Chapter 2:-

Probability

Section 2.3:- Counting Sample Points

Exercise:-

2.21 Solution:-

Number of sightseeing tours: $n_1 = 6$
Number of days: $n_2 = 3$
By the rule of multiplication: $n_1 \times n_2 = 6 \times 3 = 18$
Thus, a person can arrange to go on a sightseeing tour in **18 ways**.

2.22 Solution:-

Types of blood: $n_1 = 8$ ($AB^+, AB^-, A^+, A^-, B^+, B^-, O^+, O^-$)
level of blood pressure: $n_2 = 3$ (low, normal high)
By the rule of multiplication: $n_1 \times n_2 = 8 \times 3 = 24$
The number of ways in which a patient can be classified are **24**.

2.23 Solution:-

Since a die consists of 6 outcomes so $n_1 = 6$ and
Number of Alphabets in English are 26 so $n_2 = 26$
By the rule of multiplication: $n_1 \times n_2 = 6 \times 26 = 156$
Thus, there are **156 sample points** in the Sample Space.

2.24 Solution:-

Types of students: $n_1 = 4$ (freshman, sophomores, juniors, seniors)
Gender categories: $n_2 = 2$ (male, female)
By the rule of multiplication: $n_1 \times n_2 = 4 \times 2 = 8$
The total number of possible classifications for the students are **8**.

2.25 **Solution:-**

Number of different styles: $n_1 = 5$
Number of distinct colors: $n_2 = 4$
By the rule of multiplication: $n_1 \times n_2 = 5 \times 4 = 20$
Thus, the store will have **20 pairs** on display.

2.26 **Solution:-**

Note: The rule of multiplication considers all the possible pairs between the outcomes of 2 or more experiments. In this question we have to select 5 rules out of 7 so we will use either permutation or combination. Since there is no ordering in rules so combination will be used.

Total no. of rules: $n = 7$
No. of rules adopted: $r = 5$
Since the no. of combination of objects taken r at a time is:

$$^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

Here, we have $^7C_5 = \binom{7}{5} = \frac{7!}{5!(7-5)!} = \frac{7!}{5!\,2!}$

$$= \frac{7 \times \cancel{6}^{3} \times \cancel{5!}}{\cancel{5!} \times \cancel{2}}$$

$$= 21$$

a) If the person presently violates all 7 rules then he has to follow all 7 rules out of which 5 can be adopted in $^7C_5 =$ **21 ways**

b) If the person never drinks and always eats breakfast then it means he already follows 2 rules. Now he has to follow 5 $(7-2=5)$ rules out of which 3 $(5-2=3)$ rules can be adopted in $^5C_3$ ways.

By $^nC_r = \frac{n!}{r!(n-r)!}$, we have $^5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!\,2!}$

$$= \frac{5\times4\times3!}{3!\times2} = \textbf{10 ways.}$$

2.27 **Solution:-**

Number of home designs: $n_1 = 4$
Number of heating systems: $n_2 = 3$
Garage or carport: $n_3 = 2$
Patio or screened porch: $n_4 = 2$
By the generalized rule of multiplication: $n_1 \times n_2 \times n_3 \times n_4$
$= 4\times3\times2\times2 = 48$

So, **48 plans** are available for this buyer.

2.28 **Solution:-**

Number of manufacturers: $5 = n_1$
Forms of drug: $n_2 = 3$ (liquid, tablet, capsule)
Levels of strength of drug: $n_3 = 2$ (regular, extra)
By the generalized rule of multiplication: $n_1 \times n_2 \times n_3$
$= 5\times3\times2 = 30$.

Thus, the doctor can prescribe the drug for a patient in **30 ways**.

2.29 **Solution:-**

a

Number of race cars tested: $n_1 = 3$
Brands of gasoline: $n_2 = 5$
Number of test sites: $n_3 = 7$
Number of drivers: $n_4 = 2$
By the generalized rule of multiplication:
$n_1 \times n_2 \times n_3 \times n_4 = 3\times5\times7\times2 = 210$

Thus, **210 test runs** are needed for the fuel economy study.

b

2.30 **Solution:-**

One question can be answered in 2 ways (true-false), so $n_1 = 2$. Second question can also be answered in 2 ways (true-false) so $n_2 = 2$. Since there are 9 questions each having 2 choices, so by the generalized rule of multiplication:

$$n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^9 = 512$$

Thus, a true-false test consisting of 9 questions can be answered in **512 ways**.

2.31 **Solution:-**

By given information the lisence number contained 3 letters (RLH) and 3 digits, the first of which was a 5. So, there are 2 unknown digits in last of the lisence number.

i-e R L H 5 ? ?

Since all 3 digits are different and we have total digits 9 (0,1,2,3,4,5,6,7,8,9), so second last position can be filled in 8 ways $(9-1=8)$ (because 5 is already used)

and last position can be filled in 7 ways $(9-2=7)$ because 5 and one digit at 2nd last position is already used.

By the generalized multiplication rule:

$$n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 = 1 \times 1 \times 1 \times 1 \times 9 \times 8 = 72$$

1 way occurs only when a number is fixed e.g at first position only R can be placed.

Thus the maximum no. of automobile registrations that the police may have to check are **72**.

2.32 **Solution:-**

a Since the number of permutations of n objects taken n at a time is $n!$ so 6 people be lined up in $n! = 6! = 720$ ways.

b) Note: If the number of specific persons, insist on following each other we always consider them as one group.
Here, 3 specific persons, among 6, insist on following each other so there are only 4 persons to be arranged

e.g $\underbrace{A, B, C}_{G = 1\ \text{Group}}, \underbrace{D, E, F}_{3\ \text{persons}}$ (6 persons)

$3+1 = 4 \Rightarrow G, D, E, F$

Within group those 3 persons can be arranged in 3! ways and three individuals and 1 group can be arranged in 4! ways. All possible arrangements are $4! \times 3! = 24 \times 6 = $ **144**.

c) Similar as in (b), if 2 specific persons, among 6, insist to follow each other then there are 5 objects to be arranged. All possible arrangements are $5! \times 2! = 240$. Since we are required to find the no. of ways if 2 specific refuse to follow each other

$\underbrace{A, B}_{1\ \text{Group} = G}, \underbrace{C, D, E, F}_{4\ \text{persons}}$

then Total possible arrangements − no. of ways if 2 persons sit together, among 6

$= 6! - 240 = 480$ **ways**.

2.33 Solution:-

a) Possible answers for 1st question: $n_1 = 4$
Possible answers for 2nd question: $n_2 = 4$
Similarly for question 3, 4 and 5 we have $n_3 = n_4 = n_5 = 4$
By the generalized rule of multiplication

$$n_1 \times n_2 \times n_3 \times n_4 \times n_5 = 4 \times 4 \times 4 \times 4 \times 4 = 45 = 1024$$

Thus, a student can check off one answer to each question in **1024 ways**.

b) It is given that only 1 answer is correct, out of 4 possible answer so there are 3 wrong answers for each question.

Number of wrong answers for 1st question : $n_1 = 3$
Number of wrong answers for 2nd question : $n_2 = 3$
Similarly, for 3rd, 4th and 5th question : $n_3 = n_4 = n_5 = 3$
By the generalized rule of multiplication

$$n_1 \times n_2 \times n_3 \times n_4 \times n_5 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5 =$$

Thus, the student can get wrong answer in **243** ways.

2.34 (a) Since the word COLUMNS contains 7 letters so no. of distinct permutations will be $n! = 7! =$ **5040**

b) Fixing the first letter as M, remaining 6 letters can be arranged in $n! = 6! =$ **720** ways.

2.35 9 houses are to be placed in 9 lots so there are $n! = 9! =$ **362880** possible ways.

2.36 Solution:-

a) Zero cannot be placed at hundreds position (because 0 will make it 2 digit number) So for first place we have 6 choices and for the tens position, we have 6 choices (including zero now) having 5 digits for the units position. So there are $6 \times 6 \times 5 = 180$ three digit numbers.

No. of choices: 6 6 5 → 2 digits are already used, remaining 5 left

6: because 0 cannot be placed here
6: remaining 6 digits (including zero) after filling 1st position

b) For odd numbers, units place can be filled by using any of the 3 given odd numbers. For hundreds place, we have 5 choices (excluding zero and 1 digit used at units place) and for tens position, we have 5 choices (2 digits used at 1st and last place)

No. of choices: 5 5 3 → any of given odd no's

1, 3, 5

5: excluding zero and 1 digit used
5: excluding 2 digits used at 1st & last place

c) In given digits, maximum 3 digit number can be 666. The numbers greater than 330 will be in series of 300, 400, 500 and 600. In case 1, consider the series of 400, 500 and 600 and in case 2, consider the series of 300, greater than 330, each digit can be used only once.

Case 1

4,5,6

choices: 3 6 5

6 → one digit used at first place

5 → 2 digits used out of 7 digits

Case 2

3 4,5,6

1 3 5 → out of 7, we have used 2 digits

1 → Series starting from 340

3 → greater than 3 no's will come only

In numbers greater than 330, 4,5 or 6 can be used in hundreds positions and there remains 6 and 5 choices respectively for the tens and units positions. This gives $3 \times 6 \times 5 = 90$, three digit numbers beginning with a 4,5 or 6.
If a 3 is used in the hundreds position, then a 4,5 or 6 must be used in the tens position leaving 5 choices for the units position. In this case there are $1 \times 3 \times 5 = 15$, three digit number begin with a 3. So the total no. of digits that are greater than 330 is $90 + 15 =$ **105**.

2.37 Solution:-

The first seat must be filled by any of 5 girls and second seat must be filled by any of 4 girls. If the boys and girls must alternate then the total no. of ways to sit in row are $5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 =$ **2880**

alternate seats

G(5) G(4) G(3) G(2) G(1) → 5 girls

B(4) B(3) B(2) B(1) → 4 boys.

2.38 **Solution:-**

a) Four married couples can be seated on 8 seats in the same row with no restrictions in

$n! = 8! = 40{,}320$ **ways**

b) Four married couples on 8 seats, in such a way that each couple is to sit together, can be seated in $2^4 \times 4!$ ways $= 16 \times 24 =$ **384 ways**

One couple can sit in 2 ways (Male, Female or Female, Male)

So 4 couples can sit in $2 \times 2 \times 2 \times 2 = 2^4$ ways on 8 seats in $4!$ ways $\underbrace{\square\square}_{\text{couple } 1}\ \underbrace{\square\square}_{\text{couple } 2}\ \underbrace{\square\square}_{\text{couple } 3}\ \underbrace{\square\square}_{\text{couple } 4}$

c) 4 men can sit together to the right of all women in $4!$ ways and 4 couples can be seated in $4!$ ways. Total no. of ways $= 4! \times 4! =$ **576 ways**

2.39 **Solution:-**

a) There are 8 finalists and the possible orders in which all 8 finalists can finish the spelling bee is $n! = 8! =$ **40,320 ways**.

b) The possible orders for first 3 positions can be determined in ${}^nP_r$ ways.

$${}^nP_r = \frac{n!}{(n-r)!}, \text{ Here } n = 8,\ r = 3$$

$$\Rightarrow {}^8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}} = \textbf{336 ways}.$$

(Here we used permutation because "first" 3 positions were required - order matters)

2.40 **Solution:-**

5 starting positions on a basketball team be filled with 8 men in ${}^nP_r = \frac{n!}{(n-r)!}$

Here $n=8, r=5$

$$\Rightarrow {}^{8}P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8\times7\times6\times5\times4\times\cancel{3!}}{\cancel{3!}}$$

$$= 6720 \text{ ways}$$

(In 5 "starting" positions, order was specified that's why we used permutation)

2.41 **Solution:-**

If no teacher is assigned to more than one section (restriction, so use permutation), then 6 teachers can be assigned to 4 sections in ${}^{n}P_r$ ways.

$${}^{n}P_r = \frac{n!}{(n-r)!} \quad \text{Here } n=6,\ r=4$$

$$\Rightarrow {}^{6}P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\times5\times4\times3\times\cancel{2!}}{\cancel{2!}}$$

$$= 360 \text{ ways}$$

2.42 **Solution:-**

If each contestant holds only 1 ticket (restriction, so we use permutation) then 3 lottery tickets for first, second and third prizes can be drawn from 40 tickets in ${}^{n}P_r = {}^{40}P_3$ ways

Since $${}^{n}P_r = \frac{n!}{(n-r)!} \quad \text{Here } n=40,\ r=3$$

$${}^{40}P_3 = \frac{40!}{(40-3)!} = \frac{40!}{37!} = \frac{40\times39\times38\times\cancel{37!}}{\cancel{37!}}$$

$$= 59,280 \text{ ways.}$$

2.43 **Solution:-**

Here, trees are required to be planted in a circle so we use circular permutation $(n-1)!$. Thus 5 trees can be planted in a circle in $(n-1)! = (5-1)! = 4! = $ **24 ways**.

2.44 Solution:-

Since wagons are required to arrange in circle so by circular permutation, number of possible ways are $(n-1)!$. Here $n=8$

So $(8-1)! = 7! = $ **5040 ways**

2.45 Solution:-

Word: INFINITY

Total number of letters $= 8 = n$

Number of I $= 3 = n_1$

Number of N $= 2 = n_2$

Number of F $= 1 = n_3$

Number of T $= 1 = n_4$

Number of Y $= 1 = n_5$

Since the no. of distinct permutations of n things of which n_1 are of one kind, n_2 of second kind, ..., n_k of a k^{th} kind is:

$$\frac{n!}{n_1!\,n_2!\cdots n_k!}$$

Here, the total no. of arrangements is:

$$P = \frac{8!}{3!\,2!\,1!\,1!\,1} = \frac{8\times7\times6\times5\times\overset{2}{\cancel{4}}\times\cancel{3!}}{\cancel{3!}\times\cancel{2}}$$

$$= \mathbf{3360}$$

2.46 Solution:-

Total number of trees $= n = 8$

Number of oaks : $n_1 = 3$

Number of pines : $n_2 = 4$

Number of maples : $n_3 = 2$

The total no. of arrangements : $P = \dfrac{n!}{n_1!\,n_2!\,n_3!}$

$$= \frac{8!}{3!\,4!\,2!} = \frac{\overset{4}{\cancel{8}}\times7\times\cancel{6}\times5\times\cancel{4!}}{\cancel{4!}\times\cancel{3!}\times\cancel{2!}} = \mathbf{140\ ways}$$

2.47 Solution:-

3 candidates from 8 equally qualified recent graduates can be selected in $^nC_r = {^8C_3}$ ways

Since $^nC_r = \frac{n!}{r!(n-r)!}$, Here $n = 8$, $r = 3$

$$\Rightarrow {^8C_3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!\,5!} = \frac{8\times7\times6\times\cancel{5!}}{\cancel{3!}\times\cancel{5!}} = \textbf{56 ways}$$

(There was neither any restriction nor ranking among candidates, so we used combination)

2.48 Solution:-

There are total 365 days in a year. Number of Students is 60. Since no two students will have the same birth date so we use permutation. Total no. of ways $= {^nP_r} = {^{365}P_{60}}$ (which is a very large number).

Section 2.4: Probability of an Event

Exercise:-

2.49 Solution:-

a) We know that, sum of all probabilities must be equal to 1 i-e $P(S) = 1$ but

Here $0.19 + 0.38 + 0.29 + 0.15 = 1.01$, so

Error: Sum of probabilities exceeds 1.

b) Sum of probabilities should be equal to 1 i-e $P(S) = 1$

Here, $0.40 + 0.52 = 0.92 < 1$

Error: Sum of probabilities is less than 1.

c) A probability can never be negative but here one probability -0.25 is given which is not possible

Error: A negative probability.

d) Since no heart card is black and no black card is heart so the probability of selecting both a heart and a black card is zero.

Error: The probability of selecting both a heart and a black card is 1/8.

2.50 **Solution:-**

?

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

outcome on green die — outcome on red die — $n(S) = 36$

a) Let A be the event that sum on both dice appears greater than 8

$$A = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$$

$n(A) = 10$

The probability of an event A is:

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

b) Let C be the event that number greater than 4 appears on green die

2.46

$$C = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$n(C) = 12$

The probability of an event C is:

$$P(C) = \frac{n(C)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

c) The sample points that are common to events A and C are:

$A \cap C = \{(5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$

$n(A \cap C) = 7$

The probability of an event $A \cap C$ is:

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{7}{36}.$$

2.51 **Solution:-**

The sample space for different amounts of money is:

$S = \{\$10, \$25, \$100\}$

Now to find probabilities for each sample point:

Total envelops = 500

\$10 are in 275 envelops so associated probability with first sample point is $\frac{275}{500} = \frac{11}{20}$

\$25 are in 150 envelops so associated probability with second sample point is $\frac{150}{500} = \frac{3}{10}$

\$100 are in 75 envelops so associated probability with third sample point is $\frac{75}{500} = \frac{5}{20}$

To find the probability that the first envelop purchased contains less than \$100:

There are 2 amounts less than \$100 : \$10 & \$25, their associated probabilities are $\frac{11}{20}$ and $\frac{3}{10}$ respectively. Required probability $= \frac{11}{20} + \frac{3}{10} = \frac{11+6}{20}$

$$= \frac{17}{20}$$

2.52 Given: Total Students : $n(S) = 500$

let A be the event that Student Smokes : $n(A) = 210$

let B be the event that Student drinks alcoholic beverages : $n(B) = 258$

let C be the event that Student eats between meals : $n(C) = 216$

$A \cap B$ denotes the event that student smokes and drink alcoholic beverages : $n(A \cap B) = 122$

$A \cap C$ denotes the event that Student smokes and eat between meals : $n(A \cap C) = 97$

$B \cap C$ denotes the event that drinks alcoholic beverages and eats b/w meals : $n(B \cap C) = 83$, $n(A \cap B \cap C) = 52$

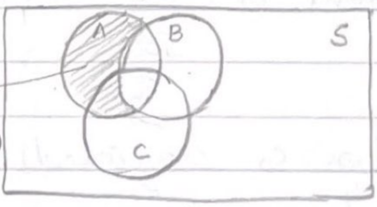
a) let $A \cap B'$ denotes the event that the student smokes but doesnot drink alcoholic beverages then $n(A \cap B')$ can be found by $n(A) - n(A \cap B)$

$= 210 - 122$

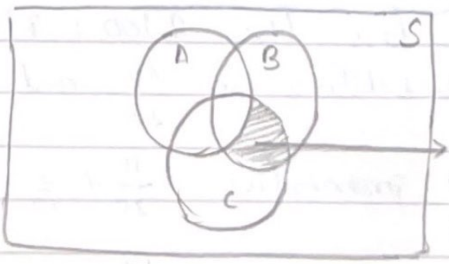
$= 88$

Required probability $= P(A \cap B') = \dfrac{n(A \cap B')}{n(S)} = \dfrac{88}{500}$

Shaded region Shows the area of $A \cap B'$ (where $B' = S - B$) can be obtained by $A - A \cap B$



b) let $C \cap B \cap A'$ denotes the event that Student eats between meals, drinks alcoholic beverages but doesnot smoke, then $n(C \cap B \cap A')$ can be found by :



Shaded region is required, can be obtained by $(C \cap B) - (A \cap B \cap C)$

$n(C \cap B \cap A') = n(C \cap B) - n(C \cap B \cap A)$

$= 83 - 52 = 31$.

c) $A \cup C$ denotes the event that student either smokes or eat between meals, so $(A \cup C)'$ denotes the event that student neither smokes nor eats b/w meals.

$$n(A \cup C)' = n(S) - n(A \cup C)$$

Since the events A, C are not mutually exclusive (disjoint)

so $n(A \cup C) = n(A) + n(C) - n(A \cap C)$

$$\Rightarrow n(A \cup C)' = 500 - (210 + 216 - 97)$$
$$= 329$$

Required probability $= P(A \cup C)' = \dfrac{n(A \cup C)'}{n(S)} = \dfrac{329}{500}$

2.53 Solution:-

let S denote the event that an industry will locate in Shanghai, China then $P(S) = 0.7$

let B denote the event that an industry will locate in Beijing, China then $P(B) = 0.4$

The probability that it will locate in either Shanghai or Beijing or both is $P(S) + P(B) + P(S \cap B) = 0.8$ which is the $P(S \cup B)$

a) we are required to find the probability that the industry will locate in both cities i.e $P(S \cap B)$.

Since $P(S \cup B) = P(S) + P(B) - P(S \cap B)$

Substituting given values

$$0.8 = 0.7 + 0.4 - P(S \cap B)$$
$$0.8 = 1.1 - P(S \cap B)$$
$$\Rightarrow P(S \cap B) = 1.1 - 0.8 = 0.3$$

b) The probability that the industry will locate in neither city i-e neither in Shanghai nor Beijing is $P(S \cup B)' = 1 - P(S \cup B)$

$$= 1 - 0.8$$
$$= 0.2$$

2.54 **Solution:-**

let T denotes the event of investing in tax-free bonds then $P(T) = 0.6$

let M denotes the event of investing in mutual funds then $P(M) = 0.3$

$T \cap M$ denotes the event of investing in both tax free and mutual fund bonds then $P(T \cap M) = 0.15$

a) The probability that a customer will invest in either tax-free or mutual funds is

$P(T \cup M) = P(T) + P(M) - P(T \cap M)$ ∴ M and T are not mutually exclusive events

$= 0.6 + 0.3 - 0.15$

$= 0.75$

b) $P(T \cup M)' = 1 - P(T \cup M) = 1 - 0.75 = 0.25$

2.55 **Solution.-**

$$\text{No. of choices} \longrightarrow \underbrace{5 \quad 25 \quad 24}_{\text{3 letters}} \quad \underbrace{6 \quad 7 \quad 8 \quad 4}_{\text{4 non-zero digits}}$$

5: only vowel can come i.e a,e,i,o,u

25: out of 26 any alphabet can come except the one used.

8: any non-zero digit can come except the one used before

4: only even no. from 1,2,3,4,5,6,7,8,9 can come.

The code that contains 3 distinct letters and 4 distinct non-zero digits can be coded in

$n(S) = \underbrace{26 \times 25 \times 24}_{\text{26 alphabets}} \times \underbrace{9 \times 8 \times 7 \times 6}_{\text{9 non-zero digits}} = 47,174,400$ possible ways

let A be the event that randomly selected code begins with first letter as vowel and last digit even then $n(A) = 5 \times 25 \times 24 \times 6 \times 7 \times 8 \times 4 = 4,032,000$

Required probability $= \dfrac{n(A)}{n(S)}$

$$= \frac{4032000}{47174400}$$

2.56 **Solution:-**

Given: The probability of defect in brake

The words "or", "either" are used in questions use law of addition i-e union between events.
when the words "and", "both" are used in questions use law of multiplication i-e intersection between events.

10

System : $P(B) = 0.25$
The probability of defect in the transmission: $P(T) = 0.18$
The probability of defect in the fuel system: $P(F) = 0.17$
The probability of defect in other area : $P(O) = 0.40$

a) The probability that the defect is in the brakes or the fueling system is:
$P(B \cup F) = P(B) + P(F) - P(B \cap F)$
Given that Prob. of defects in both systems simultaneously is $P(B \cap F) = 0.15$
$\Rightarrow P(B \cup F) = 0.25 + 0.17 - 0.15 = 0.27$.

b) The probability that there are no defects in either the brakes or the fueling system is:
$P(B \cup F)' = 1 - P(B \cup F) = 1 - 0.27 = 0.73$

2.57 Solution:-

There are 26 English alphabets : $n(S) = 26$.

a) let A be the event that randomly chosen letter is a vowel exclusive of y. Since there are 5 vowels so $n(A) = 5$.
Required probability $= \frac{n(A)}{n(S)} = \frac{5}{26}$.

b) let B be the event that chosen letter is ahead of 'j'. Since there are 16 alphabets after 'j' so $n(B) = 16$.
Required probability : $P(B) = \frac{n(B)}{n(S)} = \frac{16}{26}$.

c) let C be the event that chosen letter is after the 'g'. Since there are 19 letters that follows 'g' so $n(C) = 19$
Required probability : $P(C) = \frac{n(C)}{n(S)} = \frac{19}{26}$.

2.58 **Solution:-**

Using the sample space listed in solution of Q2.50, the $n(S) = 36$.

a) let A denote the event that total of 8 appears

$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$n(A) = 5$

Required probability : $P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$

b) let B denote the event of getting atmost (should not exceed 5) a total of 5.

$B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

$n(B) = 10$

Required probability : $P(B) = \frac{n(B)}{n(S)} = \frac{10}{36}$

2.59 **Solution:-**

Note: Poker means card games. In poker, players form sets of 5 playing cards, called hands.

5 cards out of 52 total cards can be selected in ${}^{52}C_5$ ways $\Rightarrow n(S) = {}^{52}C_5 = 2,598,960$.

let A denote the event that 3 aces are selected in those 5 cards. Since there are 4 aces so 3 aces can be selected in ${}^{4}C_3$ ways. Remaining 2 cards will be selected from 48 cards in ${}^{48}C_2$ ways so $n(A) = {}^{4}C_3 \times {}^{48}C_2 = 4512$

Required prob. of selecting 3 aces : $P(A) = \frac{n(A)}{n(S)}$

$= \frac{{}^{4}C_3 \; {}^{48}C_2}{{}^{52}C_5}$ 2 from remaining 48

3 aces out of 4

$= \frac{4512}{2598960}$ 94 54195

$= \frac{94}{54195}$

b) let B denotes the event of selecting 4 hearts and 1 club. Since there are 13 heart cards and 13 club cards, so.

$n(B) = {}^{13}C_4 \times {}^{13}C_1$

Required Probability : $P(B) = \frac{n(B)}{n(S)}$

$= \frac{{}^{13}C_4 \times {}^{13}C_1}{{}^{52}C_5}$

$= \frac{\cancel{9295}^{143}}{\cancel{2598960}_{39984}}$

$= \frac{143}{39984}$

2.60 **Solution:-**

Total no. of books are $5+3+1 = 9$ out of which 3 books can be selected in ${}^9C_3$ ways

So $n(S) = {}^9C_3 = 84$

a) let A denote the event that out of 3 randomly picked books, the dictionary is selected then remaining 2 books will be selected from other 8 books so $n(A) = \underbrace{{}^1C_1}_{\text{1 dictionary selected}} \times \underbrace{{}^8C_2}_{\text{2 books from novels and poems } (5+3=8)}$

Required Probability : $P(A) = \frac{n(A)}{n(S)} = \frac{{}^1C_1 \times {}^8C_2}{{}^9C_3}$

$= \frac{\cancel{28}^1}{\cancel{84}_3}$

$= \frac{1}{3}$

b) let B denote the event that 2 novels and 1 book of poems are selected. Since there are 5 novels out of which 2 can be selected in ${}^5C_2$ ways and 1 book of poem out of 3 can be selected in ${}^3C_1$ ways so, $n(B) = {}^5C_2 \times {}^3C_1$

Required Probability : $P(B) = \frac{n(B)}{n(S)}$

$$= \frac{{}^5C_2 \times {}^3C_1}{{}^9C_3}$$

$$= \frac{\cancel{30}^{5}}{\cancel{84}_{14}}$$

$$= \frac{5}{14}$$

2.61 **Solution:-**

Total no. of students : $n(S) = 100$

let A denote that selected student took mathematics then $n(A) = 54$

let B denote the event that student studied history then $n(B) = 69$

$A \cap B$ denotes the event that student studied both mathematics & history then $n(A \cap B) = 35$

a) $A \cup B$ denotes the event that student took mathematics or history then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ∴ Since 2 events are not ME

$= 54 + 69 - 35$

$= 88$

Required Probability : $P(A \cup B) = \frac{n(A \cup B)}{n(S)}$

$$= \frac{\cancel{88}^{22}}{\cancel{100}_{25}} = \frac{22}{25}$$

b) $(A \cup B)'$ denotes the event that student did not take any of these subjects then

$n(A \cup B)' = n(S) - n(A \cup B)$

$= 100 - 88 = 12$

Required Probability : $P(A \cup B)' = \frac{n(A \cup B)'}{n(S)} = \frac{\cancel{12}^{3}}{\cancel{100}_{25}}$

$$= \frac{3}{25}$$

c) $A' \cap B$ denotes the event that student didnot take mathematics but history then

$n(A' \cap B) = n(B) - n(A \cap B)$

$= 69 - 35$

$= 34$

Required probability : $P(A' \cap B) = \frac{n(A' \cap B)}{n(S)}$

$= \frac{\cancel{34}^{17}}{\cancel{100}_{50}}$

$= \frac{17}{50}$.

2.62 **Solution:-**

Types of crusts : $n_1 = 3$

Types of Sauces : $n_2 = 3$

a) By the rule of multiplication : $n_1 \times n_2 = 3 \times 3 = 9$
So there are 9 combinations of crusts and sauces.

b) let A be the event that judge will get a combination of thin crust with standard sauce then $n(A) = 1$.
Since there are 9 total combinations so $n(S) = 9$
Required probability : $P(A) = \frac{n(A)}{n(S)} = \frac{1}{9}$

2.63 **Solution:-**

a) let A denote the event that PC is in bedroom then $P(A) = 0.03 + 0.15 + 0.14 = 0.32$

b) let A' denote the event that it is not in bedroom then by the law of complement

$P(A') = 1 - P(A)$ $\therefore$ Since $P(A) + P(A') = 1$

$\Rightarrow P(A') = 1 - 0.32 = 0.68$

c) Office or den would be most likely to have a PC because it has the highest Probability of 0.40.

2.64 Solution:-

The probability that component survives for more than 6000 hours is 0.42 i-e $P(A) = 0.42$

The probability that life of component is less than or equal to 6000 hours is: $P(A')$

Since $P(A') = 1 - P(A)$

$\Rightarrow P(A') = 1 - 0.42 = 0.58$

The probability that the component survives no longer than 4000 hours (means ≤ 4000) is 0.04 i-e $P(B) = 0.04$. Now,

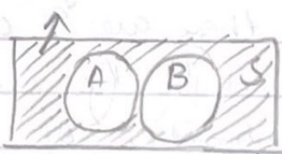
The probability that the life is greater than 4000 hours is: $P(B') = 1 - P(B)$

$= 1 - 0.04 = 0.96$

2.65 Solution.-

Given $P(A) = 0.20$, $P(B) = 0.35$

Shaded region show $A' \cap B'$



a) $P(A') = 1 - P(A) = 1 - 0.20 = 0.80$

b) $P(A' \cap B') = 1 - P(A \cup B)$

Here $P(A \cup B) = P(A) + P(B)$ $\because$ A & B are ME (failed system cannot display strain)

$= 0.20 + 0.35 = 0.55$

$\Rightarrow P(A' \cap B') = 1 - 0.55 = 0.45$

c) $P(A \cup B) = P(A) + P(B)$

$= 0.20 + 0.35 = 0.55$

2.66 Solution:-

a) The probability that the accident occured on the graveyard shift can be due to either unsafe conditions or human error so required probability is $0.2\% + 0.30\%$

$= \dfrac{0.2}{100} + \dfrac{0.30}{100}$

$= 0.32$

b) The Probability that occured due to human error can be in any of the shifts so required prob. is : $32\% + 25\% + 30\%$

$= 0.32 + 0.25 + 0.30 = 0.87$ or 87%

c) The probability That accident occured due to unsafe conditions can be in any of the shifts so required prob. is : $5\% + 6\% + 2\%$

$= 0.05 + 0.06 + 0.02$

$= 0.13 = 13\%$

d) The probability that accident occured on either the evening or the graveyard shift can be found in 2ways. (attempt only 1)

i) By law of addition

ii) By law of complement.

i) By law of addition:

$P(E \cup G) = P(E) + P(G)$ ∴ events are ME

where $P(E) = 6\% + 25\% = 0.06 + 0.25 = 0.31$

$P(G) = 2\% + 30\% = 0.02 + 0.30 = 0.32$

$\Rightarrow P(E \cup G) = 0.31 + 0.32 = 0.63 = 63\%$

ii) By law of complement:

Required Prob. = 1 − prob. of accident in Day $= 1 - P(D)$

$= 1 - (5\% + 32\%)$

$= 1 - (0.05 + 0.32)$

$= 1 - 0.37$

$= 0.63$ or 63%.

2.67 Solution: -

a) The probability that no more than 4 cars will be serviced (means less than 4 cars will be serviced) $= 0.12 + 0.19 = 0.31$

b) The Probability That he will service fewer than 8 cars = 1 − Prob. of servicing more than 8 cars

$= 1 - 0.07$

$= 0.93$

(can also be solved by law of addition)

The probability that he will service either 3 or 4 cars is $P(3\text{ cars}) + P(4\text{ cars}) = 0.12 + 0.19 = 0.31$

2.68 Solution:-

a) let E be the event that atmost 2 (i-e ≤ 2) candidates purchase electric ovens. We are required to find the probability that atleast 3 (i-e ≥ 3) candidates purchase electric ovens

1 2 3 4 5 6 → Total prob. = 1 always

Given ? → find by subtracting given from total prob i-e 1.

Given : $P(E) = 0.40$

Required prob. $= 1 - P(E)$

$= 1 - 0.40 = 0.60$

b) The probability that all 6 purchasing the electric oven or all 6 purchasing the gas oven is $0.007 + 0.104 = 0.111$. So, the probability that atleast 1 (i-e ≥ 1) of each type purchased is $1 - 0.111 = 0.889$

2.69 Solution:-

a) Given: $P(A) = 0.990$, $P(B) = 0.001$.

Since the sum of probabilities or total probability is always equal to 1 i-e

$$P(A) + P(B) + P(C) = 1$$

$$\Rightarrow P(C) = 1 - P(A) - P(B) = 1 - 0.990 - 0.001 = 0.009$$

b) $P(B) = 0.001$ is the probability that the machine underfills. The prob. that the machine doesnot underfill is $P(B') = 1 - P(B) = 1 - 0.001 = 0.999$.

c) The prob. that the machine either overfills or underfills is $P(B \cup C) = P(B) + P(C)$ (B and C are ME events)

$= 0.001 + 0.009$

$= 0.01$

2.70 leave this question, not related to probability.

2.71 **Solution:-**

a) There are 3 possibilities : weight of a product can meet the weight specification, it can be too heavy or too light.
The prob. of meeting the specification is $0.95 = P(A)$
The prob. of being too light is $0.002 = P(B)$
The prob. of being too heavy is required $= P(C)$
Since $P(A)+P(B)+P(C)=1$
$$\Rightarrow \quad P(C) = 1 - P(A) - P(B) = 1 - 0.002 - 0.95$$
$$= 0.048.$$

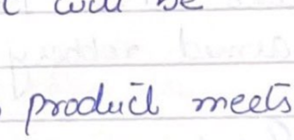
b) Cost Price = \$ 20.00 for single product
Purchase Price = \$ 25.00 for single product
Profit = Purchase Price − Cost Price
= \$ 25.00 − \$ 20.00
= \$ 5.00 for single product
So for 10,000 sold packages, profit will be
10,000 X \$5.00 = \$ 50,000.

c) It is given that the probability when product meets weight specifications is 0.95 then the prob. when product doesnot meet weight specification will be
$1 - 0.95 = 0.05$.
Number of defective packages = $10,000 \times 0.05 = 500$
(from part b)
Profit on 500 defective packages = \$5 X 500 = \$2500
Investment on 500 defective packages = \$20 X 500 = \$10,000
Thus, the profit reduced on 10,000 packages due to failure to meet weight specification is \$2500 + \$10,000
= \$12,500.

2.72 **Solution:-**

Since $A' \cap B' = (A \cup B)'$

Shaded region is required region



$$P(A' \cap B') = 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)] \quad \therefore A \,\&\, B \text{ are ME events}$$
$$= 1 - P(A) - P(B) + P(A \cap B)$$
$$= 1 + P(A \cap B) - P(A) - P(B) \quad \text{As required.}$$

Section 2.6:-

-: Conditional Probability, Independence and The Product Rule:-

Exercise:-

2.73 Solution:-

a) $P(R|D)$ expresses the probability that the convict who pushed dope also commited armed robbery.

b) $P(D'|R)$ expresses the probability that a convict who commited armed robbery didnot push dope.

c) $P(R'|D')$ expresses the probability that a convict who did not push dope also did not commit armed robbery.

2.74 Solution:-

Let 'S' denote the event that the student selected is senior and 'A' denote the event that he or she has earned an A grade.

We are required to find the probability that student is a senior given that he or she has earned an A i-e $P(S|A)$

We know, $P(S|A) = \dfrac{P(S\cap A)}{P(A)}$

$$= \frac{n(S\cap A)/\cancel{n(S)}}{n(A)/\cancel{n(S)}}$$

$$= \frac{n(S\cap A)}{n(A)}$$

The no. of students who are senior and earned 'A' grade are $10 = n(S\cap A)$

The number of Students who earned an 'A' grade are $n(A) = 3+10+5 = 18$

$$\Rightarrow P(S|A) = \frac{10}{18}$$

2.75 **Solution:-**

Let M denotes the event that the person is a male.
S denotes the event person has secondary education
C denotes the event person has college degree.

a) We are required to find the conditional probability that person is a male given that person has Secondary education i-e $P(M|S)$

$$P(M|S) = \frac{P(M\cap S)}{P(S)}$$

$$= \frac{n(M\cap S)}{n(S)}$$

$n(M\cap S)$ = No. of males who have Secondary education = 28

$n(S)$ = No. of persons having Secondary education

$= 28 + 50 = 78$

$$\Rightarrow P(M|S) = \frac{\cancel{28}^{14}}{\cancel{78}_{39}}$$

$$= \frac{14}{39}$$

We are required to find the conditional probability that the person doesnot have a college degree, given that the person is a female

i-e $P(C'|M') = \frac{P(C'\cap M')}{P(M')}$

$$= \frac{n(C'\cap M')}{n(M')}$$

$n(C'\cap M')$ = no. of persons who does not have college degree and are not males (It means females having elementary & secondary education) = $45+50 = 95$

* Note: when there are 2 categories, denote one event by A and other will be its complement A'.

$n(M')$ = no. of persons who are not males (ie females)
$= 45+50+17 = 112$
$\Rightarrow P(C'|M') = \frac{95}{112}$.

2.76 **Solution:-**

Consider the events:
H: a person is experiencing hypertension
S: a person is heavy smoker
N: a person is non smoker

a) The probability that the person is experiencing hypertension given that the person is a heavy smoker is $P(H|S)$

$$P(H|S) = \frac{P(H\cap S)}{P(S)} = \frac{n(H\cap S)}{n(S)}$$

$n(H\cap S)$ = number of persons experiencing hypertension and are heavy smokers = 30
$n(S)$ = number of persons who are heavy smokers
$= 30+19 = 49$
$\Rightarrow P(H|S) = \frac{30}{49}$

b) The probability that the person is nonsmoker, given that the person is experiencing no hypertension is $P(N|H')$.

$$P(N|H') = \frac{P(N\cap H')}{P(H')} = \frac{n(N\cap H')}{n(H')}$$

$n(N\cap H')$ = no. of persons who are non-smokers and nonhypertension = 48
$n(H')$ = no. of persons who are non hypertensive =
$= 48+26+19 = 93$

$\Rightarrow P(N|H') = \dfrac{48}{93}$

2.77 **Solution:-**

Consider the events:

P = person takes psychology

M = person takes mathematics

H = person takes history

Given: Total no. of Students: $n(S) = 100$

no. of students taking mathematics : $n(M) = 42$

" " psychology : $n(P) = 68$

" " history : $n(H) = 54$

" " both maths & history: $n(M \cap H) = 22$

" " both maths & psychology: $n(M \cap P) = 25$

" " Studying history but neither maths nor psychology $= n(H \cap M' \cap P') = 7$

no. of students studying all 3 subjects $= n(H \cap M \cap P) = 10$

no. of students not taking any of 3 $= n(H' \cap M' \cap P') = 8$

a) The probability that person enrolled in psychology takes all three subjects is $P(H \cap M \cap P / P)$

$$P(H \cap M \cap P | P) = \frac{P((H \cap M \cap P) \cap P)}{P(P)}$$

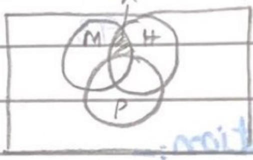
$$= \frac{P(H \cap M \cap P)}{P(P)}$$

$$= \frac{n(H \cap M \cap P)}{n(P)}$$

where $n(H \cap M \cap P) = 10$ and $n(P) = 68$ as given above

$\Rightarrow P(H \cap M \cap P | P) = \dfrac{10}{68}$

b) The probability that a person not taking psychology is taking both history and mathematics is shaded region is required region



$$P(M \cap H | P') = \frac{P(M \cap H \cap P')}{P(P')}$$

$$= \frac{n(M \cap H \cap P')}{n(P')}$$

where $n(M \cap H \cap P')$ = no. of persons taking both history and mathematics but not psychology $= n(H \cap M) - n(H \cap M \cap P)$

$= 22 - 10$

$= 12$

and $n(P')$ = no. of persons not taking psychology

$= 100 - n(P)$

$= 100 - 68 = 32$

$$\Rightarrow P(M \cap H | P') = \frac{\cancel{12}^{3}}{\cancel{32}_{8}}$$

$$= \frac{3}{8}$$

2.78 Solution:-

Rejection rate of 1st department $= 0.10 = P(A)$

Rejection rate of 2nd department $= 0.08 = P(B)$

Rejection rate of 3rd department $= 0.12 = P(C)$

a) The probability that a batch of serum survives the first inspection but is rejected by the second is $P(A' \cap B)$.

$P(A' \cap B) = P(A') \cdot P(B)$ where $P(A')$ is the acceptance rate of 1st department $= 1 - P(A)$

$= 1 - 0.10 = 0.90$

$\Rightarrow P(A' \cap B) = 0.90 \times 0.08 = 0.072$

Note: Here independence rule is used instead of conditional probability because it is given that the inspections by 3 departments are independent.

b) Since the inspections by 3 departments are sequential so first 2 departments must have accepted the serum before the rejection of 3rd department so we are required to find:

$$P(A' \cap B' \cap C) = P(A')P(B')P(C) \quad \because \text{events are independent}$$

$$= [1 - P(A)][1 - P(B)]\,P(C)$$

$$= (1-0.10)(1-0.08)\,0.12$$

$$= 0.90 \times 0.92 \times 0.12$$

$$= 0.099$$

2.79 Solution:-

a) The probability that a traveler is female who sleeps in the nude is 0.018 (see given table)

b) The probability that the traveler is male

$= 0.220 + 0.002 + 0.160 + 0.102 + 0.046 + 0.084$

$= 0.614$

c) The prob. that traveler sleeps in pajamas given that traveler is male is denoted by

$$P(P|M) = \frac{P(P \cap M)}{P(M)}$$

$$= \frac{0.102 \rightarrow \text{(see table)}}{0.614 \rightarrow \text{(from part b)}} = 0.166$$

d) The prob. that a traveler is male if the traveler sleeps in pajamas or a T-shirt

$$= \frac{\text{prob. of males in pajamas} + \text{prob. of males in T-shirt}}{\text{prob. of travelers in pajamas} + \text{prob. of travelers in T-shirt}}$$

$$= \frac{0.102 + 0.046}{0.175 + 0.134}$$

$$= \frac{0.148}{0.309}$$

$$= 0.479$$

2.80 **Solution:-**

let C be the event That oil change is needed and F be The event that oil filter is needed then $C \cap F$ denotes The event That both oil and the filter need changing.

Given $P(C) = 0.25$, $P(F) = 0.40$, $P(C \cap F) = 0.14$

a) The probability that a new filter is needed when the oil has to be changed is $P(F|C)$

$$P(F|C) = \frac{P(F \cap C)}{P(C)}$$

$$= \frac{0.14}{0.25} = 0.56$$

b) The probability that the oil has to be changed when a new filter is needed is $P(C|F)$

$$P(C|F) = \frac{P(C \cap F)}{P(F)}$$

$$= \frac{0.14}{0.40} = 0.35$$

2.81 **Solution:-**

let M be the event that man watches a show and w be the event that woman watches a show

Given That: $P(M) = 0.4$, $P(w) = 0.5$, $P(M/w) = 0.7$

a) The probability that a married couples i.e both man and woman watches the Show is $P(M \cap w)$.

Since the condition Probability of $P(M|w)$ is given, it means both events are dependent So

$$P(M \cap w) = P(M|w)P(w)$$

$$= 0.7 \times 0.5$$

$$= 0.35$$

b) The probability that a wife watches the Shows given that her husband does:

$$P(W|M) = \frac{P(W\cap M)}{P(M)}$$

from part (a) : $P(W\cap M) = 0.35$ and given $P(M) = 0.4$

$$\Rightarrow P(W|M) = \frac{0.35}{0.4} = 0.875.$$

c) Atleast one member of a married couple means either man/woman watches the show or both watches i.e we are required to find $P(M\cup W)$

$P(M\cup W) = P(M) + P(W) - P(M\cap W)$ $\because$ events are not ME

$= 0.4 + 0.5 - 0.35$

$= 0.55$

2.82 Solution:-

Let H be the event that husband will vote and W be the event that wife will vote then $H\cap W$ denotes the event that both husband and wife will vote. Given: $P(H) = 0.21$, $P(W) = 0.28$, $P(H\cap W) = 0.15$

The probability that atleast one member of married couple means either husband/wife or both will vote is $P(H\cup W)$

$P(H\cup W) = P(H) + P(W) - P(H\cap W)$ $\because$ events are not ME

$= 0.21 + 0.28 - 0.15$

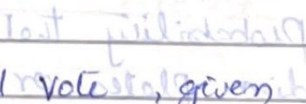
$= 0.34$

The probability that a wife will vote, given that her husband will vote is $P(W|H)$

$$P(W|H) = \frac{P(W\cap H)}{P(H)} = \frac{0.15}{0.21} = 0.714$$

The probability that a husband will vote given that his wife will not vote is: $P(H|W')$

$$P(H|W') = \frac{P(H\cap W')}{P(W')}$$



shaded region denotes $H\cap W'$

where $H\cap W' = H - (H\cap W)$ and

$W' = 1 - W$

$\Rightarrow P(H \cap W') = P(H) - P(H \cap W)$
$= 0.21 - 0.15$
$= 0.06$

& $P(W') = 1 - P(W) = 1 - 0.28 = 0.72$

Then $P(H|W') = \frac{P(H \cap W')}{P(W')} = \frac{0.06}{0.72} = 0.083$

283 Solution :-

The probability that a vehicle is with Canadian license plate is 0.12 i-e $P(N) = 0.12$, where N denotes the above event. let M be the event that vehicle is a camper then $P(M) = 0.28$ and the event that it is camper with Canadian license plate is $M \cap N$ with $P(M \cap N) = 0.09$.

a) we are required find that probability that vehicle has Canadian license plate given that it is a Camper is $P(N|M)$

$$P(N|M) = \frac{P(N \cap M)}{P(M)} = \frac{0.09}{0.28} = 0.321$$

b) The probability that a vehicle is camper given that it has Canadian license plate is $P(M|N)$

$$P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{0.09}{0.12} = 0.75$$

c) The probability that a vehicle doesnot have Canadian plates or is not a camper is $P(N' \cup M')$.

Remember,

$P(N' \cup M') = 1 - P(N \cap M) = 1 - 0.09$
$= 0.91$

Since $N' \cup M' = (N \cap M)'$
$= S - (N \cap M)$

2.84 Solution:-

let H be the event that the head of the house is home and G be the event that goods are bought from company then $P(H) = 0.4$, $P(G|H) = 0.3$.
The probability that the head of the house is home and goods are bought from company is $P(H \cap G)$.
Since conditional probability is given so events are dependent.

$$P(H \cap G) = P(G|H) P(H)$$
$$= 0.3 \times 0.4$$
$$= 0.12$$

2.85 Solution:-

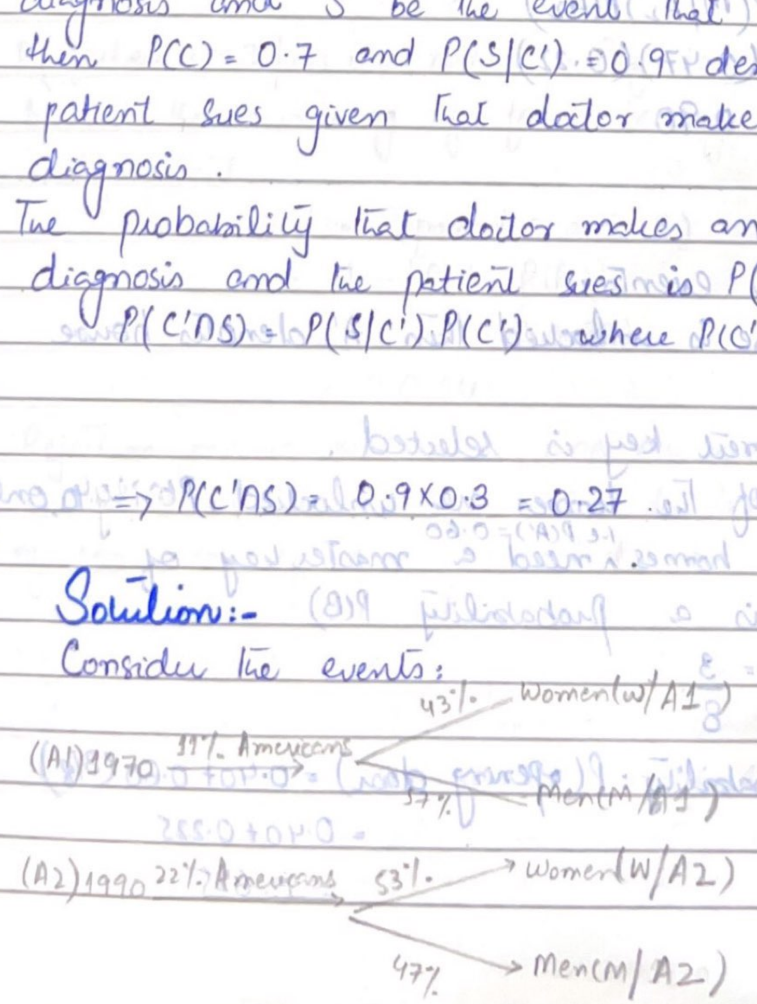
let C be the event that the doctor makes correct diagnosis and S be the event that patient sues then $P(C) = 0.7$ and $P(S|C') = 0.9$ denotes that patient sues given that doctor makes incorrect diagnosis.
The probability that doctor makes an incorrect diagnosis and the patient sues is $P(C' \cap S)$

$P(C' \cap S) = P(S|C') . P(C')$ where $P(C') = 1 - P(C)$
$= 1 - 0.7$
$= 0.3$

$$\Rightarrow P(C' \cap S) = 0.9 \times 0.3 = 0.27$$

2.86 Solution:-

Consider the events:



a) The probability that the person was a woman given that a person completed 4 years of college in 1970 is $P(W/A1)$

As shown in figure, this probability is 43% = 0.43

b) The probability that a woman finished four years of college in 1990 is denoted by

$P(A2 \cap W) = P(W/A2)\,P(A2)$ ∴ events are dependent.

$= (0.53)(0.22)$

$= 0.12$

where A2 denotes the American's completed 4 years of college in 1990.

c) The probability that a man had not finished college in 1990 is 1 - Probability that a man had finished college in 1990. ($P(A') = 1 - P(A)$ law of complement)

i.e. $1 - P(A2 \cap M)$

$P(\ ) = 1 - P(M/A_2)P(A_2)$

$= 1 - (0.47)(0.22)$

$= 0.90$

2.87 Solution:-

Consider the events:

A: the house is unlocked then A' denotes house is locked.

B: the correct key is selected.

Since 40% of the homes are unlocked, $P(A) = 0.40$, only 60% of the homes need a master key of which there is a probability $P(B)$ i.e $P(A') = 0.60$

$P(B) = \frac{3}{8}$

Required Probability: $P(\text{opening door}) = 0.40 + 0.60\,(3/8)$

$= 0.40 + 0.225$

$= 0.625$

2.88 Solution:-

Consider the events

F: failed the test

P: Passed the test

The failure rates for four testing programs are $P(F_1) = 0.01$, $P(F_2) = 0.03$, $P(F_3) = 0.02$, $P(F_4) = 0.01$, then passing rates for four testing programs will be $P(P_1) = 1 - 0.01 = 0.99$, $P(P_2) = 1 - 0.03 = 0.97$, $P(P_3) = 1 - 0.02 = 0.98$ and $P(P_4) = 1 - 0.01 = 0.99$

a) P(failed any test) = 1 - P(passed all test) (any test means failure on 1 or more tests)

Since the running of four testing programs are independent therefore used $P(A|B) = P(A)$

$$P(\text{failed any test}) = 1 - [P(P_1) \cdot P(P_2) \cdot P(P_3) \cdot P(P_4)]$$
$$= 1 - (0.99 \times 0.97 \times 0.98 \times 0.99)$$
$$= 1 - 0.93$$
$$= 0.07$$

b) P(failed 2 or 3 | CD was tested) = P(failed 2 or 3)

because the running of four testing programs are independent

$$= 1 - P(\text{passed 2 and 3})$$
$$= 1 - [P(P_2) \cdot P(P_3)]$$
$$= 1 - (0.97)(0.98)$$
$$= 0.0494$$

c) Rejection means failure on one or more tests and P(failed any test) = 0.07 so in a sample of 100, no. of CDs expected to be rejected are $100 \times 0.07 = 7$.

d) P(CD was tested | CD was defective) = P(CD was tested)

Since running programs are independent and it is given that every fourth CD is tested

So $P(\text{CD was tested}) = \frac{1}{4} = 0.25$

(05)

2.89 **Solution:-**

Let A and B represents the availability of each fire engine.

The probability that neither is available when needed is:

$P(A' \cap B') = P(A')P(B')$ (fire engines operate independently)

$= [1-P(A)][1-P(B)]$

$= (1-0.96)(1-0.96)$

$= (0.04)(0.04)$

$= 0.0016$

The probability that a fire engine is available means either A or B is available i-e

$P(A \cup B) = 1 - P(A' \cap B')$ (law of addition $P(A \cup B) = P(A) + P(B)$ cannot be used because we know the probability of 1 specific engine only)

$= 1 - 0.0016$

$= 0.9984$

Since $A \cup B = S - (A \cup B)' = S - (A' \cap B')$

2.90 **Solution:-**

Consider the events:

A: the river is polluted

B: a sample of water tested detects pollution

C: fishing is permitted

$P(A) = 0.3$, $P(B|A) = 0.75$, $P(B|A') = 0.20$,

$P(C|A \cap B) = 0.20$, $P(C|A' \cap B) = 0.15$, $P(C|A \cap B') = 0.80$ and $P(C|A' \cap B') = 0.90$

a) $P(A \cap B \cap C) = P(C|A \cap B)\,P(A \cap B)$ (for 2 events $P(A \cap B) = P(A|B)P(B)$ or $P(A \cap B) = P(B|A)P(A)$)

$= P(C|A \cap B)\,P(B|A)\,P(A)$

$= 0.20 \times 0.75 \times 0.3$

$= 0.045$

b) $P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C)$

$= P(C|A \cap B')P(A \cap B') + P(C|A' \cap B')P(A' \cap B')$

$= P(C|A \cap B')P(B'|A)P(A) + P(C|A' \cap B')P(B'|A')P(A')$

$= (0.80 \times (1-0.75) \times 0.3) + (0.90 \times (1-0.20) \times (1-0.3))$

$$= (0.80 \times 0.25 \times 0.3) + (0.90 \times 0.80 \times 0.7)$$
$$= 0.06 + 0.504$$
$$= 0.564$$

c) $P(C) = P(A \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B \cap C) + P(A' \cap B' \cap C)$

$$= 0.045 + [P(C|A \cap B') \cdot P(A \cap B')] + [P(C|A' \cap B) P(A' \cap B)] + [P(C|A' \cap B') P(A' \cap B')]$$

$$= 0.045 + [P(C|A \cap B') P(B'|A) \cdot P(A)] + [P(C|A' \cap B) P(B|A') P(A')] + P[C|A' \cap B') P(B'|A') P(A')]$$

$$= 0.045 + [0.80 \times (1-0.75) \times 0.3] + [0.15 \times 0.20 \times (1-0.3)] + [0.90 \times (1-0.20) \times (1-0.3)]$$

$$= 0.045 + 0.060 + 0.021 + 0.504$$

$$= 0.630$$

d) $P(A|B' \cap C) = \dfrac{P(A \cap B' \cap C)}{P(B' \cap C)}$

$$= \frac{P(C|A \cap B') \cdot P(A \cap B')}{P(B' \cap C)}$$

$$= \frac{P(C|A \cap B') P(B'|A) P(A)}{P(B' \cap C)}$$

$$= \frac{0.80 \times (1-0.75) \times 0.3}{0.564}$$ (from part (b) $P(B' \cap C) = 0.564$)

$$= \frac{0.80 \times 0.25 \times 0.3}{0.564}$$

$$= \frac{0.06}{0.564} = 0.106$$

2.91 Solution:-

Since $P(A_1 \cap A_2 \cap \dots \cap A_k) = P(A_1) P(A_2|A_1) P(A_3|A_1 \cap A_2) \dots P(A_k|A_1 \cap A_2 \dots \cap A_{k-1})$

For 4 Quarts of milk, Substituting $k=4$ in above theorem:

$P(Q_1 \cap Q_2 \cap Q_3 \cap Q_4) = P(Q_1)P(Q_2|Q_1)P(Q_3|Q_2 \cap Q_1)P(Q_4|Q_1 \cap Q_2 \cap Q_3)$

As a cooler contains 20 quarts of which 5 have spoiled so $P(Q_1) = P(\text{Selecting 1st good quart}) = \frac{n(Q_1)}{n(S)}$

$= \frac{15}{20}$

$\Rightarrow P(Q_1 \cap Q_2 \cap Q_3 \cap Q_4) = \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} \cdot \frac{12}{17}$

$= \frac{91}{323}$

b) Since $^nC_r = \frac{n!}{r!(n-r)!}$ and $P(A) = \frac{n(A)}{n(S)}$

Let A be the event that 4 good quarts of milk are selected, if 5 are spoiled among 20 then number of good quarts are 15.

$\Rightarrow n(A) = {}^{15}C_4$

The cooler contains 20 quarts out of which 4 are selected so

$n(S) = {}^{20}C_4$

$\Rightarrow P(A) = \frac{^{15}C_4}{^{20}C_4}$

$= \frac{\frac{15!}{4!(15-4)!}}{\frac{20!}{4!(20-4)!}}$

$= \frac{15!}{\cancel{4!}\,11!} \times \frac{\cancel{4!}\,16!}{20!}$

$= \frac{15 \times 14 \times 13 \times 12 \times \cancel{11!}}{\cancel{11!}} \times \frac{\cancel{16!}}{20 \times 19 \times 18 \times 17 \times \cancel{16!}}$

$= \frac{32760}{116280} = 0.2817$

2.92 **Solution:-**

Clearly the probability that the entire system works can be calculated as:

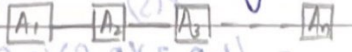
$$P(A\cap(B\cup C)\cap D) = P(A)\cdot P(B\cup C)\cdot P(D)$$
$$= P(A)\cdot[1 - P(B'\cap C')]\cdot P(D)$$
$$= P(A)[1 - P(B')P(C')]\cdot P(D)$$
$$= 0.95[1-(1-0.7)(1-0.8)]\cdot 0.9 \text{ (from diagram)}$$
$$= 0.95[1-(0.3)(0.2)]\times 0.9$$
$$= 0.95[1-0.06]\times 0.9$$
$$= 0.95(0.94)\times 0.9$$
$$= 0.893\times 0.9$$
$$= 0.8037$$

The equalities above hold because of independence among components.

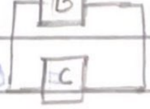
2.93 Solution:-

Note: If components are in series, where the probability of the ith component is denoted by $P(A_i)$, the probability of entire system working is

$$P = P(A_1)\cdot P(A_2)\cdots P(A_n)$$



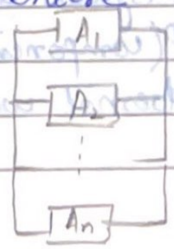
If components are in parallel, system performs if any one component remains operational as in Q2.92, we used $P(B\cup C)$



If there are n components in parallel, where the probability of the ith system working is denoted by $P(A_i)$, the probability of entire system working is

$$P = 1-(1-P(A_1))(1-P(A_2))\cdots(1-P(A_n))$$

we will use this rule in Q2.93.



Solution:-

a) This is a parallel System of 2 series Subsystems.

$P = 1 - (1-P(A\cap B))(1-P(C\cap D\cap E))$

$= 1-(1-P(A)\cdot P(B))(1-P(C)P(D)\cdot P(E))$

$= 1-(1-0.7\times 0.7)(1-0.8\times 0.8\times 0.8)$

$= 1-(1-0.49)(1-0.512)$

$= 1-(0.51)(0.488)$

$= 1-0.24888$

$= 0.75112$

The equality holds because of independence among components.

b) Required probability = P(component A doesn't work / given that the system works)

let S denotes the event that system works

then Probability $= P(A'\cap C\cap D\cap E / S)$

$= \dfrac{P((A'\cap C\cap D\cap E)\cap S)}{P(S)}$

$= \dfrac{P(A'\cap C\cap D\cap E)}{P(S)}$

$= \dfrac{P(A')P(C)P(D)P(E)}{P(S)}$

$= \dfrac{(1-0.7)(0.8)(0.8)(0.8)}{0.7512}$ ($P(S)=0.75112$ from part (a))

$= \dfrac{0.3\times 0.8\times 0.8\times 0.8}{0.75112}$

$= 0.2045$

2.94 **Solution:-**

P(system works) = P(S) = 0.75112

P(system doesnot work) = P(S') = 1 - 0.75112 = 0.24888

P(component A doesnot work known that system doesnot work) $= P(A'/S') = \dfrac{P(A'\cap S')}{P(S')}$

$$P(A'|S') = \frac{P(A' \cap S')}{P(S')} = \frac{P(A')P(S')}{P(S')}$$

$$= \frac{P(A')[1 - P(C \cap D \cap E)]}{P(S')}$$

$$= \frac{P(A')[1 - P(C)P(D)P(E)]}{P(S')}$$

$$= \frac{[1-P(A)][1 - P(C)P(D)P(E)]}{P(S')}$$

$$= \frac{(1-0.7)[1 - 0.8\times0.8\times0.8]}{0.24888}$$

$$= \frac{(0.3)(0.488)}{0.2488}$$

$$= \frac{0.1464}{0.2488}$$

$$= 0.588.$$

Section 2.7:- Bayes' Rule

Exercise:-

2.95 **Solution:-**

Consider the events:

C: an adult has cancer

D: an adult is diagnosed as having cancer

P(adult having cancer) $= P(C) = 0.05$

P(diagnosing person with cancer) $= P(D|C) = 0.78$

P(diagnosing person without cancer) $= P(D|C') = 0.06$

we are required to find probability that an adult is diagnosed as having cancer i-e $P(D)$

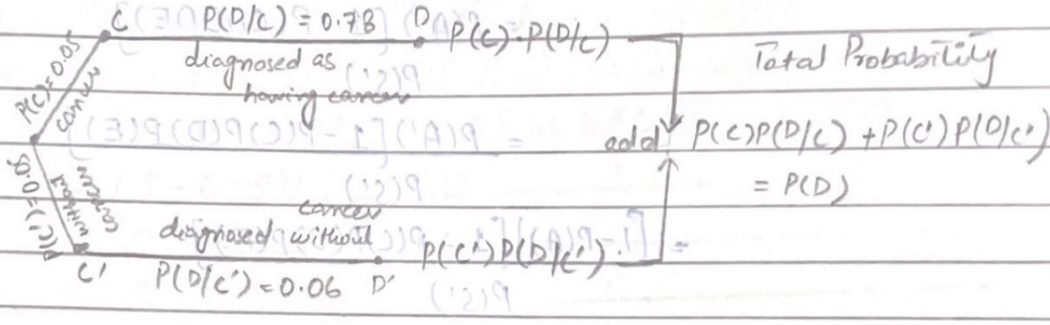
Here D is the union of 2 mutually exclusive events.

So $P(D) = P[(C \cap D) \cup (C' \cap D)] = P(C \cap D) + P(C' \cap D)$

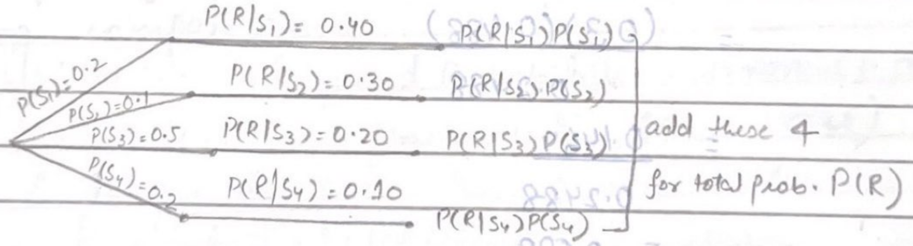
$= P(C)P(D|C) + P(C')P(D|C')$

$\Rightarrow P(D) = 0.05 \times 0.78 + (1-0.05) \times 0.06$

$= 0.096$



2.96 **Solution:-**



Let S_1, S_2, S_3 and S_4 represent the events that a person is speeding as he passes through the respective locations and let R represent the event that radar traps is operating, then the probability that he receives a speeding ticket:

$P(R) = P(R|S_1)P(S_1) + P(R|S_2)P(S_2) + P(R|S_3)P(S_3) + P(R|S_4)P(S_4)$

$= (0.40 \times 0.2) + (0.30 \times 0.1) + (0.20 \times 0.5) + (0.10 \times 0.2)$

$= 0.23$

2.97 **Solution:-**

P(Person has disease given that a person diagnosed as having cancer) = $P(C/D)$ is required

$P(C/D) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)P(D/C)}{P(D)}$

$= \frac{0.05 \times 0.78}{0.096} = 0.40625$

2.98 **Solution:-**

We are required to find P(person passed through L_2 given that he received a speeding ticket) i.e $P(S_2|R)$

$$P(S_2|R) = \frac{P(S_2 \cap R)}{P(R)}$$

$$= \frac{P(R|S_2)P(S_2)}{P(R)}$$

$$= \frac{0.3 \times 0.1}{0.23}$$

$$= \frac{0.03}{0.23}$$

$$= 0.130$$

2.99 **Solution:-**

Consider the events:

A: no expiration date

B_1: John is the inspector, $P(B_1) = 0.20$ and $P(A|B_1) = 0.005 \rightarrow 1/200$

B_2: Tom is the inspector, $P(B_2) = 0.60$ and $P(A|B_2) = 0.010$.

B_3: Jeff is the inspector, $P(B_3) = 0.15$ and $P(A|B_3) = 0.011$

B_4: Pat is the inspector, $P(B_4) = 0.05$ and $P(A|B_4) = 0.005$

P(John is inspector given that package has no exp. date) $= P(B_1/A)$ is required

Using Bayes' Rule:

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1) + P(A/B_2)P(B_2) + P(A/B_3)P(B_3) + P(A/B_4)P(B_4)}$$

$$= \frac{(0.005)(0.20)}{(0.005 \times 0.20) + (0.010 \times 0.60) + (0.011 \times 0.15) + (0.005 \times 0.05)}$$

$$= 0.1124.$$

2.100 **Solution:-**

let $\mathcal{E}$ denotes the malfunction by other human errors. A denotes the station A, B denotes the station B and C denotes the station C.

Total number of malfunctions reported at all three stations $= 2+1+1+4+3+2+5+4+2+7+7+5$

$= 43$

$$P(\text{Malfunctions at } A) = \frac{2+4+5+7}{43} = \frac{n(A)}{n(S)} = P(A)$$

$$\Rightarrow P(A) = \frac{18}{43}$$

Similarly $P(B) = \frac{15}{43}$ and $P(C) = \frac{10}{43}$

we are required to find the P(malfunction came from C given that it is caused by human errors)

i-e $P(C|\mathcal{E})$

By using Bayes' Rule:

$$P(C|\mathcal{E}) = \frac{P(\mathcal{E}|C)P(C)}{P(\mathcal{E}|C)P(C) + P(\mathcal{E}|A)P(A) + P(\mathcal{E}|B)P(B)}$$

$$= \frac{(5/10)(10/43)}{(5/10)(10/43) + (7/15)(15/43) + (7/18)(18/43)}$$

$$= \frac{0.1163}{0.4419}$$

$$= 0.2632$$

where $P(\mathcal{E}|C) = \frac{P(\mathcal{E} \cap C)}{P(C)} = \frac{n(\mathcal{E} \cap C)}{n(C)} = \frac{5}{10} \rightarrow$ from table

2.101 **Solution:-**

Consider the events:

A: a customer purchases latex paint, $P(A) = 0.75$

A': a customer purchases semigloss paint, $P(A') = 1 - 0.75$

$= 0.25$