm's management practices and philosophies. The current ratio for Engineered Industries in Figure 17–2 is above 2 (8,770,000/2,180,000 = 4.02).

Both working capital and the current ratio indicate the firm's ability to meet currently maturing obligations. However, neither describes the type of assets owned. The **acid-test ratio** or **quick ratio** becomes important when one wishes to consider the firm's ability to pay debt "instantly." The acid-test ratio is computed by dividing a firm's **quick assets** (cash, receivables, and market securities) by total current liabilities, as in Equation 17-4.

$$\text{Quick ratio} = (\text{Current assets} - \text{inventories})/\text{Current liabilities} \quad (17\text{-}4)$$

Inventories are subtracted from current assets because of the time required to sell these inventories, collect the receivables, and subsequently have the cash on hand to reduce debt. For Engineered Industries in Figure 17–2, the calculated acid-test ratio is well above 1 [(8, 770, 000 − 1, 860, 000)/2, 180, 000 = 3.17].

Working capital, current ratio, and quick ratio are all indications of the firm's financial health (status). A thorough financial evaluation would consider all three, including comparisons with values from previous periods and with broad-based industry standards. When trends extend over multiple periods, the trends may be more important than the current values.

THE INCOME STATEMENT

The **income statement** or **profit and loss statement** summarizes the firm's revenues and expenses over a month, quarter, or year. Rather than being a snapshot like the balance sheet, the income statement encompasses a *period* of business activity. The income statement is used to evaluate revenue and expenses that occur in the interval *between* consecutive balance sheet statements. The income statement reports the firm's *net income (profit)* or *loss* by subtracting expenses from revenues. If revenues minus expenses is positive in Equation 17-5, there has been a profit, if negative a loss has occurred.

$$\text{Revenues} - \text{Expenses} = \text{Net profit (Loss)} \quad (17\text{-}5)$$

To aid in analyzing performance, the income statement in Figure 17–3 separates operating and nonoperating activities and shows revenues and expenses for each. Operating revenues are made up of sales revenues (minus returns and allowances), while nonoperating revenues come from rents and interest receipts.

Operating expenses produce the products and services that generate the firm's revenue stream of cash flows. Typical operating expenses include cost of goods sold, selling and promotion costs, depreciation, general and administrative costs, and lease payments. *Cost of goods sold (COGS)* includes the labor, materials, and indirect costs of production.

Income Statement for Engineered Industries for End of Year 2018
(all amounts in $1000)

Operating revenues and expenses	
Operating revenues	
Sales	$ 18,900
(*minus*) Returns and allowances	−870
Total operating revenues	**18,030**
Operating expenses	
Cost of goods and services sold	
Labor	6140
Materials	4640
Indirect cost	2,280
Selling and promotion	930
Depreciation	450
General and administrative	2,160
Lease payments	510
Total operating expense	**17,110**
Total operating income	**920**
Nonoperating revenues and expenses	
Rents	20
Interest receipts	300
Interest payments	−120
Total nonoperating income	**200**
Net income before taxes	**1120**
Income taxes	−270
Net profit (loss) for 2018	**850**

FIGURE 17-3 Sample income statement.

Engineers design production systems, specify materials, and analyze make/buy decisions. All these items affect a firm's cost of goods sold. Good engineering design focuses not only on technical functionality but also on cost-effectiveness as the design *integrates* the entire production system. Also of interest to the engineering economist is *depreciation* (see Chapter 11)—which is the systematic "writing off " of a capital expense over a period of years. This noncash expense is important because it represents a decrease in value of the firm's capital assets.

The operating revenues and expenses are shown first, so that the firm's operating income from its products and services can be calculated. Also shown on the income statement are nonoperating expenses such as interest payments on debt.

From the data in Figure 17–3, Engineered Industries has total expenses (operating = $17, 110, 000 and nonoperating = $120, 000) of $17,230,000. Total revenues are $18, 350, 000 (= $18, 030, 000 + $20, 000 + $300, 000). The net after-tax profit for year 2018 shown in Figure 17–3 as $850,000, but it can also be calculated using Equation 17-5 as

$$\text{Net profits (Loss)} = \text{Revenues} - \text{Expenses [before taxes]}$$
$$\$1{,}120{,}000 = 18{,}350{,}000 - 17{,}230{,}000 \text{ [before taxes] and with } \$270{,}000 \text{ taxes paid}$$

thus

$$\$850{,}000 = 1{,}120{,}000 - 270{,}000 \text{ [after taxes]}$$

Financial Ratios Derived from Income Statement Data

The **profit margin** (Equation 17-6) equals net profits divided by net sales revenue. Net sales revenue equals sales minus returns and allowances.

$$\text{Profit margin} = \text{Net profit/Net sales revenue} \quad (17\text{-}6)$$

This ratio provides insight into the cost efficiency of operations as well as a firm's ability to convert sales into profits. For Engineered Industries in Figure 17–3, the profit margin is 850, 000/18, 030, 000 = 0.047 = 4.7%. Like other financial measures, the profit margin is best evaluated by comparisons with other time periods and industry benchmarks, and trends may be more significant than individual values.

Interest coverage, as given in Equation 17-7, is calculated as the ratio of total income to interest payments—where *total income* is total revenues minus all expenses except interest payments.

$$\text{Interest coverage} = \text{Total income}/\text{Interest payments} \tag{17-7}$$

The interest coverage ratio (which for industrial firms should be at least 3.0) indicates how much revenue must drop to affect the firm’s ability to finance its debt. With an interest coverage ratio of 3.0, a firm’s revenue would have to decrease by two-thirds (unlikely) before it became impossible to pay the interest on the debt. The larger the interest coverage ratio the better. Engineered Industries in Figure 17–3 has an interest coverage ratio of

$$(18{,}350{,}000 - 17{,}110{,}000)/120{,}000 = 10.3$$

Linking the Balance Sheet, Income Statement, and Capital Transactions

The balance sheet and the income statement are separate but linked documents. Understanding how the two are linked together helps clarify each. Accounting describes such links as the *articulation* between these reports.

The balance sheet shows a firm’s assets, liabilities, and equity at a particular point in time, whereas the income statement summarizes revenues and expenses over a time interval. These tabulations can be visualized as a snapshot at the period’s beginning (a balance sheet), a video summary over the period (the income statement), and a snapshot at the period’s end (another balance sheet). The income statement and changes in the balance sheets summarize the business transactions that have occurred during that period.

There are many links between these statements and the cash flows that make up business transactions, but for engineering economic analysis the following are the most important.

1. Overall profit or loss (income statement) and the starting and ending equity (balance sheets).
2. Acquisition of capital assets.
3. Depreciation of capital assets.

The overall profit or loss during the year (shown on the income statement) is reflected in the change in retained earnings between the balance sheets at the beginning and end of the year. To find the change in retained earnings (RE), one must also subtract any dividends distributed to the owners and add the value of any new capital stock sold:

$$RE_{beg} + \text{Net income/Loss} + \text{New stock} - \text{Dividends} = RE_{end}$$

When capital equipment is purchased, the balance sheet changes, but the income statement does not. If cash is paid, then the cash asset account decrease equals the increase in the capital equipment account—there is no change in total assets. If a loan is used, then the capital equipment account increases, and so does the liability item for loans. In both cases the equity accounts and the income statement are unchanged.

The depreciation of capital equipment is shown as a line on the income statement. The depreciation for that year equals the change in accumulated depreciation between the beginning and the end of the year—after subtraction of the accumulated depreciation for any asset that is sold or disposed of during that year.

Example 17–1 applies these relationships to the data in Figures 17–1 and 17–2.

EXAMPLE 17-1

For simplicity, assume that Engineered Industries will not pay dividends in 2018 and did not sell any capital equipment. It did purchase $400,000 in capital equipment. What can be said about the values on the balance sheet at the end of 2018, using the linkages just described?

SOLUTION

First, the net profit of $850,000 will be added to the retained earnings from the end of 2017 to find the new retained earnings at the end of 2018:

$$\text{RE}_{12/31/2017} = \$850{,}000 + \$8{,}745{,}000 = \$9{,}595{,}000$$

Second, the plant and equipment assets shown at the end of 2018 would increase from $6,500,000 to $6,900,000.

Third, the accumulated depreciation would increase by the $450,000 in depreciation shown in the 2018 income statement from the $2,350,000 posted in 2017. The new accumulated depreciation on the 2018 balance sheet would be $2,800,000. Combined with the change in the amount of capital equipment, the new fixed asset total for 2018 would equal:

$$\$335{,}000 + \$6{,}900{,}000 - \$2{,}800{,}000 = \$4{,}435{,}000$$

TRADITIONAL COST ACCOUNTING

A firm's *cost-accounting system* collects, analyzes, and reports operational performance data (costs, utilization rates, etc.). Cost-accounting data are used to develop product costs, to determine the mix of labor, materials, and other costs in a production setting, and to evaluate outsourcing and subcontracting possibilities.

Direct and Indirect Costs

Costs incurred to produce a product or service are traditionally classified as either **direct** or **indirect (overhead)**. Direct costs come from activities directly associated with the final product or service produced. Examples include material costs and labor costs for engineering design, component assembly, painting, and drilling.

Some organizational activities are difficult to link to specific projects, products, or services. For example, the receiving and shipping areas of a manufacturing plant are used by all incoming materials and all outgoing products. Materials and products differ in their weight, size, fragility, value, number of units, packaging, and so on, and the receiving and shipping costs depend on all these factors. Also, different materials arrive together and different products are shipped together, so these costs are intermingled and often cannot be tied directly to each product or material.

Other costs, such as the organization's management, sales, and administrative expenses, are difficult to link directly to individual products or services. These indirect or overhead expenses also include machine depreciation, engineering and technical support, and customer warranties.

Indirect Cost Allocation

To allocate indirect costs to different departments, products, and services, accountants use quantities such as direct-labor hours, direct-labor costs, material costs, and total direct cost. One of these is chosen to be the burden vehicle. The total of all indirect or overhead costs is divided by the total for the burden vehicle. For example, if direct-labor hours is the burden vehicle, then overhead will be allocated based on overhead dollar per direct-labor hour. Then each product, project, or department will *absorb* (or be allocated) overhead costs, based on the number of direct-labor hours each has.

This is the basis for calling traditional costing systems **absorption costing**. For decision making, the problem is that the absorbed costs represent average, not incremental, performance.

Four common ways of allocating overhead are direct-labor hours, direct-labor cost, direct-materials cost, and total direct cost. The first two differ significantly only if the cost per hour of labor differs for different products. Example 17–2 uses direct-labor and direct-materials cost to illustrate the different choices of burden vehicle.

EXAMPLE 17-2

Industrial Robots does not manufacture its own motors or computer chips. Its premium product differs from its standard product in having heavier-duty motors and more computer chips for greater flexibility.

As a result, Industrial Robots manufactures a higher fraction of the standard product's value itself, and it purchases a higher fraction of the premium product's value. Use the following data to allocate $850,000 in overhead on the basis of labor cost and materials cost.

	Standard	Premium
Number of units per year	750	400
Labor cost (each)	$400	$500
Materials cost (each)	50	900

SOLUTION

First, the labor and material costs for the standard product, the premium product, and in total are calculated.

	Standard	Premium	Total
Number of units per year	750	400	
Labor cost (each)	$ 400	$ 500	
Materials cost (each)	550	900	
Labor cost	300,000	200,000	$500,000
Materials cost	412,500	360,000	772,500

Then the allocated cost per labor dollar, $1.70, is found by dividing the $850,000 in overhead by the $500,000 in total labor cost. The allocated cost per material dollar, $1.100324, is found by dividing the $850,000 in overhead by the $772,500 in materials cost. Now, the $850,000 in allocated overhead is split between the two products using labor costs and material costs.

	Standard	Premium	Total
Labor cost	$300,000	$200,000	$500,000
Overhead/labor	1.70	1.70	
Allocation by labor	510,000	340,000	850,000
Material cost	412,500	360,000	0
Overhead/material	1.1003	1.1003	
Allocation by material	453,884	396,116	850,000

If labor cost is the burden vehicle, then 60% of the $850,000 in overhead is allocated to the standard product. If material cost is the burden vehicle, then 53.4% is allocated to the standard product. In both cases, the $850,000 has been split between the two products. Using total direct costs would produce another overhead allocation between these two values. However, for decision making about product mix and product prices, incremental overhead costs must be analyzed. All the allocation or burden vehicles are based on an average cost of overhead per unit of burden vehicle.

Problems with Traditional Cost Accounting

Allocation of indirect costs can distort product costs and the decisions based on those costs. To be accurate, the analyst must determine which indirect or overhead expenses will be changed because of an engineering project. In other words, what are the incremental cash flows? For example, vacation and sick leave accrual may be part of overhead, but will they change if the labor content is changed? The changes in costs incurred must be estimated. Loadings, or allocations, of overhead expenses cannot be used.

This issue has become very important because in some firms, automation has reduced direct-labor content to less than 5% of the product's cost. Yet in some of these firms, the basis for allocating overhead is still direct-labor hours or cost.

Other firms are shifting to activity-based costing (ABC), where each activity is linked to specific cost drivers, and the number of dollars allocated as overhead is minimized (see Liggett, Trevino, and Lavelle, 1992). Figure 17–4 illustrates the difference between activity-based costing and traditional overhead allocations (see Tippet and Hoekstra, 1993).

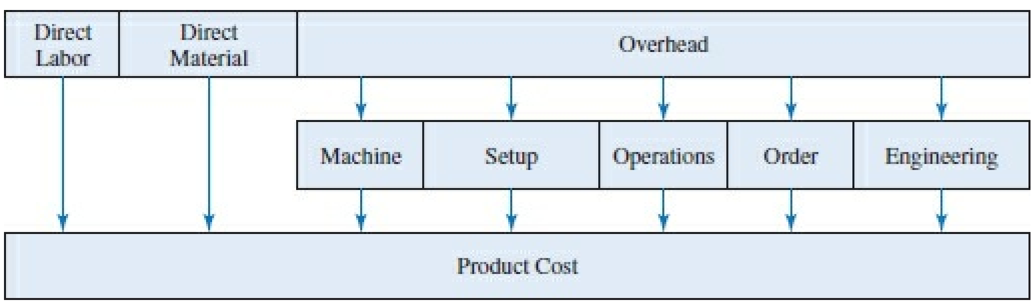


FIGURE 17-4 Activity-based costing versus traditional overhead allocation. (Based on an example by Kim LaScola Needy.)

Other Problems to Watch For

Project managers have often accused centralized accounting systems of being too slow or being "untimely." Because engineering economy is not concerned with the problem of daily project control, this is a less critical issue. However, if an organization establishes multiple files and systems so that project managers (and others) have the timely data they need, then the level of accuracy in one or all systems may be low. As a result, analysts making cost estimates will have to consider other internal data sources.

There are several cases in which data on equipment or inventory values may be questionable. When inventory is valued on a “last in, first out” basis, the remaining inventory may be valued too low. Similarly, land valued at its acquisition cost is likely to be significantly undervalued. Finally, capital equipment may be valued at either a low or a high value, depending on allowable depreciation techniques and company policy.

SUMMARY

This chapter has provided several key concepts from the accounting field. These provide additional perspective when conducting engineering economic analyses. Data from accounting records form the bases for many needed estimates. Management decisions rely on both professions to keep the firm profitable.

Engineering economic analysis: requires the estimation of future costs and benefits. These are often derived from historical data. Projects are evaluated over their life cycle using methods such as present worth, equivalent worth, and rate of return. The effects of taxes, inflation, and risk and uncertainty may be included. Analyses assist the firm in making future decisions.
Accounting: determines the economic impact of past business decisions, and thus provides important historical data for making future ones. Tools such as the balance sheet and income statement capture important measures.
Management: allocates constrained resources of the firm to promote economic growth/stability and strategic objectives. Management requires data, analysis, and communication from both the engineering economy (future view) and accounting (historical view) perspectives.

The primary accounting statements are the balance sheet and the income statement. The balance sheet provides a snapshot in time of the financial position of the firm. This tool is based on the fundamental accounting equation: Assets = Liabilities – Equity, where assets are items owned by the firm and have monetary value, liabilities represent debt obligations, and equity is the ownership value in the firm. Financial measures such as current ratio and quick ratio are formed from data on the balance sheet.

The income statement summarizes a firm’s revenue and expenses over an interval of time. This is usually a month, quarter, or year. Profit (or loss) is calculated as: Revenues – Expenses = Profit (or loss). Economic trends over time, from consecutive income statements, provide more information than one statement evaluated in isolation. Financial measures such as profit margin and interest coverage are formed from data from the income statement.

Firms’ cost accounting systems collect, analyze, and report on operational data. These are used for managing assets and operations. A traditional cost accounting method treats the firm’s direct and indirect costs differently than an activity-based method. The former spreads indirect costs (overhead) over products using very simple assumptions, and the latter relates costs to the activities that cause those costs. Firms that use traditional methods may not have accurate data for make versus buy, product mix, and other operating decisions.

By understanding accounting principles, engineering economists are better prepared to perform analyses and improve decision making for the firm.

STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

Determine retained earnings for Lavelle Manufacturing.

17-1

Current liabilities	$4,000,000
Long-term liabilities	2,000,000
Current assets	6,500,000
Fixed assets	4,000,000
Other assets	1,500,000
Common stock	2,500,000
Preferred stock	500,000

Capital surplus	1,000,000

SOLUTION

17-2

Billy Bob's Towing and Repair Service has provided the following financial information.

Cash	$ 80,000
Accounts receivable	120,000
Accounts payable	200,000
Securities	75,000
Parts inventories	42,000
Prepaid expenses	30,000
Accrued expense	15,000

Determine (a) the current ratio, (b) the quick ratio, and (c) the available working capital.

SOLUTION

17-3

Determine the current and quick ratios for Harbor Master Boats, Inc. Does the firm appear to be financially sound?

Harbor Master Boats, Inc.

Balance Sheet, January 1, 20XX

Assets		Liabilities	
Current assets		Current liabilities	
Cash	900,000	Accounts payable	2,400,000
Accounts receivable	1,100,000	Notes payable	2,000,000
Inventory	2,000,000	Accrued expense	900,000
Total current assets	4,000,000	Total current liabilities	5,300,000

Fixed assets		Long-term debt	3,000,000
Land	300,000	Total liabilities	8,300,000
Plant	2,500,000	Equity	
Equipment	6,000,000	Stock	2,000,000
Total fixed assets	8,800,000	Retained earnings	2,500,000
		Total net worth	4,500,000
Total assets	12,800,000	Total liabilities and net worth	12,800,000

SOLUTION

17-4

Rapid Delivery's financial information includes:

Acid-test ratio	1.3867
Cash on hand	$ 72,000
Accounts receivable	102,000
Market value of securities held	34,000
Inventories	143,000
Other assets	16,000
Fixed assets	215,000
Total liabilities	400,000

Determine (a) the current assets, (b) the current liabilities, (c) the total assets, and (d) the owner's equity.

SOLUTION

17-

The following information has been taken from the financial statements available for the ABC Company.

Accounts payable	$ 4,000
Accounts receivable	12,000
Income taxes	6,000

5	Owner’s equity	75,000
	Cost of goods sold	42,000
	Selling expense	10,000
	Sales revenue	80,000

Determine the net income.

SOLUTION

This financial information is from Firerock Industries’ income statement:

17-6	Revenues	
	Sales	3,200,000
	Operating revenue	2,000,000
	Nonoperating revenue	3,400,000
	Expenses	
	Total operating expenses	6,700,000
	Interest payments	500,000

Taxes paid for the year equaled $110,000. Determine (a) the net income before taxes, (b) the net profit (loss), (c) the interest coverage, and (d) the net profit ratio.

SOLUTION

17-7 Wheeler Industries had sales of $157M and returns and allowances of $21M. Operating expenses were $48M and non-operating revenues and expenses equaled the interest paid of $7M. Using the corporate tax rate from the TCJA compute the tax paid by Wheeler Industries. Construct the income statement.

SOLUTION

Wheeler Industries (problem SG17-7) had the following entries in its balance sheet at the end of last year.

Plant and equipment	$17M
(less accumulated depreciation)	9M
Retained earnings	$80M

17-8

In addition to the income statement data for this year in problem SG17-7, we also know that the firm purchased $5 million of equipment with cash and that depreciation expenses were $4 million of the $48 million in operating expenses. The firm paid no dividends and sold no stock this year. What are the entries in the balance sheet at the end of this year for:

(a) Plant and equipment?
(b) Accumulated depreciation?
(c) Retained earnings?

SOLUTION

17-9

Brown Box, Inc., manufactures shipping boxes for a wide variety of industries. Their XLLarge costs:

Direct materials costs	$0.85/unit
Direct labor costs	3.85/unit

Overhead for the entire manufacturing plant is $4,000,000 per year, and it is allocated based on direct labor costs. Total direct labor costs are $5,500,000. The demand the XLLarge is 200,000 boxes per year. Determine the cost per unit.

SOLUTION

17-10

Abby Manufacturing produces children's toys and the Dr. Dolittle Farm is a big seller. Indirect costs of $750,000 are allocated based on direct materials cost, which total $8,350,000 for the facility. The materials used in Dr. Dolittle Farm cost $7.45 per unit. The total labor (both direct and indirect) costs $9.35 per unit. The production schedule is 300,000 units for the coming year. If Abby desires a 35% profit margin, what should the Farm's wholesale price be?

SOLUTION

17-11 Denali Industries makes specialized oil well equipment. Use total direct cost as the burden vehicle, and compute the total cost per unit for each model. Total manufacturing indirect costs are $2,500,000, and there are 1000 units manufactured per year for Model A, 600 for Model K, and 500 for Model T. Use total direct cost as the burden vehicle, and compute the total cost per unit for each model.

Activity	Model A	Model K	Model T
Direct material cost	$800,000	$530,000	$630,000
Direct labor cost	$500,000	$320,000	$410,000

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Accounting

17-1 Why is it important for engineers and managers to understand accounting principles? Name a few ways that they can do so.

17-2 Explain the accounting function within a firm. What does this function do, and why is it important? What types of data does it provide?

17-3 Manipulation of financial data by the Enron Corporation was revealed in October 2001. Firm executives were sentenced to prison. Arthur Andersen, which had been one of the "Big 5" accounting firms and Enron's auditor, surrendered its licenses to practice as certified public

E accountants in August 2002. This and other scandals led to the passage of the federal Sarbanes–Oxley (SarbOx) legislation in July 2002. What are the key components of this law?

17-4 How would the information and activities in Figure 17–1 relate to potential contributions by external professional services that firms will often hire
(*a*) consulting engineers?
(*b*) management consulting firms?
(*c*) accounting firms for auditing?

17-5E Consider Figure 17–1 and external professional services that firms will often hire. What ethical questions seem likely to arise when a firms hires
(*a*) consulting engineers for design?
(*b*) management consulting firms?
(*c*) accounting firms for auditing?

17-6 Insolvency or cash flow problems in the U.S. banking industry started the financial crisis of 2007. Have significant changes in accounting standards and practices been forced through legislation? If so, what are these changes?

17-7 Accounting and finance are required topics for most management degrees, and many engineers do become managers during their careers. After graduation with a B.S. in engineering, what courses are available to you from your or another nearby university?

17-8 Using the table of contents, compare the coverage of this engineering economy text with a text for the introductory course in corporate finance (typically a junior course in BBA programs). What topics do the texts have in common? Identify the topics that seem to be covered in only one—note which topics for which course.

Balance Sheet

17-9 Develop short definitions for the following terms: balance sheet, income statement, and fundamental accounting equation.

17-10 Explain the difference between short-term and long-term liabilities.

17-11 Calculate the equity of the Gravel Construction Company if it has $10 million worth of assets. Gravel has $1.3 million in current liabilities and $2.5 million in long-term liabilities.

17-12 [A]

Matbach Industries has $930,000 in current assets and $470,000 in fixed assets less $180,000 in accumulated depreciation. The firm's current liabilities total $370,000, and the long-term liabilities $115,000.
(*a*) What is the firm's equity?
(*b*) If the firm's stock and capital surplus total $305,000, what is the value for retained earnings?

17-13

CalcTech has $1.3M in current assets and $550,000 in fixed assets less $200,000 in accumulated depreciation. The firm's current liabilities total $180,000, and the long-term liabilities $205,000.
(*a*) What is the firm's equity?
(*b*) If the firm's stock and capital surplus total $202,000, what is the value for retained earnings?

17-14 [A]

First Step Baby Monitor Company has current assets of $5.5 million and current liabilities of $2.2 million. Give the company's working capital and current ratio.

17-15

From the following data, taken from the balance sheet of Petra's Widget Factory, determine the working capital, current ratio, and quick ratio.

Cash	$110,000
Net accounts and notes receivable	325,000
Retailers' inventories	210,000
Prepaid expenses	6,000
Accounts and notes payable	300,000
(short term)	
Accrued expenses	187,000

17-16

For Gee-Whiz Devices, calculate the following: working capital, current ratio, and quick ratio.

Gee-Whiz Devices Balance Sheet Data	
Cash	$100,000
Market securities	45,000

A

Net accounts and notes receivable	150,000
Retailers' inventories	200,000
Prepaid expenses	8,000
Accounts and notes payable	315,000
(short term)	
Accrued expenses	90,000

17-17 Turbo Start has current assets totaling $1.5 million (this includes $500,000 in current inventory) and current liabilities totaling $400,000. Find the current ratio and quick ratio. Are the ratios at desirable levels? Explain.

17-18 A G

(*a*) For Evergreen Environmental Engineering (EEE), determine the working capital, current ratio, and quick ratio. Evaluate the company's economic situation with respect to its ability to pay off debt.

EEE Balance Sheet Data ($1000s)	
Cash	$120,000
Securities	40,000
Accounts receivable	110,000
Inventories	300,000
Prepaid expenses	3,000
Accounts payable	351,000
Accrued expenses	89,000

(*b*) The entries to complete EEE's balance sheet include:Construct EEE's balance sheet.

More EEE Balance Sheet Data ($1000s)	
Long-term liabilities	$ 220,000
Land	25,000
Plant and equipment	510,000
Accumulated depreciation	210,000
Stock	81,000
Capital surplus	15,000

Retained earnings	Value not given

(*c*) What are EEE's values for total assets, total liabilities, and retained earnings?

(*a*) For J&W Graphics Supply, compute the current ratio. Is this a financially healthy company? Explain.

J&W Graphics Supply Balance Sheet Data ($1000s)	
Assets	
Cash	$1740
Accounts receivable	2500
Inventories	900
Bad debt provision	−75
Liabilities	
Accounts payable	1050
Notes payable	500
Accrued expenses	125

17-19 (*b*) The entries of complete J&W's balance sheet include:

More J&W Balance Sheet Data ($1000s)	
Long-term liabilities	$950
Land	475
Plant and equipment	3100
Accumulated depreciation	1060
Stock	680
Capital surplus	45
Retained earnings	Value not given

Construct J&W's balance sheet.

(*c*) What are J&W's values for total assets, total liabilities, and total earnings?

For Sutton Manufacturing, determine the current ratio and the quick ratio. Are these values acceptable? Why or why not?

17-20 A

Assets		Liabilities	
Current assets		Current liabilities	
Cash	$ 870	Notes payable	$ 500
Accounts receivable	450	Accounts payable	600
Inventory	1200	Accruals	200
Prepaid expenses	60	Taxes payable	30
		Current portion long-term debt	100
Total current assets	2580	Total current liabilities	1430
Net fixed assets		Long-term debt	2000
Land	1200	Officer debt (subordinated)	200
Plant and equipment	3800	Total liabilities	3630
(less accumulated depreciation)	−1000	**Equity**	
Other assets		Common stock	1670
Notes receivable	200	Capital surplus	400
Intangibles	120	Retained earnings	1200
		Total equity	3270
Total assets	6900	Total liabilities and equity	6900

17-21 If a firm has a current ratio less than 2.0 and a quick ratio less than 1.0, will the company eventually go bankrupt and out of business? Explain your answer.

17-22 What is the advantage of comparing financial statements across periods or against industry benchmarks over looking at statements associated with a single date or period?

Income Statement

17-23 Laila's Surveying Inc. had revenues of $1.5 million in 2016. Expenses totaled $900,000. What was her net profit (or loss) if the tax rate is 38%?

17-24 A Bohr Paint Company has annual sales of $12 million per year. If there is a profit of $5000 per day with 7 days per week operation, what is the total yearly business expense? All calculations are on a before-tax basis.

17-25 For Magdalen Industries, compute the net income before taxes and net profit (or loss). Taxes for the year were $7.5 million.
(*a*) Calculate net profit for the year.
(*b*) Construct the income statement.
(*c*) Calculate the interest coverage and net profit ratio. Is the interest coverage acceptable? Explain why or why not.

Magdalen Industries Income Statement Data ($M)	
Revenues	
Total operating revenue	$124
(including sales of $48 million)	
Total nonoperating revenue	36
Expenses	
Total operating expenses	70
Total nonoperating expenses	35
(interest payments)	

17-26 A Find the net income of Turbo Start (Problem 17-17) given the following data from the balance sheet and income statement.

Turbo Start Data ($1000s)	
Accounts payable	$ 200
Selling expense	500
Sales revenue	5000
Owner’s equity	2400
Income taxes	315
Cost of goods sold	3000
Accounts receivable	500

17-27 The general ledger of the Fly-Buy-Nite (FBN) Engineering Company contained the following account balances. Construct an income statement. What is the net income before taxes and the net profit (or loss) after taxes? FBN has a tax rate of 21%.

	Amount ($1000s)
Administrative expenses	$ 2,750

Subcontracted services	15,000
Development expenses	900
Interest expense	200
Sales revenue	35,000
Selling expenses	4,500

17-28 A For Andrew's Electronic Instruments, calculate the interest coverage and net profit ratio. Is Andrew's business healthy?

Income Statement for Andrew's Electronics for End of Year 2010 ($1000s)	
Revenues	
Operating revenues	
Sales	$395
(*minus*) Returns	−15
Total operating revenues	380
Nonoperating revenues	
Interest receipts	50
Stock revenues	25
Total nonoperating revenues	75
Total revenues, R	455
Expenses	
Operating expenses	
Cost of goods and services sold	
Labor	200
Materials	34
Indirect cost	68
Selling and promotion	20
Depreciation	30
General and administrative	10
Lease payments	10
Total operating expenses	372

Nonoperating expenses	
Interest payments	22
Total nonoperating expenses	22
Total expenses, E	394
Net income before taxes, R − E	61
Incomes taxes	17
Net profit (loss) for the year 2010	44

Linking Balance Sheet and Income Statement

17-29 Use your own experiences or a search engine to provide specific examples where the balance sheet and income statement are linked.

17-30 G Lithium batteries are popular and widely used. Lithium is typically mined from salt flats where water is scarce, yet lithium mining requires large amounts of water. Are the environmental costs of lithium captured in the financial statements of companies that produce lithium batteries?

17-31 Sutton Manufacturing (balance sheet at the end of last year in Problem 17-20) had the following entries in this year's income statement.

Depreciation	$420,000
Profit	480,000

In addition, we also know the firm purchased $800,000 of equipment with cash. The firm paid $200,000 in dividends this year. What are the entries in the balance sheet at the end of this year for

(*a*) plant and equipment?

(*b*) accumulated depreciation?

(*c*) retained earnings?

Magdalen Industries (Problem 17-25) had the following entries in its balance sheet at the end of last year.

Plant and equipment	$15 million
(less accumulated depreciation)	8 million

Retained earnings	60 million

17-32 A In addition to the income statement data for this year in Problem 17-25, we also know that the firm purchased $3 million of equipment with cash and that depreciation expenses were $2 million of the $70 million in operating expenses listed in Problem 17-26. The firm paid no dividends this year.

What are the entries in the balance sheet at the end of this year for

(*a*) plant and equipment?
(*b*) accumulated depreciation?
(*c*) retained earnings?

Allocating Costs

17-33 Categorize each of the following costs as direct or indirect. Assume that a traditional costing system is in place.

Machine run costs	Cost to market the product
Machine depreciation	Cost of storage
Material handling costs	Insurance costs
Cost of materials	Cost of product sales force
Overtime expenses	Engineering drawings
Machine operator wages	Machine labor
Utility costs	Cost of tooling and fixtures
Support (administrative)	
staff salaries	

17-34 A RLW-II Enterprises estimated that indirect manufacturing costs for the year would be $75 million and that 15,000 machine-hours would be used.

(*a*) Compute the predetermined indirect cost application rate using machine hours as the burden vehicle.
(*b*) Determine the total cost of production for a product with direct material costs of $2 million, direct-labor costs of $2.5 million, and 200 machine-hours.

17-35 Philippe Francois Inc. produces concrete sundials—5000 were produced in a recent production run. The run required 1500 machine hours, three "set-ups" of the mixing equipment, and 100 hours of final inspection time. Costs are estimated as follows: $50 per machine hour, $3000 per "set-up," and $20 per inspection hour. Direct materials and direct labor total $95 per sundial. Indirect expenses total 45% of the direct material and labor costs.
(*a*) Determine the "cost to produce" each unit.
(*b*) At a 15% profit level what is the unit sales price?
(*c*) With six runs per year what is the total annual profit for this product?

17-36 A

LeGaroutte Industries makes industrial pipe manufacturing equipment. Use direct-labor hours as the burden vehicle, and compute the total cost per unit for each model given in the table. Total manufacturing indirect costs are $15,892,000, and there are 100,000 units manufactured per year for Model *S*, 50,000 for Model *M*, and 82,250 for Model *G*.

Item	**Model *S***	**Model *M***	**Model *G***
Direct-material costs	$3,800,000	$1,530,000	$2,105,000
Direct-labor costs	600,000	380,000	420,000
Direct-labor hours	64,000	20,000	32,000

17-37 Par Golf Equipment Company produces two types of golf bag: the standard and deluxe models. The total indirect cost to be allocated to the two bags is $35,000. Determine the net revenue that Par Golf can expect from the sale of each bag.
(*a*) Use direct-labor cost to allocate indirect costs.
(*b*) Use direct-materials cost to allocate indirect costs.

Data Item	**Standard**	**Deluxe**

Direct-labor cost	$60,000	$70,000
Direct-material cost	40,000	47,500
Selling price	70	105
Units produced	1800	1400

Minicase

17-38 G	Find a real world example where a firm has included environmental remediation costs on its balance sheet or in a note to the balance sheet. Briefly describe what was done. What percentage of the firm's assets and net profit are these costs?

CASES

The following cases from *Cases in Engineering Economy* (www.oup.com/us/newnan) are suggested as matched with this chapter.

CASE 4	**Balder-Dash Inc.**
	Standard cost and allocated costs versus true marginal cost (IEs).
CASE 52	**Aunt Allee's Jams and Jellies**
	Product costing needs activity based costs.

APPENDIX A

INTRODUCTION TO SPREADSHEETS

Computerized spreadsheets are available nearly everywhere, and they can be easily applied to economic analysis. In fact, spreadsheets were originally developed to analyze financial data, and they are often credited with initiating the explosive growth in demand for desktop computing.

A spreadsheet is a two-dimensional table, whose cells can contain numerical values, labels, or formulas. The software automatically updates the table when an entry is changed, and there are powerful tools for copying formulas, creating graphs, and formatting results.

THE ELEMENTS OF A SPREADSHEET

A spreadsheet is a two-dimensional table that labels the columns in alphabetical order A to Z, AA to AZ, BA to BZ, etc. The rows are numbered from 1 to 65,536 or higher. Thus a *cell* of the spreadsheet is specified by its column letter and row number. For example, A3 is the third row in column A and AA6 is the sixth row in the twenty-seventh column. Each cell can contain a label, a numerical value, or a formula.

A *label* is any cell where the contents should be treated as text. Arithmetic cannot be performed on labels. Labels are used for variable names, row and column headings, and explanatory notes. In Excel any cell that contains more than a simple number, such as 3.14159, is treated as a label, unless it begins with an =, which is the signal for a formula. Thus 2*3 and B1+B2 are labels. Meaningful labels can be wider than a normal column. One solution is to allow those cells to “wrap” text, which is one of the “alignment” options. The table heading row (row 8) in Example A–1 has turned this on by selecting row 8, right-clicking on the row, and selecting wrap text under the alignment tab.

A *numerical value* is any number. Acceptable formats for entry or display include percentages, currency, accounting, scientific, fractions, date, and time. In addition the number of decimal digits, the display of $ symbols, and commas for "thousands" separators can be adjusted. The format for cells can be changed by selecting a cell, a block of cells, a row, a column, or the entire spreadsheet. Then right-click on the selected area, and a menu that includes "format cells" will appear. Then number formats, alignment, borders, fonts, and patterns can be selected.

Formulas must begin with an =, such as 3*4^2 or =B1+B2. They can include many functions—financial, statistical, trigonometric, etc. (and others can be defined by the user). The formula for the "current" cell is displayed in the formula bar at the top of the spreadsheet. The value resulting from the formula is displayed in the cell in the spreadsheet.

Often the printed-out spreadsheet will be part of a report or a homework assignment and the formulas must be explained. Here is an easy way to place a copy of the formula in an adjacent or nearby cell. (1) Convert the cell with the formula to a label by inserting a space before the = sign. (2) Copy that label to an adjacent cell by using cut and paste. Do not drag the cell to copy it, as any formula ending with a number (even an address like B4) will have the number automatically incremented. (3) Convert the original formula back into a formula by deleting the space.

DEFINING VARIABLES IN A DATA BLOCK

The cell A1, top left corner, is the HOME cell for a spreadsheet. Thus, the top left area is where the data block should be placed. This data block should have every variable in the spreadsheet with an adjacent label for each. This data block supports a basic principle of good spreadsheet modeling, which is to use variables in your models.

The data block in Example A–1 contains *entered data*—the loan amount (A1), the number of payments (A2), and the interest rate (A3), and *computed data*—the payment (A4). Then instead of using the loan amount of $5000 in a formula, the cell reference A1 is used. Even if a value is referenced only once, it is better to include it in the data block. By using one location to define each variable, you can change any value at one place in the spreadsheet and have the entire spreadsheet instantly recomputed.

Even for simple homework problems you should use a data block.

1. You may be able to use it for another problem.
2. Solutions to simple problems may grow into solutions for complex problems.
3. Good habits, like using data blocks, are easy to maintain once they are established.
4. It makes the assumptions clear if you've estimated a value or for grading.

In the real world, data blocks are even more important. Most problems are solved more than once, as more and more accurate values are estimated. Often the spreadsheet is revised to add other variables, time periods, locations, etc. Without data blocks, it is hard to change a spreadsheet and the likelihood of missing a required change skyrockets.

If you want your formulas to be easier to read, you can name your variables. *Note*: In Excel, the cell's location or name is displayed at the left of the formula bar. Variable names can be entered here. They will then automatically be applied if cell addresses are entered by the point and click method. If cell addresses are entered as A2, then A2 is what is displayed. To change a displayed A2 to the name of the cell (LoanAmount), the process is to click on Insert, click on Name, click on Apply, and then select the names to be applied.

COPY COMMAND

The copy command and relative/absolute addressing make spreadsheet models easy to build. If the range of cells to be copied contains only labels, numbers, and functions, then the copy command is easy to use and understand. For example, the formula =EXP(1.9) would be copied unchanged to a new location. However, cell addresses are usually part of the range being copied, and their absolute and relative addresses are treated differently.

An *absolute address* is denoted by adding $ signs before the column and/or row. For example in Figure A–1a, $A$4 is the absolute address for the interest rate. When an absolute address is copied, the column and/or row that is fixed is copied unchanged. Thus $A$4 is completely fixed, $A4 fixes the column, and A$4 fixes the row. One common use for absolute addresses is any data block entry, such as the interest rate. When entering or editing a formula, changing between A4, $A$4, A$4, $A4, and A4 is most easily done using the F4 key, which scrolls an address through the choices.

In contrast, a *relative address* is best interpreted as directions from one cell to another. For example in Figure A–1a, the balance due in year t equals the balance due in year $t - 1$ minus the principal payment in Year t. Specifically for the balance due in Year 1, D10 contains =D9−C10. From cell D10, cell D9 is one row up and C10 is one column to the left, so the formula is really (contents of 1 up) minus (contents of 1 to the left). When a cell containing a relative address is copied to a new location, it is these directions that are copied to determine any new relative addresses. So if cell D10 is copied to cell F14, the formula is =F13−E14.

Thus to calculate a loan repayment schedule, as in Figure A–1, the row of formulas is created and then copied for the remaining years.

EXAMPLE A–1

Four repayment schedules for a loan of $5000 to be repaid over 5 years at an interest rate of 8% were shown in Table 3–1. Use a spreadsheet to calculate the amortization schedule for the constant principal payment option.

SOLUTION

The first step is to enter the loan amount, number of periods, and interest rate into a data block in the top left part of the spreadsheet. The next step is to calculate the constant principal payment amount, which was given as $1252.28 in Table 3–1. The factor approach to finding this value is given in Chapter 3 and the spreadsheet function is explained in Chapter 4.

The next step is to identify the columns for the amortization schedule. These are the year, interest owed, principal payment, and balance due. Because some of these labels are wider than a normal column, the cells are formatted so that the text wraps (row height increases automatically). The initial balance is shown in the Year-0 row.

Next, the formulas for the first year are written, as shown in Figure A–1a. The interest owed (cell B10) equals the interest rate ($A$4) times the balance due for Year 0 (D9). The principal payment (cell C10) equals the annual payment ($A$6) minus the interest owed and paid (B10). Finally, the balance due (cell D10) equals the balance due for the previous year (D9) minus the principal payment (C10). The results are shown in Figure A–1a.

Now cells A10 to D10 are selected for Year 1. By dragging down on the right corner of D10, the entire row can be copied for Years 2 through 5. Note that if you use cut and paste, then you must complete the year column separately (dragging increments the year, but cutting and pasting does not). The results are shown in Figure A–1b.

	A	B	C	D	E
1	Entered Data				
2	5000	Loan Amount			
3	5	Number of Payments			
4	8%	Interest Rate			
5	Computed Data				
6	$1,252.28	Loan Payment			
7					
8	Year	Interest Owed	Principal Payment	Balance Due	
9	0			5000.00	
10	1	400.00	852.28	4147.72	=D9−C10
11					
12		=$A$4*D9		=$A$6−B10	

(a)

	A	B	C	D	E
8	Year	Interest Owed	Principal Payment	Balance Due	
9	0			5000.00	
10	1	400.00	852.28	4147.72	
11	2	331.82	920.46	3227.25	
12	3	258.18	994.10	2233.15	
13	4	178.65	1073.63	1159.52	
14	5	92.76	1159.52	0.00	=D13−C14
15					
16		=$A$4*D13		=$A$6−B14	

(b)

FIGURE A-1 *(a)* Year 1 amortization schedule. *(b)* Completed amortization schedule.

This appendix has introduced the basics of spreadsheets. Chapter 2 uses spreadsheets and simple bar charts to draw cash flow diagrams. Chapters 4 to 15 each have spreadsheet sections. These are designed to develop spreadsheet modeling skills and to reinforce your understanding of engineering economy.

As spreadsheet packages are built around using the computer mouse to click on cells and items in charts, there is usually an intuitive connection between what you would like to do and how to do it. The best way to learn how to use the spreadsheet package is to simply play around with it. In addition, as you look at the menu choices, you will find new commands that you hadn’t thought of but will find useful.

APPENDIX B

TIME VALUE OF MONEY CALCULATIONS USING SPREADSHEETS AND CALCULATORS

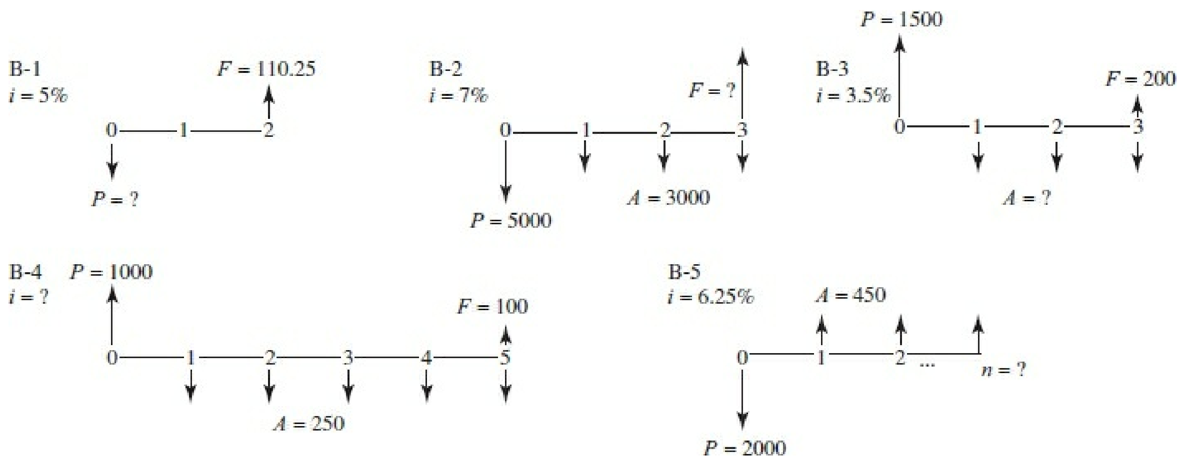
Spreadsheets and financial calculators are perfect tools for solving economic problems with financial functions that are very similar to the engineering economy factors—but more powerful. Now there are even smart phone apps with the same capabilities. Some types of programmable calculators can be used in the FE exam, making interest tables unnecessary.

This appendix focuses on the use of spreadsheets and calculators to solve simple time value of money problems. Included are the following topics:

• Basics of a 5-Button Spreadsheet Calculator
• Spreadsheets, Calculators, and the FE Exam
• Additional Spreadsheet and Calculator Capabilities
• More Complex Examples
• Possible Errors and Their Solutions

BASICS OF A 5-BUTTON SPREADSHEET CALCULATOR

Figure B-1 shows a variety of cash flow diagrams and a five button calculator template that can be used to solve economic problems quickly and easily.



For example B-1, the input variables from the cash flow diagram are placed in cells B2 through F2, with the interest rate in cell B2, the number of periods in cell C2, etc. Cell H2 calculates the PV using the appropriate cell addresses for the input variables. The equations used in column H are shown in column I. Spreadsheets such as Excel and Google docs use this type of format; others such as Open Office and Quattro Pro use a slightly different format for equivalent functions. Any of the spreadsheet functions may be copied to a new row and then new data can be inserted to solve similar problems. Thus, the five *annuity functions* in Figure B-1 are collectively a 5-button financial calculator. The same inputs may be keyed into a financial or programmable calculator to obtain identical results. When using a calculator, it is important to enter all variables if you do not know what value was entered last. You could have an unknown value residing in the calculator's memory. Additional examples of using the 5-button spreadsheet calculator can be found in Chapters 3 and 4.

EXAMPLE B1-B5

5-BUTTON SOLUTION

	A	B	C	D	E	F	G	H	I
1	Example	*i*	*N*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	B-1	5.0%	2	0		110.25	PV	-$100.00	=PV(B2,C2,D2,F2)
3	B-2	7.0%	3	-3000	-5000		FV	$15,770	=FV(B3,C3,D3,E3)
4	B-3	3.5%	3		1500	200	PMT	-$599.79	=PMT(B4,C4,E4,F4)
5	B-4		5	-250	1000	100	RATE	5.15%	=RATE(C5,D5,E5,F5)
6	B-5	6.25%		450	-2000	0	N	5.37	=NPER(B6,D6,E6,F6)

FIGURE B-1 Cash flow diagrams and the 5-button spreadsheet calculator.

EXAMPLE B-1 FACTOR SOLUTION

The present value of a cash flow of \$110.25, two years in the future, at an interest rate of 5% is −\$100.00. Note that this follows the convention of positive numbers for inflows and negative numbers for outflows. Use of the spreadsheet or calculator avoids looking up factors, which is a primary source of errors.

Using interest factors,

B-1 $P = 110.25(P/F, 5\%, 2) = 110.25(0.9070) = \$100.00$

EXAMPLE B-2 FACTOR SOLUTION

The FV function can determine the future value of both a P and an A in a single calculation. Note that negative values of PV and PMT return a positive value for FV of \$15,770.

Solving B-2 using factors and the interest tables requires two lookups, two multiplications, and one addition, because the tabulated factors can only convert from one type of cash flow to another.

B-2 $F = 3000(F/A, 7\%, 3)+5000(F/P, 7\%, 3) = 3000(3.215)+5000(1.225) = \$15,770$

EXAMPLE B-3 FACTOR SOLUTION

A uniform cash flow can be determined using the PMT function, with P and F being entered in a single calculation.

Using factors requires several steps, as shown here. If the 3.5% table were not available, this would also require interpolation between the 3% and the 4% tables. Due to the round off of the interest factors to 4 significant digits, the factor answer is slightly less precise.

B-3 $A = 1500(A/P, 3.5\%, 3) + 200(A/F, 3.5\%, 3) = 1500(0.3569) +$

200(0.3219)

= $599.73

EXAMPLE B-4 FACTOR SOLUTION

An unknown interest rate can be solved in a single computation, even though there are three types of cash flows: a P, an F, and an A. The RATE function requires at least one positive cash flow (inflow) and at least one negative cash flow (outflow). Using factors and interest tables would require solving the following equation (or a similar one):

B-4 $0 = 1000 - 250(P/A, i, 5) + 100(P/F, i, 5)$

This requires either an iterative solution or using interpolation.

$$\text{at } 5\%, 0 = 1000 - 250(4.329) + 100(0.7835) = -3.90$$

$$\text{at } 6\%, 0 = 1000 - 250(4.212) + 100(0.7473) = 21.7$$

Interpolating, $i = 5.15\%$

Spreadsheets and calculators are much more efficient than factors when solving for unknowns that require interpolation, such as the interest rate.

EXAMPLE B-5 FACTOR SOLUTION

The NPER function will display the answer in fractional periods; in this case, 5.37 periods. Using factors to solve for the number of periods again requires the use of interpolation or iterative solutions.

B-5 $0 = -2000 + 450(P/A, 6.25\%, n)$

$(P/A, 6.25\%, n) = 2000/450 = 4.444$

This will require interpolation to find two variables, one interpolation to find a set of values for n at 6.25% (because there is no 6.25% table available), and another for determining the answer. These will not be performed here, but yield an answer of 5.37 periods.

There are many advantages to using spreadsheets and financial calculators instead of, or in addition to, using the interest tables. These include:

• Time is saved because problems are entered directly without having to look up factors in tables, then copying those factors. Problems are solved with the use of a few key strokes.
• There is less chance for error because there are fewer steps. Many errors are made in looking up factors due to looking down the wrong column or row. Transcribing errors, or finding a factor but writing it down wrong, are minimized.
• There is no need to interpolate when determining an interest rate or number of periods. This greatly speeds this type of calculation.
• Students who use a spreadsheet will leave an electronic, student accessible paper trail. Once a spreadsheet is successfully built, such as the one in Figure B-1, students can access these saved files for studying, performing homework, and perhaps during a test. This can aid in students' learning and comprehension by focusing time and attention on setting up problems rather than spending time doing financial arithmetic.

SPREADSHEETS, CALCULATORS, AND THE FE EXAM

The naming of variables is slightly different between engineering economy, spreadsheets, and calculators. Table B-1 shows the typical naming conventions.

Spreadsheets use Equation B-1 to solve for the unknown variable. Equation B-2 rewrites this in standard factor notation.

$$PMT\left[\frac{1-(1+i)^{-n}}{i}\right]+FV(1+i)^{-n}+PV=0 \quad \text{(B-1)}$$

$$A(P/A,i,n)+F(P/F,i,n)+P=0 \quad \text{(B-2)}$$

Financial calculators use Equation B-3 to solve for the unknown variable. Note that Equation B-3 is written so that *i* is entered as a percentage, so 8 is entered for 8%, which is how the interest rate is entered for financial calculators. How the interest rate is entered is the single difference between how spreadsheets and calculators solve time value of money problems.

$$PMT\left[\frac{1-(1+i/100)^{-n}}{i/100}\right]+FV(1+i/100)^{-n}+PV=0 \quad \text{(B-3)}$$

Since the factors are all positive, one of *PMT*, *FV*, or *PV* (or *A*, *F*, and *P*) must be different in sign from the other two. If two positive cash flows are entered to solve for the third cash flow, then that third value must be negative to solve the equation. These equations are the foundation for the sign convention that is used in financial calculators and the spreadsheet annuity functions PV, FV, PMT, RATE, and NPER.

There are three classes of calculators that are of interest. First, there are many *financial* or *business* calculators that have buttons labeled *i*, *N*, *PMT*, *PV*, and *FV* or equivalents. The button for the uniform series cash flow, *A*, is labeled *PMT*, because one of the most important uses of these calculators is for calculating loan payments (*PMT*). Examples include the Texas Instrument BAII Plus and BAII Plus Professional, the Sharp EL-738C, and the Hewlett Packard 10bII. We no longer recommend the HP 12C for new purchases since when solving for *n*, the 12C rounds fractional values of *n* up to an integer value (see Example B–8). These calculators also typically have the capability to find the NPV of complex cash flow patterns, but we suggest using spreadsheets for more complex patterns.

TABLE B-1 Naming Conventions in Engineering Economy, Spreadsheets, and Calculators

Variable	Engineering Economy	Spreadsheets	Typical Calculator Button
Present value	*P*	PV	PV

Future value	*F*	FV	FV
Uniform series	*A*	PMT	PMT
Interest rate	*i*	RATE	I/Y
Number of periods	*n*	NPER	N

There are also *graphing* calculators that have the time value of money calculations in a menu. There are menu entries for *i*, *N*, *PMT*, *PV*, and *FV* or equivalents. Example calculators include the Texas Instrument 83 and 84 Plus.

Finally, there are programmable scientific calculators where Equation B-3 can be entered to create the menu that is built into the graphing calculators. For example, Chapter 17 of the manuals for the Hewlett Packard 33s and 35s details this. These chapters use a non-standard notation of *B* or balance for *PV* and *P* or payment for *PMT*. We suggest using standard notation and substituting *P* for *B* and *A* for *P*. Note that on the HP 35s this changes the checksum from 382E to 8DD6. The HP 33s and HP 35s calculators are currently allowed for use on the Fundamentals of Engineering (FE) exam, while the financial and graphing calculators are not.

There are too many variations of labeling and physical layout of the keys and too many financial calculators for a text to provide details. However, the website www.TVMCalcs.com provides good tutorials on financial calculators, and calculator manuals are available for download from the manufacturers.

Some calculators are shipped from the factory with a setting of 12 months per year, so that *n* will be entered as years for loans with monthly payments. We recommend that this be changed to 1 payment per "year or period" and left on this setting. Then you know to enter *n* as the number of periods or payments, and there is no confusion for problems with different length periods.

Phone apps are now widely available that perform as financial calculators, making it possible to carry the capability anywhere.

ADDITIONAL SPREADSHEET AND CALCULATOR CAPABILITIES

When initially building a spreadsheet, equations must be entered into a cell to solve the problem. In Figure B-1, the following equation is entered into cell H2 to solve for the present value:

= PV(rate, nper, pmt, [fv], [type])

The [ ] denotes that these are optional values where the default is zero. If nothing is entered for the future value, it will be assumed to have a zero value. If no value is entered for the type, periodic cash flows (*A*) will be assumed to occur at the end of the time periods.

[type] = 0 (default)	End of period cash flows
[type] > 0	Beginning period cash flows

Most financial calculators can be set to either end of period (default) or beginning of period cash flows.

The RATE function requires a combination of positive and negative inputs in order to calculate an interest rate.

= RATE(nper, pmt, pv, [fv], [type], [guess])

Note that the RATE function contains an optional input *guess*, where you can guess what the rate will be. If you do not enter a value, the default is 10%. If the RATE function does not provide an answer, change the guess. An entry is usually not required. For more see the last section of this Appendix.

MORE COMPLEX EXAMPLES

The following examples demonstrate problems that are more complex, but still make use of the basic spreadsheet annuity functions.

• Example B–6 requires conversion of annual rates and periods to monthly.
• Example B–7 is a PV problem including an initial payment and a salvage value.
• Example B–8 solves a discounted payback problem.
• Example B–9 includes cash flows at the beginning of the period.
• Example B–10 is an EAC problem with multiple cash flows.
• Example B–11 involves the doubling, tripling, and quadrupling of money.

EXAMPLE B-6

Kris is buying a used car and needs to find the monthly payment. The loan is for \$14,000, and the dealer is willing to offer financing for 5 years at 6.5% interest. What is the monthly payment?

SOLUTION

First, we need to convert the period and interest rate to months. There are 60 (= 5 × 12) monthly payments at a monthly interest rate of 6.5%/12 = 0.5417%. The required monthly payment is \$273.93.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	B-6	0.5417%	60		-14000	0	PMT	\$273.93	=PMT(B2,C2,E2,F2)
3									PMT(rate, nper, pv, [fv], [type])

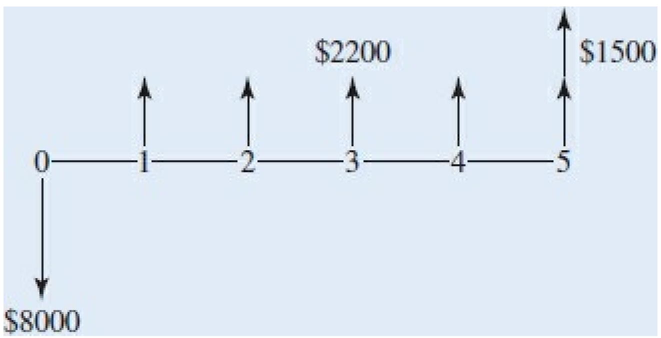
This problem can also be solved using a financial or programmable calculator. The keys on most calculators are the same as the spreadsheet variables, so $PMT(i, n, P, F) = PMT(0.5417\%, 60, -14000, 0) = \$273.93$.

Note that the calculator requires the same sign convention as spreadsheets.

EXAMPLE B-7

Automating a process will cost $8000 now, but it will save $2200 annually for 5 years. The machinery will have a salvage value of $1500. What is the present worth of the machinery if the interest rate is 10%?

SOLUTION



Let us first solve this with the tabulated factors.

$$\begin{aligned} PW &= -8000 + 2200(P/A, 10\%, 5) + 1500(P/F, 10\%, 5) \\ &= -8000 + 2200(3.791) + 1500(0.6209) \\ &= -8000 + 8340 + 931 = \$1271 \end{aligned}$$

The spreadsheet can find the combined present value of the $2200 annual savings and the salvage value of $1500 because the interest rate and number of periods are the same.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	B-7	10.0%	5	2200		1500	PV	-9271.11	=PV(B2,C2,D2,F2)
3					-8000		PV	1271.11	=-H2+E3

Notice that the present value of the future cash flows is negative when the A and F values are entered as positive numbers. This comes from solving

Equation B-1. Since in this case we want the *positive* PV of the returns to the project, we can change the sign of the answer in step 2, subtracting the $8000 initial cash flow to find the PV of $1271.

Using a calculator, $PV(i, n, A, F) = PV(10, 5, 2200, 1500) = -9271.11$. The calculator finds the combined present worth of the $2200 annual savings and the salvage value of $1500. The initial cash flow of $8000 is subtracted to find the PV of $1271.11. Like the spreadsheet, positive values for A and F produce a negative value for P.

EXAMPLE B-8

A firm might purchase a computerized quality control system. The proposed system will cost $76,000, but is expected to save $22,000 each year in reduced overtime. The firm requires that all cost reduction projects have a discounted payback of no more than 4 years with a 10% interest rate. Should the firm invest in the new system?

SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	B-8	10.0%		22000	-76000	0	NPER	4.45

The time for the system to pay for itself can be found using the NPER function; the discounted payback period is 4.45 years. Because a proposed project can have a discounted payback of no more than 4 years, the project would not be approved.

The problem can also be solved with a calculator using $N(i, A, P, F) = N(10\%, 22000, -76000, 0) = 4.45$ years. Note that the HP 12C calculator will report 5 years. In fact, the HP 12C will report $N = 5$ years when the payback period is as small as 4.005 periods.

EXAMPLE B-9

Some equipment is needed for a 4-year project. It can be leased for $15,000 annually, or it can be purchased for $60,000 at the beginning and sold for $35,000 at the end. What is the rate of return for owning the equipment rather than leasing it?

SOLUTION

Because the lease payments occur at the beginning of each period, [type] is set to any number above 0 (such as 1) to automatically shift the cash flows into *beginning* rather than *end* of period. This affects only the *A* and not the *F*.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	B-9		4	15000	-60000	35000	RATE	23.49%

A financial calculator must also be shifted into *begin* rather than *end* mode, then enter $i(n, A, P, F) = i(4, 15000, -60000, 35000) = 23.49\%$.

Without knowledge that the rate is nearly 25%, using the tabulated factors would require multiple trial solutions to solve the following 3-factor equation.

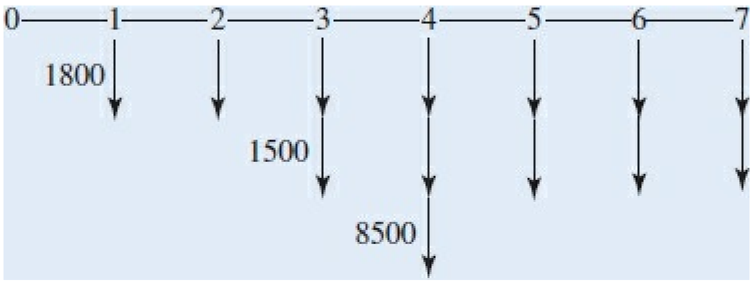
$$0 = -60{,}000 + 15{,}000(F/P, i, 1)(P/A, i, 4) + 35{,}000(P/F, i, 4)$$

EXAMPLE B-10

What is the EAC to keep a piece of new equipment operating? The warranty covers repair costs for the first 2 years, but expected repair costs are $1500 per year for the rest of the 7-year life. Normal maintenance is expected to cost $1800 annually. There is an overhaul costing $8500 at the end of year 4. The firm’s interest rate is 8%.

SOLUTION

Whether solved with tables, spreadsheets, or a financial calculator, the first step is to draw the cash flow diagram.



Let us first solve this with the tabulated factors.

$$\begin{aligned}\text{EAC} &= 1800 + 1500(F/A, 8\%, 5)(A/F, 8\%, 7) + 8500(P/F, 8\%, 4)(A/P, 8\%, 7)\\ &= 1800 + 1500(5.867)(0.1121) + 8500(0.7350)(0.1921)\\ &= 1800 + 986.5 + 1200 = \$3986.5\end{aligned}$$

With a spreadsheet, it is easiest to separately calculate the EUAC values for the repair costs and the overhaul.

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Repair	8%	5	1500	0		FV	-8799.9
3		8%	7		0	-8799.9	PMT	986.23
4								
5	Overhaul	8%	4	0		8500	PV	-6247.75
6		8%	7		-6247.75	0	PMT	1200.02
7								
8	Maint.			1800			EAC	$3,986.25

This is very similar with a financial calculator. For the repair costs, *FV* (*i*, *n*, *A*, *P*) = *FV* (8, 5, 1500, 0) = −8, 799.90. Since the *i* and *P* values are unchanged, they do not need to be re-entered. The only entry needed is *N* = 7, since the *F* was just calculated. *PMT*(*i*, *n*, *P*, *F*) = *PMT*(8%, 7, 0, −8799.90) = $986.23.

For the overhaul costs the *i* value need not be re-entered, $PV(i, n, A, F) = PV(8\%, 4, 0, 8500) = -6,247.75$. Now *N* is changed to 7 and *F* to 0, then solving for $PMT(i, n, P, F) = PMT(8\%, 7, -6247.75, 0) = \$1200.02$.

Maintenance is $1800 per year.

Adding the three values is the same as before, and the EAC is $3986.25.

EXAMPLE B-11

How long does it take for your money to double if you can earn 7% on your investment? Triple? Quadruple?

SOLUTION

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	B-11	7.0%		0	-1	2	NPER	10.24
3		7.0%		0	-1	3	NPER	16.24
4		7.0%		0	-1	4	NPER	20.49

Using a financial calculator, the setup is similar; for doubling, $n(i, A, P, F) = n(7, 0, -1, 2) = 10.24$.

The problem can also be solved using functions, but requires interpolation, or can be solved mathematically.

$$\begin{aligned} F &= P(1+i)^n \\ 2 &= 1(1.07)^n \\ \ln(2) &= n \ln(1.07) \\ n &= \ln(2)/\ln(1.07) = 0.6931/0.06766 = 10.24 \end{aligned}$$

POSSIBLE ERRORS AND THEIR

SOLUTIONS

RATE and IRR

When using RATE or IRR, the #NUM! error may occur. Both of these functions are iterative; they home in on an answer using multiple attempts or iterations. The default starting point (guess) is 10%, and if a specific answer cannot be found within 20 iterations, the error value appears. This usually happens for interest rates that are very high or very negative, but even moderate answers can cause difficulty. The problem is usually solved by trying values for [guess] that approximate the answer.

EXAMPLE B-12

An account having $800,000 will be used to make a series of 20 annual payments of $150,000 each. What interest rate is needed to fund the payments?

SOLUTION

A simple rate problem can unexpectedly result in an error message, but are usually corrected by inserting a [guess]. Notice that the guess does not need to be very accurate.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	B-12		20	150,000	-800,000	0	RATE	#NUM!	=RATE(C2,D2,E2,F2)
3			20	150,000	-800,000	0	RATE	18.10%	=RATE(C3,D3,E3,F3,0,

NPER

The function NPER is also iterative—like RATE and IRR. Unfortunately, NPER does not have a [guess] option to help. NPER sometimes returns an error if the value is very large. The answer can sometimes be found by using GOAL SEEK, as shown in Example B–13.

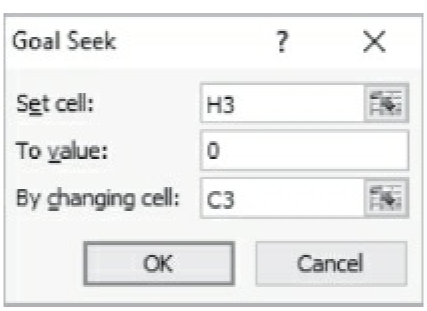
EXAMPLE B-13

An account having $600,000 will be used to make a series of annual payments of $45,000 each. The account returns 7.5% per year. How many years will payments be made?

SOLUTION

Using NPER returns an error showing that it could not solve the problem. NPER is increasing very rapidly as the interest increases by a very small amount. This can be solved by finding a value of *n* where the PV or FV = 0. GOAL SEEK returns a value of 617 plus years. SOLVER can also be used to provide the same answer.

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	B-13	8%		45,000	-600,000	0	NPER	#NUM!	=NPER(B2,D2,E2)
3		8%	617.3	45,000	-600,000	0	FV	0.00%	=FV(B5,C5,D5,E5)



STUDENT STUDY GUIDE

These questions are intended for self-study. Click the [solution] box to reveal a detailed solution.

B-1 Installing a heat exchanger in an office building will cost $750,000. It will save $65,000 annually for 20 years, when it will have a salvage value of $40,000. What is the PW of the heat exchanger at an interest rate of 5%?

SOLUTION

B-2 Suppose that a computer costs you $1500 to buy and you sell it 3 years later for $300. If your interest rate for the time value of money is 8%, what is your annual cost of ownership?

SOLUTION

B-3 How many months to pay off a credit card balance of $8000 with payments of $200? The card’s monthly interest rate is 1.5%. Assume there are no new charges.

SOLUTION

PROBLEMS

Key to icons: A = click to reveal answer; G = Green, which may include environmental ethics; E = Ethics other than green; = autograded problems that are available online in Dashboard; = The icon indicates that a spreadsheet is recommended.

Present Worth

B-1 A process redesign will cost $70,000 now, but it will save $18,000 annually for 6 years. The new machinery will have a salvage value of $12,500. What is the present worth of the machinery, if the interest rate is 12%?

B-2 A — Moving to a new facility will save $400,000 annually for 20 years. The cost of building the facility and moving is $2.5 million now. The facility will have a salvage value of $100,000. What is the facility's PW, if the interest rate is 12%?

B-3 — A lottery pays the winner $1 million in 20 equal annual payments of $50,000. The first payment will be made at the end of the second year. What is the present worth if the winner's annual interest rate is 5.25%?

Equivalent Annual Worth

B-4 A — You need to save $20,000 to buy a car in 3 years. At a 2% nominal annual interest rate, how much do you need to save each month?

B-5 — You can receive lottery winnings of either $800,000 now or $100,000 per year for the next 10 years. If your interest rate is 5% per year, which do you prefer?

B-6 A — A new road will cost $45 million to build, and $2 million annually to maintain and operate over its 50-year life. The roadbed and right-of-way are estimated to have a salvage value of $15 million. If the state highway department uses an interest rate of 5%, what is the EUAC for the road?

B-7 — A new car will cost $24,000 to buy and $5500 annually to operate. If it is sold for $9300 after 6 years, what is the EUAC? Assume that the owner's interest rate is 3% for the time value of money.

Price of a Bond

B-8 A — A $1000 bond has 15 more years to maturity, an interest rate of 6%, and it pays interest semi-annually. If the current market interest rate is 5%, what is the price of the bond?

B-9 — A $1000 bond has 12 more years to maturity, an interest rate of 6%, and it pays interest semi-annually. If the current market interest rate is 8%, what is the price of the bond?

B-10 A — An investor purchased a 10-year, $20,000 corporate bond just over 12 months ago. The bond pays 6.5% interest, due semi-annually. The current market interest rate for similar bonds has increased to 7.25%. What price will this bond sell for today?

Loan Payment

B-11 — What is the monthly payment for a 5-year car loan at a nominal interest rate of 8.5%? The loan's initial balance is $16,885.

B-12 A — A couple is buying a house, but they need a mortgage for $150,000. A 15-year loan can be obtained for 3.25% annual interest. What is the monthly payment?

Number of Periods

B-13 — A student owing $17,565 on a credit card has decided to use only a debit card in the future. The nominal annual rate on the credit card is 13.8%. If the student makes monthly payments of $250, how long is it until the credit card is paid off?

B-14 A G — A homeowner may install solar panels on the roof of her house. The total installed cost is $15,000 after federal and state credits. This solar system will meet the monthly electricity need, which costs $120 per month. The homeowner's interest rate is 3% annually.
(*a*) If electricity prices remain constant, how long will it be until the system pays for itself?
(*b*) If electricity prices increase in the future, how will this affect the installation paying for itself?

Internal Rate of Return

B-15 E — An automated storage and retrieval system will cost $135,000, but it will save $33,000 annually in labor costs. The system's salvage value is expected to be $20,000 when it is renovated in 10 years. What is the rate of return on the project? Do people need to be laid off to justify the installation of the new system?

B-16 Moving to a new facility will save $400,000 annually for 20 years. The cost of building the facility and moving is $2.5 million now. The facility will have a salvage value of $100,000. What is the rate of return on this project?
A

B-17 A new road will cost $45 million to build, and $2 million annually to maintain and operate over its 50-year life. The roadbed and right-of-way are estimated to have a salvage value of $15 million. If user benefits are estimated at $5.5 million annually, what is the rate of return on this road?

B-18 Some equipment is needed for a construction project. It can be leased for $150,000 annually, or it can be purchased for $900,000 at the beginning and sold for $225,000 at the end of 3 years. What is the rate of return for owning the equipment rather than leasing it?
A

APPENDIX C

COMPOUND INTEREST TABLES

Values of Interest Factors When n Equals Infinity

Single Payment:	Uniform Payment Series:
$(F/P, i,\infty) = \infty$	$(A/F, i,\infty) = 0$
$(P/F, i,\infty) = 0$	$(A/P, i,\infty) = i$
Arithmetic Gradient Series:	$(F/A, i,\infty) = \infty$
$(A/G, i, \infty) = 1/i$	$(P/A, i, \infty) = 1/i$
$(P/G, i, \infty) = 1/i^2$	

Compound Interest Factors

1/4%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	***n***
1	1.003	.9975	1.0000	1.0025	1.000	0.998	0.000	0.000	**1**
2	1.005	.9950	.4994	.5019	2.003	1.993	0.499	0.995	**2**
3	1.008	.9925	.3325	.3350	3.008	2.985	0.998	2.980	**3**
4	1.010	.9901	.2491	.2516	4.015	3.975	1.497	5.950	**4**
5	1.013	.9876	.1990	.2015	5.025	4.963	1.995	9.901	**5**
6	1.015	.9851	.1656	.1681	6.038	5.948	2.493	14.826	**6**
7	1.018	.9827	.1418	.1443	7.053	6.931	2.990	20.722	**7**
8	1.020	.9802	.1239	.1264	8.070	7.911	3.487	27.584	**8**
9	1.023	.9778	.1100	.1125	9.091	8.889	3.983	35.406	**9**
10	1.025	.9753	.0989	.1014	10.113	9.864	4.479	44.184	**10**
11	1.028	.9729	.0898	.0923	11.139	10.837	4.975	53.913	**11**
12	1.030	.9705	.0822	.0847	12.167	11.807	5.470	64.589	**12**
13	1.033	.9681	.0758	.0783	13.197	12.775	5.965	76.205	**13**
14	1.036	.9656	.0703	.0728	14.230	13.741	6.459	88.759	**14**
15	1.038	.9632	.0655	.0680	15.266	14.704	6.953	102.244	**15**
16	1.041	.9608	.0613	.0638	16.304	15.665	7.447	116.657	**16**
17	1.043	.9584	.0577	.0602	17.344	16.624	7.944	131.992	**17**
18	1.046	.9561	.0544	.0569	18.388	17.580	8.433	148.245	**18**
19	1.049	.9537	.0515	.0540	19.434	18.533	8.925	165.411	**19**
20	1.051	.9513	.0488	.0513	20.482	19.485	9.417	183.485	**20**
21	1.054	.9489	.0464	.0489	21.534	20.434	9.908	202.463	**21**
22	1.056	.9465	.0443	.0468	22.587	21.380	10.400	222.341	**22**
23	1.059	.9442	.0423	.0448	23.644	22.324	10.890	243.113	**23**
24	1.062	.9418	.0405	.0430	24.703	23.266	11.380	264.775	**24**
25	1.064	.9395	.0388	.0413	25.765	24.206	11.870	287.323	**25**
26	1.067	.9371	.0373	.0398	26.829	25.143	12.360	310.752	**26**
27	1.070	.9348	.0358	.0383	27.896	26.078	12.849	335.057	**27**
28	1.072	.9325	.0345	.0370	28.966	27.010	13.337	360.233	**28**
29	1.075	.9301	.0333	.0358	30.038	27.940	13.825	386.278	**29**
30	1.078	.9278	.0321	.0346	31.114	28.868	14.313	413.185	**30**
36	1.094	.9140	.0266	.0291	37.621	34.387	17.231	592.499	**36**
40	1.105	.9049	.0238	.0263	42.014	38.020	19.167	728.740	**40**
48	1.127	.8871	.0196	.0221	50.932	45.179	23.021	1 040.055	**48**
50	1.133	.8826	.0188	.0213	53.189	46.947	23.980	1 125.777	**50**
52	1.139	.8782	.0180	.0205	55.458	48.705	24.938	1 214.588	**52**
60	1.162	.8609	.0155	.0180	64.647	55.653	28.751	1 600.085	**60**
70	1.191	.8396	.0131	.0156	76.395	64.144	33.481	2 147.611	**70**
72	1.197	.8355	.0127	.0152	78.780	65.817	34.422	2 265.557	**72**
80	1.221	.8189	.0113	.0138	88.440	72.427	38.169	2 764.457	**80**
84	1.233	.8108	.0107	.0132	93.343	75.682	40.033	3 029.759	**84**
90	1.252	.7987	.00992	.0124	100.789	80.504	42.816	3 446.870	**90**
96	1.271	.7869	.00923	.0117	108.349	85.255	45.584	3 886.283	**96**
100	1.284	.7790	.00881	.0113	113.451	88.383	47.422	4 191.242	**100**
104	1.297	.7713	.00843	.0109	118.605	91.480	49.252	4 505.557	**104**
120	1.349	.7411	.00716	.00966	139.743	103.563	56.508	5 852.112	**120**
240	1.821	.5492	.00305	.00555	328.306	180.312	107.586	19 398.985	**240**
360	2.457	.4070	.00172	.00422	582.745	237.191	152.890	36 263.930	**360**
480	3.315	.3016	.00108	.00358	926.074	279.343	192.670	53 820.752	**480**

Compound Interest Factors

1/2%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	*n*
1	1.005	.9950	1.0000	1.0050	1.000	0.995	0	0	**1**
2	1.010	.9901	.4988	.5038	2.005	1.985	0.499	0.991	**2**
3	1.015	.9851	.3317	.3367	3.015	2.970	0.996	2.959	**3**
4	1.020	.9802	.2481	.2531	4.030	3.951	1.494	5.903	**4**
5	1.025	.9754	.1980	.2030	5.050	4.926	1.990	9.803	**5**
6	1.030	.9705	.1646	.1696	6.076	5.896	2.486	14.660	**6**
7	1.036	.9657	.1407	.1457	7.106	6.862	2.980	20.448	**7**
8	1.041	.9609	.1228	.1278	8.141	7.823	3.474	27.178	**8**
9	1.046	.9561	.1089	.1139	9.182	8.779	3.967	34.825	**9**
10	1.051	.9513	.0978	.1028	10.228	9.730	4.459	43.389	**10**
11	1.056	.9466	.0887	.0937	11.279	10.677	4.950	52.855	**11**
12	1.062	.9419	.0811	.0861	12.336	11.619	5.441	63.218	**12**
13	1.067	.9372	.0746	.0796	13.397	12.556	5.931	74.465	**13**
14	1.072	.9326	.0691	.0741	14.464	13.489	6.419	86.590	**14**
15	1.078	.9279	.0644	.0694	15.537	14.417	6.907	99.574	**15**
16	1.083	.9233	.0602	.0652	16.614	15.340	7.394	113.427	**16**
17	1.088	.9187	.0565	.0615	17.697	16.259	7.880	128.125	**17**
18	1.094	.9141	.0532	.0582	18.786	17.173	8.366	143.668	**18**
19	1.099	.9096	.0503	.0553	19.880	18.082	8.850	160.037	**19**
20	1.105	.9051	.0477	.0527	20.979	18.987	9.334	177.237	**20**
21	1.110	.9006	.0453	.0503	22.084	19.888	9.817	195.245	**21**
22	1.116	.8961	.0431	.0481	23.194	20.784	10.300	214.070	**22**
23	1.122	.8916	.0411	.0461	24.310	21.676	10.781	233.680	**23**
24	1.127	.8872	.0393	.0443	25.432	22.563	11.261	254.088	**24**
25	1.133	.8828	.0377	.0427	26.559	23.446	11.741	275.273	**25**
26	1.138	.8784	.0361	.0411	27.692	24.324	12.220	297.233	**26**
27	1.144	.8740	.0347	.0397	28.830	25.198	12.698	319.955	**27**
28	1.150	.8697	.0334	.0384	29.975	26.068	13.175	343.439	**28**
29	1.156	.8653	.0321	.0371	31.124	26.933	13.651	367.672	**29**
30	1.161	.8610	.0310	.0360	32.280	27.794	14.127	392.640	**30**
36	1.197	.8356	.0254	.0304	39.336	32.871	16.962	557.564	**36**
40	1.221	.8191	.0226	.0276	44.159	36.172	18.836	681.341	**40**
48	1.270	.7871	.0185	.0235	54.098	42.580	22.544	959.928	**48**
50	1.283	.7793	.0177	.0227	56.645	44.143	23.463	1 035.70	**50**
52	1.296	.7716	.0169	.0219	59.218	45.690	24.378	1 113.82	**52**
60	1.349	.7414	.0143	.0193	69.770	51.726	28.007	1 448.65	**60**
70	1.418	.7053	.0120	.0170	83.566	58.939	32.468	1 913.65	**70**
72	1.432	.6983	.0116	.0166	86.409	60.340	33.351	2 012.35	**72**
80	1.490	.6710	.0102	.0152	98.068	65.802	36.848	2 424.65	**80**
84	1.520	.6577	.00961	.0146	104.074	68.453	38.576	2 640.67	**84**
90	1.567	.6383	.00883	.0138	113.311	72.331	41.145	2 976.08	**90**
96	1.614	.6195	.00814	.0131	122.829	76.095	43.685	3 324.19	**96**
100	1.647	.6073	.00773	.0127	129.334	78.543	45.361	3 562.80	**100**
104	1.680	.5953	.00735	.0124	135.970	80.942	47.025	3 806.29	**104**
120	1.819	.5496	.00610	.0111	163.880	90.074	53.551	4 823.52	**120**
240	3.310	.3021	.00216	.00716	462.041	139.581	96.113	13 415.56	**240**
360	6.023	.1660	.00100	.00600	1 004.5	166.792	128.324	21 403.32	**360**
480	10.957	.0913	.00050	.00550	1 991.5	181.748	151.795	27 588.37	**480**

Compound Interest Factors

3/4%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* ***F/P***	Find *P* Given *F* ***P/F***	Find *A* Given *F* ***A/F***	Find *A* Given *P* ***A/P***	Find *F* Given *A* ***F/A***	Find *P* Given *A* ***P/A***	Find *A* Given *G* ***A/G***	Find *P* Given *G* ***P/G***	**n**
1	1.008	.9926	1.0000	1.0075	1.000	0.993	0	0	**1**
2	1.015	.9852	.4981	.5056	2.008	1.978	0.499	0.987	**2**
3	1.023	.9778	.3308	.3383	3.023	2.956	0.996	2.943	**3**
4	1.030	.9706	.2472	.2547	4.045	3.926	1.492	5.857	**4**
5	1.038	.9633	.1970	.2045	5.076	4.889	1.986	9.712	**5**
6	1.046	.9562	.1636	.1711	6.114	5.846	2.479	14.494	**6**
7	1.054	.9490	.1397	.1472	7.160	6.795	2.971	20.187	**7**
8	1.062	.9420	.1218	.1293	8.213	7.737	3.462	26.785	**8**
9	1.070	.9350	.1078	.1153	9.275	8.672	3.951	34.265	**9**
10	1.078	.9280	.0967	.1042	10.344	9.600	4.440	42.619	**10**
11	1.086	.9211	.0876	.0951	11.422	10.521	4.927	51.831	**11**
12	1.094	.9142	.0800	.0875	12.508	11.435	5.412	61.889	**12**
13	1.102	.9074	.0735	.0810	13.602	12.342	5.897	72.779	**13**
14	1.110	.9007	.0680	.0755	14.704	13.243	6.380	84.491	**14**
15	1.119	.8940	.0632	.0707	15.814	14.137	6.862	97.005	**15**
16	1.127	.8873	.0591	.0666	16.932	15.024	7.343	110.318	**16**
17	1.135	.8807	.0554	.0629	18.059	15.905	7.822	124.410	**17**
18	1.144	.8742	.0521	.0596	19.195	16.779	8.300	139.273	**18**
19	1.153	.8676	.0492	.0567	20.339	17.647	8.777	154.891	**19**
20	1.161	.8612	.0465	.0540	21.491	18.508	9.253	171.254	**20**
21	1.170	.8548	.0441	.0516	22.653	19.363	9.727	188.352	**21**
22	1.179	.8484	.0420	.0495	23.823	20.211	10.201	206.170	**22**
23	1.188	.8421	.0400	.0475	25.001	21.053	10.673	224.695	**23**
24	1.196	.8358	.0382	.0457	26.189	21.889	11.143	243.924	**24**
25	1.205	.8296	.0365	.0440	27.385	22.719	11.613	263.834	**25**
26	1.214	.8234	.0350	.0425	28.591	23.542	12.081	284.421	**26**
27	1.224	.8173	.0336	.0411	29.805	24.360	12.548	305.672	**27**
28	1.233	.8112	.0322	.0397	31.029	25.171	13.014	327.576	**28**
29	1.242	.8052	.0310	.0385	32.261	25.976	13.479	350.122	**29**
30	1.251	.7992	.0298	.0373	33.503	26.775	13.942	373.302	**30**
36	1.309	.7641	.0243	.0318	41.153	31.447	16.696	525.038	**36**
40	1.348	.7416	.0215	.0290	46.447	34.447	18.507	637.519	**40**
48	1.431	.6986	.0174	.0249	57.521	40.185	22.070	886.899	**48**
50	1.453	.6882	.0166	.0241	60.395	41.567	22.949	953.911	**50**
52	1.475	.6780	.0158	.0233	63.312	42.928	23.822	1 022.64	**52**
60	1.566	.6387	.0133	.0208	75.425	48.174	27.268	1 313.59	**60**
70	1.687	.5927	.0109	.0184	91.621	54.305	31.465	1 708.68	**70**
72	1.713	.5839	.0105	.0180	95.008	55.477	32.289	1 791.33	**72**
80	1.818	.5500	.00917	.0167	109.074	59.995	35.540	2 132.23	**80**
84	1.873	.5338	.00859	.0161	116.428	62.154	37.137	2 308.22	**84**
90	1.959	.5104	.00782	.0153	127.881	65.275	39.496	2 578.09	**90**
96	2.049	.4881	.00715	.0147	139.858	68.259	41.812	2 854.04	**96**
100	2.111	.4737	.00675	.0143	148.147	70.175	43.332	3 040.85	**100**
104	2.175	.4597	.00638	.0139	156.687	72.035	44.834	3 229.60	**104**
120	2.451	.4079	.00517	.0127	193.517	78.942	50.653	3 998.68	**120**
240	6.009	.1664	.00150	.00900	667.901	111.145	85.422	9 494.26	**240**
360	14.731	.0679	.00055	.00805	1 830.8	124.282	107.115	13 312.50	**360**
480	36.111	.0277	.00021	.00771	4 681.5	129.641	119.662	15 513.16	**480**

Compound Interest Factors

1%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.010	.9901	1.0000	1.0100	1.000	0.990	0	0	**1**
2	1.020	.9803	.4975	.5075	2.010	1.970	0.498	0.980	**2**
3	1.030	.9706	.3300	.3400	3.030	2.941	0.993	2.921	**3**
4	1.041	.9610	.2463	.2563	4.060	3.902	1.488	5.804	**4**
5	1.051	.9515	.1960	.2060	5.101	4.853	1.980	9.610	**5**
6	1.062	.9420	.1625	.1725	6.152	5.795	2.471	14.320	**6**
7	1.072	.9327	.1386	.1486	7.214	6.728	2.960	19.917	**7**
8	1.083	.9235	.1207	.1307	8.286	7.652	3.448	26.381	**8**
9	1.094	.9143	.1067	.1167	9.369	8.566	3.934	33.695	**9**
10	1.105	.9053	.0956	.1056	10.462	9.471	4.418	41.843	**10**
11	1.116	.8963	.0865	.0965	11.567	10.368	4.900	50.806	**11**
12	1.127	.8874	.0788	.0888	12.682	11.255	5.381	60.568	**12**
13	1.138	.8787	.0724	.0824	13.809	12.134	5.861	71.112	**13**
14	1.149	.8700	.0669	.0769	14.947	13.004	6.338	82.422	**14**
15	1.161	.8613	.0621	.0721	16.097	13.865	6.814	94.481	**15**
16	1.173	.8528	.0579	.0679	17.258	14.718	7.289	107.273	**16**
17	1.184	.8444	.0543	.0643	18.430	15.562	7.761	120.783	**17**
18	1.196	.8360	.0510	.0610	19.615	16.398	8.232	134.995	**18**
19	1.208	.8277	.0481	.0581	20.811	17.226	8.702	149.895	**19**
20	1.220	.8195	.0454	.0554	22.019	18.046	9.169	165.465	**20**
21	1.232	.8114	.0430	.0530	23.239	18.857	9.635	181.694	**21**
22	1.245	.8034	.0409	.0509	24.472	19.660	10.100	198.565	**22**
23	1.257	.7954	.0389	.0489	25.716	20.456	10.563	216.065	**23**
24	1.270	.7876	.0371	.0471	26.973	21.243	11.024	234.179	**24**
25	1.282	.7798	.0354	.0454	28.243	22.023	11.483	252.892	**25**
26	1.295	.7720	.0339	.0439	29.526	22.795	11.941	272.195	**26**
27	1.308	.7644	.0324	.0424	30.821	23.560	12.397	292.069	**27**
28	1.321	.7568	.0311	.0411	32.129	24.316	12.852	312.504	**28**
29	1.335	.7493	.0299	.0399	33.450	25.066	13.304	333.486	**29**
30	1.348	.7419	.0287	.0387	34.785	25.808	13.756	355.001	**30**
36	1.431	.6989	.0232	.0332	43.077	30.107	16.428	494.620	**36**
40	1.489	.6717	.0205	.0305	48.886	32.835	18.178	596.854	**40**
48	1.612	.6203	.0163	.0263	61.223	37.974	21.598	820.144	**48**
50	1.645	.6080	.0155	.0255	64.463	39.196	22.436	879.417	**50**
52	1.678	.5961	.0148	.0248	67.769	40.394	23.269	939.916	**52**
60	1.817	.5504	.0122	.0222	81.670	44.955	26.533	1 192.80	**60**
70	2.007	.4983	.00993	.0199	100.676	50.168	30.470	1 528.64	**70**
72	2.047	.4885	.00955	.0196	104.710	51.150	31.239	1 597.86	**72**
80	2.217	.4511	.00822	.0182	121.671	54.888	34.249	1 879.87	**80**
84	2.307	.4335	.00765	.0177	130.672	56.648	35.717	2 023.31	**84**
90	2.449	.4084	.00690	.0169	144.863	59.161	37.872	2 240.56	**90**
96	2.599	.3847	.00625	.0163	159.927	61.528	39.973	2 459.42	**96**
100	2.705	.3697	.00587	.0159	170.481	63.029	41.343	2 605.77	**100**
104	2.815	.3553	.00551	.0155	181.464	64.471	42.688	2 752.17	**104**
120	3.300	.3030	.00435	.0143	230.039	69.701	47.835	3 334.11	**120**
240	10.893	.0918	.00101	.0110	989.254	90.819	75.739	6 878.59	**240**
360	35.950	.0278	.00029	.0103	3 495.0	97.218	89.699	8 720.43	**360**
480	118.648	.00843	.00008	.0101	11 764.8	99.157	95.920	9 511.15	**480**

$1\frac{1}{4}\%$ Compound Interest Factors $1\frac{1}{4}\%$

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	*n*
1	1.013	.9877	1.0000	1.0125	1.000	0.988	0	0	**1**
2	1.025	.9755	.4969	.5094	2.013	1.963	0.497	0.976	**2**
3	1.038	.9634	.3292	.3417	3.038	2.927	0.992	2.904	**3**
4	1.051	.9515	.2454	.2579	4.076	3.878	1.485	5.759	**4**
5	1.064	.9398	.1951	.2076	5.127	4.818	1.976	9.518	**5**
6	1.077	.9282	.1615	.1740	6.191	5.746	2.464	14.160	**6**
7	1.091	.9167	.1376	.1501	7.268	6.663	2.951	19.660	**7**
8	1.104	.9054	.1196	.1321	8.359	7.568	3.435	25.998	**8**
9	1.118	.8942	.1057	.1182	9.463	8.462	3.918	33.152	**9**
10	1.132	.8832	.0945	.1070	10.582	9.346	4.398	41.101	**10**
11	1.146	.8723	.0854	.0979	11.714	10.218	4.876	49.825	**11**
12	1.161	.8615	.0778	.0903	12.860	11.079	5.352	59.302	**12**
13	1.175	.8509	.0713	.0838	14.021	11.930	5.827	69.513	**13**
14	1.190	.8404	.0658	.0783	15.196	12.771	6.299	80.438	**14**
15	1.205	.8300	.0610	.0735	16.386	13.601	6.769	92.058	**15**
16	1.220	.8197	.0568	.0693	17.591	14.420	7.237	104.355	**16**
17	1.235	.8096	.0532	.0657	18.811	15.230	7.702	117.309	**17**
18	1.251	.7996	.0499	.0624	20.046	16.030	8.166	130.903	**18**
19	1.266	.7898	.0470	.0595	21.297	16.849	8.628	145.119	**19**
20	1.282	.7800	.0443	.0568	22.563	17.599	9.088	159.940	**20**
21	1.298	.7704	.0419	.0544	23.845	18.370	9.545	175.348	**21**
22	1.314	.7609	.0398	.0523	25.143	19.131	10.001	191.327	**22**
23	1.331	.7515	.0378	.0503	26.458	19.882	10.455	207.859	**23**
24	1.347	.7422	.0360	.0485	27.788	20.624	10.906	224.930	**24**
25	1.364	.7330	.0343	.0468	29.136	21.357	11.355	242.523	**25**
26	1.381	.7240	.0328	.0453	30.500	22.081	11.803	260.623	**26**
27	1.399	.7150	.0314	.0439	31.881	22.796	12.248	279.215	**27**
28	1.416	.7062	.0300	.0425	32.280	23.503	12.691	298.284	**28**
29	1.434	.6975	.0288	.0413	34.696	24.200	13.133	317.814	**29**
30	1.452	.6889	.0277	.0402	36.129	24.889	13.572	337.792	**30**
36	1.564	.6394	.0222	.0347	45.116	28.847	16.164	466.297	**36**
40	1.644	.6084	.0194	.0319	51.490	31.327	17.852	559.247	**40**
48	1.845	.5509	.0153	.0278	65.229	35.932	21.130	759.248	**48**
50	1.861	.5373	.0145	.0270	68.882	37.013	21.930	811.692	**50**
52	1.908	.5242	.0138	.0263	72.628	38.068	22.722	864.960	**52**
60	2.107	.4746	.0113	.0238	88.575	42.035	25.809	1 084.86	**60**
70	2.386	.4191	.00902	.0215	110.873	46.470	29.492	1 370.47	**70**
72	2.446	.4088	.00864	.0211	115.675	47.293	30.205	1 428.48	**72**
80	2.701	.3702	.00735	.0198	136.120	50.387	32.983	1 661.89	**80**
84	2.839	.3522	.00680	.0193	147.130	51.822	34.326	1 778.86	**84**
90	3.059	.3269	.00607	.0186	164.706	53.846	36.286	1 953.85	**90**
96	3.296	.3034	.00545	.0179	183.643	55.725	38.180	2 127.55	**96**
100	3.463	.2887	.00507	.0176	197.074	56.901	39.406	2 242.26	**100**
104	3.640	.2747	.00474	.0172	211.190	58.021	40.604	2 355.90	**104**
120	4.440	.2252	.00363	.0161	275.220	61.983	45.119	2 796.59	**120**
240	19.716	.0507	.00067	.0132	1 497.3	75.942	67.177	5 101.55	**240**
360	87.543	.0114	.00014	.0126	6 923.4	79.086	75.840	5 997.91	**360**
480	388.713	.00257	.00003	.0125	31 017.1	79.794	78.762	6 284.74	**480**

1½% Compound Interest Factors 1½%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	*n*
1	1.015	.9852	1.0000	1.0150	1.000	0.985	0	0	**1**
2	1.030	.9707	.4963	.5113	2.015	1.956	0.496	0.970	**2**
3	1.046	.9563	.3284	.3434	3.045	2.912	0.990	2.883	**3**
4	1.061	.9422	.2444	.2594	4.091	3.854	1.481	5.709	**4**
5	1.077	.9283	.1941	.2091	5.152	4.783	1.970	9.422	**5**
6	1.093	.9145	.1605	.1755	6.230	5.697	2.456	13.994	**6**
7	1.110	.9010	.1366	.1516	7.323	6.598	2.940	19.400	**7**
8	1.126	.8877	.1186	.1336	8.433	7.486	3.422	25.614	**8**
9	1.143	.8746	.1046	.1196	9.559	8.360	3.901	32.610	**9**
10	1.161	.8617	.0934	.1084	10.703	9.222	4.377	40.365	**10**
11	1.178	.8489	.0843	.0993	11.863	10.071	4.851	48.855	**11**
12	1.196	.8364	.0767	.0917	13.041	10.907	5.322	58.054	**12**
13	1.214	.8240	.0702	.0852	14.237	11.731	5.791	67.943	**13**
14	1.232	.8118	.0647	.0797	15.450	12.543	6.258	78.496	**14**
15	1.250	.7999	.0599	.0749	16.682	13.343	6.722	89.694	**15**
16	1.269	.7880	.0558	.0708	17.932	14.131	7.184	101.514	**16**
17	1.288	.7764	.0521	.0671	19.201	14.908	7.643	113.937	**17**
18	1.307	.7649	.0488	.0638	20.489	15.673	8.100	126.940	**18**
19	1.327	.7536	.0459	.0609	21.797	16.426	8.554	140.505	**19**
20	1.347	.7425	.0432	.0582	23.124	17.169	9.005	154.611	**20**
21	1.367	.7315	.0409	.0559	24.470	17.900	9.455	169.241	**21**
22	1.388	.7207	.0387	.0537	25.837	18.621	9.902	184.375	**22**
23	1.408	.7100	.0367	.0517	27.225	19.331	10.346	199.996	**23**
24	1.430	.6995	.0349	.0499	28.633	20.030	10.788	216.085	**24**
25	1.451	.6892	.0333	.0483	30.063	20.720	11.227	232.626	**25**
26	1.473	.6790	.0317	.0467	31.514	21.399	11.664	249.601	**26**
27	1.495	.6690	.0303	.0453	32.987	22.068	12.099	266.995	**27**
28	1.517	.6591	.0290	.0440	34.481	22.727	12.531	284.790	**28**
29	1.540	.6494	.0278	.0428	35.999	23.376	12.961	302.972	**29**
30	1.563	.6398	.0266	.0416	37.539	24.016	13.388	321.525	**30**
36	1.709	.5851	.0212	.0362	47.276	27.661	15.901	439.823	**36**
40	1.814	.5513	.0184	.0334	54.268	29.916	17.528	524.349	**40**
48	2.043	.4894	.0144	.0294	69.565	34.042	20.666	703.537	**48**
50	2.105	.4750	.0136	.0286	73.682	35.000	21.428	749.955	**50**
52	2.169	.4611	.0128	.0278	77.925	35.929	22.179	796.868	**52**
60	2.443	.4093	.0104	.0254	96.214	39.380	25.093	988.157	**60**
70	2.835	.3527	.00817	.0232	122.363	43.155	28.529	1 231.15	**70**
72	2.921	.3423	.00781	.0228	128.076	43.845	29.189	1 279.78	**72**
80	3.291	.3039	.00655	.0215	152.710	46.407	31.742	1 473.06	**80**
84	3.493	.2863	.00602	.0210	166.172	47.579	32.967	1 568.50	**84**
90	3.819	.2619	.00532	.0203	187.929	49.210	34.740	1 709.53	**90**
96	4.176	.2395	.00472	.0197	211.719	50.702	36.438	1 847.46	**96**
100	4.432	.2256	.00437	.0194	228.802	51.625	37.529	1 937.43	**100**
104	4.704	.2126	.00405	.0190	246.932	52.494	38.589	2 025.69	**104**
120	5.969	.1675	.00302	.0180	331.286	55.498	42.518	2 359.69	**120**
240	35.632	.0281	.00043	.0154	2 308.8	64.796	59.737	3 870.68	**240**
360	212.700	.00470	.00007	.0151	14 113.3	66.353	64.966	4 310.71	**360**
480	1 269.7	.00079	.00001	.0150	84 577.8	66.614	66.288	4 415.74	**480**

1 3/4% Compound Interest Factors 1 3/4%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	***n***
1	1.018	.9828	1.0000	1.0175	1.000	0.983	0	0	**1**
2	1.035	.9659	.4957	.5132	2.018	1.949	0.496	0.966	**2**
3	1.053	.9493	.3276	.3451	3.053	2.898	0.989	2.865	**3**
4	1.072	.9330	.2435	.2610	4.106	3.831	1.478	5.664	**4**
5	1.091	.9169	.1931	.2106	5.178	4.748	1.965	9.332	**5**
6	1.110	.9011	.1595	.1770	6.269	5.649	2.450	13.837	**6**
7	1.129	.8856	.1355	.1530	7.378	6.535	2.931	19.152	**7**
8	1.149	.8704	.1175	.1350	8.508	7.405	3.409	25.245	**8**
9	1.169	.8554	.1036	.1211	9.656	8.261	3.885	32.088	**9**
10	1.189	.8407	.0924	.1099	10.825	9.101	4.357	39.655	**10**
11	1.210	.8263	.0832	.1007	12.015	9.928	4.827	47.918	**11**
12	1.231	.8121	.0756	.0931	13.225	10.740	5.294	56.851	**12**
13	1.253	.7981	.0692	.0867	14.457	11.538	5.758	66.428	**13**
14	1.275	.7844	.0637	.0812	15.710	12.322	6.219	76.625	**14**
15	1.297	.7709	.0589	.0764	16.985	13.093	6.677	87.417	**15**
16	1.320	.7576	.0547	.0722	18.282	13.851	7.132	98.782	**16**
17	1.343	.7446	.0510	.0685	19.602	14.595	7.584	110.695	**17**
18	1.367	.7318	.0477	.0652	20.945	15.327	8.034	123.136	**18**
19	1.390	.7192	.0448	.0623	22.311	16.046	8.481	136.081	**19**
20	1.415	.7068	.0422	.0597	23.702	16.753	8.924	149.511	**20**
21	1.440	.6947	.0398	.0573	25.116	17.448	9.365	163.405	**21**
22	1.465	.6827	.0377	.0552	26.556	18.130	9.804	177.742	**22**
23	1.490	.6710	.0357	.0532	28.021	18.801	10.239	192.503	**23**
24	1.516	.6594	.0339	.0514	29.511	19.461	10.671	207.671	**24**
25	1.543	.6481	.0322	.0497	31.028	20.109	11.101	223.225	**25**
26	1.570	.6369	.0307	.0482	32.571	20.746	11.528	239.149	**26**
27	1.597	.6260	.0293	.0468	34.141	21.372	11.952	255.425	**27**
28	1.625	.6152	.0280	.0455	35.738	21.987	12.373	272.036	**28**
29	1.654	.6046	.0268	.0443	37.363	22.592	12.791	288.967	**29**
30	1.683	.5942	.0256	.0431	39.017	23.186	13.206	306.200	**30**
36	1.867	.5355	.0202	.0377	49.566	26.543	15.640	415.130	**36**
40	2.002	.4996	.0175	.0350	57.234	28.594	17.207	492.017	**40**
48	2.300	.4349	.0135	.0310	74.263	32.294	20.209	652.612	**48**
50	2.381	.4200	.0127	.0302	78.903	33.141	20.932	693.708	**50**
52	2.465	.4057	.0119	.0294	83.706	33.960	21.644	735.039	**52**
60	2.832	.3531	.00955	.0271	104.676	36.964	24.389	901.503	**60**
70	3.368	.2969	.00739	.0249	135.331	40.178	27.586	1 108.34	**70**
72	3.487	.2868	.00704	.0245	142.127	40.757	28.195	1 149.12	**72**
80	4.006	.2496	.00582	.0233	171.795	42.880	30.533	1 309.25	**80**
84	4.294	.2329	.00531	.0228	188.246	43.836	31.644	1 387.16	**84**
90	4.765	.2098	.00465	.0221	215.166	45.152	33.241	1 500.88	**90**
96	5.288	.1891	.00408	.0216	245.039	46.337	34.756	1 610.48	**96**
100	5.668	.1764	.00375	.0212	266.753	47.062	35.721	1 681.09	**100**
104	6.075	.1646	.00345	.0209	290.028	47.737	36.652	1 749.68	**104**
120	8.019	.1247	.00249	.0200	401.099	50.017	40.047	2 003.03	**120**
240	64.308	.0156	.00028	.0178	3 617.6	56.254	53.352	3 001.27	**240**
360	515.702	.00194	.00003	.0175	29 411.5	57.032	56.443	3 219.08	**360**
480	4 135.5	.00024		.0175	236 259.0	57.129	57.027	3 257.88	**480**

2% Compound Interest Factors 2%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.020	.9804	1.0000	1.0200	1.000	0.980	0	0	**1**
2	1.040	.9612	.4951	.5151	2.020	1.942	0.495	0.961	**2**
3	1.061	.9423	.3268	.3468	3.060	2.884	0.987	2.846	**3**
4	1.082	.9238	.2426	.2626	4.122	3.808	1.475	5.617	**4**
5	1.104	.9057	.1922	.2122	5.204	4.713	1.960	9.240	**5**
6	1.126	.8880	.1585	.1785	6.308	5.601	2.442	13.679	**6**
7	1.149	.8706	.1345	.1545	7.434	6.472	2.921	18.903	**7**
8	1.172	.8535	.1165	.1365	8.583	7.325	3.396	24.877	**8**
9	1.195	.8368	.1025	.1225	9.755	8.162	3.868	31.571	**9**
10	1.219	.8203	.0913	.1113	10.950	8.983	4.337	38.954	**10**
11	1.243	.8043	.0822	.1022	12.169	9.787	4.802	46.996	**11**
12	1.268	.7885	.0746	.0946	13.412	10.575	5.264	55.669	**12**
13	1.294	.7730	.0681	.0881	14.680	11.348	5.723	64.946	**13**
14	1.319	.7579	.0626	.0826	15.974	12.106	6.178	74.798	**14**
15	1.346	.7430	.0578	.0778	17.293	12.849	6.631	85.200	**15**
16	1.373	.7284	.0537	.0737	18.639	13.578	7.080	96.127	**16**
17	1.400	.7142	.0500	.0700	20.012	14.292	7.526	107.553	**17**
18	1.428	.7002	.0467	.0667	21.412	14.992	7.968	119.456	**18**
19	1.457	.6864	.0438	.0638	22.840	15.678	8.407	131.812	**19**
20	1.486	.6730	.0412	.0612	24.297	16.351	8.843	144.598	**20**
21	1.516	.6598	.0388	.0588	25.783	17.011	9.276	157.793	**21**
22	1.546	.6468	.0366	.0566	27.299	17.658	9.705	171.377	**22**
23	1.577	.6342	.0347	.0547	28.845	18.292	10.132	185.328	**23**
24	1.608	.6217	.0329	.0529	30.422	18.914	10.555	199.628	**24**
25	1.641	.6095	.0312	.0512	32.030	19.523	10.974	214.256	**25**
26	1.673	.5976	.0297	.0497	33.671	20.121	11.391	229.196	**26**
27	1.707	.5859	.0283	.0483	35.344	20.707	11.804	244.428	**27**
28	1.741	.5744	.0270	.0470	37.051	21.281	12.214	259.936	**28**
29	1.776	.5631	.0258	.0458	38.792	21.844	12.621	275.703	**29**
30	1.811	.5521	.0247	.0447	40.568	22.396	13.025	291.713	**30**
36	2.040	.4902	.0192	.0392	51.994	25.489	15.381	392.036	**36**
40	2.208	.4529	.0166	.0366	60.402	27.355	16.888	461.989	**40**
48	2.587	.3865	.0126	.0326	79.353	30.673	19.755	605.961	**48**
50	2.692	.3715	.0118	.0318	84.579	31.424	20.442	642.355	**50**
52	2.800	.3571	.0111	.0311	90.016	32.145	21.116	678.779	**52**
60	3.281	.3048	.00877	.0288	114.051	34.761	23.696	823.692	**60**
70	4.000	.2500	.00667	.0267	149.977	37.499	26.663	999.829	**70**
72	4.161	.2403	.00633	.0263	158.056	37.984	27.223	1 034.050	**72**
80	4.875	.2051	.00516	.0252	193.771	39.744	29.357	1 166.781	**80**
84	5.277	.1895	.00468	.0247	213.865	40.525	30.361	1 230.413	**84**
90	5.943	.1683	.00405	.0240	247.155	41.587	31.793	1 322.164	**90**
96	6.693	.1494	.00351	.0235	284.645	42.529	33.137	1 409.291	**96**
100	7.245	.1380	.00320	.0232	312.230	43.098	33.986	1 464.747	**100**
104	7.842	.1275	.00292	.0229	342.090	43.624	34.799	1 518.082	**104**
120	10.765	.0929	.00205	.0220	488.255	45.355	37.711	1 710.411	**120**
240	115.887	.00863	.00017	.0202	5 744.4	49.569	47.911	2 374.878	**240**
360	1 247.5	.00080	.00002	.0200	62 326.8	49.960	49.711	2 483.567	**360**
480	13 429.8	.00007		.0200	671 442.0	49.996	49.964	2 498.027	**480**

2 1/2 % Compound Interest Factors 2 1/2 %

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	*n*
1	1.025	.9756	1.0000	1.0250	1.000	0.976	0	0	**1**
2	1.051	.9518	.4938	.5188	2.025	1.927	0.494	0.952	**2**
3	1.077	.9286	.3251	.3501	3.076	2.856	0.984	2.809	**3**
4	1.104	.9060	.2408	.2658	4.153	3.762	1.469	5.527	**4**
5	1.131	.8839	.1902	.2152	5.256	4.646	1.951	9.062	**5**
6	1.160	.8623	.1566	.1816	6.388	5.508	2.428	13.374	**6**
7	1.189	.8413	.1325	.1575	7.547	6.349	2.901	18.421	**7**
8	1.218	.8207	.1145	.1395	8.736	7.170	3.370	24.166	**8**
9	1.249	.8007	.1005	.1255	9.955	7.971	3.835	30.572	**9**
10	1.280	.7812	.0893	.1143	11.203	8.752	4.296	37.603	**10**
11	1.312	.7621	.0801	.1051	12.483	9.514	4.753	45.224	**11**
12	1.345	.7436	.0725	.0975	13.796	10.258	5.206	53.403	**12**
13	1.379	.7254	.0660	.0910	15.140	10.983	5.655	62.108	**13**
14	1.413	.7077	.0605	.0855	16.519	11.691	6.100	71.309	**14**
15	1.448	.6905	.0558	.0808	17.932	12.381	6.540	80.975	**15**
16	1.485	.6736	.0516	.0766	19.380	13.055	6.977	91.080	**16**
17	1.522	.6572	.0479	.0729	20.865	13.712	7.409	101.595	**17**
18	1.560	.6412	.0447	.0697	22.386	14.353	7.838	112.495	**18**
19	1.599	.6255	.0418	.0668	23.946	14.979	8.262	123.754	**19**
20	1.639	.6103	.0391	.0641	25.545	15.589	8.682	135.349	**20**
21	1.680	.5954	.0368	.0618	27.183	16.185	9.099	147.257	**21**
22	1.722	.5809	.0346	.0596	28.863	16.765	9.511	159.455	**22**
23	1.765	.5667	.0327	.0577	30.584	17.332	9.919	171.922	**23**
24	1.809	.5529	.0309	.0559	32.349	17.885	10.324	184.638	**24**
25	1.854	.5394	.0293	.0543	34.158	18.424	10.724	197.584	**25**
26	1.900	.5262	.0278	.0528	36.012	18.951	11.120	210.740	**26**
27	1.948	.5134	.0264	.0514	37.912	19.464	11.513	224.088	**27**
28	1.996	.5009	.0251	.0501	39.860	19.965	11.901	237.612	**28**
29	2.046	.4887	.0239	.0489	41.856	20.454	12.286	251.294	**29**
30	2.098	.4767	.0228	.0478	43.903	20.930	12.667	265.120	**30**
31	2.150	.4651	.0217	.0467	46.000	21.395	13.044	279.073	**31**
32	2.204	.4538	.0208	.0458	48.150	24.849	13.417	293.140	**32**
33	2.259	.4427	.0199	.0449	50.354	22.292	13.786	307.306	**33**
34	2.315	.4319	.0190	.0440	52.613	22.724	14.151	321.559	**34**
35	2.373	.4214	.0182	.0432	54.928	23.145	14.512	335.886	**35**
40	2.685	.3724	.0148	.0398	67.402	25.103	16.262	408.221	**40**
45	3.038	.3292	.0123	.0373	81.516	26.833	17.918	480.806	**45**
50	3.437	.2909	.0103	.0353	97.484	28.362	19.484	552.607	**50**
55	3.889	.2572	.00865	.0337	115.551	29.714	20.961	622.827	**55**
60	4.400	.2273	.00735	.0324	135.991	30.909	22.352	690.865	**60**
65	4.978	.2009	.00628	.0313	159.118	31.965	23.660	756.280	**65**
70	5.632	.1776	.00540	.0304	185.284	32.898	24.888	818.763	**70**
75	6.372	.1569	.00465	.0297	214.888	33.723	26.039	878.114	**75**
80	7.210	.1387	.00403	.0290	248.382	34.452	27.117	934.217	**80**
85	8.157	.1226	.00349	.0285	286.278	35.096	28.123	987.026	**85**
90	9.229	.1084	.00304	.0280	329.154	35.666	29.063	1 036.54	**90**
95	10.442	.0958	.00265	.0276	377.663	36.169	29.938	1 082.83	**95**
100	11.814	.0846	.00231	.0273	432.548	36.614	30.752	1 125.97	**100**

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*	*n*
1	1.030	.9709	1.0000	1.0300	1.000	0.971	0	0	**1**
2	1.061	.9426	.4926	.5226	2.030	1.913	0.493	0.943	**2**
3	1.093	.9151	.3235	.3535	3.091	2.829	0.980	2.773	**3**
4	1.126	.8885	.2390	.2690	4.184	3.717	1.463	5.438	**4**
5	1.159	.8626	.1884	.2184	5.309	4.580	1.941	8.889	**5**
6	1.194	.8375	.1546	.1846	6.468	5.417	2.414	13.076	**6**
7	1.230	.8131	.1305	.1605	7.662	6.230	2.882	17.955	**7**
8	1.267	.7894	.1125	.1425	8.892	7.020	3.345	23.481	**8**
9	1.305	.7664	.0984	.1284	10.159	7.786	3.803	29.612	**9**
10	1.344	.7441	.0872	.1172	11.464	8.530	4.256	36.309	**10**
11	1.384	.7224	.0781	.1081	12.808	9.253	4.705	43.533	**11**
12	1.426	.7014	.0705	.1005	14.192	9.954	5.148	51.248	**12**
13	1.469	.6810	.0640	.0940	15.618	10.635	5.587	59.419	**13**
14	1.513	.6611	.0585	.0885	17.086	11.296	6.021	68.014	**14**
15	1.558	.6419	.0538	.0838	18.599	11.938	6.450	77.000	**15**
16	1.605	.6232	.0496	.0796	20.157	12.561	6.874	86.348	**16**
17	1.653	.6050	.0460	.0760	21.762	13.166	7.294	96.028	**17**
18	1.702	.5874	.0427	.0727	23.414	13.754	7.708	106.014	**18**
19	1.754	.5703	.0398	.0698	25.117	14.324	8.118	116.279	**19**
20	1.806	.5537	.0372	.0672	26.870	14.877	8.523	126.799	**20**
21	1.860	.5375	.0349	.0649	28.676	15.415	8.923	137.549	**21**
22	1.916	.5219	.0327	.0627	30.537	15.937	9.319	148.509	**22**
23	1.974	.5067	.0308	.0608	32.453	16.444	9.709	159.656	**23**
24	2.033	.4919	.0290	.0590	34.426	16.936	10.095	170.971	**24**
25	2.094	.4776	.0274	.0574	36.459	17.413	10.477	182.433	**25**
26	2.157	.4637	.0259	.0559	38.553	17.877	10.853	194.026	**26**
27	2.221	.4502	.0246	.0546	40.710	18.327	11.226	205.731	**27**
28	2.288	.4371	.0233	.0533	42.931	18.764	11.593	217.532	**28**
29	2.357	.4243	.0221	.0521	45.219	19.188	11.956	229.413	**29**
30	2.427	.4120	.0210	.0510	47.575	19.600	12.314	241.361	**30**
31	2.500	.4000	.0200	.0500	50.003	20.000	12.668	253.361	**31**
32	2.575	.3883	.0190	.0490	52.503	20.389	13.017	265.399	**32**
33	2.652	.3770	.0182	.0482	55.078	20.766	13.362	277.464	**33**
34	2.732	.3660	.0173	.0473	57.730	21.132	13.702	289.544	**34**
35	2.814	.3554	.0165	.0465	60.462	21.487	14.037	301.627	**35**
40	3.262	.3066	.0133	.0433	75.401	23.115	15.650	361.750	**40**
45	3.782	.2644	.0108	.0408	92.720	24.519	17.156	420.632	**45**
50	4.384	.2281	.00887	.0389	112.797	25.730	18.558	477.480	**50**
55	5.082	.1968	.00735	.0373	136.072	26.774	19.860	531.741	**55**
60	5.892	.1697	.00613	.0361	163.053	27.676	21.067	583.052	**60**
65	6.830	.1464	.00515	.0351	194.333	28.453	22.184	631.201	**65**
70	7.918	.1263	.00434	.0343	230.594	29.123	23.215	676.087	**70**
75	9.179	.1089	.00367	.0337	272.631	29.702	24.163	717.698	**75**
80	10.641	.0940	.00311	.0331	321.363	30.201	25.035	756.086	**80**
85	12.336	.0811	.00265	.0326	377.857	30.631	25.835	791.353	**85**
90	14.300	.0699	.00226	.0323	443.349	31.002	26.567	823.630	**90**
95	16.578	.0603	.00193	.0319	519.272	31.323	27.235	853.074	**95**
100	19.219	.0520	.00165	.0316	607.287	31.599	27.844	879.854	**100**

Compound Interest Factors

3 1/2 %

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	***n***
1	1.035	.9662	1.0000	1.0350	1.000	0.966	0	0	**1**
2	1.071	.9335	.4914	.5264	2.035	1.900	0.491	0.933	**2**
3	1.109	.9019	.3219	.3569	3.106	2.802	0.977	2.737	**3**
4	1.148	.8714	.2373	.2723	4.215	3.673	1.457	5.352	**4**
5	1.188	.8420	.1865	.2215	5.362	4.515	1.931	8.719	**5**
6	1.229	.8135	.1527	.1877	6.550	5.329	2.400	12.787	**6**
7	1.272	.7860	.1285	.1635	7.779	6.115	2.862	17.503	**7**
8	1.317	.7594	.1105	.1455	9.052	6.874	3.320	22.819	**8**
9	1.363	.7337	.0964	.1314	10.368	7.608	3.771	28.688	**9**
10	1.411	.7089	.0852	.1202	11.731	8.317	4.217	35.069	**10**
11	1.460	.6849	.0761	.1111	13.142	9.002	4.657	41.918	**11**
12	1.511	.6618	.0685	.1035	14.602	9.663	5.091	49.198	**12**
13	1.564	.6394	.0621	.0971	16.113	10.303	5.520	56.871	**13**
14	1.619	.6178	.0566	.0916	17.677	10.921	5.943	64.902	**14**
15	1.675	.5969	.0518	.0868	19.296	11.517	6.361	73.258	**15**
16	1.734	.5767	.0477	.0827	20.971	12.094	6.773	81.909	**16**
17	1.795	.5572	.0440	.0790	22.705	12.651	7.179	90.824	**17**
18	1.857	.5384	.0408	.0758	24.500	13.190	7.580	99.976	**18**
19	1.922	.5202	.0379	.0729	26.357	13.710	7.975	109.339	**19**
20	1.990	.5026	.0354	.0704	28.280	14.212	8.365	118.888	**20**
21	2.059	.4856	.0330	.0680	30.269	14.698	8.749	128.599	**21**
22	2.132	.4692	.0309	.0659	32.329	15.167	9.128	138.451	**22**
23	2.206	.4533	.0290	.0640	34.460	15.620	9.502	148.423	**23**
24	2.283	.4380	.0273	.0623	36.666	16.058	9.870	158.496	**24**
25	2.363	.4231	.0257	.0607	38.950	16.482	10.233	168.652	**25**
26	2.446	.4088	.0242	.0592	41.313	16.890	10.590	178.873	**26**
27	2.532	.3950	.0229	.0579	43.759	17.285	10.942	189.143	**27**
28	2.620	.3817	.0216	.0566	46.291	17.667	11.289	199.448	**28**
29	2.712	.3687	.0204	.0554	48.911	18.036	11.631	209.773	**29**
30	2.807	.3563	.0194	.0544	51.623	18.392	11.967	220.105	**30**
31	2.905	.3442	.0184	.0534	54.429	18.736	12.299	230.432	**31**
32	3.007	.3326	.0174	.0524	57.334	19.069	12.625	240.742	**32**
33	3.112	.3213	.0166	.0516	60.341	19.390	12.946	251.025	**33**
34	3.221	.3105	.0158	.0508	63.453	19.701	13.262	261.271	**34**
35	3.334	.3000	.0150	.0500	66.674	20.001	13.573	271.470	**35**
40	3.959	.2526	.0118	.0468	84.550	21.355	15.055	321.490	**40**
45	4.702	.2127	.00945	.0445	105.781	22.495	16.417	369.307	**45**
50	5.585	.1791	.00763	.0426	130.998	23.456	17.666	414.369	**50**
55	6.633	.1508	.00621	.0412	160.946	24.264	18.808	456.352	**55**
60	7.878	.1269	.00509	.0401	196.516	24.945	19.848	495.104	**60**
65	9.357	.1069	.00419	.0392	238.762	25.518	20.793	530.598	**65**
70	11.113	.0900	.00346	.0385	288.937	26.000	21.650	562.895	**70**
75	13.199	.0758	.00287	.0379	348.529	26.407	22.423	592.121	**75**
80	15.676	.0638	.00238	.0374	419.305	26.749	23.120	618.438	**80**
85	18.618	.0537	.00199	.0370	503.365	27.037	23.747	642.036	**85**
90	22.112	.0452	.00166	.0367	603.202	27.279	24.308	663.118	**90**
95	26.262	.0381	.00139	.0364	721.778	27.483	24.811	681.890	**95**
100	31.191	.0321	.00116	.0362	862.608	27.655	25.259	698.554	**100**

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.040	.9615	1.0000	1.0400	1.000	0.962	0	0	**1**
2	1.082	.9246	.4902	.5302	2.040	1.886	0.490	0.925	**2**
3	1.125	.8890	.3203	.3603	3.122	2.775	0.974	2.702	**3**
4	1.170	.8548	.2355	.2755	4.246	3.630	1.451	5.267	**4**
5	1.217	.8219	.1846	.2246	5.416	4.452	1.922	8.555	**5**
6	1.265	.7903	.1508	.1908	6.633	5.242	2.386	12.506	**6**
7	1.316	.7599	.1266	.1666	7.898	6.002	2.843	17.066	**7**
8	1.369	.7307	.1085	.1485	9.214	6.733	3.294	22.180	**8**
9	1.423	.7026	.0945	.1345	10.583	7.435	3.739	27.801	**9**
10	1.480	.6756	.0833	.1233	12.006	8.111	4.177	33.881	**10**
11	1.539	.6496	.0741	.1141	13.486	8.760	4.609	40.377	**11**
12	1.601	.6246	.0666	.1066	15.026	9.385	5.034	47.248	**12**
13	1.665	.6006	.0601	.1001	16.627	9.986	5.453	54.454	**13**
14	1.732	.5775	.0547	.0947	18.292	10.563	5.866	61.962	**14**
15	1.801	.5553	.0499	.0899	20.024	11.118	6.272	69.735	**15**
16	1.873	.5339	.0458	.0858	21.825	11.652	6.672	77.744	**16**
17	1.948	.5134	.0422	.0822	23.697	12.166	7.066	85.958	**17**
18	2.026	.4936	.0390	.0790	25.645	12.659	7.453	94.350	**18**
19	2.107	.4746	.0361	.0761	27.671	13.134	7.834	102.893	**19**
20	2.191	.4564	.0336	.0736	29.778	13.590	8.209	111.564	**20**
21	2.279	.4388	.0313	.0713	31.969	14.029	8.578	120.341	**21**
22	2.370	.4220	.0292	.0692	34.248	14.451	8.941	129.202	**22**
23	2.465	.4057	.0273	.0673	36.618	14.857	9.297	138.128	**23**
24	2.563	.3901	.0256	.0656	39.083	15.247	9.648	147.101	**24**
25	2.666	.3751	.0240	.0640	41.646	15.622	9.993	156.104	**25**
26	2.772	.3607	.0226	.0626	44.312	15.983	10.331	165.121	**26**
27	2.883	.3468	.0212	.0612	47.084	16.330	10.664	174.138	**27**
28	2.999	.3335	.0200	.0600	49.968	16.663	10.991	183.142	**28**
29	3.119	.3207	.0189	.0589	52.966	16.984	11.312	192.120	**29**
30	3.243	.3083	.0178	.0578	56.085	17.292	11.627	201.062	**30**
31	3.373	.2965	.0169	.0569	59.328	17.588	11.937	209.955	**31**
32	3.508	.2851	.0159	.0559	62.701	17.874	12.241	218.792	**32**
33	3.648	.2741	.0151	.0551	66.209	18.148	12.540	227.563	**33**
34	3.794	.2636	.0143	.0543	69.858	18.411	12.832	236.260	**34**
35	3.946	.2534	.0136	.0536	73.652	18.665	13.120	244.876	**35**
40	4.801	.2083	.0105	.0505	95.025	19.793	14.476	286.530	**40**
45	5.841	.1712	.00826	.0483	121.029	20.720	15.705	325.402	**45**
50	7.107	.1407	.00655	.0466	152.667	21.482	16.812	361.163	**50**
55	8.646	.1157	.00523	.0452	191.159	22.109	17.807	393.689	**55**
60	10.520	.0951	.00420	.0442	237.990	22.623	18.697	422.996	**60**
65	12.799	.0781	.00339	.0434	294.968	23.047	19.491	449.201	**65**
70	15.572	.0642	.00275	.0427	364.290	23.395	20.196	472.479	**70**
75	18.945	.0528	.00223	.0422	448.630	23.680	20.821	493.041	**75**
80	23.050	.0434	.00181	.0418	551.244	23.915	21.372	511.116	**80**
85	28.044	.0357	.00148	.0415	676.089	24.109	21.857	526.938	**85**
90	34.119	.0293	.00121	.0412	827.981	24.267	22.283	540.737	**90**
95	41.511	.0241	.00099	.0410	1012.8	24.398	22.655	552.730	**95**
100	50.505	.0198	.00081	.0408	1237.6	24.505	22.980	563.125	**100**

$4^1/_2$ % Compound Interest Factors $4^1/_2$ %

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	*n*
1	1.045	.9569	1.0000	1.0450	1.000	0.957	0	0	**1**
2	1.092	.9157	.4890	.5340	2.045	1.873	0.489	0.916	**2**
3	1.141	.8763	.3188	.3638	3.137	2.749	0.971	2.668	**3**
4	1.193	.8386	.2337	.2787	4.278	3.588	1.445	5.184	**4**
5	1.246	.8025	.1828	.2278	5.471	4.390	1.912	8.394	**5**
6	1.302	.7679	.1489	.1939	6.717	5.158	2.372	12.233	**6**
7	1.361	.7348	.1247	.1697	8.019	5.893	2.824	16.642	**7**
8	1.422	.7032	.1066	.1516	9.380	6.596	3.269	21.564	**8**
9	1.486	.6729	.0926	.1376	10.802	7.269	3.707	26.948	**9**
10	1.553	.6439	.0814	.1264	12.288	7.913	4.138	32.743	**10**
11	1.623	.6162	.0722	.1172	13.841	8.529	4.562	38.905	**11**
12	1.696	.5897	.0647	.1097	15.464	9.119	4.978	45.391	**12**
13	1.772	.5643	.0583	.1033	17.160	9.683	5.387	52.163	**13**
14	1.852	.5400	.0528	.0978	18.932	10.223	5.789	59.182	**14**
15	1.935	.5167	.0481	.0931	20.784	10.740	6.184	66.416	**15**
16	2.022	.4945	.0440	.0890	22.719	11.234	6.572	73.833	**16**
17	2.113	.4732	.0404	.0854	24.742	11.707	6.953	81.404	**17**
18	2.208	.4528	.0372	.0822	26.855	12.160	7.327	89.102	**18**
19	2.308	.4333	.0344	.0794	29.064	12.593	7.695	96.901	**19**
20	2.412	.4146	.0319	.0769	31.371	13.008	8.055	104.779	**20**
21	2.520	.3968	.0296	.0746	33.783	13.405	8.409	112.715	**21**
22	2.634	.3797	.0275	.0725	36.303	13.784	8.755	120.689	**22**
23	2.752	.3634	.0257	.0707	38.937	14.148	9.096	128.682	**23**
24	2.876	.3477	.0240	.0690	41.689	14.495	9.429	136.680	**24**
25	3.005	.3327	.0224	.0674	44.565	14.828	9.756	144.665	**25**
26	3.141	.3184	.0210	.0660	47.571	15.147	10.077	152.625	**26**
27	3.282	.3047	.0197	.0647	50.711	15.451	10.391	160.547	**27**
28	3.430	.2916	.0185	.0635	53.993	15.743	10.698	168.420	**28**
29	3.584	.2790	.0174	.0624	57.423	16.022	10.999	176.232	**29**
30	3.745	.2670	.0164	.0614	61.007	16.289	11.295	183.975	**30**
31	3.914	.2555	.0154	.0604	64.752	16.544	11.583	191.640	**31**
32	4.090	.2445	.0146	.0596	68.666	16.789	11.866	199.220	**32**
33	4.274	.2340	.0137	.0587	72.756	17.023	12.143	206.707	**33**
34	4.466	.2239	.0130	.0580	77.030	17.247	12.414	214.095	**34**
35	4.667	.2143	.0123	.0573	81.497	17.461	12.679	221.380	**35**
40	5.816	.1719	.00934	.0543	107.030	18.402	13.917	256.098	**40**
45	7.248	.1380	.00720	.0522	138.850	19.156	15.020	287.732	**45**
50	9.033	.1107	.00560	.0506	178.503	19.762	15.998	316.145	**50**
55	11.256	.0888	.00439	.0494	227.918	20.248	16.860	341.375	**55**
60	14.027	.0713	.00345	.0485	289.497	20.638	17.617	363.571	**60**
65	17.481	.0572	.00273	.0477	366.237	20.951	18.278	382.946	**65**
70	21.784	.0459	.00217	.0472	461.869	21.202	18.854	399.750	**70**
75	27.147	.0368	.00172	.0467	581.043	21.404	19.354	414.242	**75**
80	33.830	.0296	.00137	.0464	729.556	21.565	19.785	426.680	**80**
85	42.158	.0237	.00109	.0461	914.630	21.695	20.157	437.309	**85**
90	52.537	.0190	.00087	.0459	1 145.3	21.799	20.476	446.359	**90**
95	65.471	.0153	.00070	.0457	1 432.7	21.883	20.749	454.039	**95**
100	81.588	.0123	.00056	.0456	1 790.9	21.950	20.981	460.537	**100**

Compound Interest Factors

5%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.050	.9524	1.0000	1.0500	1.000	0.952	0	0	**1**
2	1.102	.9070	.4878	.5378	2.050	1.859	0.488	0.907	**2**
3	1.158	.8638	.3172	.3672	3.152	2.723	0.967	2.635	**3**
4	1.216	.8227	.2320	.2820	4.310	3.546	1.439	5.103	**4**
5	1.276	.7835	.1810	.2310	5.526	4.329	1.902	8.237	**5**
6	1.340	.7462	.1470	.1970	6.802	5.076	2.358	11.968	**6**
7	1.407	.7107	.1228	.1728	8.142	5.786	2.805	16.232	**7**
8	1.477	.6768	.1047	.1547	9.549	6.463	3.244	20.970	**8**
9	1.551	.6446	.0907	.1407	11.027	7.108	3.676	26.127	**9**
10	1.629	.6139	.0795	.1295	12.578	7.722	4.099	31.652	**10**
11	1.710	.5847	.0704	.1204	14.207	8.306	4.514	37.499	**11**
12	1.796	.5568	.0628	.1128	15.917	8.863	4.922	43.624	**12**
13	1.886	.5303	.0565	.1065	17.713	9.394	5.321	49.988	**13**
14	1.980	.5051	.0510	.1010	19.599	9.899	5.713	56.553	**14**
15	2.079	.4810	.0463	.0963	21.579	10.380	6.097	63.288	**15**
16	2.183	.4581	.0423	.0923	23.657	10.838	6.474	70.159	**16**
17	2.292	.4363	.0387	.0887	25.840	11.274	6.842	77.140	**17**
18	2.407	.4155	.0355	.0855	28.132	11.690	7.203	84.204	**18**
19	2.527	.3957	.0327	.0827	30.539	12.085	7.557	91.327	**19**
20	2.653	.3769	.0302	.0802	33.066	12.462	7.903	98.488	**20**
21	2.786	.3589	.0280	.0780	35.719	12.821	8.242	105.667	**21**
22	2.925	.3419	.0260	.0760	38.505	13.163	8.573	112.846	**22**
23	3.072	.3256	.0241	.0741	41.430	13.489	8.897	120.008	**23**
24	3.225	.3101	.0225	.0725	44.502	13.799	9.214	127.140	**24**
25	3.386	.2953	.0210	.0710	47.727	14.094	9.524	134.227	**25**
26	3.556	.2812	.0196	.0696	51.113	14.375	9.827	141.258	**26**
27	3.733	.2678	.0183	.0683	54.669	14.643	10.122	148.222	**27**
28	3.920	.2551	.0171	.0671	58.402	14.898	10.411	155.110	**28**
29	4.116	.2429	.0160	.0660	62.323	15.141	10.694	161.912	**29**
30	4.322	.2314	.0151	.0651	66.439	15.372	10.969	168.622	**30**
31	4.538	.2204	.0141	.0641	70.761	15.593	11.238	175.233	**31**
32	4.765	.2099	.0133	.0633	75.299	15.803	11.501	181.739	**32**
33	5.003	.1999	.0125	.0625	80.063	16.003	11.757	188.135	**33**
34	5.253	.1904	.0118	.0618	85.067	16.193	12.006	194.416	**34**
35	5.516	.1813	.0111	.0611	90.320	16.374	12.250	200.580	**35**
40	7.040	.1420	.00828	.0583	120.799	17.159	13.377	229.545	**40**
45	8.985	.1113	.00626	.0563	159.699	17.774	14.364	255.314	**45**
50	11.467	.0872	.00478	.0548	209.347	18.256	15.223	277.914	**50**
55	14.636	.0683	.00367	.0537	272.711	18.633	15.966	297.510	**55**
60	18.679	.0535	.00283	.0528	353.582	18.929	16.606	314.343	**60**
65	23.840	.0419	.00219	.0522	456.795	19.161	17.154	328.691	**65**
70	30.426	.0329	.00170	.0517	588.525	19.343	17.621	340.841	**70**
75	38.832	.0258	.00132	.0513	756.649	19.485	18.018	351.072	**75**
80	49.561	.0202	.00103	.0510	971.222	19.596	18.353	359.646	**80**
85	63.254	.0158	.00080	.0508	1 245.1	19.684	18.635	366.800	**85**
90	80.730	.0124	.00063	.0506	1 594.6	19.752	18.871	372.749	**90**
95	103.034	.00971	.00049	.0505	2 040.7	19.806	19.069	377.677	**95**
100	131.500	.00760	.00038	.0504	2 610.0	19.848	19.234	381.749	**100**

6% Compound Interest Factors 6%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.060	.9434	1.0000	1.0600	1.000	0.943	0	0	**1**
2	1.124	.8900	.4854	.5454	2.060	1.833	0.485	0.890	**2**
3	1.191	.8396	.3141	.3741	3.184	2.673	0.961	2.569	**3**
4	1.262	.7921	.2286	.2886	4.375	3.465	1.427	4.945	**4**
5	1.338	.7473	.1774	.2374	5.637	4.212	1.884	7.934	**5**
6	1.419	.7050	.1434	.2034	6.975	4.917	2.330	11.459	**6**
7	1.504	.6651	.1191	.1791	8.394	5.582	2.768	15.450	**7**
8	1.594	.6274	.1010	.1610	9.897	6.210	3.195	19.841	**8**
9	1.689	.5919	.0870	.1470	11.491	6.802	3.613	24.577	**9**
10	1.791	.5584	.0759	.1359	13.181	7.360	4.022	29.602	**10**
11	1.898	.5268	.0668	.1268	14.972	7.887	4.421	34.870	**11**
12	2.012	.4970	.0593	.1193	16.870	8.384	4.811	40.337	**12**
13	2.133	.4688	.0530	.1130	18.882	8.853	5.192	45.963	**13**
14	2.261	.4423	.0476	.1076	21.015	9.295	5.564	51.713	**14**
15	2.397	.4173	.0430	.1030	23.276	9.712	5.926	57.554	**15**
16	2.540	.3936	.0390	.0990	25.672	10.106	6.279	63.459	**16**
17	2.693	.3714	.0354	.0954	28.213	10.477	6.624	69.401	**17**
18	2.854	.3503	.0324	.0924	30.906	10.828	6.960	75.357	**18**
19	3.026	.3305	.0296	.0896	33.760	11.158	7.287	81.306	**19**
20	3.207	.3118	.0272	.0872	36.786	11.470	7.605	87.230	**20**
21	3.400	.2942	.0250	.0850	39.993	11.764	7.915	93.113	**21**
22	3.604	.2775	.0230	.0830	43.392	12.042	8.217	98.941	**22**
23	3.820	.2618	.0213	.0813	46.996	12.303	8.510	104.700	**23**
24	4.049	.2470	.0197	.0797	50.815	12.550	8.795	110.381	**24**
25	4.292	.2330	.0182	.0782	54.864	12.783	9.072	115.973	**25**
26	4.549	.2198	.0169	.0769	59.156	13.003	9.341	121.468	**26**
27	4.822	.2074	.0157	.0757	63.706	13.211	9.603	126.860	**27**
28	5.112	.1956	.0146	.0746	68.528	13.406	9.857	132.142	**28**
29	5.418	.1846	.0136	.0736	73.640	13.591	10.103	137.309	**29**
30	5.743	.1741	.0126	.0726	79.058	13.765	10.342	142.359	**30**
31	6.088	.1643	.0118	.0718	84.801	13.929	10.574	147.286	**31**
32	6.453	.1550	.0110	.0710	90.890	14.084	10.799	152.090	**32**
33	6.841	.1462	.0103	.0703	97.343	14.230	11.017	156.768	**33**
34	7.251	.1379	.00960	.0696	104.184	14.368	11.228	161.319	**34**
35	7.686	.1301	.00897	.0690	111.435	14.498	11.432	165.743	**35**
40	10.286	.0972	.00646	.0665	154.762	15.046	12.359	185.957	**40**
45	13.765	.0727	.00470	.0647	212.743	15.456	13.141	203.109	**45**
50	18.420	.0543	.00344	.0634	290.335	15.762	13.796	217.457	**50**
55	24.650	.0406	.00254	.0625	394.171	15.991	14.341	229.322	**55**
60	32.988	.0303	.00188	.0619	533.126	16.161	14.791	239.043	**60**
65	44.145	.0227	.00139	.0614	719.080	16.289	15.160	246.945	**65**
70	59.076	.0169	.00103	.0610	967.928	16.385	15.461	253.327	**70**
75	79.057	.0126	.00077	.0608	1 300.9	16.456	15.706	258.453	**75**
80	105.796	.00945	.00057	.0606	1 746.6	16.509	15.903	262.549	**80**
85	141.578	.00706	.00043	.0604	2 343.0	16.549	16.062	265.810	**85**
90	189.464	.00528	.00032	.0603	3 141.1	16.579	16.189	268.395	**90**
95	253.545	.00394	.00024	.0602	4 209.1	16.601	16.290	270.437	**95**
100	339.300	.00295	.00018	.0602	5 638.3	16.618	16.371	272.047	**100**

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.070	.9346	1.0000	1.0700	1.000	0.935	0	0	**1**
2	1.145	.8734	.4831	.5531	2.070	1.808	0.483	0.873	**2**
3	1.225	.8163	.3111	.3811	3.215	2.624	0.955	2.506	**3**
4	1.311	.7629	.2252	.2952	4.440	3.387	1.416	4.795	**4**
5	1.403	.7130	.1739	.2439	5.751	4.100	1.865	7.647	**5**
6	1.501	.6663	.1398	.2098	7.153	4.767	2.303	10.978	**6**
7	1.606	.6227	.1156	.1856	8.654	5.389	2.730	14.715	**7**
8	1.718	.5820	.0975	.1675	10.260	5.971	3.147	18.789	**8**
9	1.838	.5439	.0835	.1535	11.978	6.515	3.552	23.140	**9**
10	1.967	.5083	.0724	.1424	13.816	7.024	3.946	27.716	**10**
11	2.105	.4751	.0634	.1334	15.784	7.499	4.330	32.467	**11**
12	2.252	.4440	.0559	.1259	17.888	7.943	4.703	37.351	**12**
13	2.410	.4150	.0497	.1197	20.141	8.358	5.065	42.330	**13**
14	2.579	.3878	.0443	.1143	22.551	8.745	5.417	47.372	**14**
15	2.759	.3624	.0398	.1098	25.129	9.108	5.758	52.446	**15**
16	2.952	.3387	.0359	.1059	27.888	9.447	6.090	57.527	**16**
17	3.159	.3166	.0324	.1024	30.840	9.763	6.411	62.592	**17**
18	3.380	.2959	.0294	.0994	33.999	10.059	6.722	67.622	**18**
19	3.617	.2765	.0268	.0968	37.379	10.336	7.024	72.599	**19**
20	3.870	.2584	.0244	.0944	40.996	10.594	7.316	77.509	**20**
21	4.141	.2415	.0223	.0923	44.865	10.836	7.599	82.339	**21**
22	4.430	.2257	.0204	.0904	49.006	11.061	7.872	87.079	**22**
23	4.741	.2109	.0187	.0887	53.436	11.272	8.137	91.720	**23**
24	5.072	.1971	.0172	.0872	58.177	11.469	8.392	96.255	**24**
25	5.427	.1842	.0158	.0858	63.249	11.654	8.639	100.677	**25**
26	5.807	.1722	.0146	.0846	68.677	11.826	8.877	104.981	**26**
27	6.214	.1609	.0134	.0834	74.484	11.987	9.107	109.166	**27**
28	6.649	.1504	.0124	.0824	80.698	12.137	9.329	113.227	**28**
29	7.114	.1406	.0114	.0814	87.347	12.278	9.543	117.162	**29**
30	7.612	.1314	.0106	.0806	94.461	12.409	9.749	120.972	**30**
31	8.145	.1228	.00980	.0798	102.073	12.532	9.947	124.655	**31**
32	8.715	.1147	.00907	.0791	110.218	12.647	10.138	128.212	**32**
33	9.325	.1072	.00841	.0784	118.934	12.754	10.322	131.644	**33**
34	9.978	.1002	.00780	.0778	128.259	12.854	10.499	134.951	**34**
35	10.677	.0937	.00723	.0772	138.237	12.948	10.669	138.135	**35**
40	14.974	.0668	.00501	.0750	199.636	13.332	11.423	152.293	**40**
45	21.002	.0476	.00350	.0735	285.750	13.606	12.036	163.756	**45**
50	29.457	.0339	.00246	.0725	406.530	13.801	12.529	172.905	**50**
55	41.315	.0242	.00174	.0717	575.930	13.940	12.921	180.124	**55**
60	57.947	.0173	.00123	.0712	813.523	14.039	13.232	185.768	**60**
65	81.273	.0123	.00087	.0709	1 146.8	14.110	13.476	190.145	**65**
70	113.990	.00877	.00062	.0706	1 614.1	14.160	13.666	193.519	**70**
75	159.877	.00625	.00044	.0704	2 269.7	14.196	13.814	196.104	**75**
80	224.235	.00446	.00031	.0703	3 189.1	14.222	13.927	198.075	**80**
85	314.502	.00318	.00022	.0702	4 478.6	14.240	14.015	199.572	**85**
90	441.105	.00227	.00016	.0702	6 287.2	14.253	14.081	200.704	**90**
95	618.673	.00162	.00011	.0701	8 823.9	14.263	14.132	201.558	**95**
100	867.720	.00115	.00008	.0701	12 381.7	14.269	14.170	202.200	**100**

8% Compound Interest Factors 8%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.080	.9259	1.0000	1.0800	1.000	0.926	0	0	**1**
2	1.166	.8573	.4808	.5608	2.080	1.783	0.481	0.857	**2**
3	1.260	.7938	.3080	.3880	3.246	2.577	0.949	2.445	**3**
4	1.360	.7350	.2219	.3019	4.506	3.312	1.404	4.650	**4**
5	1.469	.6806	.1705	.2505	5.867	3.993	1.846	7.372	**5**
6	1.587	.6302	.1363	.2163	7.336	4.623	2.276	10.523	**6**
7	1.714	.5835	.1121	.1921	8.923	5.206	2.694	14.024	**7**
8	1.851	.5403	.0940	.1740	10.637	5.747	3.099	17.806	**8**
9	1.999	.5002	.0801	.1601	12.488	6.247	3.491	21.808	**9**
10	2.159	.4632	.0690	.1490	14.487	6.710	3.871	25.977	**10**
11	2.332	.4289	.0601	.1401	16.645	7.139	4.240	30.266	**11**
12	2.518	.3971	.0527	.1327	18.977	7.536	4.596	34.634	**12**
13	2.720	.3677	.0465	.1265	21.495	7.904	4.940	39.046	**13**
14	2.937	.3405	.0413	.1213	24.215	8.244	5.273	43.472	**14**
15	3.172	.3152	.0368	.1168	27.152	8.559	5.594	47.886	**15**
16	3.426	.2919	.0330	.1130	30.324	8.851	5.905	52.264	**16**
17	3.700	.2703	.0296	.1096	33.750	9.122	6.204	56.588	**17**
18	3.996	.2502	.0267	.1067	37.450	9.372	6.492	60.843	**18**
19	4.316	.2317	.0241	.1041	41.446	9.604	6.770	65.013	**19**
20	4.661	.2145	.0219	.1019	45.762	9.818	7.037	69.090	**20**
21	5.034	.1987	.0198	.0998	50.423	10.017	7.294	73.063	**21**
22	5.437	.1839	.0180	.0980	55.457	10.201	7.541	76.926	**22**
23	5.871	.1703	.0164	.0964	60.893	10.371	7.779	80.673	**23**
24	6.341	.1577	.0150	.0950	66.765	10.529	8.007	84.300	**24**
25	6.848	.1460	.0137	.0937	73.106	10.675	8.225	87.804	**25**
26	7.396	.1352	.0125	.0925	79.954	10.810	8.435	91.184	**26**
27	7.988	.1252	.0114	.0914	87.351	10.935	8.636	94.439	**27**
28	8.627	.1159	.0105	.0905	95.339	11.051	8.829	97.569	**28**
29	9.317	.1073	.00962	.0896	103.966	11.158	9.013	100.574	**29**
30	10.063	.0994	.00883	.0888	113.283	11.258	9.190	103.456	**30**
31	10.868	.0920	.00811	.0881	123.346	11.350	9.358	106.216	**31**
32	11.737	.0852	.00745	.0875	134.214	11.435	9.520	108.858	**32**
33	12.676	.0789	.00685	.0869	145.951	11.514	9.674	111.382	**33**
34	13.690	.0730	.00630	.0863	158.627	11.587	9.821	113.792	**34**
35	14.785	.0676	.00580	.0858	172.317	11.655	9.961	116.092	**35**
40	21.725	.0460	.00386	.0839	259.057	11.925	10.570	126.042	**40**
45	31.920	.0313	.00259	.0826	386.506	12.108	11.045	133.733	**45**
50	46.902	.0213	.00174	.0817	573.771	12.233	11.411	139.593	**50**
55	68.914	.0145	.00118	.0812	848.925	12.319	11.690	144.006	**55**
60	101.257	.00988	.00080	.0808	1 253.2	12.377	11.902	147.300	**60**
65	148.780	.00672	.00054	.0805	1 847.3	12.416	12.060	149.739	**65**
70	218.607	.00457	.00037	.0804	2 720.1	12.443	12.178	151.533	**70**
75	321.205	.00311	.00025	.0802	4 002.6	12.461	12.266	152.845	**75**
80	471.956	.00212	.00017	.0802	5 887.0	12.474	12.330	153.800	**80**
85	693.458	.00144	.00012	.0801	8 655.7	12.482	12.377	154.492	**85**
90	1 018.9	.00098	.00008	.0801	12 724.0	12.488	12.412	154.993	**90**
95	1 497.1	.00067	.00005	.0801	18 701.6	12.492	12.437	155.352	**95**
100	2 199.8	.00045	.00004	.0800	27 484.6	12.494	12.455	155.611	**100**

9% Compound Interest Factors 9%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find F Given P	Find P Given F	Find A Given F	Find A Given P	Find F Given A	Find P Given A	Find A Given G	Find P Given G	
n	F/P	P/F	A/F	A/P	F/A	P/A	A/G	P/G	n
1	1.090	.9174	1.0000	1.0900	1.000	0.917	0	0	**1**
2	1.188	.8417	.4785	.5685	2.090	1.759	0.478	0.842	**2**
3	1.295	.7722	.3051	.3951	3.278	2.531	0.943	2.386	**3**
4	1.412	.7084	.2187	.3087	4.573	3.240	1.393	4.511	**4**
5	1.539	.6499	.1671	.2571	5.985	3.890	1.828	7.111	**5**
6	1.677	.5963	.1329	.2229	7.523	4.486	2.250	10.092	**6**
7	1.828	.5470	.1087	.1987	9.200	5.033	2.657	13.375	**7**
8	1.993	.5019	.0907	.1807	11.028	5.535	3.051	16.888	**8**
9	2.172	.4604	.0768	.1668	13.021	5.995	3.431	20.571	**9**
10	2.367	.4224	.0658	.1558	15.193	6.418	3.798	24.373	**10**
11	2.580	.3875	.0569	.1469	17.560	6.805	4.151	28.248	**11**
12	2.813	.3555	.0497	.1397	20.141	7.161	4.491	32.159	**12**
13	3.066	.3262	.0436	.1336	22.953	7.487	4.818	36.073	**13**
14	3.342	.2992	.0384	.1284	26.019	7.786	5.133	39.963	**14**
15	3.642	.2745	.0341	.1241	29.361	8.061	5.435	43.807	**15**
16	3.970	.2519	.0303	.1203	33.003	8.313	5.724	47.585	**16**
17	4.328	.2311	.0270	.1170	36.974	8.544	6.002	51.282	**17**
18	4.717	.2120	.0242	.1142	41.301	8.756	6.269	54.886	**18**
19	5.142	.1945	.0217	.1117	46.019	8.950	6.524	58.387	**19**
20	5.604	.1784	.0195	.1095	51.160	9.129	6.767	61.777	**20**
21	6.109	.1637	.0176	.1076	56.765	9.292	7.001	65.051	**21**
22	6.659	.1502	.0159	.1059	62.873	9.442	7.223	68.205	**22**
23	7.258	.1378	.0144	.1044	69.532	9.580	7.436	71.236	**23**
24	7.911	.1264	.0130	.1030	76.790	9.707	7.638	74.143	**24**
25	8.623	.1160	.0118	.1018	84.701	9.823	7.832	76.927	**25**
26	9.399	.1064	.0107	.1007	93.324	9.929	8.016	79.586	**26**
27	10.245	.0976	.00973	.0997	102.723	10.027	8.191	82.124	**27**
28	11.167	.0895	.00885	.0989	112.968	10.116	8.357	84.542	**28**
29	12.172	.0822	.00806	.0981	124.136	10.198	8.515	86.842	**29**
30	13.268	.0754	.00734	.0973	136.308	10.274	8.666	89.028	**30**
31	14.462	.0691	.00669	.0967	149.575	10.343	8.808	91.102	**31**
32	15.763	.0634	.00610	.0961	164.037	10.406	8.944	93.069	**32**
33	17.182	.0582	.00556	.0956	179.801	10.464	9.072	94.931	**33**
34	18.728	.0534	.00508	.0951	196.983	10.518	9.193	96.693	**34**
35	20.414	.0490	.00464	.0946	215.711	10.567	9.308	98.359	**35**
40	31.409	.0318	.00296	.0930	337.883	10.757	9.796	105.376	**40**
45	48.327	.0207	.00190	.0919	525.860	10.881	10.160	110.556	**45**
50	74.358	.0134	.00123	.0912	815.085	10.962	10.430	114.325	**50**
55	114.409	.00874	.00079	.0908	1 260.1	11.014	10.626	117.036	**55**
60	176.032	.00568	.00051	.0905	1 944.8	11.048	10.768	118.968	**60**
65	270.847	.00369	.00033	.0903	2 998.3	11.070	10.870	120.334	**65**
70	416.731	.00240	.00022	.0902	4 619.2	11.084	10.943	121.294	**70**
75	641.193	.00156	.00014	.0901	7 113.3	11.094	10.994	121.965	**75**
80	986.555	.00101	.00009	.0901	10 950.6	11.100	11.030	122.431	**80**
85	1 517.9	.00066	.00006	.0901	16 854.9	11.104	11.055	122.753	**85**
90	2 335.5	.00043	.00004	.0900	25 939.3	11.106	11.073	122.976	**90**
95	3 593.5	.00028	.00003	.0900	39 916.8	11.108	11.085	123.129	**95**
100	5 529.1	.00018	.00002	.0900	61 422.9	11.109	11.093	123.233	**100**

10% Compound Interest Factors 10%

n	Single Payment: Compound Amount Factor, Find F Given P, F/P	Single Payment: Present Worth Factor, Find P Given F, P/F	Uniform Payment Series: Sinking Fund Factor, Find A Given F, A/F	Uniform Payment Series: Capital Recovery Factor, Find A Given P, A/P	Uniform Payment Series: Compound Amount Factor, Find F Given A, F/A	Uniform Payment Series: Present Worth Factor, Find P Given A, P/A	Arithmetic Gradient: Gradient Uniform Series, Find A Given G, A/G	Arithmetic Gradient: Gradient Present Worth, Find P Given G, P/G	n
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10
11	2.853	.3505	.0540	.1540	18.531	6.495	4.064	26.396	11
12	3.138	.3186	.0468	.1468	21.384	6.814	4.388	29.901	12
13	3.452	.2897	.0408	.1408	24.523	7.103	4.699	33.377	13
14	3.797	.2633	.0357	.1357	27.975	7.367	4.996	36.801	14
15	4.177	.2394	.0315	.1315	31.772	7.606	5.279	40.152	15
16	4.595	.2176	.0278	.1278	35.950	7.824	5.549	43.416	16
17	5.054	.1978	.0247	.1247	40.545	8.022	5.807	46.582	17
18	5.560	.1799	.0219	.1219	45.599	8.201	6.053	49.640	18
19	6.116	.1635	.0195	.1195	51.159	8.365	6.286	52.583	19
20	6.728	.1486	.0175	.1175	57.275	8.514	6.508	55.407	20
21	7.400	.1351	.0156	.1156	64.003	8.649	6.719	58.110	21
22	8.140	.1228	.0140	.1140	71.403	8.772	6.919	60.689	22
23	8.954	.1117	.0126	.1126	79.543	8.883	7.108	63.146	23
24	9.850	.1015	.0113	.1113	88.497	8.985	7.288	65.481	24
25	10.835	.0923	.0102	.1102	98.347	9.077	7.458	67.696	25
26	11.918	.0839	.00916	.1092	109.182	9.161	7.619	69.794	26
27	13.110	.0763	.00826	.1083	121.100	9.237	7.770	71.777	27
28	14.421	.0693	.00745	.1075	134.210	9.307	7.914	73.650	28
29	15.863	.0630	.00673	.1067	148.631	9.370	8.049	75.415	29
30	17.449	.0573	.00608	.1061	164.494	9.427	8.176	77.077	30
31	19.194	.0521	.00550	.1055	181.944	9.479	8.296	78.640	31
32	21.114	.0474	.00497	.1050	201.138	9.526	8.409	80.108	32
33	23.225	.0431	.00450	.1045	222.252	9.569	8.515	81.486	33
34	25.548	.0391	.00407	.1041	245.477	9.609	8.615	82.777	34
35	28.102	.0356	.00369	.1037	271.025	9.644	8.709	83.987	35
40	45.259	.0221	.00226	.1023	442.593	9.779	9.096	88.953	40
45	72.891	.0137	.00139	.1014	718.905	9.863	9.374	92.454	45
50	117.391	.00852	.00086	.1009	1 163.9	9.915	9.570	94.889	50
55	189.059	.00529	.00053	.1005	1 880.6	9.947	9.708	96.562	55
60	304.482	.00328	.00033	.1003	3 034.8	9.967	9.802	97.701	60
65	490.371	.00204	.00020	.1002	4 893.7	9.980	9.867	98.471	65
70	789.748	.00127	.00013	.1001	7 887.5	9.987	9.911	98.987	70
75	1 271.9	.00079	.00008	.1001	12 709.0	9.992	9.941	99.332	75
80	2 048.4	.00049	.00005	.1000	20 474.0	9.995	9.961	99.561	80
85	3 299.0	.00030	.00003	.1000	32 979.7	9.997	9.974	99.712	85
90	5 313.0	.00019	.00002	.1000	53 120.3	9.998	9.983	99.812	90
95	8 556.7	.00012	.00001	.1000	85 556.9	9.999	9.989	99.877	95
100	13 780.6	.00007	.00001	.1000	137 796.3	9.999	9.993	99.920	100

12% Compound Interest Factors 12%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.120	.8929	1.0000	1.1200	1.000	0.893	0	0	**1**
2	1.254	.7972	.4717	.5917	2.120	1.690	0.472	0.797	**2**
3	1.405	.7118	.2963	.4163	3.374	2.402	0.925	2.221	**3**
4	1.574	.6355	.2092	.3292	4.779	3.037	1.359	4.127	**4**
5	1.762	.5674	.1574	.2774	6.353	3.605	1.775	6.397	**5**
6	1.974	.5066	.1232	.2432	8.115	4.111	2.172	8.930	**6**
7	2.211	.4523	.0991	.2191	10.089	4.564	2.551	11.644	**7**
8	2.476	.4039	.0813	.2013	12.300	4.968	2.913	14.471	**8**
9	2.773	.3606	.0677	.1877	14.776	5.328	3.257	17.356	**9**
10	3.106	.3220	.0570	.1770	17.549	5.650	3.585	20.254	**10**
11	3.479	.2875	.0484	.1684	20.655	5.938	3.895	23.129	**11**
12	3.896	.2567	.0414	.1614	24.133	6.194	4.190	25.952	**12**
13	4.363	.2292	.0357	.1557	28.029	6.424	4.468	28.702	**13**
14	4.887	.2046	.0309	.1509	32.393	6.628	4.732	31.362	**14**
15	5.474	.1827	.0268	.1468	37.280	6.811	4.980	33.920	**15**
16	6.130	.1631	.0234	.1434	42.753	6.974	5.215	36.367	**16**
17	6.866	.1456	.0205	.1405	48.884	7.120	5.435	38.697	**17**
18	7.690	.1300	.0179	.1379	55.750	7.250	5.643	40.908	**18**
19	8.613	.1161	.0158	.1358	63.440	7.366	5.838	42.998	**19**
20	9.646	.1037	.0139	.1339	72.052	7.469	6.020	44.968	**20**
21	10.804	.0926	.0122	.1322	81.699	7.562	6.191	46.819	**21**
22	12.100	.0826	.0108	.1308	92.503	7.645	6.351	48.554	**22**
23	13.552	.0738	.00956	.1296	104.603	7.718	6.501	50.178	**23**
24	15.179	.0659	.00846	.1285	118.155	7.784	6.641	51.693	**24**
25	17.000	.0588	.00750	.1275	133.334	7.843	6.771	53.105	**25**
26	19.040	.0525	.00665	.1267	150.334	7.896	6.892	54.418	**26**
27	21.325	.0469	.00590	.1259	169.374	7.943	7.005	55.637	**27**
28	23.884	.0419	.00524	.1252	190.699	7.984	7.110	56.767	**28**
29	26.750	.0374	.00466	.1247	214.583	8.022	7.207	57.814	**29**
30	29.960	.0334	.00414	.1241	241.333	8.055	7.297	58.782	**30**
31	33.555	.0298	.00369	.1237	271.293	8.085	7.381	59.676	**31**
32	37.582	.0266	.00328	.1233	304.848	8.112	7.459	60.501	**32**
33	42.092	.0238	.00292	.1229	342.429	8.135	7.530	61.261	**33**
34	47.143	.0212	.00260	.1226	384.521	8.157	7.596	61.961	**34**
35	52.800	.0189	.00232	.1223	431.663	8.176	7.658	62.605	**35**
40	93.051	.0107	.00130	.1213	767.091	8.244	7.899	65.116	**40**
45	163.988	.00610	.00074	.1207	1 358.2	8.283	8.057	66.734	**45**
50	289.002	.00346	.00042	.1204	2 400.0	8.304	8.160	67.762	**50**
55	509.321	.00196	.00024	.1202	4 236.0	8.317	8.225	68.408	**55**
60	897.597	.00111	.00013	.1201	7 471.6	8.324	8.266	68.810	**60**
65	1 581.9	.00063	.00008	.1201	13 173.9	8.328	8.292	69.058	**65**
70	2 787.8	.00036	.00004	.1200	23 223.3	8.330	8.308	69.210	**70**
75	4 913.1	.00020	.00002	.1200	40 933.8	8.332	8.318	69.303	**75**
80	8 658.5	.00012	.00001	.1200	72 145.7	8.332	8.324	69.359	**80**
85	15 259.2	.00007	.00001	.1200	127 151.7	8.333	8.328	69.393	**85**
90	26 891.9	.00004		.1200	224 091.1	8.333	8.330	69.414	**90**
95	47 392.8	.00002		.1200	394 931.4	8.333	8.331	69.426	**95**
100	83 522.3	.00001		.1200	696 010.5	8.333	8.332	69.434	**100**

15% Compound Interest Factors 15%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find F Given P	Find P Given F	Find A Given F	Find A Given P	Find F Given A	Find P Given A	Find A Given G	Find P Given G	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	**n**
1	1.150	.8696	1.0000	1.1500	1.000	0.870	0	0	**1**
2	1.322	.7561	.4651	.6151	2.150	1.626	0.465	0.756	**2**
3	1.521	.6575	.2880	.4380	3.472	2.283	0.907	2.071	**3**
4	1.749	.5718	.2003	.3503	4.993	2.855	1.326	3.786	**4**
5	2.011	.4972	.1483	.2983	6.742	3.352	1.723	5.775	**5**
6	2.313	.4323	.1142	.2642	8.754	3.784	2.097	7.937	**6**
7	2.660	.3759	.0904	.2404	11.067	4.160	2.450	10.192	**7**
8	3.059	.3269	.0729	.2229	13.727	4.487	2.781	12.481	**8**
9	3.518	.2843	.0596	.2096	16.786	4.772	3.092	14.755	**9**
10	4.046	.2472	.0493	.1993	20.304	5.019	3.383	16.979	**10**
11	4.652	.2149	.0411	.1911	24.349	5.234	3.655	19.129	**11**
12	5.350	.1869	.0345	.1845	29.002	5.421	3.908	21.185	**12**
13	6.153	.1625	.0291	.1791	34.352	5.583	4.144	23.135	**13**
14	7.076	.1413	.0247	.1747	40.505	5.724	4.362	24.972	**14**
15	8.137	.1229	.0210	.1710	47.580	5.847	4.565	26.693	**15**
16	9.358	.1069	.0179	.1679	55.717	5.954	4.752	28.296	**16**
17	10.761	.0929	.0154	.1654	65.075	6.047	4.925	29.783	**17**
18	12.375	.0808	.0132	.1632	75.836	6.128	5.084	31.156	**18**
19	14.232	.0703	.0113	.1613	88.212	6.198	5.231	32.421	**19**
20	16.367	.0611	.00976	.1598	102.444	6.259	5.365	33.582	**20**
21	18.822	.0531	.00842	.1584	118.810	6.312	5.488	34.645	**21**
22	21.645	.0462	.00727	.1573	137.632	6.359	5.601	35.615	**22**
23	24.891	.0402	.00628	.1563	159.276	6.399	5.704	36.499	**23**
24	28.625	.0349	.00543	.1554	184.168	6.434	5.798	37.302	**24**
25	32.919	.0304	.00470	.1547	212.793	6.464	5.883	38.031	**25**
26	37.857	.0264	.00407	.1541	245.712	6.491	5.961	38.692	**26**
27	43.535	.0230	.00353	.1535	283.569	6.514	6.032	39.289	**27**
28	50.066	.0200	.00306	.1531	327.104	6.534	6.096	39.828	**28**
29	57.575	.0174	.00265	.1527	377.170	6.551	6.154	40.315	**29**
30	66.212	.0151	.00230	.1523	434.745	6.566	6.207	40.753	**30**
31	76.144	.0131	.00200	.1520	500.957	6.579	6.254	41.147	**31**
32	87.565	.0114	.00173	.1517	577.100	6.591	6.297	41.501	**32**
33	100.700	.00993	.00150	.1515	664.666	6.600	6.336	41.818	**33**
34	115.805	.00864	.00131	.1513	765.365	6.609	6.371	42.103	**34**
35	133.176	.00751	.00113	.1511	881.170	6.617	6.402	42.359	**35**
40	267.864	.00373	.00056	.1506	1 779.1	6.642	6.517	43.283	**40**
45	538.769	.00186	.00028	.1503	3 585.1	6.654	6.583	43.805	**45**
50	1 083.7	.00092	.00014	.1501	7 217.7	6.661	6.620	44.096	**50**
55	2 179.6	.00046	.00007	.1501	14 524.1	6.664	6.641	44.256	**55**
60	4 384.0	.00023	.00003	.1500	29 220.0	6.665	6.653	44.343	**60**
65	8 817.8	.00011	.00002	.1500	58 778.6	6.666	6.659	44.390	**65**
70	17 735.7	.00006	.00001	.1500	118 231.5	6.666	6.663	44.416	**70**
75	35 672.9	.00003		.1500	237 812.5	6.666	6.665	44.429	**75**
80	71 750.9	.00001		.1500	478 332.6	6.667	6.666	44.436	**80**
85	144 316.7	.00001		.1500	962 104.4	6.667	6.666	44.440	**85**

n	Single Payment: Compound Amount Factor, Find F Given P, F/P	Single Payment: Present Worth Factor, Find P Given F, P/F	Uniform Payment Series: Sinking Fund Factor, Find A Given F, A/F	Uniform Payment Series: Capital Recovery Factor, Find A Given P, A/P	Uniform Payment Series: Compound Amount Factor, Find F Given A, F/A	Uniform Payment Series: Present Worth Factor, Find P Given A, P/A	Arithmetic Gradient: Gradient Uniform Series, Find A Given G, A/G	Arithmetic Gradient: Gradient Present Worth, Find P Given G, P/G	n
1	1.180	.8475	1.0000	1.1800	1.000	0.847	0	0	1
2	1.392	.7182	.4587	.6387	2.180	1.566	0.459	0.718	2
3	1.643	.6086	.2799	.4599	3.572	2.174	0.890	1.935	3
4	1.939	.5158	.1917	.3717	5.215	2.690	1.295	3.483	4
5	2.288	.4371	.1398	.3198	7.154	3.127	1.673	5.231	5
6	2.700	.3704	.1059	.2859	9.442	3.498	2.025	7.083	6
7	3.185	.3139	.0824	.2624	12.142	3.812	2.353	8.967	7
8	3.759	.2660	.0652	.2452	15.327	4.078	2.656	10.829	8
9	4.435	.2255	.0524	.2324	19.086	4.303	2.936	12.633	9
10	5.234	.1911	.0425	.2225	23.521	4.494	3.194	14.352	10
11	6.176	.1619	.0348	.2148	28.755	4.656	3.430	15.972	11
12	7.288	.1372	.0286	.2086	34.931	4.793	3.647	17.481	12
13	8.599	.1163	.0237	.2037	42.219	4.910	3.845	18.877	13
14	10.147	.0985	.0197	.1997	50.818	5.008	4.025	20.158	14
15	11.974	.0835	.0164	.1964	60.965	5.092	4.189	21.327	15
16	14.129	.0708	.0137	.1937	72.939	5.162	4.337	22.389	16
17	16.672	.0600	.0115	.1915	87.068	5.222	4.471	23.348	17
18	19.673	.0508	.00964	.1896	103.740	5.273	4.592	24.212	18
19	23.214	.0431	.00810	.1881	123.413	5.316	4.700	24.988	19
20	27.393	.0365	.00682	.1868	146.628	5.353	4.798	25.681	20
21	32.324	.0309	.00575	.1857	174.021	5.384	4.885	26.300	21
22	38.142	.0262	.00485	.1848	206.345	5.410	4.963	26.851	22
23	45.008	.0222	.00409	.1841	244.487	5.432	5.033	27.339	23
24	53.109	.0188	.00345	.1835	289.494	5.451	5.095	27.772	24
25	62.669	.0160	.00292	.1829	342.603	5.467	5.150	28.155	25
26	73.949	.0135	.00247	.1825	405.272	5.480	5.199	28.494	26
27	87.260	.0115	.00209	.1821	479.221	5.492	5.243	28.791	27
28	102.966	.00971	.00177	.1818	566.480	5.502	5.281	29.054	28
29	121.500	.00823	.00149	.1815	669.447	5.510	5.315	29.284	29
30	143.370	.00697	.00126	.1813	790.947	5.517	5.345	29.486	30
31	169.177	.00591	.00107	.1811	934.317	5.523	5.371	29.664	31
32	199.629	.00501	.00091	.1809	1 103.5	5.528	5.394	29.819	32
33	235.562	.00425	.00077	.1808	1 303.1	5.532	5.415	29.955	33
34	277.963	.00360	.00065	.1806	1 538.7	5.536	5.433	30.074	34
35	327.997	.00305	.00055	.1806	1 816.6	5.539	5.449	30.177	35
40	750.377	.00133	.00024	.1802	4 163.2	5.548	5.502	30.527	40
45	1 716.7	.00058	.00010	.1801	9 531.6	5.552	5.529	30.701	45
50	3 927.3	.00025	.00005	.1800	21 813.0	5.554	5.543	30.786	50
55	8 984.8	.00011	.00002	.1800	49 910.1	5.555	5.549	30.827	55
60	20 555.1	.00005	.00001	.1800	114 189.4	5.555	5.553	30.846	60
65	47 025.1	.00002		.1800	261 244.7	5.555	5.554	30.856	65
70	107 581.9	.00001		.1800	597 671.7	5.556	5.555	30.860	70

Compound Interest Factors

20%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.200	.8333	1.0000	1.2000	1.000	0.833	0	0	**1**
2	1.440	.6944	.4545	.6545	2.200	1.528	0.455	0.694	**2**
3	1.728	.5787	.2747	.4747	3.640	2.106	0.879	1.852	**3**
4	2.074	.4823	.1863	.3863	5.368	2.589	1.274	3.299	**4**
5	2.488	.4019	.1344	.3344	7.442	2.991	1.641	4.906	**5**
6	2.986	.3349	.1007	.3007	9.930	3.326	1.979	6.581	**6**
7	3.583	.2791	.0774	.2774	12.916	3.605	2.290	8.255	**7**
8	4.300	.2326	.0606	.2606	16.499	3.837	2.576	9.883	**8**
9	5.160	.1938	.0481	.2481	20.799	4.031	2.836	11.434	**9**
10	6.192	.1615	.0385	.2385	25.959	4.192	3.074	12.887	**10**
11	7.430	.1346	.0311	.2311	32.150	4.327	3.289	14.233	**11**
12	8.916	.1122	.0253	.2253	39.581	4.439	3.484	15.467	**12**
13	10.699	.0935	.0206	.2206	48.497	4.533	3.660	16.588	**13**
14	12.839	.0779	.0169	.2169	59.196	4.611	3.817	17.601	**14**
15	15.407	.0649	.0139	.2139	72.035	4.675	3.959	18.509	**15**
16	18.488	.0541	.0114	.2114	87.442	4.730	4.085	19.321	**16**
17	22.186	.0451	.00944	.2094	105.931	4.775	4.198	20.042	**17**
18	26.623	.0376	.00781	.2078	128.117	4.812	4.298	20.680	**18**
19	31.948	.0313	.00646	.2065	154.740	4.843	4.386	21.244	**19**
20	38.338	.0261	.00536	.2054	186.688	4.870	4.464	21.739	**20**
21	46.005	.0217	.00444	.2044	225.026	4.891	4.533	22.174	**21**
22	55.206	.0181	.00369	.2037	271.031	4.909	4.594	22.555	**22**
23	66.247	.0151	.00307	.2031	326.237	4.925	4.647	22.887	**23**
24	79.497	.0126	.00255	.2025	392.484	4.937	4.694	23.176	**24**
25	95.396	.0105	.00212	.2021	471.981	4.948	4.735	23.428	**25**
26	114.475	.00874	.00176	.2018	567.377	4.956	4.771	23.646	**26**
27	137.371	.00728	.00147	.2015	681.853	4.964	4.802	23.835	**27**
28	164.845	.00607	.00122	.2012	819.223	4.970	4.829	23.999	**28**
29	197.814	.00506	.00102	.2010	984.068	4.975	4.853	24.141	**29**
30	237.376	.00421	.00085	.2008	1 181.9	4.979	4.873	24.263	**30**
31	284.852	.00351	.00070	.2007	1 419.3	4.982	4.891	24.368	**31**
32	341.822	.00293	.00059	.2006	1 704.1	4.985	4.906	24.459	**32**
33	410.186	.00244	.00049	.2005	2 045.9	4.988	4.919	24.537	**33**
34	492.224	.00203	.00041	.2004	2 456.1	4.990	4.931	24.604	**34**
35	590.668	.00169	.00034	.2003	2 948.3	4.992	4.941	24.661	**35**
40	1 469.8	.00068	.00014	.2001	7 343.9	4.997	4.973	24.847	**40**
45	3 657.3	.00027	.00005	.2001	18 281.3	4.999	4.988	24.932	**45**
50	9 100.4	.00011	.00002	.2000	45 497.2	4.999	4.995	24.970	**50**
55	22 644.8	.00004	.00001	.2000	113 219.0	5.000	4.998	24.987	**55**
60	56 347.5	.00002		.2000	281 732.6	5.000	4.999	24.994	**60**

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	***n***
1	1.250	.8000	1.0000	1.2500	1.000	0.800	0	0	**1**
2	1.563	.6400	.4444	.6944	2.250	1.440	0.444	0.640	**2**
3	1.953	.5120	.2623	.5123	3.813	1.952	0.852	1.664	**3**
4	2.441	.4096	.1734	.4234	5.766	2.362	1.225	2.893	**4**
5	3.052	.3277	.1218	.3718	8.207	2.689	1.563	4.204	**5**
6	3.815	.2621	.0888	.3388	11.259	2.951	1.868	5.514	**6**
7	4.768	.2097	.0663	.3163	15.073	3.161	2.142	6.773	**7**
8	5.960	.1678	.0504	.3004	19.842	3.329	2.387	7.947	**8**
9	7.451	.1342	.0388	.2888	25.802	3.463	2.605	9.021	**9**
10	9.313	.1074	.0301	.2801	33.253	3.571	2.797	9.987	**10**
11	11.642	.0859	.0235	.2735	42.566	3.656	2.966	10.846	**11**
12	14.552	.0687	.0184	.2684	54.208	3.725	3.115	11.602	**12**
13	18.190	.0550	.0145	.2645	68.760	3.780	3.244	12.262	**13**
14	22.737	.0440	.0115	.2615	86.949	3.824	3.356	12.833	**14**
15	28.422	.0352	.00912	.2591	109.687	3.859	3.453	13.326	**15**
16	35.527	.0281	.00724	.2572	138.109	3.887	3.537	13.748	**16**
17	44.409	.0225	.00576	.2558	173.636	3.910	3.608	14.108	**17**
18	55.511	.0180	.00459	.2546	218.045	3.928	3.670	14.415	**18**
19	69.389	.0144	.00366	.2537	273.556	3.942	3.722	14.674	**19**
20	86.736	.0115	.00292	.2529	342.945	3.954	3.767	14.893	**20**
21	108.420	.00922	.00233	.2523	429.681	3.963	3.805	15.078	**21**
22	135.525	.00738	.00186	.2519	538.101	3.970	3.836	15.233	**22**
23	169.407	.00590	.00148	.2515	673.626	3.976	3.863	15.362	**23**
24	211.758	.00472	.00119	.2512	843.033	3.981	3.886	15.471	**24**
25	264.698	.00378	.00095	.2509	1 054.8	3.985	3.905	15.562	**25**
26	330.872	.00302	.00076	.2508	1 319.5	3.988	3.921	15.637	**26**
27	413.590	.00242	.00061	.2506	1 650.4	3.990	3.935	15.700	**27**
28	516.988	.00193	.00048	.2505	2 064.0	3.992	3.946	15.752	**28**
29	646.235	.00155	.00039	.2504	2 580.9	3.994	3.955	15.796	**29**
30	807.794	.00124	.00031	.2503	3 227.2	3.995	3.963	15.832	**30**
31	1 009.7	.00099	.00025	.2502	4 035.0	3.996	3.969	15.861	**31**
32	1 262.2	.00079	.00020	.2502	5 044.7	3.997	3.975	15.886	**32**
33	1 577.7	.00063	.00016	.2502	6 306.9	3.997	3.979	15.906	**33**
34	1 972.2	.00051	.00013	.2501	7 884.6	3.998	3.983	15.923	**34**
35	2 465.2	.00041	.00010	.2501	9 856.8	3.998	3.986	15.937	**35**
40	7 523.2	.00013	.00003	.2500	30 088.7	3.999	3.995	15.977	**40**
45	22 958.9	.00004	.00001	.2500	91 831.5	4.000	3.998	15.991	**45**
50	70 064.9	.00001		.2500	280 255.7	4.000	3.999	15.997	**50**
55	213 821.2			.2500	855 280.7	4.000	4.000	15.999	**55**

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*	*n*
1	1.300	.7692	1.0000	1.3000	1.000	0.769	0	0	**1**
2	1.690	.5917	.4348	.7348	2.300	1.361	0.435	0.592	**2**
3	2.197	.4552	.2506	.5506	3.990	1.816	0.827	1.502	**3**
4	2.856	.3501	.1616	.4616	6.187	2.166	1.178	2.552	**4**
5	3.713	.2693	.1106	.4106	9.043	2.436	1.490	3.630	**5**
6	4.827	.2072	.0784	.3784	12.756	2.643	1.765	4.666	**6**
7	6.275	.1594	.0569	.3569	17.583	2.802	2.006	5.622	**7**
8	8.157	.1226	.0419	.3419	23.858	2.925	2.216	6.480	**8**
9	10.604	.0943	.0312	.3312	32.015	3.019	2.396	7.234	**9**
10	13.786	.0725	.0235	.3235	42.619	3.092	2.551	7.887	**10**
11	17.922	.0558	.0177	.3177	56.405	3.147	2.683	8.445	**11**
12	23.298	.0429	.0135	.3135	74.327	3.190	2.795	8.917	**12**
13	30.287	.0330	.0102	.3102	97.625	3.223	2.889	9.314	**13**
14	39.374	.0254	.00782	.3078	127.912	3.249	2.969	9.644	**14**
15	51.186	.0195	.00598	.3060	167.286	3.268	3.034	9.917	**15**
16	66.542	.0150	.00458	.3046	218.472	3.283	3.089	10.143	**16**
17	86.504	.0116	.00351	.3035	285.014	3.295	3.135	10.328	**17**
18	112.455	.00889	.00269	.3027	371.518	3.304	3.172	10.479	**18**
19	146.192	.00684	.00207	.3021	483.973	3.311	3.202	10.602	**19**
20	190.049	.00526	.00159	.3016	630.165	3.316	3.228	10.702	**20**
21	247.064	.00405	.00122	.3012	820.214	3.320	3.248	10.783	**21**
22	321.184	.00311	.00094	.3009	1 067.3	3.323	3.265	10.848	**22**
23	417.539	.00239	.00072	.3007	1 388.5	3.325	3.278	10.901	**23**
24	542.800	.00184	.00055	.3006	1 806.0	3.327	3.289	10.943	**24**
25	705.640	.00142	.00043	.3004	2 348.8	3.329	3.298	10.977	**25**
26	917.332	.00109	.00033	.3003	3 054.4	3.330	3.305	11.005	**26**
27	1 192.5	.00084	.00025	.3003	3 971.8	3.331	3.311	11.026	**27**
28	1 550.3	.00065	.00019	.3002	5 164.3	3.331	3.315	11.044	**28**
29	2 015.4	.00050	.00015	.3001	6 714.6	3.332	3.319	11.058	**29**
30	2 620.0	.00038	.00011	.3001	8 730.0	3.332	3.322	11.069	**30**
31	3 406.0	.00029	.00009	.3001	11 350.0	3.332	3.324	11.078	**31**
32	4 427.8	.00023	.00007	.3001	14 756.0	3.333	3.326	11.085	**32**
33	5 756.1	.00017	.00005	.3001	19 183.7	3.333	3.328	11.090	**33**
34	7 483.0	.00013	.00004	.3000	24 939.9	3.333	3.329	11.094	**34**
35	9 727.8	.00010	.00003	.3000	32 422.8	3.333	3.330	11.098	**35**
40	36 118.8	.00003	.00001	.3000	120 392.6	3.333	3.332	11.107	**40**
45	134 106.5	.00001		.3000	447 018.3	3.333	3.333	11.110	**45**

35% Compound Interest Factors 35%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*	*n*
1	1.350	.7407	1.0000	1.3500	1.000	0.741	0	0	**1**
2	1.822	.5487	.4255	.7755	2.350	1.289	0.426	0.549	**2**
3	2.460	.4064	.2397	.5897	4.173	1.696	0.803	1.362	**3**
4	3.322	.3011	.1508	.5008	6.633	1.997	1.134	2.265	**4**
5	4.484	.2230	.1005	.4505	9.954	2.220	1.422	3.157	**5**
6	6.053	.1652	.0693	.4193	14.438	2.385	1.670	3.983	**6**
7	8.172	.1224	.0488	.3988	20.492	2.508	1.881	4.717	**7**
8	11.032	.0906	.0349	.3849	28.664	2.598	2.060	5.352	**8**
9	14.894	.0671	.0252	.3752	39.696	2.665	2.209	5.889	**9**
10	20.107	.0497	.0183	.3683	54.590	2.715	2.334	6.336	**10**
11	27.144	.0368	.0134	.3634	74.697	2.752	2.436	6.705	**11**
12	36.644	.0273	.00982	.3598	101.841	2.779	2.520	7.005	**12**
13	49.470	.0202	.00722	.3572	138.485	2.799	2.589	7.247	**13**
14	66.784	.0150	.00532	.3553	187.954	2.814	2.644	7.442	**14**
15	90.158	.0111	.00393	.3539	254.739	2.825	2.689	7.597	**15**
16	121.714	.00822	.00290	.3529	344.897	2.834	2.725	7.721	**16**
17	164.314	.00609	.00214	.3521	466.611	2.840	2.753	7.818	**17**
18	221.824	.00451	.00158	.3516	630.925	2.844	2.776	7.895	**18**
19	299.462	.00334	.00117	.3512	852.748	2.848	2.793	7.955	**19**
20	404.274	.00247	.00087	.3509	1 152.2	2.850	2.808	8.002	**20**
21	545.769	.00183	.00064	.3506	1 556.5	2.852	2.819	8.038	**21**
22	736.789	.00136	.00048	.3505	2 102.3	2.853	2.827	8.067	**22**
23	994.665	.00101	.00035	.3504	2 839.0	2.854	2.834	8.089	**23**
24	1 342.8	.00074	.00026	.3503	3 833.7	2.855	2.839	8.106	**24**
25	1 812.8	.00055	.00019	.3502	5 176.5	2.856	2.843	8.119	**25**
26	2 447.2	.00041	.00014	.3501	6 989.3	2.856	2.847	8.130	**26**
27	3 303.8	.00030	.00011	.3501	9 436.5	2.856	2.849	8.137	**27**
28	4 460.1	.00022	.00008	.3501	12 740.3	2.857	2.851	8.143	**28**
29	6 021.1	.00017	.00006	.3501	17 200.4	2.857	2.852	8.148	**29**
30	8 128.5	.00012	.00004	.3500	23 221.6	2.857	2.853	8.152	**30**
31	10 973.5	.00009	.00003	.3500	31 350.1	2.857	2.854	8.154	**31**
32	14 814.3	.00007	.00002	.3500	42 323.7	2.857	2.855	8.157	**32**
33	19 999.3	.00005	.00002	.3500	57 137.9	2.857	2.855	8.158	**33**
34	26 999.0	.00004	.00001	.3500	77 137.2	2.857	2.856	8.159	**34**
35	36 448.7	.00003	.00001	.3500	104 136.3	2.857	2.856	8.160	**35**

Compound Interest Factors

40%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find F Given P	Find P Given F	Find A Given F	Find A Given P	Find F Given A	Find P Given A	Find A Given G	Find P Given G	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.400	.7143	1.0000	1.4000	1.000	0.714	0	0	**1**
2	1.960	.5102	.4167	.8167	2.400	1.224	0.417	0.510	**2**
3	2.744	.3644	.2294	.6294	4.360	1.589	0.780	1.239	**3**
4	3.842	.2603	.1408	.5408	7.104	1.849	1.092	2.020	**4**
5	5.378	.1859	.0914	.4914	10.946	2.035	1.358	2.764	**5**
6	7.530	.1328	.0613	.4613	16.324	2.168	1.581	3.428	**6**
7	10.541	.0949	.0419	.4419	23.853	2.263	1.766	3.997	**7**
8	14.758	.0678	.0291	.4291	34.395	2.331	1.919	4.471	**8**
9	20.661	.0484	.0203	.4203	49.153	2.379	2.042	4.858	**9**
10	28.925	.0346	.0143	.4143	69.814	2.414	2.142	5.170	**10**
11	40.496	.0247	.0101	.4101	98.739	2.438	2.221	5.417	**11**
12	56.694	.0176	.00718	.4072	139.235	2.456	2.285	5.611	**12**
13	79.371	.0126	.00510	.4051	195.929	2.469	2.334	5.762	**13**
14	111.120	.00900	.00363	.4036	275.300	2.478	2.373	5.879	**14**
15	155.568	.00643	.00259	.4026	386.420	2.484	2.403	5.969	**15**
16	217.795	.00459	.00185	.4018	541.988	2.489	2.426	6.038	**16**
17	304.913	.00328	.00132	.4013	759.783	2.492	2.444	6.090	**17**
18	426.879	.00234	.00094	.4009	1 064.7	2.494	2.458	6.130	**18**
19	597.630	.00167	.00067	.4007	1 419.6	2.496	2.468	6.160	**19**
20	836.682	.00120	.00048	.4005	2 089.2	2.497	2.476	6.183	**20**
21	1 171.4	.00085	.00034	.4003	2 925.9	2.498	2.482	6.200	**21**
22	1 639.9	.00061	.00024	.4002	4 097.2	2.498	2.487	6.213	**22**
23	2 295.9	.00044	.00017	.4002	5 737.1	2.499	2.490	6.222	**23**
24	3 214.2	.00031	.00012	.4001	8 033.0	2.499	2.493	6.229	**24**
25	4 499.9	.00022	.00009	.4001	11 247.2	2.499	2.494	6.235	**25**
26	6 299.8	.00016	.00006	.4001	15 747.1	2.500	2.496	6.239	**26**
27	8 819.8	.00011	.00005	.4000	22 046.9	2.500	2.497	6.242	**27**
28	12 347.7	.00008	.00003	.4000	30 866.7	2.500	2.498	6.244	**28**
29	17 286.7	.00006	.00002	.4000	43 214.3	2.500	2.498	6.245	**29**
30	24 201.4	.00004	.00002	.4000	60 501.0	2.500	2.499	6.247	**30**
31	33 882.0	.00003	.00001	.4000	84 702.5	2.500	2.499	6.248	**31**
32	47 434.8	.00002	.00001	.4000	118 584.4	2.500	2.499	6.248	**32**
33	66 408.7	.00002	.00001	.4000	166 019.2	2.500	2.500	6.249	**33**
34	92 972.1	.00001		.4000	232 427.9	2.500	2.500	6.249	**34**
35	130 161.0	.00001		.4000	325 400.0	2.500	2.500	6.249	**35**

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	***n***
1	1.450	.6897	1.0000	1.4500	1.000	0.690	0	0	**1**
2	2.103	.4756	.4082	.8582	2.450	1.165	0.408	0.476	**2**
3	3.049	.3280	.2197	.6697	4.553	1.493	0.758	1.132	**3**
4	4.421	.2262	.1316	.5816	7.601	1.720	1.053	1.810	**4**
5	6.410	.1560	.0832	.5332	12.022	1.876	1.298	2.434	**5**
6	9.294	.1076	.0543	.5043	18.431	1.983	1.499	2.972	**6**
7	13.476	.0742	.0361	.4861	27.725	2.057	1.661	3.418	**7**
8	19.541	.0512	.0243	.4743	41.202	2.109	1.791	3.776	**8**
9	28.334	.0353	.0165	.4665	60.743	2.144	1.893	4.058	**9**
10	41.085	.0243	.0112	.4612	89.077	2.168	1.973	4.277	**10**
11	59.573	.0168	.00768	.4577	130.162	2.185	2.034	4.445	**11**
12	86.381	.0116	.00527	.4553	189.735	2.196	2.082	4.572	**12**
13	125.252	.00798	.00362	.4536	276.115	2.204	2.118	4.668	**13**
14	181.615	.00551	.00249	.4525	401.367	2.210	2.145	4.740	**14**
15	263.342	.00380	.00172	.4517	582.982	2.214	2.165	4.793	**15**
16	381.846	.00262	.00118	.4512	846.325	2.216	2.180	4.832	**16**
17	553.677	.00181	.00081	.4508	1 228.2	2.218	2.191	4.861	**17**
18	802.831	.00125	.00056	.4506	1 781.8	2.219	2.200	4.882	**18**
19	1 164.1	.00086	.00039	.4504	2 584.7	2.220	2.206	4.898	**19**
20	1 688.0	.00059	.00027	.4503	3 748.8	2.221	2.210	4.909	**20**
21	2 447.5	.00041	.00018	.4502	5 436.7	2.221	2.214	4.917	**21**
22	3 548.9	.00028	.00013	.4501	7 884.3	2.222	2.216	4.923	**22**
23	5 145.9	.00019	.00009	.4501	11 433.2	2.222	2.218	4.927	**23**
24	7 461.6	.00013	.00006	.4501	16 579.1	2.222	2.219	4.930	**24**
25	10 819.3	.00009	.00004	.4500	24 040.7	2.222	2.220	4.933	**25**
26	15 688.0	.00006	.00003	.4500	34 860.1	2.222	2.221	4.934	**26**
27	22 747.7	.00004	.00002	.4500	50 548.1	2.222	2.221	4.935	**27**
28	32 984.1	.00003	.00001	.4500	73 295.8	2.222	2.221	4.936	**28**
29	47 826.9	.00002	.00001	.4500	106 279.9	2.222	2.222	4.937	**29**
30	69 349.1	.00001	.00001	.4500	154 106.8	2.222	2.222	4.937	**30**
31	100 556.1	.00001		.4500	223 455.9	2.222	2.222	4.938	**31**
32	145 806.4	.00001		.4500	324 012.0	2.222	2.222	4.938	**32**
33	211 419.3			.4500	469 818.5	2.222	2.222	4.938	**33**
34	306 558.0			.4500	681 237.8	2.222	2.222	4.938	**34**
35	444 509.2			.4500	987 795.9	2.222	2.222	4.938	**35**

50% Compound Interest Factors 50%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	***F/P***	***P/F***	***A/F***	***A/P***	***F/A***	***P/A***	***A/G***	***P/G***	***n***
1	1.500	.6667	1.0000	1.5000	1.000	0.667	0	0	**1**
2	2.250	.4444	.4000	.9000	2.500	1.111	0.400	0.444	**2**
3	3.375	.2963	.2105	.7105	4.750	1.407	0.737	1.037	**3**
4	5.063	.1975	.1231	.6231	8.125	1.605	1.015	1.630	**4**
5	7.594	.1317	.0758	.5758	13.188	1.737	1.242	2.156	**5**
6	11.391	.0878	.0481	.5481	20.781	1.824	1.423	2.595	**6**
7	17.086	.0585	.0311	.5311	32.172	1.883	1.565	2.947	**7**
8	25.629	.0390	.0203	.5203	49.258	1.922	1.675	3.220	**8**
9	38.443	.0260	.0134	.5134	74.887	1.948	1.760	3.428	**9**
10	57.665	.0173	.00882	.5088	113.330	1.965	1.824	3.584	**10**
11	86.498	.0116	.00585	.5058	170.995	1.977	1.871	3.699	**11**
12	129.746	.00771	.00388	.5039	257.493	1.985	1.907	3.784	**12**
13	194.620	.00514	.00258	.5026	387.239	1.990	1.933	3.846	**13**
14	291.929	.00343	.00172	.5017	581.859	1.993	1.952	3.890	**14**
15	437.894	.00228	.00114	.5011	873.788	1.995	1.966	3.922	**15**
16	656.814	.00152	.00076	.5008	1 311.7	1.997	1.976	3.945	**16**
17	985.261	.00101	.00051	.5005	1 968.5	1.998	1.983	3.961	**17**
18	1 477.9	.00068	.00034	.5003	2 953.8	1.999	1.988	3.973	**18**
19	2 216.8	.00045	.00023	.5002	4 431.7	1.999	1.991	3.981	**19**
20	3 325.3	.00030	.00015	.5002	6 648.5	1.999	1.994	3.987	**20**
21	4 987.9	.00020	.00010	.5001	9 973.8	2.000	1.996	3.991	**21**
22	7 481.8	.00013	.00007	.5001	14 961.7	2.000	1.997	3.994	**22**
23	11 222.7	.00009	.00004	.5000	22 443.5	2.000	1.998	3.996	**23**
24	16 834.1	.00006	.00003	.5000	33 666.2	2.000	1.999	3.997	**24**
25	25 251.2	.00004	.00002	.5000	50 500.3	2.000	1.999	3.998	**25**
26	37 876.8	.00003	.00001	.5000	75 751.5	2.000	1.999	3.999	**26**
27	56 815.1	.00002	.00001	.5000	113 628.3	2.000	2.000	3.999	**27**
28	85 222.7	.00001	.00001	.5000	170 443.4	2.000	2.000	3.999	**28**
29	127 834.0	.00001		.5000	255 666.1	2.000	2.000	4.000	**29**
30	191 751.1	.00001		.5000	383 500.1	2.000	2.000	4.000	**30**
31	287 626.6			.5000	575 251.2	2.000	2.000	4.000	**31**
32	431 439.9			.5000	862 877.8	2.000	2.000	4.000	**32**

Compound Interest Factors

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find F Given P F/P	Find P Given F P/F	Find A Given F A/F	Find A Given P A/P	Find F Given A F/A	Find P Given A P/A	Find A Given G A/G	Find P Given G P/G	n
1	1.600	.6250	1.0000	1.6000	1.000	0.625	0	0	1
2	2.560	.3906	.3846	.9846	2.600	1.016	0.385	0.391	2
3	4.096	.2441	.1938	.7938	5.160	1.260	0.698	0.879	3
4	6.554	.1526	.1080	.7080	9.256	1.412	0.946	1.337	4
5	10.486	.0954	.0633	.6633	15.810	1.508	1.140	1.718	5
6	16.777	.0596	.0380	.6380	26.295	1.567	1.286	2.016	6
7	26.844	.0373	.0232	.6232	43.073	1.605	1.396	2.240	7
8	42.950	.0233	.0143	.6143	69.916	1.628	1.476	2.403	8
9	68.719	.0146	.00886	.6089	112.866	1.642	1.534	2.519	9
10	109.951	.00909	.00551	.6055	181.585	1.652	1.575	2.601	10
11	175.922	.00568	.00343	.6034	291.536	1.657	1.604	2.658	11
12	281.475	.00355	.00214	.6021	467.458	1.661	1.624	2.697	12
13	450.360	.00222	.00134	.6013	748.933	1.663	1.638	2.724	13
14	720.576	.00139	.00083	.6008	1 199.3	1.664	1.647	2.742	14
15	1 152.9	.00087	.00052	.6005	1 919.9	1.665	1.654	2.754	15
16	1 844.7	.00054	.00033	.6003	3 072.8	1.666	1.658	2.762	16
17	2 951.5	.00034	.00020	.6002	4 917.5	1.666	1.661	2.767	17
18	4 722.4	.00021	.00013	.6001	7 868.9	1.666	1.663	2.771	18
19	7 555.8	.00013	.00008	.6011	12 591.3	1.666	1.664	2.773	19
20	12 089.3	.00008	.00005	.6000	20 147.1	1.667	1.665	2.775	20
21	19 342.8	.00005	.00003	.6000	32 236.3	1.667	1.666	2.776	21
22	30 948.5	.00003	.00002	.6000	51 579.2	1.667	1.666	2.777	22
23	49 517.6	.00002	.00001	.6000	82 527.6	1.667	1.666	2.777	23
24	79 228.1	.00001	.00001	.6000	132 045.2	1.667	1.666	2.777	24
25	126 765.0	.00001		.6000	211 273.4	1.667	1.666	2.777	25
26	202 824.0			.6000	338 038.4	1.667	1.667	2.778	26
27	324 518.4			.6000	540 862.4	1.667	1.667	2.778	27
28	519 229.5			.6000	865 380.9	1.667	1.667	2.778	28

APPENDIX D

FUNDAMENTALS OF ENGINEERING (FE) EXAM PRACTICE PROBLEMS

From the NCEES website (ncees.org)

The National Council of Examiners for Engineering and Surveying (NCEES) is a national nonprofit organization dedicated to advancing professional licensure for engineers and surveyors. It develops, administers, and scores the examinations used for engineering and surveying licensure in the United States.

The Fundamentals of Engineering (FE) exam is the first step toward professional licensure. Effective January 2014, the exam transitioned to a computer-based testing (CBT) platform, administered in testing centers during four time periods each year. For more details on the exam, study materials, and examination content areas, please reference the NCEES website.

The set of homework problems in this Appendix has been developed in the multiple-choice style of the FE exam. With the new exam format, *Engineering Economy* and *Engineering Ethics* topics are found in all FE exams, although the number of questions and extent of coverage vary from discipline to discipline. Following are sections from the *NCEES Exam Specifications* for each discipline—highlighting the sections and number of questions on engineering economy and engineering ethics.

Chemical Engineering Exam

13. Process Design and Economics (8–12 questions)

A. Process flow diagrams and piping and instrumentation diagrams
B. Equipment selection (e.g., sizing and scale-up)
C. Cost estimation
D. Comparison of economic alternatives (e.g., net present value, discounted cash flow, rate of return, expected value and risk)
E. Process design and optimization (e.g., sustainability, efficiency, green engineering, inherently safer design, evaluation of specifications)

16. Ethics and Professional Practice (2–3 questions)

A. Codes of ethics (professional and technical societies)
B. Agreements and contracts
C. Ethical and legal considerations
D. Professional liability
E. Public protection issues (e.g., licensing boards)

Civil Engineering

4. Ethics and Professional Practice (4–6 questions)

A. Codes of ethics (professional and technical societies)
B. Professional liability
C. Licensure
D. Sustainability and sustainable design
E. Professional skills (e.g., public policy, management, and business)
F. Contracts and contract law

5. Engineering Economics (4–6 questions)

A. Discounted cash flow (e.g., equivalence, PW, equivalent annual worth, FW, rate of return)
B. Cost (e.g., incremental, average, sunk, estimating)
C. Analyses (e.g., breakeven, benefit-cost, life cycle)
D. Uncertainty (e.g., expected value and risk)

Electrical and Computer Engineering

3. Ethics and Professional Practice (3–5 questions)

A. Codes of ethics (professional and technical societies)
B. NCEES Model Law and Model Rules
C. Intellectual property (e.g., copyright, trade secrets, patents)

4. Engineering Economics (3–5 questions)

A. Time value of money (e.g., present value, future value, annuities)
B. Cost estimation
C. Risk identification
D. Analysis (e.g., cost-benefit, trade-off, breakeven)

Environmental Engineering

3. Ethics and Professional Practice (5–8 questions)

A. Codes of ethics (professional and technical societies)
B. Agreements and contracts
C. Ethical and legal considerations
D. Professional liability
E. Public protection issues (e.g., licensing boards)
F. Regulations (e.g., water, wastewater, air, solid/hazardous waste, groundwater/soils)

4. Engineering Economics (4–6 questions)

A. Discounted cash flow (e.g., life cycle, equivalence, PW, equivalent annual worth, FW, rate of return)
B. Cost (e.g., incremental, average, sunk, estimating)
C. Analyses (e.g., breakeven, benefit-cost)
D. Uncertainty (expected value and risk)

Industrial and Systems Engineering

3. Ethics and Professional Practice (5–8 questions)

A. Codes of ethics and licensure
B. Agreements and contracts
C. Professional, ethical, and legal responsibility

D. Public protection and regulatory issues

4. Engineering Economics (10–15 questions)

A. Discounted cash flows (PW, EAC, FW, IRR, amortization)
B. Types and breakdown of costs (e.g., fixed, variable, direct and indirect labor)
C. Cost analyses (e.g., benefit-cost, breakeven, minimum cost, overhead)
D. Accounting (financial statements and overhead cost allocation)
E. Cost estimation
F. Depreciation and taxes
G. Capital budgeting

Mechanical Engineering

4. Ethics and Professional Practice (3–5 questions)

A. Codes of ethics
B. Agreements and contracts
C. Ethical and legal considerations
D. Professional liability
E. Public health, safety, and welfare

5. Engineering Economics (3–5 questions)

A. Time value of money
B. Cost, including incremental, average, sunk, and estimating
C. Economic analyses
D. Depreciation

Other Disciplines

5. Ethics and Professional Practice (3–5 questions)

A. Codes of ethics
B. NCEES Model Law
C. Public protection issues (e.g., licensing boards)

7. Engineering Economics (7–11 questions)

A. Time value of money (e.g., present worth, annual worth, future worth, rate of return)
B. Cost (e.g., incremental, average, sunk, estimating)
C. Economic analyses (e.g., breakeven, benefit-cost, optimal economic life)
D. Uncertainty (e.g., expected value and risk)
E. Project selection (e.g., comparison of unequal life projects, lease/buy/make, depreciation, discounted cash flow)

PROBLEMS

Chapter 1

Decision Making

D-1 Engineering economic analysis provides useful input in all of the following situations except which one?
(*a*) Determining how much we should pay for a machine that will provide a savings.
(*b*) Determining the priority of investing our company’s retained earnings.
(*c*) Illustrating the economic advantages of one alternative over other feasible choices.
(*d*) Convincing management that one person should be hired over another.

D-2 Engineering economic analysis can be described by the following statement:
(*a*) Involves a systematic analysis of relevant costs and benefits.
(*b*) Involves a comparison of competing alternatives.
(*c*) Supports a rational economic decision-making objective.
(*d*) All of the above.

D- To which of the following questions does an engineering economy analysis provide useful input?
(*a*) Has the mechanical or electrical engineer chosen the most economical motor size given functional requirements?

3 (*b*) Has the civil or mechanical engineer chosen the best thickness for insulating a building?
(*c*) Has the biomedical engineer chosen the best materials for the company's artificial knee product?
(*d*) All of the above

Which of the following job functions potentially conducts and utilizes engineering economic analysis in decision making?
D-4 (*a*) Senior technical design engineer.
(*b*) Midlevel manager of business and finance.
(*c*) Senior management for new product development.
(*d*) All of the above.

Engineering economic analysis can be described by the following statement:
D-5 (*a*) Involves a systematic analysis of relevant costs and benefits.
(*b*) Involves a comparison of competing alternatives.
(*c*) Supports a rational economic decision-making objective.
(*d*) All of the above.

Engineering Ethics

Engineers are acting in the most ethical way in which of these situations?
(*a*) They are making sure the company's interests are protected.
D-6 (*b*) They feel good about the decisions that they've made.
(*c*) They act to protect the interests of society in general.
(*d*) They ensure that their own best interest is protected.

To act as an ethical engineer, you should accept fees for engineering work in which situation?
D-7 (*a*) If you need the money to keep your business open and thriving.
(*b*) If you are competent to complete all aspects of the job.
(*c*) If the contract is a cost plus contract.
(*d*) If there were no other engineers who bid on the job.

A registered professional engineer (PE) has as a primary obligation to protect which of the following entities?
D-8 (*a*) The government
(*b*) The PE's company
(*c*) The PE's country

(*d*) The general public

D-9 Engineers should act in ethical ways for what reason?
(*a*) It creates a feel-good situation for them.
(*b*) The engineering code of conduct requires it.
(*c*) They may be considered for a raise because of it.
(*d*) They may be violating the law if they don’t.

D-10 A design engineer is responsible for an important subelement of a large project at a firm. The project has fallen behind schedule, and the important client is very angry and threatening to sue. The boss is expecting the engineer’s design review to go well so that the project can be shipped by the end of the week. The engineer notices during final design review that an element of the design is wrong and will create a major safety issue for the entire system. To rework that portion of the design will take several months. The engineer should do which of the following?
(*a*) Sign off on the drawings because of the threatened lawsuit and because the project is so far behind.
(*b*) Do not sign off on the drawings, and let the boss know what is found at the final design review.
(*c*) Tell the boss that to sign off on the design now, the engineer must have an immediate raise.
(*d*) Sign off and keep an eye on how construction goes. Maybe the engineer can correct the safety issues before the project is fully operational.

D-11 As team leader for your unit, you function as both engineer and manager. One of your roles is to approve major purchases, and you have been contacted by a new supplier in your area. The new company has invited you to expensive dinners, has offered trips to vacation spots to attend “product shows,” and has recently been sending to your private address personal items such as collectible art, coins, sports tickets, and golf club memberships. You are unsure how to handle this situation. You should do which of the following?
(*a*) Accept all of the gifts because you know that everyone else is doing it and that this is your chance to get a share of the action.
(*b*) Decline the gifts and other offers that would be considered outside the scope of ordinary business or professional contact.
(*c*) Knowing that the gifts will not influence your purchasing decisions,

accept the items with no guilt.
(*d*) Accept the gifts but make sure that your boss and others on your team share in the bounty.

Cost Problems

A company produces several product lines. One of those lines generates the following annual cost and production data:

Manufacturing/Materials costs	$200,000
General/Administrative costs	50,000
Direct-labor costs	170,000
Other overhead costs	60,000
Annual production demand	10,000 units

D-12 The company adds 40% to its production cost in selling to the retailer. The retailer in turn adds a 50% profit margin when selling to its customers. How much would it cost a retailer to buy 100 units of the product?
(*a*) $4800
(*b*) $6720
(*c*) $7200
(*d*) $1008

An agribusiness is deciding what crop to plant in this area for the next growing season. The local agricultural extension office has provided the following data (in $100):

Crop	Cost per acre	Income/acre at 100% Yield	Estimated Yield(%)
A	$30	$45	80
B	45	75	65
C	15	25	90

D-

13
Using a 200-acre plot as an example (subtract 10% for unusable areas of the field), which crop should be planted this year, and what is the total profit?
(*a*) *A*; $108,000
(*b*) *C*; $135,000
(*c*) *C*; $150,000
(*d*) *B*; $540,000

Use the data below for Problems D-14 to D-17:

A textbook publisher produces a textbook for $50 per book and sells to the Campus Bookstore for $75 per unit. The bookstore sells the textbook new for $100 and used for $60. It is able to buy unlimited amounts of used books from a secondary supplier for $50. The bookstore buys a lot of 250 new books from the publisher.

D-14
What is the net profit (sales-cost) to the publisher for this order?
(*a*) $25
(*b*) (250)(75-50) = $6250
(*c*) $10,000
(*d*) $2500

D-15
If enrollment in the course is 150 for each of the first two semesters, what are the net costs to students for textbooks?
(*a*) $15,000
(*b*) $20,400
(*c*) (250)(75) + (300-250)(60) = $21,750
(*d*) $30,000

D-16
If the bookstore purchases $3000 worth of used books over 3 semesters to meet class enrollment demand, what was the total enrollment for this period of time?
(*a*) 300
(*b*) 150
(*c*) 75
(*d*) (3000)/(50) + 250 = 310

D-17
If enrollment in the course is 75 each semester for 4 semesters, what is the net profit (sales-costs) for the bookstore?
(*a*) (250)(100) + (300-250)(60) – (250)(75) – (300-250)(50) = $6750
(*b*) $22,500
(*c*) $11,200

(*d*) $28,000

Chapter 2

Cost Concepts

D-18 A firm bought a used machine 2 years ago for $1500. When new, the machine cost $8000. Today it could be sold for $500. Which of the following statements is true?
(*a*) The fixed cost for operating the machine can be ignored in any analysis.
(*b*) The $8000 purchase price is not included in the analysis.
(*c*) The $1500 paid 2 years ago is included in the analysis.
(*d*) The variable cost of ownership is the difference between what was paid and what the machine is now worth ($1500 – $500 = $1000).

D-19 When considering two alternatives that are described only in terms of the cost of ownership, the breakeven point cannot be described by which of the following statements?
(*a*) The difference in initial cost between the two alternatives.
(*b*) The level of production (or activity) of each alternative under consideration is equivalent.
(*c*) Fixed plus variable costs of each alternative are equivalent.
(*d*) A rational decision maker should be indifferent between the two alternatives.

D-20 If JMJ Industries realizes a profit of $4.00 per unit sold, what is the fixed-cost portion of their production costs? Their variable costs are $1.50 per unit, and they sell 1000 units per year at a price of $6.00 per unit.
(*a*) There are no fixed costs in this type of problem.
(*b*) $250
(*c*) $500
(*d*) $2000

Consider the following production data for Alternatives *A* and *B* in a firm that uses a 10% interest rate.

	Alt. *A*	Alt. *B*
Annual fixed cost per unit	$2 million	$3.5 million

Annual variable cost per unit	850	250

D-21 If the company is going to produce 4000 units annually, which alternative should be chosen?

(*a*) Neither alternative should be chosen because the negative cash flows are greater than the positive cash flows for both alternatives.
(*b*) This problem cannot be solved because there is not enough data given.
(*c*) Alt. *A*
(*d*) Alt. *B*

D-22 A company has annual fixed costs of $2,500,000 and variable costs of 0.15¢ per unit produced. For the firm to break even if they charge $1.85 for their product, the level of annual production is nearest to what value?

(*a*) 375,000 units
(*b*) 1,315,789 units
(*c*) 1,351,351 units
(*d*) 1,562,500 units

Estimating in Engineering Economy

D-23 Which statement is not true with respect to estimating the economic impacts of proposed engineering projects?

(*a*) Order-of-magnitude estimates are used for high-level planning.
(*b*) Order-of-magnitude estimates are the most accurate type at about –3 to 5%.
(*c*) Increasing the accuracy of estimates requires added time and resources.
(*d*) Estimators tend to underestimate the magnitude of costs and to overestimate benefits.

The Department of Transportation is accepting bids for materials to provide “signage and safety” for a new 25-mile section of a 4-lane highway. DOT estimates are:

Lane paint	$500 per mile per lane
Reflective lane markers	$6 per 25 feet

	Mile markers	\$18 per unit
D-24	Flexible roadside delineators	\$32 per unit; spaced at 1000 feet
	Emergency boxes	\$500 per unit spaced at 2.5 miles
	Signage (various messages)	\$1000 per mile; based on historical costs

The bids that DOT receives should be in what range of costs?
(*a*) Less than \$100,000
(*b*) Between \$100,000 and \$200,000
(*c*) Between \$200,000 and \$250,000
(*d*) Greater than \$250,000

D-25 A 250-gallon reactor cost \$780,000 when it was constructed 20 years ago. What would a 750-gallon model cost today if the power-sizing exponent is 0.56 and the construction cost index for such facilities has increased from 141 to 556 over the last 20 years? Choose the closest value.
(*a*) \$0.37 million
(*b*) \$1.66 million
(*c*) \$1.44 million
(*d*) \$5.69 million

D-26 A half-million-square-foot warehouse facility is being considered by your company for a location in Kansas City, KS. You have a bid for a similar type 25,000-ft^2 facility in Washington, DC, at \$4,375,000. If the warehouse construction cost index value for KS is 0.75 and for DC it is 1.34, what range should the KS bid fall into if you assume that construction costs are linear across size?
(*a*) Less than \$30 million
(*b*) Between \$30 million and \$60 million
(*c*) Between \$60 million and \$100 million
(*d*) Greater than \$100 million

D-27 A company has major clients in all 50 states. Fifteen (15) of the states have 4 clients, 10 have 3 clients, 20 have 6 clients and 5 have 10. The total number of clients the firm has is closest to what number?
(*a*) 23
(*b*) 50
(*c*) 250
(*d*) 260

Chapter 3

Simple Interest

D-28 Which of the following statements is not true?
(*a*) Simple interest is to be used only in simple decision situations.
(*b*) Compounded interest involves computing interest on top of interest.
(*c*) Simple interest is rare in practical situations of borrowing and loaning.
(*d*) If the interest is not stated as being simple or compounded, we assume the latter.

D-29 If you borrow $1000 from the bank at 5% simple interest per month due back in 2 years, what is the size of your monthly payments?
(*a*) $25
(*b*) $50
(*c*) $500
(*d*) $1200

D-30 Your quarterly payments on a loan are $500 and the interest that you are paying is 1% per quarter simple interest. The size of the principal that you have borrowed is closest to which value?
(*a*) $5000
(*b*) $12,500
(*c*) $20,000
(*d*) $50,000

D- The principal that you borrowed for a recent purchase was $15,000. You will pay the purchase off through a simple interest loan at 8% per year due in 3 years. The amount that is due at the end of 3 years is closest to what value?

31 (*a*) \$1200
(*b*) \$3600
(*c*) \$16,200
(*d*) \$18,900

If \$10,000 is borrowed today at 5% simple interest, how much is due at the end of 10 years?

D-32 (*a*) \$5000
(*b*) \$10,000
(*c*) \$15,000
(*d*) \$16,289

Compound Interest, Single Cash Flows

A savings account's value today is \$150 and it earns interest at 1% per month. How much will be in the account one year from today? Which of the following is correct to solve for the unknown value?

D-33 (*a*) $P = 150(1 + 0.01)^{12}$
(*b*) $150 = F(F/P, 12\%, 1)$
(*c*) $F = 150(1.12)^{-12}$
(*d*) $F = 150(F/P, 1\%, 12)$

To calculate how many years (n) an investment (P) must be kept in an account that earns interest at i%, in order to triple in amount, which of the following expressions should be used?

D-34 (*a*) $n = -P + F(P/F, i\%, n)$
(*b*) $n = [\log(F/P)]/[\log(1 + i\%)]$
(*c*) $n = [\ln(-P + F)]/[\ln i\%]$
(*d*) $n = -F(1 + i\%)^P$

If you invest \$40,000 in a stock whose value grows at 2% per year, your investment is nearest what value after 5 years?

D-35 (*a*) \$40,800
(*b*) \$43,296
(*c*) \$44,164
(*d*) \$64,420

An account pays interest at 1.5% per month. If you deposit \$5000 at the beginning of this year, how much could you withdraw at the end of next

D-36 year?
(*a*) $5151
(*b*) $7148
(*c*) $49,249
(*d*) $265,545

D-37 A machine will need to be replaced 15 years from today for $10,000. How much must be deposited now into an account that earns 5% per year to cover the replacement cost?
(*a*) $1486
(*b*) $4810
(*c*) $6139
(*d*) $10,000

D-38 You deposit $5000 into an account that will grow to $14,930 in 6 years. Your rate of return on this investment is closest to what value?
(*a*) 18%
(*b*) 20%
(*c*) 22%
(*d*) 25%

D-39 An investment company owns land now worth $500,000 that is increasing in value each year. If the land value doubles in 7 years, what is the yearly rate of return nearest to?
(*a*) 0.0%
(*b*) 2.0%
(*c*) 7.0%
(*d*) 10.5%

D-40 Your friend withdrew $630,315 from an account into which she had invested $350,000. If the account paid interest at 4% per year, she kept her money in the account for how many years?
(*a*) 1.8 years
(*b*) 6.5 years
(*c*) 12.5 years
(*d*) 15 years

D- Annual revenues in our company are $1.5 million this year. If they are expected to grow at a compounded rate of 20% per year, what will they be 10 years from now?
(*a*) $3.89 million

41 (*b*) $9.29 million
(*c*) $10.9 million
(*d*) $57.51 million

D-42 A student inherits $50,000 and invests it in government bonds that will average 3% annual interest. What is the value of the investment after 50 years?
(*a*) $67,195
(*b*) $219,195
(*c*) $355,335
(*d*) $5,869,545

D-43 A zero-coupon bond has a face (par) value of $10,000. The bond is sold at a discount for $6500 and held for 3 years, at which time it is sold. If an annual rate of 10% is earned over that 3-year period, how much was the bond sold for?
(*a*) $8652
(*b*) $13,310
(*c*) $16,859
(*d*) $25,937

Nominal and Effective Interest Rates

D-44 A rate of 2% per quarter compounded quarterly is closest to what annual compounded interest rate?
(*a*) 2.00%
(*b*) 8.00%
(*c*) 8.24%
(*d*) 24.00%

D-45 A nominal interest rate of "12% per year compounded yearly" is closest to:
(*a*) 1% per month effective rate
(*b*) 3% per quarter effective rate
(*c*) 6% per 6 months effective rate
(*d*) 12% per year effective rate

An interest rate expressed as "1.5% per month" is exactly the same as:
(*a*) 4.5% per quarter effective interest

D-46 (*b*) 18% effective interest per year
(*c*) 18% per year compounded monthly
(*d*) None of the above

D-47 A deposit of $50,000 is made into an account that pays 10% compounded semiannually. How much would be in the account after 10 years?
(*a*) $81,445
(*b*) $129,685
(*c*) $132,665
(*d*) $336,375

D-48 A mining firm must deposit funds in a "reclamation" account each quarter. The account must have $25 million on deposit when a project reaches its horizon in 10 years. The account pays interest at a rate of 2% per quarter. How much is the quarterly deposit?
(*a*) $41,500
(*b*) $172,575
(*c*) $228,325
(*d*) $414,000

D-49 A deposit of $1000 compounds to $2500 in 5 years. The interest on this account compounds quarterly. What is the closest nominal annual rate of return?
(*a*) 2.50%
(*b*) 4.70%
(*c*) 18.75%
(*d*) 20.11%

D-50 If your local bank indicates that it pays interest on passbook savings accounts at a rate of 2.25%, the nominal and effective interest rates are nearest which the following?
(*a*) 2.25%, 2.25%
(*b*) 2.25%, 2.28%
(*c*) 2.25%, 27%
(*d*) 27%, 2.25%

Continuous Compounding Interest Rate

How long will it take money to triple in an account if it pays interest

D-51 expressed as 8% nominal annual compounded continuously?
(*a*) 13.73 years
(*b*) 14.27 years
(*c*) Your money will never triple.
(*d*) None of the above

D-52 A deposit of $500 per year (beginning of year) is made for a period of 2 years in an account that earns 6% nominal interest compounded continuously. How much is in the account after 2 years?
(*a*) $917
(*b*) $1062
(*c*) $1092
(*d*) $1095

D-53 A firm offers to sell a zero-coupon bond (no semi-annual payments) to you today. When it matures in 5 years you will receive the par value of $10,000. If the firm pays interest at 15% compounded continuously on the bond how much would you pay for it today?
(*a*) $4724
(*b*) $4972
(*c*) $10,000
(*d*) $21,170

D-54 If $1000 is invested annually at 6% continuous compounding for each of 10 years, how much is in the account after the last deposit?
(*a*) $1822
(*b*) $10,000
(*c*) $13,181
(*d*) $13,295

Chapter 4

Uniform Cash Flow Series

D-55 You place $100 per month into an account that earns 1% per month. Which of the following expressions can be used to calculate the account's value after 3 years?
(*a*) $P = 100(P/A, 1\%, 3)$
(*b*) $F = 100(P/A, 1\%, 36)(F/P, 1\%, 36)$

(c) $F = 100[(1 + 0.01)^n - 1]/0.01$
(d) $F = 100(F/A, 12.68\%, 3)$

D-56 A machine must be replaced in 7 years at a cost of $7500. How much must be deposited at the end of each year into an account that earns 5% in order to have accumulated enough to pay for the replacement?
(*a*) $471
(*b*) $596
(*c*) $791
(*d*) $921

D-57 Winners of the PowerState Lottery can take $30 million now or payments of $2.5 million per year for the next 15 years. These are equivalent at what annual interest rate? The answer is closest to what value?
(*a*) 1%
(*b*) 2%
(*c*) 3%
(*d*) 5%

D-58 You deposit $1000 in a retirement investment account today that earns 1% per month. In addition, you deposit $50 at the end of every month starting this month and continue to do so for 30 years. The amount that has accumulated in this account at the end of 30 years is nearest to
(*a*) $35,949
(*b*) $42,027
(*c*) $174,748
(*d*) $210,698

D-59 Suppose $10,000 is deposited into an account that earns 10% per year for 5 years. At that point in time, uniform end-of-year withdrawals are made such that the account is emptied after the 15^{th} withdrawal. The size of these annual withdrawals is closest to what value?
(*a*) $2118
(*b*) $2621
(*c*) $3410
(*d*) $16,105

D-60 A manufacturer borrows $85,000 for machinery. The loan is for 10 years at 12% per year. What is the annual payment on the machinery?
(*a*) $4843
(*b*) $8500

(*c*) $13,834
(*d*) $15,045

D-61 How many years would you have to put $100 per year into an account that earns 15% annually to accumulate $6508?
(*a*) 17 years
(*b*) 21 years
(*c*) 30 years
(*d*) 65 years

D-62 A $10,000 face value municipal bond pays $1000 interest at the end of every year. If there are 12 more years of payments, at what price today would the bond yield 18% over the next 12 years?
(*a*) $1372
(*b*) $4793
(*c*) $6165
(*d*) $10,000

Gradient Cash Flows

D-63 Today $5000 is deposited in an account that earns 2.5% per quarter. Additional deposits are made at the end of every quarter for the next 20 years. The deposits start at $100 and increase by $50 each year thereafter. The amount that has accumulated in this account at the end of 20 years can be expressed as follows.
(*a*) = 5000(*P*/*F*, 2.5%, 20) + 100(*F*/*A*, 2.5%, 20%) + 50 (*P*/*G*, 2.5%, 20 − 1)
(*b*) = 5000(*F*/*P*, 10%, 80) + 150(*P*/*G*, 10%, 80) (*F* /*P* = 10%, 80)
(*c*) = 5000(*P*/*F*, 10.38%, 20) + 100(*P* /*A*, 10.38%, 20) + 50(*P*/*G*, 10.38%, 20)(*P*/*F*, 10.38%, 1)
(*d*) = 5000(*F*/*P*, 10.38%, 80) + 100(*F*/*A*, 2.5%, 80) + 50(*P*/*G*, 2.5%, 80)(*F*/*P*, 2.5%, 80)

D-64 An investment returns the following end-of-year cash flows: Year 1, $0; Year 2, $1500; Year 3, $3000; Year 4, $4500; and Year 5, $6000. Given a 10% interest rate, what is the present worth?
(*a*) $5970
(*b*) $6597
(*c*) $9357

(*d*) $10,293

D-65 A project's annual revenues will be $50,000 the first year and will decrease by $1500 per year over its 20-year life. If the firm's interest rate is 12%, what is the project's present worth?
(*a*) $305,998
(*b*) $373,450
(*c*) $384,654
(*d*) $440,902

D-66 A cash flow series is described by the following: $10,000 + $250(*t*), where *t* is the number of compounding periods. The present worth of this series at the end of five periods, where interest is 2% per *t*, is nearest what value?
(*a*) $11,250
(*b*) $50,620
(*c*) $56,432
(*d*) $60,620

D-67 Revenue from sales of a training video for the first year are estimated to be $350,000. In addition, revenue is expected to decrease by $25,000 per year over the life of the video (which is 10 years). If interest is 10%, the present worth of the revenue over the life of the video is nearest what value?
(*a*) $100,000
(*b*) $125,000
(*c*) $1,578,475
(*d*) $2,723,025

Chapter 5

Present Worth Analysis

D-68 A project is being considered that has a first cost of $12,500, creates $5000 in annual cost savings, requires $3000 in annual operating costs, and has a salvage value of $2000 after a project life of 3 years. If interest is 10% per year, which formula calculates the project's present worth?
(*a*) PW = 12,500(*P*/*F*, 10%, 1) + (− 5000 + 3000) (*P*/*A*, 10%, 3) − 2000(*F*/*P*, 10%, 3)

(*b*) PW = − 12,500 + (5000 − 3000) (*P*/*A*, 10%, 3) − 2000(*P*/*F*, 10%, 3)
(*c*) PW = 12,500(*F*/*P*, 10%, 3) + (5000 − 3000) (*F*/*A*, 10%, 3) + 2000
(*d*) PW = − 12, 500 + 5000(*P*/*A*, 10%, 3) − 3000 (*P*/*A*, 10%, 3) + 2000(*P*/*F*, 10%, 3)

A new packing machine will cost $57,000. The existing machine can be sold for $5000 now and the new machine for $7500 after its 10-year useful life. If the new machine reduces annual expenses by $5000, what
D- is the present worth at 25% of this investment?
69 (*a*) − $18,388
(*b*) − $33,340
(*c*) − $34,145
(*d*) − $38,340

A vendor is offering an extended repair contract on a machine. The firm's experience is that this will cover repair costs over the next 4 years of $200, $200, $400, and $500. At 6%, what is the extended repair contract
D- worth now?
70 (*a*) $1040
(*b*) $1089
(*c*) $1099
(*d*) $1300

Annual disbursements for maintenance of critical heavily used equipment will be $25,000 for the next 10 years, and then $35,000 into infinity. What is the present worth of the maintenance cost cash flow stream if
D- interest is 15%?
71 (*a*) $166,667
(*b*) $183,147
(*c*) $192,367
(*d*) $233,334

Manufacturing costs from a scrapped poor-quality product are $6000 per year. An investment in an employee training program can reduce this cost. Program *A* reduces the cost by 75% and requires an investment of $12,000. Program *B* reduces the cost by 95% and will cost $20,000. Based on low turnover at the plant, either program should be effective for
D- the next 5 years. If interest is 20%, the present worth of the two programs
72 is nearest what values? (Consider cost reduction a positive cash flow.)
(*a*) *A:* − $25,460; *B:* − $37,049

(*b*) *A:* \$1460; *B:* − \$2951
(*c*) *A:* \$5060; *B:* \$1609
(*d*) *A:* \$13,460; *B:* \$17,049

Chapter 6

Annual Worth Analysis

D-73 New product tracking equipment costs \$120,000 and will have a \$10,000 salvage value when disposed of in 10 years. Annual repair costs begin at \$5000 in the fifth year and increase by \$500 per year thereafter until disposed of. If interest is 10%, what is the closest equivalent annual cost of ownership?
(*a*) \$21,505
(*b*) \$21,766
(*c*) \$21,844
(*d*) \$23,109

D-74 Your company is considering two alternatives:

	Alt. I	Alt. II
First cost	\$42,500	\$70,000
Annual maintenance	6,000	4,000
Annual savings if implemented	18,500	20,000
Salvage value	12,000	25,000
Useful life of alternative	3 years	6 years

What is the annual dollar advantage of Alt. II over Alt. I at an interest rate of 15%?
(*a*) Alt. II has no annual advantage over Alt I.
(*b*) \$3020
(*c*) \$3500
(*d*) \$7436

Specialized bits (costing \$50,000) used in the mining industry have a useful life of 5000 hours of operation and can be traded in when a new bit is purchased for 10% of first cost. The drilling machine that uses the bit is
D- used 1000 hours per year. What is the equivalent uniform annual cost of

75 these bits at 2.5%?
(*a*) $8559
(*b*) $9510
(*c*) $9828
(*d*) $10,920

D-76 A new chemical remediation tank is needed. Current technology tanks, which cost $150,000, must be drained and treated every 2 years at a cost of $30,000; the tanks will last 10 years, and each will have a salvage value of 5% of first cost. A tank with new technology has just come on the market. There are no periodic maintenance costs, and a tank will last 20 years. If the new tanks cost $325,000, what minimum salvage value, as a percentage of first cost, would be required for this technology to be a better option? Use a 12% interest rate.
(*a*) 10%
(*b*) 36%
(*c*) 57%
(*d*) 72%

D-77 A beautiful bridge is being built over the river that runs through a major city in your state. The cost of the bridge is estimated at $600 million. Annual costs of the bridge will be $200,000, and the bridge is estimated to last a very long time. If accountants in city hall use 3% as the interest rate for analysis, what is the annualized cost of the bridge project?
(*a*) $18 million
(*b*) $18.2 million
(*c*) $20,000 million
(*d*) $219,500 million

Chapter 7

Rate of Return Analysis

D-78 Which of the following equations can be used to find the internal rate of return (i) for a project that has initial investment of P, net annual cash flows of A, and salvage value of S after n years?
(*a*) $0 = -P + A(P/A, i\%, n) + S(P/A, i\%, n)$
(*b*) $(P - A)(P/A, i\%, n) = S(P/F, i\%, n)$

(c) $-A = -P(A/P, i\%, n) - S(A/F, i\%, n - 1)$

(d) $0 = -P(F/P, i\%, n) + A(F/A, i\%, n) + S$

D-79 The rate of return on an investment of $1500 that doubles in value over a 4-year period, and produces a $300 annual cash flow, is nearest to which value?

(a) 15%

(b) 20%

(c) 25%

(d) 30%

D-80 The interest rate that makes Alternatives *A* and *B* equivalent is in what range?

Year	Alt. *A*	Alt. *B*
0	− $1000	− $3000
1	100	500
2	100	550
3	100	600
4	200	650
5	200	700

(a) Less than 2%

(b) Between 2 and 5%

(c) Greater than 5%

(d) There is no interest rate that equates these two cash flow series.

D-81 A corporate bond with a face (par) value of $10,000 will mature 7 years from today (it was issued 3 years ago). The bond just after the 6th interest payment is being sold for $6950. The bond's interest rate is 4% nominal annual, payable semiannually. The yield of the bond if held to maturity is in what range?

(a) Less than 4%

(b) Between 4 and 6%

(c) Between 6 and 10%

(d) Greater than 10%

A firm borrowed $50,000 from a mortgage bank. The terms of the loan specify quarterly payments for a 10-year period. If payments to the bank are $3750 per quarter, what effective annual interest rate is the firm

D-82 paying?
(*a*) Less than 1%
(*b*) 7%
(*c*) 28%
(*d*) 31%

Chapter 8

Incremental Cash Flows and Analysis

You are given the cash flow series for two projects, Alt. *A* and Alt. *B*.

Year	0	1	2	3	4	5	6
Alt. *A* ($)	–I1	X	X	X	X	X	X+S1
Alt. *B* ($)	–I2	Y	Y	Y	Y	Y	X+S2

D-83 Assume I2 > I1 and X, Y, S1, and S2 are positive; the incremental rate of return (i) on the additional investment in Alt. *B* can be calculated with the following expression.
(*a*) $0 = -I2 + Y(P/A, i\%, 6) + S2(P/F, i\%, 6)$
(*b*) $0 = -(I2 - I1) + (Y - X)(P/A, i\%, 5) + (S2 - S1)(P/F, i\%, 6)$
(*c*) $0 = -(I2 - I1)(F/P, i\%, 6) + (Y - X)(F/A, i\%, 6) + (S2 - S1)$
(*d*) $0 = -(I2 - I1) + (Y - X) + [(Y + S2) - (X + S1)]$

D-84 A firm is considering two mutually exclusive alternatives (i = 8%):

	Alt. Alpha	Alt. Omega
First cost	$10,000	$30,000
Annual maintenance	2,800	2,000
Annual savings if implemented	5,500	6,500
Salvage value	2,000	5,000
Useful life of alternative	4 years	8 years

If Alt. Alpha will be replaced with a “like alternative” at the end of 4 years, what is the present worth of the incremental cash flows associated with going from an investment in Alpha to an investment in Omega?
(*a*) − $6201

(*b*) − \$5942
(*c*) − \$5028
(*d*) \$852

D-85 Project 1 requires an initial investment on \$50,000 and has an internal rate of return (IRR) of 18%. A mutually exclusive alternative, Project 2, requires an investment of \$70,000 and has an IRR of 23%. Which of the following statements is true concerning the rate of return on the incremental \$20,000 investment?
(*a*) It is less than 18%.
(*b*) It is between 18 and 23%.
(*c*) It is greater than 23%.
(*d*) It cannot be determined from the data given.

D-86 Alternative Uno has a first cost of \$10,000 and annual expenses of \$3000, whereas Alternative Dos has a first cost of \$35,000, annual expenses of \$2000, and a recurring cost of \$5000 every 10 years. If both alternatives have an infinite life, which of the following equations can be used to solve for the rate of return on the incremental investment?
(*a*) $0 = -\$25,000 + \$1000/i - \$5000(A/F, i, 10)$
(*b*) $0 = -\$25,000 + \$1000/i + \$5000(A/P, i, 10)/i$
(*c*) $0 = -\$25,000 + \$1000/i - \$5000(A/F, i, 10)/i$
(*d*) $0 = +\$25,000 - \$1000/i + \$5000(A/F, i, 10)$

D-87 Compare two competing, mutually exclusive new machines that have only cost data given and tell which of the following statements is true regarding the present worth of the incremental investment at your investment interest rate.
(*a*) If it is greater than zero, we chose the alternative with the largest initial investment expense.
(*b*) The internal rate of return will always be equal to the investment rate of return.
(*c*) Neither machine is chosen if there is only cost data and the present worth is less than zero.
(*d*) If it is less then zero, we chose the alternative with the smallest initial investment expense.

Chapter 9

Future Worth Analysis

D-88 The future worth of a project with initial cost P, positive annual cash flows of A, salvage value S, and interest rate of i over a life of n years can be calculated using which statement?

(*a*) FW = $-P(F/P, i\%, n) + A(F/A, i\%, n) + S(F/P, i\%, n)$
(*b*) FW = $P(F/P, i\%, n) + A(F/A, i\%, n) + S$
(*c*) FW = $-P(P/F, i\%, n) + A(F/A, i\%, n) - A[(P/A, i\%, n) + S$
(*d*) FW = $-P(F/P, i\%, n) + A(F/A, i\%, n) + S$

D-89 A firm has been investing retained earnings to establish a building fund. The firm has retained \$1.2 million, \$1.0 million, and \$950,000, respectively, 3, 2, and 1 year ago. This year the firm has \$1.8 million to invest. If the firm earns 18% on invested funds, what is the value of the project that can be undertaken using the funds as a 25% down payment?

(*a*) \$6.28 million
(*b*) \$7.42 million
(*c*) \$25.1 million
(*d*) \$29.7 million

D-90 A firm is considering the purchase of a software analysis package that costs \$450,000. Annual licensing fees are \$25,000 (payable at the beginning of each year, starting in Year 1). The firm is bidding on a large 4-year government project where the new software will be used. If the firm uses an interest rate of 20%, what value for software costs should be put in the bid?

(*a*) \$514,725
(*b*) \$527,650
(*c*) \$1,067,540
(*d*) \$1,094,346

D-91 Which of the following is a true statement regarding the future worth of a single investment alternative?

(*a*) It will be equal to both the present worth and the annual worth if the same discounting interest rate is used.
(*b*) Choose to invest if the calculated amount is less than zero at the investment rate of return.
(*c*) It will yield a recommendation consistent with the present worth and annual worth methods if the same discounting interest rate is used.

(*d*) It cannot be used to evaluate single investment alternatives.

D-92 Using the data for Uno and Dos from Problem 86, where the lives of both alternatives is 10 years, give the future worth on the incremental investment if the interest rate used is 10%.

(*a*) − $20, 000
(*b*) − $20, 783
(*c*) − $43, 910
(*d*) − $53, 910

Benefit–Cost Ratio Analysis

D-93 The annual benefits associated with construction of an outer belt highway are estimated at $10.5 million by a local planning commission. The initial construction costs will be $400 million, and the project's useful life is 50 years. Annual maintenance costs are $500,000 with periodic rebuilding costs of $10 million every 10 years. If interest is 2%, the benefit–cost ratio is closest to what value?

(*a*) 0.25
(*b*) 0.75
(*c*) 1.11
(*d*) 1.35

D-94 A city needs a new pedestrian bridge over a local stream. The city uses an interest rate of 5%, and the project life is 30 years. The following data (in millions of dollars) summarizes the bids that were received.

	Bid A	*Bid B*
Construction materials costs	$4.20	$6.20
Construction labor costs	0.60	0.70
Construction overhead costs	0.35	0.03
Initial administrative and legal costs	0.60	0.01
Annual operating costs	0.05	0.075
Annual revenue from operation	Unknown	0.40
Other annual benefits to the city	0.22	0.25

What would the annual revenue of Bid *A* have to be for the two projects to be equivalent? Choose the closest value.

(*a*) 0.10
(*b*) 0.20
(*c*) 0.30
(*d*) 0.40

D-95 When using the benefit–cost method of analyzing a project, which of the following is a true statement?
(*a*) It will always produce a recommendation consistent with the simple payback period method.
(*b*) It will always produce a recommendation consistent with present worth, future worth, and annual worth methods.
(*c*) It can be used only to evaluate projects from the public sector (such as bridges and roadways).
(*d*) None of the above.

D-96 Project *A* has a first cost of $950,000 and will produce a $50,000 net annual benefit over its 50-year life. Project *B* costs $1,250,000 and produces an $85,000 net annual benefit. If interest is 3% per year, the benefit–cost ratios of Projects *A* and *B* are nearest what values?
(*a*) 0.52, 0.67
(*b*) 0.74, 0.57
(*c*) 1.35, 1.75
(*d*) 2.63, 3.40

D-97 Using the data for Projects *A* and *B* in Problem 96, the benefit-cost ratio on the incremental investment is nearest what value?
(*a*) 0.17
(*b*) 0.33
(*c*) 3.00
(*d*) 5.83

Sensitivity and Breakeven Analysis

D- BVM manufactured and sold 25,000 small statues this past year. At that volume, the firm was exactly in a breakeven situation in terms of profitability. BVM's unit costs are expected to increase by 30% next year. What additional information is needed to determine how much the production volume/sales would have to increase next year to just break even in terms of profitability?

98 (*a*) Costs per unit
(*b*) Sales price per unit and costs per unit
(*c*) Total fixed costs, sales price per unit, and costs per unit
(*d*) No data is needed, the volume increase is 25, 000 + 25, 000(0.30) = 32, 500 units.

D-99 Process *A* has fixed costs of $10,000 and unit costs of $4.50 each, and Process *B* has fixed costs of $25,000 and unit costs of $1.50 each. At what level of annual production would the two processes have the same cost?
(*a*) 50 units
(*b*) 500 units
(*c*) 5000 units
(*d*) 50,000 units

D-100 A seasonal bus tour firm has 5 buses with a capacity of 60 people each. Each customer pays $25 for a one-day tour. Records show $360,000 in fixed costs per season, incremental costs of $5 per customer, and an average daily occupancy of 80%. The number of days of operation necessary each season to break even is closest to which value?
(*a*) 50 days
(*b*) 75 days
(*c*) 100 days
(*d*) 120 days

D-101 Alternative I has a first cost of $50,000, will produce an $18,000 net annual benefit over its 10-year life and be salvaged for $5,000. Alternative II costs $150,000 and has a salvage value of $50,000 after its 10-year useful life. If interest is 15%, what is the minimum amount of annual benefit that Alternative II must produce to make it the preferred choice?
(*a*) This value can not be determined from the data given.
(*b*) $23,500
(*c*) $31,450
(*d*) $35,708

Use the table to determine which project is best if it is known for sure that annual sales will be $7 million. All values are in millions (PW and Sales) of dollers.

Estimated PW ($M)

D-102

Ann. Sales	Proj. 1	Proj. 2	Proj. 3
$ 0	− 75	− 10	0
5M	125	15	150
10M	325	40	300

(*a*) Project 1
(*b*) Project 2
(*c*) Project 3
(*d*) None, since Projects 1 and 2 have negative values.

Chapter 10

Uncertainty and Probability

D-103 An interest rate of 15% is used to evaluate a new system that has a first cost of $212,400, annual operating and maintenance costs of $41,200, annual savings of $94,600, a life of 6 years, and a salvage value of $32,500. After initial evaluation, the firm receives word from the vendor that the first cost is 5% higher than originally quoted. The percentage error in the system's present worth from this is closest to what value?
(*a*) 5%
(*b*) 15%
(*c*) 100%
(*d*) 300%

D-104 A machine has a first cost of $10,000 and annual costs of $3500. There is no salvage value, and interest is 10%. If the project's useful life is described by the following data, what is the annual worth of costs?

	Useful Life (years)			
	4	5	6	7
Prob. of life (%)	5	22	41	32

(*a*) $3500
(*b*) $5127
(*c*) $5554
(*d*) $5796

Three estimators have estimated a project with a 10-year life.

	Estimate 1	Estimate 2	Estimate 3
First cost	$10,000	$17,500	$15,000
Net annual cash flows	7,500	8,000	6,000
Salvage value	3,500	0	10,000

D-105 Use an interest rate of 20% to determine the project's expected present worth. The value is closest to which of the following?
(*a*) $16,066
(*b*) $16,612
(*c*) $31,660
(*d*) $31,607

D-106 The first cost (FC), life (*n*), and annual benefits (A) for a prospective project are uncertain. Optimistic (OP), most likely (ML), and pessimistic (PS) estimates are given. If the interest rate is 25%, what is the present worth difference between a total worst-case scenario and a total best-case scenario?

	Estimate		
Parameter	Pessimistic	Most Likely	Optimistic
First cost	$150,000	$100,000	$80,000
Annual cash flows	25,000	45,000	50,000
Project life, in years	5	7	10

(*a*) $15.8
(*b*) $42.2
(*c*) $181.3
(*d*) $282.5

D-107 Which of the following statements, related to the use of decision tree analyses to model a problem and recommend a solution, is not true?
(*a*) In modeling the decision, the sequence flows from left to right, with later outcomes and decisions shown to the right of earlier decision and outcomes.
(*b*) Branches at a decision point are "pruned off" if they maximize the benefit to the decision maker relative to other choices.
(*c*) In analyzing the best path, sequence flows from right to left as inferior branches are pruned at decision points.
(*d*) Expected value at outcome points is calculated by multiplying the effect of each branch by the probability of that branch event.

Chapter 11

Depreciation of Capital Assets

D-108 The correct percentage to use to calculate the depreciation allowance for a MACRS 3-year property for Year 2 is which of the following?

(*a*) 14.4%
(*b*) 32.0%
(*c*) 33.3%
(*d*) 44.5%

D-109 With reference to the straight-line depreciation method, which statement is false?

(*a*) An equal amount of depreciation is allocated in each year.
(*b*) The book value of the asset decrements by a fixed amount each year.
(*c*) The depreciation life (*n*) is set based on the MACRS property classes.
(*d*) The asset is depreciated down to a book value equal to the salvage value.

D-110 A 7-year MACRS property has a cost basis for depreciation of \$50,000. The estimated salvage (market) value is \$10,000 after its 10-year useful life. The depreciation charge for the 4th year and book value of the asset after the 4th year of depreciation are closest to what values?

(*a*) \$5760; \$44,240
(*b*) \$6245; \$12,496
(*c*) \$6245; \$15,620
(*d*) \$6245; \$43,755

D-111 A \$100,000 asset has a \$20,000 salvage value after its 10-year useful life. The depreciation allowance using straight-line depreciation is closest to what value?

(*a*) \$2000
(*b*) \$8000
(*c*) \$10,000
(*d*) \$12,000

D-112 The book value of an asset that is listed as a 10-year MACRS property is $49,500 after the first year. If the asset's estimated salvage (market) value is $5000 after its 15-year useful life, what was the asset's original cost basis?
(*a*) $50,000
(*b*) $52,105
(*c*) $55,000
(*d*) $61,875

Chapter 12

Calculating Income Taxes

D-113 Which of the following is true?
(*a*) Tax rates are based on two flat-rate schedules, one for individuals and one for businesses.
(*b*) When businesses subtract expenses, they always include capital costs.
(*c*) For businesses, taxable income is total income less depreciation and ordinary expenses.
(*d*) When quantifying depreciation allowance, one must always divide first cost by MACRS 3-year life.

D-114 If a corporation has taxable income of $60,000, which of the following expressions is used to calculate the federal tax owed?
(*a*) Flat 15% of taxable income
(*b*) Flat 25% of taxable income
(*c*) $7, 500 + 25% over $50,000
(*d*) $13, 750 + 34% over $75,000

D-115 This past year CLL Industries had income from operations of $8.2 million and expenses of $1.8 million. Allowances for depreciated capital expenses were $400,000. What is CLL's taxable income and federal taxes owed for operations last year? Choose the closest values.
(*a*) $6.0 million; $1.93 million
(*b*) $6.0 million; $2.04 million
(*c*) $6.4 million; $1.93 million
(*d*) $6.4 million; $2.04 million

D-116 Annual data for a firm for this tax year are:

Revenues	$45 million
Operating and maintenance costs	7 million
Labor/Salary costs	15 million
Overhead and administrative costs	3 million
Depreciation allowance	8 million

Next year the firm can increase revenue by 20% and costs will increase by 2%. If the depreciation allowance stays the same, what will be the change in firm's after-tax net profit? Choose the closest answer.
(*a*) $5.2 million
(*b*) $5.4 million
(*c*) $7.9 million
(*d*) $13.3 million

D-117 Widget Industries erected a facility costing $1.56 million on land bought for $1 million. The firm used straight-line depreciation over a 39-year period; it installed $2.5 million worth of plant and office equipment (all classified as MACRS 7-year property). Gross income from the first year of operations (excluding capital expenditures) was $8.2 million, and $5.8 million was spent on labor and materials. How much did Widget pay in federal income taxes for the first year of operation?
(*a*) $680,935
(*b*) $1,002,750
(*c*) $1,321,815
(*d*) $2,788,000

Chapter 13

Replacement Analysis

D-118 Which of the following is a valid reason to consider replacing an existing asset?
(*a*) It has become obsolete and does not perform its intended function.
(*b*) There is a newer asset available that is technologically superior.
(*c*) It has become very costly to keep the asset in working order.

(*d*) All of the above.

D-119 The replacement of a typical existing asset (defender) that is beyond its minimum economic life with a new asset (challenger) should be done when?
(*a*) It should never be done because the existing asset is working.
(*b*) The defender's marginal cost is greater than the challenger's equivalent annual cost.
(*c*) The challenger's average future cost becomes less than the existing asset.
(*d*) The defender's equivalent annual cost equals the challenger's equivalent annual cost.

D-120 A factory asset has a first cost of $100,000, annual costs of $15,000 the first year and increasing by 7.5% per year, and a salvage (market) value that decreases by 20% per year over its 5-year life. The minimum cost economic life of the asset is what value? Interest is 10%.
(*a*) 2 years
(*b*) 3 years
(*c*) 4 years
(*d*) 5 years

D-121 The minimum cost life of a new replacement machine is 6 years with a minimum equivalent annual cost of $6000. Given the existing machine's marginal cost data for the next 4 years, when should the existing machine be replaced?

Year	Total Marginal Cost
1	$5400
2	5800
3	6200
4	8000

(*a*) After Year 1
(*b*) After Year 2
(*c*) After Year 3
(*d*) After 6 years

A material handling system was purchased 3 years ago for $120,000. Two years ago it required substantial upgrading at a cost of $15,000. It once again is requiring an upgrading cost of $25,000. Alternately, a new

D-122 system can be purchased today at a cost of $200,000. The existing machine could be sold today for $50,000. In an economic analysis, what first cost should be assigned to the existing system?

(*a*) $50,000
(*b*) $65,000
(*c*) $75,000
(*d*) $80,000

Chapter 14

Inflation Effects

D-123 If the real growth of money interest rate for the past year has been 4% and the general inflation has been 2.5%, the combined (market) interest rate is closest to what value?

(*a*) 1.5%
(*b*) 6.5%
(*c*) 6.6%
(*d*) 10.0%

D-124 To convert actual (inflated) dollars to constant purchasing power dollars (where n = difference in time between today and purchasing base) that occur at the same point in time, one must:

(*a*) Multiply by (P/F, inflation rate %, n).
(*b*) Multiply by (F/P, inflation rate %, n).
(*c*) Multiply by (P/F, combined rate %, n).
(*d*) Divide by (P/F, real interest rate %, n).

D-125 The cost of a material was $2.00 per ounce 5 years ago. If prices have increased (inflated) at an average rate of 4% per year, what is the cost per ounce now?

(*a*) $2.04
(*b*) $2.08
(*c*) $2.43
(*d*) $8.00

A deposit of $1000 is made into an account that promises a minimum of 2% per year increase in purchasing power. If general price inflation is 3, 1, and 5%, respectively, over the next 3 years, the minimum value that

D-126 will be in the account at Year 3 is closest to what amount?
(*a*) $1061
(*b*) $1092
(*c*) $1157
(*d*) $1177

D-127 The cost of materials was $1000 per lot when the cost index is 145. Today the cost index is 210. What is the cost per lot?
(*a*) $690
(*b*) $1000
(*c*) $1448
(*d*) Cannot be determined with the data given.

Chapter 15

Capital Budgeting

D-128 Which of these statements can be said of projects considered to be mutually exclusive?
(*a*) The projects are equivalent or mutual in terms of their cash flows.
(*b*) Neither alternative should be chosen.
(*c*) All projects can be chosen as long as they meet minimum economic criterion.
(*d*) The selection of one in the set eliminates selection of others in that same set.

Use the following data for Problems 129–131 A firm has identified four projects for possible funding in the next budget cycle.

Project	First Cost ($M)	PW($M)
A	12	225
B	18	250
C	24	320
D	30	400

D- The projects are independent. What is the total number of possible investment strategies (combinations) that the firm can use?
(*a*) 4

129 (*b*) 6
(*c*) 10
(*d*) 15

D-130 If the projects have the following contingencies, what is the total number of possible investment strategies (combinations) that the firm can use? Project *A* can be invested in only by itself; Project *D* is chosen only if Project *B* is chosen.
(*a*) 0
(*b*) 2
(*c*) 4
(*d*) 6

D-131 If the projects are independent, and the project budget (capital limit) is $32 million, what investment combination maximizes PW?
(*a*) *A* and *C*
(*b*) *D* alone
(*c*) Invest in all projects to maximize PW.
(*d*) *A* and *B*

D-132 A firm is considering the "make vs. buy" question for a subcomponent. If the part is made in-house, the production data would be: first cost = $350, 000; annual costs for operation = $45, 000; salvage value = $15, 000; project life = 5 years; interest = 10%; and material cost per unit = $8.50. If annual production is 10,000 units, the maximum amount that the firm should be willing to pay to an outside vendor for the subcomponent is nearest?
(*a*) $10 per unit
(*b*) $16 per unit
(*c*) $22 per unit
(*d*) $28 per unit

D-133 A firm is considering whether to buy specialized equipment that would cost $200,000 and have annual costs of $15,000. After 5 years of operation, the equipment would have no salvage value. The same equipment can be leased for $50,000 per year (annual costs included in the lease), payable at the beginning of each year. If the firm uses an interest rate of 5% per year, the annual cost advantage of leasing over purchasing is nearest what value?
(*a*) $2494

(*b*) $8694
(*c*) $11,200
(*d*) $12,758

Chapter 17

Accounting

D-134 Which of the following summarizes a firm's revenues and expenses over a month, quarter, or year?
(*a*) Balance sheet
(*b*) Statement of assets and liabilities
(*c*) Income statement
(*d*) None of the above

D-135 Which of the following financial ratios provides insight into a firm's solvency over the short term by indicating its ability to cover current liabilities?
(*a*) Acid-test ratio
(*b*) Quick ratio
(*c*) Current ratio
(*d*) Net profit ratio

D-136 Which statement is most accurate related to an activity-based costing (ABC) system?
(*a*) ABC spreads the firm's indirect costs based on volume-based activities.
(*b*) ABC seeks to assign costs to the activities that drive those costs.
(*c*) ABC gives an inaccurate view of a firm's costs and should never be used.
(*d*) ABC is called ABC because it is an easy method to use.

A firm's balance sheet has the following data:

Cash on hand	$450,000
Market securities	25,000
Net accounts and notes receivable	125,000
Retailers' inventories	560,000

D-137

Prepaid expenses	48,500
Accounts and notes payable (short term)	700,000
Accumulated liabilities	120,000

The firm's current ratio and acid-test ratio are closest to what values?
(*a*) 1.42; 0.73
(*b*) 1.42; 0.79
(*c*) 1.47; 0.73
(*d*) 1.47; 0.79

D-138

A firm's income statement has the following data (in $10,000):

Total operating revenues	$1200
Total nonoperating revenues	500
Total operating expenses	925
Total nonoperating expenses	125

If the firm's incomes taxes were $60,000 what was the net profit (loss)?
(*a*) All necessary data is not given, one cannot calculate net profit (loss).
(*b*) −$53,500
(*c*) $1,150,000
(*d*) $6,440,000

D-139

Annual manufacturing cost data (1000s) for four product lines are as follows:

Data	Line 1	Line 2	Line 3	Line 4
Annual production	4000	3500	9800	675
Cost of direct materials	$800	$650	$1200	$2500
Cost of direct labor	$3500	$3750	$600	$320

Rank the product lines from lowest to highest in terms of manufacturing cost per unit. Total indirect costs of $10.8 million are allocated based on total direct cost.
(*a*) 1-2-3-4
(*b*) 3-1-2-4
(*c*) 3-2-1-4
(*d*) 3-4-1-2

APPENDIX E

SELECTED ANSWERS TO END-OF-CHAPTER PROBLEMS

Chapter 1

1-2 yes on a, b, e, & f
1-18 a & c Max output – input; b & d Minimize input
1-26 a) A:$725, B:$700, C:$750, D:$675/acre
b) A:$450, B:$350, C:$375, D:$475/acre
1-54 $86.40 individual; $88 team
1-56 8.5 feet in length & diameter
1-58 itemized $8525; Std. mileage $8,175; breakeven 16,591 miles
1-60 a) $211.81 at 60 mph; $198.00 at 70 mph
b) $177.41 at 60 mph; $174.00 at 70 mph
c) $169.27 at 60 mph; $172.50 at 70 mph
1-62 775 units/yr
1-68 a) 1^{st} = 6.2; 2^{nd} = 6.4; 3^{rd} = 5.8
b) 1^{st} = 5.8; 2^{nd} = 6.6; 3^{rd} = 7.2

Chapter 2

2-2 average $15, $14.60, $13.80, & $13.00; marginal $15, $13, $11, $11 for a/b/c/d
2-4 1 shift $472/unit; 2 shifts $465
2-6 33,333
2-8 avg./marginal a) $60.71/0, b) $53.13/$53.13, c) $53.13/$79.69, d) $56.69/$79.69
2-10 a) $63, b) $70, c) $35
2-12 a) 226,316, b) $41,500 in yr 1
2-18 A for 0 to 6249, B for 6249 to 11,667, C for 11,667 to 30,000
2-20 4124 units
2-34 a) $7000, b) $4000, c) stainless $1500 less
2-38 a) $126,000, b) Current process = $210,000/yr New process = $84,000/yr
2-44 a) $1,280,900, b) $1,793,260
2-46 a) $130/ft2, b) i. $585,000 ii. $508,950
2-48 $1.4 million
2-50 $752
2-52 a) $11,012
2-54 $134,318
2-56 $12,811
2-58 77% rate
2-60 T_1 = 716 person hours; T_{25} = 119
2-62 N = 25.12 so by 26th unit
2-64 costs reduced by 64%
2-74 $11.82M

Chapter 3

3-2 a) 2300
3-4 $54,000
3-6 a) 33.3 yrs, b) 23.45 yrs.
3-12 $7956
3-14 $2178
3-16 $112,095
3-18 a) $3285, c) $5916, e) $136,363
3-20 a) 14.02%, c) 4.47%
3-22 $4909
3-24 a) $2900, b) $2901, c) $2900
3-26 12.4%
3-28 $13 more interest
3-30 17.5 years
3-32 10.063Q
3-34 $424,925
3-36 $9438
3-38 9.4%
3-40 8.24%
3-42 0.80%/month
3-44 15.0%
3-46 nominal 16.7%, eff. 17.4%
3-48 668.7%
3-52 nominal 18%, eff. 19.72%
3-56 $36,000

Chapter 4

4-2 a) $99,272, b) $755,686
4-4 $35,460
4-6 $821
4-8 $3,436,352
4-10 29
4-12 35 months
4-14 1.995%/month
4-16 B = $634, C = $51.05, V = $228.13
4-18 $94,117
4-20 $8642
4-22 a) $2050, b) $969
4-24 $14,763
4-26 $5622
4-28 $488.78
4-30 $1772
4-32 loan $171,962, house $191,069, save $23,357
4-48 10%
4-50 $792.28
4-52 a) $2790, b) $554, c) $1194
4-54 $17,788
4-56 $230,878
4-58 18th bday $23,625; AW = $2658
4-62 $589.50
4-64 $340
4-66 $1496.91
4-74 $452,090
4-76 a) $186,154, b) $201,405
4-78 a) $147,674, b) $166,667
4-80 1st choice at $303,263; 2nd $306,820
4-88 a) $520, b) 15%, c) 16.08%
4-90 16.67%
4-92 a) 1.50%, pmt = $587, i_a = 19.56%
4-100 payment $1281.40, last yr interest $83.83

4-102 payment $393.53, last yr interest $2.93
4-110 $154,149
4-112 $116,873
4-118 27,959

Chapter 5

5-2 $245,957
5-4 automate for a PW of $62,657
5-6 max. price she can afford $18,102
5-8 $7698
5-10 $15,292
5-12 $2182
5-14 16.0%
5-16 $18,356
5-24 a) Buy A, PW = $7841
b) Select A, PW = $7841
5-26 $21,000 for Cost$_A$ is < $21,737 for B
5-28 NPW $52,914 for B is higher
5-30 Foxhill $57,957 < Quicksilver $67,027 < Almaden $69,103
5-32 2-stage $20,098,000 < 1-stage $23,962,000
5-36 A for $1154 > B for $1420
5-38 Alternative A saves $1.84M
5-40 Four 1-yr cost $79 < PW cost $85 for one 4-yr
5-42 SuperBlower $143,243 < Sno-Mover $155,728
5-48 $1,200,000
5-50 $1265.60
5-52 $142,046
5-54 Cap. Cost$_A$ $957,920 < Cap. Cost$_B$ $1,008,830
5-58 $6,777,957
5-60 $221,841
5-64 A $17,324>B $16,569>C $15,747
5-66 A $7.74, B $7.76, C $6.86, D $3.70, E $32.43,F $12.43; pick E
5-68 D »A, B »E; NPW B $926 > C $812 > D $724
5-70 12 yrs, C $93,497 >B $55,846 >A $53,255
5-74 Spartan $.14/$ invested
5-76 $19,438
5-80 $165,178
5-84 $201,234
5-86 $31,069

5-88 $1072
5-90 $3602.75
5-92 $1360
5-94 $769
5-96 $4010

Chapter 6

6-2 \$3572
6-4 \$10,236
6-6 \$234.28
6-10 \$2,759,875
6-12 \$5995
6-14 \$171.28
6-16 \$3558
6-18 $n = 50$
6-20 \$98
6-22 \$771
6-24 \$23,179
6-26 \$2734
6-28 \$42,817 quarterly (\$42,799 by spreadsheet)
6-30 EUAC = \$8429 >0
6-32 Buy for \$5718 > lease for \$5500
6-34 new payment of \$26,557 is less
6-36 $EUAW_L$ = \$2692; $EUAW_S$ = \$3064
6-38 existing EUAC is \$3957 > new of \$3552
6-40 Hydro-clean's 150,000 < \$223,003
6-42 $EUAW_A$ −\$752 better than $EUAW_B$ −\$1336
6-44 Hybrid \$6385 < Midsize \$7273
6-46 $EUAC_Y$ \$1218 < $EUAC_x$ \$1392
6-48 Rt 105 = \$14,893, Rt 205 = \$20,503, Rt 305 = \$17,178
6-50 C at \$329 < A at \$375 < B at \$520
6-52 36-month at \$28.13
6-54 A at \$504>B at \$421
6-60 pay \$843 more than receives for car
6-62 pmt = \$579.98; \$19,065 is owed after 2 yrs.
6-64 a) car price = \$7527, b) save for 5.52 months, c) 0.25% per month or 3% APR
6-66 a) \$550, b) beginning month so all principal, c) \$545.45
6-68 a) payment \$1049.21, b) 241.4 months,c) 108.2 months

6-72 $4675

Chapter 7

7-2 8.99%
7-4 23.11%
7-6 nominal 15%, effective 16.08%
7-8 19.8%
7-10 a) 9.4%, b) 4.8%
7-12 11.4%
7-14 50%
7-16 10.25% → don't purchase
7-18 13.23%
7-20 a) 8.18%
7-22 21.14%
7-24 a) 7.70%, b) 8.00%
7-28 nominal 5.64%, effective 5.7%
7-30 7.48%
7-32 a) 8.15%, b) 8.55%
7-34 8 years w/nominal 11.96%
7-36 a) 8.99%, b) 9.40%
7-38 13.0%
7-40 effective 21.6%
7-42 a) 15.97%, b) 9.65%, c) 6.94%
7-44 11.37%
7-46 9.11%
7-48 11.0%
7-52 7.98%
7-54 If 70 years used, then 6.58%
7-56 18.6%
7-62 do B since incremental B – A is 8%
7-64 do A since incremental B – A is 4.3%
7-66 do A since incremental A – B is 23.1%
7-68 incremental RFID – bar code is 1.9% so do bar code
7-70 27.9% so select Alt. A
7-72 a) lease generator since 6.6% <8%, b) 11.3%
7-74 B – A 35.7% → B, C – B 16.2% → C

7-76 8.3% → B
7-78 7.4% < MARR, so select B
7-80 \$126,348
7-82 12.24% for \$1,000,000
7-84 0% for \$8000; −21.3% for \$7000
7-86 15.19%

Appendix 7A

7A-2 unique root at 12.8%
7A-4 roots 3.9% & 100%, MIRR of 10.3%
7A-6 unique root at 6.8%
7A-8 unique root at 11.8%
7A-10 unique root at 13.5%
7A-12 unique root at −10.45%
7A-14 a) roots 10% & 30%, MIRR of 8.8%, b) Not ethical
7A-16 ignore negative root of −71.8%; IRR = 39.0%; MIRR = 16.46%
7A-18 ignore negative root of −63.7%, use root of 11.6%
7A-20 unique root of 8.4%

Chapter 8

8-2 X for 0–8.6%, Y for 8.6–16.4%, then do nothing
8-4 b) neutralization with 20.8% incremental ROR
8-6 b) A–B increment 13.4% < MARR, select B
8-8 a) 0–28.68% Thatch, 28.68–100% Slate; b) select Thatch
8-10 a) 0–12.6% New, 12.6–50% Used; b) Used radiator
8-12 a) 0–6.2% B, 6.2%–100% A; b) Select A
8-14 a) 0–10% A, 10–19.9% C, 19.9%–100% Do Nothing b) C
8-16 a) 0–6% A, 6–9.6% B, 9.6–100% C; b) Select B
8-18 a) 0–31.6% B, 31.6–100% Do nothing; b) Company B
8-20 a) 0–16.8% Sort-of, 16.8–27.3% U-Sort-M, 27.3–100 Nothing; b) Select Sort-Of
8-22 a) C for 0–4.7%, B for 4.7–6.9%, A for 6.9–8.2%, then do nothing;
b) same except, A for 6.9–100%
8-24 5-yr for 0–11.8%, 3-yr for 11.8–12.8%, 2-yr for12.8–16%, then 1-yr for all higher rates
8-26 a) C for 0–16.9%, A for 16.9%–100%; b) Select C
8-28 a) C for 0–4.2%, B for 4.2–11.0%, A for 11–100%;
b) Select A
8-30 D for 0–32.3%, A for higher rates
8-34 b) Dallas
8-38 B–A 13.7% select A, C–B 12.4% select B, D–B 14.3% select D
8-40 a) Buy for 0–7%, Loan for 7–39.94%, Lease for rest;
b) Loan

Chapter 9

9-2 \$10,465
9-4 \$223,734
9-6 \$357,526
9-8 \$38,126
9-10 £1.75×10^{27}
9-12 \$1391
9-14 \$407,768
9-16 \$6108
9-18 \$4116
9-20 \$68,969
9-22 \$122,758
9-24 a) \$494,492, b) \$424,772
9-26 \$67,195
9-28 \$83.76/share
9-30 B
9-32 5-story
9-34 A-B B/C=0.75<1, D-B B/C= 1.07, Select D
9-36 D-C B/C=1.34, A-D B/C=1.07, Select A
9-38 \$1,998,425
9-40 on-campus, incremental B/C is 1.2
9-44 1.9-year payback period < 2 year
9-46 8.08 years
9-48 a) Payback_A = 5 yrs, Payback_B = 6 yrs, b) \$240.5M, c) pick A with EUAW = \$2.08M vs. B with −\$3.4M
9-50 a) B payback 3.6, b) D with FV=\$25.33, c) A–B B/C=0.54, D–A B/C=1.47 Select D
9-52 a) 1.01, b) X has shortest payback of 3.1 yrs, c) Z–Y B/C=3.2, X–Z B/C=1.06 select X
9-54 a) C has shortest payback of 3yrs, b) B–C B/C=1.10, A–B B/C=1.03 Select A
9-60 6.9 years
9-62 15.1 years
9-64 20 days

9-66 80%
9-68 $32,243
9-70 $7.922M
9-72 12.1 years
9-74 $175 min Fiasco resale
9-76 44 cars
9-78 13.5 years
9-80 $102,241
9-82 4 weeks
9-84 a) 6 years, b) 7.8 years, c) both correct, but breakeven considers more

Appendix 9A

9A-2 a) $34,166, b) $27,308, c) employer match $2975,employee $31,191
9A-4 a) 9.7%, b) 5.7%
9A-6 a) 35.5%, b) 31.5%
9A-8 34.8 yrs.
9A-10 a) $70,423, b) $30,181, c) $1,333,144
9A-12 $674,927
9A-14 7.66%

Chapter 10

10-2 $EUAC_{12,\,500}$ \$6654, up to \$7765, down to \$5543
10-4 a) Optimistic 22.9%, Most likely 7.9% b) 13.0%
10-6 a) \$4.00M, \$3.27M, \$2.49M, b) \$3.26M
10-8 P(250) = .1, P(300) = .4, P(350) = .4
10-10

Sales	Prob.	Unit Profit	Prob.
3500	0.3	\$26.00	0.25
6000	0.55	\$33.00	0.45
9000	0.15	\$37.00	0.3

10-12 Bill Z=0.50, Al ranked higher at Z=0.72
10-14 Annual Profit, Prob.; \$91,000, 8%; \$115,500, 14%; \$129,500, 9%; \$156,000, 14%; \$198,000, 25%; \$222,000, 17%; \$234,000, 4%; \$297,000, 7%; \$333,000, 5%
10-16 a) P(\$18,000, 12 yr) = 0.033 & 8 other combinations,b) 25.4%, 7.47%, −4.55%
10-18 315 days
10-20 \$1100
10-22 E(GPA) of A is 1.88 < E(GPA) of B is 2.00
10-24 Loss of \$0.20
10-26 a) \$3.10M, b) 6.95 yrs, c) \$3.06M, d) don't match since PW nonlinear in time
10-28 \$1328
10-30 3m for \$48,120 E(EUAC)
10-32 \$355,000
10-34 \$184,965 both ways
10-36 a) \$45,900, b) first cost \$440,000, net \$86,000, PW \$45,900, c) same
10-38 a) 12.18%, b) 8.28%, c) interest formulas nonlinear so don't match
10-40 PW \$6600 of B > PW \$6400 of A
10-42 repair \$26,000 < as-is \$28,000 < replace \$30,000
10-44 repair \$22,920 < repair & spillway \$36,050 < add upstream dam \$72,500
10-46 a) E(loss) \$420, b) logical to avoid catastrophic loss or to satisfy

mortgage lender
10-48 E(PW) $8212, σ_{PW} $1158
10-50 σ_{PW} $1622
10-52 $332,619
10-54 a) $5951, b) 0.3, $65,686
10-56 PW(loss) 0.45, range −$204.5K to $265K, std. dev.$127.9K
10-58 Projects F, 5, 1, & 6 form efficient frontier
10-64 Answers may vary, mean $3126, std. dev. $6232

Appendix 10A

10A-2 a) $15,880, b) $10,851, c) employer match $2175, employee $13,705
10A-4 a) 5.8%, b) 2.9%
10A-6 35.1 yrs.
10A-8 a) $50,477, b) $455,402
10A-10 35 years; enjoy retirement

Chapter 11

11-2 Year 2 depreciation is $0
11-4 Year 2 depreciation is $0
11-6 a) $75,000, b) $150,000, c) $120,000, d) $30,000
11-8 D_2 $33,337.50 BV2 $16,665
11-10 D_1 – MACRS $7000, Bonus $35,000
11-12 D_2 a) $15,200, b) $18,240, c) $0, d) $24,320
11-14 D_2 a) $5000, b) $8000, c) $0, d) $12,245
11-16 D_4 $9500, BV_4 $78,000
11-18 D_2 a) $5000, b) $9375, c) $0, d) $12,245
11-20 A-Bonus, B-UOP, C-MACRS, D-DDB, E 150%DDB
11-22 D_2 a) $67,500, b) $100,000
11-24 D_2 $5600, BV2 $53,800
11-26 D_2 SL $15,000, DDB $22,222, Bonus $0, MACRS $24,490
11-28 PW SL $720, DDB $761
11-32 PW a) $138,991, b) $149,453, c) $190,476, $174,920
11-36 D_2 a) $2500, b) $3000, c) $4445
11-38 D_8 a) SL $1667, Bonus $0, MACRS $669, b) PW SL $11,846, Bonus $14,286, MACRS $12,634
11-40 D_3 a) $4250, b) $8100, c) $10,494
11-42 D_4, BV_8 a) $13,500, $42,000, b) $15,360, $25,166, c) $7,494, $0
11-44 a) M/C2 $27,380, b) $270,500, c) $1050, d) 11.4%, e) $19,590, f) $91,000, g) $17,490
11-46 MV_3-BV_3 1c) $40,800, 2c) MV-BV=$60,000
11-48 a) $2000 loss, b) $8000 dep recap, c) $14,257 dep recap
11-50 a) −$37,677, b) −$25,438, c) −$73,304, d) $11,524
11-52 a) −$63,333, b) −$44,297, c) −$31,487, d) −$52,475
11-54 $1462.50 allowable depletion
11-56 $3000.00 allowable depletion
11-58 a) $366,000, b)$1448.28 per million board feet
11-60 a) $200,000/yr, b) $118,750
11-62 $102,500/year

11-64 PW $1,421,716

Chapter 12

12-4 $2.31M
12-6 28.58%
12-8 13.1%
12-10 a) 2.27 yrs, b) 3.02 yrs, 28.82%
12-12 a) 15.4%, b) 11.5%, c) 10.0%
12-14 −$217,183
12-18 12.5%, move forward with the project
12-20 14.0%
12-22 7.52%, not desirable
12-24 $17,741
12-26 14.8%, investment is satisfactory
12-28 a) 34.9%, b) 28.2%
12-30 11.1%
12-32 a) 2.15 yrs, b) $159,884, 33%, desirable investment
12-34 3.08 yrs
12-36 $17,544
12-38 $62,387, a good investment
12-40 21.9%
12-42 $28,233
12-44 $274,994
12-46 $54,254 max price
12-48 214 days
12-50 $6172, a good investment
12-52 29.3%, investment is satisfactory
12-54 $9887
12-56 a) 15.4%, b) 13.95%, c) 12.2%
12-58 17.8%, investment is satisfactory
12-60 Select Alt. 2: a) $17,794, b) $2,090, c) 12.2%, d) $119,709, e) 2-1 incremental B/C 1.3
12-62 incremental A-C 117%, increm. B-A 74.3%, pick B
12-64 Alt. II has lowest PW cost at $6419

Appendix 12A

12A-2 \$24,009; \$10
12A-4 \$1800
12A-6 \$25,551.25
12A-8 a) \$762.50, b) 723.50
12A-10 1.88%
12A-14 \$3125
12A-16 \$280
12A-18 \$5776
12A-20 \$329.50 due to IRS
12A-22 \$2500 tax credit
12A-24 \$367 tax savings
12A-26 \$14,163 versus \$10,688
12A-28 \$6000
12A-32 \$52,931
12A-34 a) 70.9%, b) 17.1%, c) 12.0%, d) \$1080, e) \$82,353
12A-38 a) can afford, 26.6% < 43%, b) \$333,162

Chapter 13

13-2 3 yrs, $18,146

13-4 8 yrs, $6424

13-6 4 yrs, $8516

13-8 a) 8 yrs, $22,021; b) 5 years, $22,980

13-10 4 yrs, $16,861

13-12 6 yr, $2717

13-14 a) 7 yr, $7182; b) 5 yr, $8039

13-15 Keep the bottling machine indefinitely

13-16 8 yrs, $6784

13-26 Relevant costs are 1, 2, and 3

13-28 Each year: $6650, $6450, $6350, $6250

13-30 a) 4 yrs, $3100; b) Replace in year 2

13-32 Keep existing asset 3 more years

13-34 Upgrade $5659 versis $7840

13-36 Replace in year 3

13-38 a) 4 yrs, $2650; b) replace in year 5

13-39 First cost < $299,601

13-40 Implement today

13-42 Keep cab ownership $3496 vs $6750

13-44 Keep current line $1.983M vs. $3.544M

13-46 Keep old car $5111 vs. $6109

13-48 Keep old lift $240 vs. $822

13-50 New asset: $−76,500; Existing a) −$28,875b) −$25,500 c) −$19.875

13-52 Choose C, NPW $−1812

13-54 4 more years, $5015

13-56 11 years, $6316

Chapter 14

14-4 constant $ & real rate or inflated $ & market rate
14-6 $122
14-8 4.75%
14-10 $29,670
14-12 14.62 km/liter
14-14 11.7%
14-16 5.07%
14-18 $1.331, 2.90%
14-20 PW benefits at 11.3% equiv. *i* = $14,540 < cost
14-22 a) 0.3877%, b) 4.753%, c) $55,682
14-24 3.17, or 3 to 4 pads
14-26 $1611
14-28 1.59%
14-30 $1203
14-32 $2029, $1940
14-34 yr 4 $66,915, 0.96%
14-36 $6.20M, $5.52M
14-38 $515,353, $251,826
14-40 35.1 months
14-44 a) $78,432, b) $96,235
14-46 $388,832
14-48 $104,172; $83,558; $69,859; $60,266
14-50 NPW B at −$573,852 < A at −$600,491
14-52 $12,109 at 10.21%
14-54 C $267,848 > A $259,740 >B $242,896
14-56 C at $5688 PW, A at $6729
14-60 a) 150%, b) 66.8%, c) 31.9%, d) 27.3%, e) 6.8%
14-62 a) 7.9%, b) 4.6%, c) 3.1%, d) 2.2%
14-64 a) 2.36%, b) 275.4
14-68 Z $31,562
14-70 a) $18,116, b) $16,105
14-72 a) $89,250; $93,266; $97,463, b) 4.7%, c) $19,632, d) $391,843; e) $553,367

14-74 1.01%
14-76 a) 3.5%, b) 2.66%, c) 0.65%
14-78 a) $74,292, b) $41,134, c) $111,087
14-80 a) $67.68, b) $97.08
14-82 6.84%
14-84 Renting better $27,043 < $27,884

Chapter 15

15-4 a) 8.1%, b) 7.1%
15-6 a) 9.5%, b) 8.0%
15-8 9.3%, 8.1%
15-10 7.79%, 5.67%
15-18 243.29
15-20 Projects 1, 3, 5, and 6; 28.6%
15-22 Projects D, E, F, and G; 14.6%
15-24 12.4% > 8%; yes
15-26 19.46% < 20%; No
15-28 a) all but 4, b) do 5, 9, 10, 2, 1, and 7, c) First 6, d) 14.0%
15-30 a) 1B, 2B, 3A, & 4, b) $95K 3A, 2A, & 1B, c) last accepted is 13.7% & first foregone 11.2%, d) $95K 3A, 2A, & 1B same as b)

Chapter 16

16-12 real, use with constant value $s
16-14 A, B, C, F, and H should be funded B/C & IRR for $9M; E at 13% is opportunity cost
16-16 a) 15.8%, b) 9.5%
16-18 0.56
16-20 5.5 optimal PW
16-22 a) 521.70M Yuan, b) $76.27M
16-24 B/C 1.06, choose pumping
16-26 1.7 for low road, 1.14 for high – low; choose high
16-28 A vs. existing 1.71, B vs. A 2.17, C vs. B 2.42, pick C
16-30 B vs. nothing 1.10, A vs. B 2.18, D vs. A 1.62, pick D
16-32 now −$4,571, 2 yrs $40,172, 4 yrs $52,751, 5 yrs $50,192, repair in 4 years
16-44 a) $8,414,854, b) $5,546,628

Chapter 17

17-12 a) \$735,000, b) \$430,000
17-14 a) \$3,300,000, b) 2.5
17-16 a) \$98,000, b) 1.22, c) 0.73
17-18 a) \$130,000, Current 1.295, Quick ratio 0.614
17-20 a) 1.80, b) 0.92
17-24 \$10,175,000
17-26 \$1,185,000
17-28 a) 3.77, b) 0.115
17-32 a) \$18M, b) \$10M, c) \$69M
17-34 a) \$5000/hour, b) \$5,500,000
17-36 S \$132, M \$93, G \$84

Appendix B

B-2 \$920,290
B-4 \$539.52
B-6 \$4.39M
B-8 \$1104.65
B-10 \$19,021
B-12 \$1054
B-14 150.1 months = 12.5 years
B-16 15.07%
B-18 −14.98%, so lease

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