

SUBJECT – MATHS

QB SEM -1

ALL THE BEST BROTHERS

UNIT -1

1. Define Sets with example.

Ans - A set is a collection of distinct elements. Each element in a set is unique, and the order of elements does not matter.

Example: Let define a set **A** = {1, 2, 3, 4}. Here, **A** is a set containing the elements 1, 2, 3, and 4.

2. What is the operation of sets.

Ans - - Sets can be operated upon using various operations like union, intersection, and difference.

- **Union (U):** $A \cup B$ represents the set of elements that are in A, or in B, or in both.

- **Intersection (\cap):** $A \cap B$ represents the set of elements that are common to both A and B.

- **Difference (A - B or B - A):** $A - B$ represents the set of elements that are in A but not in B, and vice versa.

3. If $A = \{2, 5, 7, 8\}$ and $B = \{1, 4, 5, 6, 8\}$ find $A \cup B$

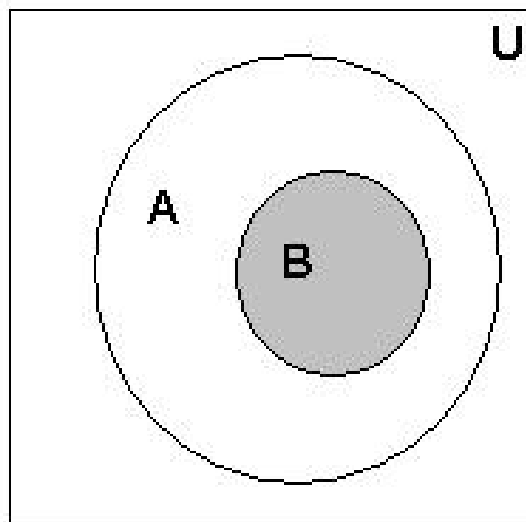
Ans - To find **$A \cup B$** (the union of sets A and B):

1. Combine all unique elements from sets A and B.

2. **$A \cup B = \{1, 2, 4, 5, 6, 7, 8\}$**

4. Draw Venn diagram of B is subset of A

Ans –



5. If $n(AB) = 28$, $n(A \cup B) = 100$ and $n(A \cap B) = 15$, then find $n(B)$.

Ans - To find $n(B)$, we can use the formula for the union of two sets:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Given that $n(A \cup B) = 100$, $n(A \cap B) = 15$, and $n(AB) = 28$, we can substitute these values into the formula:

$$100 = n(A) + n(B) - 15$$

Now, solve for $n(B)$:

$$n(B) = 100 - n(A) + n(A \cap B)$$

$$n(B) = 100 - 28 + 15$$

$$n(B) = 87$$

So, $n(B) = 87$.

6. if set $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Then write the universal set for all three sets.

Ans - The universal set for all three sets (A, B, and C) would be the set containing all unique elements from these sets:

Universal Set = $\{0, 1, 2, 3, 4, 5, 6, 8\}$

This set includes all distinct elements present in sets A, B, and C.

7. In a group of 100 people, 66 can speak English and 47 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Ans - To determine how many people can speak only English, only French, and both languages, you can use the principle of inclusion-exclusion.

Let:

- E be the number of people who can speak English,
- F be the number of people who can speak French.

The total number of people (T) is 100.

Now, you can use the formula:

$$E \cup F = E + F - E \cap F$$

Where:

- $E \cup F$ is the total number of people who can speak either English or French,
- $E \cap F$ is the number of people who can speak both English and French.

Given that 66 people can speak English ($E = 66$), 47 can speak French ($F = 47$), and the total number of people is 100 ($T = 100$), you can substitute these values into the formula:

$$66 + 47 - E \cap F = 100$$

Now, solve for $E \cap F$ (the number of people who can speak both English and French):

$$E \cap F = 66 + 47 - 100$$

$$E \cap F = 113 - 100$$

$$E \cap F = 13$$

Given that 66 people can speak English ($E = 66$), 47 can speak French ($F = 47$), and the total number of people is 100 ($T = 100$), you can substitute these values into the formula:

$$66 + 47 - E \cap F = 100$$

Now, solve for $E \cap F$ (the number of people who can speak both English and French):

$$E \cap F = 66 + 47 - 100$$

$$E \cap F = 113 - 100$$

$$E \cap F = 13$$

Now that you know $E \cap F$, you can find the number of people who can speak only English (E only), only French (F only), and both languages ($E \cap F$):

1. $E \text{ only} = E - E \cap F$

$$E \text{ only} = 66 - 13 = 53$$

2. $F \text{ only} = F - E \cap F$

$$F \text{ only} = 47 - 13 = 34$$

So, the answers are:

- The number of people who can speak only English is 53.

- The number of people who can speak only French is 34.

- The number of people who can speak both English and French is 13.

8. In If A and B are two sets such that number of elements in A is 24, number of elements in B is 22 and number of elements in both A and B is 8, find $n(A \cup B)$, $n(A \cap B)$ and $n(B - A)$

Ans- - $n(A)$ represents the number of elements in set A.

- $n(B)$ represents the number of elements in set B.

- $n(A \cap B)$ represents the number of elements in the intersection of sets A and B.

Given that:

- $n(A) = 24$,

- $n(B) = 22$,

- $n(A \cap B) = 8$.

We can find the number of elements in the union of sets A and B ($n(A \cup B)$), the number of elements in the intersection of sets A and B ($n(A \cap B)$), and the number of elements in the set difference B-A ($n(B - A)$).

1. $n(A \cup B)$ (Number of elements in the union of A and B):

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 24 + 22 - 8$$

$$n(A \cup B) = 38$$

So, $n(A \cup B)$ is 38.

2. $n(A \cap B)$ (Number of elements in the intersection of A and B) is already given as 8.

3. $n(B - A)$ (Number of elements in the set difference B-A):

$$n(B - A) = n(B) - n(A \cap B)$$

$$n(B - A) = 22 - 8$$

$$n(B - A) = 14$$

So, the answers are:


$$- n(A \cup B) = 38 ,$$

$$- n(A \cap B) = 8 ,$$

$$- n(B - A) = 14 .$$

9. There Are 100 students in a club of which 35 like drawing, 45 like music and 10 like both. Find the number of students that like either of them or neither of them.

Ans - Let Us break down the information given:

- Number of students who like drawing (D): 35
- Number of students who like music (M): 45
- Number of students who like both drawing and music ($D \cap M$): 10

Now, we can use the principle of inclusion-exclusion to find the total number of students who like either drawing or music (or both):

$$\text{Total} = D + M - D \cap M$$

$$\text{Total} = 35 + 45 - 10 = 70$$

So, there are 70 students who like either drawing or music (or both).

To find the number of students who like neither drawing nor music, subtract this total from the overall number of students in the club (100):

Neither = Total Students - Total

$$\text{Neither} = 100 - 70 = 30$$

Therefore, there are 30 students who like neither drawing nor music.

10. Let A and B be two finite sets, such that $n(A) = 16$, $n(B) = 28$, $n(A \cup B) = 56$, Find $n(A \cap B)$.

Ans - We can use the inclusion-exclusion principle to find the cardinality of the intersection of sets A and B ($n(A \cap B)$).

The inclusion-exclusion principle states:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Given that $n(A) = 16$, $n(B) = 28$, and $n(A \cup B) = 56$, we can substitute these values into the formula and solve for $n(A \cap B)$:

$$56 = 16 + 28 - n(A \cap B)$$

Now, rearrange the equation to solve for $n(A \cap B)$:



$$n(A \cap B) = 16 + 28 - 56$$

$$n(A \cap B) = 44 - 56$$

$$n(A \cap B) = -12$$

11.What is differentiation?

Ans - Differentiation means telling things apart or finding how something changes. In math, it's about figuring out how a function changes as its input (or variable) changes.

UNIT – 3 (MATRIX)

12.Define matrix. Write all types of matrix.

Ans - A matrix is a two-dimensional array of numbers, symbols, or expressions arranged in rows and columns. Matrices are commonly used in various fields of mathematics, physics, computer science, and engineering.

Types of matrices include:

1. **Row Matrix:** A matrix with only one row and multiple columns.
2. **Column Matrix:** A matrix with only one column and multiple rows.
3. **Zero Matrix (or Null Matrix):** A matrix in which all elements are zero.
4. **Scalar Matrix:** A square matrix in which all the elements of the principal diagonal are equal, and all other elements are zero.
5. **Identity Matrix:** A special scalar matrix in which all elements of the principal diagonal are 1, and all other elements are zero.
6. **Diagonal Matrix:** A matrix in which all off-diagonal elements are zero.
7. **Symmetric Matrix:** A square matrix that is equal to its transpose.
9. **Square Matrix:** A matrix with the same number of rows and columns.
10. **Rectangular Matrix:** A matrix with different numbers of rows and columns.
11. **Transpose of a Matrix:** A matrix obtained by swapping its rows with columns.

13.Define differential equation with example

Ans - Differential Equation Definition:

A differential equation is an equation involving a function and its derivatives, expressing how the rate of change of the function relates to its current value or values.

Example: $\frac{dy}{dx} = 2x$

14.What is column matrix. Give one example.

Ans - A column matrix is a matrix that has only one column and any number of rows. It is often written as a vertical arrangement of elements enclosed in brackets or parentheses.

Example - $\longrightarrow \begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix}$

15.What is determinant.Give an example.

Ans - The determinant of a matrix is a scalar value computed from its elements, often denoted as $\det(A)$ or $|A|$. For a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is calculated as $ad - bc$.

Example:

For the matrix $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$, the determinant is 2.

**16.Solve the following system of equations using Cramer's rule: $2x-y=5$,
 $x+y=4$**

Ans - Given system of equations:

$$2x - y = 5$$

$$x + y = 4$$

Step 1: Find the determinant (D) of the coefficient matrix:

$$D = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = (2 \times 1) - (-1 \times 1) = 3$$

Step 2: Find the determinant (D_x) obtained by replacing the coefficients of the x terms:

$$D_x = \begin{vmatrix} 5 & -1 \\ 4 & 1 \end{vmatrix} = (5 \times 1) - (-1 \times 4) = 9$$

Step 3: Find the determinant (D_y) obtained by replacing the coefficients of the y terms:

$$D_y = \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix} = (2 \times 4) - (5 \times 1) = 3$$

Step 4: Calculate the solutions x and y :

$$x = \frac{D_x}{D} = \frac{9}{3} = 3$$

$$y = \frac{D_y}{D} = \frac{3}{3} = 1$$

So, the solution to the system of equations is $x = 3$ and $y = 1$.

17. Solve the differential equation $dx/dt = 5x - 3$

Ans – $\frac{dx}{dt} = 5x - 3$

This is a separable differential equation. Here are the steps:

Step 1: Separate variables by bringing all terms involving x to one side and all terms involving t to the other side.

$$\frac{1}{5x-3} dx = dt$$

Step 2: Integrate both sides.

$$\int \frac{1}{5x-3} dx = \int dt$$

To integrate the left side, you can use a substitution. Let $u = 5x - 3$, then $du/dx = 5$, and $dx = du/5$.

$$\frac{1}{5} \int \frac{1}{u} du = \int dt$$

$$\frac{1}{5} \ln |u| = t + C$$

Step 3: Substitute back in terms of x by replacing u with $5x - 3$.

$$\frac{1}{5} \ln |5x - 3| = t + C$$

18.Find $A^2 + 6A + 1 = 0$ if A

Ans –

It looks like there's a formatting issue in your question, but I'll assume you're asking to solve the quadratic equation $A^2 + 6A + 1 = 0$.

To solve this quadratic equation, you can use the quadratic formula:

$$A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, the coefficients are $a = 1$, $b = 6$, and $c = 1$.

$$A = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)}$$

$$A = \frac{-6 \pm \sqrt{36 - 4}}{2}$$

$$A = \frac{-6 \pm \sqrt{32}}{2}$$

$$A = \frac{-6 \pm 4\sqrt{2}}{2}$$

Now, simplify:

$$A = -3 \pm 2\sqrt{2}$$

So, the solutions for A are:

$$A = -3 + 2\sqrt{2}$$

$$A = -3 - 2\sqrt{2}$$

19. Solve the differential equation $y' + 2xy = x$.

Ans –

Certainly! Let's solve the differential equation $y' + 2xy = x$ using the integrating factor method in easy steps:

Given differential equation: $y' + 2xy = x$

Step 1: Identify $P(x)$ and $Q(x)$:

$$P(x) = 2x, \quad Q(x) = x$$

Step 2: Find the integrating factor ($I(x)$):

$$I(x) = e^{\int P(x) dx} = e^{\int 2x dx} = e^{x^2}$$

Step 3: Multiply the entire differential equation by $I(x)$:

$$e^{x^2} y' + 2xe^{x^2} y = xe^{x^2}$$

Step 4: Recognize the left side as the derivative of $e^{x^2} y$:

$$\frac{d}{dx}(e^{x^2} y) = xe^{x^2}$$

Step 5: Integrate both sides:

$$e^{x^2} y = \int xe^{x^2} dx$$

Step 6: Solve the integral (use substitution $u = x^2$):

$$e^{x^2} y = \frac{1}{2}e^{x^2} + C$$

Step 7: Solve for y :

$$y = \frac{1}{2} + Ce^{-x^2}$$



20. To find $A^2 + 6A + I$ for the given matrix $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$

Ans –

To find $A^2 + 6A + I$ for the given matrix $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$, you can follow these steps:

1. Find A^2 : Multiply the matrix by itself.

$$A^2 = A \cdot A$$

$$A^2 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4(4) + 1(-1) & 4(1) + 1(2) \\ -1(4) + 2(-1) & -1(1) + 2(2) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

1. Find $6A$: Multiply the matrix A by 6.

$$6A = 6 \cdot \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$6A = \begin{bmatrix} 6(4) & 6(1) \\ 6(-1) & 6(2) \end{bmatrix}$$

$$6A = \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix}$$

1. Add A^2 , $6A$, and the identity matrix I :

$$A^2 + 6A + I = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix} + \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 + 6A + I = \begin{bmatrix} 15 + 24 + 1 & 6 + 6 + 0 \\ -6 - 6 + 0 & 3 + 12 + 1 \end{bmatrix}$$

$$A^2 + 6A + I = \begin{bmatrix} 40 & 12 \\ -12 & 16 \end{bmatrix}$$

So, $A^2 + 6A + I$ for the given matrix A is:

$$A^2 + 6A + I = \begin{bmatrix} 40 & 12 \\ -12 & 16 \end{bmatrix}$$

21. Find adjoint of A if $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Ans –

Given matrix:

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Step 1: Find the cofactor matrix of A . The cofactor C_{ij} for each element a_{ij} is given by $(-1)^{i+j} \cdot M_{ij}$, where M_{ij} is the determinant of the matrix obtained by removing the i -th row and j -th column.

$$C_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1 \cdot 3 - 2 \cdot 2 = -1$$

Similarly, calculate the cofactors for all elements in A .

$$C_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = -(3 \cdot 3 - 2 \cdot 1) = 5$$

$$C_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$$

$$C_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = -(1 \cdot 3 - 3 \cdot 2) = 3$$

$$C_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} = 3$$

$$C_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -(2 \cdot 2 - 1 \cdot 1) = -3$$

$$C_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5$$

$$C_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = -(2 \cdot 1 - 3 \cdot 3) = 7$$

$$C_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = -5$$

So, the cofactor matrix C is:

$$C = \begin{bmatrix} -1 & 5 & 5 \\ 3 & 3 & -3 \\ -5 & 7 & -5 \end{bmatrix}$$

Step 2: Transpose the cofactor matrix to get the adjoint of A .

$$\text{adj}(A) = C^T = \begin{bmatrix} -1 & 3 & -5 \\ 5 & 3 & 7 \\ 5 & -3 & -5 \end{bmatrix}$$

Therefore, the adjoint of the matrix A is:

$$\text{adj}(A) = \begin{bmatrix} -1 & 3 & -5 \\ 5 & 3 & 7 \\ 5 & -3 & -5 \end{bmatrix}$$

UNIT 5

22. Define complex number with example.

Ans –

A complex number is a number that can be expressed in the form $a + bi$, where a and b are real numbers, and i is the imaginary unit, defined as the square root of -1.

In this expression:

- a is the real part of the complex number,
- b is the imaginary part of the complex number,
- i is the imaginary unit.

Here's an example of a complex number:

$$3 + 4i$$

In this example:

- 3 is the real part,
- 4 is the imaginary part,
- i is the imaginary unit.

23. Define Real Number

Ans - A real number is any number that can be found on the number line, including positive and negative integers, fractions, and decimals. Examples of real numbers are 3, -1.7,

24. What is Imaginary Number.

Ans - An imaginary number is a number that, when squared, gives a negative result. It is denoted by the symbol i , where $i^2 = -1$. Imaginary numbers are often used in conjunction with real numbers to form complex numbers. In the complex number $a + bi$, where a and b are real numbers, the term bi represents the imaginary part.

For example, in the imaginary number $3i$:

- 3 is the coefficient (real part),
- i is the imaginary unit.

25. To simplify the expression $7i + 4(2 - 3i)$

Ans - To simplify the expression $7i + 4(2 - 3i)$, distribute the 4 to both terms inside the parentheses and then combine like terms:

$$7i + 4(2 - 3i) = 7i + 8 - 12i$$

Now, combine the imaginary terms and the real terms separately:

$$7i + 8 - 12i = (7i - 12i) + 8 = -5i + 8$$

So, $7i + 4(2 - 3i)$ simplifies to $-5i + 8$.

26. If $A = 3 + 12i$ and $B = 2 + 7i$ then find $A - B$

Ans - To find $A - B$, subtract the corresponding components of A and B :

$$A - B = (3 + 12i) - (2 + 7i)$$

Combine the real parts and the imaginary parts separately:

$$A - B = (3 - 2) + (12i - 7i)$$

Simplify:

$$A - B = 1 + 5i$$

Therefore, $A - B$ is $1 + 5i$.

27. Express the following $\frac{2+3i}{3+4i}$ in the form $a + bi$,

Ans –

To express $\frac{2+3i}{3+4i}$ in the form $a + bi$, where a and b are real numbers, you typically multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $3 + 4i$ is $3 - 4i$.

Here are the steps:

$$\frac{2+3i}{3+4i} \cdot \frac{3-4i}{3-4i}$$

Now, multiply the numerators and denominators:

$$= \frac{(2+3i)(3-4i)}{(3+4i)(3-4i)}$$

$$= \frac{6-8i+9i-12i^2}{9-16i^2}$$

Since $i^2 = -1$, substitute this into the expression:

$$= \frac{6+i-12(-1)}{9-16(-1)}$$

$$= \frac{6+i+12}{9+16}$$

$$= \frac{18+i}{25}$$

So, $\frac{2+3i}{3+4i}$ in the form $a + bi$ is $\frac{18+i}{25}$.

28. Express in the polar form $1 - \sqrt{3}i$

Ans –

To express the complex number $1 - \sqrt{3}i$ in polar form, we'll find its magnitude (r) and argument (θ).

The magnitude (r) is found using the formula:

$$r = \sqrt{a^2 + b^2}$$

where a is the real part and b is the imaginary part.

For $1 - \sqrt{3}i$:

$$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1 + 3} = 2$$

The argument (θ) is found using the arctangent:

$$\theta = \arctan\left(\frac{b}{a}\right)$$

where a is the real part and b is the imaginary part.

For $1 - \sqrt{3}i$:

$$\theta = \arctan\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$$

Now, we can express $1 - \sqrt{3}i$ in polar form $r(\cos(\theta) + i \sin(\theta))$:

$$1 - \sqrt{3}i = 2 \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)$$

So, the polar form of $1 - \sqrt{3}i$ is $2 \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)$.

29. Find modulus and amplitude $2 + 2\sqrt{3}i$

Ans – 1. **Modulus (r):**

$$r = |2 + 2\sqrt{3}i| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$

2. **Amplitude (θ):**

$$\theta = \arctan\left(\frac{2\sqrt{3}}{2}\right) = \arctan(\sqrt{3}) = \frac{\pi}{3}$$

Therefore, for the complex number $2 + 2\sqrt{3}i$:

- Modulus (r) is 4,
- Amplitude (θ) is $\frac{\pi}{3}$.

30. Find the square root of $-15 - 8i$

Ans –

To find the square root of the complex number $-15 - 8i$, let's express it in polar form and then apply the square root operation.

1. Convert to Polar Form:

Find the magnitude (r) and argument (θ).

$$r = \sqrt{(-15)^2 + (-8)^2} = \sqrt{225 + 64} = \sqrt{289} = 17$$

$$\theta = \arctan\left(\frac{-8}{-15}\right) = \arctan\left(\frac{8}{15}\right)$$

2. Express in Polar Form:

$$-15 - 8i = 17 (\cos(\theta) + i \sin(\theta))$$

3. Apply Square Root Operation:

The square root of a complex number in polar form is given by:

$$\sqrt{r} \left(\cos\left(\frac{\theta}{2}\right) + i \sin\left(\frac{\theta}{2}\right) \right)$$

$$\sqrt{-15 - 8i} = \sqrt{17} \left(\cos\left(\frac{\theta}{2}\right) + i \sin\left(\frac{\theta}{2}\right) \right)$$

Since we have the expression in polar form, we can now simplify $\cos\left(\frac{\theta}{2}\right)$ and $\sin\left(\frac{\theta}{2}\right)$.

31.If $A = 3-2i$ and $B= 4-1$ then find AXB , A/B and $A- B$.

Ans –

Let's perform the requested operations with the given complex numbers:

1. Multiplication (AXB):

$$AXB = (3 - 2i)(4 - 1)$$

Expand and simplify:

$$AXB = 12 - 3i - 8i + 2i^2$$

Substitute $i^2 = -1$:

$$AXB = 12 - 11i$$

2. Division (A/B):

$$\frac{A}{B} = \frac{3-2i}{4-1}$$

Multiply the numerator and denominator by the conjugate of the denominator:

$$\frac{A}{B} = \frac{(3-2i)(4+1)}{(4-1)(4+1)}$$

Simplify:

$$\frac{A}{B} = \frac{11-2i}{15}$$

3. Subtraction ($A - B$):

$$A - B = (3 - 2i) - (4 - 1)$$

Expand and simplify:

$$A - B = 3 - 2i - 4 + 1$$

Combine like terms:

$$A - B = -1 - 2i$$

So, after performing the operations:

- $AXB = 12 - 11i$,
- $\frac{A}{B} = \frac{11-2i}{15}$,
- $A - B = -1 - 2i$.

UNIT – 3

32. Define Integration with example.

Ans –Integration is a mathematical operation that involves finding the antiderivative of a function. It is denoted as $\int f(x) dx$, where $f(x)$ is the function to be integrated, and dx indicates the variable with respect to which integration is performed.

Example:

$$\int 3x^2 dx = x^3 + C$$

33. Evaluate $\int (x - 1)(x + 2) dx$

Ans –

To evaluate the integral $\int (x - 1)(x + 2) dx$, we will expand the expression and then integrate term by term:

$$\int (x - 1)(x + 2) dx$$

Expand the expression:

$$= \int (x^2 + 2x - x - 2) dx$$

Combine like terms:

$$= \int (x^2 + x - 2) dx$$

Now, integrate term by term:

$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + C$$

So, $\int (x - 1)(x + 2) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + C$, where C is the constant of integration.

34.Integrate $7x\sqrt{x}$

Ans –

$$\int 7x\sqrt{x} dx = \int 7x^{\frac{3}{2}} dx$$

Now, apply the power rule for integration:

$$= \frac{7}{\frac{5}{2}} x^{\frac{5}{2}} + C$$

Simplify:

$$= \frac{14}{5} x^{\frac{5}{2}} + C$$

So, $\int 7x\sqrt{x} dx = \frac{14}{5} x^{\frac{5}{2}} + C$, where C is the constant of integration.

35.Integrate $x^3 - 3x^2 + 6x - 4$ w.r.t.x

Ans –

To integrate the expression $x^3 - 3x^2 + 6x - 4$ with respect to x , apply the power rule for integration term by term:

$$\int (x^3 - 3x^2 + 6x - 4) dx$$

Integrate each term separately:

$$\frac{1}{4}x^4 - x^3 + 3x^2 - 4x + C$$

So, the result of integrating $x^3 - 3x^2 + 6x - 4$ with respect to x is:

$$\frac{1}{4}x^4 - x^3 + 3x^2 - 4x + C$$

36.What are the rules of integration

Ans – 1. Power Rule:

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

2. Constant Rule:

$$\int c dx = cx + C$$

3. Sum/Difference Rule:

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

4. Constant Multiple Rule:

$$\int c \cdot f(x) dx = c \cdot \int f(x) dx$$

5. Exponential Functions:

$$\int e^x dx = e^x + C$$

6. Trigonometric Functions:

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

37.To find $f(x)$ when $f(1) = 0$ given $f(x) = (2x^3 + 3)^2$

Ans –

To find $f(x)$ when $f(1) = 0$ given $f(x) = (2x^3 + 3)^2$, you can use the fact that $f(1) = 0$. This implies that when $x = 1$, $f(x)$ equals zero.

So, set $(2 \cdot 1^3 + 3)^2 = 0$:

$$(2 + 3)^2 = 0$$

$$(5)^2 = 0$$

$$25 = 0$$

38. (1) Evaluate $\int 12(5x + 2)(3x - 1) dx$

(2) Find the integral of $\sqrt{1 - \sin^2 x}$

Ans –

To evaluate $\int 12(5x + 2)(3x - 1) dx$, expand and simplify the expression, then integrate term by term:

$$\begin{aligned}\int 12(5x + 2)(3x - 1) dx \\&= \int (180x^2 - 24x - 36) dx \\&= 60x^3 - 12x^2 - 36x + C\end{aligned}$$

So, $\int 12(5x + 2)(3x - 1) dx = 60x^3 - 12x^2 - 36x + C$, where C is the constant of integration.

(2) Find the integral of $(\int \sqrt{1 - \sin^2 x}, dx$:

Recognize that $\sqrt{1 - \sin^2 x}$ is equivalent to $\cos x$. The integral becomes:

$$\int \cos x dx = \sin x + C$$

So, $\int \sqrt{1 - \sin^2 x} dx = \sin x + C$, where C is the constant of integration.

UNIT – 2

39. Find the value of

$\frac{dy}{dx}$ for the given function $y = 3x + 8$, you differentiate y with respect to x :

Ans $-\frac{dy}{dx} = \frac{d}{dx}(3x + 8)$

Using the power rule (derivative of ax is a), the derivative of $3x$ is 3, and the derivative of a constant (like 8) is zero:

$$\frac{dy}{dx} = 3$$

So, $\frac{dy}{dx} = 3$ for the given function $y = 3x + 8$.

40. find the derivative of

$$f(x) = (x + 1)(x + 2),$$

Ans - $(uv)' = u'v + uv'$

Let $u = (x + 1)$ and $v = (x + 2)$.

Now, find the derivatives:

$$u' = \frac{d}{dx}(x + 1) = 1$$

$$v' = \frac{d}{dx}(x + 2) = 1$$

Apply the product rule:

$$f'(x) = u'v + uv'$$

$$f'(x) = (1)(x + 2) + (x + 1)(1)$$

$$f'(x) = x + 2 + x + 1$$

$$f'(x) = 2x + 3$$

So, the derivative of $(x + 1)(x + 2)$ is $2x + 3$.

41. If $y = x \log(x)$, Find $\frac{dy}{dx}$

Ans – $\frac{dy}{dx} = u'v + uv'$

Let $u = x$ and $v = \log(x)$. Now, find the derivatives:

$$u' = \frac{d}{dx}(x) = 1$$

$$v' = \frac{d}{dx}(\log(x))$$

The derivative of $\log(x)$ is $\frac{1}{x}$. So,

$$v' = \frac{1}{x}$$

Now, apply the product rule:

$$\frac{dy}{dx} = u'v + uv'$$

$$\frac{dy}{dx} = (1)(\log(x)) + (x) \left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = \log(x) + 1$$

So, $\frac{dy}{dx} = \log(x) + 1$ for the function $y = x \log(x)$.

42.find the derivative of sin x cos x.

Ans $-\frac{dy}{dx} = u'v + uv'$

Let $u = \sin(x)$ and $v = \cos(x)$. Now, find the derivatives:

$$u' = \frac{d}{dx}(\sin(x)) = \cos(x)$$

$$v' = \frac{d}{dx}(\cos(x)) = -\sin(x)$$

Now, apply the product rule:

$$\frac{dy}{dx} = u'v + uv'$$

$$\frac{dy}{dx} = (\cos(x))(\cos(x)) + (\sin(x))(-\sin(x))$$

$$\frac{dy}{dx} = \cos^2(x) - \sin^2(x)$$

Now, you can use the trigonometric identity $\cos^2(x) - \sin^2(x) = \cos(2x)$ to simplify further:

$$\frac{dy}{dx} = \cos(2x)$$

So, the derivative of $\sin(x) \cos(x)$ is $\cos(2x)$.

43. (1) Find the derivative of $r = \frac{(x+1)(x+2)}{(x+3)}$:

(2) If $y = \frac{1}{2x+3}$, find $\frac{dy}{dx}$:

Ans – To find the derivative $\frac{dr}{dx}$, you can use the quotient rule. The quotient rule states that if you have a function $r = \frac{u}{v}$, then the derivative is given by:

$$\frac{dr}{dx} = \frac{u'v - uv'}{v^2}$$

Let $u = (x + 1)(x + 2)$ and $v = (x + 3)$. Now, find the derivatives:

$$u' = \frac{d}{dx}[(x + 1)(x + 2)]$$

$$u' = (x + 2) + (x + 1)$$

$$v' = \frac{d}{dx}(x + 3)$$

$$v' = 1$$

Now, apply the quotient rule:

$$\frac{dr}{dx} = \frac{u'v - uv'}{v^2}$$

$$\frac{dr}{dx} = \frac{((x+2)+(x+1))(x+3) - (x+1)(x+2)}{(x+3)^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Let $u = 2x + 3$. Now, find the derivatives:

$$\frac{du}{dx} = 2$$

$$\frac{dy}{du} = \frac{d}{du} \left(\frac{1}{u} \right)$$

$$\frac{dy}{du} = -\frac{1}{u^2}$$

Now, apply the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = -\frac{2}{(2x+3)^2}$$

So, the derivative $\frac{dy}{dx}$ is $-\frac{2}{(2x+3)^2}$ for the given function $y = \frac{1}{2x+3}$.

44. (1) If $f(x) = \frac{1}{2x+5}$, find $f'(2)$:

(2) If $y = \frac{3}{x} + 5\sqrt{x} + 7 + 7\log(x)$, find $\frac{dy}{dx}$:

Ans – To find $f'(x)$, the derivative of $f(x)$, apply the chain rule. Let $u = 2x + 5$, then:

$$f'(x) = \frac{d}{dx} \left(\frac{1}{u} \right) \cdot \frac{du}{dx}$$

$$f'(x) = -\frac{1}{u^2} \cdot \frac{d}{dx}(2x + 5)$$

$$f'(x) = -\frac{1}{(2x+5)^2} \cdot 2$$

Now, evaluate $f'(2)$:

$$f'(2) = -\frac{1}{(2 \cdot 2 + 5)^2} \cdot 2$$

To find $\frac{dy}{dx}$, differentiate each term of y with respect to x :

$$y' = \frac{d}{dx} \left(\frac{3}{x} \right) + \frac{d}{dx} (5\sqrt{x}) + \frac{d}{dx} (7) + \frac{d}{dx} (7\log(x))$$

Use the power rule, chain rule, and derivative of a logarithmic function:

$$y' = -\frac{3}{x^2} + \frac{5}{2\sqrt{x}} + 0 + \frac{7}{x}$$

Combine terms:

$$y' = -\frac{3}{x^2} + \frac{5}{2\sqrt{x}} + \frac{7}{x}$$

$$\text{So, } \frac{dy}{dx} = -\frac{3}{x^2} + \frac{5}{2\sqrt{x}} + \frac{7}{x}.$$

NOTE - सर्व प्रश्न नाही आहे फक्त जेवढे भेटले ते आहे

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हर कदम ऐसा चलो कि निशान बन जाए,
यहां जिंदगी तो हर कोई काट लेता है,
जिंदगी जियो इस कदर कि मिसाल बन जाए।