AOS 575 Problem Set 1 **Due September 20, 2022**

1. (a) Determine an $O(\Delta x^2)$ accurate one-sided finite difference approximation to the first derivative $\left.\dfrac{\partial \psi}{\partial x}\right|_j$. Use the minimum number of points to the right of x_j – i.e. x_j, x_{j+1}, x_{j+2}, x_{j+3}, …, and assume a constant grid spacing.

(b) How does the magnitude of the leading-order term in the truncation error of this one-sided approximation compare with the leading-order term in the truncation error of the second-order centered difference approximation $\left.\dfrac{\partial \psi}{\partial x}\right|_j = \dfrac{\psi_{j+1} - \psi_{j-1}}{2\Delta x} + O(\Delta x^2)$?

2. Examine the accuracy and stability of the family of 3-step time stepping schemes for the oscillator equation $\dfrac{\partial \psi}{\partial t} = i\omega\psi$, given by

$$\psi^{*} = \psi^{n} + \alpha\Delta t\left(i\omega\psi^{n}\right)$$
$$\psi^{**} = \psi^{n} + \tfrac{1}{2}\Delta t\left(i\omega\psi^{*}\right)$$
$$\psi^{n+1} = \psi^{n} + \Delta t\left(i\omega\psi^{**}\right)$$

(a) What is the value of α that gives the highest order of accuracy, and what is that order of accuracy? (Hint: Consider the error when making a prediction at a specified time T, with T independent of your choice of Δt.)
(b) What value of α gives the most efficient scheme (i.e. the one with the largest stable time step)?
(c) Give the expression for and plot the magnitude of the amplification factor as a function of $\omega\Delta t$ for the values of α that you found in steps (a) and (b).
(d) Give the expression for and plot the magnitude of the relative phase error as a function of $\omega\Delta t$ for the values of α that you found in steps (a) and (b).
(e) Are there any computational modes present with this time-stepping scheme? If there are any computational modes, describe their behavior.
Analytic derivation of the solutions is not required.

3. The Lorenz equations are a coupled set of three nonlinear ordinary differential equations derived by E. Lorenz (J. Atmos. Sci., 1963, p. 130-141) as a highly idealized model of thermal convection. In abstract form, they are

$$\frac{dx}{dt} = -3(x-y)$$
$$\frac{dy}{dt} = -xz + rx - y$$
$$\frac{dz}{dt} = xy - z$$

where *r* is a constant. These equations' fame comes from the fact that for a certain range of *r*, the solution settles into an irregular oscillation, now known as the Lorenz attractor. This was the first example of what later came to be called a strange attractor.

(a) Write a numerical model in Python, Matlab, C, Fortran, or another programming language of your choice that integrates the Lorenz equations using an appropriate time stepping scheme of your choice (other than Forward-Euler).

(b) Try three different values of *r* of your choosing (an *r* of about 25 should give a chaotic behavior). Plot the results in x-y and x-z space.

(c) Discuss briefly (in less than a page) the rationale behind your choice of a time stepping scheme, and how you chose the length of your time-step.

(d) Finally, e-mail the model code to me (Robert.Hallberg@noaa.gov).