



TÉCNICO LISBOA

Principles of Biosignals and Biomedical Imaging

3rd year, P₃ (ECTS: 3.0), LEBiom
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Outline

Edge detection

Sobel

Canny

Segmentation

Intensity

Texture

Edge detection



Gradient field

Approximation for the **first** derivatives

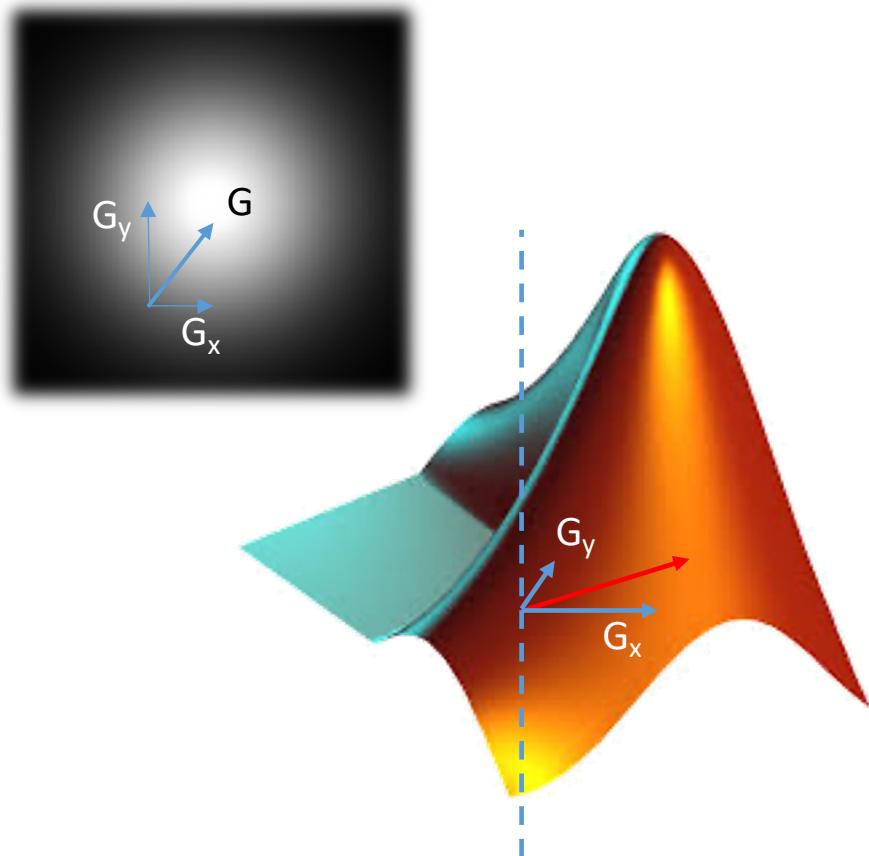
$$G_x(i,j) = I(i,j) - I(i-1,j)$$

$$G_y(i,j) = I(i,j) - I(i,j-1)$$

Gradient vector

$$G(i,j) = (G_x(i,j), G_y(i,j))$$

Contours occur in intensity transitions
where the gradient magnitude is large



Gradient magnitude and angle

$$G_x(i,j) = I(i+1,j) - I(i-1,j)$$

$$G_y(i,j) = I(i,j+1) - I(i,j-1)$$

$$w = [1 \quad 0 \quad -1]$$

$$G_x = w * I$$

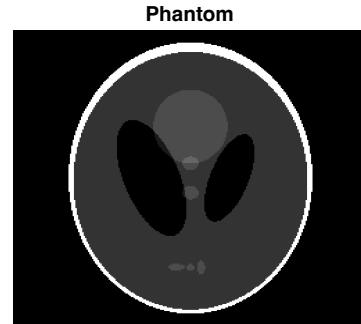
Horizontal differences

$$G_y = w' * I$$

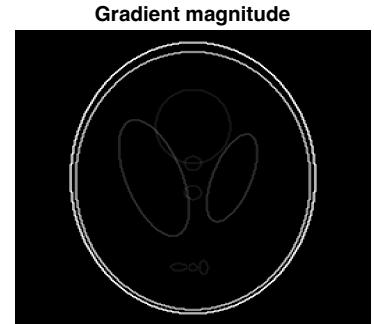
Vertical differences

$$G(i,j) = \sqrt{G_x(i,j)^2 + G_y(i,j)^2}$$

$$\theta(i,j) = \text{atan}\left(\frac{G_y(i,j)}{G_x(i,j)}\right)$$



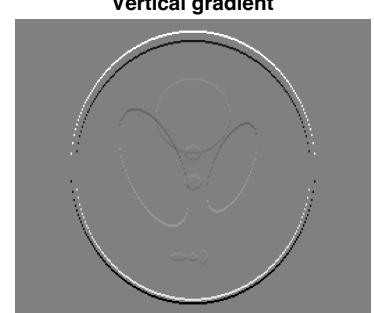
Phantom



Gradient magnitude



Horizontal gradient



Vertical gradient

Laplace Operator

Approximation for the **Second** derivatives

$$\begin{aligned}G_x^2(i,j) &= G_x(i,j) - G_x(i-1,j) \\&= [I(i,j) - I(i-1,j)] - [I(i-1,j) - I(i-2,j)] \\&= I(i,j) - 2I(i-1,j) + I(i-2,j) \\&\sim \mathbf{I}(i+1,j) - 2\mathbf{I}(i,j) + \mathbf{I}(i-1,j)\end{aligned}$$

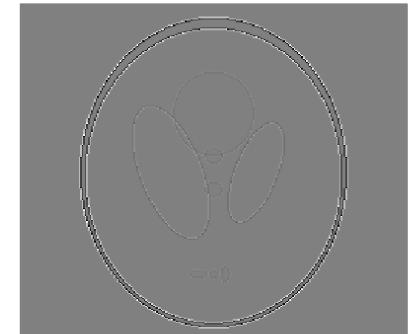
$$G_x^2 = I * [1 \quad -2 \quad 1]$$

$$\begin{aligned}G_y^2(i,j) &= G_y(i,j) - G_y(i,j-1) \\&= [I(i,j) - I(i,j-1)] - [I(i,j-1) - I(i,j-2)] \\&= I(i,j) - 2I(i,j-1) + I(i,j-2) \\&\sim \mathbf{I}(i,j+1) - 2\mathbf{I}(i,j) + \mathbf{I}(i,j-1)\end{aligned}$$

$$G_y^2 = I * [1 \quad -2 \quad 1]^T$$

$$\nabla^2 = \nabla \cdot \nabla = \left(\frac{\partial^2}{\partial x^2} \right) + \left(\frac{\partial^2}{\partial y^2} \right)$$

$$\approx \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



Sobel operator

$$W_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad W_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$G_x = W_x * I$$

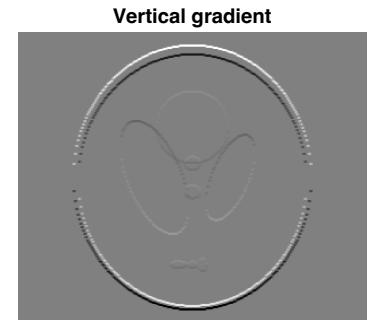
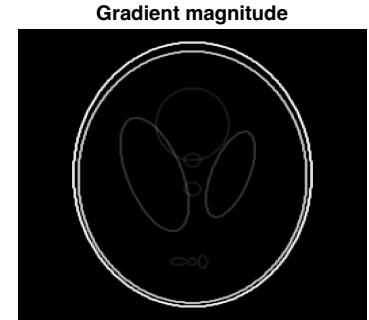
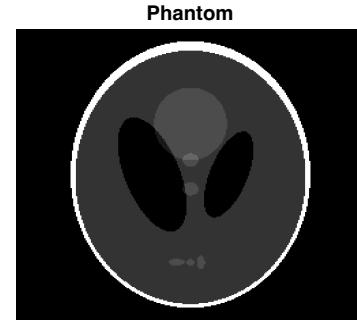
Filtered
Horizontal
differences

$$G_y = W_y * I$$

Filtered
Vertical
differences

$$G(i,j) = \sqrt{G_x(i,j)^2 + G_y(i,j)^2}$$

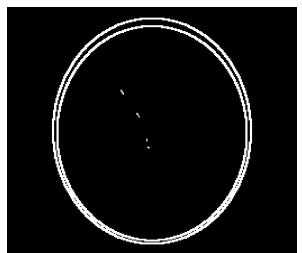
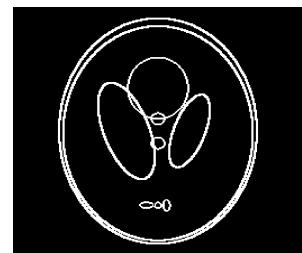
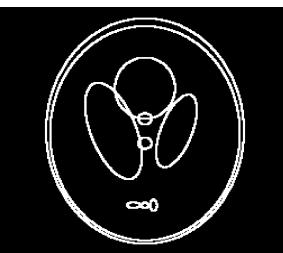
$$\theta(i,j) = \text{atan}\left(\frac{G_y(i,j)}{G_x(i,j)}\right)$$



Edge detection

$$G(i,j) = \sqrt{G_x(i,j)^2 + G_y(i,j)^2}$$

$$C(i,j) = \begin{cases} 0 & \text{if } G(i,j) < \text{thrs} \\ 1 & \text{otherwise} \end{cases}$$



Canny edge detector

1. Noise reduction
2. Gradient calculation
3. Non-maximum suppression
4. Double thresholding
5. Edge Tracking by Hysteresis

Canny Edge Detection Step by Step in Python
by Sofiane Sahir



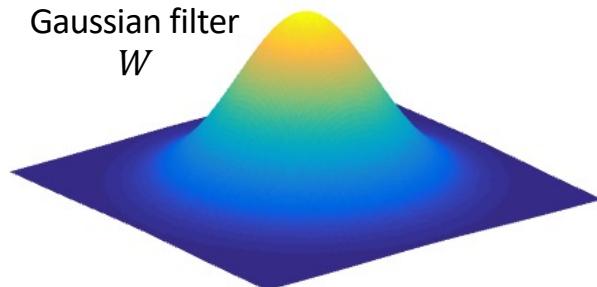
https://docs.opencv.org/4.x/d4/d22/tutorial_py_canny.html

<https://indiantechwarrior.com/hypothesis-testing/>

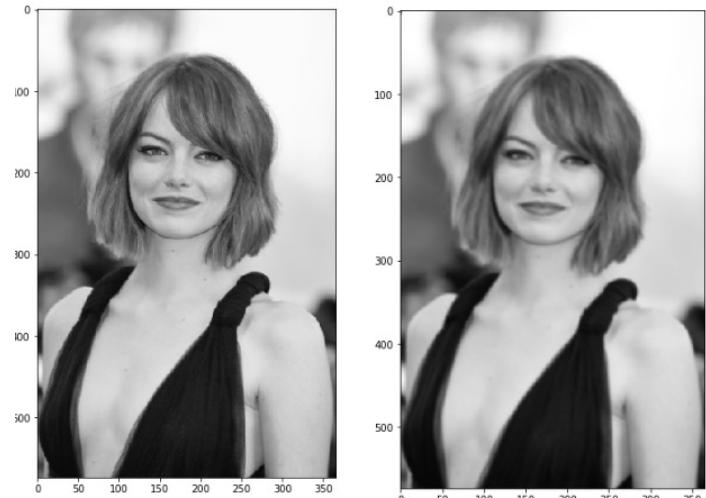
<https://towardsdatascience.com/canny-edge-detection-step-by-step-in-python-computer-vision-b49c3a2d8123>

Canny edge detector

1. Noise reduction



$$X = I * Wc$$



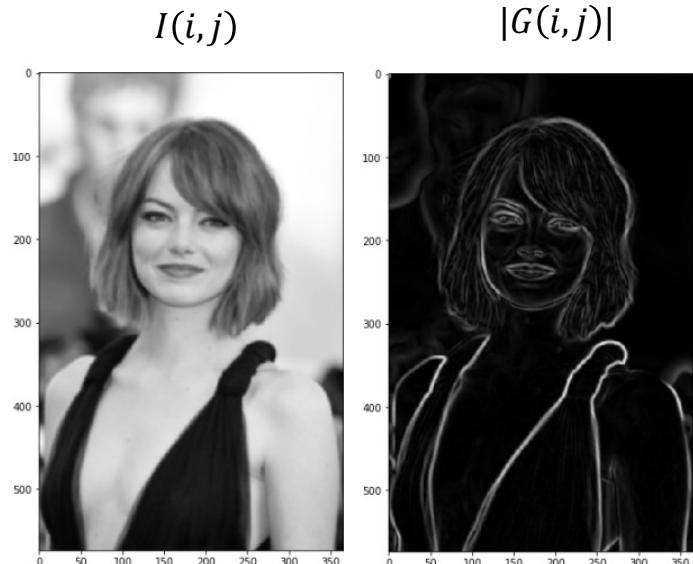
<https://towardsdatascience.com/canny-edge-detection-step-by-step-in-python-computer-vision-b49c3a2d8123>

Canny edge detector

2. Gradient computation

$$G_x = W_x * I \quad G_y = W_y * I$$

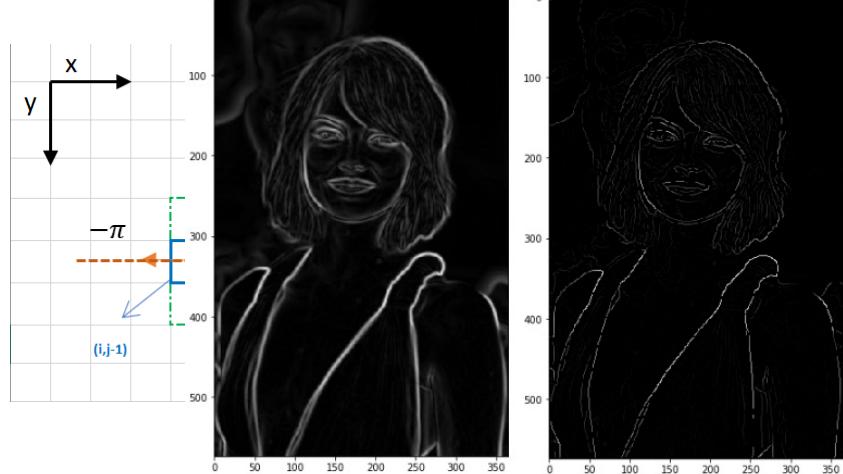
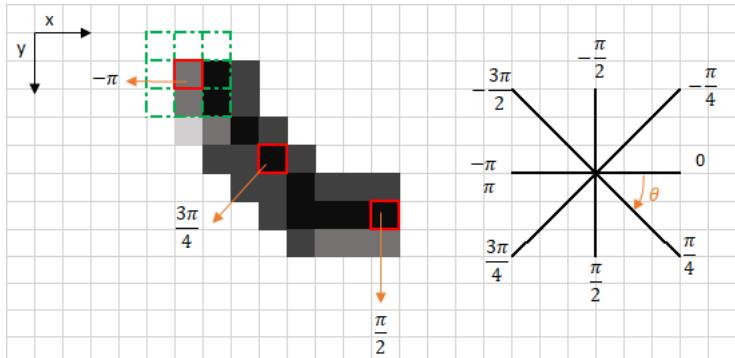
$$G(i,j) = \sqrt{G_x(i,j)^2 + G_y(i,j)^2}$$
$$\theta(i,j) = \text{atan} \left(\frac{G_y(i,j)}{G_x(i,j)} \right)$$



<https://towardsdatascience.com/canny-edge-detection-step-by-step-in-python-computer-vision-b49c3a2d8123>

Canny edge detector

3. Non-maximum suppression



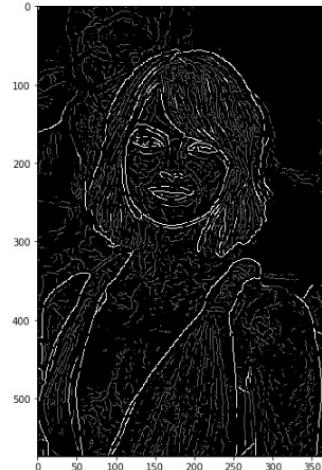
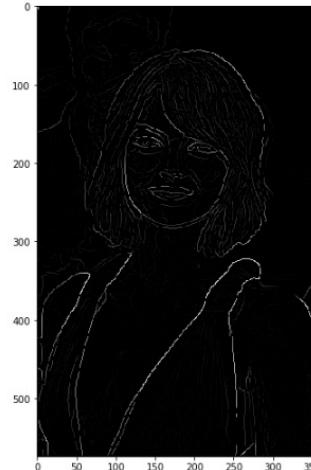
The purpose of the step is to check if the pixels on the same gradient direction are more or less intense than the ones being processed. If one those two pixels are more intense than the one being processed, then only the more intense one is kept. **The intensity value of the current pixel (i, j) is set to 0 if it is not the highest intense.**

Canny edge detector

$$C(i,j) = \begin{cases} 1 & I(i,j) > thrs_1 \\ 0.5 & if thrs_2 < I(i,j) < thrs_1 \\ 0 & if I(i,j) < thrs_2 \end{cases}$$

4. Double thresholding

- The double threshold step aims at identifying 3 kinds of pixels: **strong, weak, and non-relevant**:
- Strong pixels are pixels that have an intensity so high that we are sure they contribute to the final edge.
- Weak pixels are pixels that have an intensity value that is not enough to be considered as strong ones, but yet not small enough to be considered as non-relevant for the edge detection.
- Other pixels are considered as non-relevant for the edge.

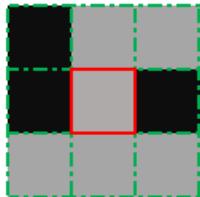


<https://towardsdatascience.com/canny-edge-detection-step-by-step-in-python-computer-vision-b49c3a2d8123>

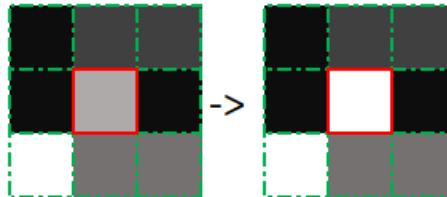
Canny edge detector

5. Edge Tracking by Hysteresis

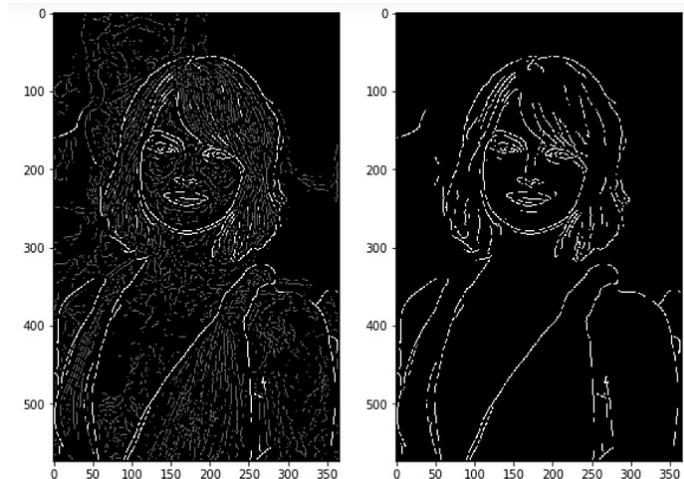
Based on the threshold results, the hysteresis consists of transforming weak pixels into strong ones, if and only if at least one of the pixels around the one being processed is a strong one, as described below



No strong pixels around



One strong pixel around



<https://towardsdatascience.com/canny-edge-detection-step-by-step-in-python-computer-vision-b49c3a2d8123>

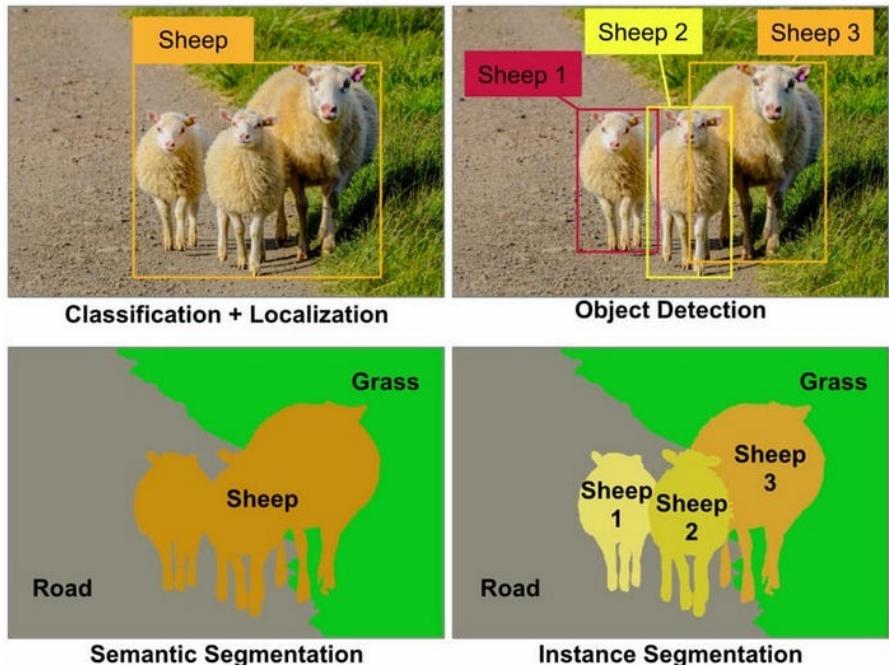
Break

Segmentation

(Chapter 14)

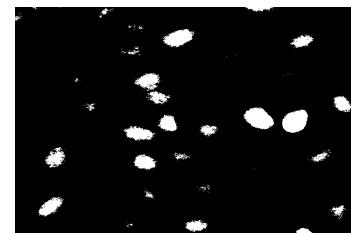
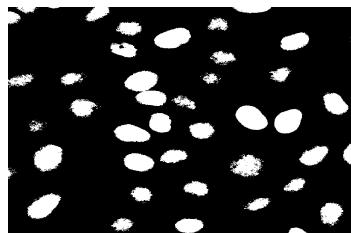
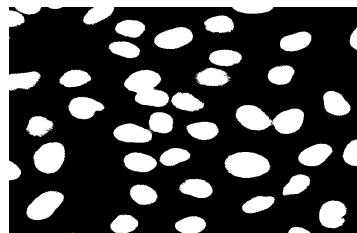
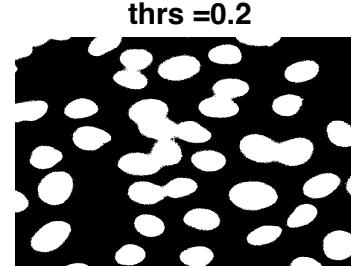
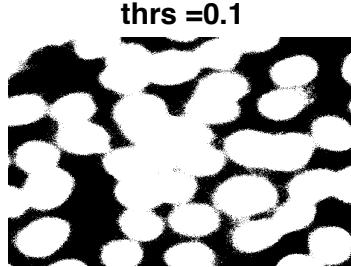
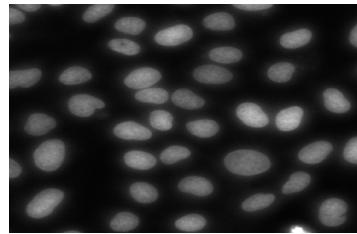
Segmentation

- Semantic segmentation
(e.g. object vs background)
- Instance Segmentation



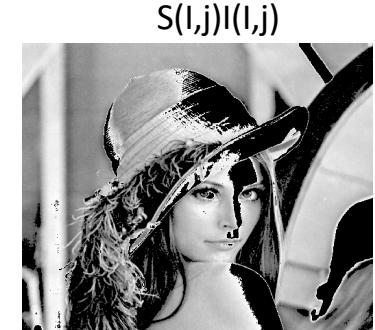
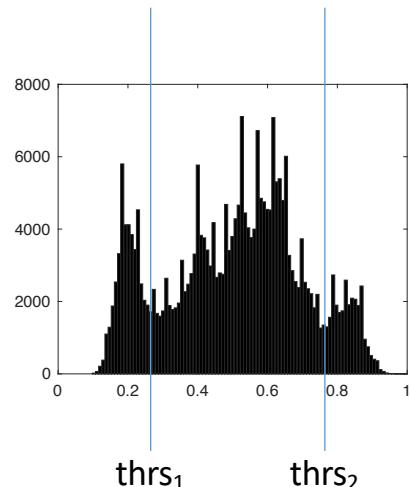
Segmentation by thresholding

$$S(i,j) = \begin{cases} 0 & \text{if } x(i,j) < \text{thrs} \\ 1 & \text{otherwise} \end{cases}$$

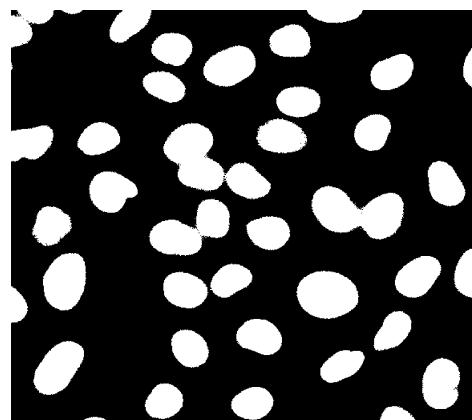
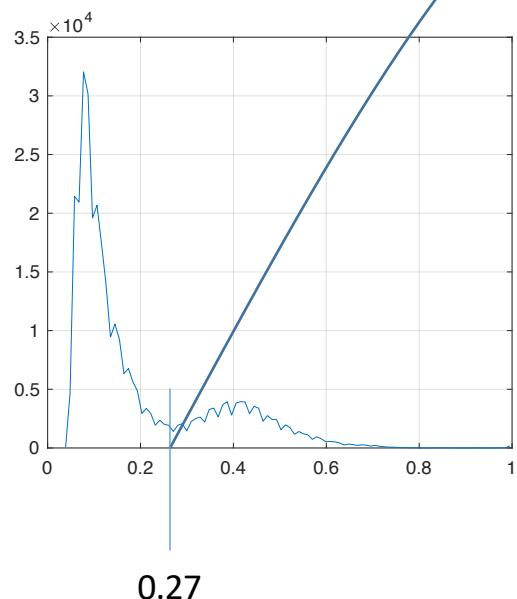
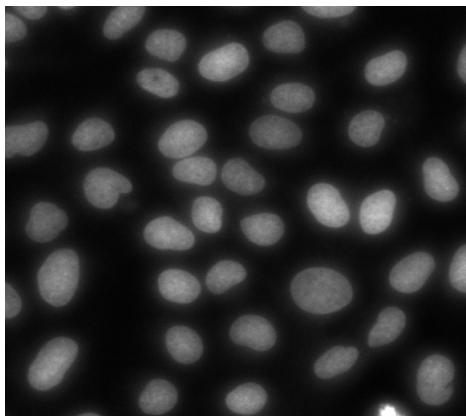


Multi-threshold segmentation

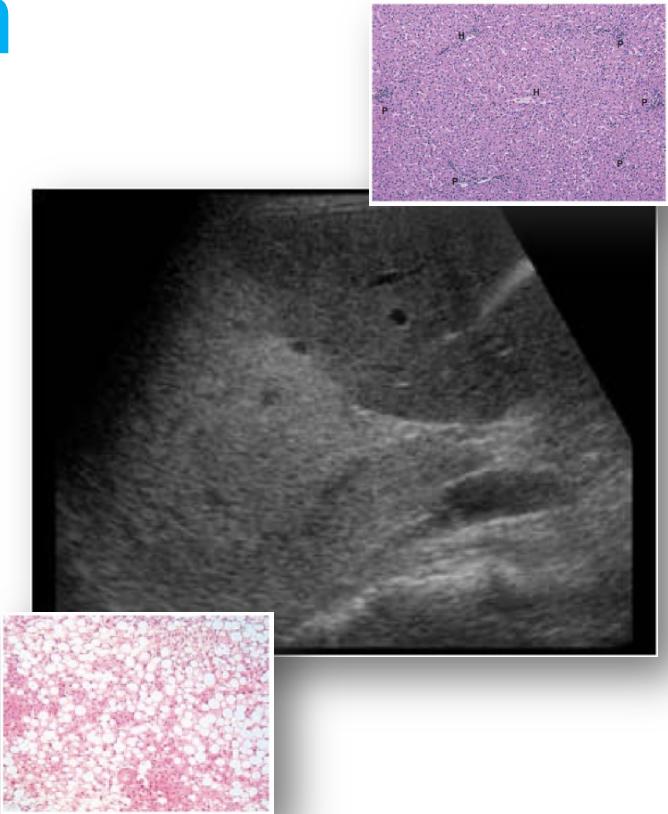
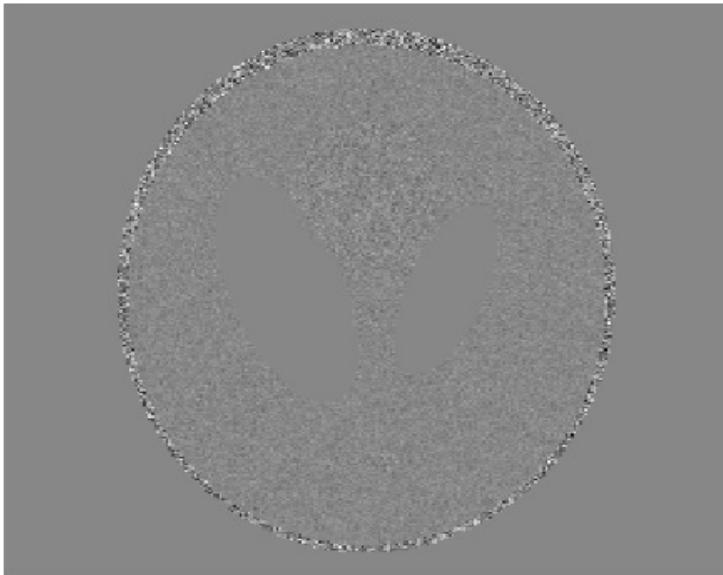
$$S(i,j) = \begin{cases} 1 & \text{if } thrs_1 < x(i,j) < thrs_2 \\ 0 & \text{otherwise} \end{cases}$$



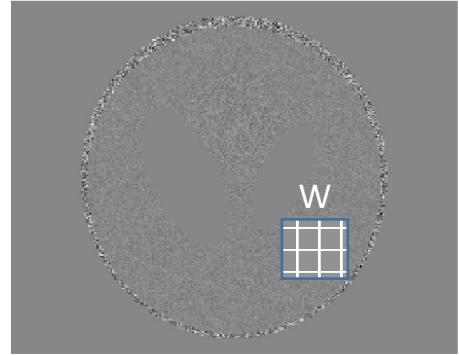
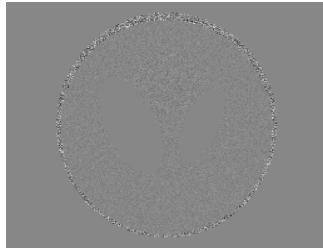
Otsu (Automatic threshold)



Texture segmentation

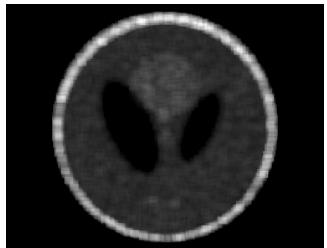


Texture segmentation

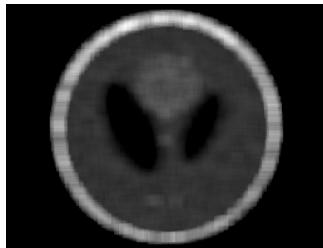


$$\mu(i,j) = \frac{1}{N_W} \sum_{i,j \in W} x(i,j)$$

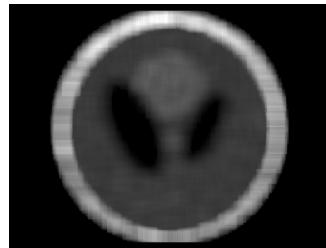
W:3



W:5



W:7



$$\sigma(i,j) = \sqrt{\frac{1}{N_W} \sum_{i,j \in W} (x(i,j) - \mu(i,j))^2}$$



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