AOS 575 Homework Set 4 **Due November 5, 2020**

1. Solve for the evolution of the tracer field shown in Figure 5.23 of Durran (2010) [or Fig. 5.19 in Durran (1999)] using Easter's split pseudocompressibility approach and the limited piecewise linear method flux scheme described in the notes for the transport in each direction. This problem is in the square domain $0 \le x \le 1$, $0 \le y \le 1$, with the tracer distribution obeying the equation
$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}(u\phi) + \frac{\partial}{\partial y}(v\phi) = 0,$$
where the velocities are given by the nondivergent, reversing flow field
$$u(x,y,t) = \sin^2(\pi x)\sin(2\pi y)\cos(\pi t/5)$$
$$v(x,y,t) = -\sin(2\pi x)\sin^2(\pi y)\cos(\pi t/5).$$
The initial tracer distribution given by
$$\phi(x,y,0) = \tfrac{1}{2}[1+\cos(\pi r)], \text{ where}$$
$$r(x,y) = \min\left[1, 4\sqrt{\left(x-\tfrac{1}{4}\right)^2 + \left(y-\tfrac{1}{4}\right)^2}\right].$$
Show the distribution at times 2.5 and 5 using horizontal grid intervals of 0.02 and 0.01. Use a time step that will give a maximum Courant number of 0.5. Is this solution monotonic? Is it conservative? Does it exhibit significant evidence of grid splitting (evaluate this by using schemes with and without Strang splitting)? Make sure to use velocities that are appropriately centered in time. Include the code or python or Matlab script that you used to solve this problem.

   (Hint: You will want to manage your memory to make this work efficiently. Do not try to store the tracer distributions at every time level, but just use 2-D arrays for the tracers. Also, get the low resolution case working, perhaps even starting with simple upwind advection, before moving on to the high resolution case.)

2. Repeat problem 1, but this time calculate the fluxes in both directions using the tracer properties at time level n, still using limited PLM fluxes, but without using the pseudocompressibility approach. Show the same distributions as for problem 1. Is the solution monotonic? Is it conservative? Comment in general on the comparison between these two solutions.

3. a) Demonstrate that the inviscid, nonlinear, flat-bottom shallow water equations, which can be written in the momentum flux form as

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + f\hat{k} \times \vec{u} = -\nabla gh$$
$$\frac{\partial h}{\partial t} + \nabla \cdot (\vec{u}h) = 0,$$

conserve energy in the form

$$\frac{\partial}{\partial t}\int_A \left[gh^2 + \tfrac{1}{2}h(\vec{u} \cdot \vec{u})\right] dA = 0$$

if appropriate boundary conditions are applied.

b) Derive the discrete *f*-plane dispersion relation for inertia gravity waves for the *linear* form of the simplest centered difference form of the B-grid shallow water equations. The discrete dispersion relation itself is given in the notes; I am asking that you show how it is derived. (On a B-grid, eastward and northward velocities are collocated and staggered 1/2 gridspace to the north and east of thickness points.) Plot this dispersion relation over the range of resolved wavenumbers in the cases when the wavevector is exactly aligned with the grid and when the wavevector is at 45° to the grid.

c) Write down a second-order centered discretization of the full *nonlinear* inviscid flat-bottom shallow water equations on a B-grid, making sure that all terms are evaluated at the appropriate staggered locations.

d) Evaluate whether the discretization in part c) has an exactly conserved discrete analog of total energy conservation by going through the discrete equivalent of continuous derivation of energy conservation from part a).