DSP Assignment

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(1) $na^n u[n]$

By differentiat^n property

$$nx[n] \xleftrightarrow{z} -z\frac{d}{dz}X(z)$$

$$\therefore n[a^n u[n]] \xleftrightarrow{z} -z\frac{d}{dz}X(a^n(u[n]))$$

$$X(a^n u[n]) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= 1 + az^{-1} + a^2z^{-2} + \cdots$$

$$= 1/(1-az^{-1})$$

ROC $\Rightarrow |az^{-1}| < 1$

$$|z^{-1}| < 1/|a|$$

$$1/|z| < 1/|a|$$

$$|z| > |a|$$

$$\therefore X(na^n u[n]) = -z\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right)$$

$$X(na^n u[n]) = \frac{az^{-1}}{(1-az^{-1})^2}$$

ROC $\Rightarrow |z| > |a|$

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(ii) $-na^n u[-n-1]$

using differentiat^n property

$$nx[n] \xleftrightarrow{z} -z\frac{d}{dz}X(z)$$

$$\therefore -na^n u[-n-1] \xleftrightarrow{z} -z\frac{d}{dz}X(-a^n u[-n-1])$$

$$X(-a^n u[-n-1]) = \sum^{\infty} -a^n u[-n-1] z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$= -\sum_{n=1}^{\infty} a^{-n} z^{n}$$

$$= 1 - \sum_{n=0}^{\infty} (a^{-1}z)^n$$

$$= 1 - \left[1 + a^{-1}z + a^{-2}z^2 + \cdots\right]$$

$$= 1 - \frac{1}{1-a^{-1}z} = 1 - \frac{a}{a-z}$$

$$= -\frac{z}{a-z} = \frac{z}{z-a} = \frac{1}{1-az^{-1}}$$

ROC $\Rightarrow |a^{-1}z| < 1$

$$|a^{-1}| < 1/|z|$$

$$1/|a| < 1/|z|$$

$$\Rightarrow |z| < |a|$$

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$$x\left(-na^n u(-n-1)\right) = -z\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right)$$

$$= -z\,\frac{az^{-2}}{(1-az^{-1})^2}$$

$$= \frac{az^{-1}}{(1-az^{-1})^2}$$

$\therefore$ ROC $\Rightarrow$ $|z| < |a|$

iii) $[\cos\omega_0 n]\,u[n]$

$$\cos\omega_0 n = \frac{e^{j\omega_0 n}}{2} + \frac{e^{-j\omega_0 n}}{2}$$

$$(\cos\omega_0 n)\,u[n] = \frac{1}{2}e^{j\omega_0 n}u[n] + \frac{1}{2}e^{-j\omega_0 n}u[n]$$

$$x\left(\frac{1}{2}e^{j\omega_0 n}u[n]\right) = \frac{1}{2}\,x\left(e^{j\omega_0 n}u[n]\right)$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\left(e^{j\omega_0}z^{-1}\right)^n$$

$$= \frac{1}{2}\times\frac{1}{1-e^{j\omega_0}z^{-1}}$$

ROC $\Rightarrow$ $|z| > |e^{j\omega_0}|$

$|z| > 1$

))ly, $x\left(\frac{1}{2}e^{-j\omega_0 n}u[n]\right) = \frac{1}{2}\,\frac{1}{1-e^{-j\omega_0}z^{-1}}$

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$$ROC_2 = |z| > |e^{-j\omega_0}|$$

$$|z| > 1$$

$$x(\cos\omega_0 n \; u[n]) = \frac{1}{2}\left[\frac{1}{1-e^{j\omega_0}z^{-1}} + \frac{1}{1-e^{-j\omega_0}z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{1-e^{-j\omega_0}z^{-1} + 1-e^{j\omega_0}z^{-1}}{(1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})}\right]$$

$$= \frac{1}{2}\left[\frac{2-z^{-1}(e^{-j\omega_0}+e^{j\omega_0})}{1-e^{-j\omega_0}z^{-1}-e^{j\omega_0}z^{-1}+z^{-2}}\right]$$

$$= \frac{1-(\cos\omega_0)z^{-1}}{1-z^{-1}(e^{-j\omega_0}+e^{j\omega_0})+z^{-2}}$$

$$= \frac{1-(\cos\omega_0)z^{-1}}{1-(2\cos\omega_0)z^{-1}+z^{-2}}$$

ROC of $\cos\omega_0 n, u[n] = ROC_1 \cap ROC_2$

$$\Rightarrow |z| > 1$$

(iv) $(\sin\omega_0 n)u[n]$

$$\sin\omega_0 n = \frac{e^{j\omega_0 n}}{2} - \frac{e^{-j\omega_0 n}}{2}$$

$$(\sin\omega_0 n)u[n] = \frac{1}{2}e^{j\omega_0 n}u[n] - \frac{1}{2}e^{-j\omega_0 n}u[n]$$

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$$X\left(\frac{1}{2} e^{j\omega_0 n} u[n]\right) = \frac{1}{2} X\left(e^{j\omega_0 n} u[n]\right)$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} e^{j\omega_0 n} u[n] z^{-n}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \left(e^{j\omega_0} z^{-1}\right)^n$$

$$= \frac{1}{2} \frac{1}{1 - e^{j\omega_0} z^{-1}}$$

$$ROC_1 = |z| > |e^{j\omega_0}|$$

$$= |z| > 1$$

Illy $X\left(\frac{1}{2} e^{-j\omega_0 n} u[n]\right) = \frac{1}{2} \frac{1}{1 - e^{-j\omega_0} z^{-1}}$

$$ROC_2 = |z| > |e^{-j\omega_0}|$$

$$|z| > 1$$

$$X\left([\sin\omega_0 n] u[n]\right) = \frac{1}{2}\left[\frac{1}{1 - e^{j\omega_0} z^{-1}} - \frac{1}{1 - e^{-j\omega_0} z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{1 - e^{j\omega_0} z^{-1} - 1 + e^{j\omega_0} z^{-1}}{(1 - e^{j\omega_0} z^{-1})(1 - e^{-j\omega_0} z^{-1})}\right]$$

$$= \frac{(\sin\omega_0) z^{-1}}{1 - z^{-1}(e^{-j\omega_0} + e^{j\omega_0}) + z^{-2}}$$

$$= \frac{\sin\omega_0 \cdot z^{-1}}{1 - (2\cos\omega_0) z^{-1} + z^{-2}}$$

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$$ROC = ROC_1 \cap ROC_2$$
$$= |z| > 1$$

(v) $[r^n \cos w_0 n] u[n]$

$$r^n \cos w_0 n = \frac{(re^{jw_0})^n}{2} + \frac{(re^{-jw_0})^n}{2}$$

$$X([r^n \cos w_0 n] u[n]) = \frac{1}{2} X[(re^{jw_0})^n u[n]] + \frac{1}{2} X[(re^{-jw_0})^n u[n]]$$

$$\frac{1}{2} X[(re^{jw_0})^n u[n]] = \frac{1}{2} \sum_{n=-\infty}^{\infty} (re^{jw_0})^n \cdot u[n] \cdot z^{-n}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} (re^{jw_0})^n z^{-n}$$

$$= \frac{1}{2} X \frac{1}{1-(re^{jw_0}) z^{-1}}$$

$$ROC_1 = |z| > |re^{jw_0}|$$
$$|z| > r$$

|||ly $\frac{1}{2} X[(re^{-jw_0})^n u[n]] = \frac{1}{2} X \frac{1}{1-(re^{-jw_0}) z^{-1}}$

$$ROC_2 = |z| > |re^{-jw_0}|$$
$$\Rightarrow |z| > r$$

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$$\therefore X\left([r^n \cos\omega_0 n]u[n]\right) = \frac{1}{2}\left[\frac{1}{1-(re^{j\omega_0})z^{-1}} + \frac{1}{1-(re^{-j\omega_0})z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{1-(re^{-j\omega_0})z^{-1} + 1-(re^{j\omega_0})z^{-1}}{1-(re^{-j\omega_0})z^{-1} - re^{j\omega_0}z^{-1} + r^2z^{-2}}\right]$$

$$= \frac{1}{2}\left[\frac{2-(re^{-j\omega_0} + re^{j\omega_0})z^{-1}}{1-(re^{-j\omega_0} + re^{j\omega_0})z^{-1} + r^2z^{-2}}\right]$$

$$= \left[\frac{1-(r\cos\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1} + r^2z^{-2}}\right]$$

$$ROC = ROC_1 \cap ROC_2 \Rightarrow |z| > r$$

(vi) $[r^n \sin\omega_0 n]u[n]$

$$r^n \sin\omega_0 n = \frac{(re^{j\omega_0})^n}{2} - \frac{(re^{-j\omega_0})^n}{2}$$

$$X\left([r^n \sin\omega_0 n]u[n]\right) = \frac{1}{2}X\left[(re^{j\omega_0})^n u[n]\right] - \frac{1}{2}X\left[(re^{-j\omega_0})^n u[n]\right]$$

$$\frac{1}{2}X\left[(re^{j\omega_0})^n u[n]\right] = \frac{1}{2}\sum_{n=-\infty}^{\infty}(re^{j\omega_0})^n u[n] z^{-n}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}(re^{j\omega_0})^n z^{-n}$$

$$= \frac{1}{2} \Big/ 1-re^{j\omega_0}z^{-1}$$

$$ROC_1 = |z| > |re^{j\omega_0}|$$

$$|z| > r$$

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Illy $\frac{1}{2} x\left([re^{-j\omega_0}]^n u[n]\right) = \frac{1}{2} \times 1 / (1 - re^{-j\omega_0} z^{-1})$

$ROC_2 = |z| > |re^{-j\omega_0}|$

$|z| > r$

$x\left[(r^n \sin\omega_0 n) u[n]\right]$

$= \frac{1}{2}\left[\frac{1}{1-(re^{j\omega_0})z^{-1}} - \frac{1}{1-(re^{-j\omega_0})z^{-1}}\right]$

$= \frac{1}{2}\left[\frac{1-(re^{-j\omega_0})z^{-1} - 1 + (re^{j\omega_0})z^{-1}}{1-(re^{-j\omega_0})z^{-1} - (re^{j\omega_0})z^{-1} + r^2 z^{-2}}\right]$

$= \frac{1}{2}\left[\frac{(re^{j\omega_0} - re^{-j\omega_0})z^{-1}}{1-(re^{j\omega_0} + re^{-j\omega_0})z^{-1} + r^2 z^{-2}}\right]$

$= \frac{(r\sin\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1} + r^2 z^{-2}}$

$ROC = ROC_1 \cap ROC_2 = |z| > r$

(vii) $\begin{cases} a^n & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$

$X(a_n) = \sum_{n=-\infty}^{\infty} a^n z^{-n} = \sum_{n=0}^{N-1} (az^{-1})^n$ Ac to Q

$= 1 + az^{-1} + a^2 z^{-2} + \cdots + a^{N-1} \cdot z^{-(N-1)}$

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$$X(a^n) = \frac{1-a^N z^{-N}}{1-az^{-1}}$$

since only finite terms are present, $X(a^n)$ will become infinite at $z=0$

ROC will be entire z-plane excluding $z=0$