CFD Homework #2

Due March 10

1. Consider the following data, which are obtained from a smooth function also known as Runge's function, $y = \frac{1}{(1+25x^2)}$:

x_i	-1.00	-0.80	-0.60	-0.40	-0.20	0.00	0.20	0.40	0.60	0.80	1.00
y_i	0.038	0.058	0.100	0.200	0.500	1.00	0.500	0.200	0.100	0.058	0.038

The Lagrange polynomial interpolation, for which the value at any point x is simply

$$P(x) = \sum_{j=0}^{n} y_j \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{x - x_i}{x_j - x_i}$$

Write a computer program for Lagrange interpolation. Test your program by verifying that *P*(0.7)=-0.226.

(a) Using the above data in the table, find the interpolated value at x=0.9. (10 pts)

(b) Use Runge's function to generate a table of 21 equally spaced data points. Interpolate these data using a Lagrange polynomial of order 20. Plot this polynomial and comment on the comparison between your result and the plot of part (a). (10 pts)

2. The concentration of a certain toxin in a system of lakes downwind of an industrial area has been monitored very accurately at intervals from 1993 to 2007 as shown in the table below. It is believed that the concentration has varied smoothly between these data points

a) Interpolate the data with the Lagrange polynomial (5 pts). Plot the polynomial and the data points (5 pts). Use the polynomial to predict the condition of the lakes in 2009 (5 pts). Discuss this prediction (5 pts).

b) Interpolation may also be used to fill "holes" in the date. Say the data from 1997 and 1999 disappeared. Predict these values using the Lagrange polynomial fitted through the other known data points (10 pts).

Year	Toxin Concentration
1993	12.0
1995	12.7
1997	13.0
1999	15.2
2001	18.2
2003	19.8
2005	24.1
2007	28.1
2009	???

3. An example of a peaky function is the Lorentz profile $\frac{1}{\left(1+\frac{x^2}{a^2}\right)}$. For large a, it is well behaved but for small a, it is strongly peaked near the origin. For $a = 0.1$ and 1, (a) plot the function from 0 to 1 (5 pts) and integrate this function from 0 to 1 using (b) Newton-Cotes methods of various orders (*n=2, 3, and 6*) with fixed number of intervals (*number of intervals = 12*) (10 pts), and (c) Trapezoidal rule ($n = 1$) with variable number of intervals (*number of intervals = 6, 12, and 24*) (10 pts). (d) Plot the error versus the order *n* for (b) and versus the number of intervals for (c) (10 pts).

4. Integrate the function of Problem 3 using Gauss quadrature of various numbers of Gauss points ($n \leq 5$) with single interval (20 pts). Plot the error versus the number of Gauss points. (15 pts)