



TÉCNICO LISBOA

# Principles of Biosignals and Biomedical Imaging

3<sup>rd</sup> year, P<sub>3</sub> (ECTS: 3.0), LEBiom  
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# Outline

## Image reconstruction

Reconstruction from noisy and missing data

Image Tomography / Radon Transform

# Image Reconstruction

# Reconstruction with missing data

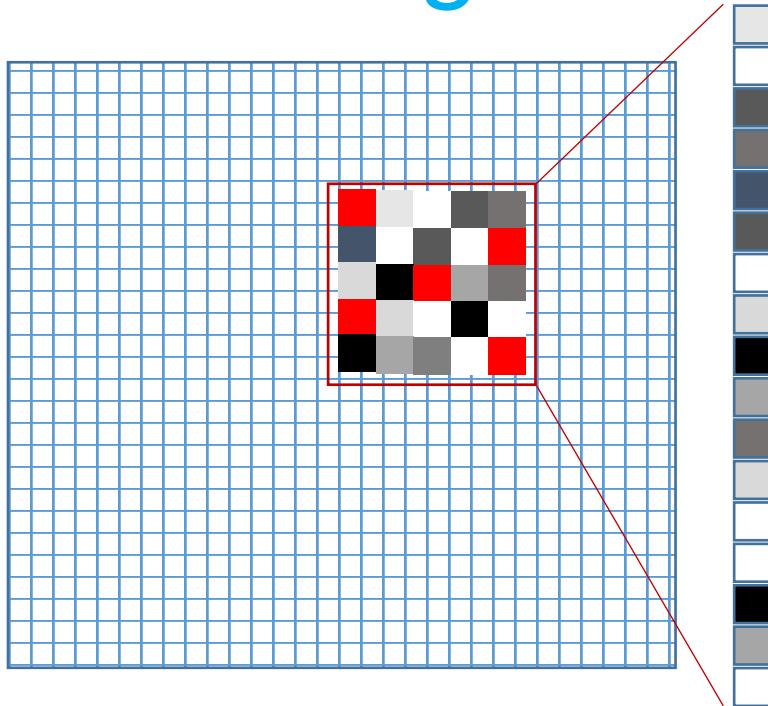
Original



Uncomplete (20%)



# Reconstruction with missing data



# Reconstruction with missing data

```
X. =double(imread('len_gray.jpg'));
X. =255*x/max(x(:));
z = zeros(size(x));

[N,M] = size(x);
thrs = 0.80; %Discarded amount
mask = rand(size(x))>thrs;

e = 0; % Noise parameter
eta = e*randn(size(x)); % Noise signal
xNoisy = x+eta; % Noisy Data

y = mask.*xNoisy; % Noisy and missing data

w=5; %Dimensão da janela
for l=w+1:N-w-1
    for J=w+1:N-w-1
        aux=y(l-w:l+w,J-w:J+w);
        q=nonzeros(aux(:));
        if (~isempty(q(:)))
            z(l,J)=median(q(:));
        end
    end
end
```

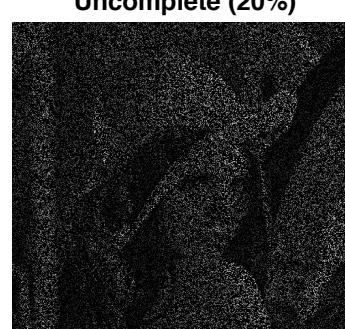
SNR=20\*log(norm(x(:))/norm(x(:)-z(:)));

```
figure(1); colormap(gray(256));
subplot(1,3,1); image(x); axis off; title('Original','fontsize', 18);
subplot(1,3,2); image(y); axis off; title(strcat('Incomplete (' ,strcat(num2str(100*(1-thrs)), '%'))), 'fontsize', 18);
subplot(1,3,3); image(z); axis off; title(strcat('SNR(median): ',num2str(SNR,\b \%5.2f)), 'fontsize', 18);
```

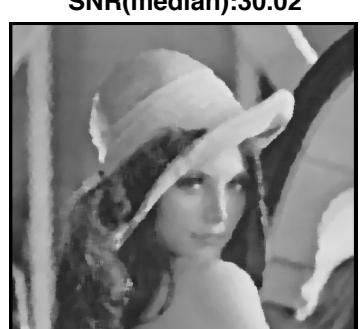
**Original**



**Uncomplete (20%)**



**SNR(median):30.02**



# Reconstruction with noisy and missing data

```
x=double(imread('len_gray.jpg'));
x=255*x/max(x(:));

[N,M] = size(x);
thrs = 0.80; %Discarded amount
mask = rand(size(x))>thrs;

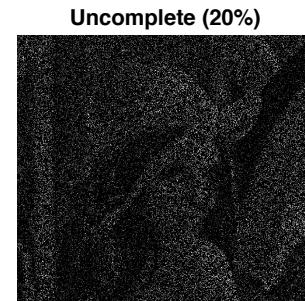
e = 50; % Noise parameter
eta = e*randn(size(x)); % Noise signal
xNoisy = x+eta; % Noisy Data

y = mask.*xNoisy; % Noisy and missing data
z = zeros(size(x));

w=5; %Dimensão da janela: 2w+1
for l=w+1:N-w-1
    for J=w+1:N-w-1
        aux=y(l-w:l+w,J-w:J+w);
        q=nonzeros(aux(:));
        if (~isempty(q(:)))
            %Test other statistics, e.g., mean, max, min
            z(l,J)=median(q(:));
        end
    end
end
```



Original



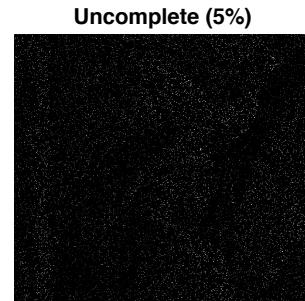
Uncomplete (20%)



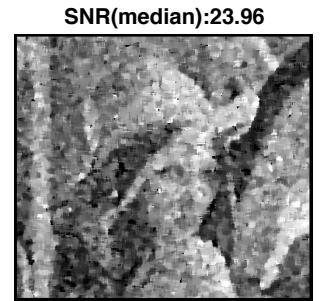
SNR(median):28.30



Original



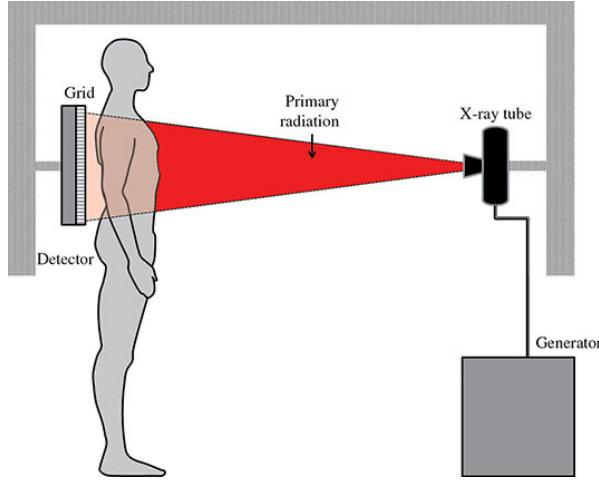
Uncomplete (5%)



SNR(median):23.96

# Break

# X Ray

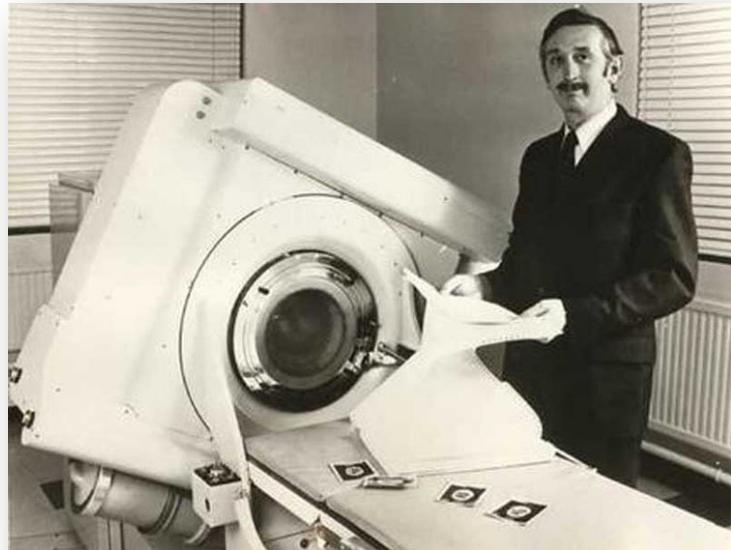


# Computer Tomography

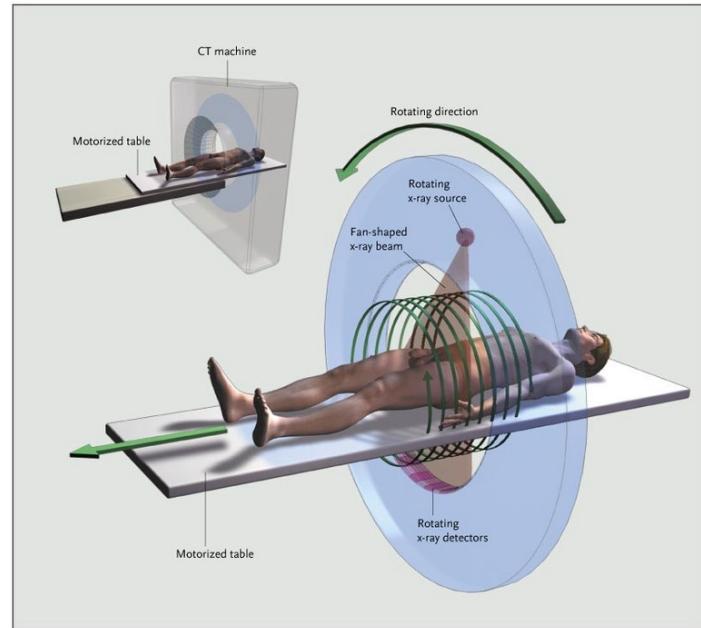
1972  
Godfrey Hounsfield

The first commercially available CT scanner was created by British engineer Godfrey Hounsfield of EMI Laboratories in 1972. He co-invented the technology with physicist Dr. Allan Cormack. Both researchers were later on jointly awarded the 1979 Nobel Prize in Physiology and Medicine.

(<https://catalinaimaging.com/history-ct-scan/>)



# Computer Tomography

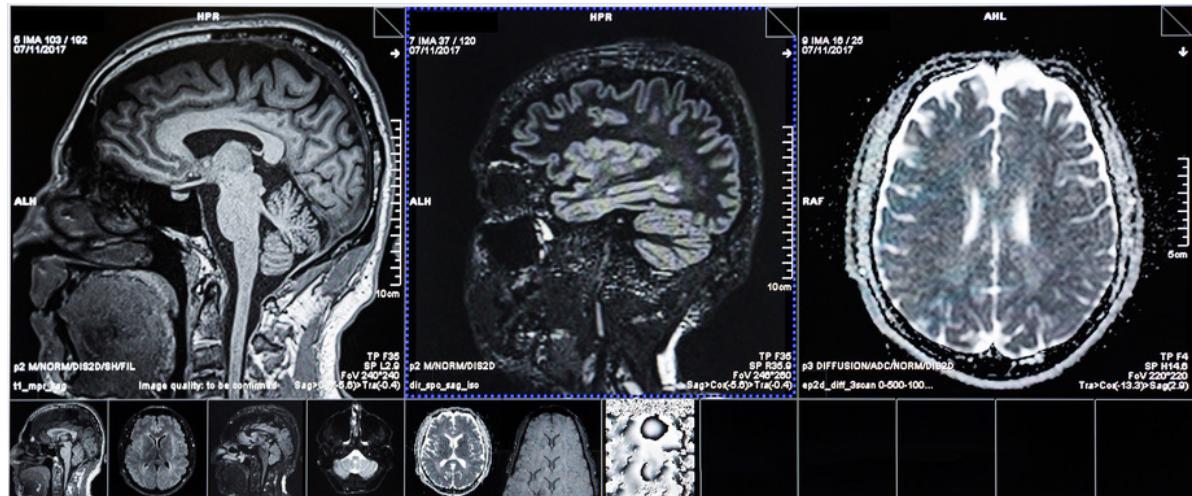


# Computer Tomography

X-Ray

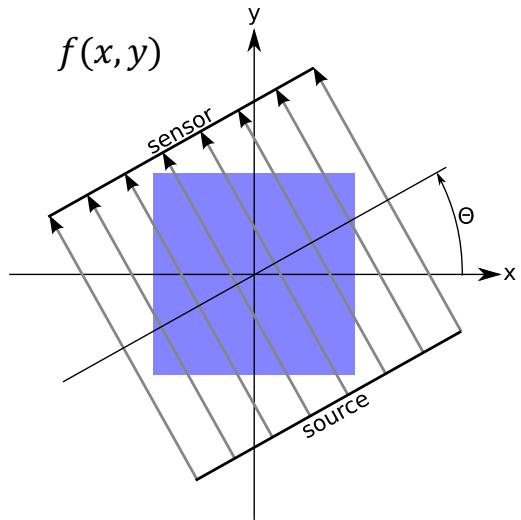


CT

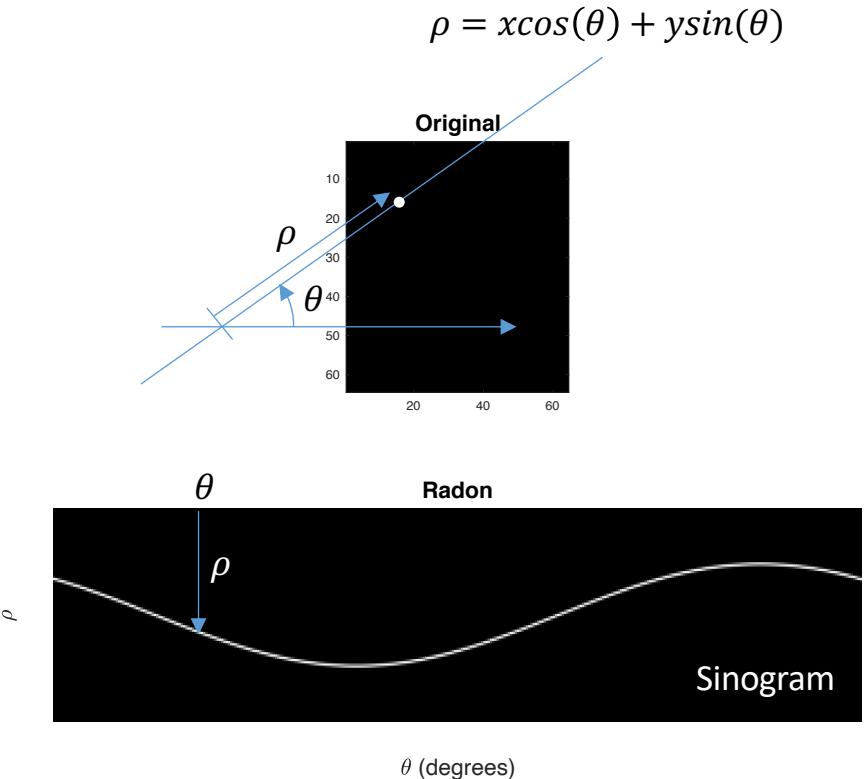


<https://two-views.com/ct/brain-scan.html>

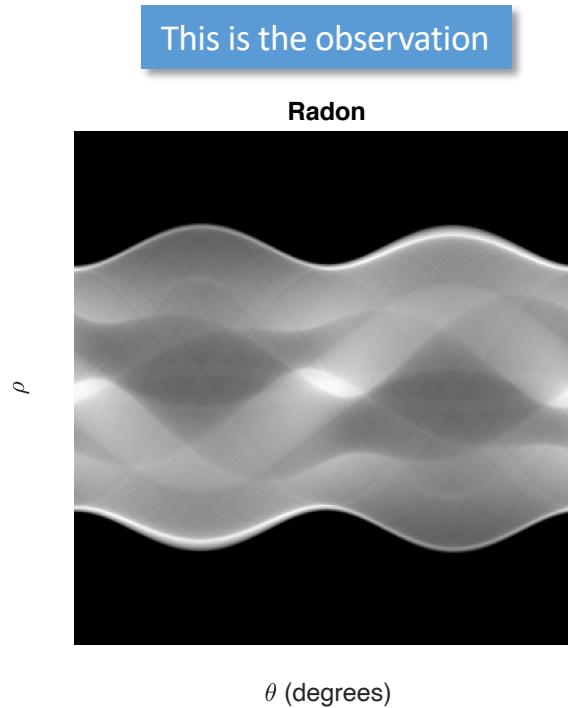
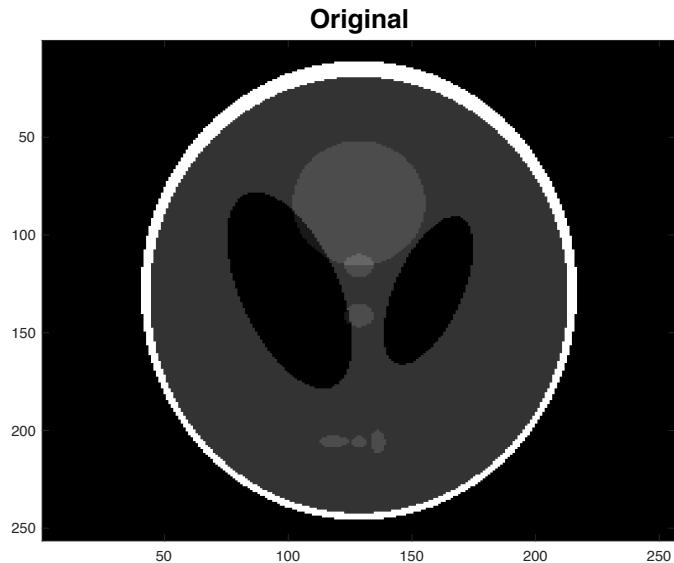
# Radon transform



$$R(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) dx dy$$

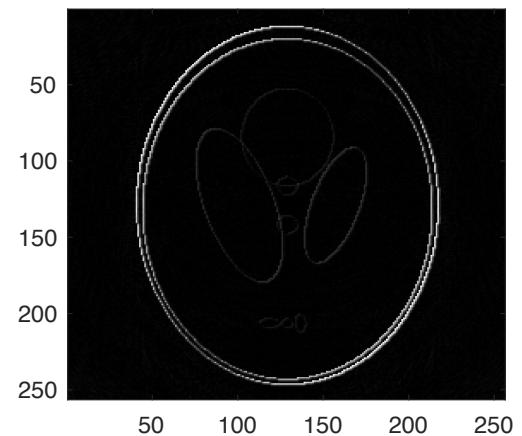
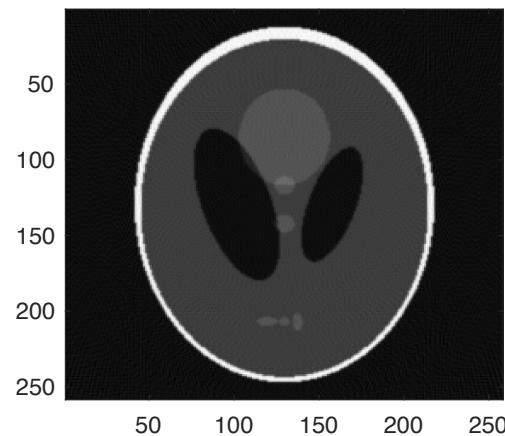
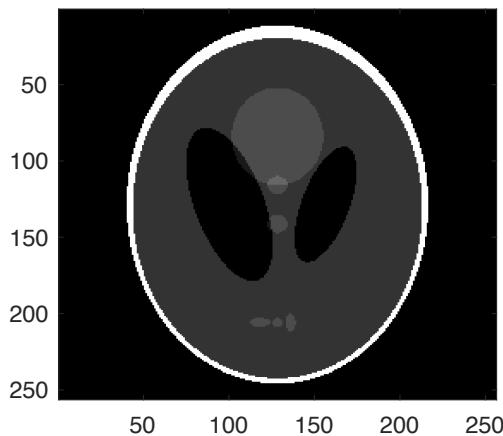


# Radon transform



# Inverse Radon

**SNR=35.01 dB**



# Inverse Radon

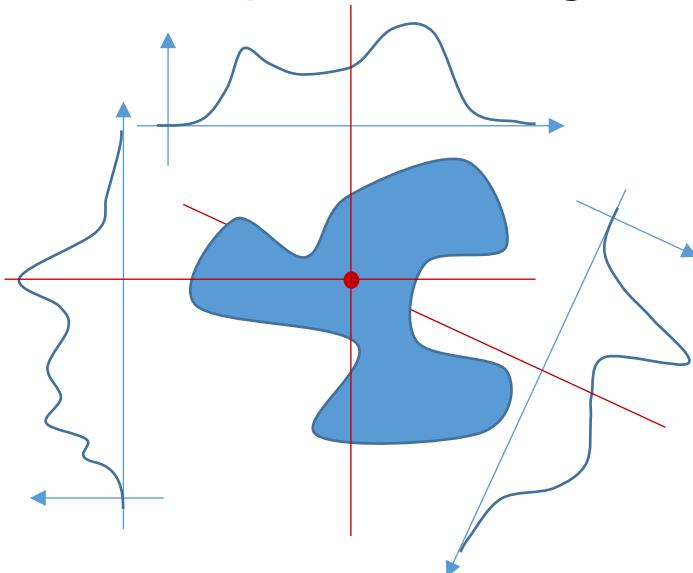
## Reconstruction Methods

- Fourier Reconstruction
- **Backprojection Filtering**
- Fourier Filtered Backprojection

# Backprojection

The value of the backprojection  $B(x,y)$  is evaluated by integrating  $R(\rho, \theta)$  over  $\theta$  for all lines that pass through that point.

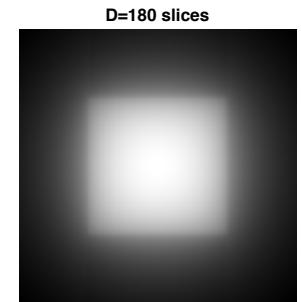
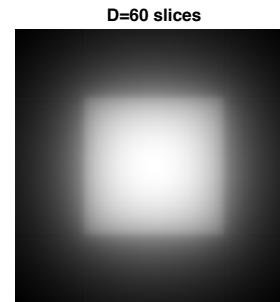
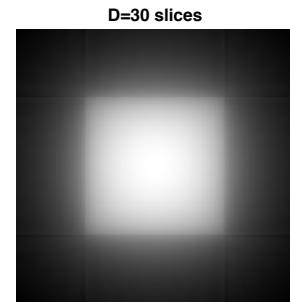
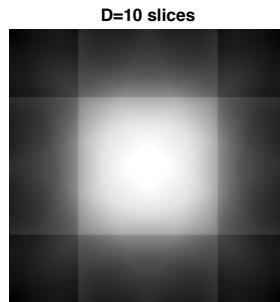
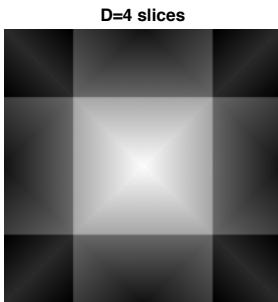
$$B(x, y) = \int_0^{\pi} R(\rho = x\cos(\theta) + y\sin(\theta), \theta) d\theta$$



# Backprojection

Discrete number of projections

$$\tilde{B}(x, y) = \sum_k R(\rho = x\cos(\theta_k) + y\sin(\theta_k), \theta_k)$$



# Lena

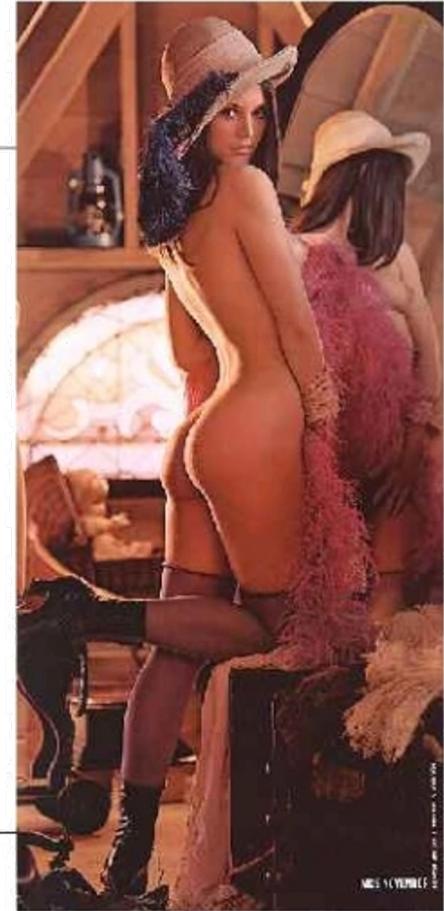


# Lena

The USC-SIPI Image Database



Top Models in CG





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