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EAS B9018
March 4, 2022

Homework 2

1. **The image formed by a digital camera on the detector array of photodiodes is 24 x 36 mm, and is equal to the size of the detector array. Assuming the individual photodiode detector elements have a size of 5 um, how many pixels will the digital camera resolve? (Express your answer in megapixels.)**

   Let w_{image} represent the image width, h_{image} represent the image height, and s_{diode} represent the size of a photodiode detector element. Therefore,

$$\begin{aligned} w_{image} &= 24\,\mathrm{mm} = 0.024\,\mathrm{m} \\ h_{image} &= 36\,\mathrm{mm} = 0.036\,\mathrm{m} \\ s_{diode} &= 5\,\mu\mathrm{m} = 5\mathrm{e}{-6}\,\mathrm{m} \end{aligned}$$

$$\begin{aligned} N &= \frac{w_{image}\,h_{image}}{s_{diode}^2} \\ &= \frac{0.024\,\mathrm{m}\cdot 0.036\,\mathrm{m}}{(5\mathrm{e}{-6}\,\mathrm{m})^2} \\ &= 3.456\mathrm{e}7 \text{ pixels} = 34.56 \text{ megapixels} \end{aligned}$$

   So, the camera will be able to resolve 34.56 megapixels.

2. **Consider a digital imaging sensor flown on an airplane. The optical system has a focal length of 7 cm. The imaging detector array is 15 cm x 15 cm.**

   Let f represent the focal length and l_{image} represent the image detector array length. Assume flat terrain at sea level elevation ($h = 0$).

   (a) **What altitude must the aircraft fly in order to image an area 10 km x 10 km?**

   Let l_{ground} represent the image area on the ground plane and H represent the altitude of the aircraft above ground.

$$\begin{aligned} f &= 7\,\mathrm{cm} = 0.07\,\mathrm{m} \\ l_{image} &= 15\,\mathrm{cm} = 0.15\,\mathrm{m} \\ l_{ground} &= 10\,\mathrm{km} = 10\mathrm{e}3\,\mathrm{m} \end{aligned}$$

$$\begin{aligned} S &= \frac{l_{image}}{l_{ground}} \\ H &= \frac{f}{S} + h = \frac{f}{\frac{l_{image}}{l_{ground}}} + h \\ &= \frac{0.07\,\mathrm{m}}{\frac{0.15\,\mathrm{m}}{10\mathrm{e}3\,\mathrm{m}}} + 0 = 4667\,\mathrm{m} \end{aligned}$$

   Therefore, the aircraft must fly at an altitude of 4667 m above ground level.

   (b) **If the pixel pitch for the sensor's detector array is 5 mm (as in Problem 1), what is the resolution of the system assuming the aircraft flies at the altitude in (a)?**

Let p_i represent the pixel pitch for the detector array. Note that at the image plane, the pixel pitch is equivalent to the spatial resolution. Let r_{ground} represent the image resolution on the ground plane.

$$p_i = r_i = 5\,\mu\text{m} = 5\text{e}{-6}\,\text{m}$$

$$\begin{aligned} r_{ground} &= \frac{r_i}{S} = \frac{r_i}{\frac{l_{image}}{l_{ground}}} \\ &= \frac{5\text{e}{-6}\,\text{m}}{\frac{0.15\,\text{m}}{10\text{e}3\,\text{m}}} = 0.33\,\text{m} \end{aligned}$$

Therefore, the resolution of the image on the ground plane is 0.33 m.

(c) **If the system photographs a 25 meter tall tower, located 550 meters along track from directly under the camera, what is the displacement of the top of the tower in the image plane (i.e. on the detector array) relative to the bottom of the building?**

Let $h_{feature}$ represent the height of the tower and R represent the distance of the feature along the flight path from the camera. Additionally, let D represent the distance corresponding to the feature top offset on the ground plane, d represent the relief displacement on the image plane, and r represent the distance on the image plane from the principal point to the offset point.

$$\begin{aligned} R &= 550\,\text{m} \\ h_{feature} &= 25\,\text{m} \end{aligned}$$

Calculate the offset distance on the ground plane:

$$D = \frac{Rh_{feature}}{H} = \frac{550\,\text{m} \cdot 25\,\text{m}}{4667\,\text{m}}$$

Use similar triangles to find the radial distance from the principal point to the feature on the image plane:

$$\frac{r}{R} = \frac{f}{H} \rightarrow r = \frac{fR}{H}$$

Calculate relief displacement on the image plane:

$$\begin{aligned} d &= \frac{rh_{feature}}{H} = \frac{\frac{fR}{H}h_{feature}}{H} \\ &= \frac{\frac{7\,\text{cm}\cdot 550\,\text{m}}{4667\,\text{m}} \cdot 25\,\text{m}}{4667\,\text{m}} = 0.000\,044\,\text{m} \end{aligned}$$

Therefore, the displacement of the top of the tower in the image plane relative to the bottom of the building is 4.4e−5 m.

(d) **Consider now that you wish to image a 15 km x 15 km area. Now what altitude must the aircraft fly? Now what is the spatial resolution of the system?**

Use the same parameters for focal length and image detector array sizes as in part (a). Let l_{ground} represent the image length on the ground plane. Again, assume flat terrain at sea-level elevation.

Calculate altitude of aircraft path:

$$f = 7\,\text{cm} = 0.07\,\text{m}$$
$$l_{image} = 15\,\text{cm} = 0.15\,\text{m}$$
$$l_{ground} = 15\,\text{km} = 15\text{e}3\,\text{m}$$

$$S = \frac{l_{image}}{l_{ground}}$$
$$H = \frac{f}{S} + h = \frac{f}{\frac{l_{image}}{l_{ground}}} + h$$
$$= \frac{0.07\,\text{m}}{\frac{0.15\,\text{m}}{15\text{e}3\,\text{m}}} + 0 = 7000\,\text{m}$$

Calculate spatial resolution of the image on the ground plane:

$$p_i = r_i = 5\,\mu\text{m} = 5\text{e}{-6}\,\text{m}$$

$$r_{ground} = \frac{r_i}{S} = \frac{r_i}{\frac{l_{image}}{l_{ground}}}$$
$$= \frac{5\text{e}{-6}\,\text{m}}{\frac{0.15\,\text{m}}{15\text{e}3\,\text{m}}} = 0.50\,\text{m}$$

Therefore, the aircraft must fly at 7000 m above ground level to capture an image of a 15 km x 15 km area at a spatial resolution of 0.50 m.

Appendix

EAS B9018 - Homework 2, Photogrammetry

In [1]:

```
import numpy as np
```

Problem 1

The image formed by a digital camera on the detector array of photodiodes is 24 x 36 mm, and is equal to the size of the detector array. Assuming the individual photodiode detector elements have a size of 5 um, how many pixels will the digital camera resolve? (Express your answer in megapixels.)

In [39]:

```
image_width = 0.024
image_height = 0.036
size = 5e-6

N = (image_width * image_height / (size ** 2))

print('Number of pixels: {0:.2f} megapixels'.format(N))
```

```
Number of pixels: 34560000.00 megapixels
```

Problem 2

Consider a digital imaging sensor flown on an airplane. The optical system has a focal length of 7 cm. The imaging detector array is 15 cm x 15 cm.

In [33]:

```
# Image focal length (m)
f = 0.07
# Image array length (m)
l_image = 0.15
# Assume sea-level terrain elevation (m)
h = 0
```

(a) What altitude must the aircraft fly in order to image an area 10 km x 10 km?

In [34]:

```
# Ground image length (m)
l_ground = 10e3
# Scale
S = l_image / l_ground
# Height above ground
H = f/S + h

print('Height above ground: {0:.2f} m'.format(H))
```

```
Height above ground: 4666.67 m
```

(b) If the pixel pitch for the sensor's detector array is 5 mm (as in problem 1), what is the resolution of the system assuming the aircraft flies at the altitude in (a)?

In [35]:

```
# Pixel pitch on detector array (m)
p_i = 5e-6
# Resolution at print positive (m)
r_i = p_i
# Ground image spatial resolution (m)
r_g = r_i / S

print('Ground image spatial resolution: {0:.2f} m'.format(r_g))
```

```
Ground image spatial resolution: 0.33 m
```

(c) If the system photographs a 25 meter tall tower, located 550 meters along track from directly under the camera, what is the displacement of the top of the tower in the image plane (i.e. on the detector array) relative to the bottom of the building?

In [36]:

```
# Distance of feature along track from the camera (m)
R = 550
# Height of feature above datum (m)
h_feature = 25
# Calculate offset distance of feature top on the datum plane (m)
D = R*h_feature/H
# Use similar triangles to find radial distance on the image from the principal point to t
r = f*R/H
# Calculate relief displacement in the image plane (m)
d = r*h_feature/H

print('Displacement of the datum top on the image plane: {0:.6f} m'.format(d))
```

```
Displacement of the datum top on the image plane: 0.000044 m
```

(d) Consider now that you wish to image a 15 km x 15 km area.

- Now what altitude must the aircraft fly?
- Now what is the spatial resolution of the system?

In [38]:

```
# Ground image length (m)
l_ground = 15e3
# Scale
S_ = l_image / l_ground
# Height above ground
H_ = f/S_ + h
# Ground image spatial resolution (m)
r_g_ = r_i / S_

print('Height above ground: {0:.2f} m'.format(H_))
print('Ground image spatial resolution: {0:.2f} m'.format(r_g_))
```

```
Height above ground: 7000.00 m
Ground image spatial resolution: 0.50 m
```