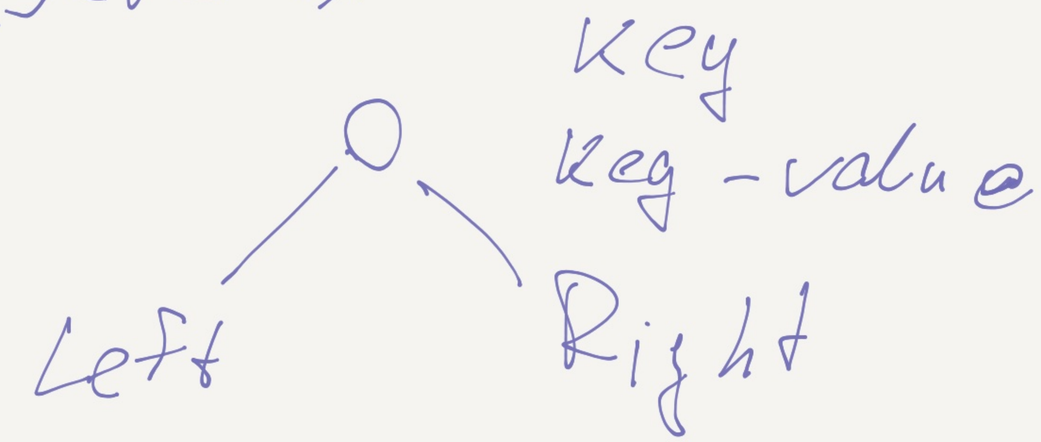
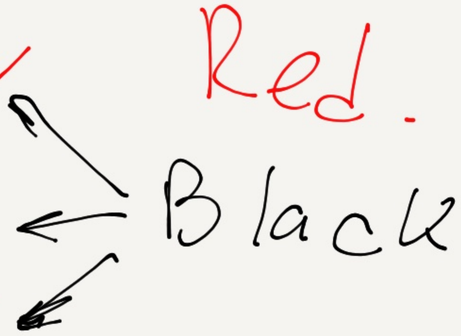
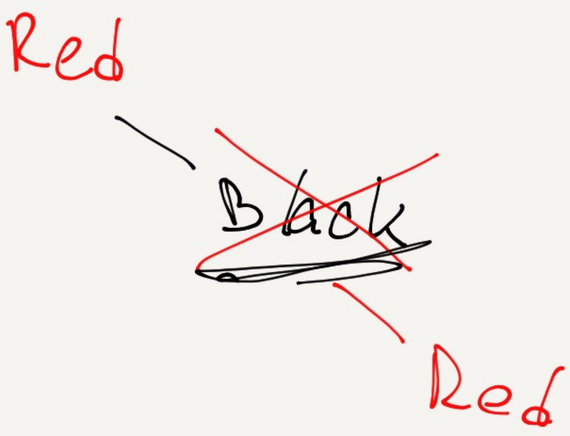
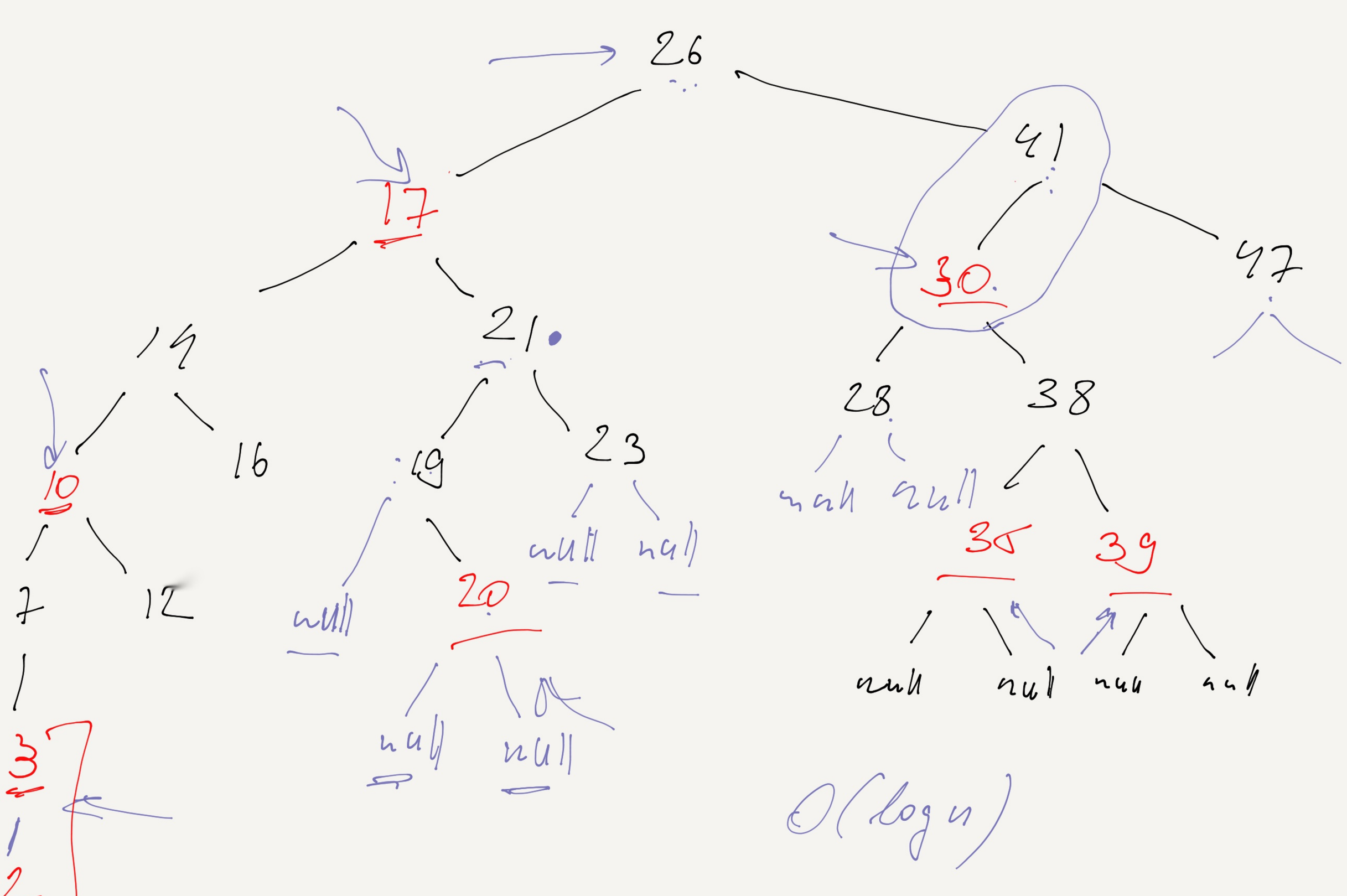
Красно-черные деревья



1) Red / Black
2) Root is Black ✓
3) Red ✗ Red ✓
4) Black Height ✓
5) Null is Black






26
17
41
30
47
14
21
28
38
10
16
19
23
7
12
20
35
39
3
1
2
O(log n)

z

y

w

null

Black

Red

x

null null

1. w — Red

z

y

w

x

P

T

null Black Red

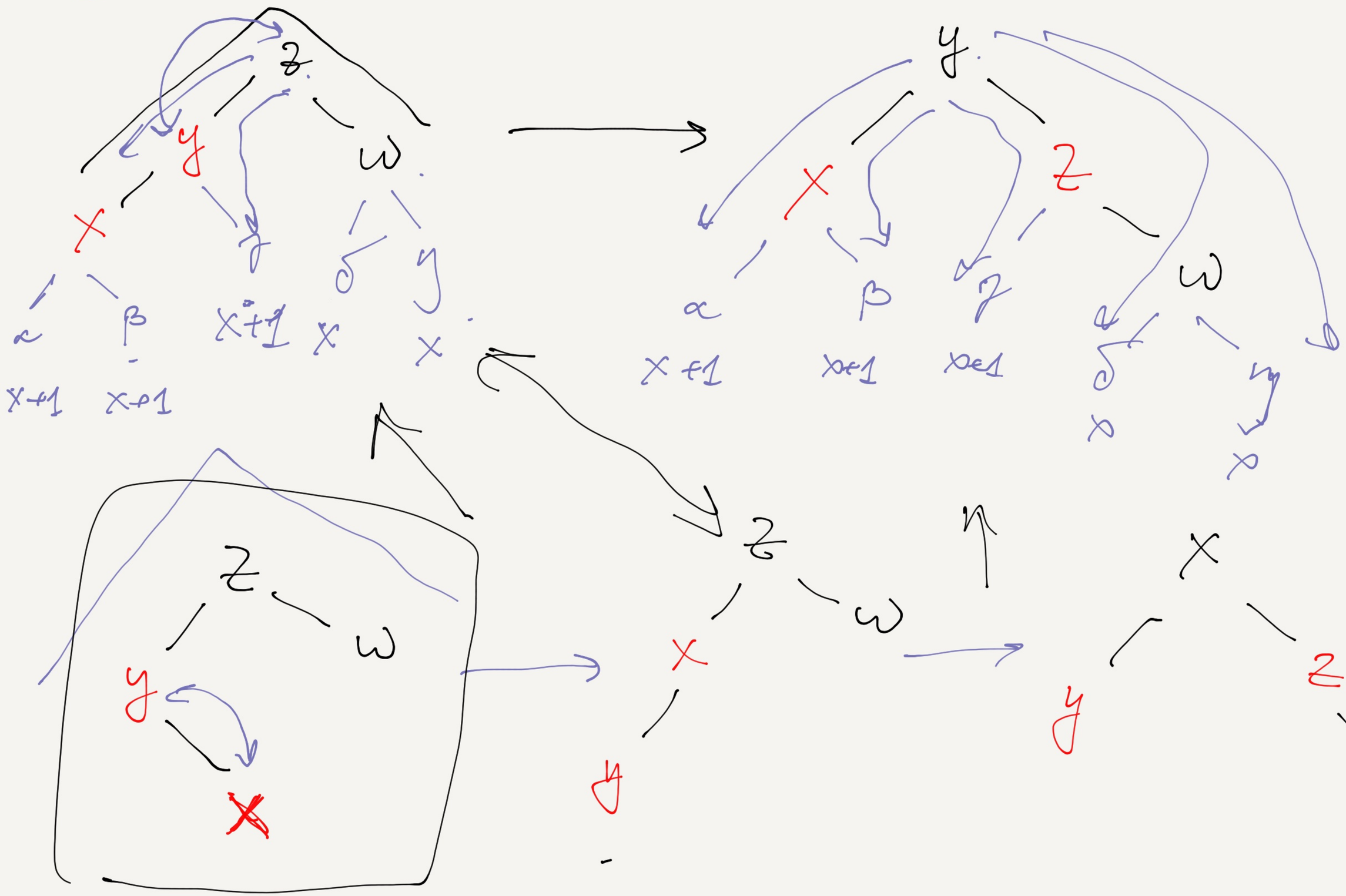
z

y

w

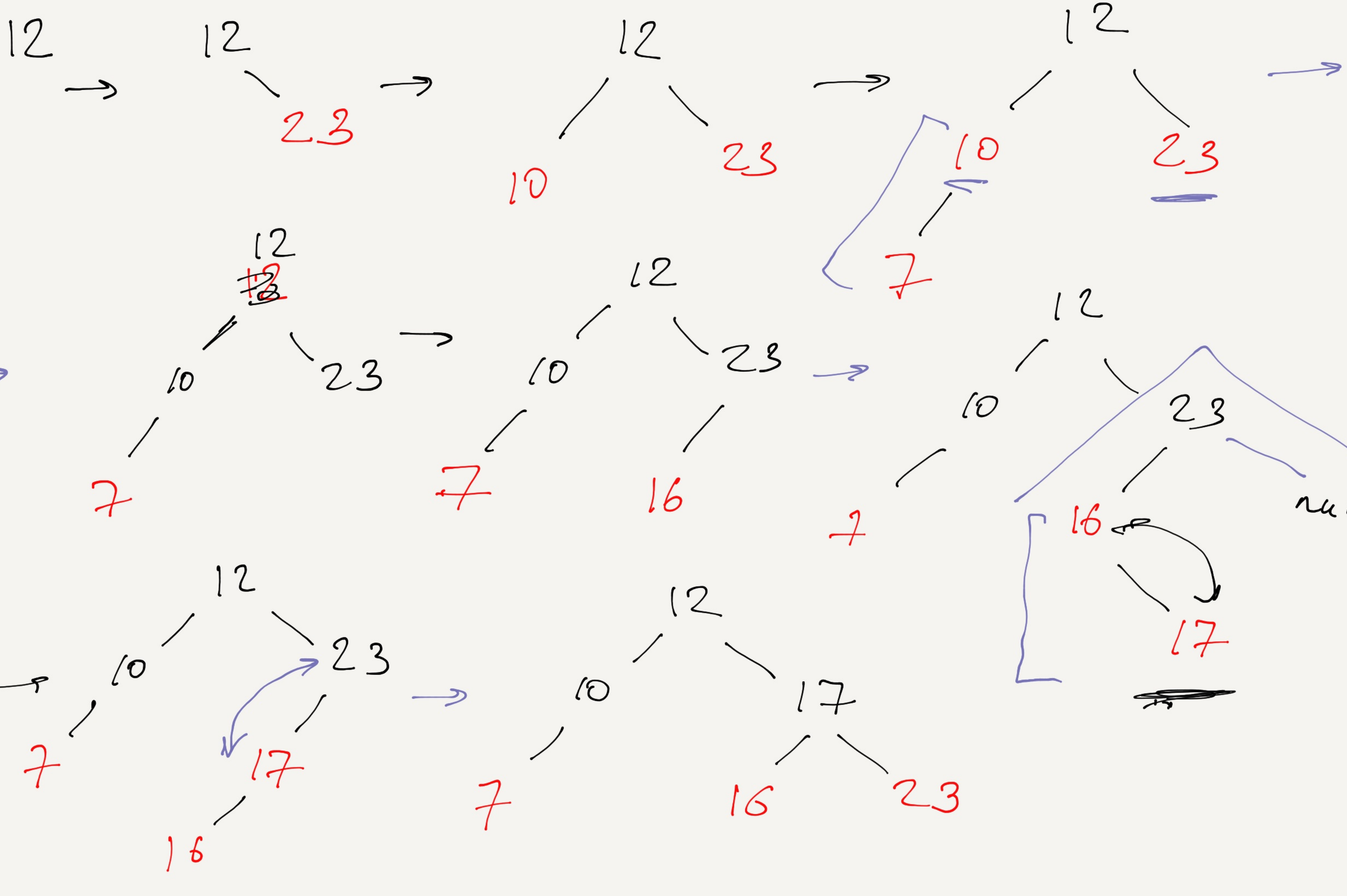
x

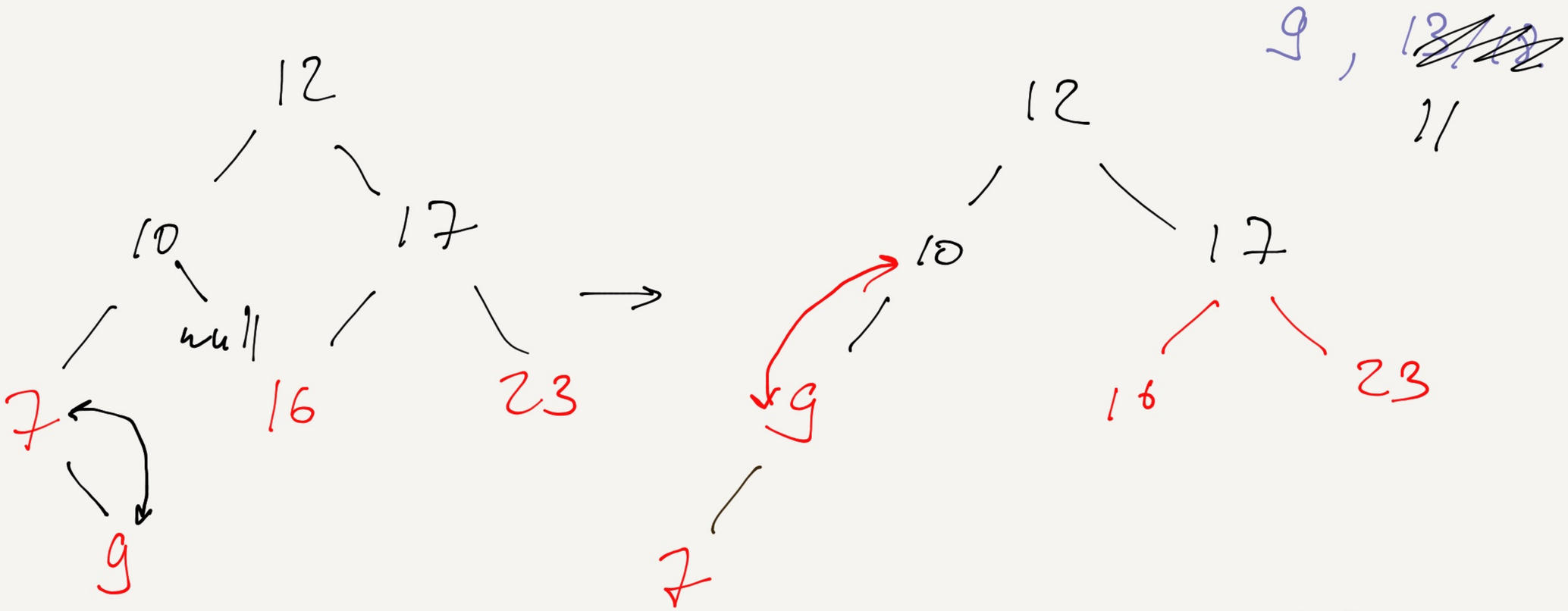
2. w — Black.

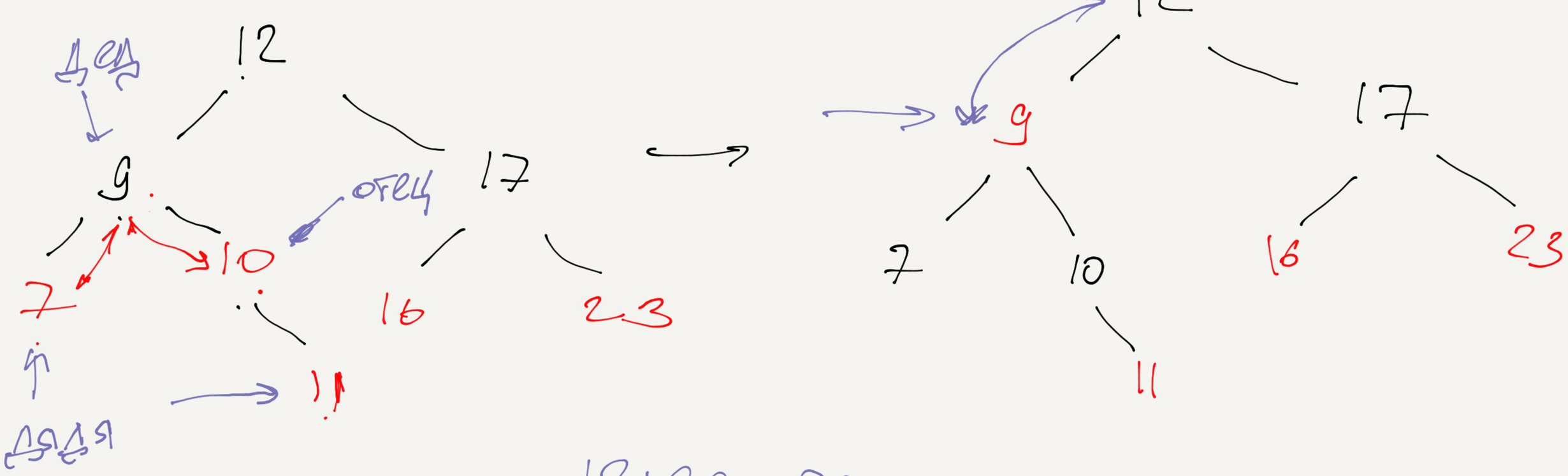


~~3, 7, 10, 12, 14, 16, 17, 21, 20, 19, 23~~

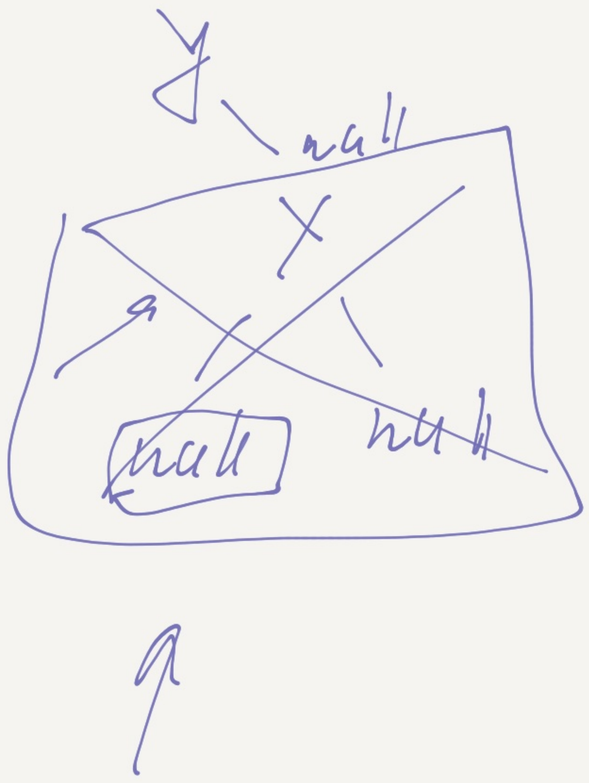
~~12, 23, 10, 7, 16, 17~~, 3, 14, 20, 19, 21



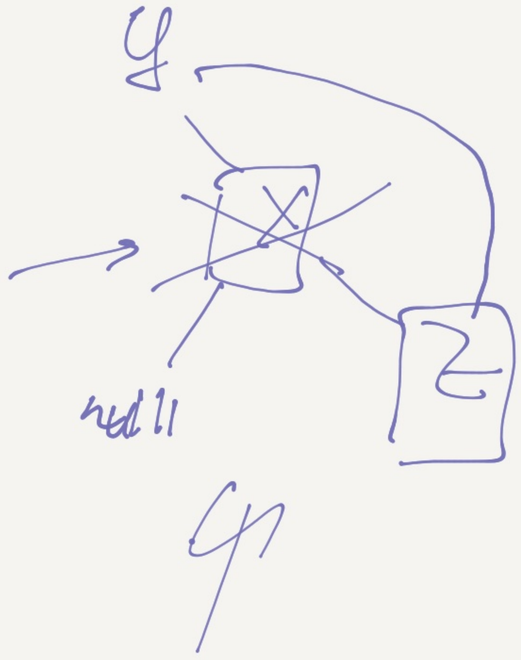




12:00 – продолжение


y
null
X
null
null

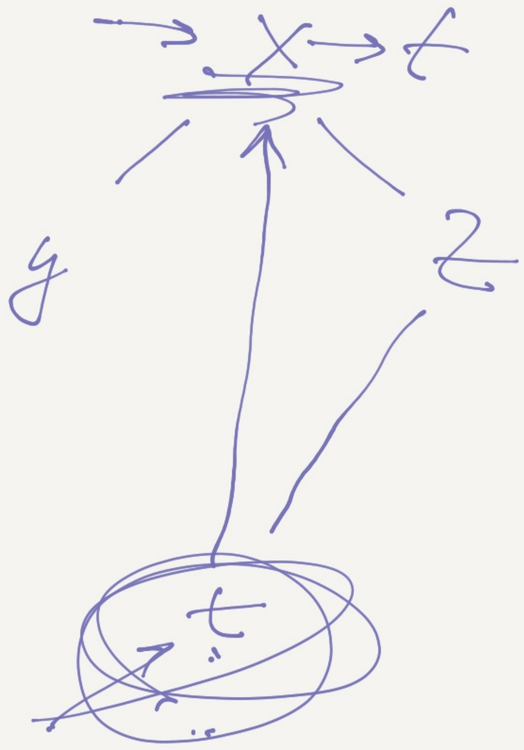
q

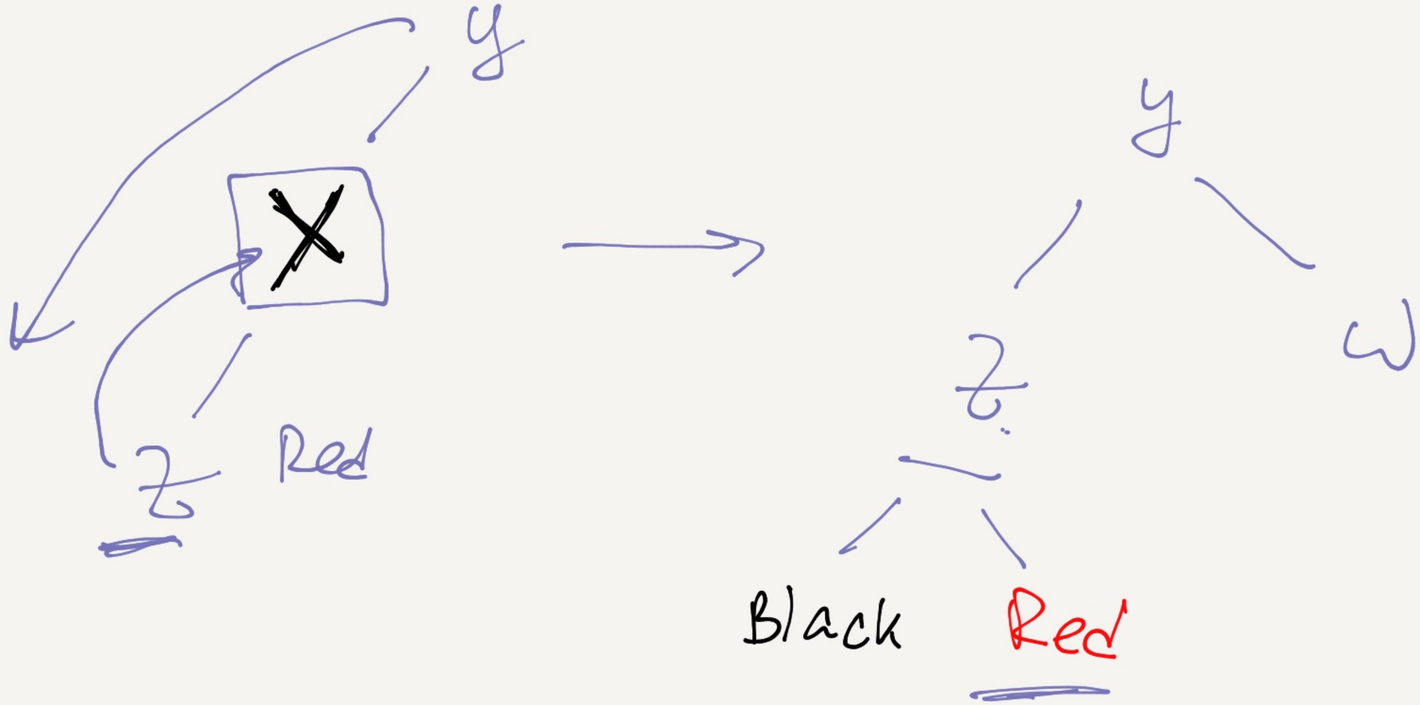

y
X
null
Z

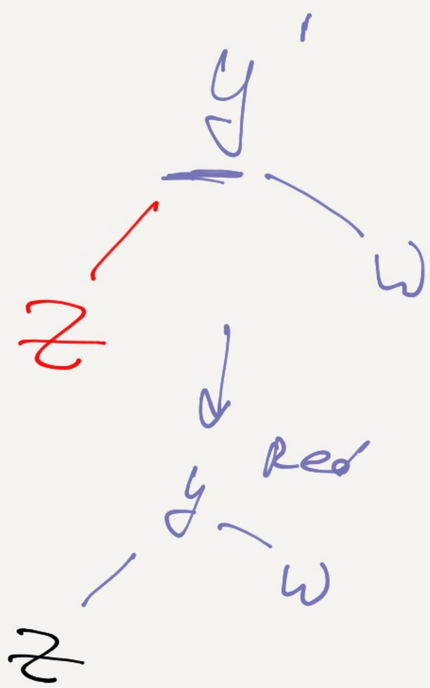
Cp

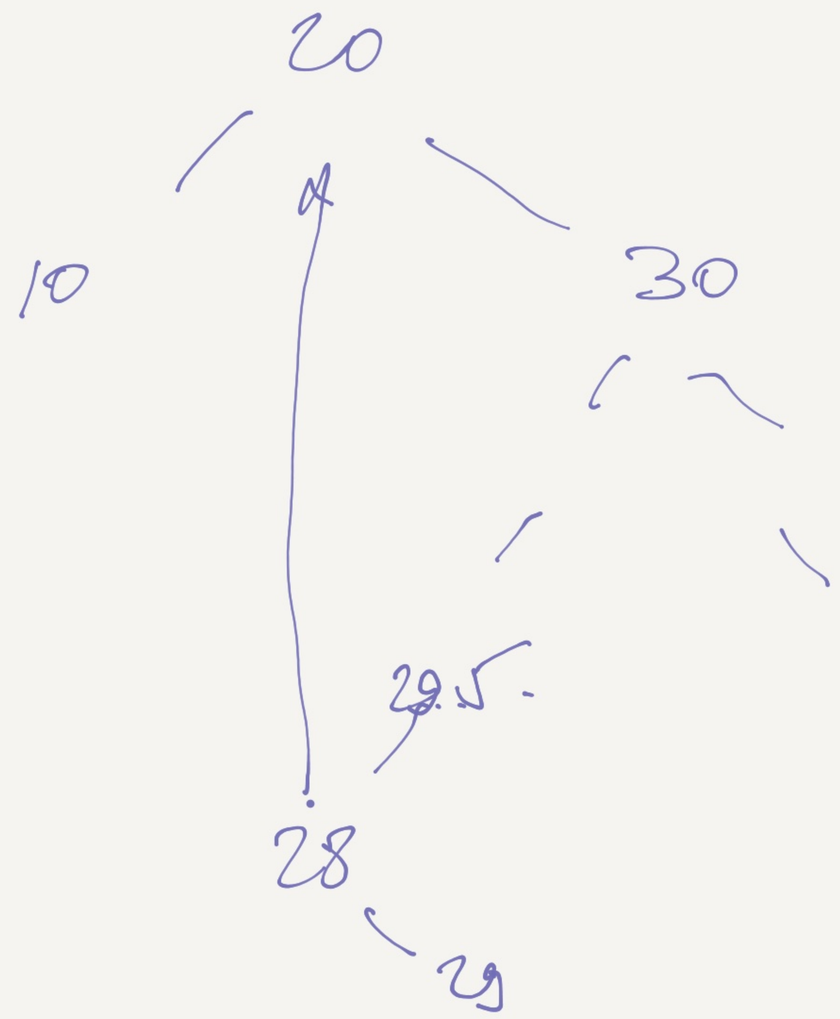
Cq

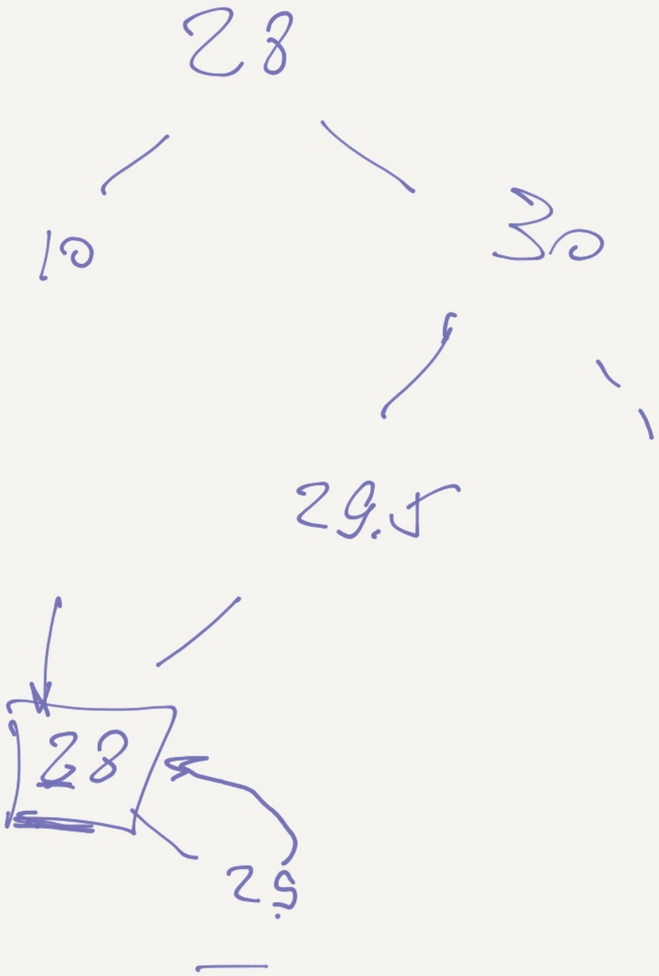
Red


X
t
y
Z
t

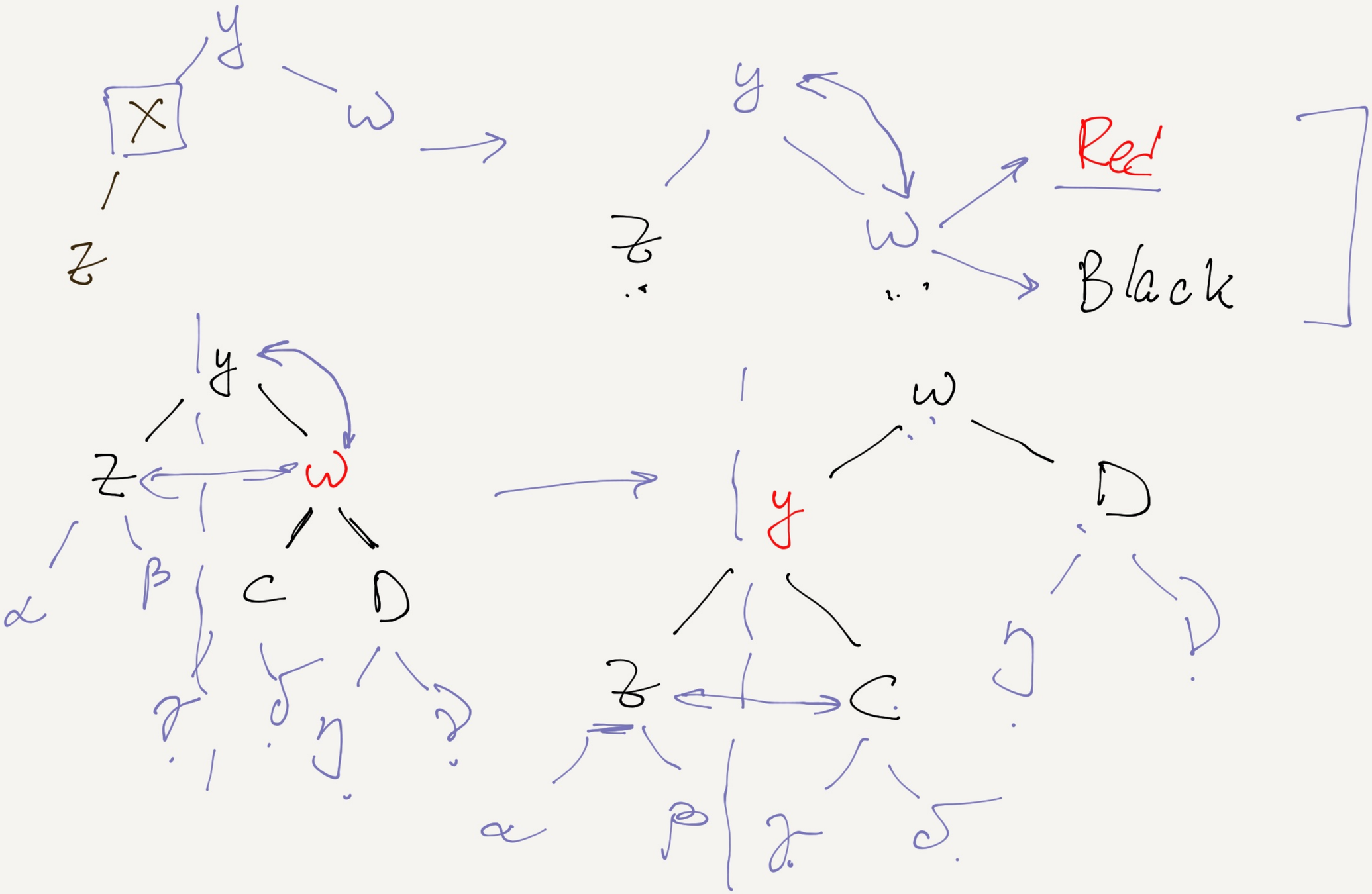

y
X
Z
Red
y
Z
w
Black
Red

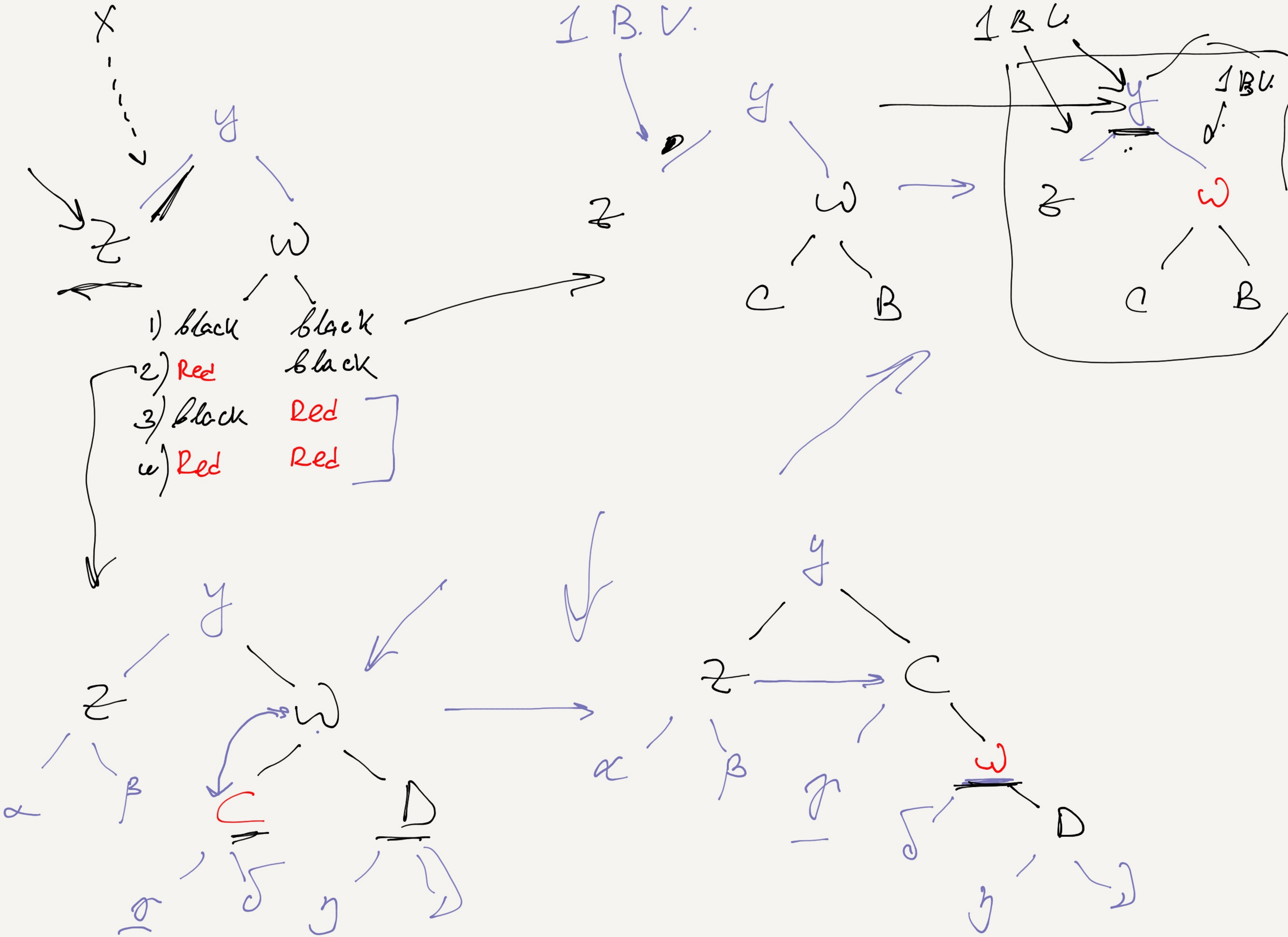

y'
w
Z
Red
y
w
Z



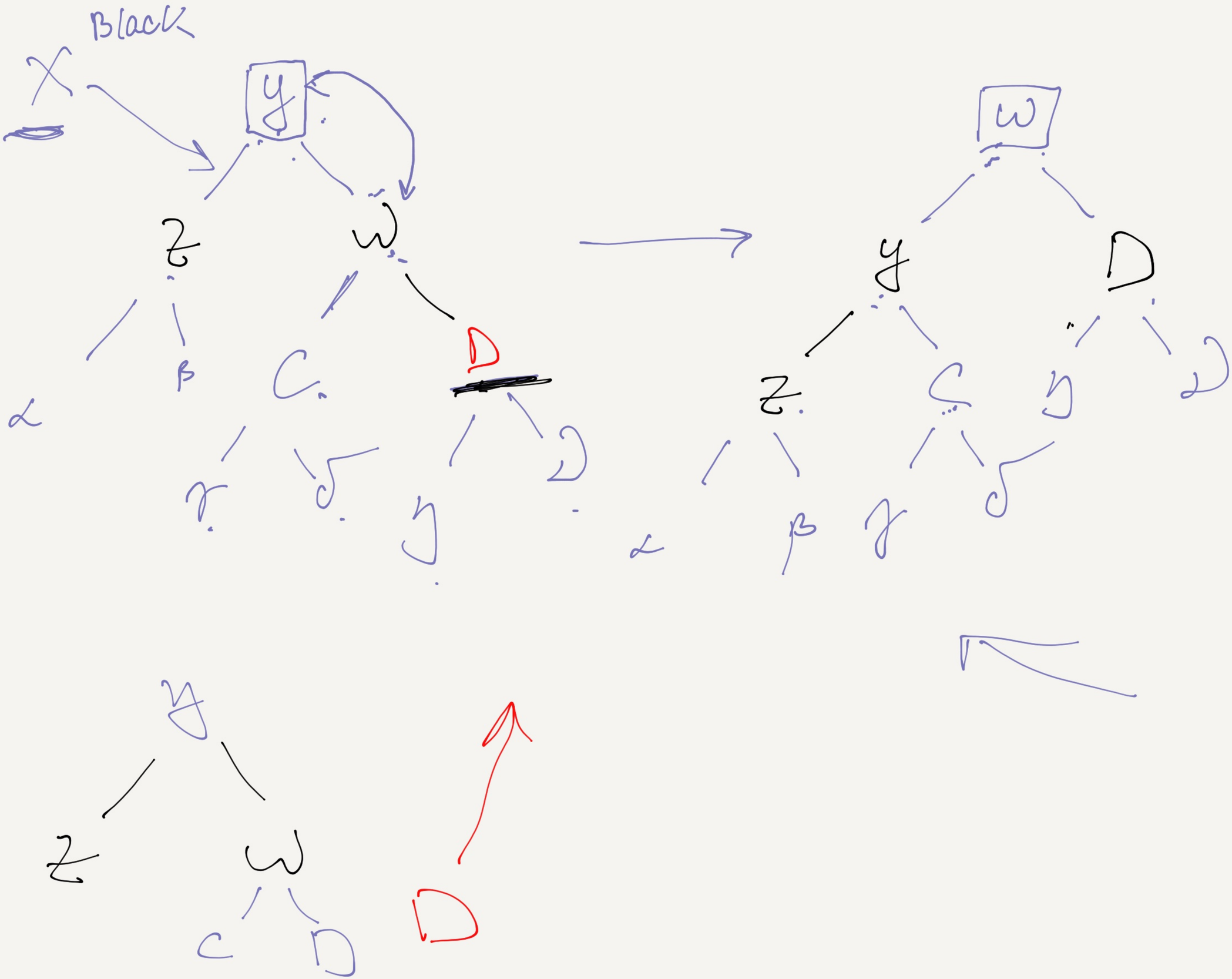


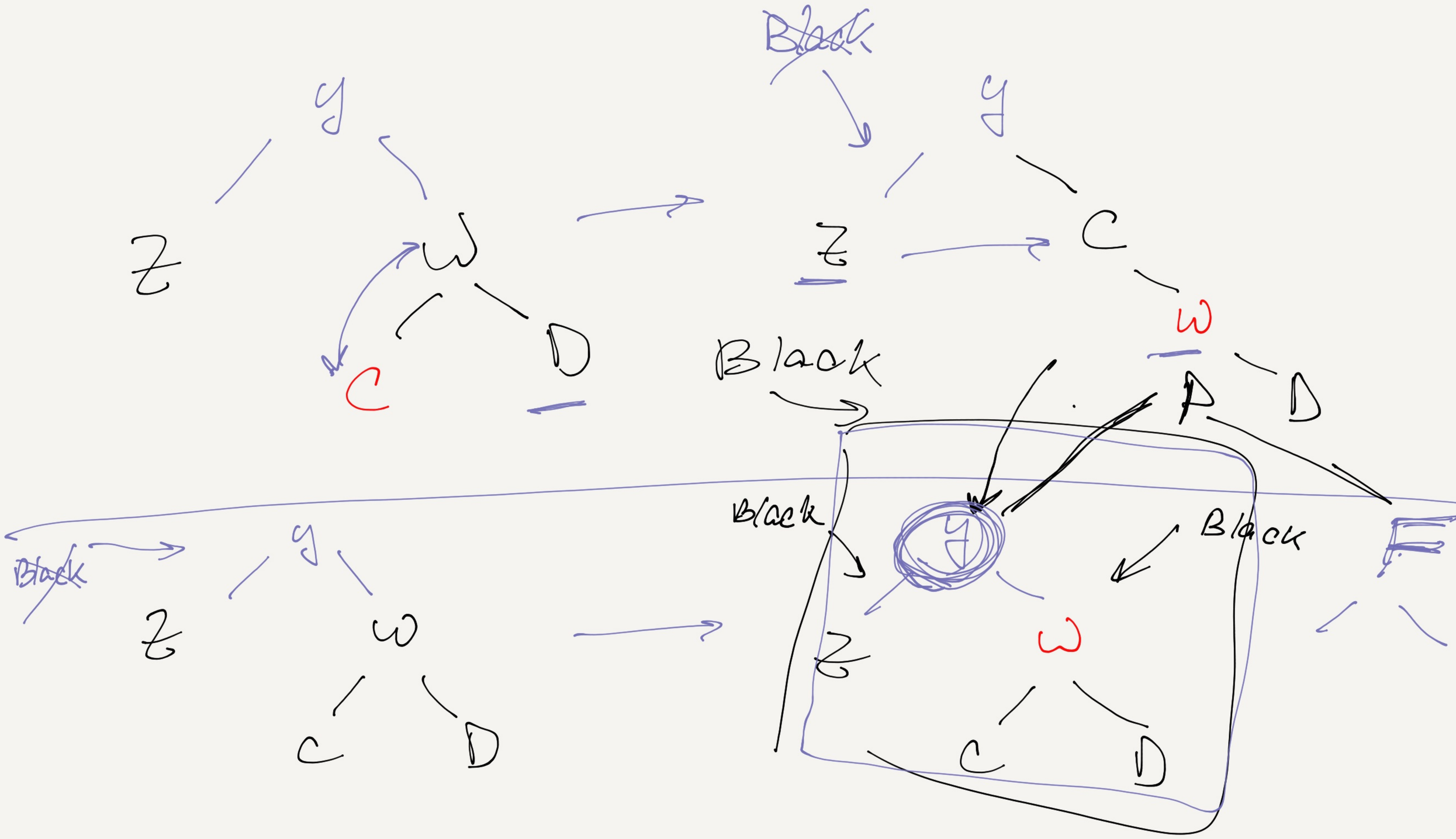
29.8

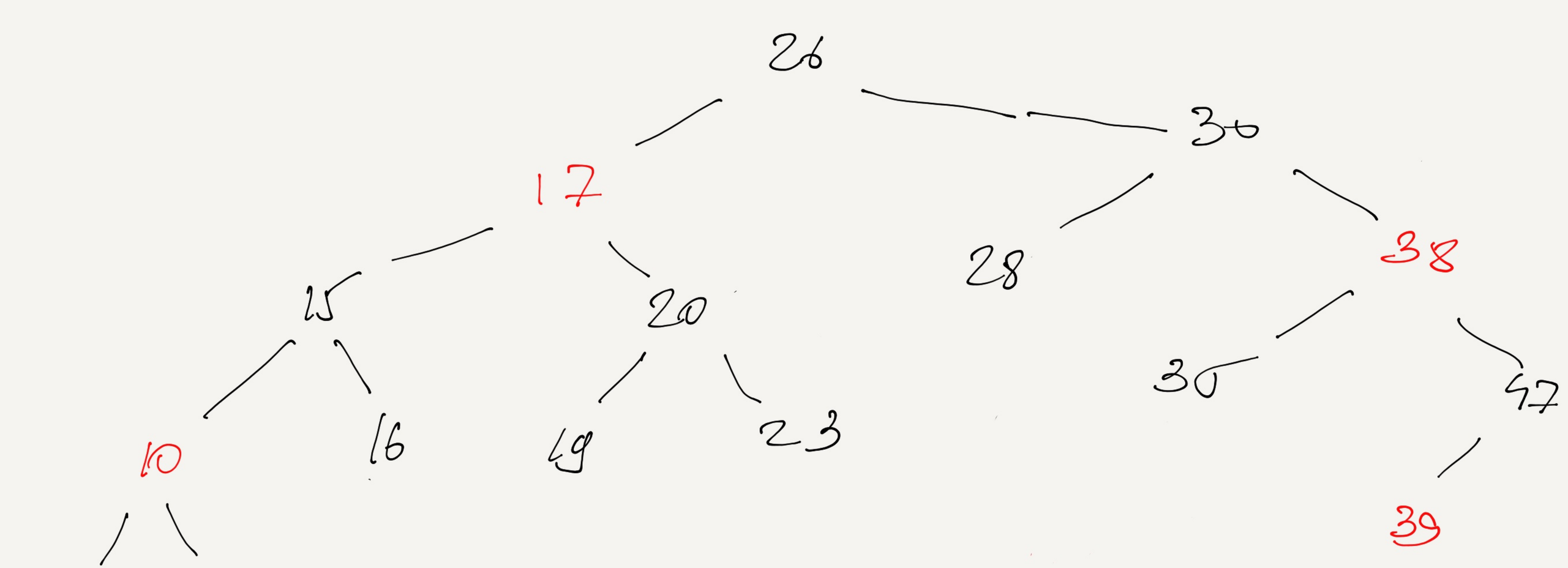

y
X
w
z
y
z
w
Red
Black
y
z
w
α
β
C
D
γ
δ
η
ζ
w
y
D
z
C
α
β
γ
δ
ε
ζ


x
y
z
w
1) black black
2) Red black
3) black Red
4) Red Red
1 B. V.
z
y
w
C
B
1 B. V.
y
1 B. V.
z
w
C
B
y
α
β
C
D
x
y
y
z
C
α
β
γ
ω
D
δ
γ

Black




y
z
w
C
D
Black
y
z
C
w
Black
P
D
Black
Black
y
z
w
C
D
z
w
C
D


26
17
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49
23
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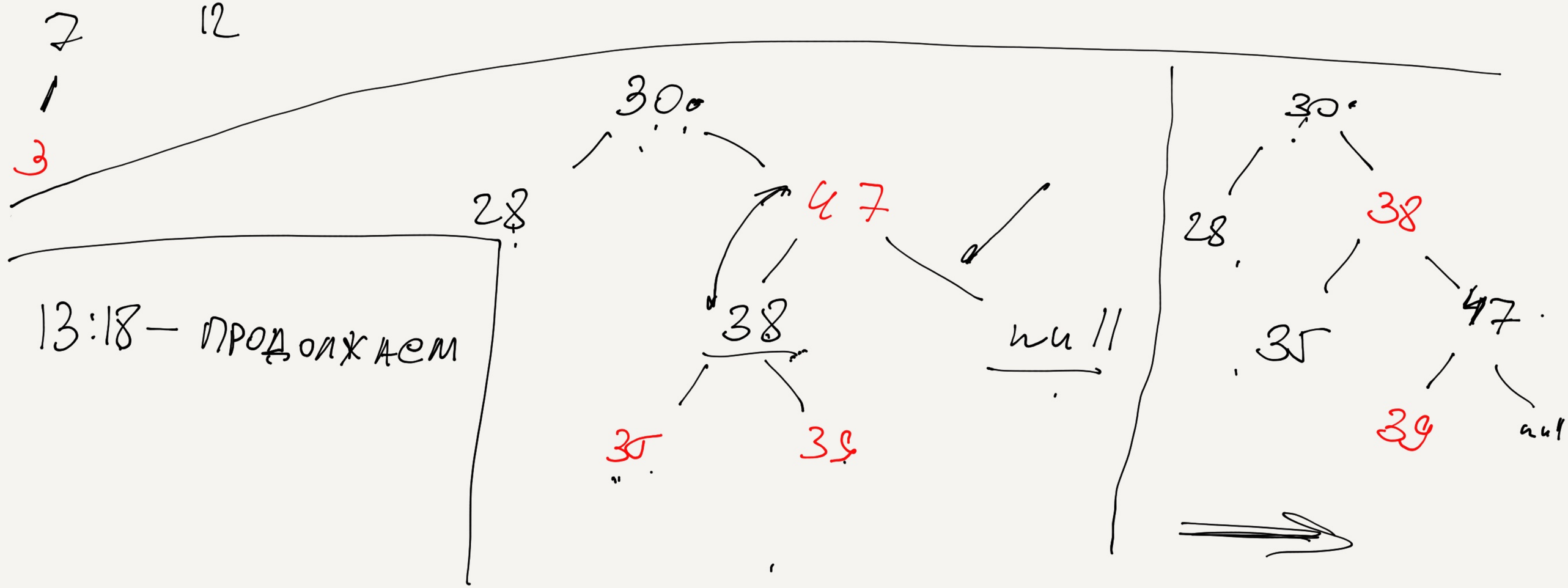
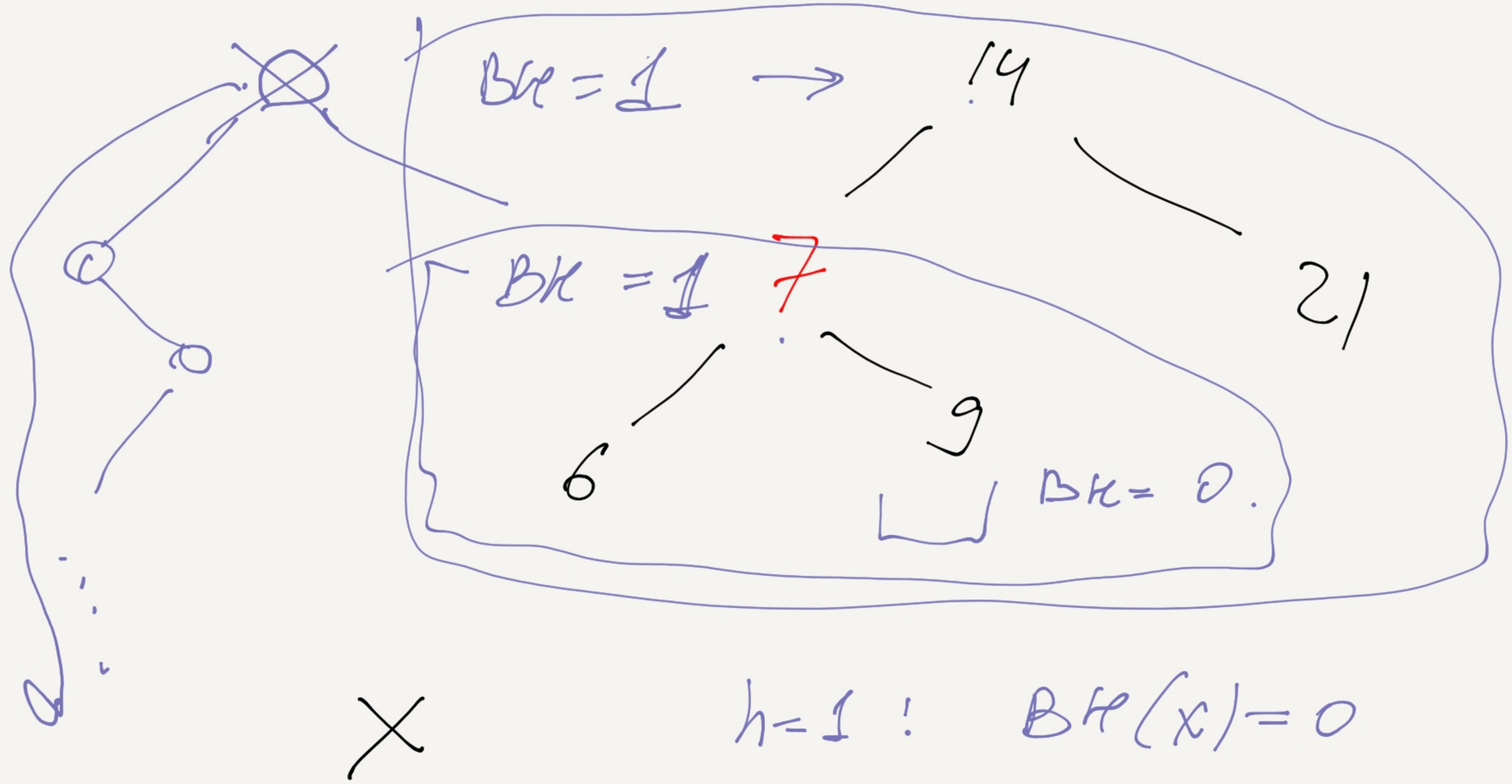
47
39

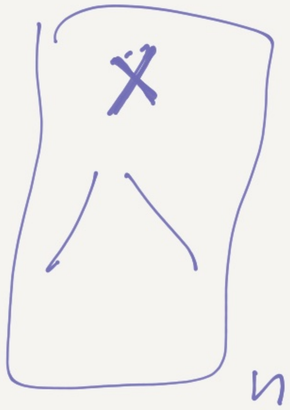

7
12
1
3
30
28
47
38
35
39
null
13:18 — продолжает
30
28
38
35
47
39

Black Height



X

$h = 1$! $BH(x) = 0$

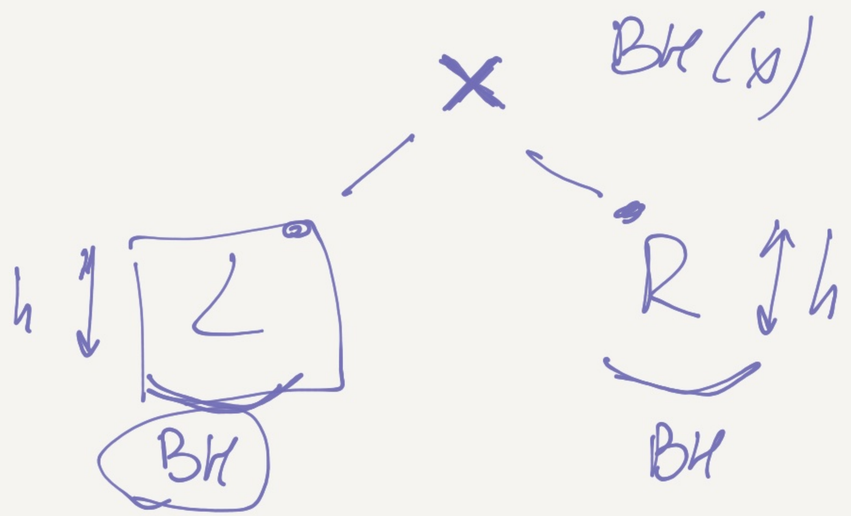


$h = 1 \geq 2^0 - 1 = 0$

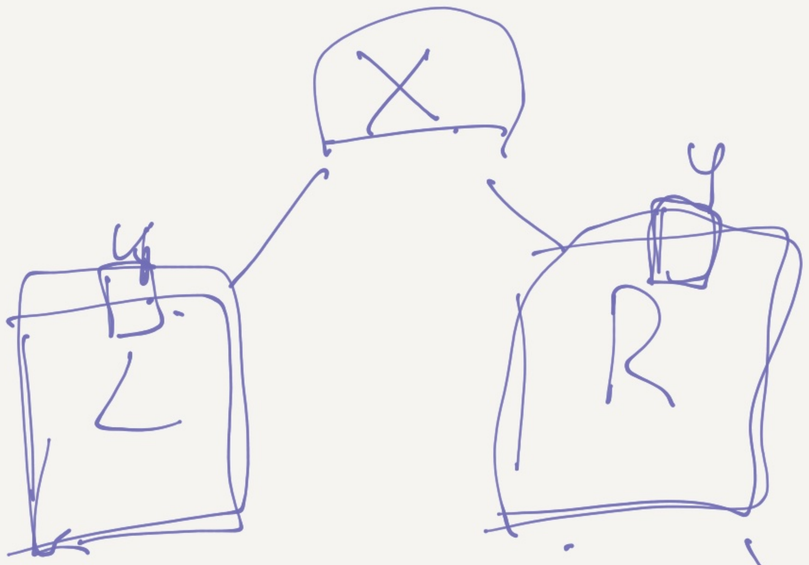
$h \quad h \geq 2^{BH(x)} - 1$

$h \geq 2^{BH(x)} - 1.$

$h+1$



2^B



$$n \geq 2^{BH(x)} - 1.$$

$$2^{BH(x)-1} - 1 + 2^{BH(x)-1} - 1 + 1 =$$

$$2 \cdot 2^{BH(x)-1} - 1 = \boxed{2^{BH(x)} - 1.}$$

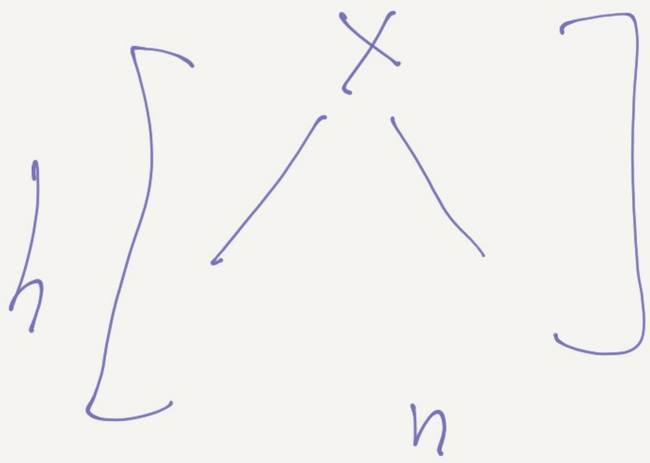
$$h(x) = 2 \cdot BH(x)$$

$$n \geq 2^{\frac{h}{2}} - 1$$

$$n+1 \geq 2^{h/2}$$

$$h/2 \leq \log(n+1)$$

$$h \leq 2\log(n+1) \Rightarrow \boxed{h = O(\log n)}$$



$BH(x)$

$n \geq 2^{BH(x)} - 1.$

$n \longleftrightarrow BH(x)$

$n \; ? \; h.$

$n+1 \geq 2^{BH(x)}$

$\frac{h}{2} \leq BH(x) \leq h$

$\frac{h}{2} \leq BH(x) \leq \log(n+1)$

$h \leq 2\log(n+1)$

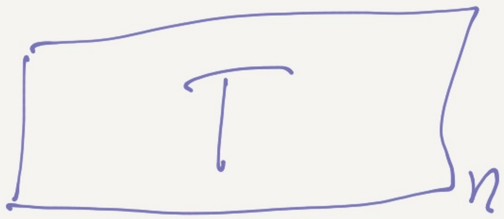
$h = O(\log(n))$

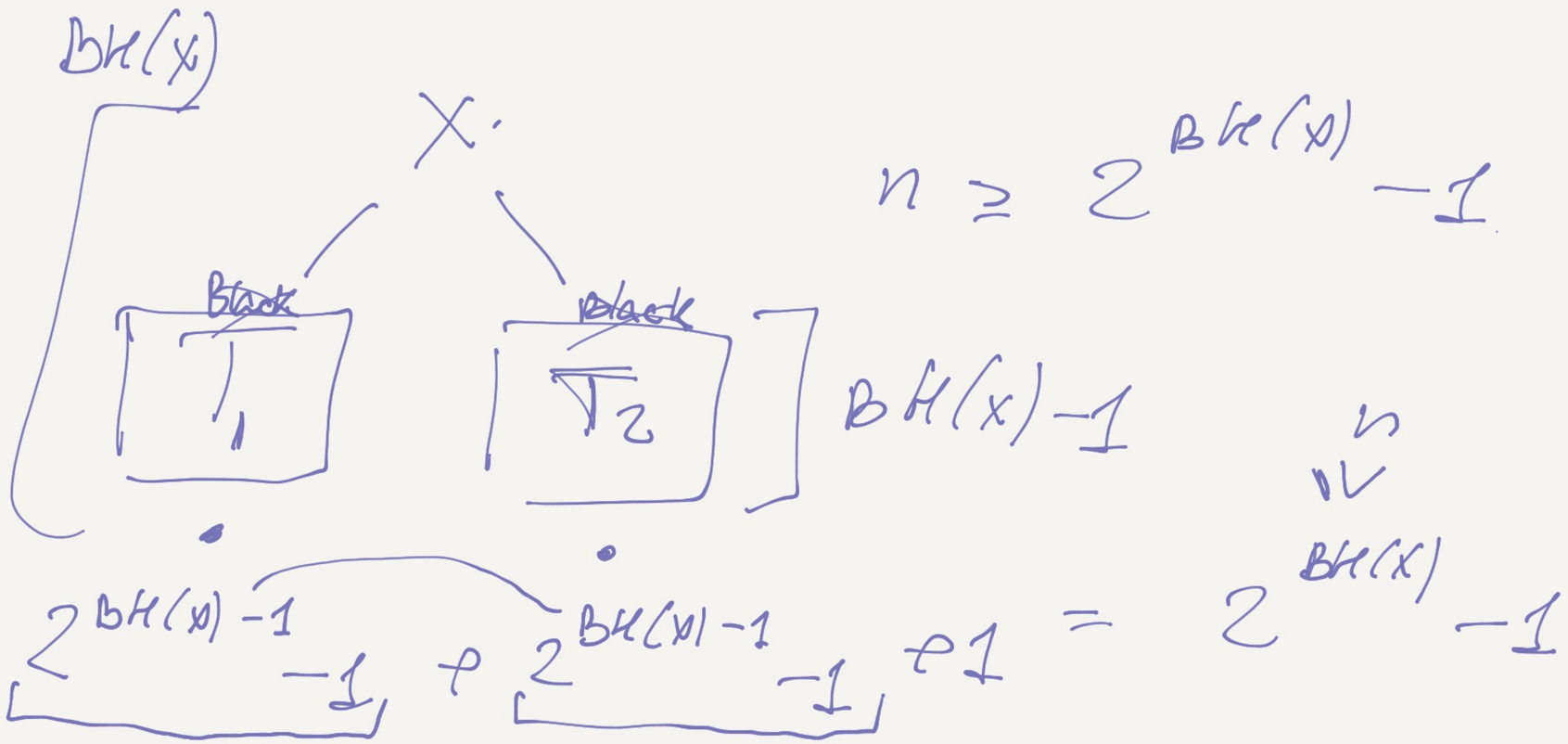
$$n \geq 2^{BH(x)} - 1.$$

X $BH(x) = 0$

null null

$1 \geq 2^0 - 1 = 1 - 1 = 0$ ∎





$n \geq 2^{BH(x)} - 1$

$n \geq 2^{BH(x)} - 1$

Декартово дерево

Aragon, Siedel

treap

Дуча

tree heap

курево

a_i

a_{2i}

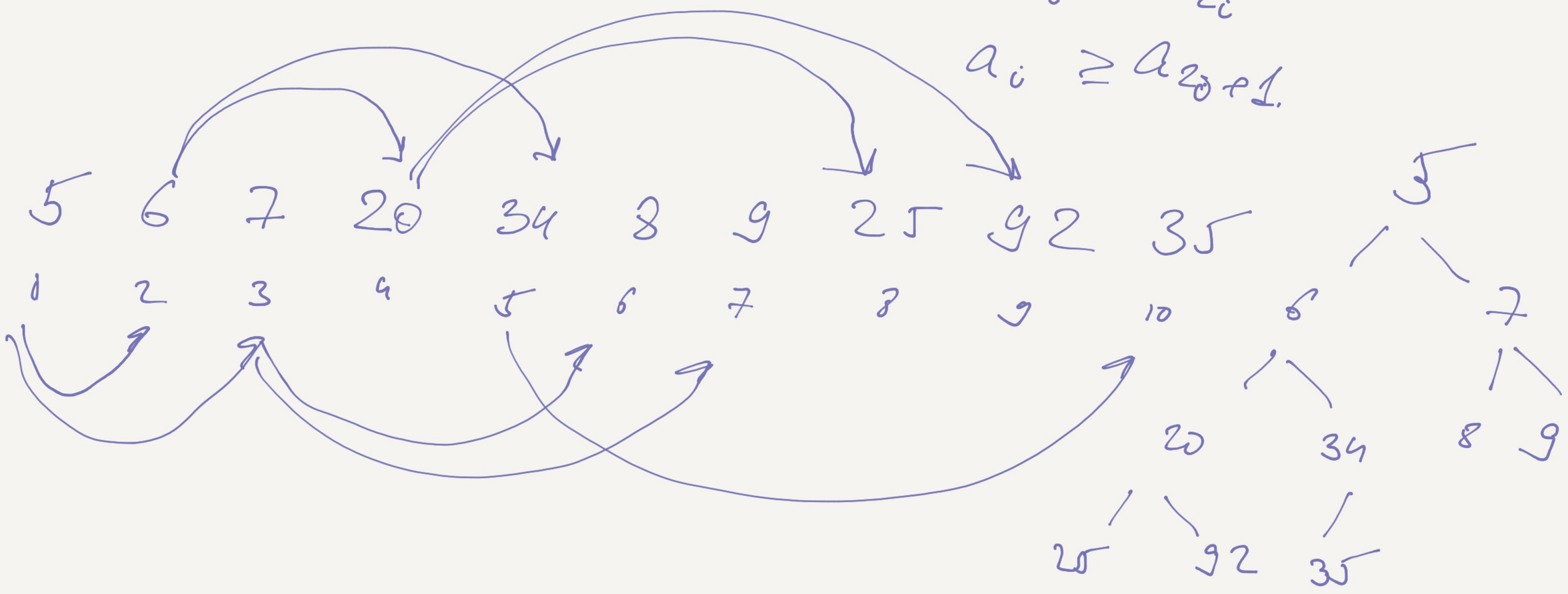
a_{2i+1}

$a_i \le a_{2i}$

$a_i \le a_{2i+1}$

$a_i \ge a_{2i}$

$a_i \ge a_{2i+1}$

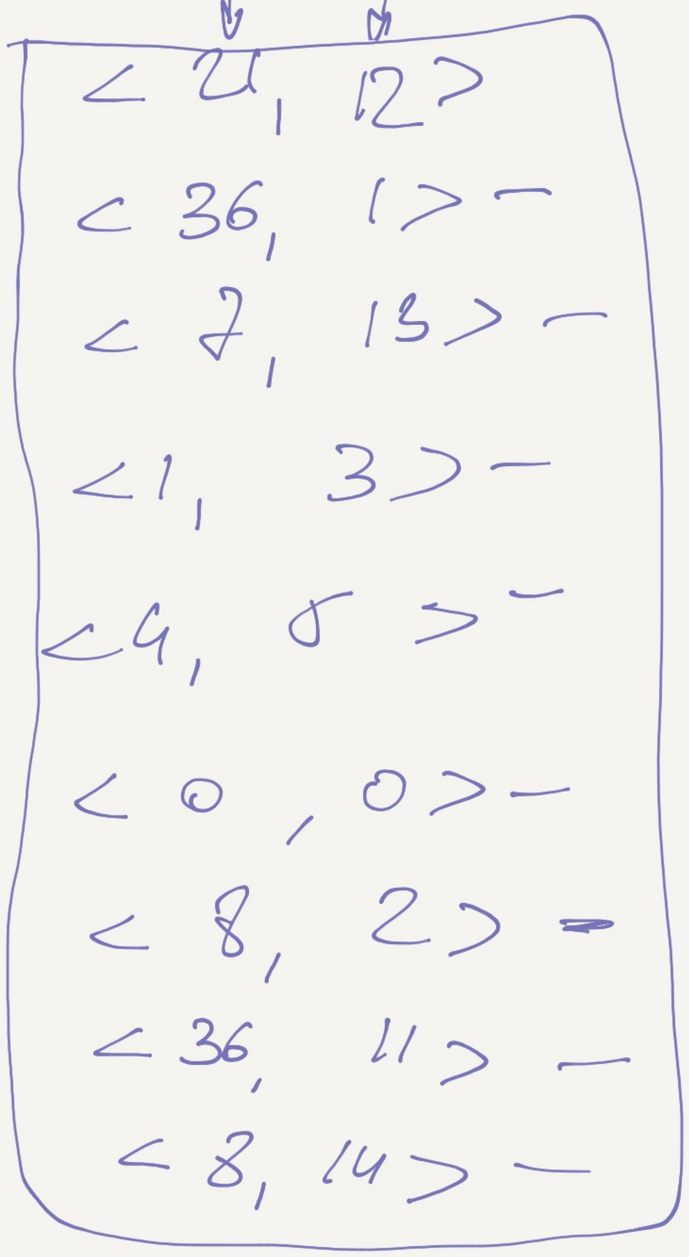


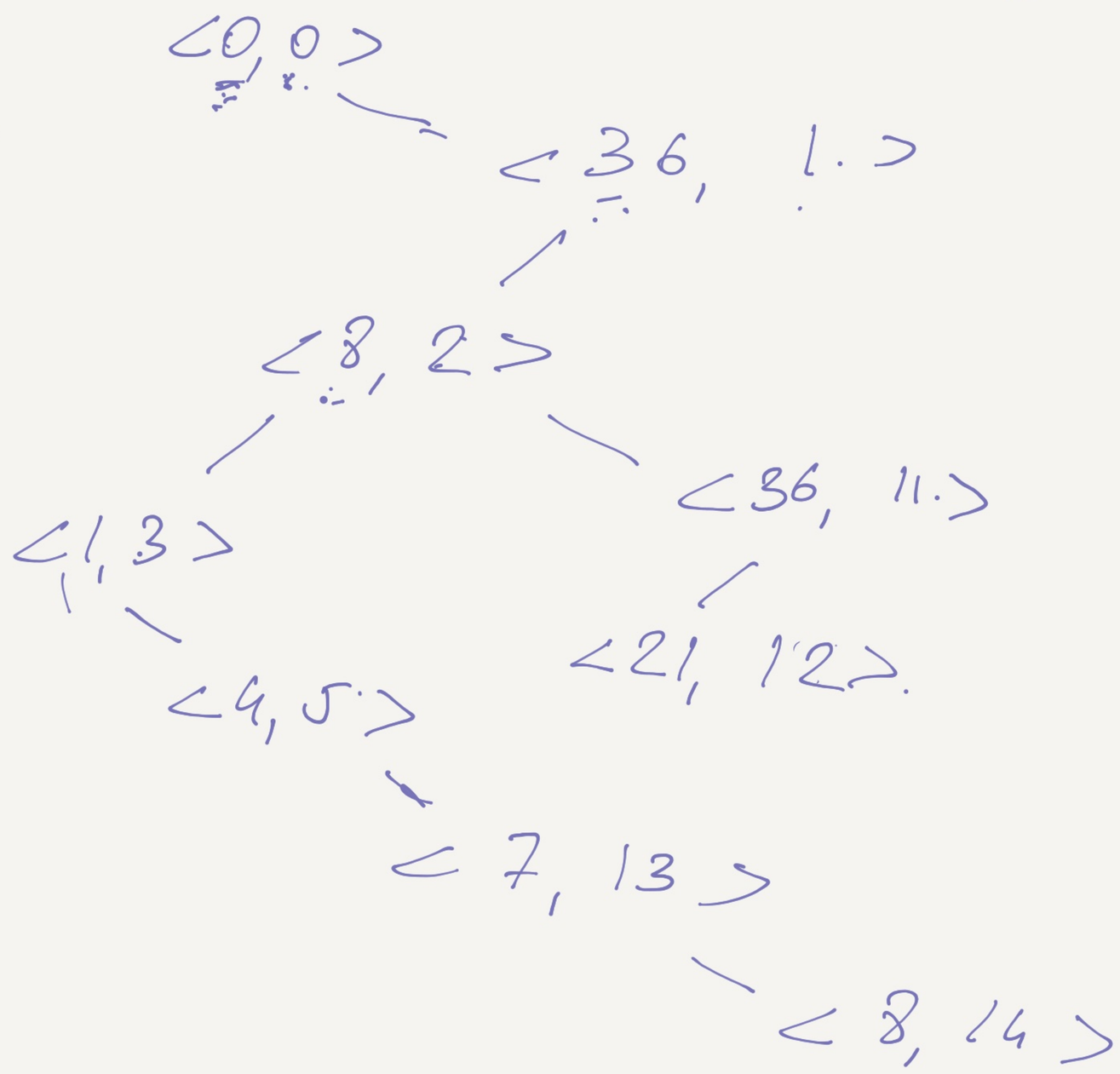
<key, priority>

BST

Heap

log (n)





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<9, 3>

<9, 3>
<3, 7>
<1, 0>
<1, -1>.

<4, 1>
<7, 2>

<9, 3>

<11, 0>

<6, 5>

<3, 7>

<11, 0>

<9, 3>

<4, 0>

<6, 5> →

<9, 3>

<3, 7>

<6, 5>

<1, -1>

<3, 7>

<1, -1>

<11, 0>

<9, 3>

<1, -1>

<6, 5>

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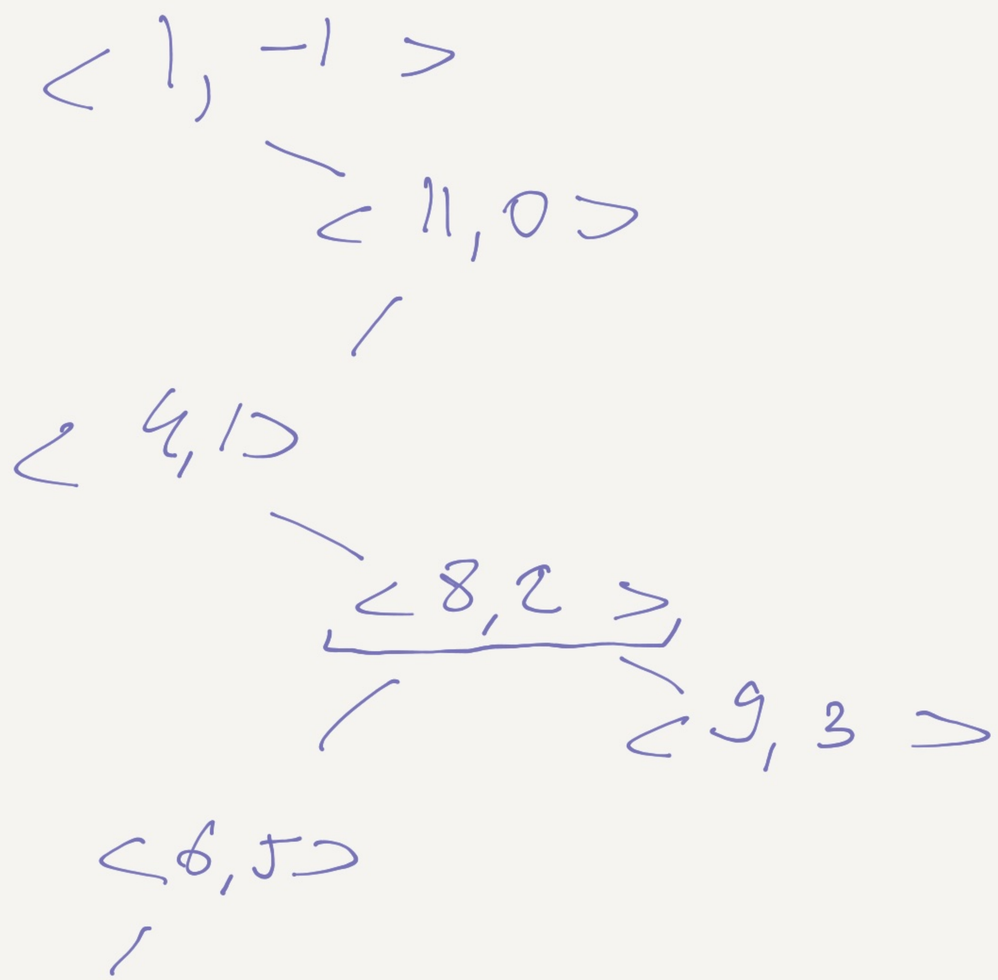
<3, 7>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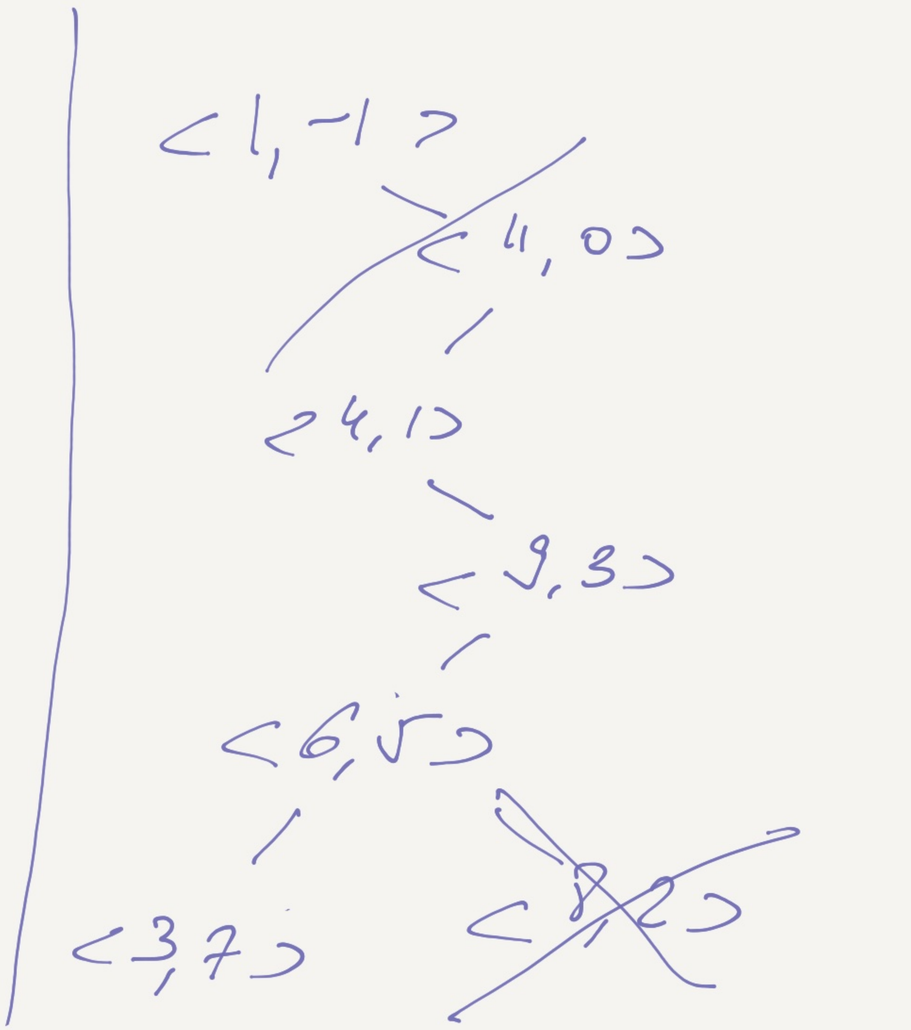
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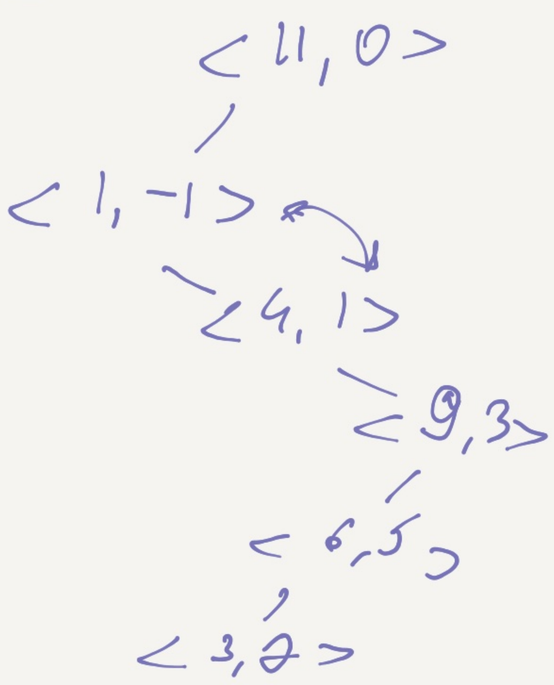
<11,0>
<1,-1>
<4,1>
<9,3>
<6,5>
<3,7>