

# Image Processing in Biomedical Applications

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- Image processing: what for?

## 2 Digital images

- Image acquisition and representation

## 3 Image enhancement

- Spatial Filtering
- Intensity transformations
- Image histograms and equalization
- Convolution filters
- Order-statistic filters



4

## Image segmentation

- Thresholding
- Texture segmentation
- Edge detection
- Active contours

5

## Image Registration

- Basics

6

## Shape analysis

- Basics

# Imaging modalities

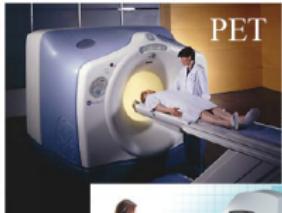
CT



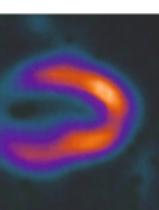
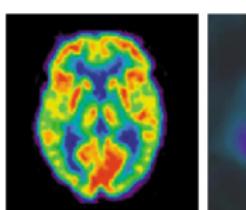
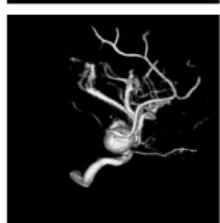
MRI / fMRI



Nuclear



Ultrasound

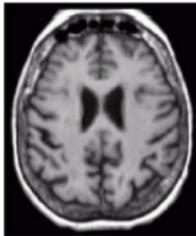
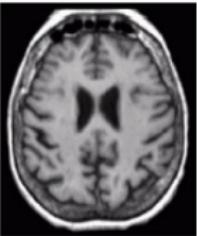
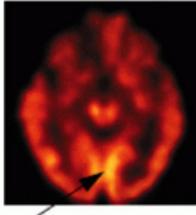


X-ray





# Anatomical vs. Functional Imaging

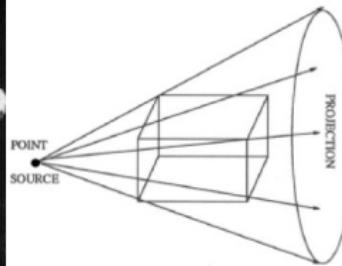
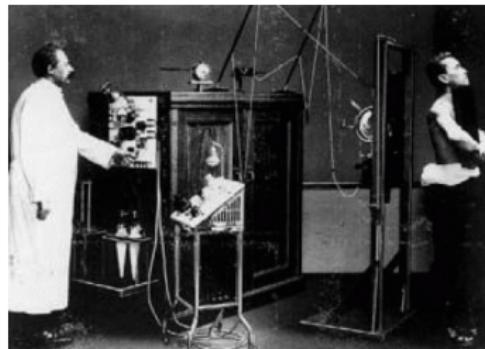
	Person alive	Person dead	
MRI scan			anatomical information
PET scan			functional information

bright spots = high brain activity

# History: X-ray

Wilhelm Conrad Röntgen

- 8 November 1895: discovers X-rays
- 22 November 1895: X-rays  
Mrs. Röntgen's hand
- 1901: receives first Nobel Prize in  
physics



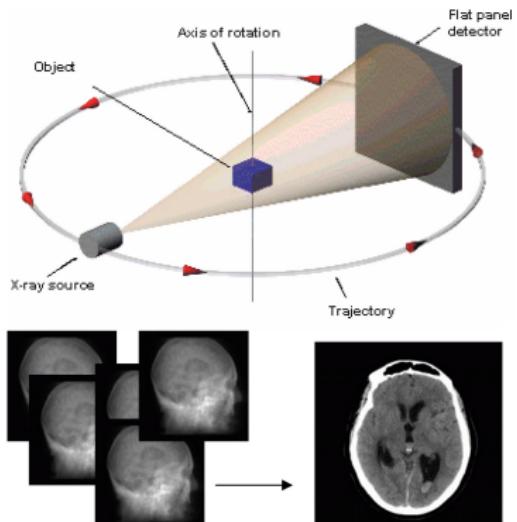
# History: Computed Tomography (CT)

## The breakthrough

- acquiring many projections around the object enables the reconstruction of the 3D object (or a cross-sectional 2D slice)

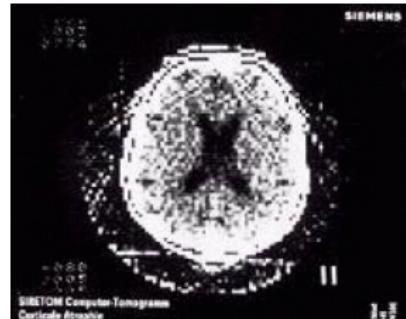
## CT pioneers

- 1917:** Johann Radon establishes the mathematical framework for tomography, now called the Radon transform
- 1963:** Allan Cormack publishes mathematical analysis of tomographic image reconstruction, unaware of Radons work
- 1972:** Godfrey Hounsfield develops first CT system, unaware of either Radon or Cormacks work, develops his own reconstruction method
- 1979:** Hounsfield and Cormack receive the Nobel Prize in Physiology or Medicine

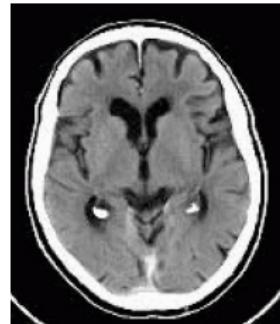


# CT: past and present

1975: Siemens SIRETON CT scanner (image size  $128 \times 128$ )



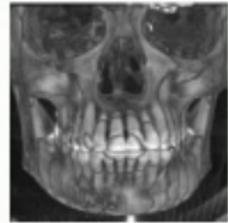
Now: Common modern CT scanner (image size  $512 \times 512$ )



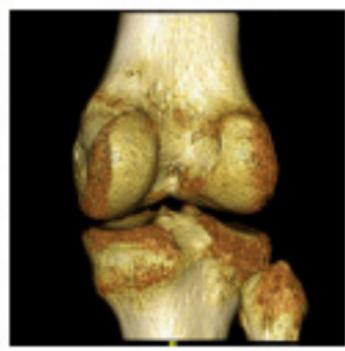
# 3D Visualization Capabilities



Extrapolate novel views of the structures

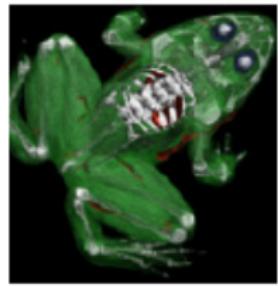
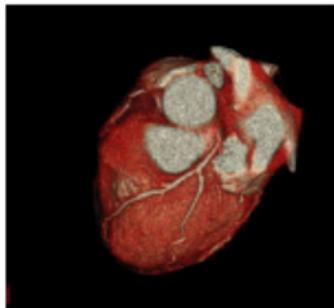
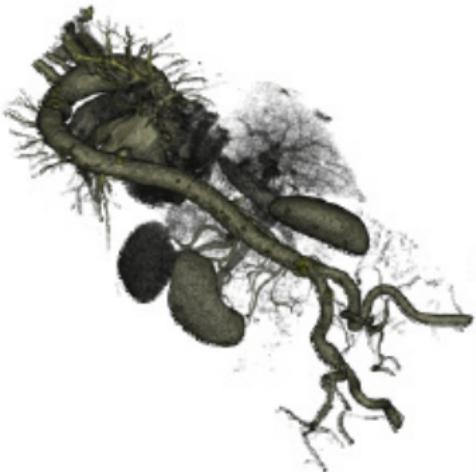


Maximum intensity visualization



Shaded structures visualization

## More visualizations





# Visualization and Virtual Medicine

Offer a **virtual reality** environment for

- Virtual examination (e.g. virtual colonoscopy)
- Surgical planning
- Medical training



# Ultrasound: past and present

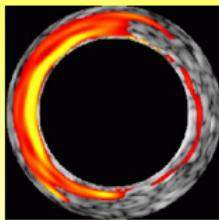
- 1942. Dr. Karl Theodore Dussik: transmission ultrasound investigation of the brain
- 1955. Holmes and Howry: Subject submerged in water tank to achieve good acoustic coupling
- 1959. Automatic scanner, Glasgow



US scanner



3D Ultrasound



Intra Vascular



Doppler

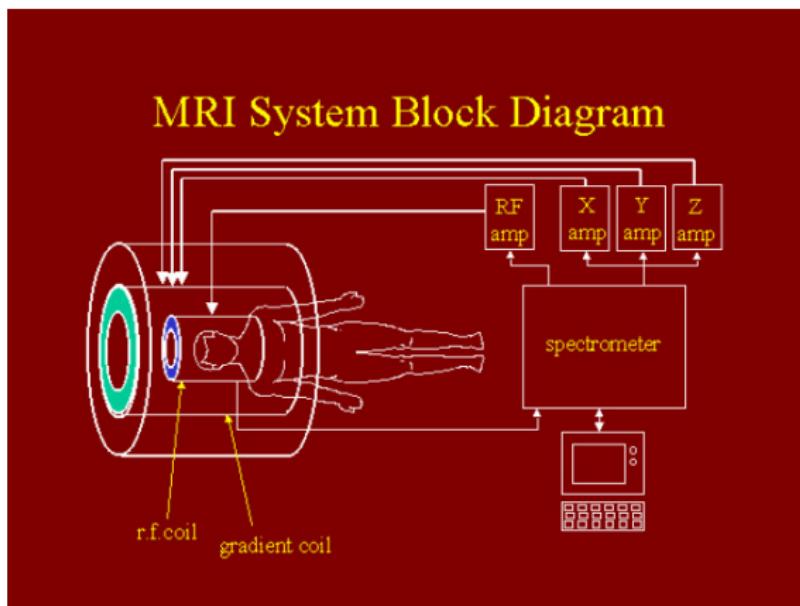


# History: MRI

- **1946:** Felix Bloch (Stanford) and Edward Purcell (Harvard) demonstrate nuclear magnetic resonance (NMR)
- **1973:** Paul Lauterbur (Stony Brook University, Nobel 2003) published first MRI (Magnetic Resonance Imaging) image in Nature
- **Late 1970s:** First human MRI images conceived
- **Early 1980s:** First commercial MRI systems available
- **1993:** Functional MRI in humans demonstrated

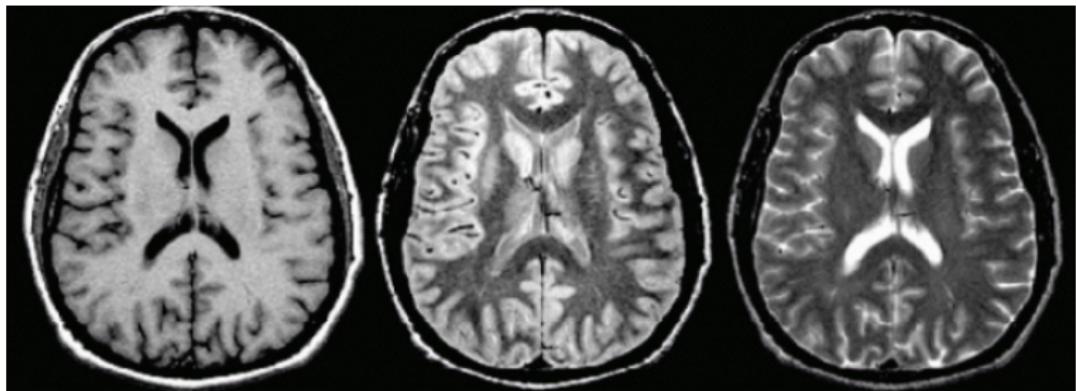
# MRI: basics

- MRI measures the effects of magnetic properties of tissue
- Effects are tissue-specific
- Also specific to blood perfusion/ oxygenization (functional MRI)
- MRI is very versatile (but also more expensive than CT)



# MRI: basics

Permits the acquisition of several kind of images:

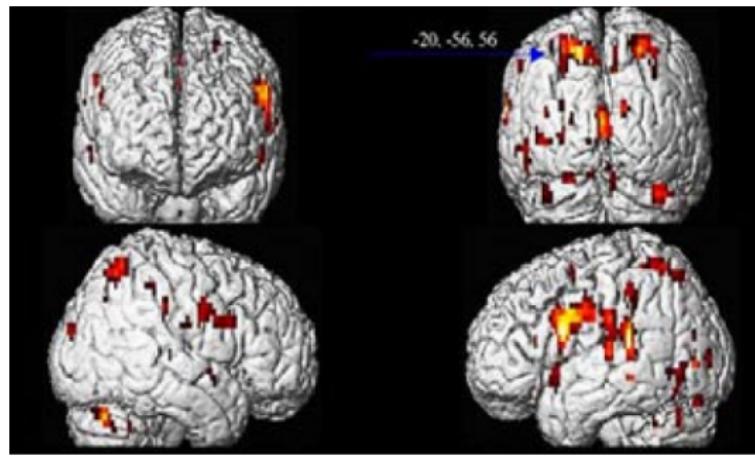


T1, density and T2 weighted MRI

# MRI: applications

## Functional MRI:

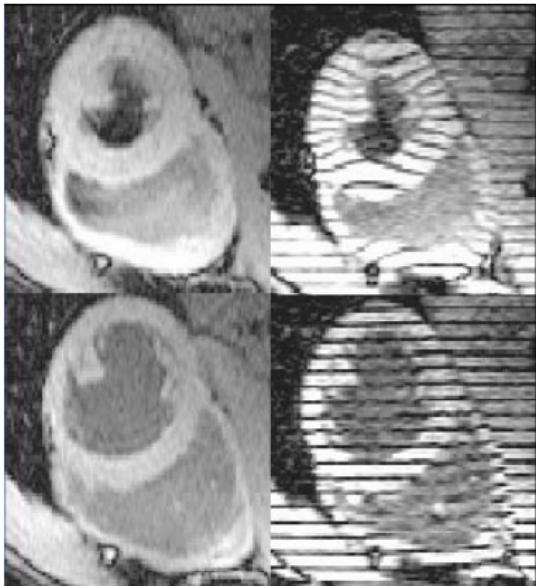
- Allows to assess brain activity during certain tasks
- Valuable for brain functional studies  
(cognitive sciences)
- Also for surgery planning and diagnosis



# MRI: recent applications

## Cardiac tagged MRI:

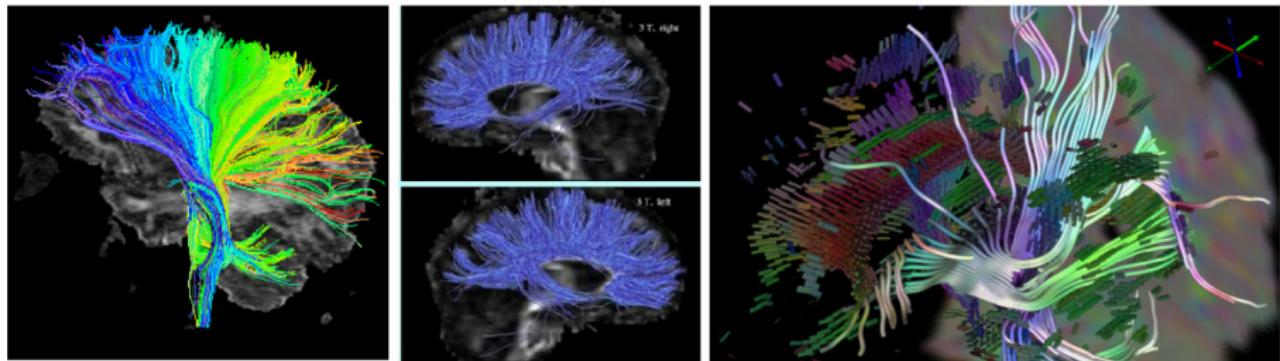
- Draw **magnetic patterns** in the matter
- Study how these patterns are distorted during heart contraction
- Infer information about heart dynamical behavior



# MRI: very recent applications

## Diffusion Tensor Imaging:

- Measures the diffusion of water
- Allows the tracking of nerve fibers in the brain (white matter)
- Visualization challenging!



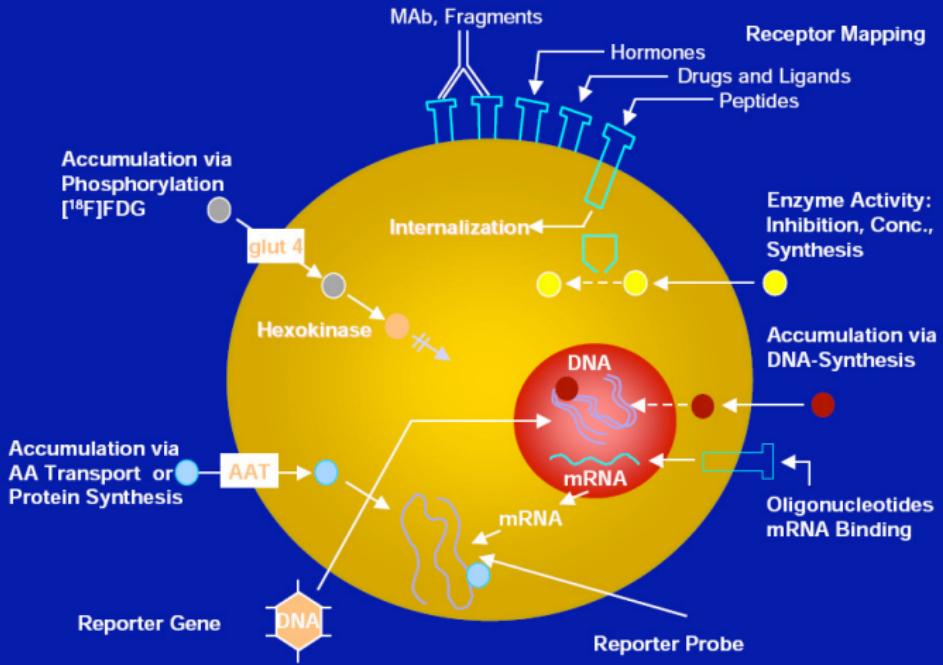


# Molecular imaging

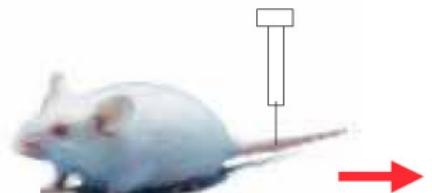
- Exotic but trendy
- Molecular imaging provides information about specific molecular processes
- Links to genomic and proteomics
- Exploits all portions of the physical spectrum in addition to sound
- No one of the previous imaging modality is ideal so combinations must often be used
- Often *in vivo*

# Molecular imaging

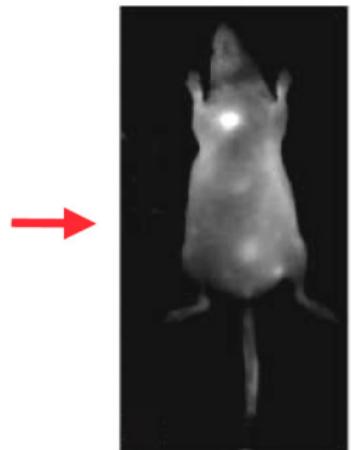
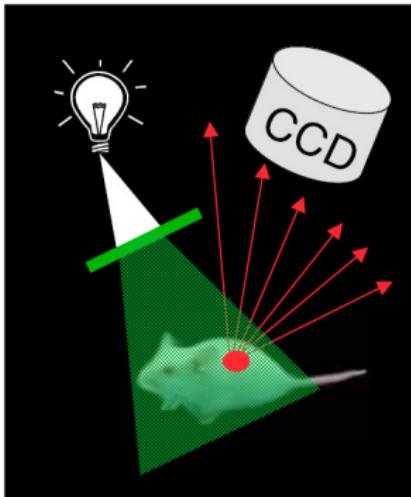
## Molecular Imaging Targets/Probes



# Molecular imaging



inject fluorophore or  
labeled biomolecule



Weissleder et al,  
Nature Biotech.  
1999; 17: 375



# Image processing: what for?

Image acquisition and reconstruction ⇒ Physics, Maths, Engineer, ...

- studies how to reconstruct efficiently meaningful images from the raw data

Visualization ⇒ Computer graphics

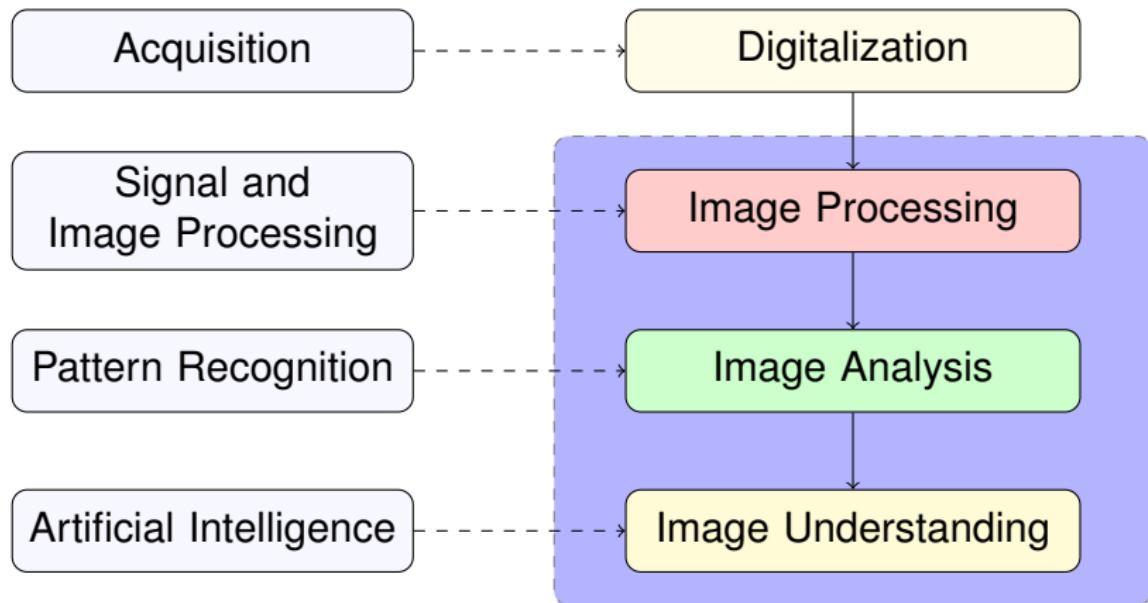
- studies how to visualize the reconstructed images in a way that is useful for human observers

Image Analysis and Understanding ⇒ Computer Vision

- studies how to emulate with a computer **perceptual and visual behaviors** similar to the biological ones
- studies models, algorithms and techniques to
  - ▶ obtain objective measurements automatically
  - ▶ recognize objects, structures and events



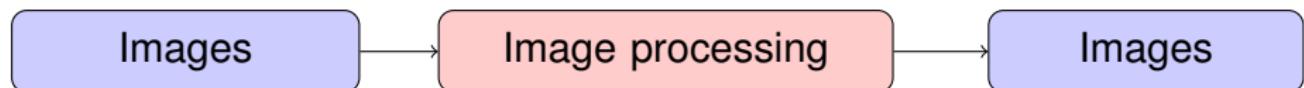
# Processes in Computer Vision



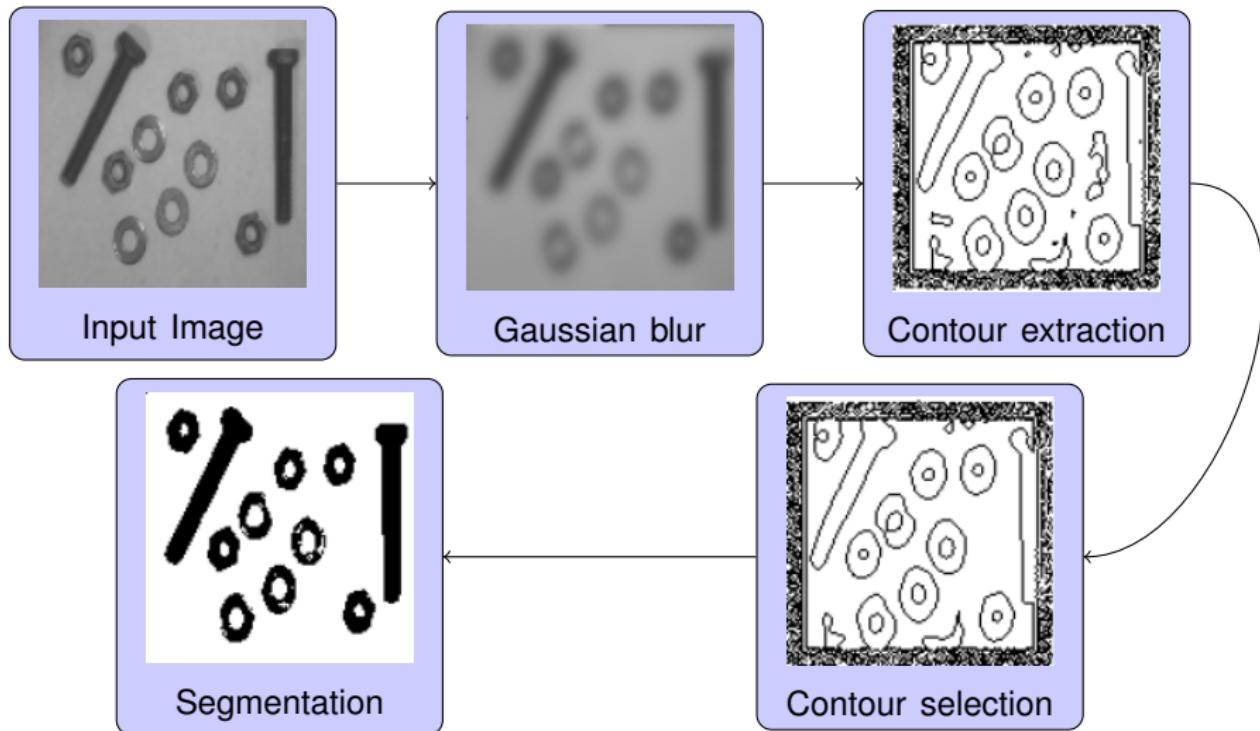


# Low-Level Vision (image processing)

Image processing processes: set of operations performed on images aiming at enhancing their quality and selecting useful information, which will be processed by humans or other algorithms



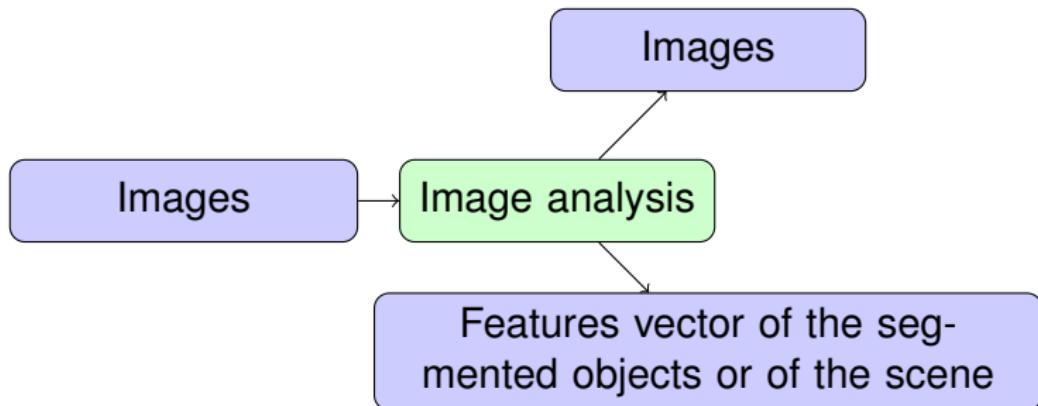
# Example of low-level processing





# Mid-Level Vision (image analysis)

- Includes extraction of symbolic information from pre-processed images and analysis techniques of the visual characteristics of the objects that are in the images

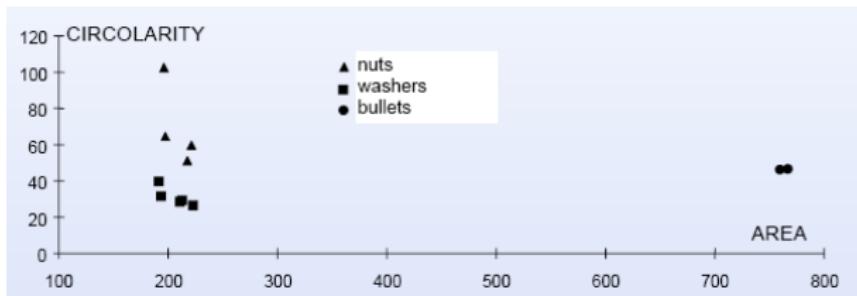


# Example of mid-level processing

- Extraction of visual primitives:
  - Area
  - Circularity



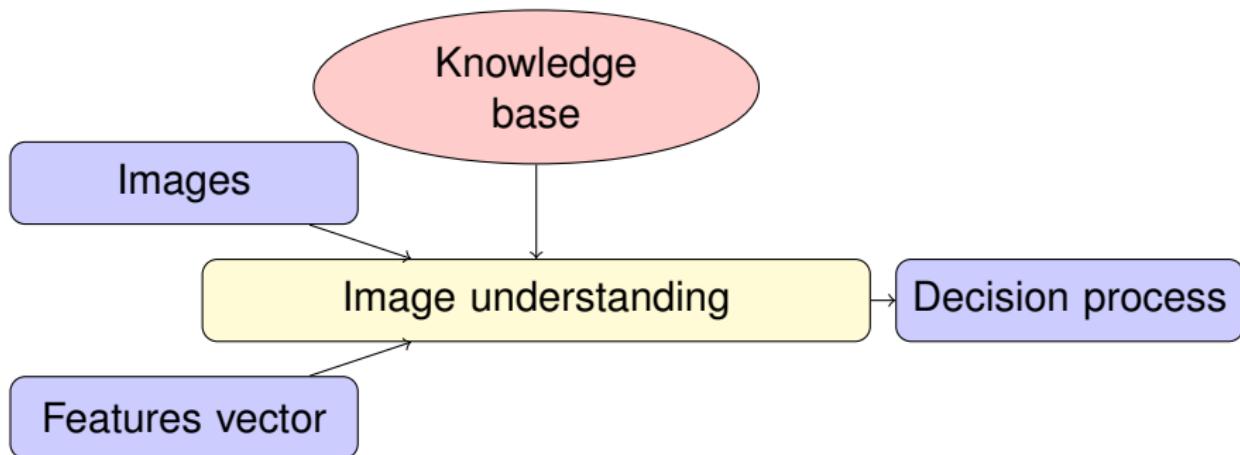
Labeling



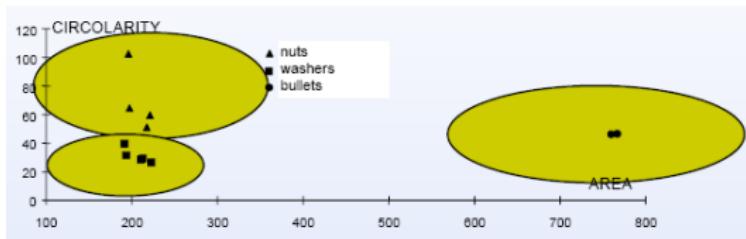
# High-Level Vision (image understanding)

Aims at obtaining some “comprehension” of the observed scene, as shape recognition or spatial relationship among objects. It includes high-level abstraction processes:

- Classification
- Identification
- Localization



# Example of high-level processing



## Clustering

- Classification method
- Non supervised



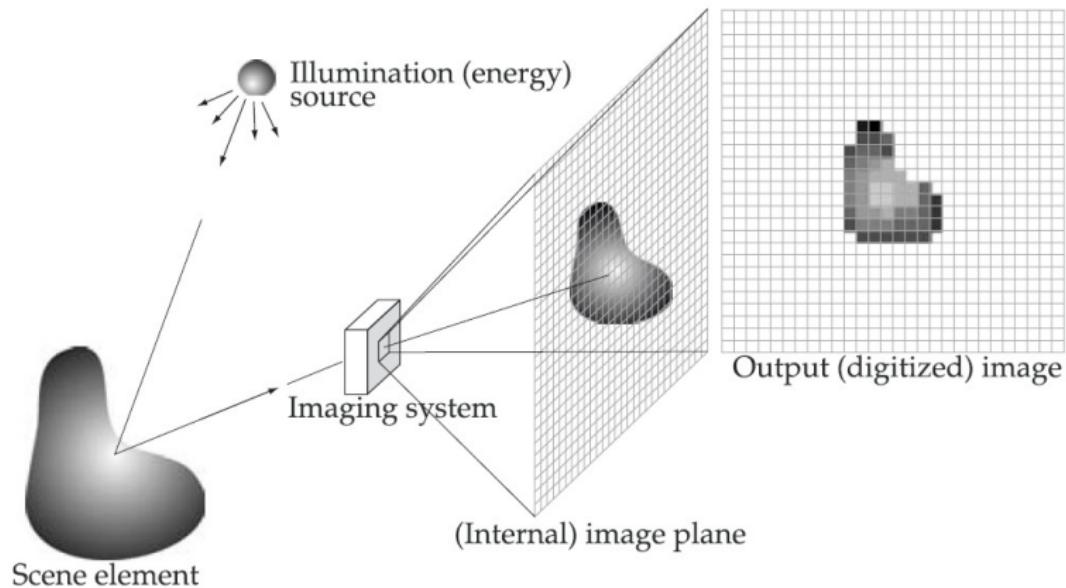


# Outline

## 2 Digital images

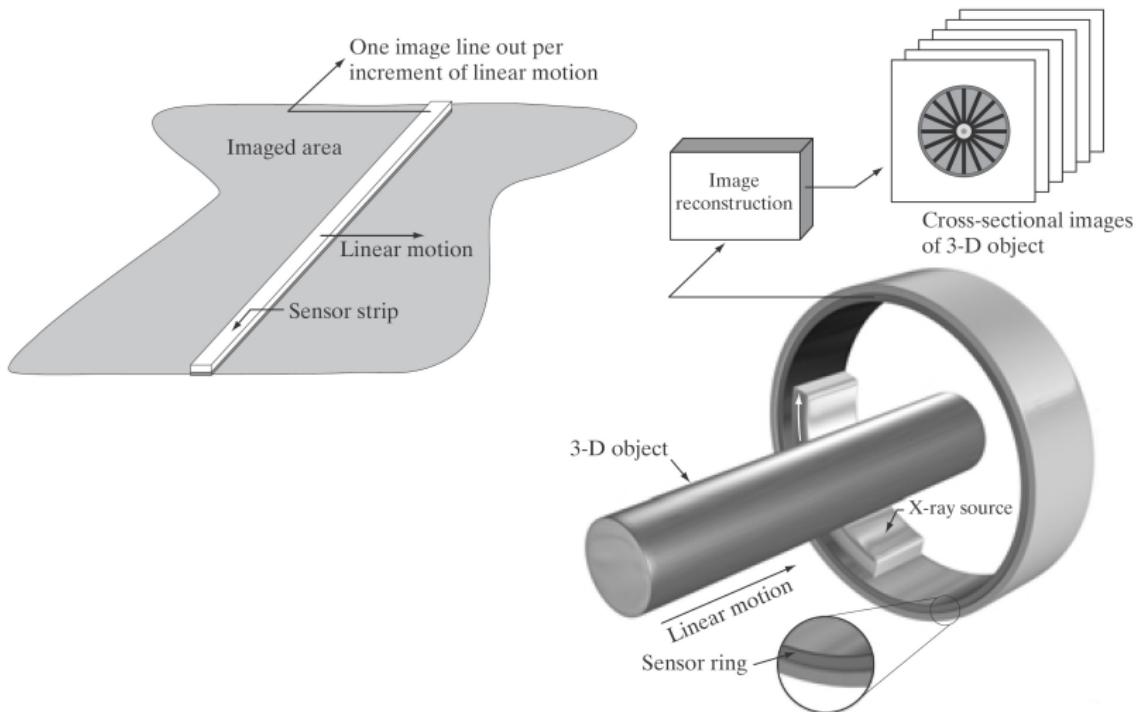
- Image acquisition and representation

# Digital Image Acquisition: sensor array



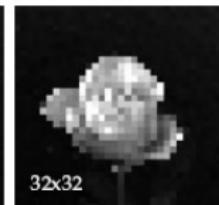
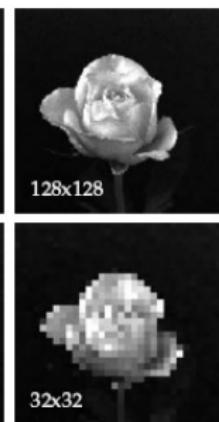
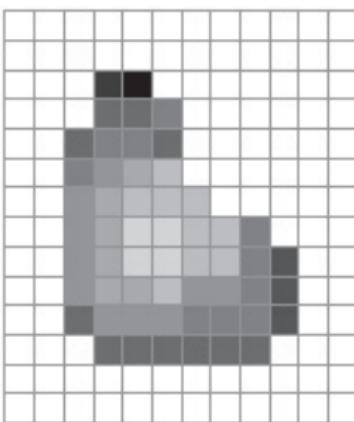
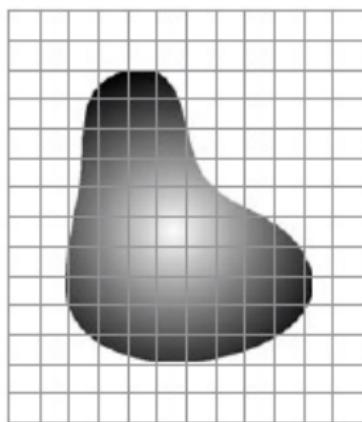
- Light source, Object reflection
- CCD sensor
- Imaging System: Lenses
- Digital Image

# Digital Image Acquisition: sensor strip



# Sampling & Quantization

- Sampling produces continuous values  $f[x, y]$
- Digitizing the Pixel Amplitude — Quantization
- $n$  bits per pixel —  $2^n$  Discrete (monochromatic)

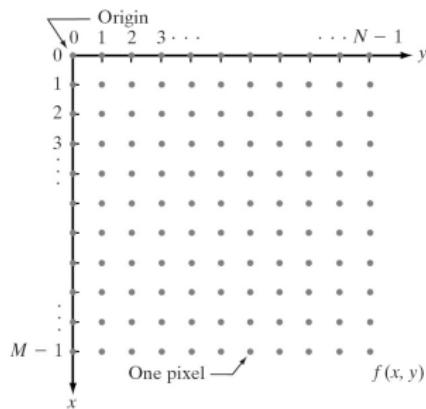




# Digital Image Representation

The result of sampling and quantization is a **matrix**:

$$f(x, y) = \begin{pmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{pmatrix}$$





# Outline

3

## Image enhancement

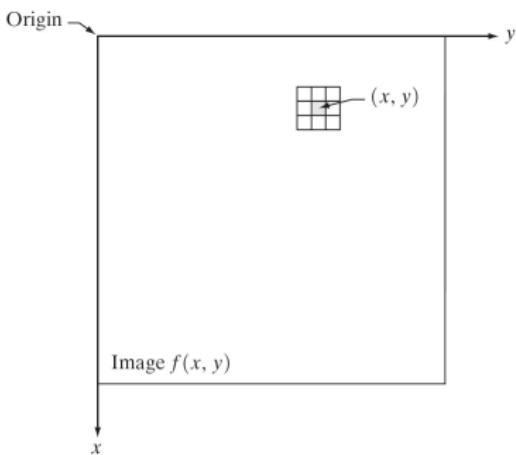
- Spatial Filtering
- Intensity transformations
- Image histograms and equalization
- Convolution filters
- Order-statistic filters

# Spatial filtering

- The simplest way to enhance an image  $f$
- Based on direct manipulation of the pixels

$$g(x, y) = T[f(x, y)]$$

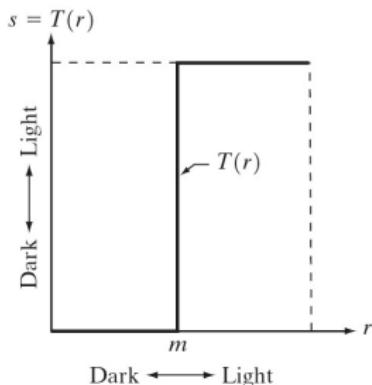
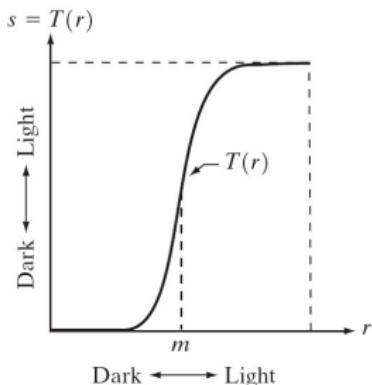
- $T$  operator on  $f$
- defined over a neighborhood of the pixel  $(x, y)$  (for example a small subimage centered at  $(x, y)$ )



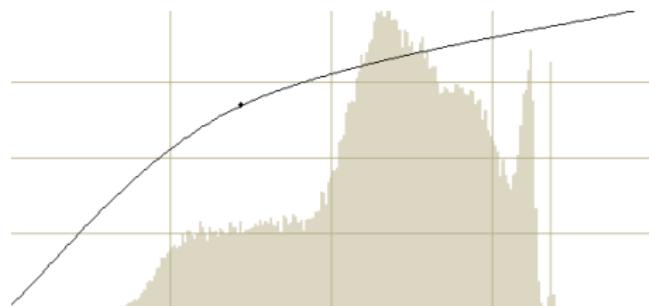
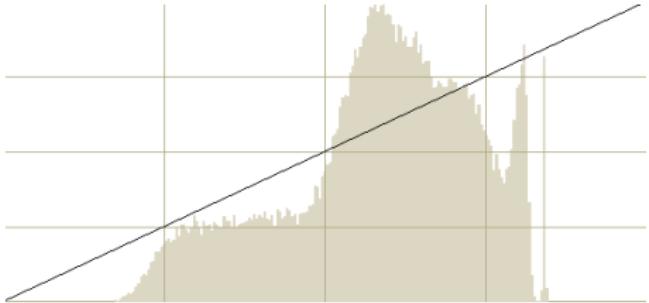
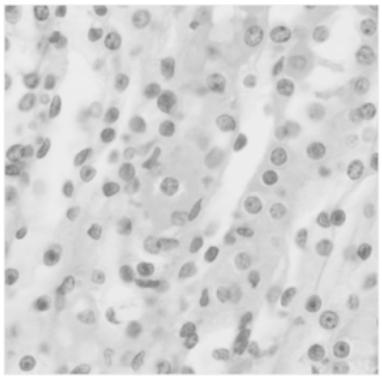
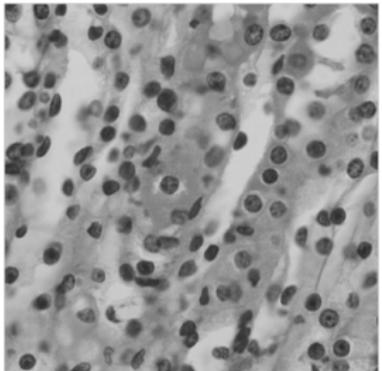
# Intensity transformations

- Spatial filtering with neighborhood size  $1 \times 1$
- In this case  $g$  depends only on the **intensity value** of  $f$  at  $(x, y)$
- It is completely described as a function between intensity values:

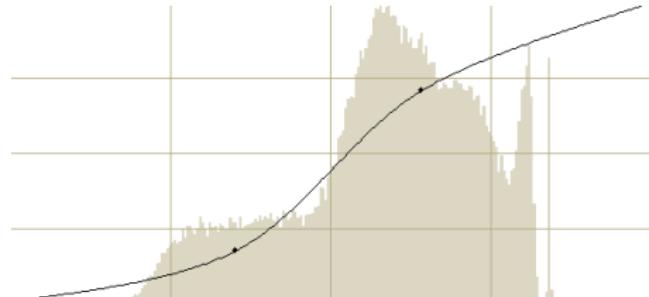
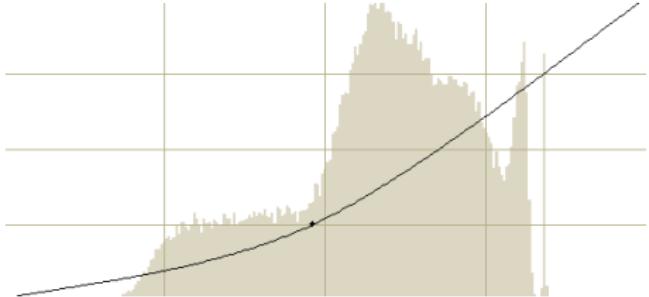
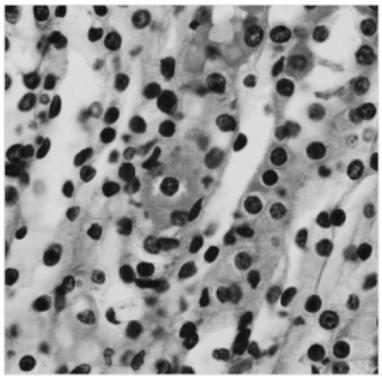
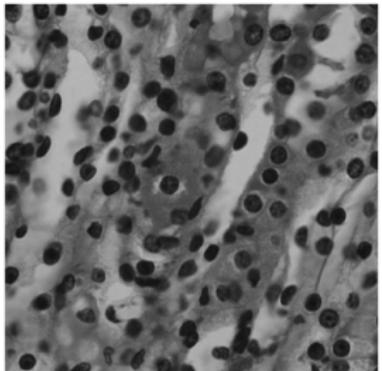
$$s = T(r)$$



# First examples

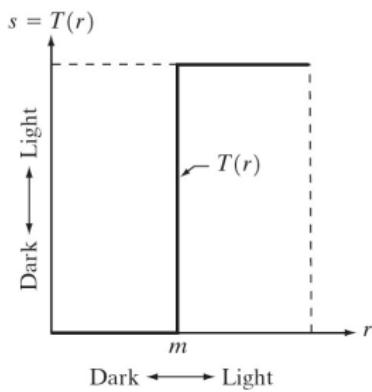
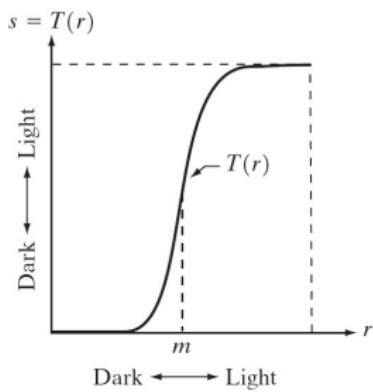


# First examples



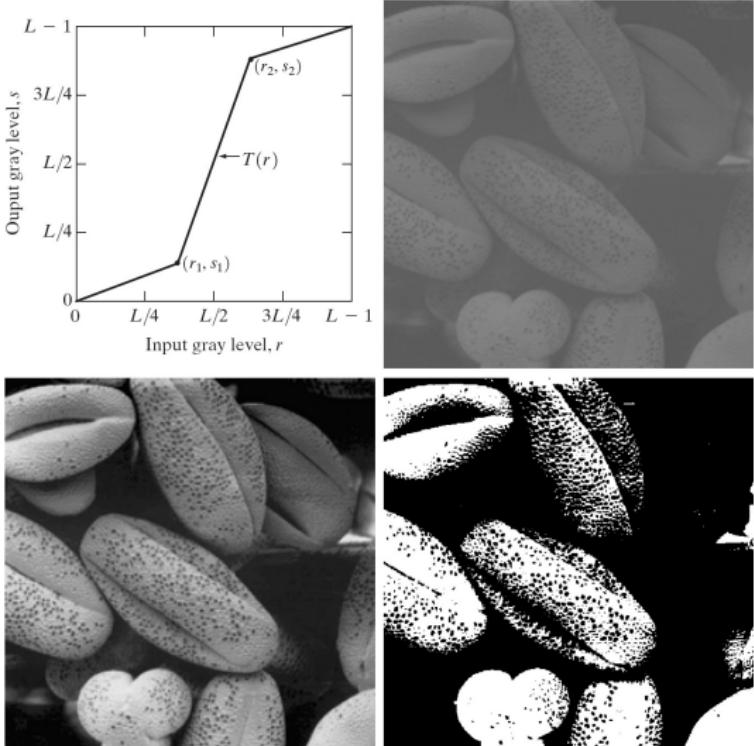
# Intensity transformations: contrast enhancement

- Left plot: **Contrast stretching**
  - ▶ Darken gray values **below**  $m$
  - ▶ Brighten gray values **above**
- Right plot: **Thresholding**
  - ▶ Limit of contrast stretching
  - ▶ Produces a binary image



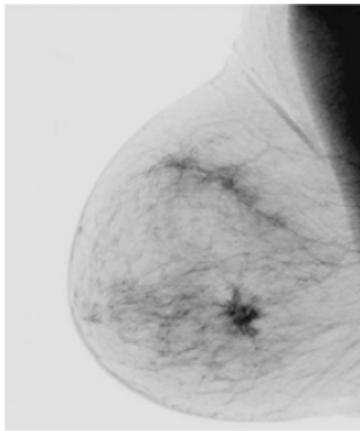
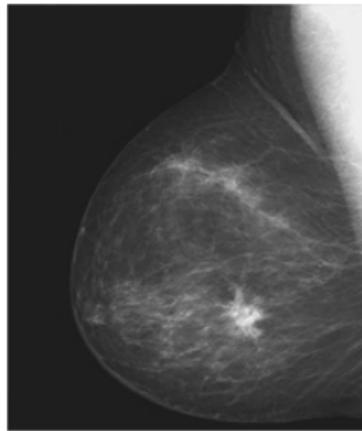
# Contrast enhancement

- a) Intensity transform
- b) Low contrast image
- c) Contrast enhanced image
- d) Thresholded image



# Negative

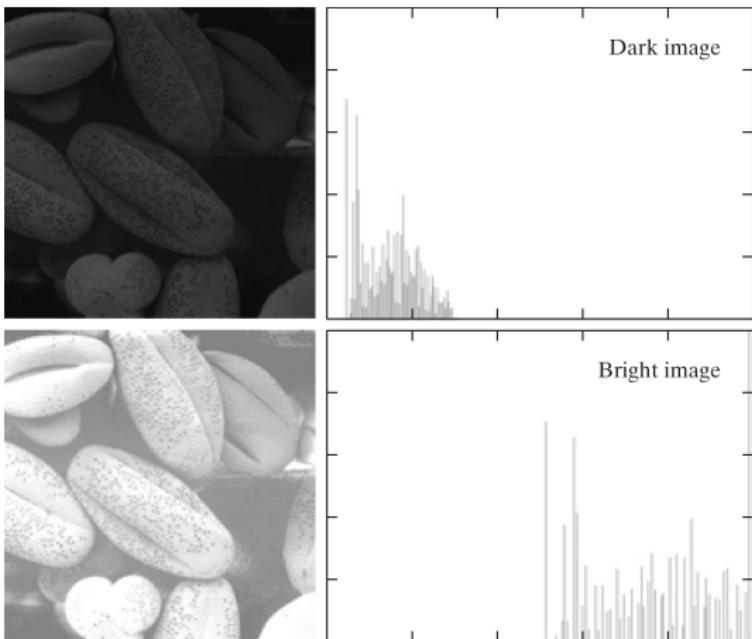
- Equivalent of photographic negative
- Enhance white or gray **details** embedded in a black background



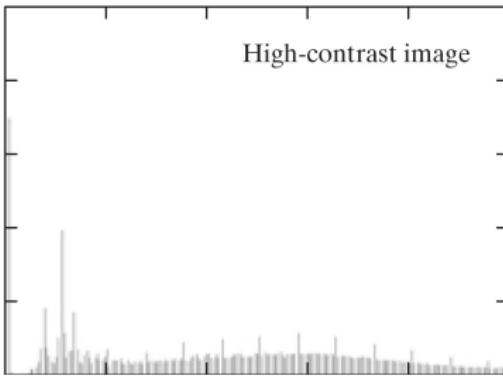
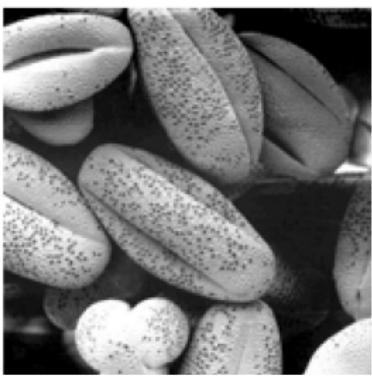
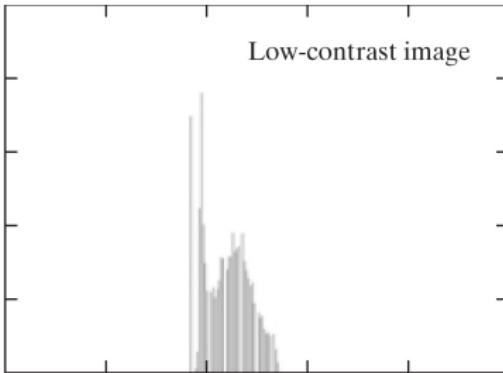
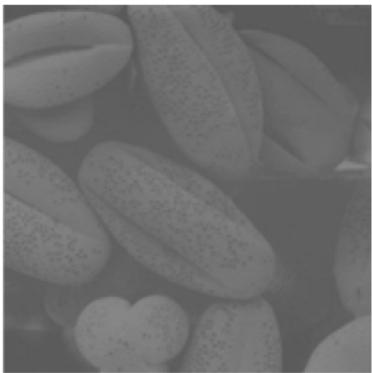
# Image histograms

## Image Histogram

Discrete function that associates to the gray level  $r$  the **number of pixels  $N_r$**  having gray value equal to  $r$



# Image histograms: example





# Histogram equalization

- An enhancement method based on intensity transformation
- Find a function  $s = T(r)$  (strictly increasing) such that the image

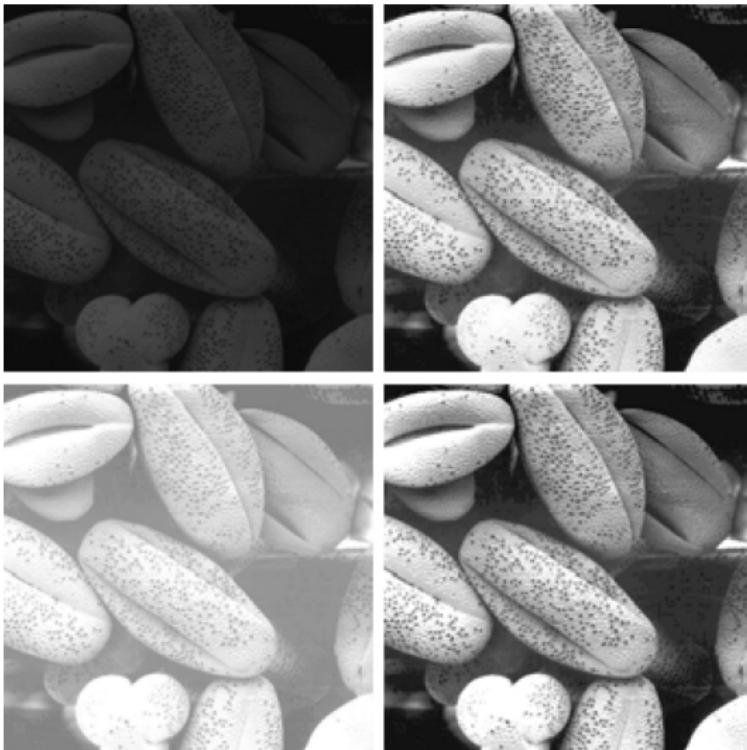
$$g(x, y) = T \circ f(x, y)$$

has **uniform** normalized histogram

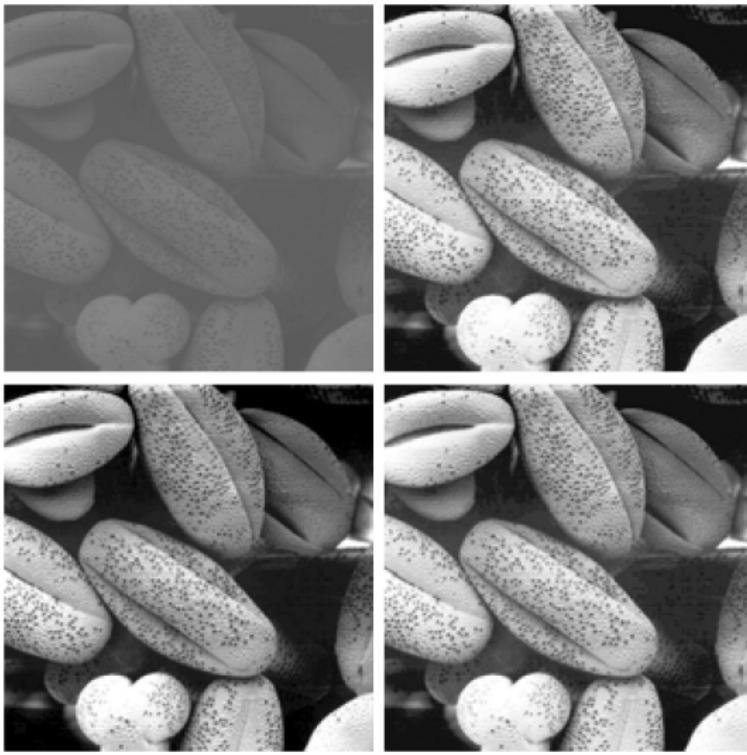
- Letting  $p(r)$  be the normalized histogram of  $f$

$$T(r) = \sum_{r' \leq r} p(r')$$

# Histogram equalization: example 1



# Histogram equalization: example 2



# Convolution filters

Linear spatial filters based on convolution kernels

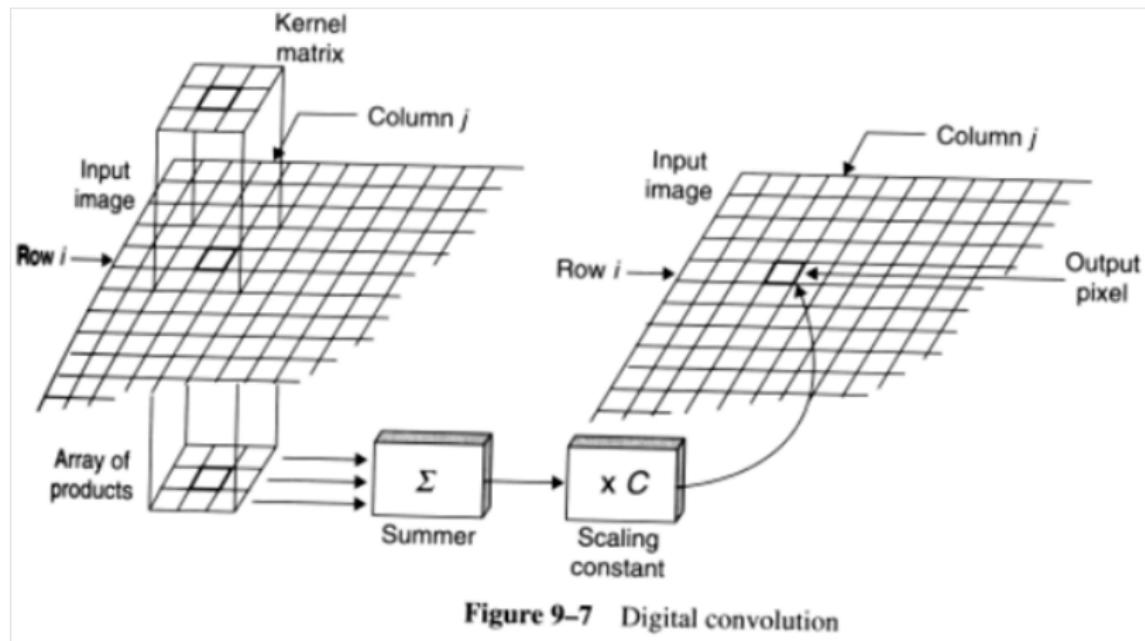


Figure 9–7 Digital convolution



# Convolution filters

Different choices of  $h$  lead to very different results:

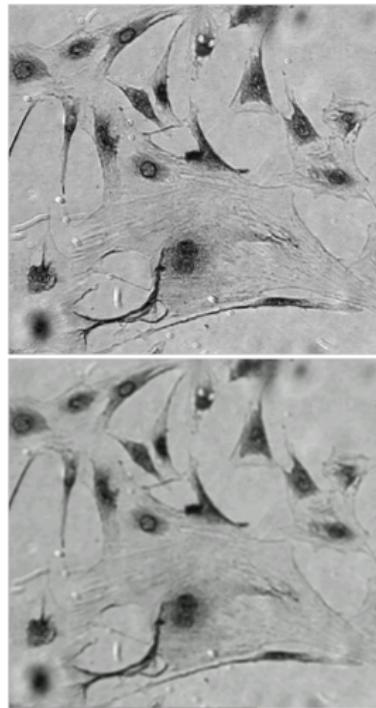
- Smoothing
- Sharpening
- Edge enhancement

# Smoothing filters

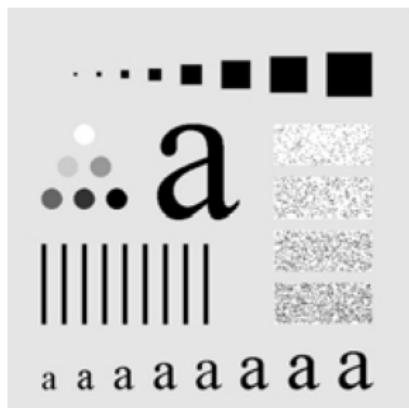
Averaging matrix kernel:

1	1	1
1	1	1
1	1	1

(size  $3 \times 3$ )



# Averaging filters



Original image

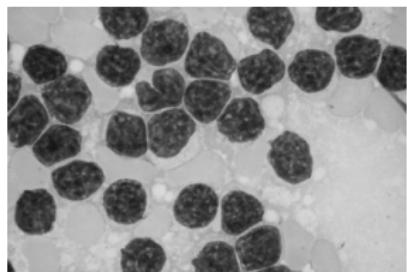


mask size =  $9 \times 9$

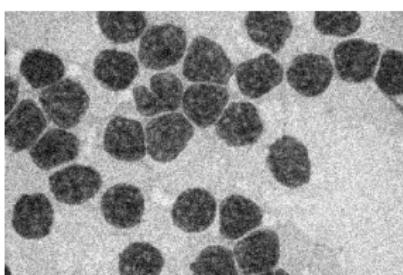


mask size =  $35 \times 35$

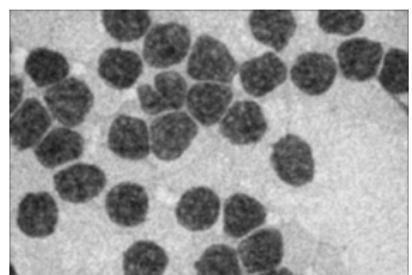
# Averaging filters



Original image



Gaussian Noise added



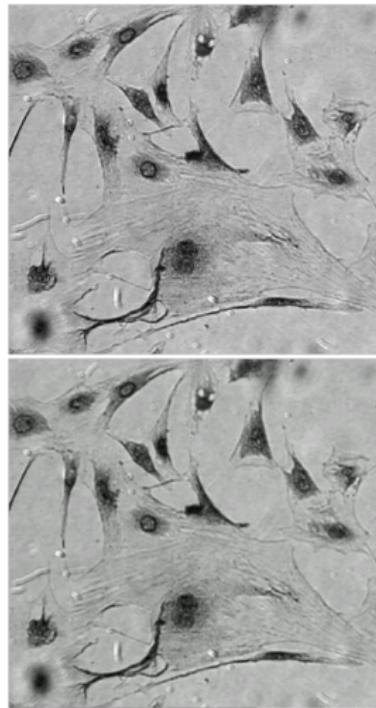
mask size =  $3 \times 3$

# Smoothing filters

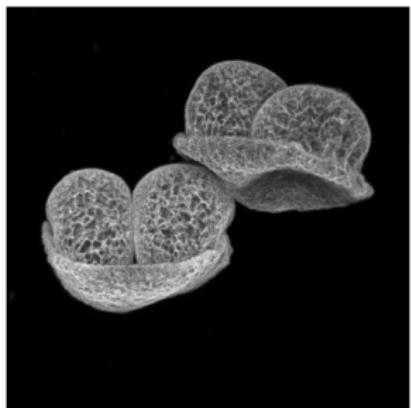
Gaussian filter:

1	2	1
2	4	2
1	2	1

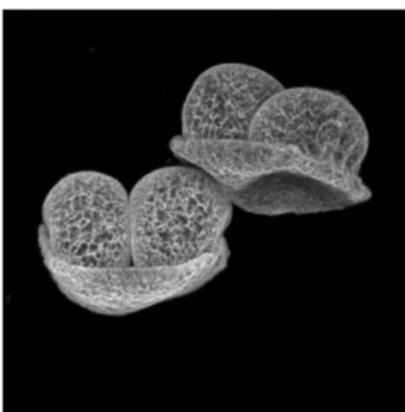
(size  $3 \times 3$ )



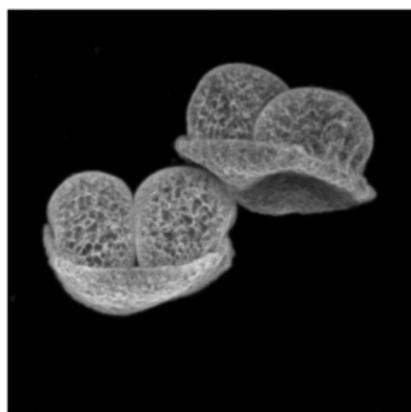
# Gaussian filters



Original image



mask size =  $3 \times 3$

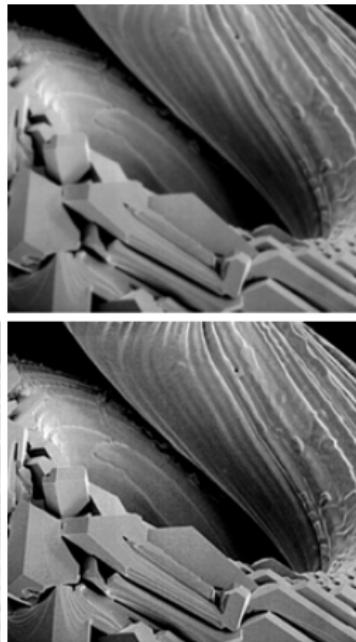


mask size =  $9 \times 9$

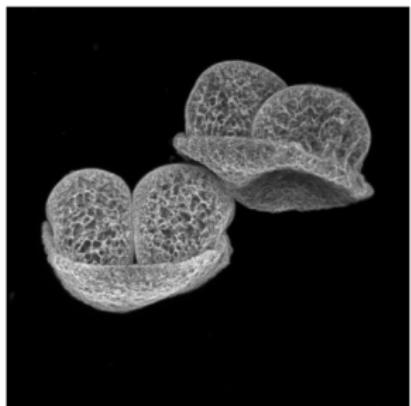
# Sharpening filters

- a) Two sharpening filters
- b) Original image
- c) Application of I filter
- d) Application of II filter

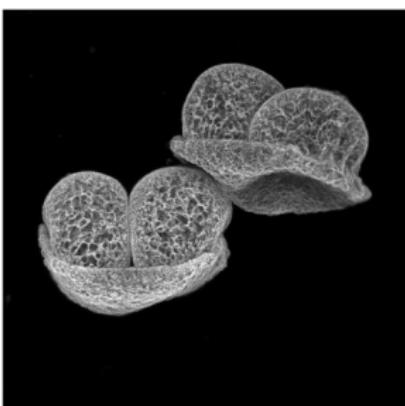
$$\begin{array}{|c|c|c|} \hline 0 & -1 & 0 \\ \hline -1 & 5 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$
$$\begin{array}{|c|c|c|} \hline -1 & -1 & -1 \\ \hline -1 & 9 & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$



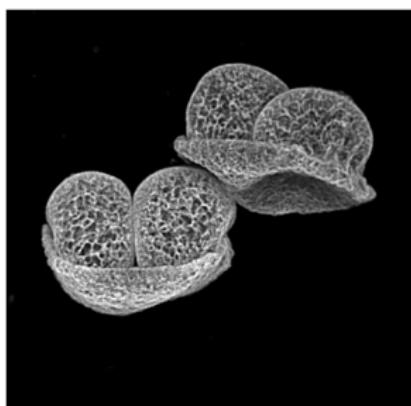
# Sharpening filters



Original image



mask size =  $3 \times 3$



mask size =  $9 \times 9$



## Edge enhancement: Definition

- An edge is a location in the image where there is a **steep intensity variation**
- Hopefully, these **discontinuities** correspond to boundaries of object of interest
- How do we enhance edges?
  - ▶ Determine a measure of intensity change in the pixels neighbourhood
  - ▶ First derivative of a two-variate function → **Gradient**



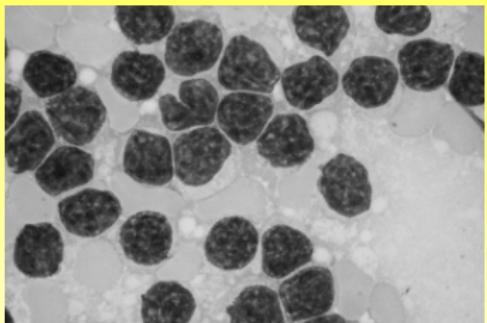
# Edge enhancement: Sobel Operator

- Estimation of  $\nabla f$  in 2 directions:

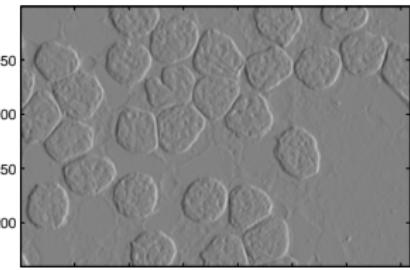
$$h_{\text{hor}} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad h_{\text{vert}} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

- It can be thought:
  - 1 First Gaussian blurring
  - 2 Then derivation
- Indeed,  $f' * g = (f * g)' = f * g'$
- Sobel operator is **separable!**

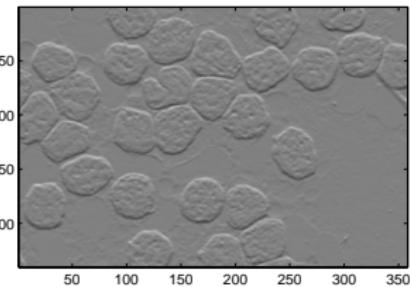
# Edge enhancement: Sobel Operator



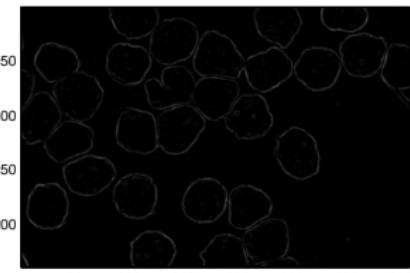
Original



Horizontal



Vertical

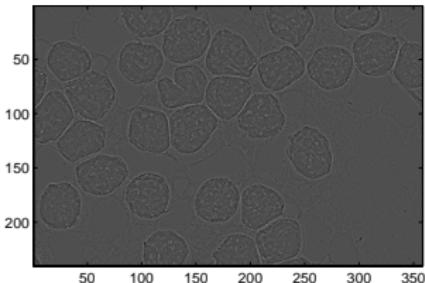
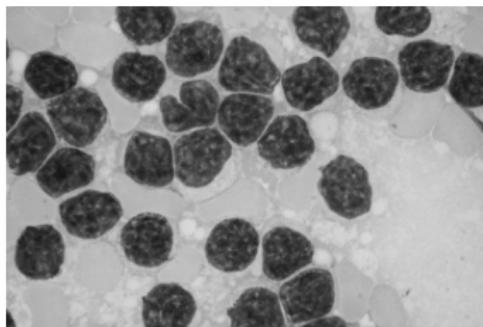


Gradient norm

# Edge enhancement: Laplace operator

$$\nabla^2 = \nabla \cdot \nabla = \left( \frac{\partial^2}{\partial x^2} \right) + \left( \frac{\partial^2}{\partial y^2} \right)$$

$$\nabla^2 \approx \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$





## Order-statistic filters

- Sort pixel values inside a  $m \times n$  neighborhood of pixel  $(x, y)$

9	7	11
6	4	5
2	5	1

$$1 \leq 2 \leq 4 \leq 5 \leq 5 \leq 6 \leq 7 \leq 9 \leq 11$$

- The response of this class of filters depends on the ordering of the pixel values



## Order-statistics filters: Min-filter

- Sort pixel values inside a  $m \times n$  neighborhood of pixel  $(x, y)$

9	7	11
6	4	5
2	5	1

$$1 \leq 2 \leq 4 \leq 5 \leq 5 \leq 6 \leq 7 \leq 9 \leq 11$$

- Elimination of **salt** noise



## Order-statistics filters: Max-filter

- Sort pixel values inside a  $m \times n$  neighborhood of pixel  $(x, y)$

9	7	11
6	4	5
2	5	1

$$1 \leq 2 \leq 4 \leq 5 \leq 5 \leq 6 \leq 7 \leq 9 \leq 11$$

- Elimination of **pepper** noise



## Order-statistics filters: Median filter

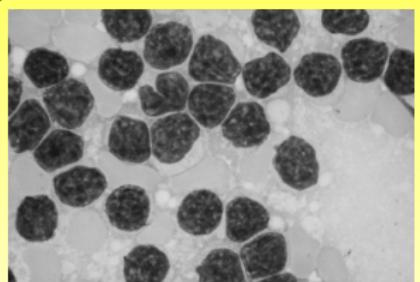
- Sort pixel values inside a  $m \times n$  neighborhood of pixel  $(x, y)$

9	7	11
6	4	5
2	5	1

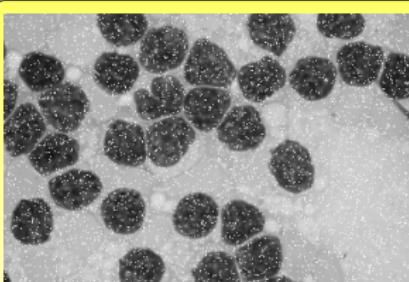
$$1 \leq 2 \leq 4 \leq 5 \leq 5 \leq 6 \leq 7 \leq 9 \leq 11$$

- Elimination of general **impulsive** noise
- Less blurring than linear smoothing filter

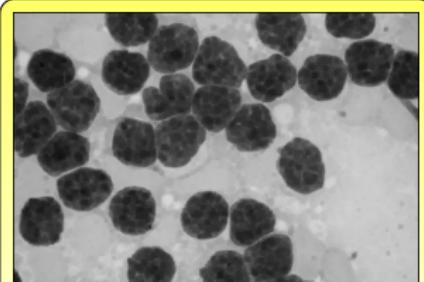
# Applications of Order-statistics filters: Min filter



Original

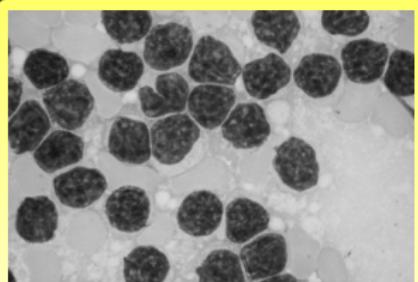


Salt added

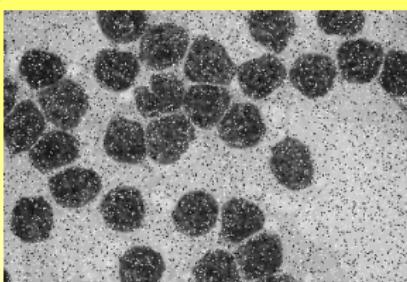


Min filter  $3 \times 3$

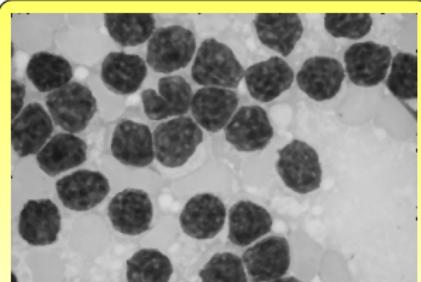
# Applications of Order-statistics filters: Median filter



Original

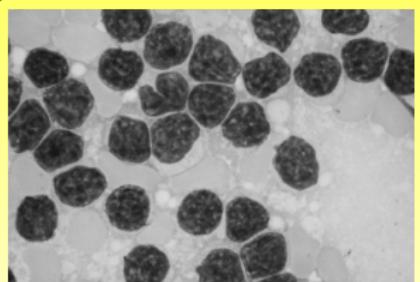


Salt & Pepper added

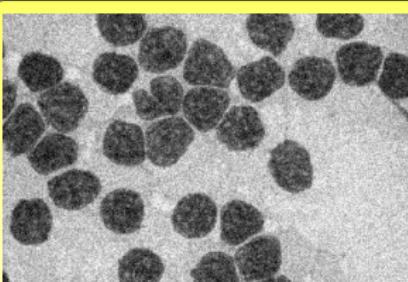


Median filter  $3 \times 3$

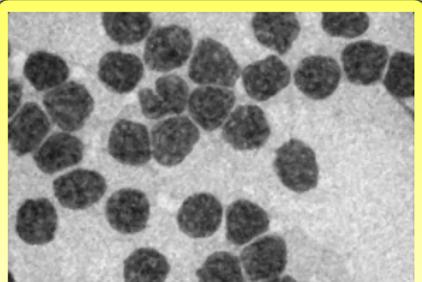
# Applications of Order-statistics filters: Median filter



Original



Gaussian noise added



Median filter  $3 \times 3$



## Further Example: Anisotropic Diffusion



Original image

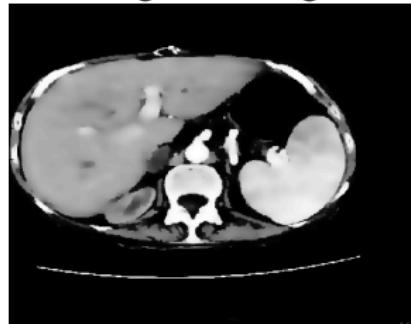
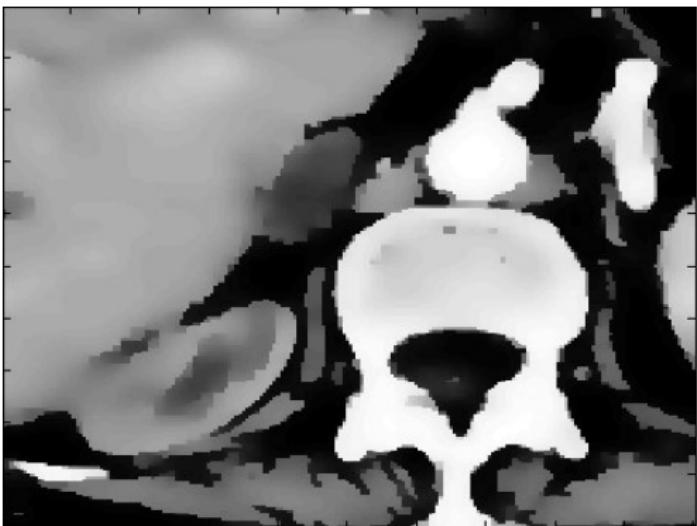


Image after evolution



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# Outline

4

## Image segmentation

- Thresholding
- Texture segmentation
- Edge detection
- Active contours



# Image segmentation: definition

- Fundamental step in many applications
- Segmentation = Partitioning of the image into homogeneous regions with respect to some visual feature (e.g. gray level value)
- Distinguishing objects from the background
- Two approaches:
  - ▶ Region based methods
  - ▶ Edge based methods



# Thresholding

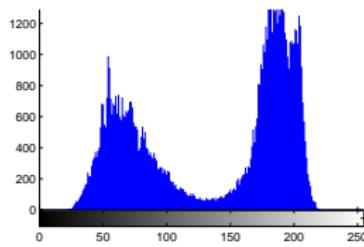
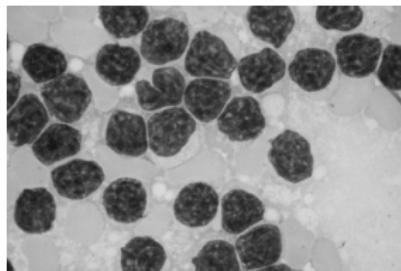
- Selection of an intensity  $T$  (called **Threshold**) capable to divide the image into two regions, corresponding to higher or lower intensity value
- Given an image  $f(x, y)$  and a threshold  $T$ , a binary image  $g(x, y)$  is produced:

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) \geq T \\ 0 & \text{otherwise} \end{cases}$$

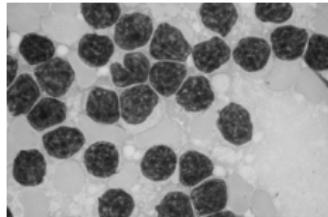
- Threshold selection depends on the intensity value of the objects of interest

# Automatic Threshold selection

- In case no a priori information is available
- Exploiting the statistical properties of the image, e.g. its histogram
  - ▶ Histogram valleys as thresholds
  - ▶ Histogram inflection points as thresholds
  - ▶ Otsu's method (1979)
    - ★ Optimal threshold selection method
    - ★ Minimize the **within-group variance**



# Otsu thresholding: Example



Original



$T = 100$

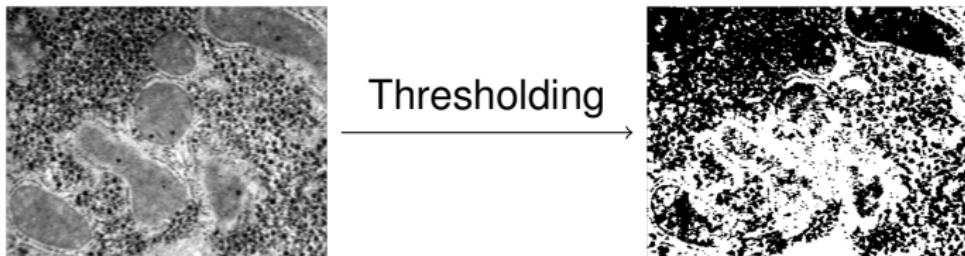


Otsu ( $T = 127$ )

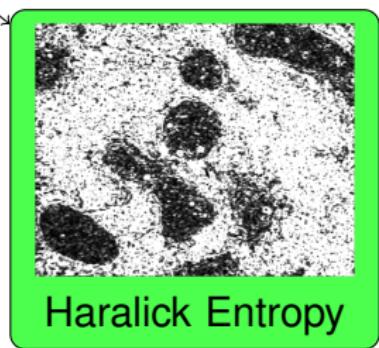
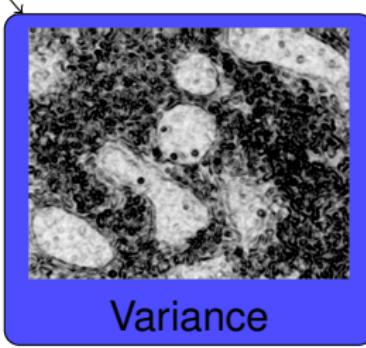
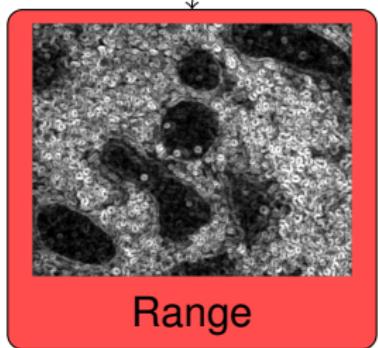


$T = 160$

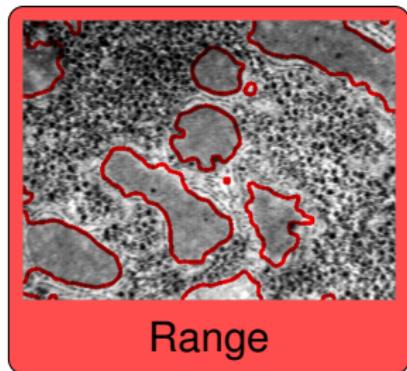
# Example: Improvement via texture features



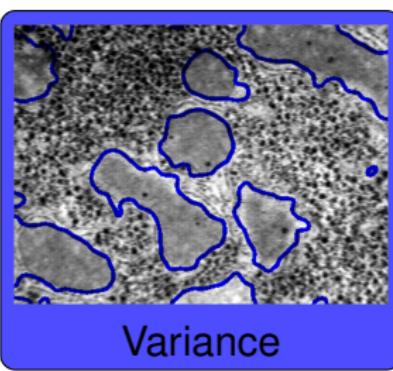
Texture Features



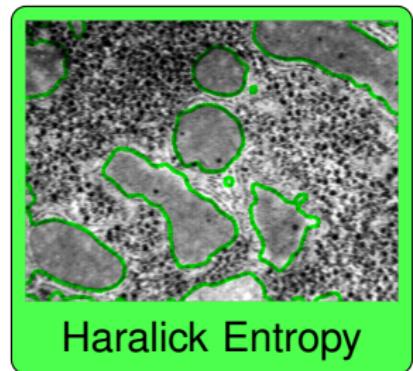
# Example: Improvement via texture features



Range



Variance

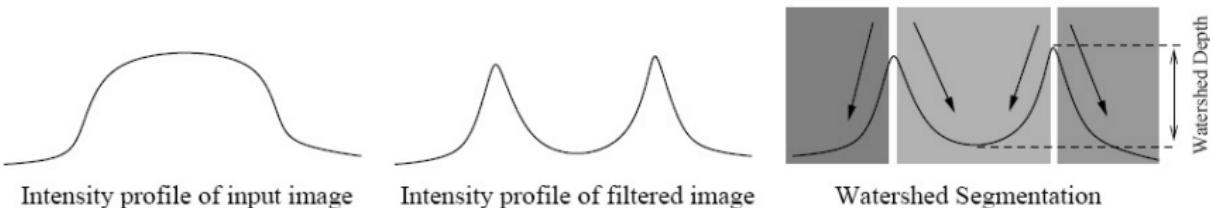


Haralick Entropy



## Further Example: Watershed segmentation

- Another popular **region based** method, like thresholding
- Boundaries are local extrema of features
- Drawback: over-segmentation



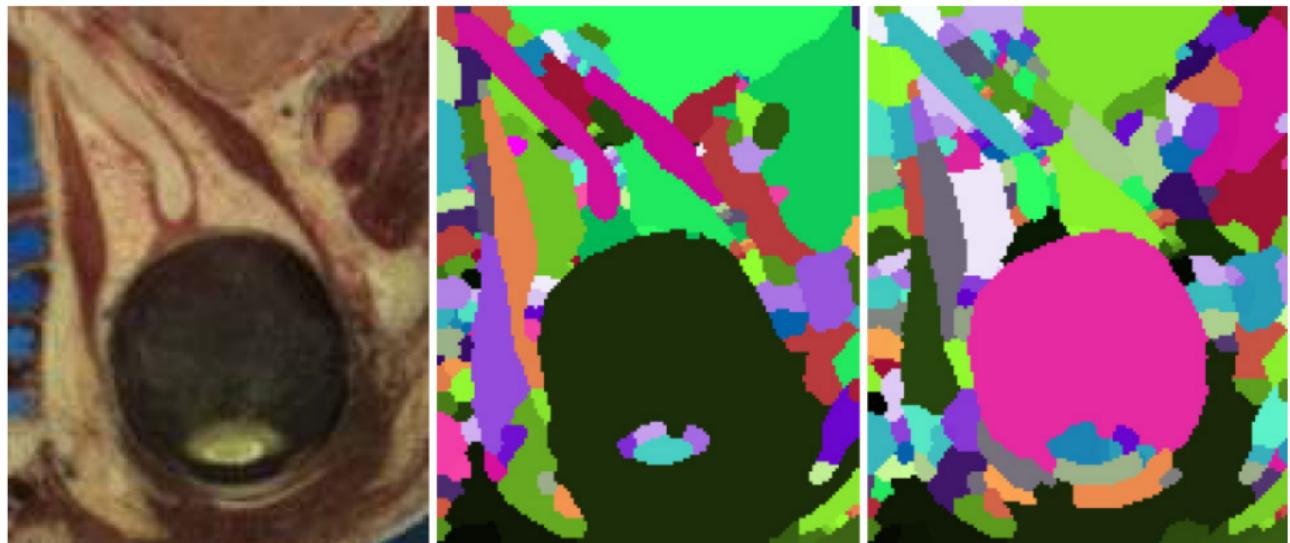
Intensity profile of input image

Intensity profile of filtered image

Watershed Segmentation

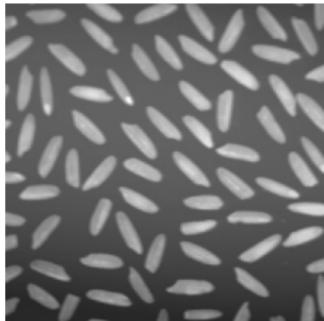
## Further Example: Watershed segmentation

- Gradient magnitude as feature



# Edge based segmentation

- This approach identifies the steep intensity variations in an image, called **edges**
- Uses **edge operators** plus **binarization** (e.g. thresholding)
- Hopefully these correspond to object boundaries
- Edges are extended or deleted so as to produce closed boundaries
- Only good for simple images
- Shape can then be used for recognition



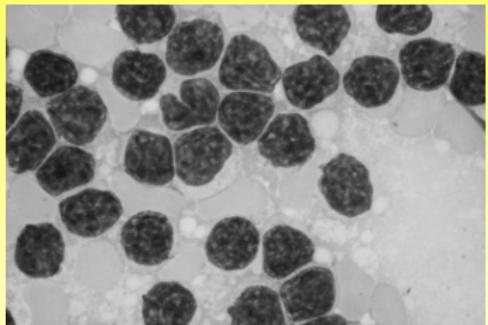
# Example:Canny Edge Detector (1986)

Popular but **powerful** edge detector

- ① Gaussian smoothing
- ② Gradient computation
- ③ Search of local maxima in the direction of the gradient, i.e. ridges in the gradient magnitude image
- ④ Non-maximal suppression
- ⑤ Thresholding of ridge points (actually with hysteresis)



# Edge detectors: examples



Original



Laplace



Canny



Sobel

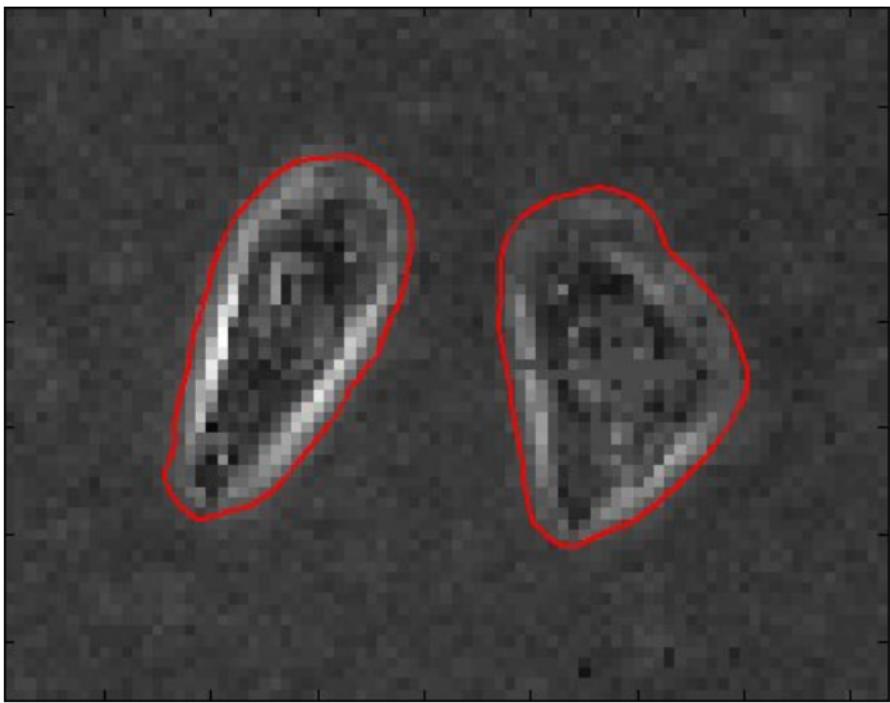


## Further Example: Active contours

- Aim: **improve** boundary detection
- Integrate information over distance
- Use cues from biological vision (Gestalt cues)
  - ▶ Smoothness
  - ▶ Closure
- Main ideas:
  - ▶ Insert an initial contour in the image domain
  - ▶ Stretch and bend it according to the **forces** defined by the **image data**
  - ▶ Keep the contour smooth during the evolution with a sort of **internal energy**
- Active contours techniques use:
  - ▶ Optimization strategies
  - ▶ Calculus of variations and PDEs



## Example



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# HEARTFAID project: Example of Activity



Slow Normal Fast Play/Pause Stop

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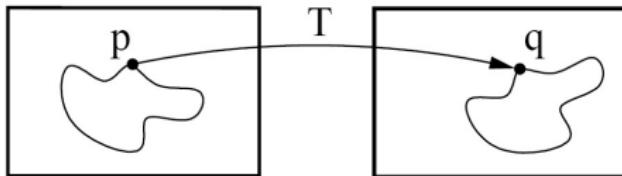
5

## Image Registration

- Basics

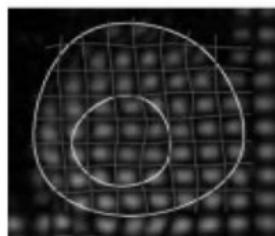
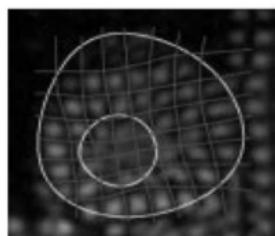
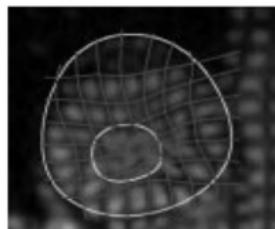
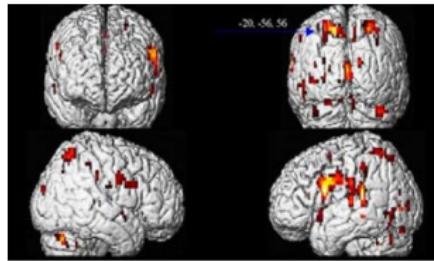
# Image Registration: definition

- Image registration is the process of determining the spatial transform that maps points from one image to homologous points on an object in the second image
- e.g. align if the images depict the same object, align the points corresponding to the same material point
- or match structures of interest (corners, edges, ...)
- Generally, the criterion is prescribed by an application relying on the registration task



# Registration purposes

- Data fusion (e.g. functional data to anatomical data)
- Construction of anatomical atlases
- Mapping to anatomical atlases
- Comparison of patients
- Content based image retrieval
- Motion analysis in image sequences
- ...





# Registration bases

- Let  $f, g$  be images,  $T$  a geometric transformation
- $D(f, g, T)$  a criterion asserting the goodness of the matching
- Then the registration problem boils down to:

$$\bar{T} = \arg \min_T D(f, g, T)$$

- Thus to develop a registration framework, we may:
  - ▶ Represent somehow the geometric transformation  $T$
  - ▶ Design a **similarity measure**  $D(f, g, T)$
  - ▶ Devise an optimization algorithm



# Representing geometric transformation

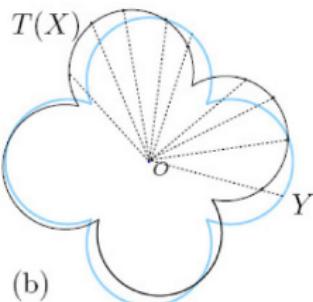
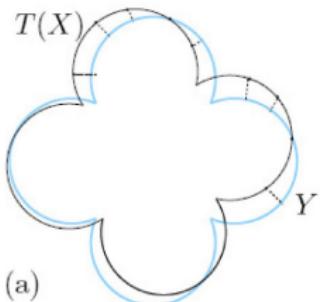
- Classical (matrix) groups
  - ▶ rotations, scaling, isometries, affine transformations,...
- Splines
- General diffeomorphisms
  - ▶ Discretized as **Free Form Deformations** (FFD)
  - ▶ i.e. we assign to every pixel (voxel)  $x$  a displacement vector  $u(x)$   
s.t.:
$$T(x) = x + u(x)$$
  - ▶ Requires some **regularization** of the solution (e.g. Tichonov regularization )

# Similarity measures

- Landmark based

- ▶ If we know that a set of points  $\{p\}$  in the first image  $f$  correspond to the points  $\{q\}$  in the second image  $g$
- ▶ Remark: such points should be characteristic points of an object (a contour, corners or other easily detectable couple of points)
- ▶ Then we may choose:

$$D(f, g, T) = \sum_{p \in \{p\}} \|Tp - q\|^2 \quad (1)$$





# Similarity measures

- Intensity based

- ▶ Uses directly the intensity functions of the images
- ▶ Remark: No point extraction or segmentation is required
- ▶ Then we may choose some functional norm, the simplest ones being:

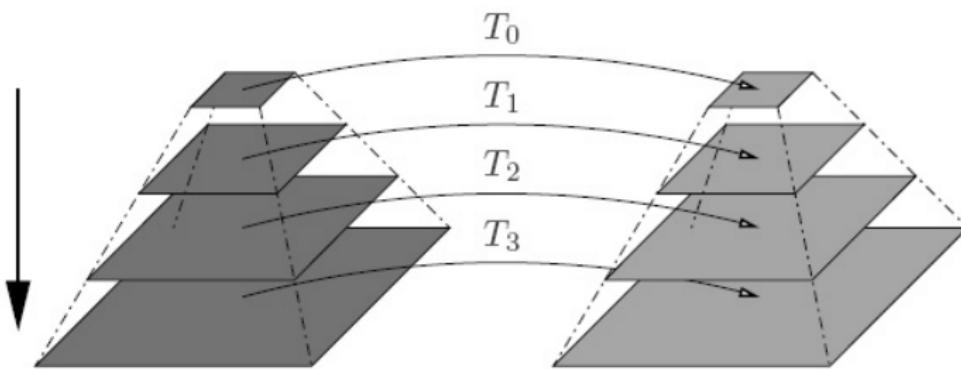
$$\text{SSD}(f, g) = \int_{\Omega} (f(x) - g(T(x)))^2 = \|f - g \circ T\|_2^2$$

$$\text{SAD}(f, g) = \int_{\Omega} |f(x) - g(T(x))| = \|f - g \circ T\|_1$$

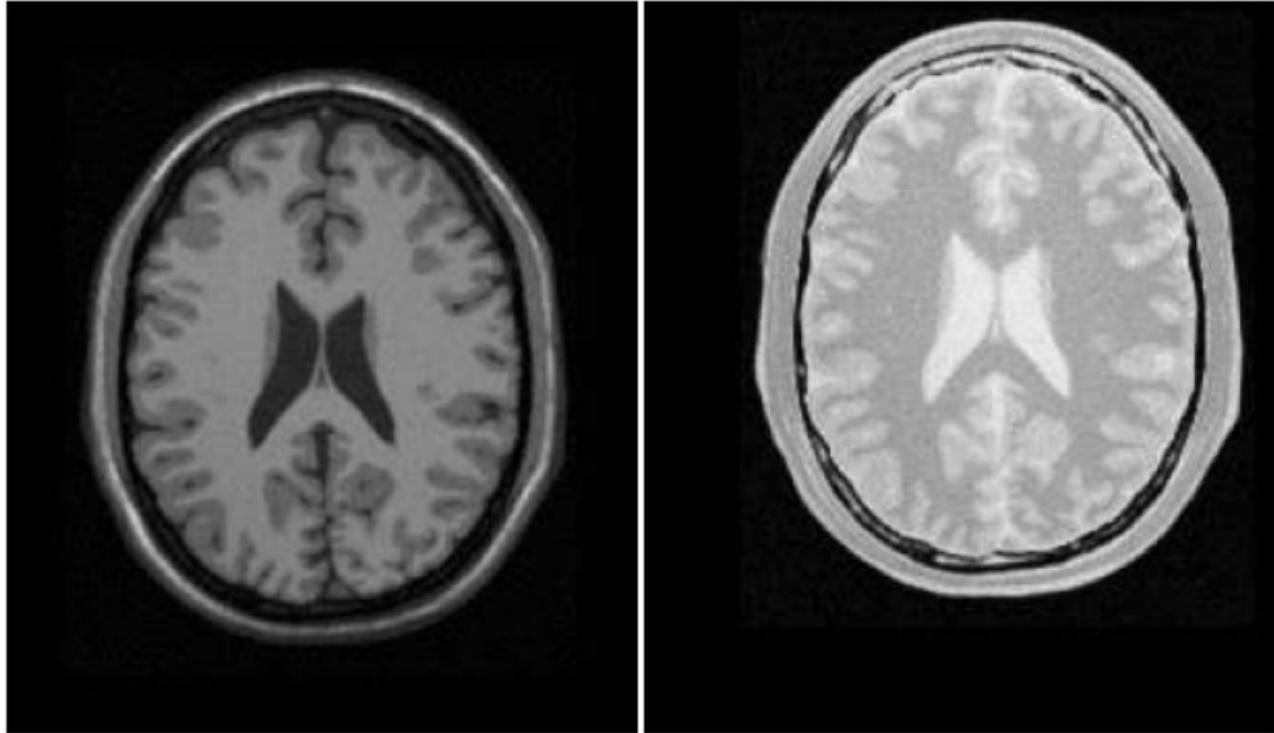
- ▶ Also in use: mutual information, cross correlation,...
- ▶ Depending on
  - ★ statistical hypothesis (e.g. gaussian noise)
  - ★ *a priori relation* between the intensity functions (identical, linear dependence, functional dependence, ...)

# Optimization

- Generally gradient descent
- Non convex functional  $\Rightarrow$  Big dependence on the initialization
- Use the pyramid trick:

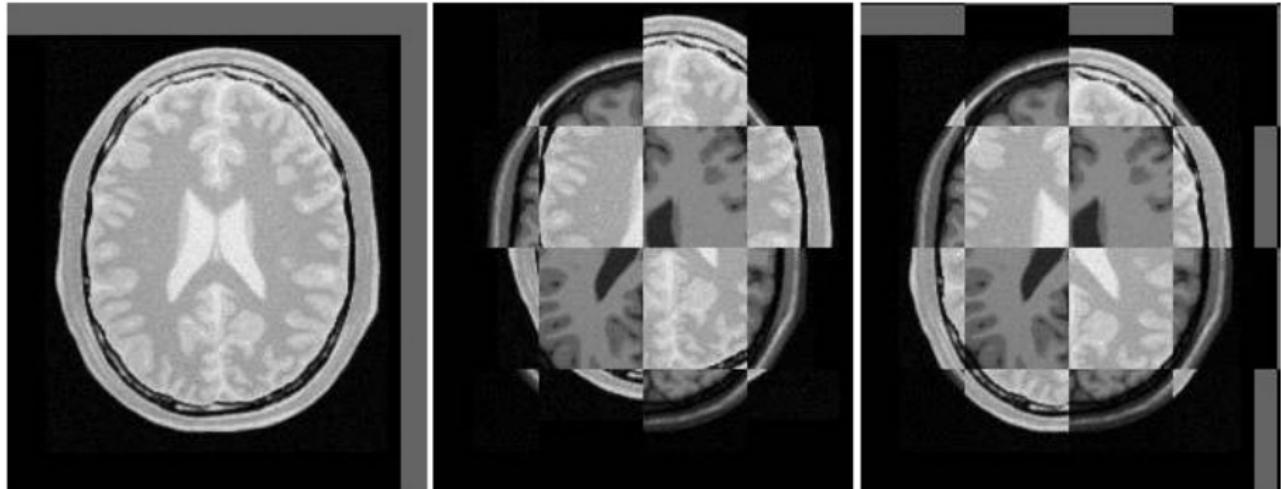


## Examples



T2 weighted and proton density brain MRI

# Examples



Moving image, initial and final alignment



6

## Shape analysis

- Basics

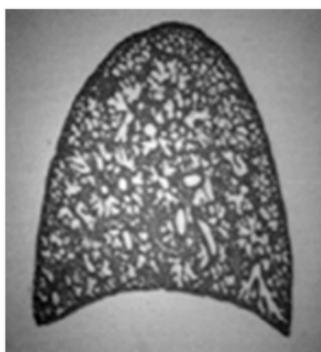


# Shape analysis

- Shape measurements are physical dimensional measures that characterize the appearance of an object
- The goal is to use the fewest necessary measures to characterize an object adequately so that it may be unambiguously classified
- The shape may not be entirely reconstructable from the descriptors, but the descriptors for different shapes should be different enough that the shapes can be discriminated

# Area

- The **area** is the number of pixels in a shape
- The convex area of an object is the area of the convex hull that encloses the object

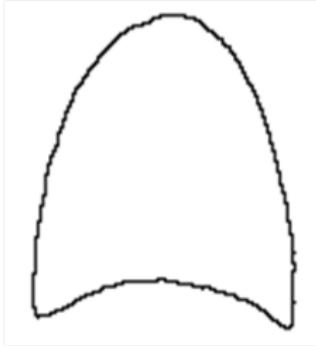


# Perimeter

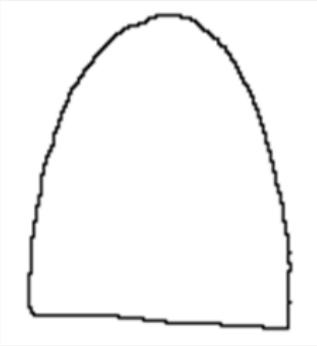
- The **perimeter** (length) is the number of pixels in the boundary of the object
- The **convex perimeter** of an object is the perimeter of the convex hull that encloses the object



Perimeter



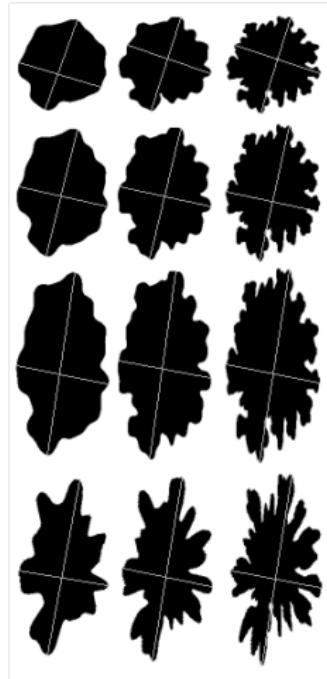
External perimeter



Convex perimeter

# Axis

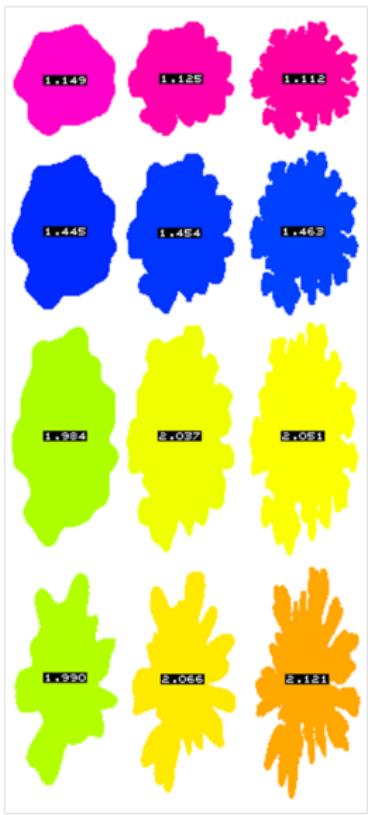
- The major axis is the longest line that can be drawn through the object
- The minor axis is the longest line that can be drawn through the object whilst remaining perpendicular with the major-axis



# Aspect Ratio

- The major axis is the longest line that can be drawn through the object
- The minor axis is the longest line that can be drawn through the object whilst remaining perpendicular with the major-axis
- The **aspect ratio** measures the ratio of the objects height to its width:

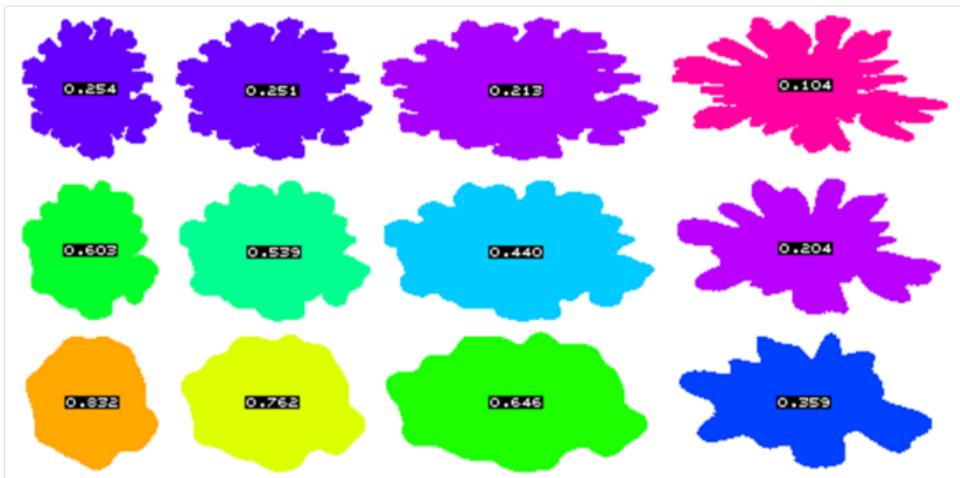
$$\text{AspectRatio} = \frac{\text{Height}}{\text{Width}}$$



# Compactness

- Compactness (also called **formfactor**) is defined as the ratio of the area of an object to the area of a circle with the same perimeter:

$$\text{Compactness} = \frac{4\pi \cdot \text{Area}}{\text{Perimeter}^2}$$



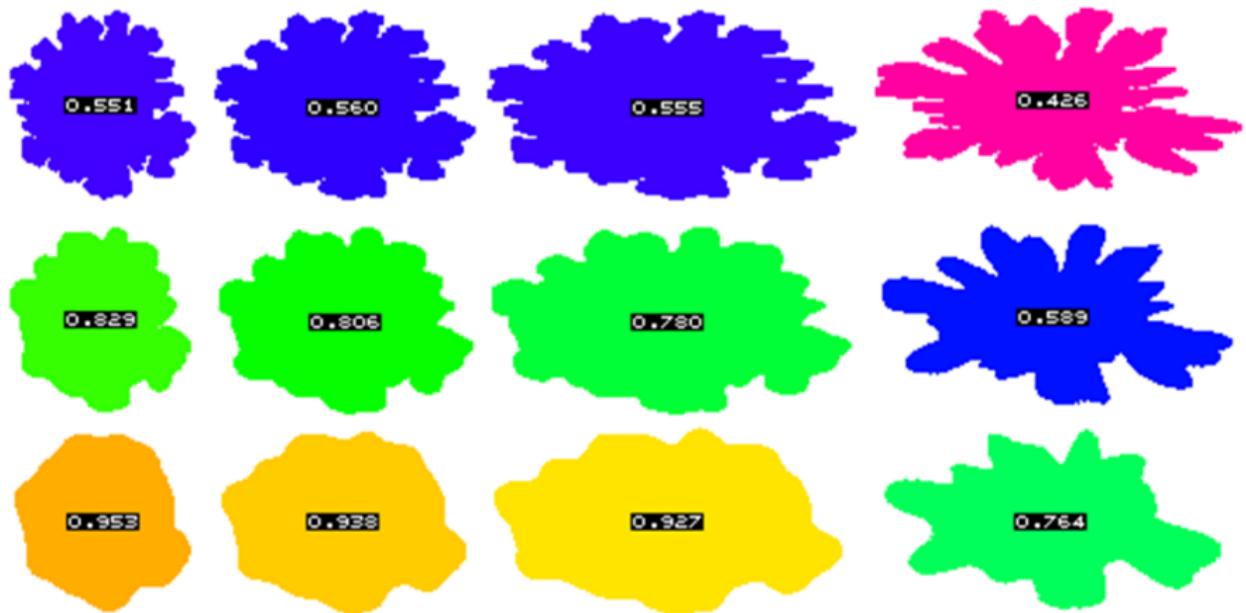


# Convexity

- **Convexity** is the relative amount that an object differs from a convex object
- A measure of convexity can be obtained by forming the ratio of the perimeter of an objects convex hull to the perimeter of the object itself:

$$\text{Convexity} = \frac{\text{Perimeter}_{\text{convex}}}{\text{Perimeter}_{\text{external}}}$$

# Convexity



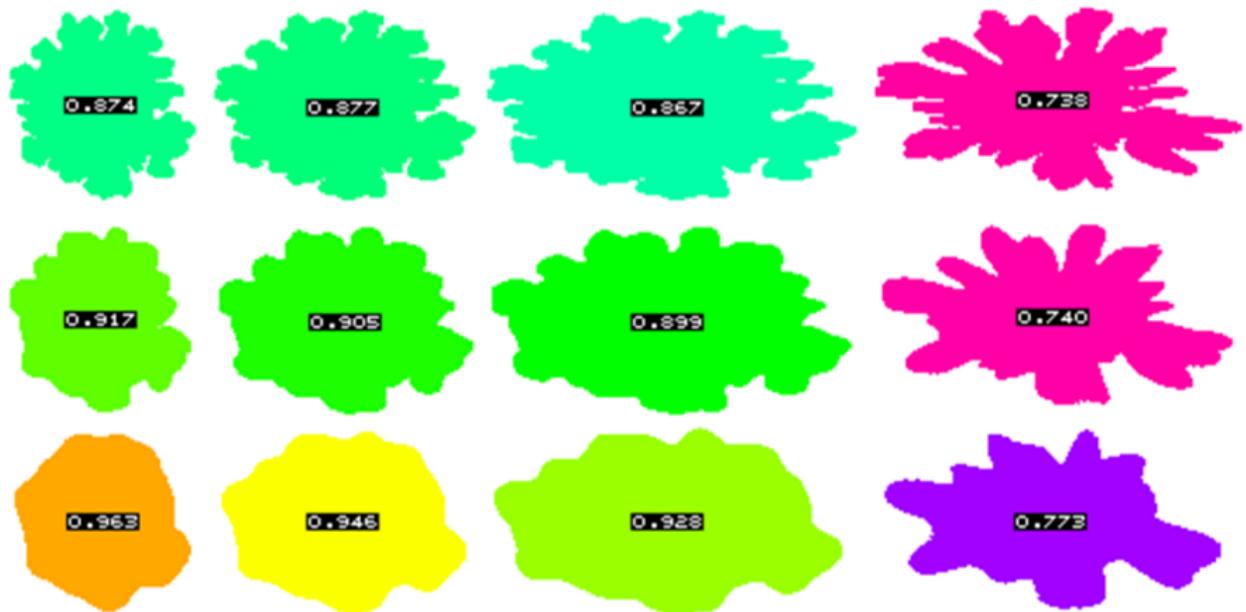


# Solidity

- **Solidity** measures the density of an object
- A measure of solidity can be obtained as the ratio of the area of an object to the area of a convex hull of the object:

$$\text{Solidity} = \frac{\text{Area}}{\text{Area}_{\text{convex}}}$$

# Solidity





## Fiber length

- Fiber length gives an estimate as to the true length of a threadlike object

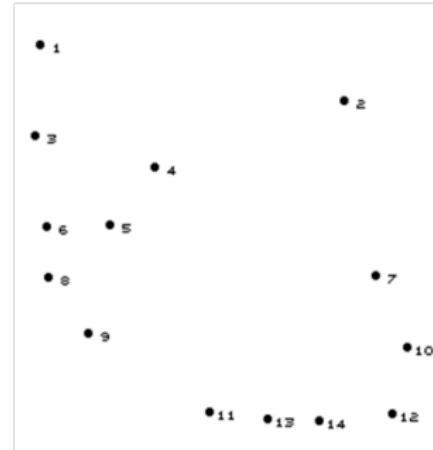
$$\text{FiberLength} = \frac{\text{Perimeter} + (\text{Perimeter}^2 - 16 \cdot \text{Area})^{1/2}}{4}$$

- The estimate is fairly accurate on threadlike objects with a formfactor that is less than 0.25 and gets worse as the formfactor increases

# Average Fiber length

- The number of skeleton end-points estimates the number  $N$  of fibers (half the number of ends)

$$\text{AverageFiberLength} = \frac{\text{TotalFiberLength}}{N}$$





# Reference Bibliography

## Books:

- 1 Gonzalez RC, Woods RE: *Digital Image processing*, Prentice Hall, 2001
- 2 C.M. Bishop: *Pattern Recognition and Machine Learning*, Springer-Verlag, 2006
- 3 E.R. Davies: *Machine Vision: Theory, Algorithms, Practicalities*, Morgan Kaufmann, 2005
- 4 R. Hartley, A. Zisserman: *Multiple View Geometry in Computer Vision*, Cambridge University Press, 2003
- 5 Sonka M, Hlavac V, Boyle R: *Image Processing analysis and machine vision*, PWS pubs, 2nd ed., 1999
- 6 Haralick RM, Shapiro LG: *Computer and Robot Vision*, Vol I-II, Addison-Wesley, Mass., USA, 1992
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- 10 Horn BKP: *Robot Vision*, MIT Press, 1986
- 11 Pratt WK: *Digital Image Processing*, John Wiley & Sons, inc., 3rd Edition, 2001
- 12 Jain AK: *Fundamentals of Digital Image Processing*, Englewood Cliffs, NJ:Prentice H

## Journals:

- 1 IEEE Transaction on Pattern Analysis and Machine Intelligence
- 2 IEEE Transaction on Image Processing International
- 3 Journal of Computer Vision Image and Vision Computing
- 4 Pattern Recognition
- 5 Journal of Mathematical Imaging and Vision
- 6 Pattern Recognition and Image



# Software tools

- Photoshop
- GIMP
- ImageJ
- Mathematica and Digital Image Processing Toolbox
- MATLAB: Image Processing Toolbox
- ITK
- VTK
- AVS



# Credits

Some materials for this presentation have been drawn from public resources available on the World Wide Web.

In particular, most historical data were taken from Klaus Mueller presentation (<http://www.cs.sunysb.edu/~mueller/>).

Some images and data were taken from:

- Gonzalez RC, Woods RE: *Digital Image processing*
- Sam Gahmbir  
([http://mips.stanford.edu/public/video\\_lectures/index.adp](http://mips.stanford.edu/public/video_lectures/index.adp))
- Guy Gilboa, Nir Sochen and Yehoshua Y. Zeevi ([http://www.math.ucla.edu/~gilboa/PDE-based\\_image\\_filtering.html](http://www.math.ucla.edu/~gilboa/PDE-based_image_filtering.html))
- Chunming Li, Chenyang Xu, Changfeng Gui and Martin D. Fox  
(<http://www.engr.uconn.edu/~cmlis/code/>)
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