Chapter 4:-

-: Mathematical Expectation:-

4.1 Solution:-

X denotes the number of imperfections. The average number of imperfections is:

$$\mu = E(X) = \Sigma x f(x) \quad (\because X \text{ is discrete variable})$$

x	$f(x)$	$xf(x)$
0	0.41	0
1	0.37	0.37
2	0.16	0.32
3	0.05	0.15
4	0.01	0.04
		0.88

$\Rightarrow \mu = E(X) = 0.88$ is the average number of imperfections.

4.2 X is the discrete random variable

$$f(x) = \binom{3}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{3-x}, \quad x = 0, 1, 2, 3$$

So, the mean of x is: $\mu = E(X) = \Sigma x f(x)$

x	$f(x)$	$xf(x)$
0	${}^3C_0(0.25)^0(0.75)^3 = 0.422$	0
1	${}^3C_1(0.25)^1(0.75)^2 = 0.422$	0.422
2	${}^3C_2(0.25)^2(0.75)^1 = 0.141$	0.282
3	${}^3C_3(0.25)^3(0.75)^0 = 0.016$	0.048
		0.752

$\Rightarrow \mu = E(X) = 0.752$

4.3 Solution:-

Note: 1 Dime = 10 cents or 10 pennies

1 Nickel = 5 cents or 5 pennies

If 3 coins are selected at random without replacement then possible selections are NND, NDD and DDD because there are 2 nickels, 4 dimes.

Here T represents the total ¢ of the 3 coins. The prob. dist is

Coins	T	P(T)	T·P(T)
NND	5+5+10 = 20	$({}^2C_2\,{}^4C_1)/{}^6C_3 = 0.2$	4
NDD	5+10+10 = 25	$({}^2C_1\,{}^4C_2)/{}^6C_3 = 0.6$	15
DDD	10+10+10 = 30	$({}^2C_0\,{}^4C_3)/{}^6C_3 = 0.2$	6

The mean of the random variable T is:

$$\mu = E(T) = \sum T.P(T)$$
$$= 4+15+6$$
$$= 25 \text{ cents}$$

4.4 **Solution:-**

When the coin is tossed twice

$S = \{HH, HT, TH, TT\}$

let X denotes the expected number of tails then $X = 0, 1, 2$. Since the head is 3 times as likely to occur as tail so $P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$

The probability dist of X is:

X	Sample Points	f(x)
0	HH	$P(HH) = 3/4 \cdot 3/4 = 9/16$
1	HT+TH	$P(HT)+P(TH) = 3/4 \cdot 1/4 + 1/4 \cdot 3/4 = 3/16 + 3/16 = 6/16$
2	TT	$P(TT) = 1/4 \cdot 1/4 = 1/16$

The expected number of tails is:

$$\mu = E(X) = \sum x f(x)$$
$$= 0 \cdot \frac{9}{16} + 1 \cdot \frac{6}{16} + \frac{1}{16} \cdot 2$$
$$= 0 + \frac{6}{16} + \frac{2}{16}$$
$$= \frac{8}{16}$$
$$= 0.5$$

Solution:-

Let c be the amount she pays to play and 'y' be the amount she wins.

Using the given information, prob. dist. of y is:

y	$f(y)$	$y \cdot f(y)$
$5-c$	$4/52 + 4/52 = 8/52$	$(5-c).8/52$
$3-c$	$4/52 + 4/52 = 8/52$	$(3-c).8/52$
$0-c$	$36/52$	$(0-c).36/52$

Since the game is fair so the expected amount she will win must be greater than and equal to zero ie $E(y) \geq 0$

$$\Rightarrow E(y) = 0 \quad (\text{for finding } c)$$

$$\Sigma y \cdot f(y) = 0$$

$$(5-c)\frac{8}{52} + (3-c)\frac{8}{52} + (0-c)\frac{36}{52} = 0$$

$$\frac{40}{52} - \frac{8c}{52} + \frac{24}{52} - \frac{8c}{52} - \frac{36c}{52} = 0$$

$$\frac{64}{52} - \frac{52c}{52} = 0$$

$$\frac{64}{52} = \frac{\cancel{52}c}{\cancel{52}}$$

$$\Rightarrow c = 64/52$$

$$\Rightarrow c = \frac{64}{52} = 1.23$$

Thus, she should pay \$1.23 to play.

4.6 **Solution:-**

Given:

Probabilities	1/12	1/12	1/4	1/4	1/6	1/6
Earnings	7	9	11	13	15	17

Let E denote the earnings

$\mathcal{E}$	$P(\mathcal{E})$	$\mathcal{E}.P(\mathcal{E})$
7	1/12	$7/12 = 0.583$
9	1/12	$9/12 = 0.75$
11	1/4	$11/4 = 2.75$
13	1/4	$13/4 = 3.25$
15	1/6	$15/6 = 2.5$
17	1/6	$17/6 = 2.83$

The attendant's expected earning is,

$$E(\mathcal{E}) = \Sigma \mathcal{E}.P(\mathcal{E})$$
$$= 0.58 + 0.75 + 2.75 + 3.25 + 2.5 + 2.83$$
$$= \$12.67.$$

4.7 **Solution:-**

Given:

Investment	Probability
4000 (profit)	0.3
-1000 (loss)	0.7

Let G, denote the person's gain then expected gain is $E(G) = \Sigma GP(G)$

G	P(G)	GP(G)
4000	0.3	1200
-1000	0.7	-700

$$E(G) = 1200 - 700$$
$$= \$500$$

4.8 **Solution:-**

Given: Let S denote the Price

Sale Price (S)	P(S)	S.P(S)
250 (Profit)	0.22	55
150 (profit)	0.36	54
0 (breakeven)	0.28	0
-150 (loss)	0.14	-21

The expected Profit is:

$E(S) = \Sigma S P(S)$

$= 55 + 54 + 0 - 21$

$= \$88.$

1 let X be the amount of loss

X	P(X)	X.P(X)
200,000	0.002	400
+ 100,000 (loss)	0.01	+ 1000
+ 50,000 (loss)	0.1	+ 5000

$E(X) = \Sigma x P(x) = 400 + 1000 + 5000$

$= \$6400$

The insurance company should charge a premium of $\$6400 + \$500 = \$69,00$

11 Solution:-

$$f(x) = \begin{cases} \dfrac{4}{\pi(1+x^2)} & , 0<x<1 \\ 0 & , \text{elsewhere} \end{cases}$$

Since X is a continuous variable

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx.$$

$$= \int_0^1 x \cdot \frac{4}{\pi(1+x^2)}\, dx.$$

$$= \frac{4}{\pi} \int_0^1 x(1+x^2)^{-1} dx.$$

$$= \frac{4}{\pi} \cdot \frac{1}{2} \int_0^1 \frac{2x}{(1+x^2)}\, dx.$$

$$= \frac{2}{\pi} \Big| \ln(1+x^2) \Big|_0^1$$

$$= \frac{2}{\pi} \left[\ln(1+1) - \ln(1+0) \right] = \frac{2}{\pi} \left[\ln 2 - \ln 1 \right] = \frac{2}{\pi} \ln 2$$

4.12 **Solution:-**

$$f(x) = \begin{cases} 2(1-x), & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$$

$$E(x) = \int_0^1 x \cdot f(x)\,dx$$

$$= \int_0^1 x \cdot 2(1-x)\,dx$$

$$= \int_0^1 (2x - 2x^2)\,dx = \int_0^1 2x\,dx - \int_0^1 2x^2\,dx$$

$$= 2\left|\frac{x^2}{2}\right|_0^1 - 2\left|\frac{x^3}{3}\right|_0^1$$

$= (1-0) - \frac{2}{3}(1-0) = 1 - \frac{2}{3} = \frac{1}{3}$, Since dealer profit is \$5000, the average profit per automobile is $\frac{1}{3} \times 5000 =$ \$1666.67

4.13 **Solution:-**

$$f(x) = \begin{cases} x, & 0<x<1 \\ 2-x, & 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$= \int_0^1 x \cdot f(x)\,dx + \int_1^2 x \cdot f(x)\,dx$$

$$= \int_0^1 x \cdot x\,dx + \int_1^2 x \cdot (2-x)\,dx$$

$$= \int_0^1 x^2\,dx + \int_1^2 (2x - x^2)\,dx$$

$$= \left|\frac{x^3}{3}\right|_0^1 + \left[\int_1^2 2x\,dx - \int_1^2 x^2\,dx\right]$$

$$= \frac{1}{3}(1-0) + \left[\frac{2}{2}\left|x^2\right|_1^2 - \left|\frac{x^3}{3}\right|_1^2\right]$$

$$= \frac{1}{3} + \left[(4-1) - \frac{1}{3}(8-1)\right] = \frac{1}{3} + \left[\frac{3}{1} - \frac{7}{3}\right]$$

$$= \frac{1}{3} + \left[\frac{9-7}{3}\right] = \frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$$

the average no. of hours per year, in units of 100 hours is $100E(X) = 100(1) = 100$ hours

(4)

4.14

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$$

$$\mu = E(X) = \int_0^1 x f(x)\, dx$$

$$= \int_0^1 x \cdot \frac{2(x+2)}{5} dx$$

$$= \frac{2}{5}\int_0^1 (x^2+2x)\, dx$$

$$= \frac{2}{5}\left[\int_0^1 x^2 dx + 2\int_0^1 x\, dx\right]$$

$$= \frac{2}{5}\left[\left|\frac{x^3}{3}\right|_0^1 + \frac{2}{2}\left|x^2\right|_0^1\right]$$

$$= \frac{2}{5}\left[\frac{1}{3}(1-0) + \frac{\cancel{2}}{\cancel{2}}(1-0)\right] = \frac{2}{5}\left[\frac{1}{3}+1\right] = \frac{2}{5}\left[\frac{4}{3}\right]$$

$$= \frac{8}{15}$$

4.20 **Solution:-**

$$f(x) = \begin{cases} e^{-x}, & x>0 \\ 0, & \text{elsewhere} \end{cases}$$

$$E[g(x)] = \int_0^\infty g(x) \cdot f(x)\, dx$$

$$= \int_0^\infty e^{2x/3} \cdot e^{-x}\, dx$$

$$= \int_0^\infty e^{\frac{2x}{3}-\frac{x}{1}} dx = \int_0^\infty e^{\frac{2x-3x}{3}} dx$$

$$= \int_0^\infty e^{-\frac{x}{3}} dx = \left|\frac{e^{-x/3}}{-1/3}\right|_0^\infty = -3\left[e^{-\infty} - e^{-0}\right]$$

$$= -3[0-1] = +3$$

Solution:-

$$f(x) = \begin{cases} \frac{32}{(x+4)^3}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Y denotes the hospitalization period $Y = X+4$

we have to find $E(Y)$.

$$E(Y) = \int_{-\infty}^{\infty} Y . f(Y)\, dy$$

$$= \int_0^{\infty} (X+4) \cdot \frac{32}{(x+4)^3}\, dx$$

$$= 32 \int_0^{\infty} \frac{1}{(x+4)^2}\, dx$$

$$= 32 \int_0^{\infty} (x+4)^{-2} dx = 32 \left| \frac{(x+4)^{-1}}{-1} \right|_0^{\infty}$$

$$= -32 \left[0 - (0+4)^{-1} \right]$$

$$= -32 \left(-\frac{1}{4} \right) = +8.$$

7.30 Solution:-

$$f(y) = \begin{cases} 1/4\, e^{-y/4}, & y \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$E(y) = \int_0^{\infty} y \cdot f(y)\, dy$$

$$= \frac{1}{4} \int_0^{\infty} y \cdot e^{-y/4}\, dy$$

$$= \frac{1}{4} \left[y \int_0^{\infty} e^{-y/4} dy - \int_0^{\infty} \frac{d}{dy}(y) \int e^{-y/4} dy \right]$$

$$= \frac{1}{4} \left[\left. \frac{y e^{-y/4}}{-1/4} \right|_0^{\infty} - \int_0^{\infty} 1 \cdot \left. \frac{e^{-y/4}}{-1/4}\, dy \right| \right]$$

$$= \frac{1}{4} \left[-4(0-0) + 4 \left| \frac{e^{-y/4}}{-1/4} \right|_0^{\infty} \right]$$

$= \frac{1}{4}\left[-16(0-1)\right] = \frac{+16}{4} = 4$

7 Solution:-

Given: $g(x) = (2x+1)^2$

we have to find $\mu_{g(x)}$

$\mu_{g(x)} = E[g(x)] = \sum g(x) \cdot f(x)$

$= \sum (2x+1)^2 \cdot f(x)$

x	$(2x+1)^2$	$f(x)$	$(2x+1)^2 \cdot f(x)$
-3	$(2(-3)+1)^2 = 25$	$1/6$	$25 \times 1/6 = 25/6$
6	169	$1/2$	$169 \times 1/2 = 169/2$
9	361	$1/3$	$361 \times 1/3 = 361/3$

So $\mu_{g(x)} = E[g(x)] = \frac{25}{6} + \frac{169}{2} + \frac{361}{3}$

$= \frac{25+507+722}{6} = \frac{1254}{6} = 209$

18 Solution:-

Given: $g(x) = x^2$

$E[g(x)] = \sum g(x) \cdot f(x) = \sum x^2 \cdot f(x)$

The prob. dist of Exercise 4.2 is:

x	$f(x)$	x^2	$x^2 \cdot f(x)$
0	0.422	0	$0 \times 0.422 = 0$
1	0.422	1	0.422
2	0.141	4	0.564
3	0.016	9	0.144
			1.13

$\Rightarrow E[g(x)] = 1.13.$

4.19 Solution:-

Here, x denotes the no. of word processors we are required to find the expected value that firm can spend. Let the expected cost is denoted by $E[g(x)]$

where cost $= g(x) = 1200X - 50X^2$

1200 per unit refunded amount

$E[g(x)] = \sum g(x) \cdot f(x)$

$= \sum (1200x - 50x^2) \cdot f(x)$

The probability dist of x is:

x	$g(x) = 1200x - 50x^2$	$f(x)$	$f(x) \cdot g(x)$
0	$1200(0) - 50(0)^2 = 0$	$1/10$	0
1	1150	$3/10$	$3450/10$
2	2200	$2/5$	$4400/5$
3	3150	$1/5$	$3150/5$

So $E[g(x)] = 0 + \frac{3450}{10} + \frac{4400}{5} + \frac{3150}{5}$

$= 345 + 880 + 630$

$= \$1855$

.21 we are required to find $E[g(x)]$ where $g(x) = x^2$ and $f(x) = \begin{cases} 2(1-x), & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$

So $E[g(x)] = \int_{-\infty}^{\infty} g(x) \cdot f(x)\, dx$

$= \int_0^1 x^2 \cdot 2(1-x)\, dx$

$= 2 \int_0^1 (x^2 - x^3)\, dx$

$= 2 \left| \frac{x^3}{3} - \frac{x^4}{4} \right|_0^1$

$= 2\left[\left(\frac{1}{3} - \frac{1}{4}\right) - (0-0)\right]$

$= 2\left(\frac{4-3}{12}\right) = \frac{1}{6}$

So the average profit per automobile is
$\$5000 \times \frac{1}{6} = \$833.33$.

4.27 **Solution:**

In Exercise 3.27,

$$f(x) = \begin{cases} \frac{1}{2000} e^{-x/2000}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The mean no. of hours to failure of the component is $E(x)$.

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{0}^{\infty} x \cdot \frac{1}{2000} e^{-x/2000}\, dx.$$

$$= \frac{1}{2000} \int_{0}^{\infty} x e^{-x/2000}\, dx$$

$$= \frac{1}{2000} \left[x \cdot \int_{0}^{\infty} e^{-x/2000} dx - \int_{0} \frac{d}{dx}(x) \int e^{-x/2000} dx \right]$$

$$= \frac{1}{2000} \left[\frac{x e^{-x/2000}}{-1/2000} \Big|_{0}^{\infty} - \int_{0}^{\infty} \frac{1 \cdot e^{-x/2000}}{-1/2000} dx \right]$$

$$= \frac{1}{2000} \left[-\left(\infty e^{-\infty/2000} - 0 e^{-0/2000} \right) \frac{1}{1/2000} + 2000 \left| \frac{e^{-x/2000}}{-1/2000} \right|_{0}^{\infty} \right]$$

$$= \frac{1}{2000} \left[-\left(\infty \times 0 - 0 \right) \frac{1}{1/2000} - (2000)^2 \{ e^{-\infty} - e^{-0} \} \right]$$

$$= \frac{1}{2000} \left[0 - (2000)^2 (0 - 1) \right] = \frac{(2000)^{2}}{2000} = 2000.$$

4.28 **Solution:**

$$f(x) = \begin{cases} \frac{2}{5}, & 23.75 \leq x \leq 26.25 \\ 0, & \text{elsewhere} \end{cases}$$

(b) $E(x) = \int_{23.75}^{26.25} x \cdot f(x)\, dx$

$= \int_{23.75}^{26.25} x \cdot \frac{2}{5}\, dx$

$= \frac{2}{5} \left| \frac{x^2}{2} \right|_{23.75}^{26.25}$

$= \frac{1}{5}\left((26.25)^2 - (23.73)^2\right)$

$= 125/5 = 25$

4.29 Solution:-

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & \text{elsewhere} \end{cases}$$

(b) $E(x) = \int_{1}^{\infty} x \cdot f(x)\, dx$

$= \int_{1}^{\infty} x \cdot 3x^{-4}\, dx$

$= 3\int_{1}^{\infty} x^{-3}\, dx = 3 \left| \frac{x^{-3+1}}{-3+1} \right|_{1}^{\infty}$

$= -\frac{3}{2}\left[\infty^{-2} - 1^{-2}\right]$

$= -1.5(0-1) = +1.5$

45 **4.31** Solution:-

$$f(y) = \begin{cases} 5(1-y)^4, & 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

$E(y) = \int_{0}^{1} y \cdot f(y)\, dy$

$= \int_{0}^{1} 5(1-y)^4 \cdot y\, dy$

$= 5\int_{0}^{1} y(1-y)^4\, dy$

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$$= 5\left[\frac{y(1-y)^5}{-5}\Big|_0^1 - \int_0^1 \frac{d}{dy}(y)\int (1-y)^4 dy \right]$$

$$= 5\left[\frac{1}{-5}\{1(1-1)^5 - 0(1-0)\} - \int_0^1 1 \cdot \frac{(1-y)^5}{-5}\,dy\right]$$

$$= 5\left[\frac{-1}{5}(0) + \frac{1}{5} \left| \frac{(1-y)^6}{-6}\right|_0^1 \right]$$

$$= 5\left[-\frac{1}{30}\left((1-1)^6 - (1-0)^6 \right)\right]$$

$$= \cancel{5}\left[-\frac{1}{\cancel{30}_{6}}\left(0 - 1\right)\right] = +\frac{1}{6} = 0.167$$

P(company exceeds the mean) = $P(X > 0.167)$

$$= \int_{0.167}^{1} 5(1-y)^4\,dy$$

$$= \cancel{5}\left| \frac{(1-y)^5}{-\cancel{5}} \right|_{0.167}^{1}$$

$$= (-)\left[(1-1)^5 - (1-0.167)^5\right]$$

$$= -\left[0 - (0.4019)\right]$$

$$= +0.4019$$

:32 Solution:-

x	$f(x)$	$xf(x)$	x^2	$x^2f(x)$
0	0.41	0	0	0
1	0.37	0.37	1	0.37
2	0.16	0.32	4	0.64
3	0.05	0.15	9	0.45
4	0.01	0.04	16	0.16
		0.88		1.62

b) The expected number of imperfections is

$\mu = E(x) = \Sigma x f(x) = 0.88.$

c) $E(x^2) = \Sigma x^2 f(x) = 1.62.$

Definition 4.3: $\sigma^2 = E(X-\mu)^2 = \Sigma(x-\mu)^2 f(x)$

The prob. dist. of X in Exercise 4.7 is:

X	$f(X)$	$xf(x)$
4000	0.3	1200
-1000	0.7	-700
		500

$\mu = E(X) = \Sigma x f(x) = 500\$$

For variance σ^2:

$X-\mu$	$(X-\mu)^2$	$(X-\mu)^2 f(x)$
$4000-500=3500$	$(3500)^2$	3675000
$-1000-500=-1500$	$(-1500)^2$	1575000

$\Rightarrow \sigma^2 = E(X-\mu)^2 = \Sigma(x-\mu)^2 f(x) = 3675000+1575000$
$= 5250,000$

.34 **Solution:-**

x	$f(x)$	$xf(x)$	x^2	$x^2 f(x)$
-2	0.3	-0.6	4	1.2
3	0.2	0.6	9	1.8
5	0.5	2.5	25	12.5
		2.5		15.5

$\sigma^2 = E(X^2) - E(X)^2$

where $E(X^2) = \Sigma x^2 f(x) = 15.5$

and $E(X) = \Sigma x f(x) = 2.5$

$\Rightarrow \sigma^2 = 15.5 - (2.5)^2 = 9.25$

Since Standard deviation is the square root of variance

$\Rightarrow \sigma = \sqrt{9.25} = 3.041$

4.35 **Solution:-**

Theorem 4.2: $\sigma^2 = E(X^2) - \mu^2$.

x	$f(x)$	x^2	$x^2 f(x)$	$x f(x)$
2	0.01	4	0.04	0.02
3	0.25	9	2.25	0.75
4	0.4	16	6.4	1.6
5	0.3	25	7.5	1.5
6	0.04	36	1.44	0.24
			17.63	4.11

$E(X^2) = \sum x^2 f(x) = 17.63$

$\mu = E(X) = \sum x f(x) = 4.11$

So $\sigma^2 = E(X^2) - \mu^2$

$= 17.63 - (4.11)^2$

$= 0.7379$

4.36 **Solution:-**

The prob. dist. of random variable X is:

x	$f(x)$	$x f(x)$	x^2	$x^2 f(x)$
0	0.4	0	0	0
1	0.3	0.3	1	0.3
2	0.2	0.4	4	0.8
3	0.1	0.3	9	0.9
		1		2

Mean: $\mu = E(X) = \sum x f(x) = 1$

Variance: $\sigma^2 = E(X^2) - \mu^2$

where $E(X^2) = \sum x^2 f(x) = 2$ and $\mu = 1$

$\Rightarrow \sigma^2 = 2 - 1^2 = 1$

4.37 **Solution:-**

Density function in Ex. 4.12 is:

$$f(x) = \begin{cases} 2(1-x), & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$$

for which mean $= E(X) = 1/3$ (see solution 4.12)

we are required to find the variance.

$\sigma^2 = E(X^2) - \mu^2$

where $E(X^2) = \int_0^1 x^2 . f(x)\, dx$

$= \int_0^1 x^2 . 2(1-x)\, dx$

$= 2\int_0^1 (x^2 - x^3)\, dx$

$= 2\left[\frac{x^3}{3}\Big|_0^1 - \frac{x^4}{4}\Big|_0^1 \right]$

$= 2\left[\frac{1}{3}(1^3 - 0^3) - \frac{1}{4}(1^4 - 0^4) \right]$

$= 2\left[\frac{1}{3} - \frac{1}{4} \right] = \cancel{2}\left[\frac{4-3}{\cancel{12}_6} \right] = \frac{1}{6}$

So,

$\sigma^2 = E(X^2) - \mu^2 = \frac{1}{6} - \left(\frac{1}{3}\right)^2 = \frac{1}{6} - \frac{1}{9} = \frac{3-2}{18} = \frac{1}{18}$

The variance for dealer's profit in units of \$5000 is $(5000)^2 . \frac{1}{18}$ $\quad \therefore V(aX) = a^2 V(X)$

$= \$1388,888.889$

4.38 Solution:-

The density function of X in Ex. 4.14 is:

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$$

with mean $= \mu = E(X) = \frac{8}{15}$ (See Solution Ex. 4.14)

we are required to find variance.

$\sigma^2 = E(X^2) - \mu^2$

where $E(X^2) = \int_0^1 x^2 . f(x)\, dx = \int_0^1 x^2 . \frac{2}{5}(x+2)\, dx$

$= \frac{2}{5}\int_0^1 (x^3 + 2x^2)\,dx$

$= \frac{2}{5}\left| \frac{x^4}{4} + \frac{2x^3}{3} \right|_0^1$

$= \frac{2}{5}\left[\left\{ \frac{1^4}{4} + \frac{2}{3}(1)^3 \right\} - \left\{ \frac{0}{4} + \frac{0}{3} \right\} \right]$

$= \frac{2}{5}\left[0.25 + 0.67 \right] = 0.367$

$\Rightarrow \sigma^2 = E(X^2) - \mu^2$

$= 0.367 - (8/15)^2$

$= 0.367 - 0.284$

$= 0.083$

4.39 **Solution:-**

The density function given in Ex. 4.13 is:

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

with $\mu = E(X) = 1$ (See Solution Ex 4.13)

we are required to find the variance of X

$\sigma^2 = E(X^2) - \mu^2$

where $E(X^2) = \int_0^2 x^2 f(x)\,dx$

$= \int_0^1 x^2 f(x)\,dx + \int_1^2 x^2 \cdot f(x)\,dx$

$= \int_0^1 x^2 \cdot x\,dx + \int_1^2 x^2 \cdot (2-x)\,dx$

$= \int_0^1 x^3\,dx + \int_1^2 (2x^2 - x^3)\,dx$

$= \frac{x^4}{4}\Big|_0^1 + \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_1^2$

$= \frac{1}{4}(1^4 - 0^4) + \left[\left\{ \frac{2}{3}(2)^3 - \frac{(2)^4}{4} \right\} - \left\{ \frac{2}{3}(1)^3 - \frac{(1)^4}{4} \right\} \right]$

$$= \frac{1}{4} + \left[\left(\frac{2}{3}(8) - \frac{16}{4}\right) - \left(\frac{2}{3} - \frac{1}{4}\right)\right]$$

$$+ \left[\left(\frac{16}{3} - \frac{16}{4}\right) - \left(\frac{2}{3} - \frac{1}{4}\right)\right]$$

$$= 1.167$$

So, $\sigma^2 = E(X^2) - \mu^2$

$$= 1.167 - 1^2$$

$$= 1.167.$$

4.41 Solution:-

The prob. dist. for Exercise 4.17 is:

x	$(2x+1)^2$	$f(x)$	$(2x+1)^2 f(x)$	$(2x+1)^4$	$(2x+1)^4 f(x)$
-3	25	1/6	25/6	625	625/6
6	169	1/2	169/2	28561	28561/2
9	361	1/3	361/3	130321	130321/3

and $\mu = E[g(x)] = 209$ (see solution 4.17)

we are required to find the standard deviation of $g(x)$ i.e $(2x+1)^2$.

Since $\sigma^2_{g(x)} = E[g(x)^2] - E[g(x)]^2$

Here $E[g(x)^2] = \Sigma\, g(x)^2 \cdot f(x)$

$$= \Sigma\, [(2x+1)^2]^2 f(x)$$

$$= \Sigma\, (2x+1)^4 f(x)$$

$$= \frac{625}{6} + \frac{28561}{2} + \frac{130321}{3}$$

$$= 104.17 + 14280.5 + 43440.33$$

$$= 57825.003$$

$\Rightarrow \sigma^2_{g(x)} = 57825.003 - (209)^2 = 14144.003$

and $\sigma_{g(x)} = \sqrt{14144.003} = 118.93$

Solution:-

The density function is:

$$f(x) = \begin{cases} 2(1-x), & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$$ (10)

Using the results of Exercise 4.21, we have

$$E[g(X)] = E(X^2) = \frac{1}{6}$$

Now, to find the variance of $g(X) = X^2$

$$\sigma^2_{g(x)} = E[g(X)^2] - E[g(X)]^2$$

where $E[g(X)^2] = \int_0^1 g(X)^2 \cdot f(x)\,dx$

$$= \int_0^1 (X^2)^2 \cdot f(x)\,dx$$

$$= \int_0^1 x^4 \cdot 2(1-x)\,dx = 2\int_0^1 (x^4 - x^5)\,dx.$$

$$= 2\left|\frac{x^5}{5} - \frac{x^6}{6}\right|_0^1 = 2\left[\left\{\frac{1}{5} - \frac{1}{6}\right\} - \left\{\frac{0}{5} - \frac{0}{6}\right\}\right]$$

$$= 2\left[\frac{6-5}{30}\right] = \frac{1}{15}$$

Hence, $\sigma^2_{g(x)} = E[g(X)^2] - E[g(X)]^2$

$$= \frac{1}{15} - \left(\frac{1}{6}\right)^2$$

$$= 0.067 - 0.0278$$

$$= 0.039$$

4.43 ## Solution:-

$$f(x) = \begin{cases} 1/4\, e^{-x/4}, & x>0 \\ 0 & \text{elsewhere} \end{cases}$$

The mean of random variable $Y = 3X - 2$ is

$$E(Y) = \int_0^\infty Y \cdot f(y)\,dy$$

where $f(y) = f(x)$

$$= \int_0^\infty (3X-2)\,\frac{1}{4} e^{-x/4}\,dx$$

$$= \frac{1}{4}\int_0^\infty (3x-2)e^{-x/4}\,dx$$

$$= \frac{1}{4}\left[3\int_0^\infty xe^{-x/4}dx - 2\int_0^\infty e^{-x/4}dx\right]$$

$$= \frac{1}{4}\left[3\left\{\frac{xe^{-x/4}}{-1/4}\Big|_0^\infty - \int_0^\infty \frac{d}{dx}(x)\int e^{-x/4}dx\right\} - 2\left|\frac{e^{-x/4}}{-1/4}\right|_0^\infty\right]$$

$$= \frac{1}{4}\left[3\left\{\frac{\infty e^{-\infty/4}}{-1/4} - \frac{0e^{-0/4}}{-1/4} - \int_0^\infty \frac{e^{-x/4}}{-1/4}dx\right\} + 8(e^{-\infty} - e^{-0})\right]$$

$$= \frac{1}{4}\left[3\left\{(0-0) + 4\left|\frac{e^{-x/4}}{-1/4}\right|_0^\infty\right\} + 8(0-1)\right]$$

$$= \frac{1}{4}\left[-48(0-1) - 8\right]$$

$$= \frac{1}{4}\left[48-8\right] = \frac{40}{4} = 10.$$

$$\Rightarrow \boxed{E(Y) = 10}$$

Now for variance:

$$\sigma_y^2 = E(Y^2) - E(Y)^2$$

where $E(Y^2) = E[(3X-2)^2] = \int_0^\infty (3x-2)^2 \cdot \frac{1}{4}e^{-x/4}\,dx.$

$$= \frac{1}{4}\int_0^\infty (9x^2+4-12x)e^{-x/4}dx.$$

$$= \frac{1}{4}\int_0^\infty (9x^2e^{-x/4} - 12xe^{-x/4} + 4e^{-x/4})dx.$$

$$= \frac{1}{4}\left[9\int_0^\infty x^2e^{-x/4}dx - 12\int_0^\infty xe^{-x/4} + 4\int_0^\infty e^{-x/4}dx\right]$$

let $A = \int_0^\infty x^2e^{-x/4}dx$, $B = \int_0^\infty xe^{-x/4}dx$, $C = \int_0^\infty e^{-x/4}dx$.

$$\Rightarrow E(Y^2) = \frac{1}{4}\left[9A - 12B + 4C\right] \quad \text{—(1)}$$

Consider $A = \int_0^\infty x^2e^{-x/4}dx.$

$$= \frac{x^2e^{-x/4}}{-1/4}\Big|_0^\infty - \int_0^\infty \frac{d}{dx}(x^2)\int e^{-x/4}dx\,dx.$$

$$= (0-0) - \int_0^\infty 2x \frac{e^{-x/4}}{-1/4} dx.$$

$$= +8 \left\{ x\frac{e^{-x/4}}{-1/4}\Big|_0^\infty - \int 1 \cdot \frac{e^{-x/4}}{-1/4} dx \right\}$$

$$= 8\left[(0-0) + 4 \left| \frac{e^{-x/4}}{-1/4} \right|_0^\infty \right]$$

$$= 8\left[-16(e^{-\infty} - e^{-0}) \right] = 8[-16(0-1)] = +128$$

Consider $B = \int_0^\infty x e^{-x/4} dx.$

$$= \frac{xe^{-x/4}}{-1/4}\Big|_0^\infty - \int_0 \frac{1e^{-x/4}}{-1/4} dx.$$

$$= (0-0) + 4 \left| \frac{e^{-x/4}}{-1/4} \right|_0^\infty = -16(e^{-\infty} - e^{-0}) = -16(0-1)$$
$$= +16$$

Now $C = \int_0^\infty e^{-x/4} dx = \frac{e^{-x/4}}{-1/4}\Big|_0^\infty = -4(e^{-\infty} - e^{-0})$
$= -4(0-1) = +4$

Substituting the values of A, B and C in eq 1

$$\Rightarrow E(y^2) = \frac{1}{4}\left[9(128) - 12(16) + 4(4) \right]$$

$$= 976/4 = 244$$

Since $\sigma_y^2 = E(y^2) - E(y)^2$

$$\Rightarrow \sigma_y^2 = 244 - (10)^2$$
$$= 244 - 100$$
$$= 144$$

19 Solution:-

x	$f(x)$	$xf(x)$	x^2	$x^2f(x)$
0	0.41	0	0	0
1	0.37	0.37	1	0.37
2	0.16	0.32	4	0.64
3	0.05	0.15	9	0.45
4	0.01	0.04	16	0.16
		0.88		1.62

Here $\mu = E(X) = \Sigma x f(x) = 0.88$.

$E(X^2) = \Sigma x^2 f(x) = 1.62$.

Since $\sigma_X^2 = E(X^2) - E(X)^2$

$= 1.62 - (0.88)^2$

$= 0.8456$ is the required variance.

$\sigma_x = \sqrt{\sigma^2 x} = \sqrt{0.8456} = 0.9196$ is the required Standard deviation.

4.50 Solution:-

$$f(x) = \begin{cases} 2(1-x), & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$$

Variance $= \sigma_X^2 = E(X^2) - E(X)^2$

where $E(X^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 2(1-x) \cdot x^2\,dx$.

$= 2\int_0^1 (x^2 - x^3)\,dx = 2\left|\frac{x^3}{3} - \frac{x^4}{4}\right|_0^1$

$= 2\left[\left\{\frac{1}{3} - \frac{1}{4}\right\} - \left\{\frac{0}{3} - \frac{0}{4}\right\}\right] = \cancel{2}\left[\frac{4-3}{\cancel{12}_6}\right] = \frac{1}{6}$.

and $E(X) = \int_0^1 x \cdot f(x)\,dx = \int_0^1 x\, 2(1-x)\,dx$.

$= 2\int_0^1 (x - x^2)\,dx = 2\left[\left|\frac{x^2}{2} - \frac{x^3}{3}\right|_0^1\right]$

$= 2\left[\left\{\frac{1}{2} - \frac{1}{3}\right\} - \left\{\frac{0}{2} - \frac{0}{3}\right\}\right] = \cancel{2}\left[\frac{3-2}{\cancel{6}_3}\right] = \frac{1}{3}$.

$\Rightarrow \sigma_x^2 = \frac{1}{6} - \left(\frac{1}{3}\right)^2 = 0.167 - 0.111$

$= 0.056$

Standard deviation $= \sqrt{\text{Variance}}$

$\Rightarrow \sigma = \sqrt{\sigma_x^2} = \sqrt{0.056} = 0.236$

Solution:

In Exercise 4.35, we had:

$\mu = E(X) = 4.11$

$\sigma^2 = E(X^2) - \mu^2 = 0.7379$ and $E(X^2) = 17.63$.

Now we have to find the mean and variance of $Z = 3X - 2$.

Mean:

$$Z = 3X - 2$$

Applying expectation on both sides

$$E(Z) = E(3X - 2)$$
$$E(Z) = E(3X) - E(2)$$
$$= 3E(X) - 2 \qquad \because E(aX) = aE(X) \; \& \; E(a) = a.$$

Substituting $E(X) = 4.11$ in above equation

$$\Rightarrow E(Z) = 3(4.11) - 2 = 10.33$$

Thus Mean of random variable $Z = 10.33$

Variance:

$$Z = 3X - 2$$

Applying variance on both sides.

$$V(Z) = V(3X - 2)$$
$$= V(3X) + V(2)$$
$$= 3^2 V(X) + 0 \qquad \because V(aX) = a^2 V(X) \; \& \; V(a) = 0$$
$$= 9V(X)$$

Substituting $V(X) = \sigma^2 = 0.7379$

$$\Rightarrow V(Z) = 9 \times 0.7379 = 6.641$$

Thus variance of random variable $Z = 6.641$

4.54 **Solution:**

Theorem 4.5 states: $E(aX + b) = aE(X) + b$.

Corollary 4.6: $\sigma^2_{aX+c} = a^2 \sigma^2_X = a^2 \sigma^2$.

or $V(aX + C) = a^2 V(X) + V(C)$

$$= a^2 V(X) + 0$$
$$= a^2 V(X)$$

From Exercise 4.36 Solution we have:

$\mu = E(X) = 1$

$\sigma^2 = E(X^2) - \mu^2 = 1$

we have to find the mean & variance of $Z = 5X+3$

for Mean:

$Z = 5X+3$

Applying expectation on both sides.

$E(Z) = E(5X+3)$

$E(Z) = 5E(X) + E(3)$

$E(Z) = 5E(X) + 3$ (Theorem 4.5)

Since $E(X) = 1$

$\Rightarrow E(Z) = 5(1) + 3 = 8$.

For Variance:

$Z = 5X+3$

Applying variance on both sides.

$V(Z) = V(5X+3)$

$= V(5X) + V(3)$

$= 25V(X) + 0$ (Corollary 4.6)

$\Rightarrow V(Z) = 25V(X)$

Since $V(X) = \sigma^2 = 1$.

$\Rightarrow V(Z) = 25(1) = 25$.

4.56 Solution:-

In Exercise 4.43, $Y = 3X - 2$.

For Mean:

$Y = 3X - 2$

Applying expectation on both sides

$E(Y) = E(3X-2)$ (using Theorem 4.5)

$E(Y) = E(3X) - E(2)$

$E(Y) = 3E(X) - 2$ — (1)

Now we have to find $E(X)$.

Given: $f(x) = \begin{cases} \frac{1}{4} e^{-x/4}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$

$E(X) = \int_0^\infty x \cdot f(x)\, dx$

$= \int_0^\infty x \cdot \frac{1}{4} e^{-x/4}\, dx$

$= \frac{1}{4}\int_0^\infty x e^{-x/4}\, dx$

$= \frac{1}{4}\left[\frac{x e^{-x/4}}{-1/4}\Big|_0^\infty - \int_0^\infty 1 \cdot \frac{e^{-x/4}}{-1/4}\, dx \right]$

$= \frac{1}{4}\left[(0-0) + 4 \left| \frac{e^{-x/4}}{-1/4} \right|_0^\infty \right]$

$= \frac{1}{4}\left[-16 (e^{-\infty} - e^{-0}) \right]$

$= \frac{1}{4}\left[-16(0-1) \right] = +4$

Substituting $E(X)$ in eq ①

$\Rightarrow \quad E(Y) = 3(4) - 2 = 12 - 2 = 10$

For variance:

$Y = 3X - 2$

Applying variance on both sides

$V(Y) = V(3X-2) = V(3X) + V(2) = 3^2 V(X) + 0$

$\Rightarrow V(Y) = 9V(X)$ — ② (Using Corollary 4.6)

Now we have to find $V(X)$

$V(X) = E(X^2) - E(X)^2$

where $E(X^2) = \int_0^\infty x^2 f(x)\, dx$

$= \int_0^\infty x^2 \cdot \frac{1}{4} e^{-x/4}\, dx$

$= \frac{1}{4}\int_0^\infty x^2 e^{-x/4}\, dx$

$= \frac{1}{4}\left[\frac{x^2 e^{-x/4}}{-1/4}\Big|_0^\infty - \int_0^\infty \frac{2x e^{-x/4}}{-1/4}\, dx \right]$

$= \frac{1}{4}\left[(0-0) + 8\left\{ \frac{x e^{-x/4}}{-1/4}\Big|_0^\infty - \int_0^\infty \frac{e^{-x/4}}{-1/4}\, dx \right\} \right]$

$$= \frac{1}{4}\left[8\left\{ (0-0) + 4 \left| \frac{e^{-x/4}}{-1/4} \right|_0^\infty \right\} \right]$$

$$= \frac{1}{4}\left[8(0) - 8(16)(e^{-\infty} - e^{-0}) \right]$$

$$= \frac{1}{4}\left[0 - 128(0-1) \right] = \frac{+128}{4} = 32$$

So, $V(X) = 32 - (4)^2$

$= 32 - 16 = 16.$

Substituting $V(X)$ in eq 2

$\Rightarrow V(Y) = 9(16) = 144.$

4.57 Solution:-

x	$f(x)$	$xf(x)$	$x^2 f(x)$	x^2
-3	$1/6$	$-3/6$	$9/6$	9
6	$1/2$	$6/2$	$36/2$	36
9	$1/3$	$9/3$	$81/3$	81

$$E(X) = \Sigma x f(x) = \frac{-3}{6} + \frac{6}{2} + \frac{9}{3} = -0.5 + 3 + 3$$
$$= 5.5$$

$$E(X^2) = \Sigma x^2 f(x) = \frac{9}{6} + \frac{36}{2} + \frac{81}{3} = 1.5 + 18 + 27$$
$$= 46.5$$

using these values we have to evaluate $E[(2X+1)^2]$

$$E[(2x+1)^2] = E[(2X)^2 + (1)^2 + 2(2X)(1)]$$
$$= E[4X^2 + 1 + 4X]$$
$$= E(4X^2) + E(1) + E(4X)$$
$$= 4E(X^2) + 1 + 4E(X)$$

Substituting values of $E(X^2)$ and $E(X)$

$$\Rightarrow E[(2X+1)^2] = 4(46.5) + 1 + 4(5.5)$$
$$= 209$$

Solution:-

$E[(X-1)^2] = 10$ and $E[(X-2)^2] = 6$.

Consider $E[(X-1)^2] = 10$

$E[X^2 + 1^2 - 2X] = 10$

$E(X^2) + E(1^2) - 2E(X) = 10$

$E(X^2) + 1 - 2E(X) = 10$

$E(X^2) - 2E(X) = 10 - 1$

$\Rightarrow E(X^2) - 2E(X) = 9$ — ①

Consider $E[(X-2)^2] = 6$

$E[X^2 + 2^2 - 4X] = 6$

$E(X^2) + E(4) - 4E(X) = 6$

$E(X^2) + 4 - 4E(X) = 6$

$E(X^2) - 4E(X) = 6 - 4$

$E(X^2) - 4E(X) = 2$ — ②

Subtracting eq ① from eq ②.

$$\begin{array}{r} \cancel{E(X^2)} - 4E(X) = 2 \\ \underline{-\cancel{E(X^2)} \mp 2E(X) = -9} \\ -2E(X) = -7 \end{array}$$

$\Rightarrow E(X) = +7/2 = 3.5$

Thus $\mu = E(X) = 3.5$

Now to find σ^2, we need $E(X^2)$.

Substitute $E(X) = 3.5$ in eq ① or ②.

Eq ① $\Rightarrow E(X^2) - 2(3.5) = 9$

$E(X^2) = 9 + 7 = 16$.

Since $\sigma^2 = E(X^2) - \mu^2$

$\Rightarrow \sigma^2 = 16 - (3.5)^2$

$= 16 - 12.25$.

Thus $\sigma^2 = 3.75$.

4.62 Solution:-

Given: $\sigma_X^2 = 5$, $\sigma_Y^2 = 3$, X & Y are independent random variables.

we have to find the variance of $Z = -2X + 4Y - 3$

Applying variance on both sides of Z.

$$V(Z) = V(-2X + 4Y - 3)$$

Since X & Y are independent so covariance term will be zero.

$$V(Z) = (-2)^2 V(X) + 4^2 V(Y) + V(3)$$

$$= 4V(X) + 16V(Y) + 0.$$

Since $V(X) = \sigma_X^2 = 5$ and $V(Y) = \sigma_Y^2 = 3$

$$\Rightarrow V(Z) = 4(5) + 16(3) = 20 + 48$$

$$= 68$$

4.63 Solution:-

Now X and Y are not independent so covariance term will not be zero on applying variance.

Given: $Cov(X,Y) = \sigma_{XY} = 1$.

$$Z = -2X + 4Y - 3.$$

$$V(Z) = V(-2X) + V(4Y) + V(3) + 2(-2)(4)\ Cov(X,Y)$$

(see Theorem 4.9)

$$V(Z) = (-2)^2 V(X) + 4^2 V(Y) + 0 - 16\,Cov(X,Y)$$

$$V(Z) = 4V(X) + 16V(Y) - 16\,Cov(X,Y)$$

Since $V(X) = \sigma_X^2 = 5$, $V(Y) = \sigma_Y^2 = 3$

and $Cov(X,Y) = \sigma_{XY} = 1$

$$\Rightarrow V(Z) = 4(5) + 16(3) - 16(1)$$

$$= 20 + 48 - 16$$

$$= 52$$

4.65 Solution:-

X represents the no. that occurs when a red die is tossed so the probability dist. of X is:

x	$f(x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

$$E(x) = \Sigma x f(x)$$
$$= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$
$$= (1/6)[1+2+3+4+5+6]$$
$$= \frac{1}{6}(21)$$
$$= 3.5$$

Similarly the prob. dist of Y, the no. that occurs when a green die is tossed is:

y	$f(y)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6.

$$E(y) = \Sigma y f(y) = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$$
$$= (1/6)[1+2+3+4+5+6]$$
$$= (1/6) \times 21$$
$$= 3.5.$$

a) $E(X+Y) = E(X) + E(Y) = 3.5 + 3.5 = 7$

b) $E(X-Y) = E(X) - E(Y) = 3.5 - 3.5 = 0$

c) $E(XY) = E(X)E(Y) = (3.5)(3.5) = 12.25$

4.66 **Solution:-**

X represents the no. that occurs when a green die is tossed, the probability dist will be same as given in Solution of Exercise 4.65. for both X and Y.

x	$f(x)$	$xf(x)$	x^2	$x^2f(x)$
1	1/6	1/6	1	1/6
2	1/6	2/6	4	4/6
3	1/6	3/6	9	9/6
4	1/6	4/6	16	16/6
5	1/6	5/6	25	25/6
6	1/6	6/6	36	36/6
		3.5		15.167

$\mu_x = E(X) = \Sigma x f(x) = 3.5$

$E(X^2) = \Sigma x^2 f(x) = 15.167$

so $\sigma_x^2 = V(X) = E(X^2) - E(X)^2$

$= 15.167 - (3.5)^2$

$= 2.92$

The dist of Y will be same, giving

$\mu_y = E(Y) = \Sigma y f(y) = 3.5$

$E(Y^2) = \Sigma y^2 f(y) = 15.167$

and $\sigma_y^2 = V(Y) = 2.92$.

a) $2X - Y$.

let $Z = 2X - Y$

$V(Z) = V(2X - Y)$

The outcomes on two dice are always independent so covariance term will be zero.

$\Rightarrow V(Z) = V(2X) + V(Y)$ $\quad \because V(X-Y) = V(X) + V(Y)$

$= 4V(X) + V(Y)$

$= 4(2.92) + 2.92$

$= 14.6$

$X + 3Y - 5$

let $Z = X + 3Y - 5$.

$V(Z) = V(X + 3Y - 5)$

$= V(X) + V(3Y) + V(5)$

$= V(X) + 3^2 V(Y) + 0$

$= V(X) + 9V(Y)$

$= 2.92 + 9(2.92)$

$= 29.2$

Solution:-

$$f(y) = \begin{cases} \frac{1}{4} e^{-y/4}, & 0 \le y \le \infty. \\ 0, & \text{elsewhere} \end{cases}$$

) The mean time to reflex is $E(Y) = \mu$

$$\mu = E(Y) = \int_0^\infty y \cdot f(y)\, dy$$

$$= \int_0^\infty y \cdot \frac{1}{4} e^{-y/4}\, dy$$

$$= \frac{1}{4}\left[\frac{y e^{-y/4}}{-1/4}\Big|_0^\infty - \int_0^\infty 1 \cdot \frac{e^{-y/4}}{-1/4}\, dy \right]$$

$$= \frac{1}{4}\left[(0 - 0) + 4 \left| \frac{e^{-y/4}}{-1/4} \right|_0^\infty \right]$$

$$= \frac{1}{4}\left[-16 (e^{-\infty} - e^{-0}) \right]$$

$$= \frac{1}{4}\left[-16 (0 - 1) \right] = +4$$

b) $$E(Y^2) = \int_0^\infty y^2 \cdot \frac{1}{4} e^{-y/4}\, dy$$

$$= \frac{1}{4} \int_0^\infty y^2 e^{-y/4}\, dy$$

$$= \frac{1}{4}\left[\frac{y^2 e^{-y/4}}{-1/4}\Big|_0^\infty - \int_0^\infty \frac{2y e^{-y/4}}{-1/4}\, dy \right]$$

$= \frac{1}{4}\left[(0-0) + 8\int_0^{\infty} y e^{-y/4} dy \right]$

$= \frac{8}{4}\left[\frac{y e^{-y/4}}{-1/4} \Big|_0^{\infty} - \int_0^{\infty} \frac{1 . e^{-y/4}}{-1/4} dy \right]$

$= 2\left[(0-0) + 4 \frac{e^{-y/4}}{-1/4} \Big|_0^{\infty} \right]$

$= 2\left[-16 (e^{-\infty} - e^{-0}) \right]$

$= 2[-16(0-1)]$

$= 2(+16)$

$= 32.$

$V(Y) = E(Y^2) - E(Y)^2$

$= 32 - 4^2$

$= 32 - 16$

$= 16.$

4.72 **Solution:-**

$f(y) = \begin{cases} 1 & , 7 \le y \le 8 \\ 0 & , \text{elsewhere} \end{cases}$

(b) $E(Y) = \int_7^8 y f(y)\, dy$

$= \int_7^8 y . 1\, dy = \frac{y^2}{2} \Big|_7^8 = \frac{1}{2}(8^2 - 7^2)$

$= \frac{1}{2}(64 - 49) = \frac{15}{2} = 7.5$

$E(Y^2) = \int_7^8 y^2 f(y)\, dy = \int_7^8 y^2 . 1\, dy = \frac{y^3}{3} \Big|_7^8$

$= \frac{1}{3}(8^3 - 7^3) = \frac{1}{3}(512 - 343) = \frac{169}{3} = 56.33$

$V(Y) = E(Y^2) - E(Y)^2$

$= 56.33 - (7.5)^2$

$= 0.0833$