$3D^2 + 6D + D + 2$
$3D^2 + D + 6D + 2$

M T W T F S — D·E

Ch#8

System Of Diff. Eq:-

Ex 8.1 (Elimination / operator method)

Example:-

$$\begin{cases} 3\frac{dx}{dt} + dy/dt + x + 3y = 0 \\ dx/dt - 2dy/dt + 2x - y = 0 \end{cases}$$

$x(t) = ?$
$y(t) = ?$

Solution

$$\left(3\frac{d}{dt} + 1\right)x + \left(\frac{d}{dt} + 3\right)y$$

$$(d/dt + 2)x - \left(2\frac{d}{dt} + 1\right)y = 0$$

Let $\frac{d}{dt} = D$

$$\begin{cases} (3D+1)x + (D+3)y & \text{(i)} \\ (D+2)x - (2D+1)y = 0 & \text{(ii)} \end{cases}$$

$\times$(i) by $(D+2)$ and $\times$(ii) by $(3D+1)$

→ By doing this cofficient of x become same and x will be cancelled out.

M T W T F S

تاریخ: ___/___/___

$$(3D^2+D+6\cancel{D}+2)x+(D^2+3D+2D+6)y=0$$
$$\pm(3D^2+6\cancel{D}+D+2)x+(6D^2+3D+2D+1)y=0$$

$$(7D^2+10D+7)y=0$$
$$7D^2y+10Dy+7y$$
$$7\frac{d^2y}{dt^2}+10\frac{dy}{dt}+7y=0$$

By 4.3 concept.

$$y=e^{mx} \longrightarrow y=e^{mt}.$$
$$7m^2+7m+7=0$$

$$m_1,m_2=\left[-\frac{5}{7}\pm\frac{2\sqrt{6}}{7}i\right]$$

Q1

$$\frac{dx}{dt}=2x-y$$

$$dy/dt=x$$

Let $d/dt=D$

$$Dx-2x+y=0 \quad \text{(i)}$$
$$Dy-x=0 \quad \text{(ii)}$$

تاریخ: ___/___/___

$$\left.\begin{array}{l}(D-2)x + y \\ -x + Dy\end{array}\right\} \begin{array}{l}\text{-(i)} \\ \text{(ii)}\end{array}$$

(For y)

$\times \cdot (D-2)$ by (ii)

$(D-2)x + y = 0$

$-(D-2)(-x) + (D-2)(Dy) = 0$

$$\begin{array}{l} (D-2)x + y = 0 \\ \underline{\pm (D-2)x + (D^2-2D)y = 0} \\ \qquad (D^2-2D+1)y = 0 \end{array}$$

$D^2y - 2Dy + y = 0$

$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0$

$y = e^{mt}$

$m^2 - 2m + 1 = 0$

$m = 1, +1$

$\boxed{y = c_1 e^t + c_2 t e^t}$ -(y)

M T W T F S

تاریخ: ___/___/___

(For x)

$$D\begin{cases}(D-2)x + y = 0 & \text{(i)} \\ -x + Dy = 0 & \text{(ii)}\end{cases}$$

$$(D^2 - 2D)x + \cancel{Dy} = 0$$
$$\pm x \pm \cancel{Dy} = 0$$

$$\frac{d^2x}{dt^2} - \frac{2dx}{dt} + x = 0$$

let $x = e^{mt}$

$$m^2 - 2m + 1 = 0$$

$$m = 1, 1$$

$$\boxed{x = c_3 e^t + c_4 te^t} - (x)$$

Put x, y in (ii)

$$-(c_3 e^t + c_4 te^t) + D(c_1 e^t + c_2 te^t) = 0$$
$$-c_3 e^t + c_4 te^t + c_1 e^t + c_2(t \cdot e^t + e^t \cdot 1).$$
$$-c_3 e^t - c_4 te^t + c_1 e^t + c_2 te^t + c_2 e^t = 0$$
$$(-c_3 + c_1 + c_2)e^t + (-c_4 + c_2)te^t = 0$$

M T W T F S تاریخ: ___/___/___

Now comparing coff.

te^t, e^t coff. L.H.S = R.H.S

$$\begin{cases} -c_3+c_1+c_2=0 \\ -c_4+c_2=0 \end{cases} \quad \Big| \quad \begin{array}{l} \rightarrow c_3=c_1+c_2 \\ \rightarrow c_4=c_2 \end{array}$$

$$\boxed{\left.\begin{array}{l} x=(c_1+c_2)e^t+c_2e^t \\ y=(c_1e^t+c_2te^t) \end{array}\right\}}$$ Solution

Q3 $\frac{dx}{dt}=-y+t$, $\frac{dy}{dt}=x-t$

$Dx+y=t$ (i) , $Dy-x=-t$ (ii)

for (x)

$$\begin{array}{l} \cancel{Dx}+y=Dt=t \\ -\cancel{Dx}+D^2y=-Dt=-1 \\ \hline D^2y+y=t-1 \end{array}$$

$$\boxed{\frac{d^2y}{dt^2}+y=t-1} \quad (2)$$

as it is non-homo

so $y_G=y_p+y_c$

M T W T F S
تاریخ: ___/___/___

$y = e^{mt}$

$y_c = m^2 + 1$

$m = \pm i = \overset{\alpha}{0} \pm \overset{\beta}{i \cdot 1}$

$y_c = c_1 \cos t + c_2 \sin t$ $\quad \times \quad (c_1 \cos \beta x + c_2 \sin \beta x) e^{\alpha}$

y_p By UC method:

$y_p = At + B$

$y_p' = A$

$y_p'' = 0$

~~By~~ Put in (2)

$\cancel{\frac{dt}{dy}} \quad \frac{dy}{dt^2} + y_p = t - 1$

$\frac{d^2 y_p}{dt^2} + At + B = t - 1$

$0 + At + B = t - 1$

$y_p = At + B$

$\therefore y_p = t - 1$

$y = y_c + y_p$

(A) — $\boxed{y = c_1 \cos t + c_2 \sin t + t - 1}$

(Now for y)

$D \times \begin{cases} Dx + y = t & \text{— (i)} \\ 1 \times -x + Dy = -t & \text{— (ii)} \end{cases}$

$D^2x + \cancel{Dy} = 1$

$\underline{\mp x \pm \cancel{Dy} = -t}$

$D^2x + x = 1 - t$

$\boxed{\frac{d^2x}{dt^2} + x = 1 - t}$ — (y)

non-hom so By UC method

$y = y_c + y_p \longrightarrow x = x_c + x_p$

$y_c = e^{mt}$

$= m^2 + 1$

$m^2 + 1$

$m^2 = -1$, $m = \pm i$, $0 \pm i \cdot 1$ (α, β)

$\boxed{x_c = c_3 \cos t + c_4 \sin t}$

M T W T F S   تاريخ: ___/___/___

$x_p = At + B \quad , x_p' = A \quad , x_p'' = 0$

$0 + At + B = t + 1$

compare coff.

$A = 1 , B = 1$

$x_p = At + B \rightarrow (t+1) = x_p$

$\boxed{x = c_3 \cos t + c_4 \sin t + t + 1}$ (x)

put $x(t)$ & $y(t)$ in 2

$D(c_3 \cos t + c_4 \sin t + t + 1)$
$+ (c_1 \cos t + c_2 \sin t + t - 1) = t$

$-c_3 \sin t + c_4 \cos t + 1 + 0 + c_1 \cos t + c_2 \sin t$
$\cancel{+t} \cancel{-1} \cancel{= t}$

$(-c_3 + c_2) \sin t + (c_4 + c_1) \cos t = 0$

compare

$\Rightarrow \quad -c_3 + c_2 = 0 \quad \} \quad c_3 = c_2$
$c_4 + c_1 = 0 \quad \quad c_4 = c_1$

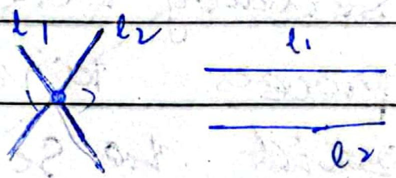
(D·E) (4x 8·6)

Solution System of D·E using matrix

$$\begin{cases} a x_1 + b_1 y = c_1 \\ a x_2 + b_2 y = c_2 \end{cases}$$

non-homogeneous eq.

case 1



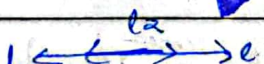
(In homo) ← (AX = 0)

(y/x) slope $\tan\theta$

$$a x_1 + b_1 y_1 = 0$$
$$a x_2 + b_2 y_2 = 0$$

→ $\left\{ \begin{matrix} x = 0 \\ y = 0 \end{matrix} \right\}$ Trivial sol.

→ non-trivial

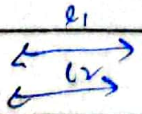


this case

$|A| = 0 \leftrightarrow$ (A) singular

infinite many sol. (overlap) (Non-Sing)

or



← No Solution (when Singular)

M T W T F S

تاریخ: ___/___/___

Ex

$dx/dt = a_{11}x + a_{12}y$

$dy/dt = a_{21}x + a_{22}y$

change into matrix form

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \overset{(X)}{\begin{bmatrix} x \\ y \end{bmatrix}}$$

Let $X = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$\frac{dX}{dt} = AX \rightarrow \boxed{X' = AX} \text{ -(i)}$$

Now:-

Let:- $X = e^{mt}$

column matrix so we have to make right hand column matrix

$$X = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} e^{mt}$$

$\boxed{X = ke^{mt}}$ **Solution:-**

$\boxed{X' = mk\, e^{mt}}$ or $\boxed{X' = \lambda k e^{\lambda t}}$

put in (i)

$$\lambda k \cancel{e^{\lambda t}} = A\, k \cancel{e^{\lambda t}}$$

$\boxed{AK = \lambda K}$ -(ii) $\quad |A - \lambda I| = 0$

eigen value

M T W T F S تاریخ: _________

$|A - \lambda I| = 0$ ← Eigen Value (λ value)

$\lambda = \lambda_1, \lambda_2, \lambda_3 \ldots 0$ ↓ Put λ here.

$(A - \lambda I)X = 0$ ← eigen Vectors

By (ii)

$\boxed{AK = \lambda K}$

∵ λ = Eigen values of A

∵ K = corresponding eigen vectors

if

$\lambda = \overset{\downarrow K}{\lambda_1}, \overset{\downarrow P}{\lambda_2} \ldots$

$$\boxed{\begin{aligned} &(A - I\lambda_1)K = 0 \Rightarrow K = ? \\ &(A - I\lambda_2)P = 0 \Rightarrow P = ? \end{aligned}} \text{ — (iii)}$$

$X_1 = Ke^{\lambda_1 t}$

$X_2 = pe^{\lambda_2 t}$

$\boxed{X = C_1X_1 + C_2X_2}$ General Sol.

note

$X' = AX$ — (i) ← given

$X = Ke^{\lambda t}$ — (ii) ← sol. make

$|A - \lambda I| = 0$ — (iii) ← find λ

$(A - \lambda I)K = 0$ — (iv) ← find K

Ex#1 (Book)

$$\frac{dx}{dt} = 2x + 3y$$

$$dy/dt = 2x + y$$

Sol

$\Rightarrow X' = AX$ — (i)

$A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Let $\boxed{X = Ke^{\lambda t}}$ — (ii) , $X_1 = Ke^{\lambda_1 t}$ — (A) $X_2 = Pe^{\lambda_2 t}$ — (B)

∵ λ eigenvalue of A

∵ K eigenvectors of A

For finding λ we have $\boxed{|A - \lambda I|} = 0$ — (iii)

Directly

∵ identity matr.

$$\begin{vmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(1-\lambda) - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda+1)(\lambda-4) = 0$$

$\lambda = -1$; $4 = \lambda$

$\boxed{\lambda = \lambda_1 = -1}$

$\boxed{\lambda = \lambda_2 = 4}$

M T W T F S　　　تاریخ: ___/___/___

① First we need To take :- $\lambda_1 = -1$

$(A - \lambda_1 I) k = 0$ —— (iv) .

→ finding k. eigen vectors

$\lambda_1 = -1$ so;

$(A + I) k = 0$

$$\begin{bmatrix} 2+1 & 3 \\ 2 & 1+1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$$ Now by echalonform

$$\left[\begin{array}{cc|c} 3 & 3 & 0 \\ 2 & 2 & 0 \end{array}\right]$$

$\frac{1}{3} R_1$ $$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 2 & 0 \end{array}\right]$$

$R_2 - 2R_1$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$k_1 + k_2 = 0$

$k_1 = -k_2$

if $k_2 = r$ so, $k_1 = -r$

$k = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -r \\ r \end{bmatrix}$ if $r = 1$

$k = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Scanned with CamScanner

M T W T F S — تاریخ: __/__/__

put in (A) $X_1 = Ke^{\lambda_1 t}$

$$\boxed{X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t}}$$

(2) $\lambda = -2$ Now finding eigenvector k for $\lambda = -2$

~~$|A - \lambda_2 I| = 0$~~

$$(A - \lambda_2 I)P = 0 \quad \text{(V)}$$

$$(A - 4I)P = 0$$

$$\begin{bmatrix} 2-4 & 3 \\ 2 & 1-4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -2 & 3 & 0 \\ 2 & -3 & 0 \end{array}\right]$$ ✓ Augmented matrix

Reduce to echolonform:-

$$R_2 + R_1 \sim \left[\begin{array}{cc|c} -2 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$-1/2 R_1 \sim \left[\begin{array}{cc|c} 1 & -3/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Now,

$$P_1 - P_2 \, 3/2 = 0$$

$$P_1 = P_2 \, 3/2$$

if $P_2 = r \quad r \neq 0, \; P_1 = r\,3/2$

$$P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 3/2\,r \\ r \end{bmatrix}$$

if $r = 2$

$$P = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Now Put in (B) $X_2 = P e^{\lambda_2 t}$

$$\boxed{X_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}}$$

$$X = X_1 C_1 + X_2 C_2$$

Put in this we get

$$\boxed{X = C_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + C_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}}$$ Ans.

(D·E)

Exp. 5

Eigenvalue of Multiplicity of 2.

$$X' = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 2 \\ 2 & -2 & 1 \end{pmatrix} X$$

Given. $X' = AX$ — 1

Sol. $X = Ke^{\lambda t}$ — 2

Eigenvalue $|A - \lambda I| = 0$ — 3

Eigen vector $(A - \lambda I)K = 0$ — 4

λ By eq (3)

$$\begin{vmatrix} 1-\lambda & -2 & 2 \\ -2 & 1-\lambda & -2 \\ 2 & -2 & 1-\lambda \end{vmatrix} = 0$$

expand R_1

$$(1-\lambda)\begin{vmatrix} 1-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} - (-2)\begin{vmatrix} -2 & -2 \\ 2 & 1-\lambda \end{vmatrix}$$

$$+ 2\begin{vmatrix} -2 & 1-\lambda \\ 2 & -2 \end{vmatrix} = 0$$

M T W T F S   تاريخ: __/__/__

$$(1-\lambda)\left[(1-\lambda)^2-4\right]+2\left[-2+2\lambda+4\right]$$

$$+2(4-2+2\lambda)=0$$

$$(1-\lambda)(1+\lambda^2-2\lambda-4)+2(2+2\lambda)$$

$$+2(2+2\lambda)=0$$

$$(1-\lambda)(\lambda^2-2\lambda-3)+8(1+\lambda)=0$$

$$(1-\lambda)(\lambda+1)(\lambda-3)+8(1+\lambda)=0$$

$$(\lambda+1)\left[(1-\lambda)(\lambda-3)+8\cancel{(1+\lambda)}\right]=0$$

$$(\lambda+1)\left[\lambda-3-\lambda^2+3\lambda+8\right]=0$$

$$-(\lambda+1)(\lambda^2-4\lambda-5)=0$$

$$(\lambda+1)(\lambda+1)(\lambda-5)=0$$

$$\lambda=-1,-1,5$$

$$\lambda_1=-1,\ \lambda_2=-1,\ \lambda_3=5$$

So, 3 eigenvectors.

- So, eigen vector for λ_1 k:-

M T W T F S

تاریخ: ___/___/___

$(A - \lambda_1 I) k = 0$

$(A + I) k = 0$

$$\begin{bmatrix} 2 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ -2 & 2 & -2 & 0 \\ 2 & -2 & 2 & 0 \end{array}\right]$$

$\left.\begin{array}{l} R_3 - R_1 \\ R_2 + 1 \end{array}\right\}$ $\left[\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\frac{1}{2} R_1$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$k_1 - k_2 + k_3 = 0$

$k_1 = k_2 - k_3$

if $\lambda_1 = k_2$, $k_3 = s$

$-\lambda \therefore -(-1)$

$+1$

$\{$

M T W T F S

تاریخ: ___/___/___

$k_1 = r - s$

$$K = \begin{bmatrix} r-s \\ r \\ s \end{bmatrix}$$

Let $r = 1$, $s = 0$

$$K = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Put in (2) :-

$X_1 = K e^{\lambda_1 t}$

$$\boxed{X_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{-t}} \text{ — (i)}$$

Same for $\lambda_2 = -1$ just P differ

$$\cancel{\boxed{X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{-t}}}$$

$X_2 = P e^{\lambda_2 t}$

$$P = \begin{bmatrix} r-s \\ r \\ s \end{bmatrix}$$

Let

$r = 0$, $s = 1$

$$P = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} e^{-t}} \text{ — (ii)}$$

Now for $\lambda_3 = 5$

By using (4)

$$(A - \lambda_3 I)q = 0$$

$$(A - 5I)q = 0$$

$$\begin{bmatrix} -4 & -2 & 2 \\ -2 & -4 & -2 \\ 2 & -2 & -4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augementrin matrin,

$$\left[\begin{array}{ccc|c} -4 & -2 & 2 & 0 \\ -2 & -4 & -2 & 0 \\ 2 & -2 & -4 & 0 \end{array}\right]$$

reduce tie echolon form.

$R_1 \leftrightarrow R_3$ or R_{13}

$$\sim \left[\begin{array}{ccc|c} 2 & -2 & -4 & 0 \\ 4 & -2 & 2 & 0 \\ 4 & -2 & 2 & 0 \end{array}\right]$$

$\frac{1}{2}R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ -2 & -4 & -2 & 0 \\ -4 & -2 & -2 & 0 \end{array}\right]$$

M T W T F S تاریخ: ___/___/___

$$R_2+2R_1 \left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -4 & -2 & 2 & 0 \end{array}\right]$$

$$\left.\begin{array}{l} R_2+2R_1 \\ R_3+4R_1 \end{array}\right\} \left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & -6 & -6 & 0 \\ 0 & -6 & -6 & 0 \end{array}\right]$$

$$-\tfrac{1}{6}R_2 \left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -6 & -6 & 0 \end{array}\right]$$

$$\left.\begin{array}{l} R_3+6R_2 \\ R_1+R_2 \end{array}\right\} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$q_1 - q_3 = 0$$
$$q_2 + q_3 = 0$$
$$q_1 = q_3$$
$$q_2 = -q_3$$

if $q_3 = 1$

$$q_1 = 1$$
$$q_2 = -1$$

$\left[\frac{1}{2}+2\right.$ $\quad \frac{1+4}{2}$

$\frac{1}{2}+1 \quad \frac{1+2}{2}$

$\frac{1}{5}+5$

M T W T F S
تاریخ: ___/___/___

$$a = \begin{bmatrix} \lambda \\ -\lambda \\ \lambda \end{bmatrix} \quad \text{if } \lambda = 1$$

$$a \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad X_3 = a e^{\lambda_3 t}$$

$$\boxed{X_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} e^{5t}} \quad \text{— (iii)}$$

put (i), (ii) and (iii) in

$$X = C_1 X_1 + C_2 X_2 + C_3 X_3$$

Mutipicity Avoidence :-

To avoid multipicity we do

$$\boxed{(A - \lambda_2 I)\, p = K} \rightarrow \text{non-homo}$$

$$\left[\begin{array}{c|c} \text{———} & K_1 \\ \text{———} & K_2 \\ \text{———} & K_3 \end{array}\right]$$

so solution is also changes

$$\boxed{\begin{aligned} X_2 &= t K e^{\lambda_1 t} + p e^{\lambda_2 t} \\ &\quad t X_1 + p e^{\lambda_2 t} \end{aligned}}$$

Now if third value is same

$$(A - \lambda I_3) Q = P$$

$$Q \left[\begin{array}{c|c} \text{—} & P_1 \\ \text{—} & P_2 \\ \text{—} & P_3 \end{array} \right]$$

$$\boxed{X_3 = \frac{t^2}{2} K e^{\lambda_1 t} + t P e^{\lambda_2 t} + Q e^{\lambda_3 t}}$$

Exp 7

$$X' = \begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix} X$$

λ: $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 6 \\ 0 & 2-\lambda & 5 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$$

$(2-\lambda)^3 = 0$

$(2-\lambda)(2-\lambda)(2-\lambda)$

$\lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 2$

M T W T F S

تاریخ: ___/___/___

Take $\lambda = \lambda_1 = 2$

$$(A - \lambda_1 I)K = 0.$$

$$(A - 2I)K = 0$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 6 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\frac{1}{5}R_2 \left[\begin{array}{ccc|c} 0 & 1 & 6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_1 - 6R_2 \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$k_1 + k_2 + k_3 = 0$$

$$k_2 = 0$$

$$k_3 = 0$$

$$k_1 = \lambda$$

$$K = \begin{bmatrix} \lambda \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ if } 1$$

$$\boxed{X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t}}$$

تاریخ: ___/___/___

M T W T F S

$\lambda - \lambda_2 = 2$

$(A - \lambda_2 I)P = K$

$(A - 2I)P = K$

$$\left[\begin{array}{ccc|c} 0 & 1 & 6 & 1 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{1}{5}R_2$ $$\left[\begin{array}{ccc|c} 0 & 1 & 6 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 - 6R_2$ $$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$P_2 = 1$

$P_3 = 0$

$P_1 = x$

if $x = 1$

$P = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$$x_2 = -tke^{\lambda_1 t} + pe^{\lambda_2 t}$$

$$\boxed{x_2 = t\begin{bmatrix}1\\0\\0\end{bmatrix}e^{2t} + \begin{bmatrix}1\\1\\2\end{bmatrix}e^{2t}}$$

Now

$$\lambda = \lambda_3 = 2$$

$$(A - \lambda_3 I)q = P$$

$$(A - 2I)Q = P$$

$$= \left[\begin{array}{ccc|c} 0 & 1 & 6 & 1 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\frac{1}{5}R_2 \left[\begin{array}{ccc|c} 0 & 1 & 6 & 1 \\ 0 & 0 & 1 & 1/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_1 - 6R_2 \left[\begin{array}{ccc|c} 0 & 1 & 0 & -1/5 \\ 0 & 0 & 1 & 1/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$q_2 = -1/5$,

$q_3 = 1/5$, $q_1 = 1$

Ex 8.6 (29-38)

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$$Q = \begin{bmatrix} 1 \\ -1/5 \\ 1/5 \end{bmatrix}$$

$$X_3 = \frac{t^2}{2} K e^{\lambda_1 t} + t P e^{\lambda_2 t} + Q e^{\lambda_3 t}$$

$$\boxed{X_3 = \frac{t^2}{2}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{2t} + \begin{bmatrix} 1 \\ -1/5 \\ 1/5 \end{bmatrix} e^{2t}}$$

Now put in general

$$X = C_1 X_1 + C_2 X_2 + C_3 X_3$$

$$\boxed{\begin{aligned} X = {} & C_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + C_2\left\{ t\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{2t} \right\} \\ & + C_3\left\{ \frac{t^2}{2}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{2t} + \begin{bmatrix} 1 \\ -1/5 \\ 1/5 \end{bmatrix} e^{2t} \right\} \end{aligned}}$$

Ans.