

IT COS 50
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Digital & Logic Design

Assignment # 03

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(3.2)

a) $F(x, y, z) = \Sigma(2, 3, 4, 5, 7)$

$x\backslash y^z$	00	01	11	10
0	0	0	11	1
1	1	1	1	0

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$$= xy' + x'y + yz$$

b) $F(x, y, z) = \Sigma(0, 1, 4, 5, 6, 7)$

$x\backslash y^z$	00	01	11	10
0	1	1	0	0
1	1	1	1	1

$$= x + y'$$

(2)

$$x = 1 + \cos \omega_0$$

c) $F(x, y, z) = \Sigma(0, 1, 2, 4, 5, 6)$

x	y	z	00	01	11	10
0	0	0	1	0	0	1
1	1	1	1	0	0	1
			4	5	7	6

$$= z' + y'$$

d) $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$

x	y	z	00	01	11	10
0	0	0	0	1	1	1
1	1	1	0	1	1	1
			4	5	7	6

$$= z + y$$

e) $F(x, y, z) = \Sigma(1, 3, 5, 7)$

x	y	z	00	01	11	10
0	0	0	0	1	1	0
1	0	1	0	1	1	0
			4	5	7	6

$$= z$$

f) $F(x, y, z) = \Sigma(0, 1, 2, 3, 5, 7)$

a)

$x \backslash yz$	00	01	11	10
0	1	1	1	1
1	0	1	1	0

$$= x' + z$$

(3·4)

a) $F(w, x, y, z) = \Sigma(1, 4, 5, 8, 12)$

$wx \backslash yz$	00	01	11	10
00	0	1	0	0
01	1	1	0	0
11	1	0	0	0
10	1	0	0	0

$$= wy'z' + w'xy' + w'y'z$$

b) $F(w, x, y, z) = \Sigma(4, 6, 9, 11, 12, 14)$

wx	y^2	00	01	11	10
00		0	0	0	0
01		1	0	0	1
11		1	0	0	1
10		0	1	1	0
		4	5	13	15
		12	14	11	10

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$$= xz' + wx'z$$

c) $F(w, x, y, z) = \Sigma(1, 3, 5, 6, 7, 9, 11)$

wx	y^2	00	01	11	10
00		0	1	1	0
01		0	1	1	1
11		0	0	0	0
10		0	1	1	0
		4	5	13	15
		12	14	11	10

$$= x'z + w'x + w'xy$$

d) $F(w, x, y, z) = \Sigma(2, 6, 7, 10, 14, 15)$

$wx \backslash yz$	00	01	11	10
00	0	0	0	1
01	0 ₄	0 ₅	1 ₇	1 ₆
11	0 ₁₂	0 ₁₃	1 ₁₅	1 ₁₄
10	0 ₂	0 ₉	0 ₁₁	1 ₁₀

$$= xy + yz'$$

e) $F(w, x, y, z) = \Sigma(9, 10, 11, 12, 13, 15)$

$wx \backslash yz$	00	01	11	10
00	0	0	0	0
01	0 ₄	0 ₅	0 ₇	0 ₆
11	1 ₁₂	1 ₁₃	1 ₁₅	0 ₁₄
10	0 ₂	1 ₉	1 ₁₁	1 ₁₀

$$= wx'y' + wz + wx'y$$

f) $F(w, x, y, z) = \Sigma(0, 2, 4, 6, 9, 11, 13, 15)$

$wx\backslash yz$	00	01	11	10
00	1 ₀	0 ₁	0 ₃	1 ₂
01	1 ₄	0 ₅	0 ₇	1 ₆
11	0 ₁₂	1 ₁₃	1 ₁₅	0 ₁₄
10	0 ₈	1 ₉	1 ₁₁	0 ₁₀

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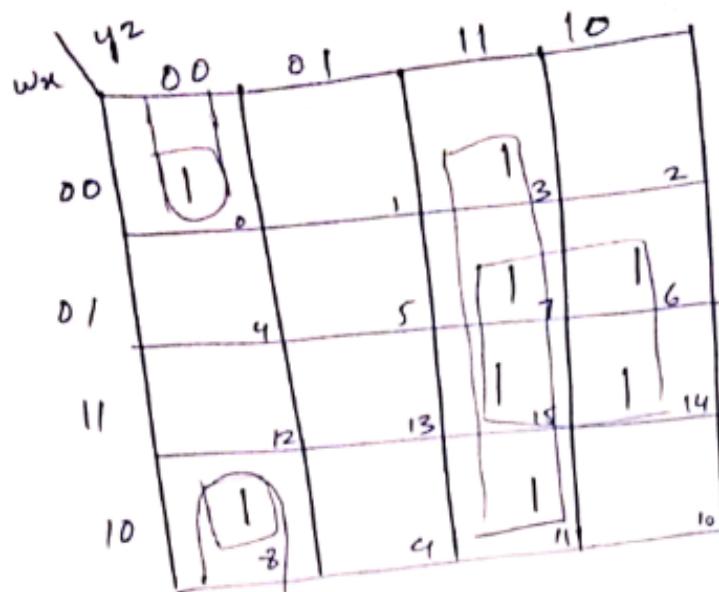
$$= w'x' + wx = 1$$

g) $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 6, 8, 9)$

$wx\backslash yz$	00	01	11	10
00	1 ₀	1 ₁	3	1 ₂
01	1 ₄	5	7	1 ₆
11	1 ₂	0	15	14
10	1 ₈	1 ₉	11	10

$$= w'y' + w'z'$$

$$h) F(w, x, y, z) = \Sigma(0, 3, 6, 7, 8, 11, 14, 15)$$



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$$= x'y'z' + yz + xy$$

(3.6)

$$a) ABC'D' + A'B'C + B'C'D' + AB'CD + B'C'D$$



$$= AB' + AC'D' + B'C'$$

$$b) w'x'y' + w'z'y_2 + x'y'z + xy_2 + y'z'$$

$wx \backslash yz$	00	01	11	10
00	1	0	1	0
01	1	0	0	0
11	1	0	{0}	0
10	1	{0}	0	0

$$= y'z' + w'x'z + x'y'$$

$$c) A'B'CD + A'BC + C'D + ABCD + AB'C$$

$AB \backslash CD$	00	01	11	10
00	0	0	1	0
01	0	0	1	1
11	0	0	1	0
10	0	0	{1}	1

$\left. \begin{matrix} B= \\ B=1 \end{matrix} \right\}$

$A=1 \quad \left. \begin{matrix} 11 \\ 10 \end{matrix} \right\}$

$$= CD + A'BC + AB'C$$

7) q) $A'B'C' + A'BD + A'BC'D' + BC'D + ABCD$ (3.6)

	AB	CD	00	01	11	10
00	1	1			0	0
01	1	1			0	
11	0	1	1		0	
10	0	0	0	0	0	0

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$$= A'C' + BD$$

(3.8)

a) $w'x + xz + wyz$

	yz	00	01	11	10
wx	0	1	1	3	2
01	1	1	1	1	0
11	12	13	1	15	14
10	7	9	1	11	10

$$= \sum(4, 5, 6, 7, 11, 13, 15)$$

b) $ABD' + BC'D + CD$

AB\CD	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	1	1	1	1
10	0	0	1	0

$$= \sum(3, 4, 5, 6, 7, 11, 12, 13, 14, 15)$$

c) $x'y'z' + xy + y'z$

x\y\z	00	01	11	10
0	1	1	0	0
1	0	1	1	1

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d) $= \sum(0, 1, 5, 6, 7)$

$AB + B'C'D' + BCD + A'C'D'$

AB\CD	00	01	11	10
00	1	0	0	2
01	1	0	1	0
11	1	1	1	1
10	1	0	0	0

$$= \sum(0, 4, 6, 7, 8, 12, 13, 14, 15)$$

a) (3.10)

$$F(w, x, y, z) = \Sigma(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$$

wz'	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	1	1	1	1
10	1	0	0	1

Essential: wz , ~~$w'z' x'z'$~~ , wx

boolean expression: $wz + wx + x'z'$

b) $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 10, 12, 13, 14, 15)$

CD	00	01	11	10
00	1		1	1
01		1	1	1
11	1	1	1	1
10	1	1	1	1

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Essential: $B'D'$, AB , BD , $A'B'C$

boolean expression: $AB + B'D' + BD + A'B'C$

c) $F(A, B, C, D) = \Sigma(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	0	0
11	1	1	1	1
10	0	0	1	1

Essentials: $AB, AC, A'B'D$

boolean expression: $A'C'D + AB + AC + A'B'D$

Expression: $AC + A'B'D + BC'$

$AB, A'B'D, B'CD, AC'D$

BC', AC

$BC' + AC + A'B'D$

non-Essential: $A'C'D, B'CD, AB, A'B'D$

d) $F(w, x, y, z) = \Sigma(0, 1, 4, 5, 6, 7, 9, 11, 14, 15)$

wx \ yz	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	0	1	1
10	0	1	1	0

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$$F = w'x + w'y' + xy + wx'z$$

Essential $\Rightarrow w'y', xy$

non-Essential $\Rightarrow x'y'z, wyz, w'x, wx'z$

$$e) F(A, B, C, D) = \Sigma(0, 1, 3, 8, 9, 10, 13, 15)$$

AB\CD	00	01	11	10
00	1	1	1	0
01	0	0	1	0
11	0	1	1	0
10	1	1	0	1

$$F = B'C' + AB'D' + ABD + A'CD$$

Essential = $B'C'$, ~~ABD~~ , $AB'D'$

non-essential = $AC'D$, ABD , BCD , $A'CD$, $A'B'D$,

$$f) F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 7, 10, 15) \quad \text{BSE-133}$$

wz\xy	00	01	11	10
00	1	1	1	0
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$F = w'y' + w'x + xy2 + w'y2'$$

Essential = $w'y'$, $w'y2$, $w'y2'$

non-Essential = $w'x$, $w'x'z'$,

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(3.12)

a) $F(A, B, C, D) = \pi(0, 1, 6, 7, 8, 10, 12, 14)$

		CD	00	01	11	10
		AB	00	01	11	10
CD	AB	00	10	0	1	1
		01	11	1	0	0
CD	AB	11	0	1	1	0
		10	0	1	1	0

$$F = \overline{AD} + A'B'C + A'B'C'$$

$$F = A'B'C + A'BC' + AD$$

b) $F(A, B, C, D) = \pi(0, 2, 5, 8, 10, 15)$

		CD	00	01	11	10
		AB	00	01	11	10
CD	AB	00	0	1	2	0
		01	4	5	7	6
CD	AB	11	12	13	15	14
		10	9	8	11	10

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$$= (\underline{B + FD'}) A' \quad \text{In POS form}$$

$$F = (B + D)(AB' + CD')(A'B' + C'D')$$

(3.14)

$$F = A'B'C' + ABCD + A'C'D' + A'BD + ABC + AB'C$$

① By K-map: (in SOP)

		CD	00	01	11	10
		AB	00	1.	1.	1.
		01	1.	1.	1.	1.
		11	1.	0.	1.	1.
		10	0.	0.	1.	1.

$$F = A'C' + CD + AC$$

② By K-map: (in POS) BSE-133

Considering zero pairs:
(Presented by dots)

$$= (A' + C)(A + C'D)$$

③ By simplification:

$$\begin{aligned}
 &= A'B'C' + A'CD + A'C'D' + A'BD + ABC + ABC \\
 &= C'(A'B' + AB) + A'(CD + C'D') + A'BD + ABC \\
 &= A' + C' + A'BD + ABC \\
 &= A'(1 + BD) + C' + ABC \\
 &= A'C' + CD + AC
 \end{aligned}$$

(3.16)

a) $F(A, B, C, D) = A'B'C + A'BC + ABC$

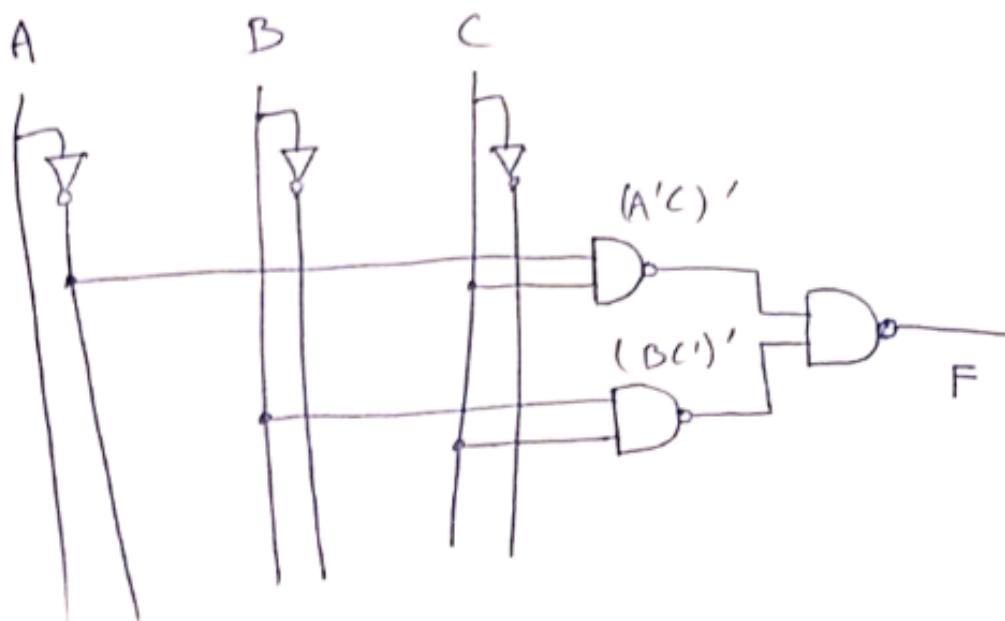
AB\CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	1	1
10	0	0	0	0

~~$A'C + ABC$~~

~~or~~
 $F = A'C + BC$

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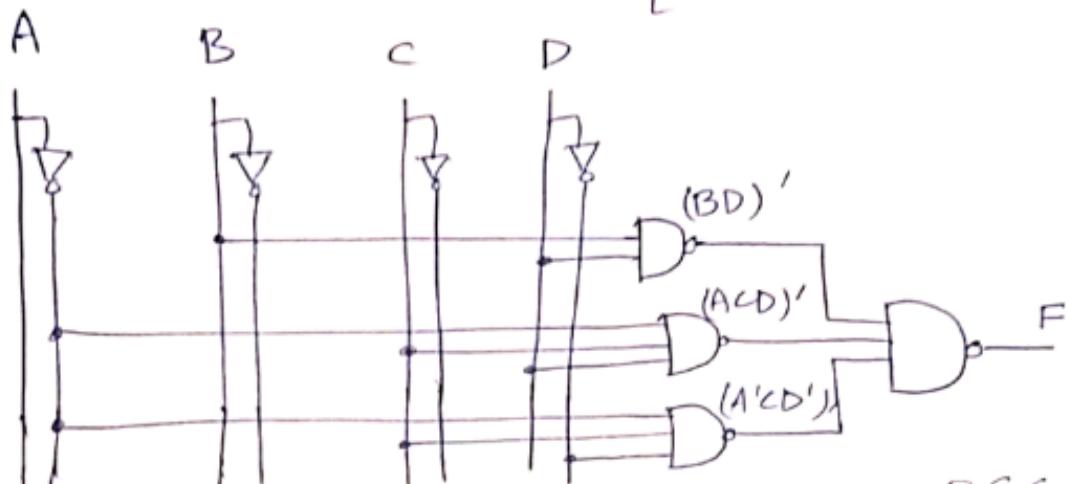
$$= [(A'C + BC)'']'$$
$$((A'C)'.(BC)')'$$



b) $F(A, B, C, D) = A'CD' + A'BD + AB'D + AB'C'D$

AB \ CD		00	01	11	10
00	0	0	0	1	-
01	0	1	1	1	0
11	0	1	1	1	0
10	0	0	1	1	0

$$F = BD + ACD + A'CD' = [(BD)' \cdot (ACD)' \cdot (A'CD')]'$$

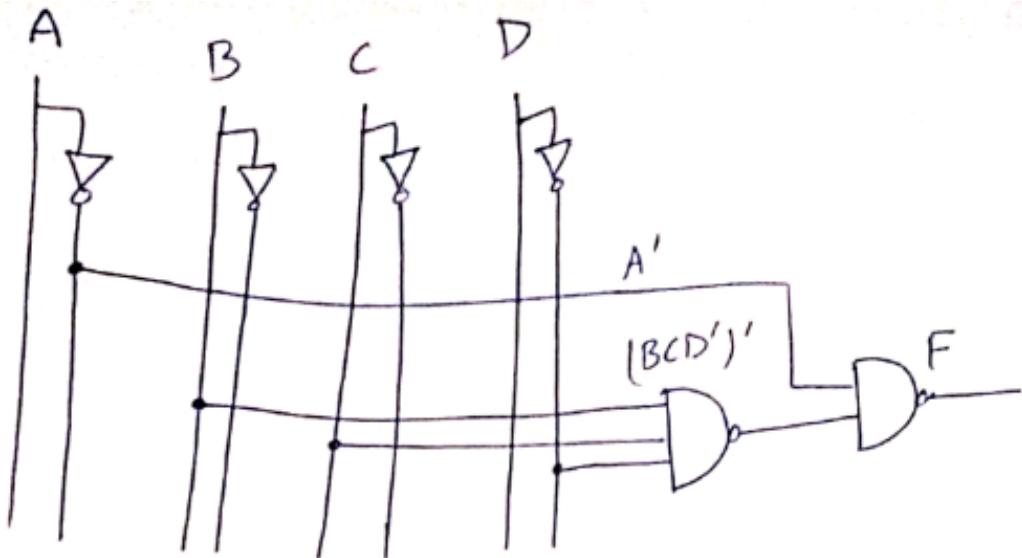


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c) $F(A, B, C, D) = (A' + B' + C)(A' + B' + C')(B' + C' + D)$

AB \ CD		00	01	11	10
00	1	1	1	1	1
01	1	1	1	0	0
11	D ₁₂	D ₁₃	0	0	0
10	1	1	1	1	1

$$\begin{aligned}
 F &= (A'B)(B' + C'D) \\
 &= (A'B')(B' + C'D) \\
 (F')' &= ((A'B') \cdot (B' + C'D))' \\
 &= (A + B + BCD')' \\
 &= ((A + BCD')')' \\
 &= (A' + (BCD')')'
 \end{aligned}$$



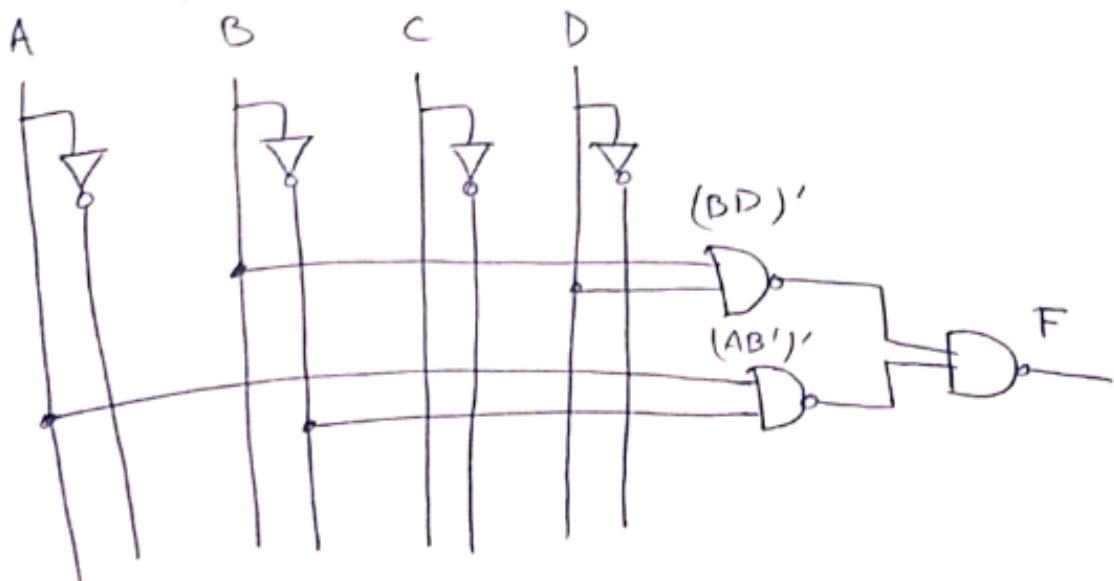
$$d) F(A, B, C, D) = AB'D' + AD + A'BD$$

AB	CD	00	01	11	10
00	0	0	0	0	0
01	0	1	1	0	0
11	0	1	1	0	0
10	1	1	1	1	1

$$F = BD + AB'$$

$$(F')' = [(BD)' \cdot (AB')']'$$

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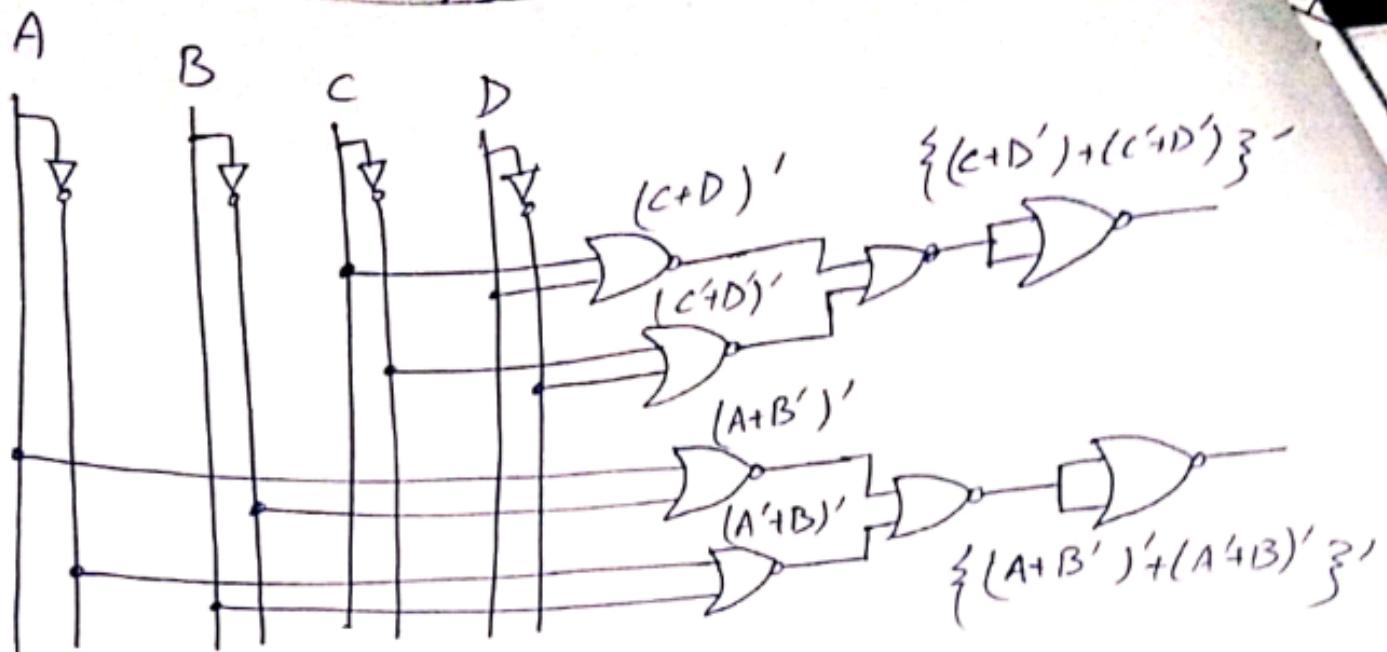
a)

(3.18)

$$\begin{aligned}
 & (A \oplus B)'(C \oplus D) \\
 = & (A'B + AB')'(CD' + C'D) \\
 = & (A'B)'(AB')'(C'D + CD') \\
 = & (A + B')(A' + B)(C'D + CD') \\
 = & (AB + A'B')(C'D + CD') \\
 = & ABC'D + ABCD' + A'B'C'D + A'B'CD' \quad \text{Q}
 \end{aligned}$$

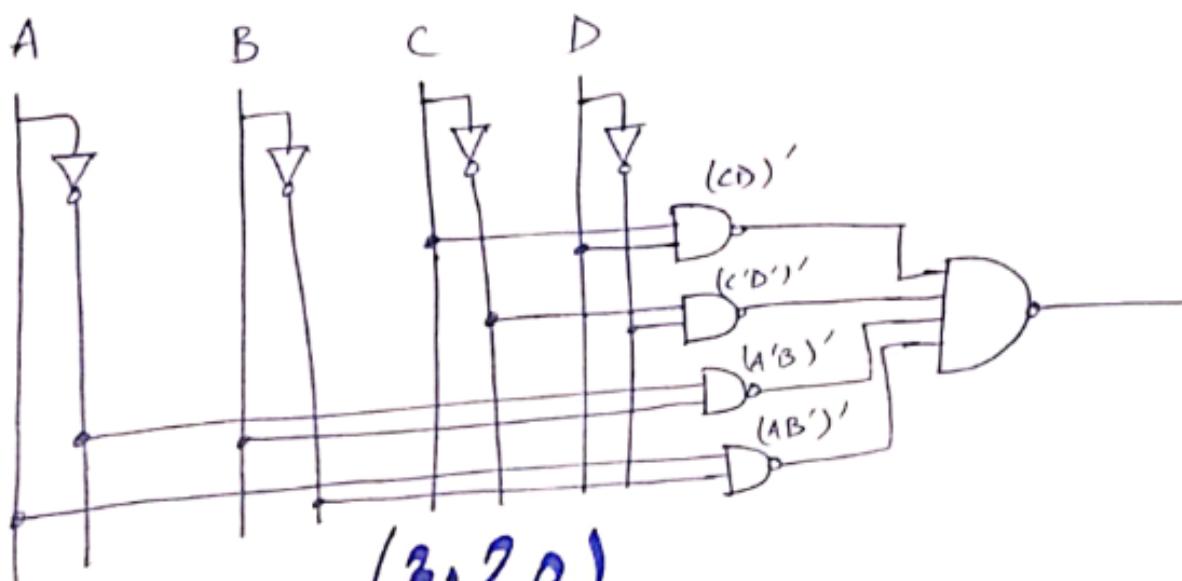
	CD	00	01	11	10
AB	00	0	1	0	1
	01	0	0	0	0
	11	0	1	0	1
	10	0	0	0	0

$$\begin{aligned}
 F &= CD + C'D' + A'B + AB' \\
 (F') &= [(C'D' + CD + A'B + AB')']' \\
 &= [(C+D)(C'+D')(A+B')(A'+B)]' \\
 &= [(C+D)' + (C'+D')' + (A+B')' + (A'+B)]' \\
 &= [\{(C+D)' + (C'+D')\} \cdot \{(A+B')' + (A'+B)\}]' \\
 &= [[(C+D') + (C'+D')]']' + [\{(A+B')' + (A'+B)\}]'
 \end{aligned}$$



b)

$$F = [(CD)' + (C'D')'(A'B')(AB')']'$$



(3.20)

By multiple NOR:

$$F = CD(B+C)A + (BC' + DE')$$

$$F' = [[CD(B+C)A + (BC' + DE')]]'$$

$$= [[\{(CD(B+C)A)'\} + \{((BC' + DE')\}]]'$$

$$= [[\{(ABC'D + ACD)\}' + \{ - \}']]'$$

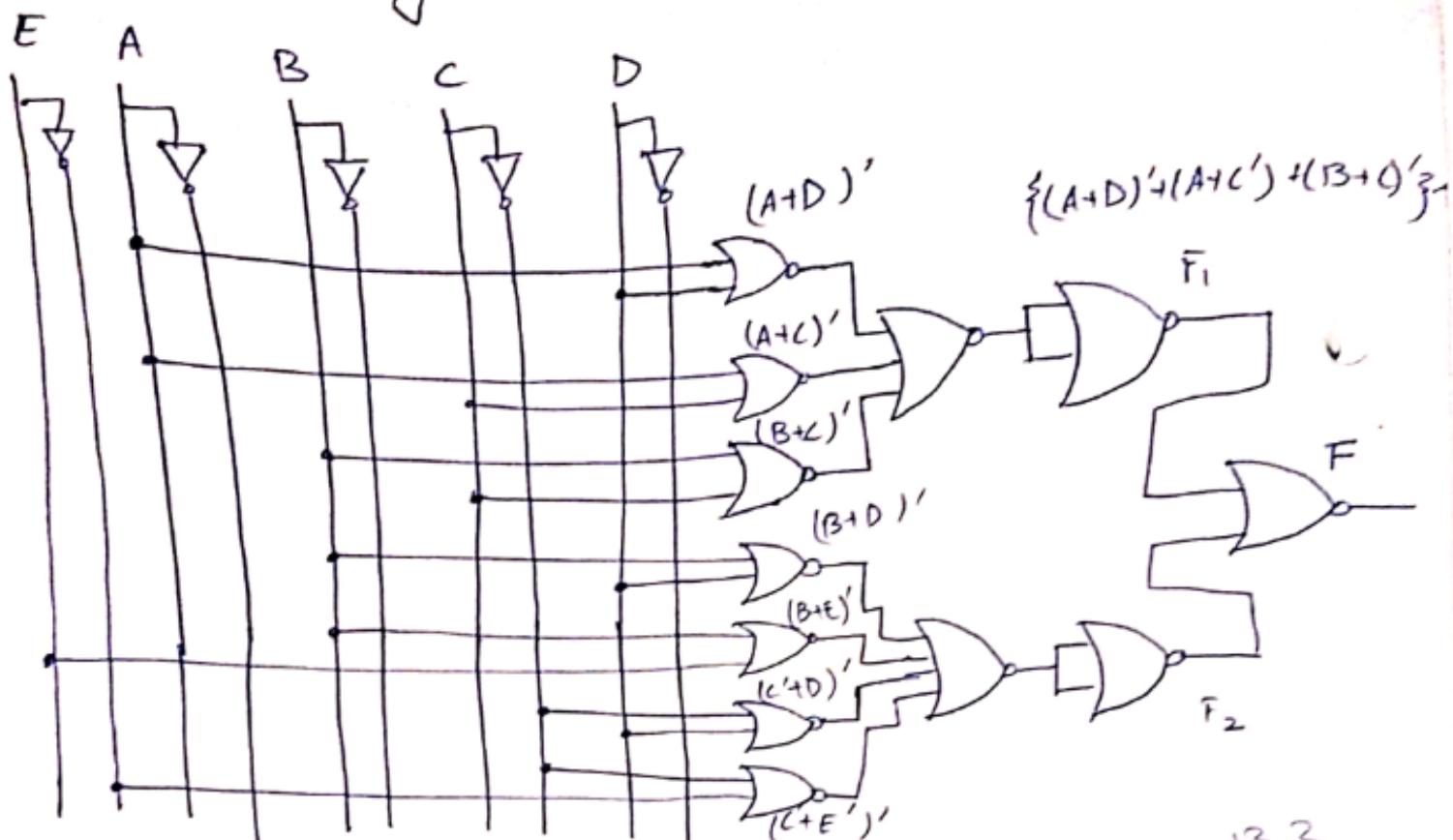
$$= [[\{(CD + (B+C) + A)\}' + \{ - \}']]'$$

$$= [[\{(A+D)(A+C)(B+C)\}' + \{(B+D)' + (B+E)' \\ (C'+D)(C+E)\}']]'$$

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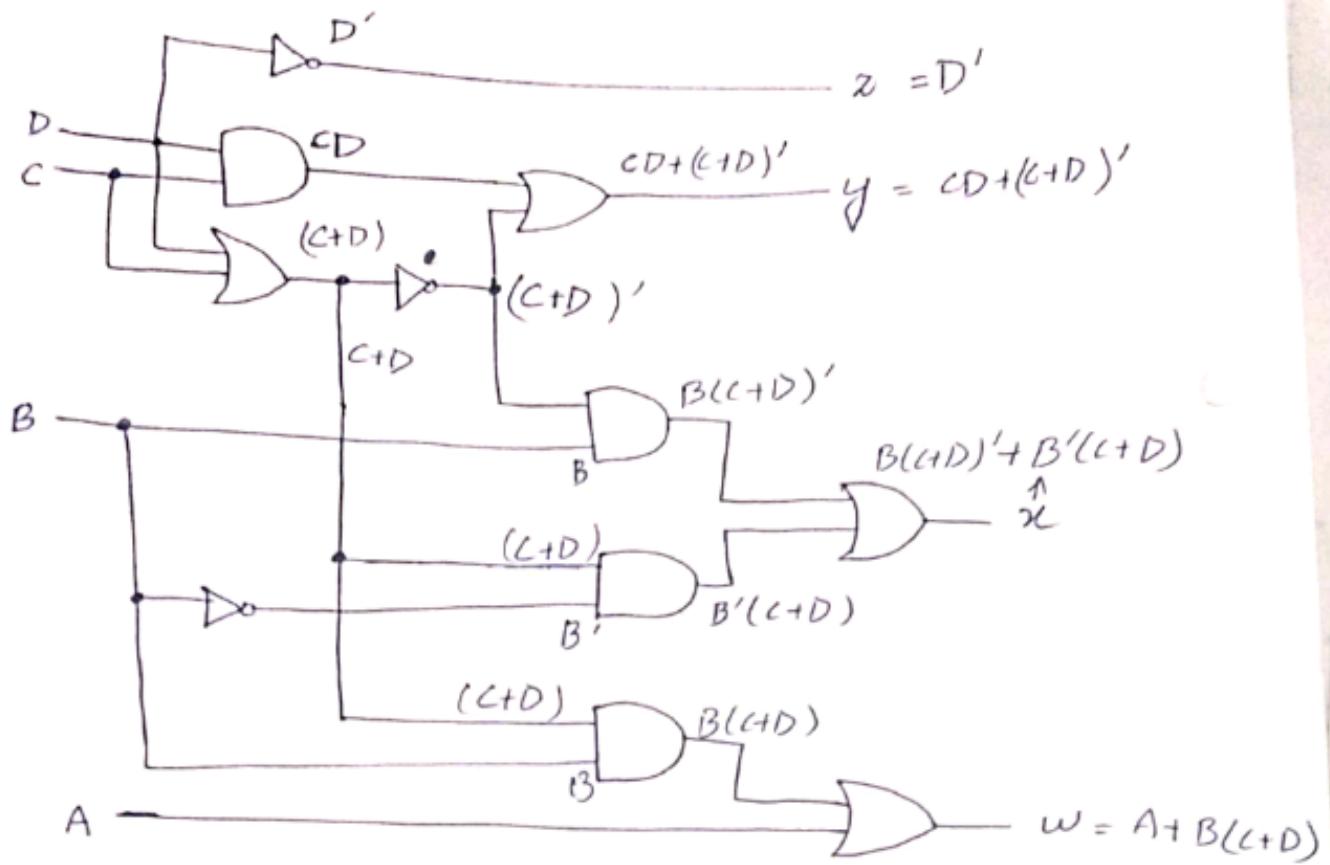
$$F' = \left[\left[\left\{ (A+D)' + (A+C)' + (B+C)' \right\}' + \left\{ (B+D)' + (B+E)' + (C'+E)' \right\}' \right]' \right]'$$

Logic Diagram:



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Apply two level NAND gate to fig 4.4



$$\begin{aligned} Y &= CD + (C+D)' \\ &= [(CD + (C+D)')']' \\ &= [(CD)' \cdot (C+D)'''']' \end{aligned}$$

$$Y' = [(CD)' \cdot (C'D)']' \Rightarrow [(CD')' \cdot (C'D)']'$$

$$x = [(B(C+D') + B'(C+D))']'$$

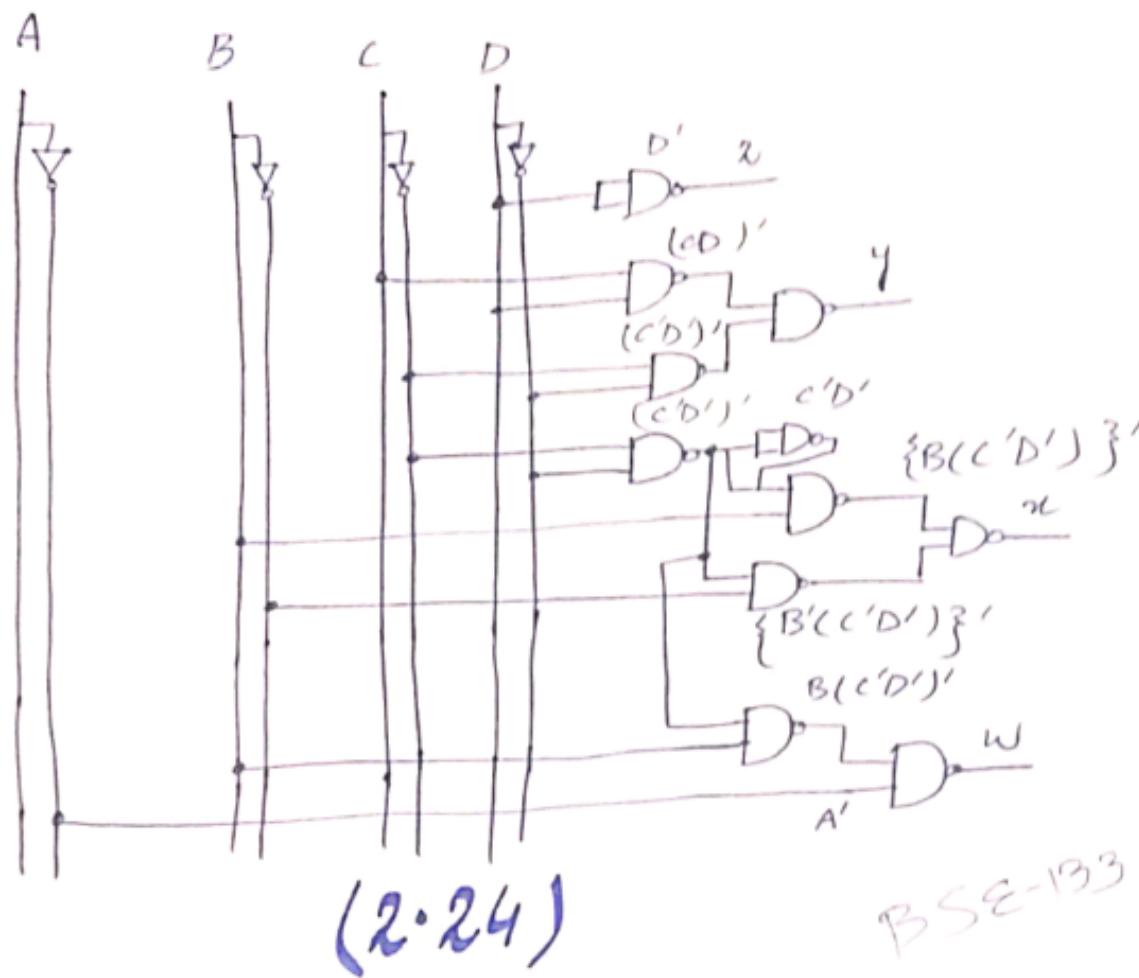
$$= [(B(C+D'))' \cdot (B'(C+D))']'$$

$$= [B'(C'D') \cdot B(C'D')]'$$

$$w = \left[\{A + B(C + D)\} \right]' = \left[\{A + B(C'D')\}' \right]'$$

$$= \left[\{A': B(C'D')\}' \right]'$$

$$z = D'$$



(2.24)

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Implement
NAND, AND, AND-NOR, OR-NAND, NOR-OR

$$F(A, B, C, D) = \sum(0, 4, 8, 9, 10, 11, 12, 14)$$

		CD	00	01	11	10
		AB	00	01	11	10
AB	CD	00	1	0	0	0
		01	1	0	0	0
AB	CD	11	1	1	1	1
		10	1	1	1	1

$$F' = A'D + A'C + BD$$

$$F \Rightarrow \{A'D + A'C + BD\}' \rightarrow b$$

removing complement

$$\Rightarrow (A'D)' \cdot (A'C)' + (BD)' \rightarrow q$$

OR consider 1's:

$$F = C'D' + AB' + AD'$$

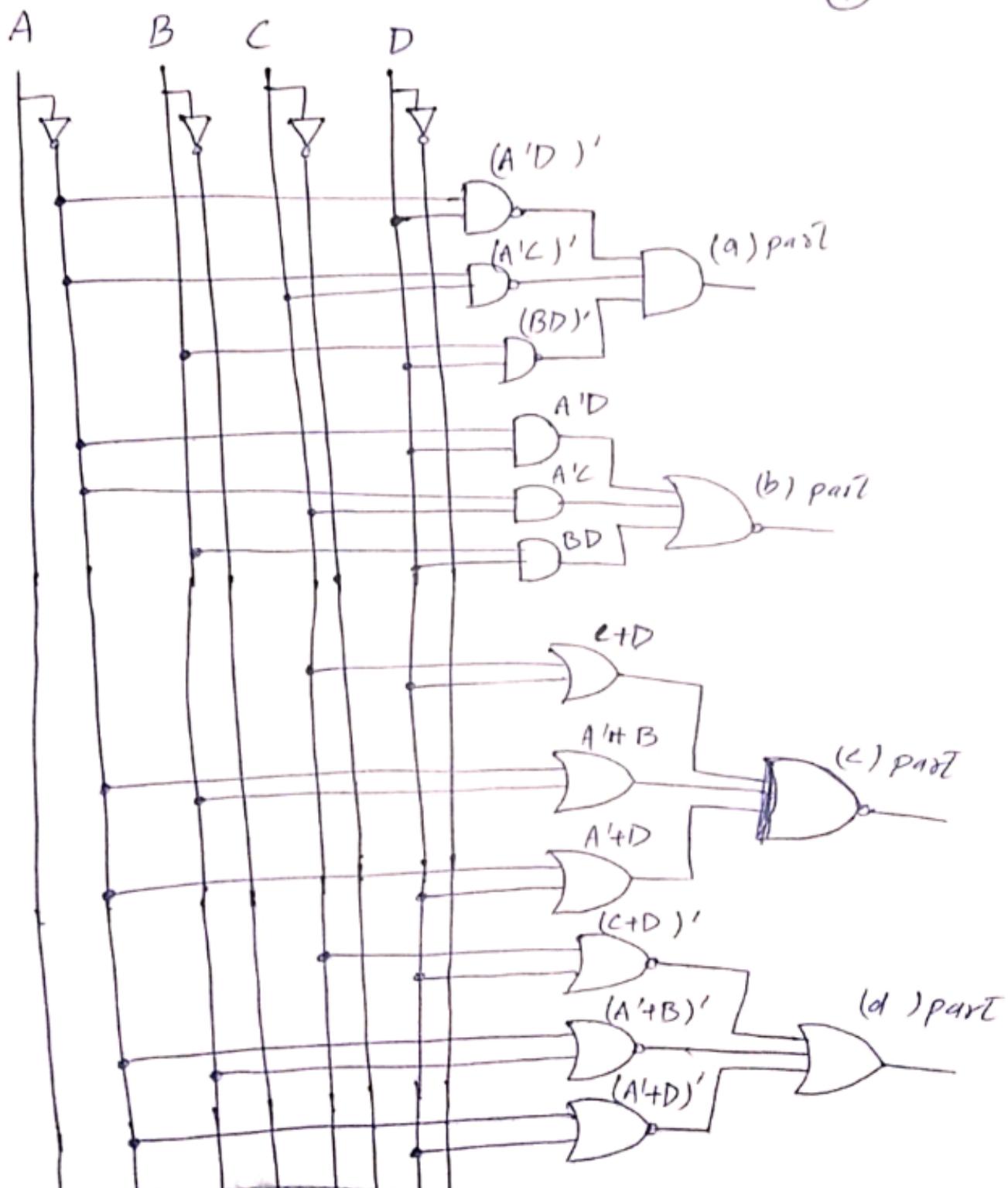
$$\Rightarrow [\{C'D' + AB' + AD'\}']'$$

$$\begin{aligned}
 F &= \left[\{C'D' + AB' + AD'\}' \right]' \\
 &= \left[(C'D')' \cdot (AB')' \cdot (AD')' \right]' \\
 &= \left[(C+D)(A'+B)(A'+D) \right]' \rightarrow \text{OR-NAND}
 \end{aligned}$$

$$F = C'D' + AB' + AD'$$

$$(C+D)' + (A'+B)' + (A'+D)' \rightarrow \text{NOR-OR}$$

(d)



(2.26)

Find SOP from $F = fg$

$$f = abc' + c'd + a'cd' + b'cd'$$

$$g = (a+b+c'+d')(b'+c'+d)(a'+c+d')$$

By first converting 'g' to SOP

$$g' = \bar{a}\bar{b}'\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d}' + \bar{a}\bar{c}\bar{d}$$

Now sketching K map of f & g'

		cd					
		00	01	11	10		
ab	00	0.	1.	0.	1.		
	01	0.	1.	0.	1.		
11	1.	1.	0.	0.	0.		
10	0.	1.	0.	1.	0.		

		cd					
		00	01	11	10		
ab	00	1.	1.	0.	1.		
	01	1.	1.	1.	0.		
11	1.	0.	1.	0.	0.		
10	1.	0.	1.	1.	1.		

In case of complement consider '1' as zeroes.

Take common pairs:

$$F = fg \Rightarrow a'c'd + abc'd' + b'cd'$$

(2.28)

3 bit parity generator using odd parity

A	B	C	P
0	0	0	1.
0	0	1	0.
0	1	0	0.
0	1	1	1.
1	0	0	0.
1	0	1	0.
1	1	0	1.
1	1	1	0.

0 - odd
1 - even

		BC					
		00	01	11	10		
A	0	1.	0.	1.	0.		
	1	0.	1.	0.	1.		

$$\begin{aligned}
 P &= A'B'C' + A'BC + AB'C + ABC \\
 &= A'(B'C' + BC) + A(B'C + BC') \\
 &= A'(B \oplus C) + A(B \oplus C')
 \end{aligned}$$

$$P = A \oplus B \oplus C - 1$$

4 bit parity checker using odd parity

A	B	C	P	P Checker
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	1	0
1	0	1	0	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

0-add
1-even

		00	01	11	10	
		AB	00	01	11	
		00	1_{00}	0_{01}	1_{11}	0_{10}
		01	0_{04}	1_{05}	0_{17}	1_{16}
		11	1_{12}	0_{13}	1_{15}	0_{14}
		10	0_{28}	1_{29}	0_{31}	1_{30}

$$\begin{aligned}
 &= A'B'C'D' + A'B'CP + A'B'C'P + A'BCP' + \\
 &+ ABC'P' + ABCP + AB'C'P + A'B'CP, \\
 &= A'B'(C'D' + CP) + A'B(C'D' + CP') + AB(CD' + CP) \\
 &+ AP(CD' + CP'), \\
 &= A'B'(COP) + A'B(C\oplus P) + AB(COP) + AB'(C\oplus P)
 \end{aligned}$$

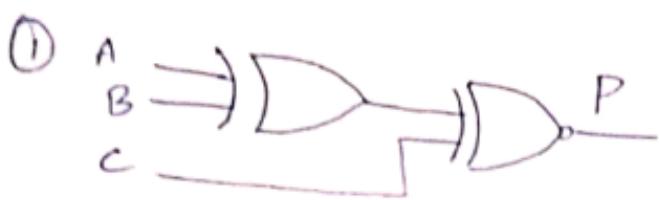
$$\begin{aligned}
 &= (A'B' + AB)(COP) + (A'B + AB')(COP) \\
 &= (AOB)(COP) + (A\oplus B)(COP) \\
 C &= (AOB) \odot (COP) - ②
 \end{aligned}$$

Equations:

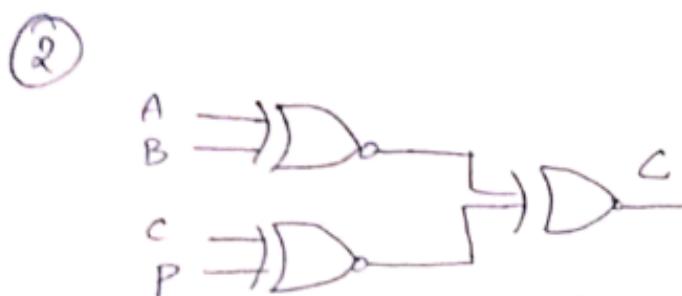
$$P = A \oplus B \odot C$$

$$C = (AOB) \odot (COP)$$

$$\begin{aligned}
 &\textcircled{1} A'B' + AB \\
 &AOB \\
 &\textcircled{2} \downarrow XNOR \\
 &A'B + AB' \\
 &A \oplus B \\
 &\downarrow XOR
 \end{aligned}$$



BSE-B3



(3.30)

Implement XOR & AND gates:

$$F = AB'CD' + A'BCD' + A'B'C'D + A'B'C'D'$$

Taking Common

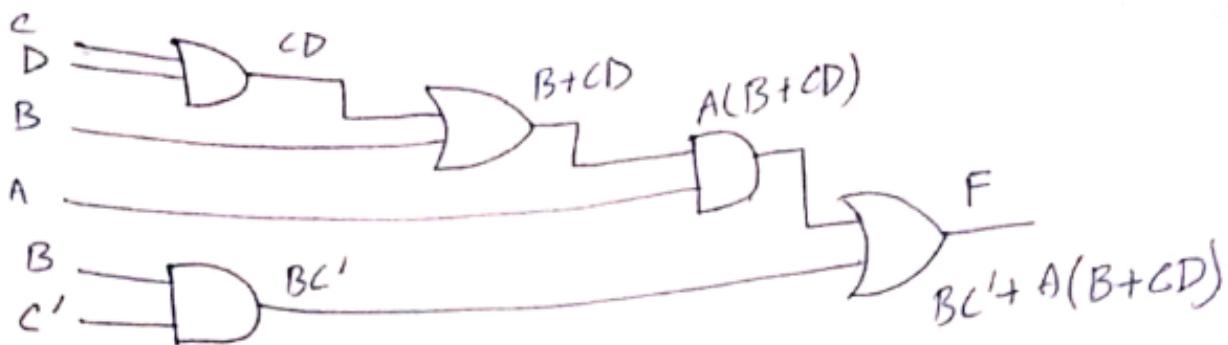
$$= AB'(CD' + C'D) + A'B(CD' + C'D)$$

$$= (AB' + A'B)(CD' + C'D)$$

$$= (A \oplus B)(C \oplus D)$$

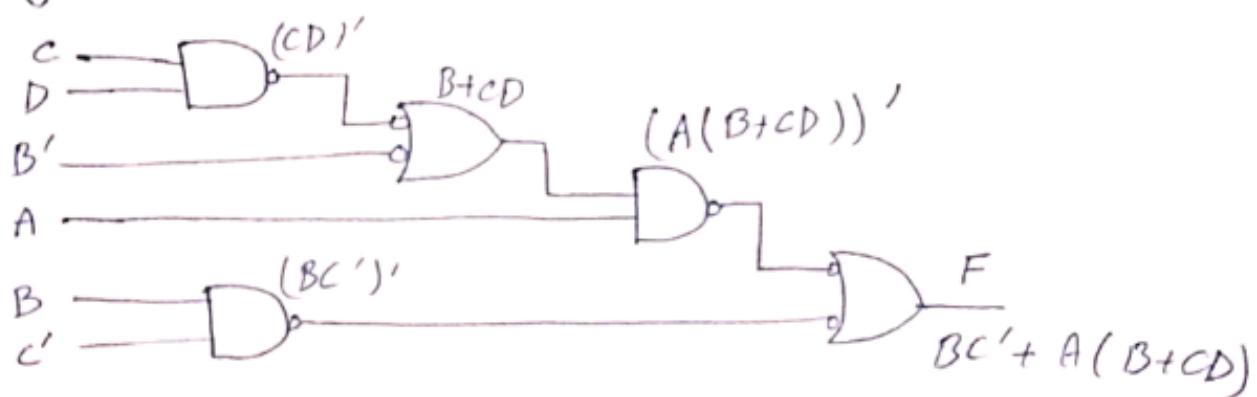
a) Fig 3.32

Write equation (boolean)



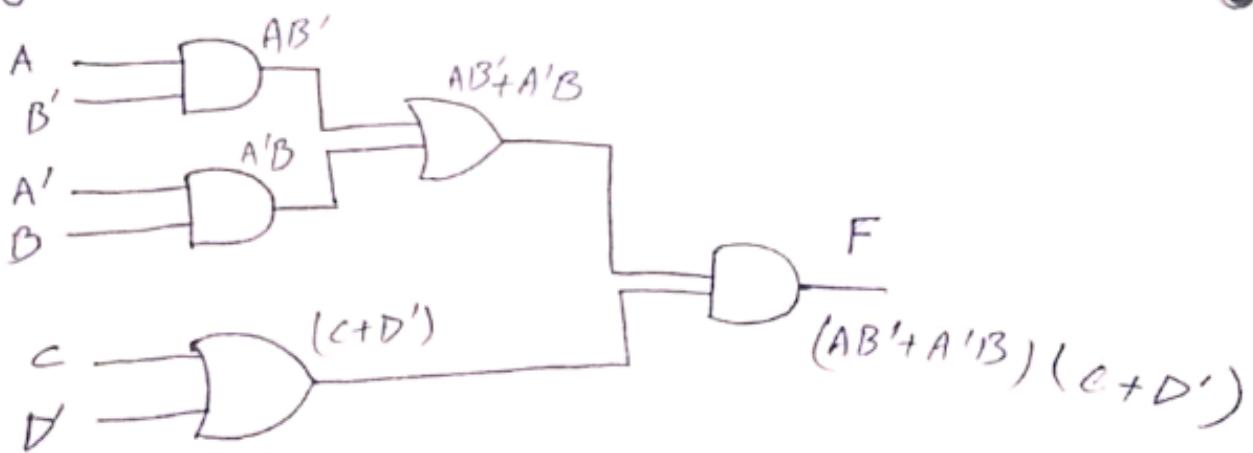
Boolean Expression: $BC' + A(B+CD)$

b) Fig 3.20 (b)



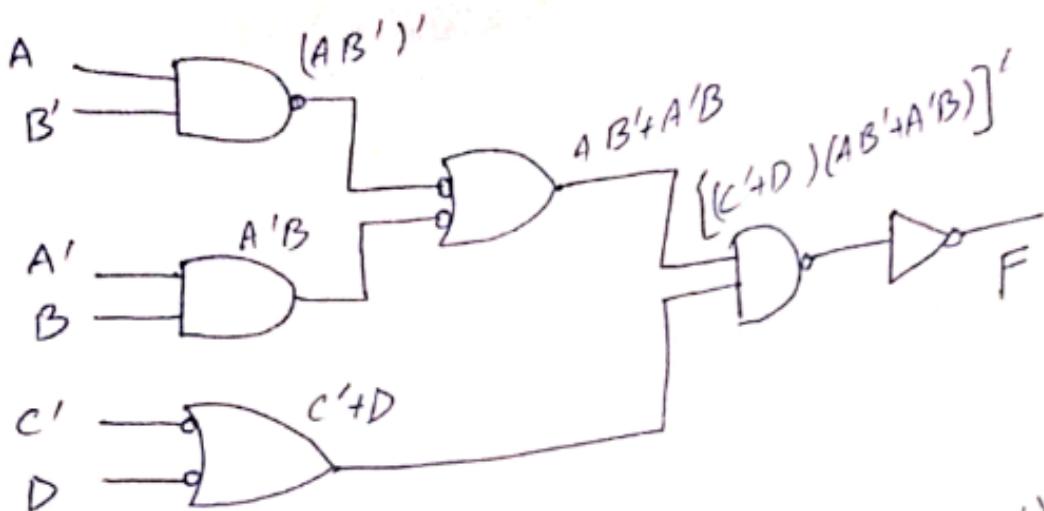
Boolean Expression: $BC' + A(B+CD)$

c) Fig 3.21 (a)



Boolean Expression: $(AB' + A'B)(C+D')$

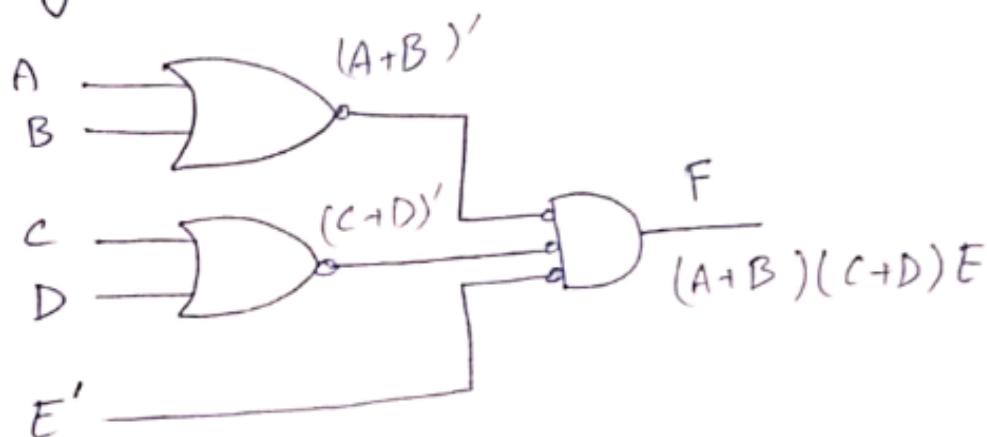
d) Fig 3.21 (b)



Boolean expressions: $(AB' + A'B)(C'D)$

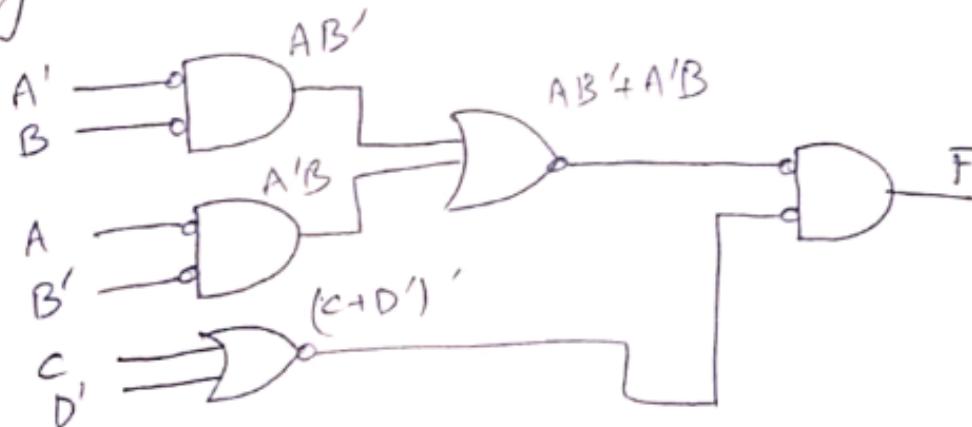
e) Fig 3.24

BSSE-133



Boolean Expression: $(A+B)(C+D)E$

f) Fig 3.25



Boolean Expression: $(AB' + A'B)(C+D')$

3.34

a) $(A+B')C'(C+D)$

simplify using K-map

		CD	00	01	11	10
		AB	00	01	11	10
00	00	0	0	1	1	1
		1	0	0	0	0
01	01	0	1	0	1	0
		1	0	1	1	0
11	11	0	0	1	1	1
		1	0	0	1	1
10	10	0	0	1	1	1
		1	0	1	1	1

BSE-133

$$= C' (A+B')$$

b) $(C'D + BCD + CD')(A'+B)$

$$= A'C'D + A'BCD + A'CD' + BC'D + BCD + BCD'$$

$$\Rightarrow BD + A'C'D + A'CD'$$

		CD	00	01	11	10
		AB	00	01	11	10
00	00	0	1	0	1	1
		1	0	1	1	1
01	01	0	1	1	1	1
		1	0	1	1	1
11	11	0	1	1	1	1
		1	0	1	1	1
10	10	0	1	0	1	1
		1	0	0	1	1

		CD	00	01	11	10
		AB	00	01	11	10
00	00	0	1	0	1	1
		1	0	1	1	1
01	01	0	1	1	1	1
		1	0	1	1	1
11	11	0	1	1	1	0
		1	0	1	1	0
10	10	0	0	0	0	0
		1	0	0	0	0

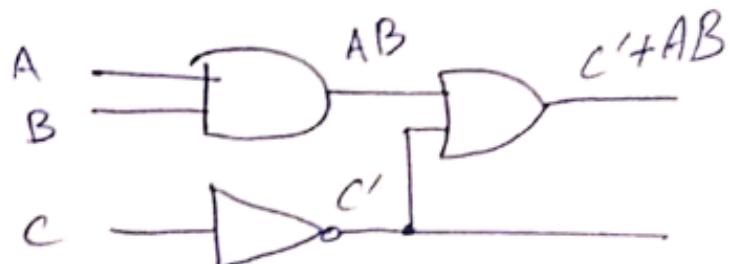
c) $(AB + C)D + B'C$

$\sim AB D + CD + B'C$

Already simplified

(3.40)

Write equation from logic diagram



BSE-133

Boolean Expression: $C' + AB$